Computer algebra independent integration tests

1-Algebraic-functions/1.3-Miscellaneous/1.3.1-Rational-functions

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [494]. This is test number [51].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 99.39 (491)	% 0.61 (3)
Mathematica	% 100. (494)	% 0. (0)
Maple	% 98.99 (489)	% 1.01 (5)
Maxima	% 72.87 (360)	% 27.13 (134)
Fricas	% 87.04 (430)	% 12.96 (64)
Sympy	% 87.45 (432)	% 12.55 (62)
Giac	% 83. (410)	% 17. (84)

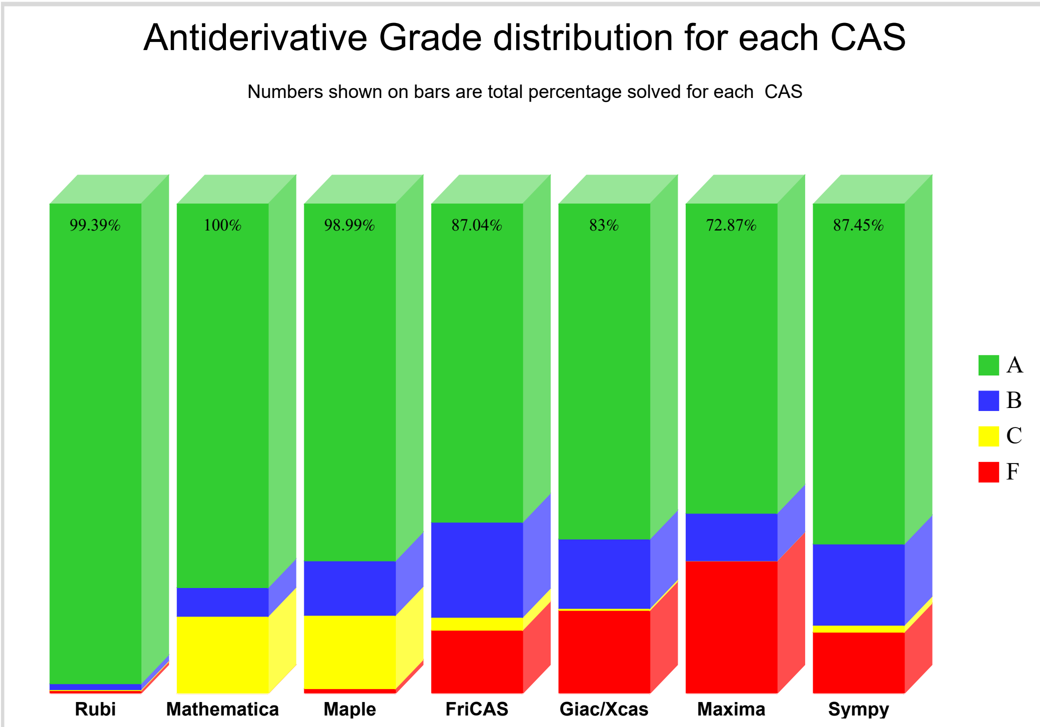
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

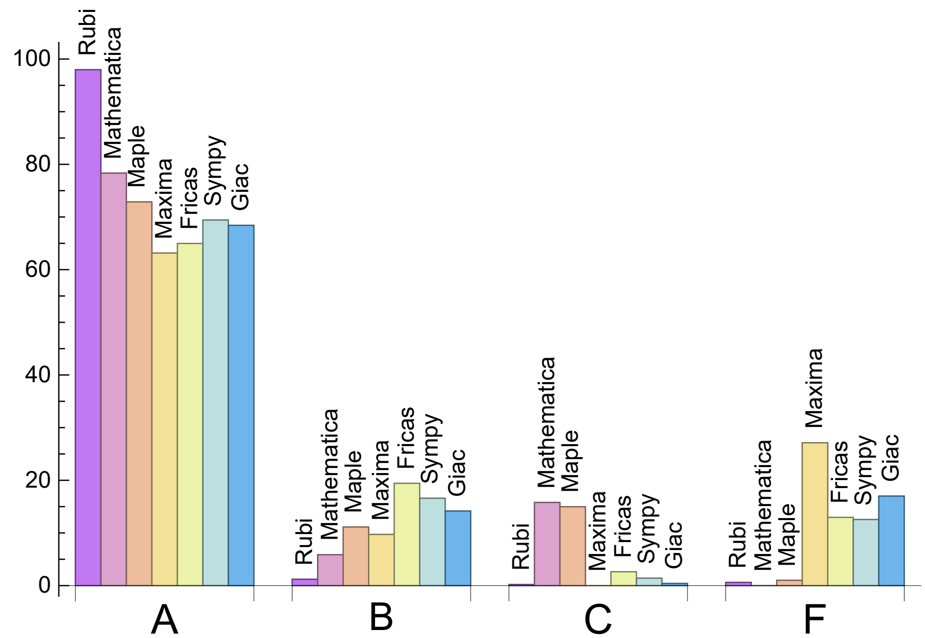
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	97.98	1.21	0.2	0.61
Mathematica	78.34	5.87	15.79	0.
Maple	72.87	11.13	14.98	1.01
Maxima	63.16	9.72	0.	27.13
Fricas	64.98	19.43	2.63	12.96
Sympy	69.43	16.6	1.42	12.55
Giac	68.42	14.17	0.4	17.

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.18	119.38	1.04	33.	1.
Mathematica	0.05	80.7	1.24	37.	1.
Maple	0.01	208.48	6.08	34.	0.91
Maxima	1.27	120.08	2.52	35.	1.18
Fricas	2.02	779.99	7.68	100.5	3.09
Sympy	1.6	116.94	2.22	39.	0.89
Giac	1.19	169.61	2.93	41.	1.25

1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica $\{31, 32, 151, 152, 153, 154, 155, 156, 157\}$

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

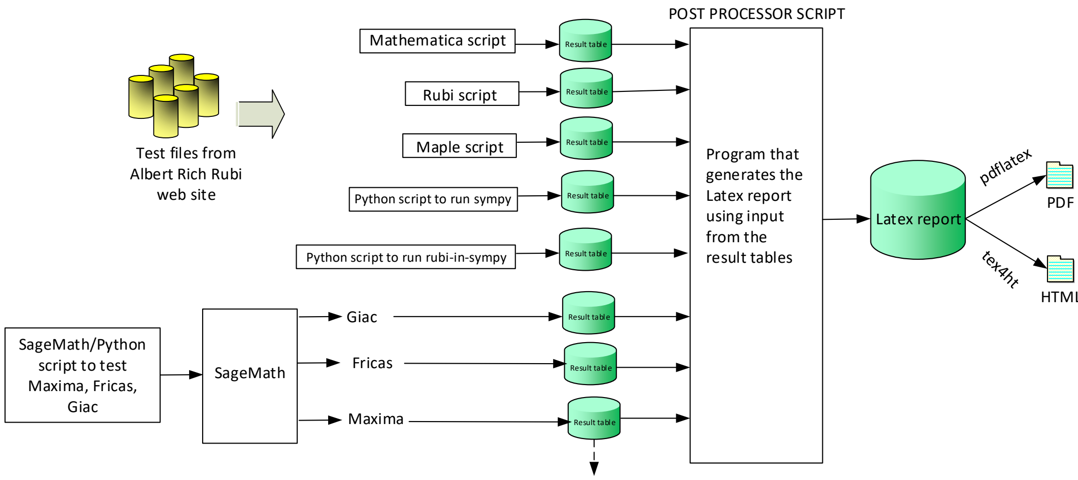
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

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June 22, 2018

Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492 }

B grade: { 65, 77, 221, 222, 233, 424 }

C grade: { 174 }

F grade: { 393, 493, 494}

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 33, 34, 35, 36, 39, 40, 41, 42, 45, 46, 47, 48, 51, 52, 53, 54, 57, 58, 59, 60, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 93, 94, 96, 97, 98, 99, 100, 101, 102, 106, 113, 116, 117, 118, 119, 123, 124, 125, 126, 130, 131, 132, 133, 158, 159, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 177, 178, 179, 180, 181, 182, 183, 186, 187, 188, 189, 190, 191, 193, 194, 208, 209, 216, 217, 218, 219, 220, 223, 224, 225, 226, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 492, 493, 494 }

B grade: { 65, 92, 95, 160, 161, 162, 192, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 210, 211, 212, 213, 214, 215, 221, 222, 421 }

C grade: { 12, 13, 14, 32, 37, 38, 43, 44, 49, 50, 55, 56, 61, 62, 103, 104, 105, 107, 108, 109, 110, 111, 112, 114, 115, 120, 121, 122, 127, 128, 129, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 173, 174, 175, 176, 184, 185, 227, 250, 251, 252, 253, 254, 255, 256, 257, 387, 388, 389, 390, 391, 392, 393, 491 }

F grade: { }

2.1.3 Maple

A grade: { 2, 5, 6, 7, 8, 10, 11, 16, 17, 18, 19, 21, 22, 23, 24, 25, 26, 27, 28, 33, 34, 35, 36, 39, 40, 41, 42, 45, 46, 47, 48, 51, 52, 53, 54, 57, 58, 59, 60, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 93, 94, 96, 97, 98, 99, 100, 117, 118, 119, 123, 124, 125, 126, 130, 131, 132, 133, 158, 159, 164, 165, 166, 167, 172, 173, 175, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 208, 209, 216, 217, 218, 219, 220, 223, 224, 225, 226, 228, 229, 230, 231, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 394, 395, 396, 397, 398, 399, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 422, 423, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 492 }

B grade: { 3, 4, 9, 15, 20, 63, 64, 65, 92, 95, 101, 102, 116, 160, 161, 162, 163, 168, 169, 170, 171, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 210, 211, 212, 213, 214, 215, 221, 222, 227, 232, 337, 338, 400, 421, 424, 443, 491, 494 }

C grade: { 1, 12, 13, 14, 37, 38, 43, 44, 49, 50, 55, 56, 61, 62, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 120, 121, 122, 127, 128, 129, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 174, 250, 251, 252, 253, 254, 255, 256, 257, 387, 388, 389, 390, 391, 392, 393, 493 }

F grade: { 29, 30, 31, 32, 176 }

2.1.4 Maxima

A grade: { 2, 5, 6, 10, 11, 15, 16, 17, 18, 21, 22, 23, 24, 25, 28, 33, 34, 35, 36, 39, 40, 41, 42, 45, 46, 47, 48, 51, 52, 53, 54, 57, 58, 59, 60, 69, 70, 71, 72, 73, 75, 76, 77, 89, 90, 93, 96, 97, 98, 99, 100, 116, 117, 118, 119, 123, 124, 125, 126, 130, 131, 132, 133, 158, 159, 160, 164, 165, 166, 168, 173, 174, 175, 176, 177, 178, 182, 185, 188, 189, 190, 191, 192, 193, 195, 196, 198, 199, 208, 209, 216, 217, 218, 219, 220, 223, 224, 225, 226, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 339, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 415, 416, 417, 418, 419, 420, 422, 423, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 492

B grade: { 3, 4, 7, 8, 9, 19, 20, 63, 64, 65, 66, 67, 68, 74, 88, 91, 92, 94, 95, 161, 162, 167, 169, 170, 171, 194, 197, 200, 201, 202, 203, 204, 205, 206, 207, 210, 211, 212, 213, 214, 215, 221, 222, 421, 424, 477, 493, 494 }

C grade: { }

F grade: { 1, 12, 13, 14, 26, 27, 29, 30, 31, 32, 37, 38, 43, 44, 49, 50, 55, 56, 61, 62, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 120, 121, 122, 127, 128, 129, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 163, 172, 179, 180, 181, 183, 184, 186, 187, 227, 228, 250, 251, 252, 253, 254, 255, 256, 257, 334, 335, 336, 337, 338, 340, 341, 342, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 459, 489, 490, 491 }

2.1.5 FriCAS

A grade: { 1, 2, 5, 6, 10, 11, 16, 17, 18, 21, 22, 23, 24, 25, 26, 27, 28, 33, 34, 35, 36, 39, 40, 41, 42, 45, 46, 47, 48, 51, 52, 53, 54, 57, 58, 59, 60, 69, 70, 71, 72, 73, 78, 79, 80, 81, 82, 83, 84, 85, 86, 88, 89, 90, 96, 98, 99, 100, 101, 102, 106, 113, 117, 118, 119, 123, 124, 125, 126, 130, 131, 132, 133, 158, 159, 164, 165, 166, 167, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 208, 209, 216, 217, 218, 219, 220, 224, 225, 226, 228, 230, 231, 232, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 269, 270, 271, 272, 273, 274, 275, 276, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 335, 336, 339, 340, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 397, 404, 411, 415, 416, 417, 418, 419, 420, 422, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 454, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 478, 479, 480, 481, 482, 485, 486, 487, 488, 489, 490, 491, 492 }

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C grade: { 49, 50, 55, 56, 104, 105, 107, 108, 109, 334, 341, 368, 393 }

F grade: { 19, 20, 29, 30, 31, 32, 61, 62, 103, 110, 111, 112, 114, 115, 127, 128, 129, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 147, 148, 149, 150, 151, 152, 153, 155, 156, 157, 227, 337, 338, 342, 391, 392, 394, 395, 396, 398, 399, 400, 401, 402, 403, 405, 406, 407, 408, 409, 410, 412, 413, 414, 483 }

2.1.6 Sympy

A grade: { 1, 5, 10, 11, 12, 13, 14, 16, 17, 21, 22, 23, 24, 25, 27, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 69, 70, 71, 72, 73, 75, 76, 93, 96, 97, 98, 99, 103, 104, 105, 106, 110, 111, 112, 113, 116, 117, 118, 119, 120, 123, 124, 125, 126, 127, 130, 131, 132, 133, 138, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 164, 165, 166, 167, 172, 177, 178, 187, 188, 189, 191, 208, 219, 220, 224, 225, 226, 228, 229, 230, 231, 232, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 255, 256, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 401, 402, 403, 404, 408, 409, 410, 411, 415, 416, 417, 418, 419, 420, 422, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 491, 492 }

B grade: { 3, 4, 6, 7, 8, 9, 15, 26, 63, 64, 65, 66, 67, 68, 74, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 92, 94, 95, 107, 121, 128, 134, 135, 160, 161, 162, 168, 179, 184, 185, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 210, 211, 212, 213, 214, 215, 216, 221, 222, 223, 233, 254, 268, 339, 340, 421, 423, 424, 459, 477, 489, 490, 493, 494 }

C grade: { 32, 89, 90, 91, 257, 279, 458 }

F grade: { 2, 18, 19, 20, 28, 29, 30, 31, 100, 101, 102, 108, 109, 114, 115, 122, 129, 136, 137, 139, 140, 141, 142, 163, 169, 170, 171, 173, 174, 175, 176, 180, 181, 182, 183, 186, 190, 209, 217, 218, 227, 234, 235, 236, 237, 238, 239, 240, 241, 337, 338, 341, 342, 398, 399, 400, 405, 406, 407, 412, 413, 414 }

2.1.7 Giac

A grade: { 1, 5, 6, 7, 8, 10, 11, 12, 16, 17, 21, 22, 23, 24, 25, 26, 27, 28, 33, 34, 35, 36, 37, 39, 40, 41, 42, 45, 46, 47, 48, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 66, 67, 68, 69, 70, 71, 72, 73, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 93, 94, 96, 97, 98, 99, 100, 101, 102, 106, 113, 117, 118, 119, 123, 124, 125, 126, 130, 131, 132, 133, 158, 164, 165, 166, 168, 169, 170, 172, 177, 178, 179, 184, 185, 189, 208, 216, 217, 218, 219, 220, 223, 224, 225, 226, 228, 229, 230, 231, 232, 233, 242, 243, 244, 245, 246, 247, 248, 249, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 340, 341, 342, 343, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 394, 395, 396, 397, 398, 400, 401, 402, 403, 404, 405, 407, 408, 409, 410, 411, 412, 414, 415, 416, 417, 418, 419, 420, 422, 423, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 486, 487, 488, 489, 490, 492 }

B grade: { 2, 3, 4, 9, 13, 14, 15, 63, 64, 65, 74, 92, 95, 116, 159, 160, 161, 162, 163, 173, 175, 180, 181, 182, 183, 186, 187, 188, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 209, 210, 211, 212, 213, 214, 215, 221, 222, 234, 235, 236, 237, 257, 268, 282, 303, 324, 344, 359, 421, 424, 485, 494 }

C grade: { 337, 338 }

F grade: { 18, 19, 20, 29, 30, 31, 32, 38, 43, 44, 49, 50, 61, 62, 103, 104, 105, 107, 108, 109, 110, 111, 112, 114, 115, 120, 121, 122, 127, 128, 129, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 167, 171, 174, 176, 227, 238, 239, 240, 241, 250, 251, 252, 253, 254, 255, 256, 339, 387, 388, 389, 390, 391, 392, 393, 399, 406, 413, 491, 493 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	143	43	0	184	60	73
normalized size	1	1.	1.86	0.56	0.	2.39	0.78	0.95
time (sec)	N/A	0.134	0.048	0.007	0.	1.388	0.444	1.097

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	23	46	34	90	0	99
normalized size	1	1.	0.77	1.53	1.13	3.	0.	3.3
time (sec)	N/A	0.018	0.055	0.002	1.177	1.255	0.	1.098

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	98	292	212	107	131
normalized size	1	1.	1.	7.	20.86	15.14	7.64	9.36
time (sec)	N/A	0.007	0.001	0.002	1.169	1.114	0.084	1.061

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	65	134	128	66	86
normalized size	1	1.	1.	4.64	9.57	9.14	4.71	6.14
time (sec)	N/A	0.007	0.001	0.	1.162	1.08	0.073	1.062

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	35	32	42	66	32	42
normalized size	1	1.	1.	0.91	1.2	1.89	0.91	1.2
time (sec)	N/A	0.007	0.	0.001	1.09	1.083	0.062	1.084

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	32	49	26	16
normalized size	1	1.	1.	0.93	2.29	3.5	1.86	1.14
time (sec)	N/A	0.009	0.003	0.003	1.17	1.229	0.325	1.082

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	77	116	61	16
normalized size	1	1.	1.	0.93	5.5	8.29	4.36	1.14
time (sec)	N/A	0.009	0.003	0.003	1.195	1.188	0.494	1.088

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	122	185	97	16
normalized size	1	1.	1.	0.93	8.71	13.21	6.93	1.14
time (sec)	N/A	0.008	0.003	0.003	1.138	1.213	0.73	1.075

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	159	295	275	375	175	224
normalized size	1	1.	1.89	3.51	3.27	4.46	2.08	2.67
time (sec)	N/A	0.125	0.019	0.001	1.715	1.227	0.093	1.076

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	82	84	126	177	87	112
normalized size	1	1.	1.46	1.5	2.25	3.16	1.55	2.
time (sec)	N/A	0.068	0.008	0.001	1.25	1.141	0.079	1.07

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	29	38	63	31	38
normalized size	1	1.	1.	0.91	1.19	1.97	0.97	1.19
time (sec)	N/A	0.006	0.	0.	1.146	1.152	0.06	1.067

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	63	57	0	878	53	281
normalized size	1	1.	0.34	0.3	0.	4.67	0.28	1.49
time (sec)	N/A	0.312	0.016	0.003	0.	1.414	0.375	1.112

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	245	112	136	0	1536	192	551
normalized size	1	1.	0.46	0.56	0.	6.27	0.78	2.25
time (sec)	N/A	0.249	0.054	0.007	0.	1.447	1.491	1.168

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	305	305	149	276	0	2731	474	817
normalized size	1	1.	0.49	0.9	0.	8.95	1.55	2.68
time (sec)	N/A	0.302	0.08	0.014	0.	1.466	5.974	1.285

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	361	361	653	861	622	2136	1018	1311
normalized size	1	1.	1.81	2.39	1.72	5.92	2.82	3.63
time (sec)	N/A	0.658	0.205	0.001	1.203	1.123	0.216	1.108

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	241	188	243	774	345	467
normalized size	1	1.	1.25	0.97	1.26	4.01	1.79	2.42
time (sec)	N/A	0.231	0.078	0.002	1.091	1.1	0.124	1.083

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	76	51	68	163	63	73
normalized size	1	1.	1.36	0.91	1.21	2.91	1.12	1.3
time (sec)	N/A	0.016	0.	0.	1.223	1.084	0.068	1.069

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	80	87	151	230	0	0
normalized size	1	1.	0.93	1.01	1.76	2.67	0.	0.
time (sec)	N/A	0.073	0.048	0.008	1.045	28.686	0.	0.

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	232	398	2830	0	0	0
normalized size	1	1.	0.99	1.7	12.09	0.	0.	0.
time (sec)	N/A	0.405	0.653	0.05	1.85	0.	0.	0.

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	495	495	490	1076	14857	0	0	0
normalized size	1	1.	0.99	2.17	30.01	0.	0.	0.
time (sec)	N/A	1.462	1.281	0.032	4.525	0.	0.	0.

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	26	69	19	27
normalized size	1	1.	1.	0.8	1.04	2.76	0.76	1.08
time (sec)	N/A	0.015	0.008	0.004	1.594	1.806	0.119	1.245

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	26	34	81	24	35
normalized size	1	1.	1.	0.84	1.1	2.61	0.77	1.13
time (sec)	N/A	0.02	0.008	0.006	1.554	1.831	0.132	1.239

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	11	19	8	11
normalized size	1	1.	1.	0.9	1.1	1.9	0.8	1.1
time (sec)	N/A	0.001	0.	0.002	1.006	1.498	0.063	1.224

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	29	38	61	19	41
normalized size	1	1.	1.	1.04	1.36	2.18	0.68	1.46
time (sec)	N/A	0.015	0.004	0.007	1.147	1.229	0.436	1.265

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	27	49	15	32
normalized size	1	1.	1.	0.95	1.23	2.23	0.68	1.45
time (sec)	N/A	0.012	0.005	0.006	1.035	1.288	0.188	1.188

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	61	62	0	494	564	84
normalized size	1	1.	0.98	1.	0.	7.97	9.1	1.35
time (sec)	N/A	0.055	0.069	0.007	0.	1.311	1.931	1.127

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	89	91	0	749	20	151
normalized size	1	1.	0.77	0.79	0.	6.51	0.17	1.31
time (sec)	N/A	0.063	0.027	0.004	0.	1.222	0.145	1.156

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	17	23	31	0	22
normalized size	1	1.	1.	1.06	1.44	1.94	0.	1.38
time (sec)	N/A	0.004	0.002	0.002	1.131	1.371	0.	1.14

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	53	0	0	0	0	0
normalized size	1	1.	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.019	0.012	0.033	0.	0.	0.	0.

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	59	61	0	0	0	0	0
normalized size	1	1.11	1.15	0.	0.	0.	0.	0.
time (sec)	N/A	0.023	0.012	0.033	0.	0.	0.	0.

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	132	132	157	0	0	0	0	0
normalized size	1	1.	1.19	0.	0.	0.	0.	0.
time (sec)	N/A	0.158	0.284	0.02	0.	0.	0.	0.

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	35	44	196	0	0	0	34	0
normalized size	1	1.26	5.6	0.	0.	0.	0.97	0.
time (sec)	N/A	0.009	0.151	0.029	0.	0.	16.009	0.

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	270	285	392	502	684	299	374
normalized size	1	1.	1.06	1.45	1.86	2.53	1.11	1.39
time (sec)	N/A	0.539	0.036	0.003	1.181	1.037	0.119	1.143

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	171	231	277	383	180	224
normalized size	1	1.	1.	1.35	1.62	2.24	1.05	1.31
time (sec)	N/A	0.092	0.017	0.001	1.111	1.177	0.092	1.112

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	92	84	127	192	95	112
normalized size	1	1.	1.	0.91	1.38	2.09	1.03	1.22
time (sec)	N/A	0.043	0.008	0.	1.091	1.121	0.078	1.142

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	29	38	63	31	38
normalized size	1	1.	1.	0.91	1.19	1.97	0.97	1.19
time (sec)	N/A	0.006	0.	0.	1.049	1.105	0.066	1.119

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	529	529	71	64	0	1770	88	471
normalized size	1	1.	0.13	0.12	0.	3.35	0.17	0.89
time (sec)	N/A	0.896	0.026	0.059	0.	1.388	0.936	1.322

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	746	746	182	232	0	6965	427	0
normalized size	1	1.	0.24	0.31	0.	9.34	0.57	0.
time (sec)	N/A	1.328	0.103	0.014	0.	1.828	5.815	0.

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	295	295	345	500	517	884	366	436
normalized size	1	1.	1.17	1.69	1.75	3.	1.24	1.48
time (sec)	N/A	0.532	0.049	0.003	1.026	1.136	0.124	1.117

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	207	288	289	482	218	252
normalized size	1	1.	1.02	1.42	1.42	2.37	1.07	1.24
time (sec)	N/A	0.123	0.025	0.001	1.162	1.156	0.1	1.145

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	109	100	136	225	112	122
normalized size	1	1.	1.02	0.93	1.27	2.1	1.05	1.14
time (sec)	N/A	0.051	0.013	0.	1.122	1.387	0.08	1.13

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	34	45	72	36	41
normalized size	1	1.	1.	0.92	1.22	1.95	0.97	1.11
time (sec)	N/A	0.007	0.	0.	1.152	1.263	0.063	1.145

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	71	67	0	2630	122	0
normalized size	1	1.	0.46	0.44	0.	17.19	0.8	0.
time (sec)	N/A	0.254	0.022	0.067	0.	1.695	1.454	0.

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	B	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	342	342	234	288	0	12891	580	0
normalized size	1	1.	0.68	0.84	0.	37.69	1.7	0.
time (sec)	N/A	0.532	0.18	0.015	0.	2.36	14.049	0.

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	96	85	113	281	94	113
normalized size	1	1.	1.	0.89	1.18	2.93	0.98	1.18
time (sec)	N/A	0.03	0.002	0.	1.047	1.111	0.074	1.164

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	74	65	86	190	71	86
normalized size	1	1.	1.	0.88	1.16	2.57	0.96	1.16
time (sec)	N/A	0.023	0.001	0.002	1.166	1.128	0.064	1.111

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	45	59	116	49	59
normalized size	1	1.	1.	0.83	1.09	2.15	0.91	1.09
time (sec)	N/A	0.017	0.001	0.001	1.134	1.068	0.059	1.108

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	20	26	45	19	26
normalized size	1	1.	1.	0.87	1.13	1.96	0.83	1.13
time (sec)	N/A	0.003	0.	0.001	1.161	1.082	0.053	1.125

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	268	268	45	41	0	4797	41	0
normalized size	1	1.	0.17	0.15	0.	17.9	0.15	0.
time (sec)	N/A	0.396	0.008	0.003	0.	10.65	0.8	0.

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	357	357	113	83	0	8227	71	0
normalized size	1	1.	0.32	0.23	0.	23.04	0.2	0.
time (sec)	N/A	0.397	0.016	0.007	0.	12.607	0.916	0.

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	97	78	104	254	94	104
normalized size	1	1.	1.	0.8	1.07	2.62	0.97	1.07
time (sec)	N/A	0.028	0.001	0.001	1.151	1.119	0.068	1.113

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	69	58	77	163	66	77
normalized size	1	1.	1.	0.84	1.12	2.36	0.96	1.12
time (sec)	N/A	0.02	0.001	0.	1.16	1.126	0.065	1.125

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	45	38	50	96	42	50
normalized size	1	1.	1.	0.84	1.11	2.13	0.93	1.11
time (sec)	N/A	0.015	0.001	0.002	1.781	1.09	0.059	1.121

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	23	42	19	23
normalized size	1	1.	1.	0.86	1.1	2.	0.9	1.1
time (sec)	N/A	0.003	0.	0.001	1.093	1.091	0.053	1.143

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	C	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	47	41	0	2303	36	374
normalized size	1	1.	0.2	0.18	0.	9.84	0.15	1.6
time (sec)	N/A	0.315	0.015	0.004	0.	8.501	0.724	1.159

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	C	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	317	317	108	79	0	4027	71	463
normalized size	1	1.	0.34	0.25	0.	12.7	0.22	1.46
time (sec)	N/A	0.335	0.023	0.008	0.	9.997	0.875	1.293

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	104	85	113	321	100	113
normalized size	1	1.	1.	0.82	1.09	3.09	0.96	1.09
time (sec)	N/A	0.034	0.002	0.003	1.204	1.233	0.077	1.117

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	76	65	86	209	73	86
normalized size	1	1.	1.	0.86	1.13	2.75	0.96	1.13
time (sec)	N/A	0.024	0.002	0.	1.159	1.256	0.068	1.137

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	52	45	59	122	49	59
normalized size	1	1.	1.	0.87	1.13	2.35	0.94	1.13
time (sec)	N/A	0.016	0.001	0.	1.34	1.332	0.063	1.116

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	32	61	27	32
normalized size	1	1.	1.	0.83	1.07	2.03	0.9	1.07
time (sec)	N/A	0.004	0.	0.002	1.196	1.241	0.052	1.128

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	263	263	55	49	0	0	41	0
normalized size	1	1.	0.21	0.19	0.	0.	0.16	0.
time (sec)	N/A	0.493	0.01	0.005	0.	0.	0.852	0.

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	366	366	128	96	0	0	76	0
normalized size	1	1.	0.35	0.26	0.	0.	0.21	0.
time (sec)	N/A	0.508	0.019	0.007	0.	0.	0.957	0.

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	164	799	402	185	220
normalized size	1	1.	1.	11.71	57.07	28.71	13.21	15.71
time (sec)	N/A	0.017	0.001	0.001	1.195	1.591	0.106	1.14

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	109	308	230	114	146
normalized size	1	1.	1.	7.79	22.	16.43	8.14	10.43
time (sec)	N/A	0.018	0.001	0.001	1.159	1.535	0.087	1.102

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	61	61	54	72	116	60	72
normalized size	1	4.36	4.36	3.86	5.14	8.29	4.29	5.14
time (sec)	N/A	0.014	0.	0.001	1.178	1.484	0.071	1.118

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	62	92	49	16
normalized size	1	1.	1.	0.93	4.43	6.57	3.5	1.14
time (sec)	N/A	0.018	0.004	0.004	1.206	1.698	0.444	1.096

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	136	212	109	16
normalized size	1	1.	1.	0.93	9.71	15.14	7.79	1.14
time (sec)	N/A	0.017	0.004	0.003	1.263	1.624	0.841	1.134

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	211	370	168	16
normalized size	1	1.	1.	0.93	15.07	26.43	12.	1.14
time (sec)	N/A	0.018	0.004	0.003	1.67	1.764	1.551	1.146

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	31	41	100	29	42
normalized size	1	1.	1.	0.82	1.08	2.63	0.76	1.11
time (sec)	N/A	0.025	0.007	0.007	2.706	1.8	0.142	1.145

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	84	65	86	266	80	86
normalized size	1	1.	1.	0.77	1.02	3.17	0.95	1.02
time (sec)	N/A	0.086	0.002	0.002	1.754	1.547	0.072	1.14

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	63	50	66	180	60	66
normalized size	1	1.	1.	0.79	1.05	2.86	0.95	1.05
time (sec)	N/A	0.076	0.002	0.001	1.205	1.515	0.069	1.117

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	44	35	46	108	41	46
normalized size	1	1.	1.	0.8	1.05	2.45	0.93	1.05
time (sec)	N/A	0.069	0.001	0.	1.146	1.526	0.062	1.123

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	26	53	22	26
normalized size	1	1.	1.	0.8	1.04	2.12	0.88	1.04
time (sec)	N/A	0.003	0.	0.	1.172	1.437	0.054	1.124

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	62	42	73	149	63	84
normalized size	1	1.	2.	1.35	2.35	4.81	2.03	2.71
time (sec)	N/A	0.023	0.013	0.01	1.733	1.632	0.14	1.128

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	103	84	120	467	104	131
normalized size	1	1.	1.16	0.94	1.35	5.25	1.17	1.47
time (sec)	N/A	0.061	0.051	0.02	1.749	1.779	1.103	1.12

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	137	126	161	849	134	151
normalized size	1	1.	0.85	0.78	1.	5.27	0.83	0.94
time (sec)	N/A	0.119	0.084	0.021	1.855	1.757	1.241	1.156

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	205	132	116	154	589	296	181
normalized size	1	2.25	1.45	1.27	1.69	6.47	3.25	1.99
time (sec)	N/A	0.13	0.088	0.019	1.957	1.788	1.112	1.112

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	73	127	0	466	209	104
normalized size	1	1.	0.94	1.63	0.	5.97	2.68	1.33
time (sec)	N/A	0.063	0.05	0.005	0.	1.834	0.76	1.119

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	54	89	0	377	153	73
normalized size	1	1.	1.08	1.78	0.	7.54	3.06	1.46
time (sec)	N/A	0.039	0.028	0.002	0.	1.78	0.577	1.173

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	38	54	0	333	124	58
normalized size	1	1.	0.93	1.32	0.	8.12	3.02	1.41
time (sec)	N/A	0.021	0.014	0.003	0.	1.831	0.23	1.142

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	28	0	205	54	23
normalized size	1	1.	1.	1.33	0.	9.76	2.57	1.1
time (sec)	N/A	0.008	0.003	0.003	0.	1.749	0.175	1.123

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	48	72	0	385	738	84
normalized size	1	1.	0.81	1.22	0.	6.53	12.51	1.42
time (sec)	N/A	0.035	0.033	0.006	0.	1.863	1.708	1.124

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	81	123	0	533	1620	158
normalized size	1	1.	1.03	1.56	0.	6.75	20.51	2.
time (sec)	N/A	0.086	0.044	0.007	0.	1.921	4.434	1.116

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	106	198	0	811	3284	263
normalized size	1	1.	0.88	1.64	0.	6.7	27.14	2.17
time (sec)	N/A	0.125	0.129	0.01	0.	1.926	7.246	1.16

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	34	0	243	61	32
normalized size	1	1.	1.	1.1	0.	7.84	1.97	1.03
time (sec)	N/A	0.024	0.009	0.006	0.	1.671	0.202	1.13

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	60	86	0	555	117	88
normalized size	1	1.	0.95	1.37	0.	8.81	1.86	1.4
time (sec)	N/A	0.033	0.025	0.004	0.	1.812	0.727	1.113

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	75	147	0	1231	257	139
normalized size	1	1.	0.82	1.62	0.	13.53	2.82	1.53
time (sec)	N/A	0.048	0.06	0.005	0.	1.948	1.652	1.1

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	35	42	89	576	92	41
normalized size	1	1.	1.	1.2	2.54	16.46	2.63	1.17
time (sec)	N/A	0.035	0.014	0.006	1.658	2.115	0.198	1.146

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	24	26	24	14
normalized size	1	1.	1.	1.1	2.4	2.6	2.4	1.4
time (sec)	N/A	0.003	0.004	0.003	1.561	1.686	0.159	1.127

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	31	59	69	134	56	55
normalized size	1	1.	0.84	1.59	1.86	3.62	1.51	1.49
time (sec)	N/A	0.01	0.012	0.004	1.458	1.8	0.587	1.155

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	52	94	155	347	146	99
normalized size	1	1.	0.87	1.57	2.58	5.78	2.43	1.65
time (sec)	N/A	0.016	0.015	0.003	1.48	1.748	1.241	1.114

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	32	26	34	61	22	36
normalized size	1	1.	3.2	2.6	3.4	6.1	2.2	3.6
time (sec)	N/A	0.003	0.005	0.007	0.959	1.742	0.16	1.116

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	45	52	76	208	53	76
normalized size	1	1.	1.15	1.33	1.95	5.33	1.36	1.95
time (sec)	N/A	0.013	0.02	0.008	1.055	1.716	0.596	1.15

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	65	78	165	498	141	119
normalized size	1	1.	1.02	1.22	2.58	7.78	2.2	1.86
time (sec)	N/A	0.022	0.027	0.008	1.014	1.696	1.307	1.14

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	15	12	15	39	10	18
normalized size	1	1.	3.75	3.	3.75	9.75	2.5	4.5
time (sec)	N/A	0.002	0.002	0.005	1.007	1.668	0.093	1.117

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	26	24	34	99	22	36
normalized size	1	1.	0.96	0.89	1.26	3.67	0.81	1.33
time (sec)	N/A	0.007	0.016	0.007	1.168	1.671	0.11	1.115

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	37	36	59	170	44	53
normalized size	1	1.	0.82	0.8	1.31	3.78	0.98	1.18
time (sec)	N/A	0.012	0.017	0.009	1.182	1.746	0.138	1.142

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	64	64	73	130	58	84
normalized size	1	1.	1.08	1.08	1.24	2.2	0.98	1.42
time (sec)	N/A	0.055	0.021	0.003	1.13	1.77	0.312	1.123

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	11	12	15	32	10	15
normalized size	1	1.	1.1	1.2	1.5	3.2	1.	1.5
time (sec)	N/A	0.011	0.006	0.002	1.114	1.643	0.082	1.13

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	51	35	49	84	0	31
normalized size	1	1.	1.16	0.8	1.11	1.91	0.	0.7
time (sec)	N/A	0.026	0.021	0.002	1.651	1.769	0.	1.111

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	55	152	0	211	0	74
normalized size	1	1.	0.82	2.27	0.	3.15	0.	1.1
time (sec)	N/A	0.049	0.069	0.016	0.	1.874	0.	1.16

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	51	146	0	165	0	95
normalized size	1	1.	0.81	2.32	0.	2.62	0.	1.51
time (sec)	N/A	0.04	0.057	0.01	0.	1.744	0.	1.191

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	132	108	0	0	238	0
normalized size	1	1.	0.56	0.46	0.	0.	1.02	0.
time (sec)	N/A	0.372	0.047	0.006	0.	0.	2.524	0.

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	81	74	0	10654	158	0
normalized size	1	1.	0.39	0.35	0.	50.73	0.75	0.
time (sec)	N/A	0.228	0.029	0.002	0.	8.125	0.953	0.

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	79	72	0	4685	83	0
normalized size	1	1.	0.44	0.4	0.	26.03	0.46	0.
time (sec)	N/A	0.156	0.022	0.001	0.	7.861	0.66	0.

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	116	71	0	1079	26	211
normalized size	1	1.	0.83	0.51	0.	7.71	0.19	1.51
time (sec)	N/A	0.107	0.03	0.003	0.	1.374	0.25	1.133

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	238	119	105	0	9806	559	0
normalized size	1	1.06	0.53	0.47	0.	43.78	2.5	0.
time (sec)	N/A	0.482	0.048	0.007	0.	8.207	17.941	0.

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	314	312	173	144	0	18572	0	0
normalized size	1	0.99	0.55	0.46	0.	59.15	0.	0.
time (sec)	N/A	0.545	0.09	0.007	0.	11.262	0.	0.

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	393	393	244	217	0	29965	0	0
normalized size	1	1.	0.62	0.55	0.	76.25	0.	0.
time (sec)	N/A	0.604	0.143	0.01	0.	53.037	0.	0.

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	356	356	106	97	0	0	374	0
normalized size	1	1.	0.3	0.27	0.	0.	1.05	0.
time (sec)	N/A	0.426	0.042	0.012	0.	0.	3.399	0.

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	318	318	106	97	0	0	274	0
normalized size	1	1.	0.33	0.31	0.	0.	0.86	0.
time (sec)	N/A	0.312	0.033	0.003	0.	0.	2.546	0.

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	261	104	95	0	0	131	0
normalized size	1	1.	0.4	0.36	0.	0.	0.5	0.
time (sec)	N/A	0.264	0.027	0.003	0.	0.	0.902	0.

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	161	94	0	443	26	196
normalized size	1	1.	0.73	0.43	0.	2.	0.12	0.89
time (sec)	N/A	0.185	0.071	0.003	0.	1.557	0.285	1.137

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	393	393	163	139	0	0	0	0
normalized size	1	1.	0.41	0.35	0.	0.	0.	0.
time (sec)	N/A	0.466	0.063	0.008	0.	0.	0.	0.

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	496	496	238	188	0	0	0	0
normalized size	1	1.	0.48	0.38	0.	0.	0.	0.
time (sec)	N/A	0.894	0.124	0.01	0.	0.	0.	0.

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	195	264	259	624	199	296
normalized size	1	1.	1.59	2.15	2.11	5.07	1.62	2.41
time (sec)	N/A	0.24	0.028	0.002	1.131	1.262	0.112	1.252

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	114	138	161	335	114	173
normalized size	1	1.	0.95	1.15	1.34	2.79	0.95	1.44
time (sec)	N/A	0.063	0.013	0.002	1.192	1.283	0.086	1.117

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	66	63	88	165	65	88
normalized size	1	1.	0.92	0.88	1.22	2.29	0.9	1.22
time (sec)	N/A	0.03	0.007	0.001	1.082	1.34	0.068	1.113

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	23	30	54	22	30
normalized size	1	1.	1.	0.88	1.15	2.08	0.85	1.15
time (sec)	N/A	0.004	0.	0.	1.067	1.276	0.057	1.104

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	57	49	0	1254	66	0
normalized size	1	1.	0.64	0.55	0.	14.09	0.74	0.
time (sec)	N/A	0.087	0.014	0.013	0.	1.547	0.705	0.

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	B	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	150	158	0	5716	292	0
normalized size	1	1.	0.89	0.93	0.	33.82	1.73	0.
time (sec)	N/A	0.291	0.054	0.01	0.	1.798	4.605	0.

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	252	254	398	0	13437	0	0
normalized size	1	1.	1.01	1.58	0.	53.32	0.	0.
time (sec)	N/A	0.532	0.119	0.016	0.	1.902	0.	0.

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	204	267	246	655	212	300
normalized size	1	1.	0.97	1.27	1.17	3.12	1.01	1.43
time (sec)	N/A	0.226	0.026	0.001	1.103	1.315	0.148	1.148

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	130	143	153	367	128	180
normalized size	1	1.	0.97	1.07	1.14	2.74	0.96	1.34
time (sec)	N/A	0.14	0.015	0.001	1.118	1.305	0.125	1.131

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	75	66	80	180	70	92
normalized size	1	1.	0.95	0.84	1.01	2.28	0.89	1.16
time (sec)	N/A	0.076	0.008	0.001	1.08	1.34	0.072	1.129

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	35	28	36	68	29	36
normalized size	1	1.	1.	0.8	1.03	1.94	0.83	1.03
time (sec)	N/A	0.01	0.001	0.001	1.084	1.3	0.058	1.131

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	59	50	0	0	155	0
normalized size	1	1.	0.51	0.43	0.	0.	1.34	0.
time (sec)	N/A	0.083	0.016	0.003	0.	0.	2.577	0.

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F(-1)	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	166	162	0	0	539	0
normalized size	1	1.	0.72	0.7	0.	0.	2.33	0.
time (sec)	N/A	0.241	0.06	0.01	0.	0.	16.462	0.

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	349	349	284	405	0	0	0	0
normalized size	1	1.	0.81	1.16	0.	0.	0.	0.
time (sec)	N/A	0.369	0.119	0.016	0.	0.	0.	0.

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	204	267	246	671	219	300
normalized size	1	1.	0.97	1.27	1.17	3.2	1.04	1.43
time (sec)	N/A	0.164	0.027	0.002	1.032	1.304	0.117	1.103

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	132	143	153	379	134	180
normalized size	1	1.	0.96	1.04	1.11	2.75	0.97	1.3
time (sec)	N/A	0.121	0.015	0.001	1.002	1.254	0.091	1.134

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	73	66	80	185	73	92
normalized size	1	1.	0.92	0.84	1.01	2.34	0.92	1.16
time (sec)	N/A	0.077	0.008	0.001	1.088	1.275	0.073	1.14

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	35	28	36	68	29	36
normalized size	1	1.	1.	0.8	1.03	1.94	0.83	1.03
time (sec)	N/A	0.009	0.002	0.	1.133	1.299	0.057	1.142

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	61	52	0	0	172	0
normalized size	1	1.	0.62	0.53	0.	0.	1.74	0.
time (sec)	N/A	0.087	0.015	0.003	0.	0.	3.884	0.

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F(-1)	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	182	160	0	0	559	0
normalized size	1	1.	0.81	0.71	0.	0.	2.48	0.
time (sec)	N/A	0.213	0.061	0.01	0.	0.	18.002	0.

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	545	545	99	93	0	0	0	0
normalized size	1	1.	0.18	0.17	0.	0.	0.	0.
time (sec)	N/A	1.476	0.062	0.01	0.	0.	0.	0.

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	487	487	99	93	0	0	0	0
normalized size	1	1.	0.2	0.19	0.	0.	0.	0.
time (sec)	N/A	0.756	0.049	0.003	0.	0.	0.	0.

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	334	334	97	93	0	0	167	0
normalized size	1	1.	0.29	0.28	0.	0.	0.5	0.
time (sec)	N/A	0.47	0.041	0.003	0.	0.	28.181	0.

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	469	469	95	91	0	0	0	0
normalized size	1	1.	0.2	0.19	0.	0.	0.	0.
time (sec)	N/A	0.684	0.045	0.003	0.	0.	0.	0.

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	522	522	99	90	0	0	0	0
normalized size	1	1.	0.19	0.17	0.	0.	0.	0.
time (sec)	N/A	0.861	0.059	0.003	0.	0.	0.	0.

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	563	563	157	134	0	0	0	0
normalized size	1	1.	0.28	0.24	0.	0.	0.	0.
time (sec)	N/A	1.157	0.096	0.008	0.	0.	0.	0.

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	645	640	163	133	0	0	0	0
normalized size	1	0.99	0.25	0.21	0.	0.	0.	0.
time (sec)	N/A	1.38	0.12	0.007	0.	0.	0.	0.

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	395	395	61	56	0	0	70	0
normalized size	1	1.	0.15	0.14	0.	0.	0.18	0.
time (sec)	N/A	1.438	0.016	0.007	0.	0.	0.218	0.

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	377	377	61	56	0	0	65	0
normalized size	1	1.	0.16	0.15	0.	0.	0.17	0.
time (sec)	N/A	0.91	0.013	0.006	0.	0.	0.231	0.

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	361	361	61	56	0	0	61	0
normalized size	1	1.	0.17	0.16	0.	0.	0.17	0.
time (sec)	N/A	0.515	0.013	0.006	0.	0.	0.22	0.

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	248	59	56	0	4757	48	0
normalized size	1	1.	0.24	0.23	0.	19.18	0.19	0.
time (sec)	N/A	0.323	0.012	0.005	0.	8.111	0.174	0.

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	361	361	57	54	0	0	61	0
normalized size	1	1.	0.16	0.15	0.	0.	0.17	0.
time (sec)	N/A	0.548	0.012	0.005	0.	0.	0.216	0.

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	377	377	62	53	0	0	65	0
normalized size	1	1.	0.16	0.14	0.	0.	0.17	0.
time (sec)	N/A	0.721	0.012	0.004	0.	0.	0.225	0.

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	415	415	103	75	0	0	82	0
normalized size	1	1.	0.25	0.18	0.	0.	0.2	0.
time (sec)	N/A	0.902	0.018	0.007	0.	0.	0.367	0.

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	448	448	109	74	0	0	70	0
normalized size	1	1.	0.24	0.17	0.	0.	0.16	0.
time (sec)	N/A	1.102	0.019	0.007	0.	0.	0.269	0.

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1064	1064	167	122	0	0	112	0
normalized size	1	1.	0.16	0.11	0.	0.	0.11	0.
time (sec)	N/A	2.504	0.041	0.01	0.	0.	0.372	0.

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1005	1005	167	122	0	0	112	0
normalized size	1	1.	0.17	0.12	0.	0.	0.11	0.
time (sec)	N/A	2.403	0.03	0.01	0.	0.	0.382	0.

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	677	677	167	122	0	0	112	0
normalized size	1	1.	0.25	0.18	0.	0.	0.17	0.
time (sec)	N/A	1.553	0.045	0.01	0.	0.	0.358	0.

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	A	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	682	682	167	122	0	8841	104	0
normalized size	1	1.	0.24	0.18	0.	12.96	0.15	0.
time (sec)	N/A	1.235	0.025	0.008	0.	9.747	0.282	0.

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	850	850	167	122	0	0	112	0
normalized size	1	1.	0.2	0.14	0.	0.	0.13	0.
time (sec)	N/A	1.47	0.035	0.009	0.	0.	0.369	0.

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	873	873	167	122	0	0	112	0
normalized size	1	1.	0.19	0.14	0.	0.	0.13	0.
time (sec)	N/A	1.916	0.027	0.009	0.	0.	0.371	0.

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	986	986	167	122	0	0	112	0
normalized size	1	1.	0.17	0.12	0.	0.	0.11	0.
time (sec)	N/A	1.927	0.034	0.009	0.	0.	0.362	0.

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	28	47	22	28
normalized size	1	1.	1.	0.88	1.12	1.88	0.88	1.12
time (sec)	N/A	0.055	0.002	0.	1.099	1.258	0.082	1.196

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	79	114	142	223	90	493
normalized size	1	1.	0.84	1.21	1.51	2.37	0.96	5.24
time (sec)	N/A	0.127	0.039	0.004	1.055	1.243	0.45	1.487

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	172	155	18	387	175	208
normalized size	1	1.	11.47	10.33	1.2	25.8	11.67	13.87
time (sec)	N/A	0.018	0.006	0.001	0.988	1.216	0.116	1.17

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	182	157	211	402	182	211
normalized size	1	1.	11.38	9.81	13.19	25.12	11.38	13.19
time (sec)	N/A	0.055	0.006	0.002	1.028	1.091	0.122	1.211

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	186	157	211	408	185	211
normalized size	1	1.	11.62	9.81	13.19	25.5	11.56	13.19
time (sec)	N/A	0.05	0.006	0.001	1.019	1.188	0.126	1.153

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	230	0	657	0	255
normalized size	1	1.	1.	10.95	0.	31.29	0.	12.14
time (sec)	N/A	0.031	0.115	0.038	0.	1.615	0.	1.868

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	9	9	14	24	8	15
normalized size	1	1.	0.9	0.9	1.4	2.4	0.8	1.5
time (sec)	N/A	0.004	0.004	0.	1.018	1.303	0.285	1.212

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	18	39	12	24
normalized size	1	1.	1.	0.93	1.2	2.6	0.8	1.6
time (sec)	N/A	0.026	0.005	0.004	1.036	1.257	0.321	1.222

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	18	39	12	20
normalized size	1	1.	1.	0.93	1.2	2.6	0.8	1.33
time (sec)	N/A	0.027	0.006	0.005	1.045	1.362	0.334	1.197

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	18	63	61	29	0
normalized size	1	1.	1.	1.2	4.2	4.07	1.93	0.
time (sec)	N/A	0.025	0.012	0.013	1.059	1.547	1.418	0.

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	14	177	18	171	87	18
normalized size	1	1.	0.93	11.8	1.2	11.4	5.8	1.2
time (sec)	N/A	0.004	0.022	0.	1.075	1.352	3.977	1.238

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	197	109	177	0	20
normalized size	1	1.	1.	12.31	6.81	11.06	0.	1.25
time (sec)	N/A	0.023	0.029	0.018	1.1	1.405	0.	1.349

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	197	109	177	0	20
normalized size	1	1.	1.	12.31	6.81	11.06	0.	1.25
time (sec)	N/A	0.023	0.036	0.013	1.13	1.318	0.	1.433

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	203	826	328	0	0
normalized size	1	1.	1.	9.67	39.33	15.62	0.	0.
time (sec)	N/A	0.032	0.172	0.049	1.239	2.469	0.	0.

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	17	24	0	53	46	50
normalized size	1	1.	0.89	1.26	0.	2.79	2.42	2.63
time (sec)	N/A	0.005	0.009	0.	0.	1.336	0.56	1.259

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	97	31	47	72	0	73
normalized size	1	1.	3.59	1.15	1.74	2.67	0.	2.7
time (sec)	N/A	0.023	0.075	0.004	1.299	1.425	0.	1.182

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	C	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	116	97	142	47	74	0	0
normalized size	1	4.3	3.59	5.26	1.74	2.74	0.	0.
time (sec)	N/A	0.102	0.027	0.114	1.214	1.315	0.	0.

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	99	33	47	74	0	78
normalized size	1	1.	3.41	1.14	1.62	2.55	0.	2.69
time (sec)	N/A	0.024	0.076	0.003	1.593	1.392	0.	1.174

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	108	0	53	101	0	0
normalized size	1	1.	3.	0.	1.47	2.81	0.	0.
time (sec)	N/A	0.086	0.148	0.07	1.369	1.468	0.	0.

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	27	35	66	29	35
normalized size	1	1.	1.	0.84	1.09	2.06	0.91	1.09
time (sec)	N/A	0.034	0.002	0.	1.022	1.253	0.08	1.13

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	14	23	8	14
normalized size	1	1.	1.	0.92	1.17	1.92	0.67	1.17
time (sec)	N/A	0.049	0.001	0.	1.019	1.241	0.082	1.244

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	32	0	225	124	42
normalized size	1	1.	1.	0.76	0.	5.36	2.95	1.
time (sec)	N/A	0.057	0.014	0.004	0.	1.336	0.287	1.226

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	23	26	0	85	0	122
normalized size	1	1.	0.92	1.04	0.	3.4	0.	4.88
time (sec)	N/A	0.024	0.014	0.004	0.	1.398	0.	1.297

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	21	34	0	74	0	92
normalized size	1	1.	0.88	1.42	0.	3.08	0.	3.83
time (sec)	N/A	0.018	0.047	0.005	0.	1.496	0.	1.165

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	24	26	53	74	0	88
normalized size	1	1.	0.96	1.04	2.12	2.96	0.	3.52
time (sec)	N/A	0.021	0.02	0.003	1.201	1.355	0.	1.193

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	0	63	0	72
normalized size	1	1.	1.	1.05	0.	3.15	0.	3.6
time (sec)	N/A	0.01	0.009	0.003	0.	1.469	0.	1.188

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	106	26	0	53	73	50
normalized size	1	1.	5.58	1.37	0.	2.79	3.84	2.63
time (sec)	N/A	0.009	0.065	0.003	0.	1.641	12.45	1.204

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	108	23	42	55	76	53
normalized size	1	1.	4.91	1.05	1.91	2.5	3.45	2.41
time (sec)	N/A	0.008	0.035	0.003	1.2	1.632	58.611	1.153

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	21	23	0	69	0	82
normalized size	1	1.	0.95	1.05	0.	3.14	0.	3.73
time (sec)	N/A	0.018	0.012	0.003	0.	1.625	0.	1.177

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	19	28	0	58	53	58
normalized size	1	1.	0.9	1.33	0.	2.76	2.52	2.76
time (sec)	N/A	0.012	0.011	0.003	0.	1.294	0.763	1.285

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	22	28	43	61	56	69
normalized size	1	1.	0.92	1.17	1.79	2.54	2.33	2.88
time (sec)	N/A	0.023	0.013	0.001	1.174	1.423	5.156	1.122

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	23	43	61	53	55
normalized size	1	1.	1.	1.05	1.95	2.77	2.41	2.5
time (sec)	N/A	0.009	0.01	0.003	1.166	1.431	5.103	1.156

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	21	23	43	69	0	82
normalized size	1	1.	0.95	1.05	1.95	3.14	0.	3.73
time (sec)	N/A	0.013	0.006	0.002	1.152	1.542	0.	1.261

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	19	28	43	58	53	58
normalized size	1	1.	0.9	1.33	2.05	2.76	2.52	2.76
time (sec)	N/A	0.012	0.005	0.002	1.172	1.417	0.774	1.291

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	143	25686	26	4385	1771	2641
normalized size	1	1.	6.81	1223.14	1.24	208.81	84.33	125.76
time (sec)	N/A	0.125	0.146	0.005	1.002	1.159	0.345	1.221

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	18	5596	24	1118	469	670
normalized size	1	1.	0.9	279.8	1.2	55.9	23.45	33.5
time (sec)	N/A	0.052	0.026	0.003	1.001	1.205	0.164	1.173

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	18	5596	595	1118	469	670
normalized size	1	1.	0.95	294.53	31.32	58.84	24.68	35.26
time (sec)	N/A	0.069	0.011	0.002	0.979	1.124	0.154	1.143

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	127	2185	19	1071	483	656
normalized size	1	1.	7.94	136.56	1.19	66.94	30.19	41.
time (sec)	N/A	0.024	0.057	0.002	0.987	1.175	0.151	1.213

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	98	89	18	197	97	119
normalized size	1	1.	6.53	5.93	1.2	13.13	6.47	7.93
time (sec)	N/A	0.013	0.003	0.002	1.01	1.126	0.089	1.19

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	98	89	119	197	97	119
normalized size	1	1.	6.12	5.56	7.44	12.31	6.06	7.44
time (sec)	N/A	0.028	0.002	0.	0.986	1.208	0.083	1.323

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	115	2205	22	1085	484	659
normalized size	1	1.	6.39	122.5	1.22	60.28	26.89	36.61
time (sec)	N/A	0.044	0.053	0.002	0.985	1.136	0.152	1.211

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	98	89	20	198	97	119
normalized size	1	1.	5.76	5.24	1.18	11.65	5.71	7.
time (sec)	N/A	0.025	0.003	0.002	0.978	1.131	0.091	1.31

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	98	89	119	198	97	119
normalized size	1	1.	7.	6.36	8.5	14.14	6.93	8.5
time (sec)	N/A	0.227	0.003	0.003	0.974	1.077	0.092	1.274

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	98	89	119	198	97	119
normalized size	1	1.	7.	6.36	8.5	14.14	6.93	8.5
time (sec)	N/A	0.011	0.003	0.003	0.982	1.069	0.094	1.28

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	115	2205	618	1085	484	659
normalized size	1	1.	6.39	122.5	34.33	60.28	26.89	36.61
time (sec)	N/A	0.06	0.011	0.002	0.996	1.042	0.152	1.197

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	98	89	119	198	97	119
normalized size	1	1.	7.	6.36	8.5	14.14	6.93	8.5
time (sec)	N/A	0.022	0.003	0.001	1.158	1.08	0.089	1.183

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	98	89	119	198	97	119
normalized size	1	1.	5.44	4.94	6.61	11.	5.39	6.61
time (sec)	N/A	0.019	0.003	0.003	1.012	1.039	0.088	1.197

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	98	89	119	198	97	119
normalized size	1	1.	7.	6.36	8.5	14.14	6.93	8.5
time (sec)	N/A	0.002	0.002	0.002	1.038	1.068	0.082	1.21

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	80	67	89	158	70	89
normalized size	1	1.	2.86	2.39	3.18	5.64	2.5	3.18
time (sec)	N/A	0.027	0.005	0.002	1.009	1.11	0.081	1.236

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	108	325	252	470	194	281
normalized size	1	1.	3.48	10.48	8.13	15.16	6.26	9.06
time (sec)	N/A	0.031	0.034	0.002	1.021	1.14	0.107	1.173

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	31	70	112	230	41
normalized size	1	1.	1.	0.91	2.06	3.29	6.76	1.21
time (sec)	N/A	0.008	0.055	0.003	1.696	1.393	87.108	1.128

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	35	33	73	126	0	107
normalized size	1	1.	1.	0.94	2.09	3.6	0.	3.06
time (sec)	N/A	0.009	0.054	0.003	1.665	1.378	0.	1.211

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	93	78	104	197	87	104
normalized size	1	1.	3.1	2.6	3.47	6.57	2.9	3.47
time (sec)	N/A	0.02	0.006	0.002	1.01	1.114	0.09	1.222

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	140	618	378	710	314	393
normalized size	1	1.	4.52	19.94	12.19	22.9	10.13	12.68
time (sec)	N/A	0.038	0.046	0.002	1.069	1.167	0.13	1.238

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	98	81	108	213	90	108
normalized size	1	1.	2.88	2.38	3.18	6.26	2.65	3.18
time (sec)	N/A	0.033	0.008	0.003	0.975	1.229	0.095	1.172

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	146	646	390	747	321	402
normalized size	1	1.	3.56	15.76	9.51	18.22	7.83	9.8
time (sec)	N/A	0.045	0.05	0.002	0.988	1.123	0.146	1.201

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	244	1523	390	782	323	417
normalized size	1	1.	5.3	33.11	8.48	17.	7.02	9.07
time (sec)	N/A	0.047	0.062	0.002	1.056	1.099	0.146	1.182

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	248	4284	1044	2256	930	1253
normalized size	1	1.	5.28	91.15	22.21	48.	19.79	26.66
time (sec)	N/A	0.091	0.117	0.003	1.013	1.062	0.236	1.218

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	36	31	73	112	201	41
normalized size	1	1.	1.06	0.91	2.15	3.29	5.91	1.21
time (sec)	N/A	0.009	0.062	0.001	1.651	1.354	175.955	1.178

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	42	37	96	131	0	49
normalized size	1	1.	0.95	0.84	2.18	2.98	0.	1.11
time (sec)	N/A	0.01	0.083	0.003	1.701	1.395	0.	1.15

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	49	43	112	171	0	57
normalized size	1	1.	0.98	0.86	2.24	3.42	0.	1.14
time (sec)	N/A	0.009	0.188	0.003	1.736	1.373	0.	1.196

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	33	30	23	70	29	39
normalized size	1	1.	1.74	1.58	1.21	3.68	1.53	2.05
time (sec)	N/A	0.009	0.002	0.001	1.052	1.118	0.058	1.198

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	21	18	19	42	17	23
normalized size	1	1.	1.31	1.12	1.19	2.62	1.06	1.44
time (sec)	N/A	0.007	0.001	0.001	0.985	1.108	0.054	1.196

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	96	96	87	116	263	94	116
normalized size	1	2.91	2.91	2.64	3.52	7.97	2.85	3.52
time (sec)	N/A	0.198	0.006	0.002	0.993	1.146	0.075	1.227

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	96	96	87	116	263	94	116
normalized size	1	2.91	2.91	2.64	3.52	7.97	2.85	3.52
time (sec)	N/A	0.154	0.005	0.001	0.986	1.123	0.083	1.262

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	16	151	56	16
normalized size	1	1.	1.	0.93	1.14	10.79	4.	1.14
time (sec)	N/A	0.008	0.005	0.001	1.021	1.307	0.208	1.237

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	18	35	12	19
normalized size	1	1.	1.	0.93	1.2	2.33	0.8	1.27
time (sec)	N/A	0.009	0.005	0.003	1.011	1.256	0.084	1.184

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	20	14	20	38	14	24
normalized size	1	1.	1.18	0.82	1.18	2.24	0.82	1.41
time (sec)	N/A	0.01	0.006	0.	1.071	1.19	0.095	1.125

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	23	28	31	50	19	32
normalized size	1	1.	0.57	0.7	0.78	1.25	0.48	0.8
time (sec)	N/A	0.097	0.062	0.009	1.054	1.247	30.034	1.899

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	605	605	98	2105	0	0	0	0
normalized size	1	1.	0.16	3.48	0.	0.	0.	0.
time (sec)	N/A	4.535	0.075	0.047	0.	0.	0.	0.

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	55	82	0	193	58	78
normalized size	1	1.	0.87	1.3	0.	3.06	0.92	1.24
time (sec)	N/A	0.066	0.027	0.024	0.	1.483	0.114	1.201

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	30	97	20	18
normalized size	1	1.	1.	0.93	2.14	6.93	1.43	1.29
time (sec)	N/A	0.05	0.007	0.006	1.049	1.494	0.092	1.164

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	24	27	43	119	29	31
normalized size	1	1.	0.86	0.96	1.54	4.25	1.04	1.11
time (sec)	N/A	0.028	0.012	0.005	1.008	1.494	0.104	1.283

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	28	30	51	86	34	38
normalized size	1	1.	0.47	0.51	0.86	1.46	0.58	0.64
time (sec)	N/A	0.091	0.012	0.006	1.011	1.207	0.181	1.195

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	41	15	24	8	15
normalized size	1	1.	1.	3.73	1.36	2.18	0.73	1.36
time (sec)	N/A	0.007	0.007	0.009	1.022	1.351	0.119	1.244

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	205	132	116	154	585	272	181
normalized size	1	2.25	1.45	1.27	1.69	6.43	2.99	1.99
time (sec)	N/A	0.149	0.089	0.016	1.515	1.318	1.041	1.238

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	23	26	59	85	0	134
normalized size	1	1.	0.92	1.04	2.36	3.4	0.	5.36
time (sec)	N/A	0.025	0.336	0.007	1.381	3.317	0.	1.617

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	21	24	53	82	0	120
normalized size	1	1.	0.91	1.04	2.3	3.57	0.	5.22
time (sec)	N/A	0.082	0.234	0.008	1.326	1.684	0.	1.788

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	21	24	53	82	0	120
normalized size	1	1.	0.91	1.04	2.3	3.57	0.	5.22
time (sec)	N/A	0.059	0.178	0.006	1.204	1.424	0.	1.366

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	19	22	50	80	0	117
normalized size	1	1.	0.9	1.05	2.38	3.81	0.	5.57
time (sec)	N/A	0.044	0.12	0.006	1.216	1.465	0.	1.286

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	17	20	45	74	0	0
normalized size	1	1.	0.89	1.05	2.37	3.89	0.	0.
time (sec)	N/A	0.043	0.01	0.004	1.294	1.42	0.	0.

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	21	24	49	77	0	0
normalized size	1	1.	0.91	1.04	2.13	3.35	0.	0.
time (sec)	N/A	0.034	0.175	0.004	1.276	1.61	0.	0.

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	21	24	49	80	0	0
normalized size	1	1.	0.91	1.04	2.13	3.48	0.	0.
time (sec)	N/A	0.035	0.182	0.007	1.314	1.824	0.	0.

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	21	24	49	80	0	0
normalized size	1	1.	0.91	1.04	2.13	3.48	0.	0.
time (sec)	N/A	0.036	0.213	0.005	1.322	2.043	0.	0.

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	83	74	99	262	97	99
normalized size	1	1.	0.86	0.76	1.02	2.7	1.	1.02
time (sec)	N/A	0.138	0.037	0.007	1.5	1.235	0.213	1.138

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	78	69	92	246	92	92
normalized size	1	1.	0.87	0.77	1.02	2.73	1.02	1.02
time (sec)	N/A	0.123	0.023	0.006	1.55	1.402	0.21	1.217

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	72	62	82	220	78	82
normalized size	1	1.	0.94	0.81	1.06	2.86	1.01	1.06
time (sec)	N/A	0.121	0.03	0.004	1.645	1.507	0.204	1.202

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	69	57	76	212	75	76
normalized size	1	1.	0.96	0.79	1.06	2.94	1.04	1.06
time (sec)	N/A	0.093	0.022	0.005	1.628	1.387	0.2	1.243

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	65	56	74	205	75	74
normalized size	1	1.	0.92	0.79	1.04	2.89	1.06	1.04
time (sec)	N/A	0.078	0.017	0.005	1.473	1.431	0.195	1.192

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	69	60	80	220	78	81
normalized size	1	1.	0.92	0.8	1.07	2.93	1.04	1.08
time (sec)	N/A	0.135	0.02	0.006	1.504	1.491	0.25	1.234

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	78	65	86	250	87	88
normalized size	1	1.	0.93	0.77	1.02	2.98	1.04	1.05
time (sec)	N/A	0.152	0.033	0.007	1.511	1.41	0.276	1.198

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	82	70	93	281	94	95
normalized size	1	1.	0.9	0.77	1.02	3.09	1.03	1.04
time (sec)	N/A	0.157	0.055	0.008	1.488	1.305	0.289	1.133

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	307	307	109	74	0	5511	61	0
normalized size	1	1.	0.36	0.24	0.	17.95	0.2	0.
time (sec)	N/A	0.578	0.019	0.007	0.	9.7	0.802	0.

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	269	101	67	0	5019	53	0
normalized size	1	1.	0.38	0.25	0.	18.66	0.2	0.
time (sec)	N/A	0.392	0.016	0.006	0.	9.302	0.725	0.

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	230	230	94	62	0	4852	48	0
normalized size	1	1.	0.41	0.27	0.	21.1	0.21	0.
time (sec)	N/A	0.357	0.014	0.004	0.	9.299	0.731	0.

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	90	58	0	4867	46	0
normalized size	1	1.	0.45	0.29	0.	24.58	0.23	0.
time (sec)	N/A	0.194	0.013	0.005	0.	9.758	0.717	0.

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	245	101	67	0	5003	6967	0
normalized size	1	1.	0.41	0.27	0.	20.42	28.44	0.
time (sec)	N/A	0.473	0.017	0.008	0.	9.442	14.651	0.

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	281	109	72	0	5700	65	0
normalized size	1	1.	0.39	0.26	0.	20.28	0.23	0.
time (sec)	N/A	0.467	0.019	0.008	0.	9.749	2.415	0.

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	317	317	116	77	0	6164	70	0
normalized size	1	1.	0.37	0.24	0.	19.44	0.22	0.
time (sec)	N/A	0.539	0.018	0.008	0.	9.78	1.673	0.

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	87	75	0	171	44	117
normalized size	1	1.	4.58	3.95	0.	9.	2.32	6.16
time (sec)	N/A	0.105	0.045	0.107	0.	1.416	1.048	3.987

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	57	34	45	144	36	46
normalized size	1	1.	1.33	0.79	1.05	3.35	0.84	1.07
time (sec)	N/A	0.067	0.023	0.01	1.537	1.47	0.154	1.142

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	20	49	14	24
normalized size	1	1.	1.	0.94	1.18	2.88	0.82	1.41
time (sec)	N/A	0.037	0.005	0.006	0.975	1.443	0.116	1.412

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	28	68	20	32
normalized size	1	1.	1.	0.88	1.12	2.72	0.8	1.28
time (sec)	N/A	0.037	0.005	0.007	0.989	1.398	0.116	1.145

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	20	19	24	73	14	38
normalized size	1	1.	0.91	0.86	1.09	3.32	0.64	1.73
time (sec)	N/A	0.013	0.01	0.007	0.986	1.227	0.102	1.172

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	38	22	28	76	26	30
normalized size	1	1.	1.41	0.81	1.04	2.81	0.96	1.11
time (sec)	N/A	0.039	0.011	0.006	1.519	1.516	0.126	1.149

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	23	46	15	23
normalized size	1	1.	1.	0.86	1.1	2.19	0.71	1.1
time (sec)	N/A	0.015	0.004	0.001	1.474	1.437	0.077	1.099

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	31	72	22	31
normalized size	1	1.	1.	0.89	1.15	2.67	0.81	1.15
time (sec)	N/A	0.026	0.007	0.004	1.495	1.402	0.098	1.193

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	26	34	96	26	39
normalized size	1	1.	1.	0.67	0.87	2.46	0.67	1.
time (sec)	N/A	0.02	0.006	0.007	0.975	1.263	0.211	1.161

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	23	19	24	49	17	26
normalized size	1	1.	1.05	0.86	1.09	2.23	0.77	1.18
time (sec)	N/A	0.015	0.004	0.003	0.984	1.369	0.071	1.123

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	18	39	14	18
normalized size	1	1.	1.	0.93	1.2	2.6	0.93	1.2
time (sec)	N/A	0.029	0.005	0.003	1.528	1.507	0.084	1.151

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	24	21	27	80	17	30
normalized size	1	1.	2.	1.75	2.25	6.67	1.42	2.5
time (sec)	N/A	0.021	0.011	0.007	0.986	1.223	0.093	1.118

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	28	63	19	31
normalized size	1	1.	1.	0.88	1.12	2.52	0.76	1.24
time (sec)	N/A	0.036	0.006	0.007	1.02	1.261	0.095	1.206

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	15	41	10	15
normalized size	1	1.	1.	0.92	1.15	3.15	0.77	1.15
time (sec)	N/A	0.028	0.007	0.003	1.504	1.345	0.104	1.11

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	35	31	47	150	36	41
normalized size	1	1.	1.	0.89	1.34	4.29	1.03	1.17
time (sec)	N/A	0.034	0.017	0.008	1.495	1.442	0.136	1.158

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	23	57	17	27
normalized size	1	1.	1.	0.78	1.	2.48	0.74	1.17
time (sec)	N/A	0.037	0.006	0.007	0.97	1.348	0.123	1.112

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	20	26	58	19	27
normalized size	1	1.	1.	0.87	1.13	2.52	0.83	1.17
time (sec)	N/A	0.054	0.007	0.004	0.994	1.445	0.099	1.383

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	25	24	31	90	20	31
normalized size	1	1.	0.86	0.83	1.07	3.1	0.69	1.07
time (sec)	N/A	0.014	0.011	0.006	1.489	1.458	0.111	1.112

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	44	42	55	186	49	58
normalized size	1	1.	1.	0.95	1.25	4.23	1.11	1.32
time (sec)	N/A	0.243	0.025	0.008	1.485	1.416	0.187	1.21

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	41	51	132	51	51
normalized size	1	1.	1.	0.89	1.11	2.87	1.11	1.11
time (sec)	N/A	0.133	0.02	0.006	1.516	1.301	0.187	1.139

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	34	59	186	39	59
normalized size	1	1.	1.	1.03	1.79	5.64	1.18	1.79
time (sec)	N/A	0.166	0.019	0.01	1.054	1.372	0.177	1.117

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	12	15	49	10	15
normalized size	1	1.	1.	0.71	0.88	2.88	0.59	0.88
time (sec)	N/A	0.007	0.006	0.006	1.607	1.461	0.124	1.126

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	22	21	27	62	34	27
normalized size	1	1.	0.92	0.88	1.12	2.58	1.42	1.12
time (sec)	N/A	0.018	0.009	0.003	1.506	1.51	0.305	1.294

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	23	18	23	62	17	27
normalized size	1	1.	1.53	1.2	1.53	4.13	1.13	1.8
time (sec)	N/A	0.06	0.006	0.008	1.49	1.665	0.123	1.262

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	18	23	62	19	23
normalized size	1	1.	1.	0.9	1.15	3.1	0.95	1.15
time (sec)	N/A	0.012	0.009	0.006	1.544	1.67	0.133	1.089

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	33	28	36	138	29	81
normalized size	1	1.	0.89	0.76	0.97	3.73	0.78	2.19
time (sec)	N/A	0.04	0.023	0.005	1.513	1.609	0.144	1.168

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	22	28	69	26	28
normalized size	1	1.	1.	0.85	1.08	2.65	1.	1.08
time (sec)	N/A	0.011	0.008	0.003	1.459	1.633	0.087	1.223

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	14	43	10	15
normalized size	1	1.	1.	0.92	1.17	3.58	0.83	1.25
time (sec)	N/A	0.032	0.004	0.004	0.99	1.515	0.091	1.132

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	23	54	15	27
normalized size	1	1.	1.	0.86	1.1	2.57	0.71	1.29
time (sec)	N/A	0.03	0.007	0.007	0.993	1.57	0.118	1.316

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	27	95	17	27
normalized size	1	1.	1.	0.95	1.23	4.32	0.77	1.23
time (sec)	N/A	0.015	0.008	0.003	1.463	1.516	0.103	1.206

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	21	27	92	19	27
normalized size	1	1.	1.	0.88	1.12	3.83	0.79	1.12
time (sec)	N/A	0.016	0.01	0.005	1.511	1.594	0.112	1.137

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	36	34	45	113	39	45
normalized size	1	1.	1.	0.94	1.25	3.14	1.08	1.25
time (sec)	N/A	0.026	0.015	0.003	1.452	1.51	0.167	1.189

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	28	36	100	29	36
normalized size	1	1.	1.	0.76	0.97	2.7	0.78	0.97
time (sec)	N/A	0.024	0.008	0.004	1.466	1.556	0.153	1.235

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	22	28	69	22	31
normalized size	1	1.	1.	0.76	0.97	2.38	0.76	1.07
time (sec)	N/A	0.017	0.006	0.006	1.074	1.496	0.102	1.134

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	20	43	14	22
normalized size	1	1.	1.	0.84	1.05	2.26	0.74	1.16
time (sec)	N/A	0.013	0.004	0.001	0.962	1.438	0.075	1.177

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	35	46	112	46	46
normalized size	1	1.	1.	0.85	1.12	2.73	1.12	1.12
time (sec)	N/A	0.028	0.016	0.003	1.483	1.463	0.104	1.234

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	39	32	42	95	34	42
normalized size	1	1.	0.95	0.78	1.02	2.32	0.83	1.02
time (sec)	N/A	0.028	0.01	0.004	1.458	1.562	0.111	1.121

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	24	21	27	73	24	31
normalized size	1	1.	0.8	0.7	0.9	2.43	0.8	1.03
time (sec)	N/A	0.058	0.012	0.007	0.968	1.539	0.135	1.126

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	35	28	36	90	31	41
normalized size	1	1.	1.	0.8	1.03	2.57	0.89	1.17
time (sec)	N/A	0.04	0.01	0.008	0.963	1.531	0.132	1.124

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	32	25	32	112	27	46
normalized size	1	1.	0.94	0.74	0.94	3.29	0.79	1.35
time (sec)	N/A	0.057	0.017	0.007	0.985	1.537	0.133	1.153

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	35	45	130	36	45
normalized size	1	1.	1.	0.83	1.07	3.1	0.86	1.07
time (sec)	N/A	0.021	0.036	0.007	1.464	1.476	0.122	1.149

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	38	50	163	46	50
normalized size	1	1.	1.	0.78	1.02	3.33	0.94	1.02
time (sec)	N/A	0.157	0.014	0.007	1.473	1.672	0.196	1.121

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	20	26	70	24	30
normalized size	1	1.	1.	0.69	0.9	2.41	0.83	1.03
time (sec)	N/A	0.053	0.007	0.009	0.972	1.561	0.13	1.154

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	43	34	45	115	41	46
normalized size	1	1.	0.93	0.74	0.98	2.5	0.89	1.
time (sec)	N/A	0.044	0.016	0.004	1.477	1.529	0.127	1.222

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	14	15	19	49	10	20
normalized size	1	1.	0.88	0.94	1.19	3.06	0.62	1.25
time (sec)	N/A	0.025	0.008	0.007	0.969	1.545	0.089	1.123

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	14	16	23	43	15	16
normalized size	1	1.	0.67	0.76	1.1	2.05	0.71	0.76
time (sec)	N/A	0.019	0.003	0.004	0.998	1.459	0.084	1.131

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	31	51	19	41
normalized size	1	1.	1.	1.07	2.07	3.4	1.27	2.73
time (sec)	N/A	0.027	0.012	0.004	0.971	1.478	0.111	1.114

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	29	38	103	3	39
normalized size	1	1.	1.	0.94	1.23	3.32	0.1	1.26
time (sec)	N/A	0.042	0.012	0.005	1.47	1.536	0.116	1.142

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	26	68	19	30
normalized size	1	1.	1.	0.8	1.04	2.72	0.76	1.2
time (sec)	N/A	0.042	0.007	0.007	0.985	1.626	0.124	1.162

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	29	25	32	109	20	35
normalized size	1	1.	0.97	0.83	1.07	3.63	0.67	1.17
time (sec)	N/A	0.03	0.016	0.007	1.004	1.897	0.092	1.159

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	23	65	17	24
normalized size	1	1.	1.	0.78	1.	2.83	0.74	1.04
time (sec)	N/A	0.038	0.005	0.005	1.493	1.837	0.122	1.154

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	93	73	104	387	88	100
normalized size	1	1.	0.9	0.71	1.01	3.76	0.85	0.97
time (sec)	N/A	0.476	0.044	0.012	1.726	1.746	0.462	1.145

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	30	28	34	97	24	34
normalized size	1	1.	0.91	0.85	1.03	2.94	0.73	1.03
time (sec)	N/A	0.017	0.011	0.006	1.622	1.564	0.116	1.385

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	28	39	130	27	41
normalized size	1	1.	1.	0.85	1.18	3.94	0.82	1.24
time (sec)	N/A	0.041	0.019	0.007	1.629	1.563	0.133	1.189

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	24	31	55	17	35
normalized size	1	1.	1.	0.96	1.24	2.2	0.68	1.4
time (sec)	N/A	0.041	0.005	0.006	1.053	1.588	0.089	1.147

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	36	32	42	111	36	42
normalized size	1	1.	1.	0.89	1.17	3.08	1.	1.17
time (sec)	N/A	0.115	0.015	0.004	1.685	1.536	0.171	1.169

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	22	28	105	22	28
normalized size	1	1.	1.	0.76	0.97	3.62	0.76	0.97
time (sec)	N/A	0.107	0.018	0.007	1.62	1.529	0.16	1.203

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	36	47	128	46	49
normalized size	1	1.	1.	0.78	1.02	2.78	1.	1.07
time (sec)	N/A	0.058	0.026	0.007	1.578	1.494	0.146	1.226

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	26	26	19	24	58	17	27
normalized size	1	1.18	1.18	0.86	1.09	2.64	0.77	1.23
time (sec)	N/A	0.015	0.006	0.007	1.025	1.46	0.09	1.125

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	20	51	15	24
normalized size	1	1.	1.	0.94	1.18	3.	0.88	1.41
time (sec)	N/A	0.039	0.006	0.007	1.082	1.489	0.118	1.446

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	19	50	14	22
normalized size	1	1.	1.	1.07	1.36	3.57	1.	1.57
time (sec)	N/A	0.025	0.004	0.006	1.13	1.461	0.103	1.211

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	25	18	23	58	17	27
normalized size	1	1.	1.32	0.95	1.21	3.05	0.89	1.42
time (sec)	N/A	0.027	0.007	0.007	1.11	1.477	0.117	1.14

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	23	69	19	23
normalized size	1	1.	1.	0.78	1.	3.	0.83	1.
time (sec)	N/A	0.029	0.009	0.004	1.568	1.466	0.159	1.143

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	57	51	68	216	68	72
normalized size	1	1.	0.9	0.81	1.08	3.43	1.08	1.14
time (sec)	N/A	0.088	0.027	0.009	1.557	1.568	0.314	1.319

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	86	67	54	80	312	63	80
normalized size	1	1.25	0.97	0.78	1.16	4.52	0.91	1.16
time (sec)	N/A	0.099	0.046	0.013	1.54	1.496	0.197	1.404

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	18	54	12	18
normalized size	1	1.	1.	0.82	1.06	3.18	0.71	1.06
time (sec)	N/A	0.013	0.006	0.004	1.562	1.521	0.099	1.227

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	18	23	54	15	23
normalized size	1	1.	1.	0.95	1.21	2.84	0.79	1.21
time (sec)	N/A	0.027	0.005	0.001	1.46	1.462	0.091	1.289

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	9	32	7	38
normalized size	1	1.	1.	0.89	1.	3.56	0.78	4.22
time (sec)	N/A	0.075	0.006	0.004	1.589	1.605	0.122	1.225

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	16	38	12	16
normalized size	1	1.	1.	1.08	1.33	3.17	1.	1.33
time (sec)	N/A	0.016	0.005	0.001	1.045	1.525	0.08	1.121

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	58	41	69	182	99	74
normalized size	1	1.	0.89	0.63	1.06	2.8	1.52	1.14
time (sec)	N/A	0.062	0.037	0.007	1.673	1.499	0.497	1.203

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	29	38	123	24	38
normalized size	1	1.	1.	1.04	1.36	4.39	0.86	1.36
time (sec)	N/A	0.022	0.01	0.004	1.68	1.543	0.121	1.224

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	27	35	74	29	35
normalized size	1	1.	1.	0.84	1.09	2.31	0.91	1.09
time (sec)	N/A	0.04	0.024	0.003	1.718	1.442	0.09	1.431

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	41	39	24	31	84	26	36
normalized size	1	1.32	1.26	0.77	1.	2.71	0.84	1.16
time (sec)	N/A	0.049	0.007	0.007	1.004	1.468	0.17	1.125

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	23	30	74	19	31
normalized size	1	1.	1.	0.96	1.25	3.08	0.79	1.29
time (sec)	N/A	0.167	0.009	0.007	1.486	1.461	0.142	1.131

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	19	32	90	20	26
normalized size	1	1.	1.	0.83	1.39	3.91	0.87	1.13
time (sec)	N/A	0.025	0.011	0.004	1.67	1.468	0.123	1.183

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	19	32	90	20	26
normalized size	1	1.	1.	0.83	1.39	3.91	0.87	1.13
time (sec)	N/A	0.039	0.005	0.005	1.496	1.459	0.123	1.153

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	18	38	10	18
normalized size	1	1.	1.	1.08	1.38	2.92	0.77	1.38
time (sec)	N/A	0.049	0.006	0.003	1.032	1.431	0.093	1.14

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	206	193	236	0	9873	138	302
normalized size	1	1.	0.94	1.15	0.	47.93	0.67	1.47
time (sec)	N/A	0.274	0.116	0.006	0.	9.598	1.303	1.192

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	45	41	0	132	44	51
normalized size	1	1.	1.	0.91	0.	2.93	0.98	1.13
time (sec)	N/A	0.067	0.025	0.008	0.	1.483	0.148	1.117

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	44	41	0	180	46	57
normalized size	1	1.	0.75	0.69	0.	3.05	0.78	0.97
time (sec)	N/A	0.064	0.021	0.009	0.	1.54	0.182	1.11

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	234	616	0	0	0	9341
normalized size	1	1.	1.12	2.95	0.	0.	0.	44.69
time (sec)	N/A	0.371	0.255	0.044	0.	0.	0.	3.1

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	245	633	0	0	0	9196
normalized size	1	1.	1.09	2.83	0.	0.	0.	41.05
time (sec)	N/A	0.389	0.269	0.027	0.	0.	0.	3.069

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	56	57	81	128	190	0
normalized size	1	1.	1.	1.02	1.45	2.29	3.39	0.
time (sec)	N/A	0.047	0.035	0.005	1.235	1.457	1.041	0.

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	73	87	0	354	1355	115
normalized size	1	1.	0.76	0.91	0.	3.69	14.11	1.2
time (sec)	N/A	0.108	0.042	0.007	0.	1.771	7.01	1.187

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	264	264	228	336	0	11777	0	432
normalized size	1	1.	0.86	1.27	0.	44.61	0.	1.64
time (sec)	N/A	0.472	0.095	0.006	0.	10.988	0.	1.205

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	417	417	370	422	0	0	0	575
normalized size	1	1.	0.89	1.01	0.	0.	0.	1.38
time (sec)	N/A	0.547	0.25	0.013	0.	0.	0.	1.203

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	24	21	27	80	19	28
normalized size	1	1.	1.5	1.31	1.69	5.	1.19	1.75
time (sec)	N/A	0.008	0.009	0.006	1.54	1.408	0.1	1.151

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	27	24	31	90	20	41
normalized size	1	1.	1.42	1.26	1.63	4.74	1.05	2.16
time (sec)	N/A	0.013	0.011	0.008	1.171	1.438	0.11	1.129

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	85	90	101	320	92	104
normalized size	1	1.	0.88	0.93	1.04	3.3	0.95	1.07
time (sec)	N/A	0.071	0.047	0.012	1.87	1.513	0.329	1.159

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	18	43	12	18
normalized size	1	1.	1.	0.93	1.2	2.87	0.8	1.2
time (sec)	N/A	0.106	0.008	0.004	1.919	1.472	0.109	1.15

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	15	41	10	15
normalized size	1	1.	1.	0.92	1.15	3.15	0.77	1.15
time (sec)	N/A	0.094	0.007	0.005	2.283	1.416	0.105	1.135

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	25	32	86	29	32
normalized size	1	1.	1.	0.86	1.1	2.97	1.	1.1
time (sec)	N/A	0.118	0.014	0.004	2.14	1.449	0.165	1.327

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	19	51	10	19
normalized size	1	1.	1.	1.07	1.36	3.64	0.71	1.36
time (sec)	N/A	0.011	0.009	0.005	2.365	1.478	0.123	1.168

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	14	30	7	15
normalized size	1	1.	1.	0.92	1.17	2.5	0.58	1.25
time (sec)	N/A	0.011	0.003	0.005	0.983	1.427	0.084	1.143

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	12	35	8	14
normalized size	1	1.	1.	0.91	1.09	3.18	0.73	1.27
time (sec)	N/A	0.034	0.005	0.005	1.532	1.475	0.114	1.157

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	14	32	8	15
normalized size	1	1.	1.	0.92	1.17	2.67	0.67	1.25
time (sec)	N/A	0.021	0.003	0.006	1.1	1.46	0.084	1.163

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	16	39	12	19
normalized size	1	1.	1.	1.08	1.33	3.25	1.	1.58
time (sec)	N/A	0.018	0.003	0.004	1.008	1.448	0.093	1.225

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	20	55	15	22
normalized size	1	1.	1.	0.94	1.18	3.24	0.88	1.29
time (sec)	N/A	0.037	0.005	0.004	1.497	1.785	0.115	1.129

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	18	39	10	19
normalized size	1	1.	1.	1.08	1.38	3.	0.77	1.46
time (sec)	N/A	0.035	0.009	0.006	1.678	1.781	0.116	1.118

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	22	21	27	82	20	30
normalized size	1	1.	0.79	0.75	0.96	2.93	0.71	1.07
time (sec)	N/A	0.011	0.014	0.008	1.018	1.721	0.104	1.154

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	27	35	107	26	58
normalized size	1	1.	1.	0.84	1.09	3.34	0.81	1.81
time (sec)	N/A	0.026	0.018	0.007	1.059	1.737	0.122	1.114

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	28	20	26	65	19	27
normalized size	1	1.	1.22	0.87	1.13	2.83	0.83	1.17
time (sec)	N/A	0.034	0.008	0.005	1.644	1.778	0.121	1.248

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	22	21	27	115	20	63
normalized size	1	1.	0.92	0.88	1.12	4.79	0.83	2.62
time (sec)	N/A	0.035	0.015	0.007	1.554	1.408	0.124	1.124

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	40	53	146	44	53
normalized size	1	1.	1.	0.82	1.08	2.98	0.9	1.08
time (sec)	N/A	0.142	0.015	0.005	1.702	1.51	0.198	1.114

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	19	20	26	62	19	30
normalized size	1	1.	0.76	0.8	1.04	2.48	0.76	1.2
time (sec)	N/A	0.055	0.008	0.005	1.165	1.518	0.133	1.134

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	54	48	63	236	63	81
normalized size	1	1.	0.9	0.8	1.05	3.93	1.05	1.35
time (sec)	N/A	0.246	0.056	0.01	1.651	1.547	0.225	1.097

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	12	18	5	12
normalized size	1	1.	1.	0.91	1.09	1.64	0.45	1.09
time (sec)	N/A	0.007	0.	0.	1.105	1.369	0.054	1.137

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	22	28	68	22	31
normalized size	1	1.	1.	0.76	0.97	2.34	0.76	1.07
time (sec)	N/A	0.016	0.005	0.006	1.05	1.433	0.096	1.102

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	45	37	50	165	37	50
normalized size	1	1.	1.	0.82	1.11	3.67	0.82	1.11
time (sec)	N/A	0.025	0.012	0.006	1.655	1.41	0.128	1.112

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	29	38	158	29	39
normalized size	1	1.	1.	0.91	1.19	4.94	0.91	1.22
time (sec)	N/A	0.254	0.018	0.009	1.634	1.513	0.222	1.158

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	159	110	178	635	146	165
normalized size	1	1.	1.07	0.74	1.2	4.29	0.99	1.11
time (sec)	N/A	0.136	0.069	0.012	1.682	1.587	0.42	1.34

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	C	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	101	79	134	2269	61	124
normalized size	1	1.	0.9	0.71	1.2	20.26	0.54	1.11
time (sec)	N/A	0.114	0.051	0.006	1.714	9.765	0.782	1.281

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	16	36	10	19
normalized size	1	1.	1.	0.93	1.14	2.57	0.71	1.36
time (sec)	N/A	0.016	0.003	0.004	1.097	1.412	0.089	1.198

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	14	34	8	16
normalized size	1	1.	1.	0.92	1.17	2.83	0.67	1.33
time (sec)	N/A	0.029	0.004	0.005	1.055	1.715	0.089	1.122

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	20	55	14	23
normalized size	1	1.	1.	0.94	1.18	3.24	0.82	1.35
time (sec)	N/A	0.03	0.004	0.006	1.097	1.725	0.097	1.111

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	20	47	14	23
normalized size	1	1.	1.	0.94	1.18	2.76	0.82	1.35
time (sec)	N/A	0.029	0.004	0.004	1.063	1.161	0.09	1.15

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	22	66	15	31
normalized size	1	1.	1.	0.94	1.22	3.67	0.83	1.72
time (sec)	N/A	0.03	0.005	0.005	1.659	1.228	0.097	1.129

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	14	41	7	15
normalized size	1	1.	1.	1.1	1.4	4.1	0.7	1.5
time (sec)	N/A	0.019	0.005	0.005	1.096	1.184	0.081	1.145

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	31	41	108	36	45
normalized size	1	1.	1.	0.74	0.98	2.57	0.86	1.07
time (sec)	N/A	0.04	0.007	0.009	1.13	1.264	0.131	1.132

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	41	51	132	51	51
normalized size	1	1.	1.	0.89	1.11	2.87	1.11	1.11
time (sec)	N/A	0.125	0.017	0.	1.679	1.223	0.182	1.086

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	20	47	14	23
normalized size	1	1.	1.	0.84	1.05	2.47	0.74	1.21
time (sec)	N/A	0.062	0.003	0.006	1.086	1.239	0.099	1.159

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	31	41	108	34	43
normalized size	1	1.	1.	0.78	1.02	2.7	0.85	1.08
time (sec)	N/A	0.021	0.005	0.006	1.291	1.185	0.107	1.13

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	26	34	85	27	36
normalized size	1	1.	1.	0.79	1.03	2.58	0.82	1.09
time (sec)	N/A	0.02	0.004	0.005	1.02	1.248	0.106	1.246

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	21	27	66	20	30
normalized size	1	1.	1.	0.81	1.04	2.54	0.77	1.15
time (sec)	N/A	0.015	0.004	0.007	1.049	1.148	0.106	1.285

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	23	54	17	26
normalized size	1	1.	1.	0.86	1.1	2.57	0.81	1.24
time (sec)	N/A	0.007	0.003	0.006	0.999	1.258	0.099	1.159

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	23	50	15	26
normalized size	1	1.	1.	0.86	1.1	2.38	0.71	1.24
time (sec)	N/A	0.01	0.003	0.004	1.009	1.252	0.096	1.103

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	22	28	70	24	32
normalized size	1	1.	1.	0.81	1.04	2.59	0.89	1.19
time (sec)	N/A	0.016	0.004	0.006	0.998	1.222	0.127	1.14

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	27	35	90	31	39
normalized size	1	1.	1.	0.79	1.03	2.65	0.91	1.15
time (sec)	N/A	0.03	0.004	0.006	1.	1.263	0.143	1.122

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	32	42	117	36	46
normalized size	1	1.	1.	0.78	1.02	2.85	0.88	1.12
time (sec)	N/A	0.034	0.005	0.008	0.981	1.258	0.153	1.145

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	48	37	49	138	41	53
normalized size	1	1.	1.	0.77	1.02	2.88	0.85	1.1
time (sec)	N/A	0.041	0.005	0.007	1.009	1.3	0.161	1.246

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	61	54	0	4887	41	0
normalized size	1	1.	0.39	0.34	0.	31.13	0.26	0.
time (sec)	N/A	0.168	0.017	0.009	0.	8.713	0.206	0.

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	61	56	0	4898	41	0
normalized size	1	1.	0.39	0.36	0.	31.2	0.26	0.
time (sec)	N/A	0.112	0.018	0.009	0.	8.615	0.202	0.

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	61	54	0	8759	39	0
normalized size	1	1.	0.32	0.29	0.	46.59	0.21	0.
time (sec)	N/A	0.276	0.013	0.007	0.	9.791	0.189	0.

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	61	56	0	8735	39	0
normalized size	1	1.	0.32	0.3	0.	46.46	0.21	0.
time (sec)	N/A	0.156	0.013	0.006	0.	7.331	0.188	0.

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	663	663	63	69	0	0	133	0
normalized size	1	1.	0.1	0.1	0.	0.	0.2	0.
time (sec)	N/A	1.108	0.038	0.072	0.	0.	2.988	0.

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	663	663	63	69	0	0	133	0
normalized size	1	1.	0.1	0.1	0.	0.	0.2	0.
time (sec)	N/A	0.648	0.032	0.002	0.	0.	3.01	0.

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	C	C	F	C	A	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	168	0	95	67	0	14804	42	0
normalized size	1	0.	0.57	0.4	0.	88.12	0.25	0.
time (sec)	N/A	0.38	0.068	0.253	0.	10.243	1.671	0.

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	320	320	322	314	0	0	384	420
normalized size	1	1.	1.01	0.98	0.	0.	1.2	1.31
time (sec)	N/A	0.256	0.242	0.009	0.	0.	3.697	1.201

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	291	291	243	292	0	0	277	385
normalized size	1	1.	0.84	1.	0.	0.	0.95	1.32
time (sec)	N/A	0.198	0.103	0.003	0.	0.	2.561	1.115

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	184	151	0	0	124	290
normalized size	1	1.	0.84	0.69	0.	0.	0.57	1.32
time (sec)	N/A	0.174	0.058	0.003	0.	0.	0.791	1.182

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	134	128	0	306	20	242
normalized size	1	1.	0.72	0.69	0.	1.65	0.11	1.31
time (sec)	N/A	0.098	0.018	0.002	0.	1.413	0.145	1.21

Problem 398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	416	416	404	433	0	0	0	517
normalized size	1	1.	0.97	1.04	0.	0.	0.	1.24
time (sec)	N/A	0.428	0.153	0.007	0.	0.	0.	1.179

Problem 399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	552	552	524	866	0	0	0	0
normalized size	1	1.	0.95	1.57	0.	0.	0.	0.
time (sec)	N/A	0.808	0.706	0.01	0.	0.	0.	0.

Problem 400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	680	680	738	1201	0	0	0	1266
normalized size	1	1.	1.09	1.77	0.	0.	0.	1.86
time (sec)	N/A	0.95	0.986	0.013	0.	0.	0.	1.602

Problem 401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	349	349	347	390	0	0	350	462
normalized size	1	1.	0.99	1.12	0.	0.	1.	1.32
time (sec)	N/A	0.298	0.386	0.005	0.	0.	6.909	1.277

Problem 402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	322	322	321	362	0	0	318	436
normalized size	1	1.	1.	1.12	0.	0.	0.99	1.35
time (sec)	N/A	0.269	0.325	0.004	0.	0.	3.24	1.159

Problem 403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	241	224	188	0	0	155	325
normalized size	1	1.	0.93	0.78	0.	0.	0.64	1.35
time (sec)	N/A	0.189	0.199	0.004	0.	0.	1.284	1.111

Problem 404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	183	143	0	414	39	262
normalized size	1	1.	0.91	0.71	0.	2.05	0.19	1.3
time (sec)	N/A	0.116	0.107	0.004	0.	0.996	0.469	1.119

Problem 405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	855	855	558	1122	0	0	0	1038
normalized size	1	1.	0.65	1.31	0.	0.	0.	1.21
time (sec)	N/A	0.849	0.422	0.025	0.	0.	0.	1.297

Problem 406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1141	1141	807	1636	0	0	0	0
normalized size	1	1.	0.71	1.43	0.	0.	0.	0.
time (sec)	N/A	1.659	0.983	0.022	0.	0.	0.	0.

Problem 407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1384	1384	996	2121	0	0	0	1993
normalized size	1	1.	0.72	1.53	0.	0.	0.	1.44
time (sec)	N/A	1.963	1.414	0.026	0.	0.	0.	1.705

Problem 408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	394	394	388	470	0	0	413	525
normalized size	1	1.	0.98	1.19	0.	0.	1.05	1.33
time (sec)	N/A	0.346	0.356	0.006	0.	0.	9.529	1.211

Problem 409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	360	360	358	419	0	0	374	481
normalized size	1	1.	0.99	1.16	0.	0.	1.04	1.34
time (sec)	N/A	0.326	0.298	0.005	0.	0.	4.518	1.261

Problem 410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	266	249	222	0	0	192	351
normalized size	1	1.	0.94	0.83	0.	0.	0.72	1.32
time (sec)	N/A	0.249	0.191	0.006	0.	0.	2.3	1.217

Problem 411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	200	158	0	545	63	275
normalized size	1	1.	0.91	0.72	0.	2.49	0.29	1.26
time (sec)	N/A	0.142	0.084	0.004	0.	1.014	1.226	1.149

Problem 412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1352	1352	835	2098	0	0	0	1705
normalized size	1	1.	0.62	1.55	0.	0.	0.	1.26
time (sec)	N/A	1.412	0.763	0.023	0.	0.	0.	1.37

Problem 413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1830	1830	1115	2769	0	0	0	0
normalized size	1	1.	0.61	1.51	0.	0.	0.	0.
time (sec)	N/A	2.781	1.693	0.027	0.	0.	0.	0.

Problem 414	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2204	2204	1338	3334	0	0	0	2869
normalized size	1	1.	0.61	1.51	0.	0.	0.	1.3
time (sec)	N/A	3.165	3.048	0.035	0.	0.	0.	1.833

Problem 415	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	33	29	38	90	34	38
normalized size	1	1.	1.03	0.91	1.19	2.81	1.06	1.19
time (sec)	N/A	0.019	0.009	0.003	1.465	1.443	0.101	1.113

Problem 416	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	33	29	38	90	34	38
normalized size	1	1.	1.03	0.91	1.19	2.81	1.06	1.19
time (sec)	N/A	0.029	0.005	0.003	1.451	1.406	0.102	1.13

Problem 417	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	31	29	35	90	36	35
normalized size	1	1.	0.97	0.91	1.09	2.81	1.12	1.09
time (sec)	N/A	0.017	0.009	0.003	1.457	0.985	0.104	1.114

Problem 418	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	31	29	35	90	36	35
normalized size	1	1.	0.97	0.91	1.09	2.81	1.12	1.09
time (sec)	N/A	0.035	0.005	0.001	1.462	0.976	0.108	1.135

Problem 419	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	42	29	47	128	39	59
normalized size	1	1.	0.93	0.64	1.04	2.84	0.87	1.31
time (sec)	N/A	0.012	0.02	0.003	1.484	0.983	0.101	1.134

Problem 420	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	42	29	47	128	39	59
normalized size	1	1.	0.93	0.64	1.04	2.84	0.87	1.31
time (sec)	N/A	0.021	0.005	0.003	1.459	0.959	0.108	1.116

Problem 421	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	23	18	23	51	15	20
normalized size	1	1.	3.83	3.	3.83	8.5	2.5	3.33
time (sec)	N/A	0.002	0.003	0.004	0.958	0.967	0.084	1.131

Problem 422	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	23	18	23	51	15	26
normalized size	1	1.	1.1	0.86	1.1	2.43	0.71	1.24
time (sec)	N/A	0.004	0.002	0.003	0.965	0.965	0.089	1.115

Problem 423	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	11	10	12	53	22	12
normalized size	1	1.	0.85	0.77	0.92	4.08	1.69	0.92
time (sec)	N/A	0.002	0.002	0.002	0.952	0.924	0.121	1.121

Problem 424	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	81	11	58	77	53	22	77
normalized size	1	6.23	0.85	4.46	5.92	4.08	1.69	5.92
time (sec)	N/A	0.012	0.002	0.003	0.965	1.011	0.277	1.103

Problem 425	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	78	67	89	232	85	92
normalized size	1	1.	1.13	0.97	1.29	3.36	1.23	1.33
time (sec)	N/A	0.111	0.014	0.01	1.517	1.028	0.221	1.115

Problem 426	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	78	67	89	232	85	92
normalized size	1	1.	1.13	0.97	1.29	3.36	1.23	1.33
time (sec)	N/A	0.124	0.006	0.002	1.46	0.986	0.227	1.107

Problem 427	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	31	21	27	55	20	28
normalized size	1	1.	1.29	0.88	1.12	2.29	0.83	1.17
time (sec)	N/A	0.019	0.006	0.003	1.043	0.966	0.073	1.16

Problem 428	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	25	21	27	54	19	28
normalized size	1	1.	0.96	0.81	1.04	2.08	0.73	1.08
time (sec)	N/A	0.016	0.004	0.002	1.014	0.964	0.07	1.085

Problem 429	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	22	16	20	45	15	20
normalized size	1	1.	1.29	0.94	1.18	2.65	0.88	1.18
time (sec)	N/A	0.006	0.	0.002	0.969	0.901	0.053	1.136

Problem 430	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	22	20	27	45	15	27
normalized size	1	1.	0.92	0.83	1.12	1.88	0.62	1.12
time (sec)	N/A	0.005	0.001	0.	0.985	0.878	0.054	1.107

Problem 431	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	24	23	30	76	20	31
normalized size	1	1.	1.09	1.05	1.36	3.45	0.91	1.41
time (sec)	N/A	0.019	0.005	0.006	1.482	1.	0.117	1.106

Problem 432	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	27	16	20	51	15	24
normalized size	1	1.	1.59	0.94	1.18	3.	0.88	1.41
time (sec)	N/A	0.013	0.004	0.007	0.974	1.015	0.117	1.106

Problem 433	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	14	18	42	12	22
normalized size	1	1.	1.	0.74	0.95	2.21	0.63	1.16
time (sec)	N/A	0.021	0.004	0.004	1.054	0.96	0.087	1.108

Problem 434	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	17	11	14	27	8	20
normalized size	1	1.	1.42	0.92	1.17	2.25	0.67	1.67
time (sec)	N/A	0.006	0.003	0.002	0.993	0.948	0.083	1.112

Problem 435	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	11	11	14	24	8	22
normalized size	1	1.	1.1	1.1	1.4	2.4	0.8	2.2
time (sec)	N/A	0.009	0.006	0.002	0.965	0.978	0.274	1.098

Problem 436	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	20	53	15	24
normalized size	1	1.	1.	0.94	1.18	3.12	0.88	1.41
time (sec)	N/A	0.018	0.006	0.004	0.972	0.999	0.111	1.186

Problem 437	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	23	63	17	24
normalized size	1	1.	1.	0.78	1.	2.74	0.74	1.04
time (sec)	N/A	0.021	0.004	0.004	1.461	1.015	0.114	1.145

Problem 438	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	18	32	10	18
normalized size	1	1.	1.	0.93	1.2	2.13	0.67	1.2
time (sec)	N/A	0.009	0.004	0.002	0.962	0.97	0.086	1.112

Problem 439	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	23	61	20	27
normalized size	1	1.	1.	0.86	1.1	2.9	0.95	1.29
time (sec)	N/A	0.021	0.005	0.006	0.975	0.963	0.118	1.107

Problem 440	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	9	14	12	19	7	12
normalized size	1	1.	0.9	1.4	1.2	1.9	0.7	1.2
time (sec)	N/A	0.01	0.005	0.004	0.977	0.941	0.081	1.102

Problem 441	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	18	23	66	20	27
normalized size	1	1.	1.	0.72	0.92	2.64	0.8	1.08
time (sec)	N/A	0.021	0.005	0.005	0.962	0.977	0.12	1.16

Problem 442	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	19	43	12	19
normalized size	1	1.	1.	1.07	1.36	3.07	0.86	1.36
time (sec)	N/A	0.025	0.004	0.001	1.47	0.976	0.093	1.136

Problem 443	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	14	30	15	35	17	15
normalized size	1	1.	1.08	2.31	1.15	2.69	1.31	1.15
time (sec)	N/A	0.007	0.007	0.01	0.968	0.937	0.131	1.206

Problem 444	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	25	32	35	54	20	36
normalized size	1	1.	0.96	1.23	1.35	2.08	0.77	1.38
time (sec)	N/A	0.045	0.008	0.003	0.964	0.962	0.301	1.13

Problem 445	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	4	5	5	16	3	7
normalized size	1	1.	0.67	0.83	0.83	2.67	0.5	1.17
time (sec)	N/A	0.008	0.001	0.	1.005	0.955	0.058	1.104

Problem 446	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	27	68	19	36
normalized size	1	1.	1.	0.95	1.35	3.4	0.95	1.8
time (sec)	N/A	0.015	0.005	0.006	0.982	0.969	0.098	1.116

Problem 447	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	40	28	36	96	31	38
normalized size	1	1.	1.05	0.74	0.95	2.53	0.82	1.
time (sec)	N/A	0.025	0.01	0.007	1.481	0.985	0.126	1.122

Problem 448	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	18	16	20	41	14	22
normalized size	1	1.	1.06	0.94	1.18	2.41	0.82	1.29
time (sec)	N/A	0.008	0.003	0.002	0.969	0.964	0.07	1.131

Problem 449	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	36	22	28	80	22	30
normalized size	1	1.	1.16	0.71	0.9	2.58	0.71	0.97
time (sec)	N/A	0.012	0.005	0.004	1.478	1.005	0.138	1.111

Problem 450	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	23	47	17	23
normalized size	1	1.	1.	0.84	1.21	2.47	0.89	1.21
time (sec)	N/A	0.017	0.006	0.005	0.981	0.967	0.107	1.144

Problem 451	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	9	9	12	28	8	15
normalized size	1	1.	0.82	0.82	1.09	2.55	0.73	1.36
time (sec)	N/A	0.005	0.004	0.001	1.001	0.974	0.084	1.123

Problem 452	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	13	16	53	14	16
normalized size	1	1.	1.	0.72	0.89	2.94	0.78	0.89
time (sec)	N/A	0.014	0.009	0.006	1.505	1.008	0.128	1.139

Problem 453	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	44	39	62	239	46	70
normalized size	1	1.	0.96	0.85	1.35	5.2	1.	1.52
time (sec)	N/A	0.023	0.016	0.01	1.024	0.989	0.176	1.154

Problem 454	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	10	14	11	24	7	12
normalized size	1	1.	0.83	1.17	0.92	2.	0.58	1.
time (sec)	N/A	0.002	0.001	0.002	0.971	0.935	0.072	1.147

Problem 455	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	27	28	31	90	20	34
normalized size	1	1.	1.29	1.33	1.48	4.29	0.95	1.62
time (sec)	N/A	0.003	0.007	0.008	1.063	0.952	0.1	1.187

Problem 456	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	20	55	12	20
normalized size	1	1.	1.	0.84	1.05	2.89	0.63	1.05
time (sec)	N/A	0.004	0.008	0.004	1.567	0.949	0.093	1.223

Problem 457	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	11	24	7	12
normalized size	1	1.	1.	0.9	1.1	2.4	0.7	1.2
time (sec)	N/A	0.001	0.001	0.	1.133	1.192	0.052	1.234

Problem 458	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	14	20	20	14
normalized size	1	1.	1.	1.1	1.4	2.	2.	1.4
time (sec)	N/A	0.003	0.002	0.003	1.682	1.319	0.102	1.202

Problem 459	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	16	0	151	53	20
normalized size	1	1.	1.	0.67	0.	6.29	2.21	0.83
time (sec)	N/A	0.007	0.004	0.002	0.	1.288	0.121	1.109

Problem 460	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	17	22	58	26	22
normalized size	1	1.	1.	0.89	1.16	3.05	1.37	1.16
time (sec)	N/A	0.011	0.006	0.003	1.602	1.23	0.097	1.12

Problem 461	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	22	41	17	22
normalized size	1	1.	1.	0.77	1.	1.86	0.77	1.
time (sec)	N/A	0.007	0.001	0.	1.041	1.114	0.052	1.12

Problem 462	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	22	39	15	22
normalized size	1	1.	1.	0.77	1.	1.77	0.68	1.
time (sec)	N/A	0.005	0.001	0.	1.104	1.039	0.052	1.105

Problem 463	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	19	35	10	19
normalized size	1	1.	1.	0.94	1.19	2.19	0.62	1.19
time (sec)	N/A	0.005	0.001	0.003	1.043	1.203	0.066	1.109

Problem 464	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	31	41	97	39	41
normalized size	1	1.	1.	0.84	1.11	2.62	1.05	1.11
time (sec)	N/A	0.021	0.01	0.004	1.523	1.159	0.106	1.083

Problem 465	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	16	28	10	16
normalized size	1	1.	1.	0.93	1.14	2.	0.71	1.14
time (sec)	N/A	0.004	0.	0.001	1.048	1.011	0.049	1.126

Problem 466	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	12	20	7	12
normalized size	1	1.	1.	0.91	1.09	1.82	0.64	1.09
time (sec)	N/A	0.004	0.	0.	1.054	1.138	0.054	1.106

Problem 467	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	26	70	20	27
normalized size	1	1.	1.	0.8	1.04	2.8	0.8	1.08
time (sec)	N/A	0.01	0.006	0.004	1.606	1.362	0.117	1.153

Problem 468	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	26	69	19	27
normalized size	1	1.	1.	0.8	1.04	2.76	0.76	1.08
time (sec)	N/A	0.01	0.006	0.005	1.62	1.177	0.113	1.106

Problem 469	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	26	69	19	27
normalized size	1	1.	1.	0.8	1.04	2.76	0.76	1.08
time (sec)	N/A	0.017	0.006	0.004	1.657	1.278	0.109	1.118

Problem 470	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	7	7	8	9	31	5	11
normalized size	1	0.78	0.78	0.89	1.	3.44	0.56	1.22
time (sec)	N/A	0.006	0.003	0.003	1.135	1.172	0.07	1.146

Problem 471	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	22	65	20	22
normalized size	1	1.	1.	0.77	1.	2.95	0.91	1.
time (sec)	N/A	0.011	0.012	0.007	1.647	1.274	0.133	1.107

Problem 472	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	31	41	104	44	41
normalized size	1	1.	1.	0.84	1.11	2.81	1.19	1.11
time (sec)	N/A	0.022	0.014	0.004	1.656	1.169	0.112	1.139

Problem 473	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	20	49	14	22
normalized size	1	1.	1.	0.93	1.33	3.27	0.93	1.47
time (sec)	N/A	0.004	0.002	0.006	1.108	1.19	0.084	1.163

Problem 474	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	18	41	10	19
normalized size	1	1.	1.	0.93	1.2	2.73	0.67	1.27
time (sec)	N/A	0.004	0.001	0.006	1.094	1.208	0.07	1.15

Problem 475	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	18	31	8	18
normalized size	1	1.	1.	1.09	1.64	2.82	0.73	1.64
time (sec)	N/A	0.011	0.003	0.004	1.082	1.249	0.088	1.126

Problem 476	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	19	18	23	41	17	23
normalized size	1	1.	0.86	0.82	1.05	1.86	0.77	1.05
time (sec)	N/A	0.004	0.001	0.	1.116	1.13	0.051	1.143

Problem 477	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	26	47	19	12
normalized size	1	1.	1.	0.91	2.36	4.27	1.73	1.09
time (sec)	N/A	0.001	0.001	0.002	1.109	1.061	0.053	1.102

Problem 478	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	16	14	18	35	12	19
normalized size	1	1.	1.23	1.08	1.38	2.69	0.92	1.46
time (sec)	N/A	0.01	0.003	0.002	1.152	1.222	0.071	1.121

Problem 479	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	19	50	14	22
normalized size	1	1.	1.	0.94	1.19	3.12	0.88	1.38
time (sec)	N/A	0.006	0.002	0.006	1.114	1.209	0.093	1.082

Problem 480	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	26	24	34	99	22	36
normalized size	1	1.	0.96	0.89	1.26	3.67	0.81	1.33
time (sec)	N/A	0.011	0.014	0.009	1.085	1.236	0.101	1.084

Problem 481	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	18	16	20	38	12	22
normalized size	1	1.	1.06	0.94	1.18	2.24	0.71	1.29
time (sec)	N/A	0.008	0.003	0.001	1.092	1.209	0.069	1.096

Problem 482	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	22	51	15	23
normalized size	1	1.	1.	0.94	1.22	2.83	0.83	1.28
time (sec)	N/A	0.006	0.001	0.004	1.09	1.293	0.074	1.11

Problem 483	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F(-2)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	28	19	0	14	19
normalized size	1	1.	1.	1.56	1.06	0.	0.78	1.06
time (sec)	N/A	0.01	0.001	0.	1.642	0.	0.052	1.129

Problem 484	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	22	46	117	31	34
normalized size	1	1.	1.	0.96	2.	5.09	1.35	1.48
time (sec)	N/A	0.018	0.01	0.005	1.097	1.197	0.105	1.134

Problem 485	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	12	13	16	50	10	43
normalized size	1	1.	0.75	0.81	1.	3.12	0.62	2.69
time (sec)	N/A	0.012	0.006	0.003	1.693	1.232	0.102	1.124

Problem 486	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	30	26	34	65	24	35
normalized size	1	1.	1.03	0.9	1.17	2.24	0.83	1.21
time (sec)	N/A	0.02	0.008	0.003	1.322	0.943	0.075	1.092

Problem 487	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	16	31	10	16
normalized size	1	1.	1.	0.81	1.	1.94	0.62	1.
time (sec)	N/A	0.006	0.	0.001	1.264	0.945	0.056	1.075

Problem 488	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	23	72	14	16
normalized size	1	1.	1.	0.94	1.35	4.24	0.82	0.94
time (sec)	N/A	0.015	0.012	0.005	1.833	0.935	0.115	1.186

Problem 489	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	47	43	0	431	151	65
normalized size	1	1.	1.	0.91	0.	9.17	3.21	1.38
time (sec)	N/A	0.066	0.025	0.006	0.	1.026	0.268	1.152

Problem 490	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	61	61	0	547	294	81
normalized size	1	1.	1.07	1.07	0.	9.6	5.16	1.42
time (sec)	N/A	0.082	0.028	0.006	0.	1.024	0.323	1.196

Problem 491	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	39	308	0	771	24	0
normalized size	1	1.	0.21	1.64	0.	4.1	0.13	0.
time (sec)	N/A	0.189	0.031	0.077	0.	1.085	0.458	0.

Problem 492	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	27	28	88	147	61	41
normalized size	1	1.	0.45	0.47	1.47	2.45	1.02	0.68
time (sec)	N/A	0.136	0.015	0.009	1.671	0.938	0.244	1.127

Problem 493	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	C	B	B	B	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	27	0	27	250	88	147	61	0
normalized size	1	0.	1.	9.26	3.26	5.44	2.26	0.
time (sec)	N/A	0.311	0.009	0.024	1.324	0.923	0.317	0.

Problem 494	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	B	B	B	B	B
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	27	0	27	112	88	147	61	150
normalized size	1	0.	1.	4.15	3.26	5.44	2.26	5.56
time (sec)	N/A	0.433	0.011	0.016	1.312	0.94	0.286	1.146

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [412] had the largest ratio of [0.8824]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	2	1.	24	0.083
2	A	3	3	1.	29	0.103
3	A	2	2	1.	29	0.069
4	A	2	2	1.	29	0.069
5	A	1	0	1.	27	0.
6	A	2	2	1.	29	0.069
7	A	2	2	1.	29	0.069
8	A	2	2	1.	29	0.069
9	A	3	2	1.	27	0.074
10	A	3	2	1.	27	0.074
11	A	1	0	1.	25	0.
12	A	7	7	1.	27	0.259
13	A	8	8	1.	27	0.296
14	A	9	8	1.	27	0.296
15	A	3	2	1.	46	0.043

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
16	A	3	2	1.	46	0.043
17	A	1	0	1.	44	0.
18	A	2	1	1.	46	0.022
19	A	2	1	1.	46	0.022
20	A	2	1	1.	46	0.022
21	A	5	4	1.	11	0.364
22	A	5	4	1.	17	0.235
23	A	2	2	1.	7	0.286
24	A	3	2	1.	13	0.154
25	A	5	5	1.	11	0.454
26	A	7	7	1.	16	0.438
27	A	6	6	1.	9	0.667
28	A	2	2	1.	7	0.286
29	A	3	3	1.	13	0.231
30	A	3	3	1.11	11	0.273
31	A	3	3	1.	16	0.188
32	A	2	2	1.26	9	0.222
33	A	3	2	1.	29	0.069
34	A	2	1	1.	29	0.034
35	A	2	1	1.	29	0.034
36	A	1	0	1.	27	0.
37	A	10	6	1.	29	0.207
38	A	11	7	1.	29	0.241
39	A	3	2	1.	32	0.062
40	A	2	1	1.	32	0.031
41	A	2	1	1.	32	0.031
42	A	1	0	1.	30	0.
43	A	4	3	1.	32	0.094
44	A	5	4	1.	32	0.125
45	A	2	1	1.	17	0.059
46	A	2	1	1.	17	0.059
47	A	2	1	1.	17	0.059

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
48	A	1	0	1.	15	0.
49	A	16	9	1.	17	0.529
50	A	18	11	1.	17	0.647
51	A	2	1	1.	17	0.059
52	A	2	1	1.	17	0.059
53	A	2	1	1.	17	0.059
54	A	1	0	1.	15	0.
55	A	15	9	1.	17	0.529
56	A	17	11	1.	17	0.647
57	A	2	1	1.	22	0.045
58	A	2	1	1.	22	0.045
59	A	2	1	1.	22	0.045
60	A	1	0	1.	20	0.
61	A	16	9	1.	22	0.409
62	A	18	11	1.	22	0.5
63	A	2	2	1.	51	0.039
64	A	2	2	1.	51	0.039
65	B	1	0	4.36	49	0.
66	A	2	2	1.	51	0.039
67	A	2	2	1.	51	0.039
68	A	2	2	1.	51	0.039
69	A	6	5	1.	13	0.385
70	A	5	3	1.	19	0.158
71	A	5	3	1.	19	0.158
72	A	5	3	1.	19	0.158
73	A	1	0	1.	17	0.
74	A	5	2	1.	19	0.105
75	A	7	3	1.	19	0.158
76	A	10	3	1.	19	0.158
77	B	15	7	2.25	17	0.412
78	A	6	5	1.	15	0.333
79	A	6	5	1.	15	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
80	A	4	4	1.	13	0.308
81	A	2	2	1.	11	0.182
82	A	6	6	1.	15	0.4
83	A	7	6	1.	15	0.4
84	A	7	6	1.	15	0.4
85	A	2	2	1.	13	0.154
86	A	3	3	1.	13	0.231
87	A	4	3	1.	13	0.231
88	A	2	2	1.	19	0.105
89	A	2	2	1.	11	0.182
90	A	3	3	1.	11	0.273
91	A	4	3	1.	11	0.273
92	A	2	2	1.	13	0.154
93	A	3	3	1.	13	0.231
94	A	4	3	1.	13	0.231
95	A	2	2	1.	11	0.182
96	A	3	3	1.	11	0.273
97	A	4	3	1.	11	0.273
98	A	3	2	1.	15	0.133
99	A	4	3	1.	13	0.231
100	A	4	4	1.	17	0.235
101	A	4	4	1.	19	0.21
102	A	4	4	1.	17	0.235
103	A	11	10	1.	17	0.588
104	A	9	9	1.	17	0.529
105	A	7	7	1.	15	0.467
106	A	7	7	1.	13	0.538
107	A	11	10	1.06	17	0.588
108	A	11	10	0.99	17	0.588
109	A	11	10	1.	17	0.588
110	A	16	12	1.	17	0.706
111	A	14	10	1.	17	0.588

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
112	A	14	10	1.	15	0.667
113	A	10	7	1.	13	0.538
114	A	18	13	1.	17	0.765
115	A	18	13	1.	17	0.765
116	A	3	2	1.	22	0.091
117	A	2	1	1.	22	0.045
118	A	2	1	1.	22	0.045
119	A	1	0	1.	20	0.
120	A	4	3	1.	22	0.136
121	A	5	4	1.	22	0.182
122	A	6	5	1.	22	0.227
123	A	2	1	1.	24	0.042
124	A	2	1	1.	24	0.042
125	A	2	1	1.	24	0.042
126	A	2	1	1.	22	0.045
127	A	8	7	1.	24	0.292
128	A	10	9	1.	24	0.375
129	A	12	10	1.	24	0.417
130	A	2	1	1.	26	0.038
131	A	2	1	1.	26	0.038
132	A	2	1	1.	26	0.038
133	A	2	1	1.	24	0.042
134	A	9	8	1.	26	0.308
135	A	11	10	1.	26	0.385
136	A	14	5	1.	46	0.109
137	A	14	5	1.	46	0.109
138	A	8	3	1.	46	0.065
139	A	14	5	1.	44	0.114
140	A	14	5	1.	42	0.119
141	A	14	5	1.	46	0.109
142	A	14	5	0.99	46	0.109
143	A	14	6	1.	26	0.231

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
144	A	14	6	1.	26	0.231
145	A	14	6	1.	26	0.231
146	A	8	4	1.	26	0.154
147	A	14	6	1.	24	0.25
148	A	14	6	1.	22	0.273
149	A	14	6	1.	26	0.231
150	A	14	6	1.	26	0.231
151	A	23	7	1.	26	0.269
152	A	23	7	1.	26	0.269
153	A	14	6	1.	26	0.231
154	A	17	5	1.	26	0.192
155	A	23	7	1.	26	0.269
156	A	23	7	1.	26	0.269
157	A	23	7	1.	26	0.269
158	A	2	1	1.	52	0.019
159	A	4	3	1.	52	0.058
160	A	1	1	1.	18	0.056
161	A	3	3	1.	23	0.13
162	A	3	3	1.	23	0.13
163	A	3	3	1.	29	0.103
164	A	1	1	1.	18	0.056
165	A	4	3	1.	20	0.15
166	A	4	3	1.	20	0.15
167	A	4	3	1.	22	0.136
168	A	1	1	1.	18	0.056
169	A	3	3	1.	23	0.13
170	A	3	3	1.	23	0.13
171	A	3	3	1.	29	0.103
172	A	1	1	1.	18	0.056
173	A	1	1	1.	25	0.04
174	C	7	3	4.3	38	0.079
175	A	1	1	1.	27	0.037

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
176	A	1	1	1.	31	0.032
177	A	2	1	1.	54	0.019
178	A	3	1	1.	54	0.019
179	A	5	4	1.	54	0.074
180	A	1	1	1.	30	0.033
181	A	1	1	1.	29	0.034
182	A	1	1	1.	28	0.036
183	A	1	1	1.	21	0.048
184	A	1	1	1.	20	0.05
185	A	1	1	1.	21	0.048
186	A	1	1	1.	26	0.038
187	A	1	1	1.	25	0.04
188	A	2	2	1.	26	0.077
189	A	1	1	1.	24	0.042
190	A	1	1	1.	24	0.042
191	A	1	1	1.	23	0.043
192	A	1	1	1.	30	0.033
193	A	1	1	1.	29	0.034
194	A	1	1	1.	28	0.036
195	A	1	1	1.	21	0.048
196	A	1	1	1.	20	0.05
197	A	2	2	1.	21	0.095
198	A	1	1	1.	26	0.038
199	A	1	1	1.	25	0.04
200	A	2	2	1.	26	0.077
201	A	1	1	1.	22	0.045
202	A	1	1	1.	24	0.042
203	A	2	2	1.	23	0.087
204	A	1	1	1.	23	0.043
205	A	1	1	1.	19	0.053
206	A	2	1	1.	22	0.045
207	A	2	1	1.	23	0.043

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
208	A	2	1	1.	22	0.045
209	A	2	1	1.	23	0.043
210	A	2	1	1.	24	0.042
211	A	2	1	1.	25	0.04
212	A	2	1	1.	31	0.032
213	A	2	1	1.	32	0.031
214	A	2	1	1.	35	0.029
215	A	2	1	1.	36	0.028
216	A	2	1	1.	24	0.042
217	A	2	1	1.	31	0.032
218	A	2	1	1.	35	0.029
219	A	1	1	1.	22	0.045
220	A	1	1	1.	18	0.056
221	B	3	2	2.91	26	0.077
222	B	2	1	2.91	28	0.036
223	A	1	1	1.	18	0.056
224	A	1	1	1.	20	0.05
225	A	1	1	1.	21	0.048
226	A	3	3	1.	52	0.058
227	A	9	5	1.	38	0.132
228	A	3	2	1.	32	0.062
229	A	4	3	1.	33	0.091
230	A	3	2	1.	34	0.059
231	A	6	6	1.	43	0.14
232	A	1	1	1.	16	0.062
233	B	15	7	2.25	25	0.28
234	A	1	1	1.	56	0.018
235	A	1	1	1.	51	0.02
236	A	1	1	1.	49	0.02
237	A	1	1	1.	46	0.022
238	A	2	2	1.	48	0.042
239	A	1	1	1.	49	0.02

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
240	A	1	1	1.	48	0.021
241	A	1	1	1.	48	0.021
242	A	10	5	1.	35	0.143
243	A	10	5	1.	35	0.143
244	A	10	5	1.	35	0.143
245	A	10	5	1.	33	0.152
246	A	10	5	1.	32	0.156
247	A	13	6	1.	35	0.171
248	A	13	6	1.	35	0.171
249	A	13	6	1.	35	0.171
250	A	13	6	1.	35	0.171
251	A	13	6	1.	35	0.171
252	A	11	6	1.	33	0.182
253	A	9	5	1.	32	0.156
254	A	13	6	1.	35	0.171
255	A	13	6	1.	35	0.171
256	A	13	6	1.	35	0.171
257	A	2	2	1.	40	0.05
258	A	6	5	1.	20	0.25
259	A	3	2	1.	20	0.1
260	A	3	2	1.	20	0.1
261	A	2	1	1.	16	0.062
262	A	5	4	1.	22	0.182
263	A	3	2	1.	21	0.095
264	A	6	5	1.	26	0.192
265	A	2	1	1.	20	0.05
266	A	2	1	1.	11	0.091
267	A	4	3	1.	22	0.136
268	A	3	2	1.	21	0.095
269	A	3	2	1.	25	0.08
270	A	6	6	1.	22	0.273
271	A	5	5	1.	31	0.161

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
272	A	3	2	1.	21	0.095
273	A	4	3	1.	33	0.091
274	A	4	4	1.	14	0.286
275	A	7	6	1.	33	0.182
276	A	7	6	1.	29	0.207
277	A	6	5	1.	44	0.114
278	A	3	2	1.	15	0.133
279	A	5	4	1.	15	0.267
280	A	4	3	1.	18	0.167
281	A	3	2	1.	20	0.1
282	A	5	4	1.	26	0.154
283	A	3	2	1.	13	0.154
284	A	3	2	1.	18	0.111
285	A	2	1	1.	26	0.038
286	A	5	5	1.	19	0.263
287	A	5	5	1.	24	0.208
288	A	8	6	1.	20	0.3
289	A	8	6	1.	18	0.333
290	A	5	3	1.	19	0.158
291	A	4	3	1.	13	0.231
292	A	6	5	1.	22	0.227
293	A	6	5	1.	24	0.208
294	A	2	1	1.	29	0.034
295	A	2	1	1.	30	0.033
296	A	2	1	1.	19	0.053
297	A	4	4	1.	16	0.25
298	A	10	5	1.	36	0.139
299	A	2	1	1.	21	0.048
300	A	5	4	1.	16	0.25
301	A	2	1	1.	24	0.042
302	A	2	1	1.	21	0.048
303	A	2	1	1.	24	0.042

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
304	A	6	5	1.	26	0.192
305	A	3	2	1.	25	0.08
306	A	2	1	1.	29	0.034
307	A	6	5	1.	20	0.25
308	A	14	10	1.	32	0.312
309	A	4	4	1.	23	0.174
310	A	6	5	1.	26	0.192
311	A	4	3	1.	26	0.115
312	A	8	4	1.	25	0.16
313	A	6	3	1.	23	0.13
314	A	7	6	1.	23	0.261
315	A	5	3	1.18	20	0.15
316	A	3	2	1.	25	0.08
317	A	3	2	1.	22	0.091
318	A	2	1	1.	24	0.042
319	A	7	6	1.	24	0.25
320	A	6	5	1.	43	0.116
321	A	7	5	1.25	50	0.1
322	A	3	2	1.	16	0.125
323	A	6	5	1.	15	0.333
324	A	5	4	1.	20	0.2
325	A	3	2	1.	24	0.083
326	A	5	3	1.	27	0.111
327	A	5	5	1.	26	0.192
328	A	3	2	1.	16	0.125
329	A	11	8	1.32	22	0.364
330	A	5	4	1.	24	0.167
331	A	4	3	1.	26	0.115
332	A	5	3	1.	36	0.083
333	A	4	3	1.	26	0.115
334	A	10	9	1.	20	0.45
335	A	6	6	1.	27	0.222

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
336	A	7	7	1.	20	0.35
337	A	8	7	1.	25	0.28
338	A	8	7	1.	22	0.318
339	A	2	1	1.	18	0.056
340	A	5	4	1.	20	0.2
341	A	10	9	1.	20	0.45
342	A	16	12	1.	20	0.6
343	A	3	2	1.	14	0.143
344	A	2	2	1.	20	0.1
345	A	14	8	1.	20	0.4
346	A	4	3	1.	26	0.115
347	A	4	3	1.	24	0.125
348	A	6	4	1.	30	0.133
349	A	4	4	1.	21	0.19
350	A	2	1	1.	15	0.067
351	A	4	3	1.	18	0.167
352	A	3	2	1.	22	0.091
353	A	3	2	1.	16	0.125
354	A	6	5	1.	16	0.312
355	A	3	2	1.	25	0.08
356	A	3	2	1.	19	0.105
357	A	2	1	1.	23	0.043
358	A	5	4	1.	23	0.174
359	A	5	4	1.	21	0.19
360	A	10	6	1.	28	0.214
361	A	2	1	1.	24	0.042
362	A	6	5	1.	26	0.192
363	A	2	1	1.	14	0.071
364	A	5	3	1.	16	0.188
365	A	5	5	1.	16	0.312
366	A	7	5	1.	43	0.116
367	A	17	13	1.	26	0.5

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
368	A	18	13	1.	16	0.812
369	A	3	2	1.	15	0.133
370	A	3	2	1.	15	0.133
371	A	3	2	1.	17	0.118
372	A	3	2	1.	15	0.133
373	A	4	3	1.	15	0.2
374	A	4	3	1.	18	0.167
375	A	3	2	1.	20	0.1
376	A	7	6	1.	29	0.207
377	A	4	3	1.	22	0.136
378	A	6	4	1.	16	0.25
379	A	6	4	1.	16	0.25
380	A	5	4	1.	14	0.286
381	A	4	3	1.	12	0.25
382	A	4	3	1.	16	0.188
383	A	6	5	1.	16	0.312
384	A	4	3	1.	16	0.188
385	A	4	3	1.	16	0.188
386	A	4	3	1.	16	0.188
387	A	8	5	1.	17	0.294
388	A	8	5	1.	19	0.263
389	A	8	5	1.	15	0.333
390	A	8	5	1.	17	0.294
391	A	16	10	1.	23	0.435
392	A	17	10	1.	21	0.476
393	F	0	0	N/A	0	N/A
394	A	15	11	1.	17	0.647
395	A	13	9	1.	17	0.529
396	A	13	9	1.	15	0.6
397	A	9	6	1.	9	0.667
398	A	17	12	1.	17	0.706
399	A	17	12	1.	17	0.706

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
400	A	17	12	1.	17	0.706
401	A	16	12	1.	17	0.706
402	A	14	10	1.	17	0.588
403	A	14	10	1.	15	0.667
404	A	10	7	1.	9	0.778
405	A	31	14	1.	17	0.824
406	A	31	14	1.	17	0.824
407	A	31	14	1.	17	0.824
408	A	15	11	1.	17	0.647
409	A	15	10	1.	17	0.588
410	A	15	10	1.	15	0.667
411	A	11	7	1.	9	0.778
412	A	46	15	1.	17	0.882
413	A	46	15	1.	17	0.882
414	A	46	15	1.	17	0.882
415	A	4	4	1.	14	0.286
416	A	5	5	1.	13	0.385
417	A	4	4	1.	16	0.25
418	A	5	5	1.	18	0.278
419	A	3	2	1.	14	0.143
420	A	4	3	1.	16	0.188
421	A	2	2	1.	11	0.182
422	A	1	0	1.	17	0.
423	A	1	1	1.	11	0.091
424	B	1	0	6.23	73	0.
425	A	11	7	1.	13	0.538
426	A	13	9	1.	19	0.474
427	A	3	2	1.	15	0.133
428	A	3	2	1.	11	0.182
429	A	1	0	1.	11	0.
430	A	1	0	1.	11	0.
431	A	5	4	1.	16	0.25

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
432	A	2	1	1.	16	0.062
433	A	4	3	1.	15	0.2
434	A	1	1	1.	15	0.067
435	A	1	1	1.	20	0.05
436	A	3	2	1.	15	0.133
437	A	6	5	1.	13	0.385
438	A	1	1	1.	22	0.045
439	A	3	2	1.	18	0.111
440	A	3	3	1.	20	0.15
441	A	3	2	1.	16	0.125
442	A	6	6	1.	17	0.353
443	A	1	1	1.	17	0.059
444	A	4	3	1.	25	0.12
445	A	2	2	1.	20	0.1
446	A	3	2	1.	18	0.111
447	A	6	5	1.	15	0.333
448	A	2	1	1.	11	0.091
449	A	5	5	1.	13	0.385
450	A	3	2	1.	20	0.1
451	A	2	1	1.	16	0.062
452	A	4	3	1.	16	0.188
453	A	2	1	1.	16	0.062
454	A	1	1	1.	9	0.111
455	A	2	2	1.	7	0.286
456	A	2	2	1.	11	0.182
457	A	1	1	1.	7	0.143
458	A	1	1	1.	9	0.111
459	A	1	1	1.	9	0.111
460	A	2	2	1.	10	0.2
461	A	2	1	1.	13	0.077
462	A	2	1	1.	11	0.091
463	A	2	1	1.	14	0.071

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
464	A	4	4	1.	16	0.25
465	A	2	1	1.	7	0.143
466	A	2	1	1.	11	0.091
467	A	5	5	1.	13	0.385
468	A	5	5	1.	13	0.385
469	A	5	4	1.	14	0.286
470	A	2	1	0.78	13	0.077
471	A	3	2	1.	20	0.1
472	A	4	4	1.	18	0.222
473	A	2	1	1.	12	0.083
474	A	2	1	1.	10	0.1
475	A	3	2	1.	16	0.125
476	A	2	1	1.	11	0.091
477	A	1	1	1.	7	0.143
478	A	2	1	1.	15	0.067
479	A	2	1	1.	12	0.083
480	A	3	3	1.	16	0.188
481	A	2	1	1.	11	0.091
482	A	2	1	1.	17	0.059
483	A	2	1	1.	29	0.034
484	A	2	1	1.	18	0.056
485	A	3	2	1.	14	0.143
486	A	2	1	1.	24	0.042
487	A	2	1	1.	11	0.091
488	A	3	2	1.	18	0.111
489	A	3	3	1.	15	0.2
490	A	3	3	1.	16	0.188
491	A	10	7	1.	15	0.467
492	A	5	2	1.	50	0.04
493	F	0	0	N/A	0	N/A
494	F	0	0	N/A	0	N/A

Chapter 3

Listing of integrals

3.1 $\int \frac{1}{2\sqrt{3}b^{3/2}-9bx+9x^3}\,dx$

Optimal. Leaf size=77

$$\frac{1}{3\sqrt{3}\sqrt{b}\left(\sqrt{3}\sqrt{b}-3x\right)}-\frac{\log\left(\sqrt{b}-\sqrt{3}x\right)}{27b}+\frac{\log\left(2\sqrt{b}+\sqrt{3}x\right)}{27b}$$

```
[Out] 1/(3*Sqrt[3]*Sqrt[b]*(Sqrt[3]*Sqrt[b] - 3*x)) - Log[Sqrt[b] - Sqrt[3]*x]/(2
7*b) + Log[2*Sqrt[b] + Sqrt[3]*x]/(27*b)
```

Rubi [A] time = 0.13389, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2063, 44}

$$\frac{1}{3\sqrt{3}\sqrt{b}\left(\sqrt{3}\sqrt{b}-3x\right)}-\frac{\log\left(\sqrt{b}-\sqrt{3}x\right)}{27b}+\frac{\log\left(2\sqrt{b}+\sqrt{3}x\right)}{27b}$$

Antiderivative was successfully verified.

```
[In] Int[(2*Sqrt[3]*b^(3/2) - 9*b*x + 9*x^3)^(-1),x]
```

```
[Out] 1/(3*Sqrt[3]*Sqrt[b]*(Sqrt[3]*Sqrt[b] - 3*x)) - Log[Sqrt[b] - Sqrt[3]*x]/(2
7*b) + Log[2*Sqrt[b] + Sqrt[3]*x]/(27*b)
```

Rule 2063

```
Int[((a_.) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_Symbol] :> Dist[1/(3^(3*p)*
a^(2*p)), Int[(3*a - b*x)^p*(3*a + 2*b*x)^(2*p), x], x] /; FreeQ[{a, b, d},
 x] && EqQ[4*b^3 + 27*a^2*d, 0] && IntegerQ[p]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{1}{2\sqrt{3}b^{3/2}-9bx+9x^3}\,dx = \left(324b^3\right)\int \frac{1}{\left(6\sqrt{3}b^{3/2}-18bx\right)^2\left(6\sqrt{3}b^{3/2}+9bx\right)}\,dx$$

$$= \left(324b^3\right)\int \left(\frac{1}{324\sqrt{3}b^{7/2}\left(\sqrt{3}\sqrt{b}-3x\right)^2}+\frac{1}{2916b^4\left(\sqrt{3}\sqrt{b}-3x\right)}+\frac{1}{2916b^4\left(2\sqrt{3}\sqrt{b}+3x\right)}\right)dx$$

$$= \frac{1}{3\sqrt{3}\sqrt{b}\left(\sqrt{3}\sqrt{b}-3x\right)}-\frac{\log\left(\sqrt{b}-\sqrt{3}x\right)}{27b}+\frac{\log\left(2\sqrt{b}+\sqrt{3}x\right)}{27b}$$

Mathematica [A] time = 0.0484474, size = 143, normalized size = 1.86

$$-\frac{\left(3x-\sqrt{3}\sqrt{b}\right)\left(2\sqrt{3}\sqrt{b}+3x\right)\left(\left(3x-\sqrt{3}\sqrt{b}\right)\log\left(3x-\sqrt{3}\sqrt{b}\right)+\left(\sqrt{3}\sqrt{b}-3x\right)\log\left(2\sqrt{3}\sqrt{b}+3x\right)+3\sqrt{3}\sqrt{b}\right)}{81b\left(2\sqrt{3}b^{3/2}-9bx+9x^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*Sqrt[3]*b^(3/2) - 9*b*x + 9*x^3)^(-1),x]
```

```
[Out] -((-(Sqrt[3]*Sqrt[b]) + 3*x)*(2*Sqrt[3]*Sqrt[b] + 3*x)*(3*Sqrt[3]*Sqrt[b] +
 (-(Sqrt[3]*Sqrt[b]) + 3*x)*Log[-(Sqrt[3]*Sqrt[b]) + 3*x] + (Sqrt[3]*Sqrt[b
] - 3*x)*Log[2*Sqrt[3]*Sqrt[b] + 3*x]))/(81*b*(2*Sqrt[3]*b^(3/2) - 9*b*x +
9*x^3))
```

Maple [C] time = 0.007, size = 43, normalized size = 0.6

$$\frac{1}{9} \sum_{\_R=\mathrm{RootOf}\left(-9\,b\_Z+9\_Z^3+2\,b^{3/2}\sqrt{3}\right)} \frac{\ln\left(x-\_R\right)}{3\_R^2-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-9*b*x+9*x^3+2*b^(3/2)*3^(1/2)),x)
```

```
[Out] 1/9*sum(1/(3*_R^2-b)*ln(x-_R),_R=RootOf(-9*b*_Z+9*_Z^3+2*b^(3/2)*3^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{9\,x^3+2\,\sqrt{3}b^{\frac{3}{2}}-9\,bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-9*b*x+9*x^3+2*b^(3/2)*3^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(9*x^3 + 2*sqrt(3)*b^(3/2) - 9*b*x), x)
```

Fricas [A] time = 1.38785, size = 184, normalized size = 2.39

$$-\frac{3\,\sqrt{3}\sqrt{b}x-\left(3\,x^2-b\right)\log\left(2\,\sqrt{3}\sqrt{b}+3\,x\right)+\left(3\,x^2-b\right)\log\left(-\sqrt{3}\sqrt{b}+3\,x\right)+3\,b}{27\left(3\,bx^2-b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-9*b*x+9*x^3+2*b^(3/2)*3^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/27*(3*sqrt(3)*sqrt(b)*x - (3*x^2 - b)*log(2*sqrt(3)*sqrt(b) + 3*x) + (3*
x^2 - b)*log(-sqrt(3)*sqrt(b) + 3*x) + 3*b)/(3*b*x^2 - b^2)
```

Sympy [A] time = 0.443516, size = 60, normalized size = 0.78

$$-\frac{3\sqrt{3}}{81\sqrt{b}x-27\sqrt{3}b}+\frac{-\frac{\log\left(-\frac{\sqrt{3}\sqrt{b}}{3}+x\right)}{27}+\frac{\log\left(\frac{2\sqrt{3}\sqrt{b}}{3}+x\right)}{27}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-9*b*x+9*x**3+2*b**(3/2)*3**(1/2)),x)
```

```
[Out] -3*sqrt(3)/(81*sqrt(b)*x - 27*sqrt(3)*b) + (-log(-sqrt(3)*sqrt(b)/3 + x)/27
 + log(2*sqrt(3)*sqrt(b)/3 + x)/27)/b
```

Giac [A] time = 1.09659, size = 73, normalized size = 0.95

$$\frac{\log\left(\left|\sqrt{3}x+2\sqrt{b}\right|\right)}{27\,b}-\frac{\log\left(\left|-\sqrt{3}x+\sqrt{b}\right|\right)}{27\,b}-\frac{1}{9\left(\sqrt{3}x-\sqrt{b}\right)\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-9*b*x+9*x^3+2*b^(3/2)*3^(1/2)),x, algorithm="giac")
```

```
[Out] 1/27*log(abs(sqrt(3)*x + 2*sqrt(b)))/b - 1/27*log(abs(-sqrt(3)*x + sqrt(b))
)/b - 1/9/((sqrt(3)*x - sqrt(b))*sqrt(b))
```

3.2 $\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^p\,dx$

Optimal. Leaf size=30

$$\frac{\left(\frac{a}{b}+x\right)\left(b^3\left(\frac{a}{b}+x\right)^3\right)^p}{3p+1}$$

```
[Out] ((a/b + x)*(b^3*(a/b + x)^3)^p)/(1 + 3*p)
```

Rubi [A] time = 0.0177064, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2067, 15, 30}

$$\frac{\left(\frac{a}{b}+x\right)\left(b^3\left(\frac{a}{b}+x\right)^3\right)^p}{3p+1}$$

Antiderivative was successfully verified.

```
[In] Int[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^p,x]
```

```
[Out] ((a/b + x)*(b^3*(a/b + x)^3)^p)/(1 + 3*p)
```

Rule 2067

```
Int[(P3_)^(p_), x_Symbol] :> With[{a = Coeff[P3, x, 0], b = Coeff[P3, x, 1]
, c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Subst[Int[Simp[(2*c^3 - 9*b*c*
d + 27*a*d^2)/(27*d^2) - ((c^2 - 3*b*d)*x)/(3*d) + d*x^3, x]^p, x], x, x +
c/(3*d)] /; NeQ[c, 0]] /; FreeQ[p, x] && PolyQ[P3, x, 3]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^p\,dx &= \text{Subst}\left(\int \left(b^3x^3\right)^p\,dx, x, \frac{a}{b}+x\right)\\ &= \left(\left(\frac{a}{b}+x\right)^{-3p}\left(b^3\left(\frac{a}{b}+x\right)^3\right)^p\right)\text{Subst}\left(\int x^{3p}\,dx, x, \frac{a}{b}+x\right)\\ &= \frac{(a+bx)\left((a+bx)^3\right)^p}{b(1+3p)}\end{aligned}$$

Mathematica [A] time = 0.054513, size = 23, normalized size = 0.77

$$\frac{(a+bx)\left((a+bx)^3\right)^p}{3bp+b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^p,x]
```

```
[Out] ((a + b*x)*((a + b*x)^3)^p)/(b + 3*b*p)
```

Maple [A] time = 0.002, size = 46, normalized size = 1.5

$$\frac{(bx+a)\left(b^3x^3+3\,ab^2x^2+3\,xa^2b+a^3\right)^p}{b\left(1+3\,p\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^p,x)
```

```
[Out] (b*x+a)/b/(1+3*p)*(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^p
```

Maxima [A] time = 1.17738, size = 34, normalized size = 1.13

$$\frac{(bx+a)(bx+a)^{3\,p}}{b\left(3\,p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^p,x, algorithm="maxima")
```

```
[Out] (b*x + a)*(b*x + a)^(3*p)/(b*(3*p + 1))
```

Fricas [A] time = 1.25522, size = 90, normalized size = 3.

$$\frac{(bx+a)\left(b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3\right)^p}{3\,bp+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^p,x, algorithm="fricas")
```

```
[Out] (b*x + a)*(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)^p/(3*b*p + b)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**3*x**3+3*a*b**2*x**2+3*a**2*b*x+a**3)**p,x)
```

```
[Out] Exception raised: TypeError
```

Giac [B] time = 1.0984, size = 99, normalized size = 3.3

$$\frac{\left(b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3\right)^p bx+\left(b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3\right)^p a}{3\,bp+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^p,x, algorithm="giac")
```

```
[Out] ((b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)^p*b*x + (b^3*x^3 + 3*a*b^2*x^2 +
 3*a^2*b*x + a^3)^p*a)/(3*b*p + b)
```

3.3 $\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^3\,dx$

Optimal. Leaf size=14

$$\frac{(a+bx)^{10}}{10b}$$

```
[Out] (a + b*x)^10/(10*b)
```

Rubi [A] time = 0.0074009, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2059, 32}

$$\frac{(a+bx)^{10}}{10b}$$

Antiderivative was successfully verified.

```
[In] Int[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^3,x]
```

```
[Out] (a + b*x)^10/(10*b)
```

Rule 2059

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[u^p, x] /;  !SumQ[Non
freeFactors[u, x]]] /; PolyQ[P, x] && IntegerQ[p]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^3\,dx &= \int (a+bx)^9\,dx\\ &= \frac{(a+bx)^{10}}{10b}\end{aligned}$$

Mathematica [A] time = 0.0009253, size = 14, normalized size = 1.

$$\frac{(a+bx)^{10}}{10b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^3,x]
```

```
[Out] (a + b*x)^10/(10*b)
```

Maple [B] time = 0.002, size = 98, normalized size = 7.

$$\frac{b^9x^{10}}{10}+ab^8x^9+\frac{9\,a^2b^7x^8}{2}+12\,a^3b^6x^7+21\,a^4b^5x^6+\frac{126\,a^5b^4x^5}{5}+21\,a^6b^3x^4+12\,a^7b^2x^3+\frac{9\,a^8bx^2}{2}+a^9x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^3,x)
```

```
[Out] 1/10*b^9*x^10+a*b^8*x^9+9/2*a^2*b^7*x^8+12*a^3*b^6*x^7+21*a^4*b^5*x^6+126/5
*a^5*b^4*x^5+21*a^6*b^3*x^4+12*a^7*b^2*x^3+9/2*a^8*b*x^2+a^9*x
```

Maxima [B] time = 1.16856, size = 292, normalized size = 20.86

$$\frac{1}{10}\,b^9x^{10}+ab^8x^9+\frac{27}{8}\,a^2b^7x^8+\frac{27}{7}\,a^3b^6x^7+\frac{27}{4}\,a^6b^3x^4+a^9x+\frac{3}{4}\left(b^3x^4+4\,ab^2x^3+6\,a^2bx^2\right)a^6+\frac{9}{10}\left(5\,b^3x^6+18\,ab^2x^5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^3,x, algorithm="maxima")
```

```
[Out] 1/10*b^9*x^10 + a*b^8*x^9 + 27/8*a^2*b^7*x^8 + 27/7*a^3*b^6*x^7 + 27/4*a^6*
b^3*x^4 + a^9*x + 3/4*(b^3*x^4 + 4*a*b^2*x^3 + 6*a^2*b*x^2)*a^6 + 9/10*(5*b
^3*x^6 + 18*a*b^2*x^5)*a^4*b^2 + 3/70*(10*b^6*x^7 + 70*a*b^5*x^6 + 126*a^2*
b^4*x^5 + 210*a^4*b^2*x^3 + 21*(4*b^3*x^5 + 15*a*b^2*x^4)*a^2*b)*a^3 + 9/56
*(7*b^6*x^8 + 48*a*b^5*x^7 + 84*a^2*b^4*x^6)*a^2*b
```

Fricas [B] time = 1.11428, size = 212, normalized size = 15.14

$$\frac{1}{10}x^{10}b^9 + x^9b^8a + \frac{9}{2}x^8b^7a^2 + 12x^7b^6a^3 + 21x^6b^5a^4 + \frac{126}{5}x^5b^4a^5 + 21x^4b^3a^6 + 12x^3b^2a^7 + \frac{9}{2}x^2ba^8 + xa^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^3,x, algorithm="fricas")
```

```
[Out] 1/10*x^10*b^9 + x^9*b^8*a + 9/2*x^8*b^7*a^2 + 12*x^7*b^6*a^3 + 21*x^6*b^5*a
^4 + 126/5*x^5*b^4*a^5 + 21*x^4*b^3*a^6 + 12*x^3*b^2*a^7 + 9/2*x^2*b*a^8 +
x*a^9
```

Sympy [B] time = 0.083674, size = 107, normalized size = 7.64

$$a^9x + \frac{9a^8bx^2}{2} + 12a^7b^2x^3 + 21a^6b^3x^4 + \frac{126a^5b^4x^5}{5} + 21a^4b^5x^6 + 12a^3b^6x^7 + \frac{9a^2b^7x^8}{2} + ab^8x^9 + \frac{b^9x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**3*x**3+3*a*b**2*x**2+3*a**2*b*x+a**3)**3,x)
```

```
[Out] a**9*x + 9*a**8*b*x**2/2 + 12*a**7*b**2*x**3 + 21*a**6*b**3*x**4 + 126*a**5
*b**4*x**5/5 + 21*a**4*b**5*x**6 + 12*a**3*b**6*x**7 + 9*a**2*b**7*x**8/2 +
 a*b**8*x**9 + b**9*x**10/10
```

Giac [B] time = 1.06108, size = 131, normalized size = 9.36

$$\frac{1}{10}b^9x^{10} + ab^8x^9 + \frac{9}{2}a^2b^7x^8 + 12a^3b^6x^7 + 21a^4b^5x^6 + \frac{126}{5}a^5b^4x^5 + 21a^6b^3x^4 + 12a^7b^2x^3 + \frac{9}{2}a^8bx^2 + a^9x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^3,x, algorithm="giac")
```

```
[Out] 1/10*b^9*x^10 + a*b^8*x^9 + 9/2*a^2*b^7*x^8 + 12*a^3*b^6*x^7 + 21*a^4*b^5*x
^6 + 126/5*a^5*b^4*x^5 + 21*a^6*b^3*x^4 + 12*a^7*b^2*x^3 + 9/2*a^8*b*x^2 +
a^9*x
```

3.4 $\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^2 \, dx$

Optimal. Leaf size=14

$$\frac{(a+bx)^7}{7b}$$

```
[Out] (a + b*x)^7/(7*b)
```

Rubi [A] time = 0.007188, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2059, 32}

$$\frac{(a+bx)^7}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^2,x]
```

```
[Out] (a + b*x)^7/(7*b)
```

Rule 2059

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[u^p, x] /;  !SumQ[Non
freeFactors[u, x]]] /; PolyQ[P, x] && IntegerQ[p]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^2 \, dx &= \int (a+bx)^6 \, dx\\ &= \frac{(a+bx)^7}{7b}\end{aligned}$$

Mathematica [A] time = 0.0010834, size = 14, normalized size = 1.

$$\frac{(a+bx)^7}{7b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^2,x]
```

```
[Out] (a + b*x)^7/(7*b)
```

Maple [B] time = 0., size = 65, normalized size = 4.6

$$\frac{b^6x^7}{7}+ab^5x^6+3\,a^2b^4x^5+5\,a^3b^3x^4+5\,a^4b^2x^3+3\,a^5bx^2+a^6x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^2,x)
```

```
[Out] 1/7*b^6*x^7+a*b^5*x^6+3*a^2*b^4*x^5+5*a^3*b^3*x^4+5*a^4*b^2*x^3+3*a^5*b*x^2
+a^6*x
```

Maxima [B] time = 1.16221, size = 134, normalized size = 9.57

$$\frac{1}{7}\,b^6x^7+ab^5x^6+\frac{9}{5}\,a^2b^4x^5+3\,a^4b^2x^3+a^6x+\frac{1}{2}\left(b^3x^4+4\,ab^2x^3+6\,a^2bx^2\right)a^3+\frac{3}{10}\left(4\,b^3x^5+15\,ab^2x^4\right)a^2b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^2,x, algorithm="maxima")
```

```
[Out] 1/7*b^6*x^7 + a*b^5*x^6 + 9/5*a^2*b^4*x^5 + 3*a^4*b^2*x^3 + a^6*x + 1/2*(b^
3*x^4 + 4*a*b^2*x^3 + 6*a^2*b*x^2)*a^3 + 3/10*(4*b^3*x^5 + 15*a*b^2*x^4)*a^
2*b
```

Fricas [B] time = 1.08018, size = 128, normalized size = 9.14

$$\frac{1}{7}x^7b^6+x^6b^5a+3x^5b^4a^2+5x^4b^3a^3+5x^3b^2a^4+3x^2ba^5+xa^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^2,x, algorithm="fricas")
```

```
[Out] 1/7*x^7*b^6 + x^6*b^5*a + 3*x^5*b^4*a^2 + 5*x^4*b^3*a^3 + 5*x^3*b^2*a^4 + 3
*x^2*b*a^5 + x*a^6
```

Sympy [B] time = 0.073416, size = 66, normalized size = 4.71

$$a^6x + 3a^5bx^2 + 5a^4b^2x^3 + 5a^3b^3x^4 + 3a^2b^4x^5 + ab^5x^6 + \frac{b^6x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**3*x**3+3*a*b**2*x**2+3*a**2*b*x+a**3)**2,x)
```

```
[Out] a**6*x + 3*a**5*b*x**2 + 5*a**4*b**2*x**3 + 5*a**3*b**3*x**4 + 3*a**2*b**4*
x**5 + a*b**5*x**6 + b**6*x**7/7
```

Giac [B] time = 1.06185, size = 86, normalized size = 6.14

$$\frac{1}{7}\,b^6x^7 + ab^5x^6 + 3\,a^2b^4x^5 + 5\,a^3b^3x^4 + 5\,a^4b^2x^3 + 3\,a^5bx^2 + a^6x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^2,x, algorithm="giac")
```

```
[Out] 1/7*b^6*x^7 + a*b^5*x^6 + 3*a^2*b^4*x^5 + 5*a^3*b^3*x^4 + 5*a^4*b^2*x^3 + 3
*a^5*b*x^2 + a^6*x
```

3.5 $\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)\,dx$

Optimal. Leaf size=35

$$\frac{3}{2}a^2bx^2+a^3x+ab^2x^3+\frac{b^3x^4}{4}$$

```
[Out] a^3*x + (3*a^2*b*x^2)/2 + a*b^2*x^3 + (b^3*x^4)/4
```

Rubi [A] time = 0.0068023, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\frac{3}{2}a^2bx^2+a^3x+ab^2x^3+\frac{b^3x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3,x]
```

```
[Out] a^3*x + (3*a^2*b*x^2)/2 + a*b^2*x^3 + (b^3*x^4)/4
```

Rubi steps

$$\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)\,dx=a^3x+\frac{3}{2}a^2bx^2+ab^2x^3+\frac{b^3x^4}{4}$$

Mathematica [A] time = 0.0000398, size = 35, normalized size = 1.

$$\frac{3}{2}a^2bx^2+a^3x+ab^2x^3+\frac{b^3x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3,x]
```

```
[Out] a^3*x + (3*a^2*b*x^2)/2 + a*b^2*x^3 + (b^3*x^4)/4
```

Maple [A] time = 0.001, size = 32, normalized size = 0.9

$$a^3x + \frac{3\,a^2bx^2}{2} + ab^2x^3 + \frac{b^3x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3,x)
```

```
[Out] a^3*x+3/2*a^2*b*x^2+a*b^2*x^3+1/4*b^3*x^4
```

Maxima [A] time = 1.09014, size = 42, normalized size = 1.2

$$\frac{1}{4}\,b^3x^4 + ab^2x^3 + \frac{3}{2}\,a^2bx^2 + a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3,x, algorithm="maxima")
```

```
[Out] 1/4*b^3*x^4 + a*b^2*x^3 + 3/2*a^2*b*x^2 + a^3*x
```

Fricas [A] time = 1.08271, size = 66, normalized size = 1.89

$$\frac{1}{4}x^4b^3 + x^3b^2a + \frac{3}{2}x^2ba^2 + xa^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3,x, algorithm="fricas")
```

```
[Out] 1/4*x^4*b^3 + x^3*b^2*a + 3/2*x^2*b*a^2 + x*a^3
```

Sympy [A] time = 0.061812, size = 32, normalized size = 0.91

$$a^3x + \frac{3a^2bx^2}{2} + ab^2x^3 + \frac{b^3x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b**3*x**3+3*a*b**2*x**2+3*a**2*b*x+a**3,x)
```

```
[Out] a**3*x + 3*a**2*b*x**2/2 + a*b**2*x**3 + b**3*x**4/4
```

Giac [A] time = 1.08373, size = 42, normalized size = 1.2

$$\frac{1}{4}b^3x^4 + ab^2x^3 + \frac{3}{2}a^2bx^2 + a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3,x, algorithm="giac")
```

```
[Out] 1/4*b^3*x^4 + a*b^2*x^3 + 3/2*a^2*b*x^2 + a^3*x
```

3.6 $\int \frac{1}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx$

Optimal. Leaf size=14

$$-\frac{1}{2b(a+bx)^2}$$

```
[Out] -1/(2*b*(a + b*x)^2)
```

Rubi [A] time = 0.0085853, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2058, 32}

$$-\frac{1}{2b(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^(-1),x]
```

```
[Out] -1/(2*b*(a + b*x)^2)
```

Rule 2058

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx &= \int \frac{1}{(a+bx)^3}\,dx\\ &= -\frac{1}{2b(a+bx)^2}\end{aligned}$$

Mathematica [A] time = 0.0027345, size = 14, normalized size = 1.

$$-\frac{1}{2b(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^(-1),x]
```

```
[Out] -1/(2*b*(a + b*x)^2)
```

Maple [A] time = 0.003, size = 13, normalized size = 0.9

$$-\frac{1}{2\,b\,(bx+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3),x)
```

```
[Out] -1/2/b/(b*x+a)^2
```

Maxima [A] time = 1.1701, size = 32, normalized size = 2.29

$$-\frac{1}{2\left(b^3x^2+2\,ab^2x+a^2b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3),x, algorithm="maxima")
```

```
[Out] -1/2/(b^3*x^2 + 2*a*b^2*x + a^2*b)
```

Fricas [A] time = 1.22929, size = 49, normalized size = 3.5

$$-\frac{1}{2\left(b^3x^2+2\,ab^2x+a^2b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3),x, algorithm="fricas")
```

```
[Out] -1/2/(b^3*x^2 + 2*a*b^2*x + a^2*b)
```

Sympy [B] time = 0.324639, size = 26, normalized size = 1.86

$$-\frac{1}{2a^2b + 4ab^2x + 2b^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**3*x**3+3*a*b**2*x**2+3*a**2*b*x+a**3),x)
```

```
[Out] -1/(2*a**2*b + 4*a*b**2*x + 2*b**3*x**2)
```

Giac [A] time = 1.08152, size = 16, normalized size = 1.14

$$-\frac{1}{2\,(bx + a)^2 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3),x, algorithm="giac")
```

```
[Out] -1/2/((b*x + a)^2*b)
```

3.7 $\int \frac{1}{\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^2}\,dx$

Optimal. Leaf size=14

$$-\frac{1}{5b(a+bx)^5}$$

```
[Out] -1/(5*b*(a + b*x)^5)
```

Rubi [A] time = 0.0086401, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2058, 32}

$$-\frac{1}{5b(a+bx)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^(-2),x]
```

```
[Out] -1/(5*b*(a + b*x)^5)
```

Rule 2058

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^2}\,dx &= \int \frac{1}{(a+bx)^6}\,dx\\ &= -\frac{1}{5b(a+bx)^5}\end{aligned}$$

Mathematica [A] time = 0.0034624, size = 14, normalized size = 1.

$$-\frac{1}{5b(a+bx)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^(-2),x]
```

```
[Out] -1/(5*b*(a + b*x)^5)
```

Maple [A] time = 0.003, size = 13, normalized size = 0.9

$$-\frac{1}{5\,b\,(bx+a)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^2,x)
```

```
[Out] -1/5/b/(b*x+a)^5
```

Maxima [B] time = 1.19541, size = 77, normalized size = 5.5

$$-\frac{1}{5\left(b^6x^5+5\,ab^5x^4+10\,a^2b^4x^3+10\,a^3b^3x^2+5\,a^4b^2x+a^5b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^2,x, algorithm="maxima")
```

```
[Out] -1/5/(b^6*x^5 + 5*a*b^5*x^4 + 10*a^2*b^4*x^3 + 10*a^3*b^3*x^2 + 5*a^4*b^2*x
 + a^5*b)
```

Fricas [B] time = 1.18818, size = 116, normalized size = 8.29

$$-\frac{1}{5\left(b^6x^5+5\,ab^5x^4+10\,a^2b^4x^3+10\,a^3b^3x^2+5\,a^4b^2x+a^5b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^2,x, algorithm="fricas")
```

```
[Out] -1/5/(b^6*x^5 + 5*a*b^5*x^4 + 10*a^2*b^4*x^3 + 10*a^3*b^3*x^2 + 5*a^4*b^2*x
 + a^5*b)
```

Sympy [B] time = 0.494435, size = 61, normalized size = 4.36

$$-\frac{1}{5a^5b + 25a^4b^2x + 50a^3b^3x^2 + 50a^2b^4x^3 + 25ab^5x^4 + 5b^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**3*x**3+3*a*b**2*x**2+3*a**2*b*x+a**3)**2,x)
```

```
[Out] -1/(5*a**5*b + 25*a**4*b**2*x + 50*a**3*b**3*x**2 + 50*a**2*b**4*x**3 + 25*
a*b**5*x**4 + 5*b**6*x**5)
```

Giac [A] time = 1.08833, size = 16, normalized size = 1.14

$$-\frac{1}{5\,(bx + a)^5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^2,x, algorithm="giac")
```

```
[Out] -1/5/((b*x + a)^5*b)
```

3.8 $\int \frac{1}{\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^3}\,dx$

Optimal. Leaf size=14

$$-\frac{1}{8b(a+bx)^8}$$

```
[Out] -1/(8*b*(a + b*x)^8)
```

Rubi [A] time = 0.0081347, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2058, 32}

$$-\frac{1}{8b(a+bx)^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^(-3),x]
```

```
[Out] -1/(8*b*(a + b*x)^8)
```

Rule 2058

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^3}\,dx &= \int \frac{1}{(a+bx)^9}\,dx\\ &= -\frac{1}{8b(a+bx)^8}\end{aligned}$$

Mathematica [A] time = 0.0029336, size = 14, normalized size = 1.

$$-\frac{1}{8b(a+bx)^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^(-3),x]
```

```
[Out] -1/(8*b*(a + b*x)^8)
```

Maple [A] time = 0.003, size = 13, normalized size = 0.9

$$-\frac{1}{8\,b\,(bx+a)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^3,x)
```

```
[Out] -1/8/b/(b*x+a)^8
```

Maxima [B] time = 1.13839, size = 122, normalized size = 8.71

$$-\frac{1}{8\left(b^9x^8+8\,ab^8x^7+28\,a^2b^7x^6+56\,a^3b^6x^5+70\,a^4b^5x^4+56\,a^5b^4x^3+28\,a^6b^3x^2+8\,a^7b^2x+a^8b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^3,x, algorithm="maxima")
```

```
[Out] -1/8/(b^9*x^8 + 8*a*b^8*x^7 + 28*a^2*b^7*x^6 + 56*a^3*b^6*x^5 + 70*a^4*b^5*
x^4 + 56*a^5*b^4*x^3 + 28*a^6*b^3*x^2 + 8*a^7*b^2*x + a^8*b)
```

Fricas [B] time = 1.21306, size = 185, normalized size = 13.21

$$-\frac{1}{8\left(b^9x^8+8\,ab^8x^7+28\,a^2b^7x^6+56\,a^3b^6x^5+70\,a^4b^5x^4+56\,a^5b^4x^3+28\,a^6b^3x^2+8\,a^7b^2x+a^8b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^3,x, algorithm="fricas")
```

```
[Out] -1/8/(b^9*x^8 + 8*a*b^8*x^7 + 28*a^2*b^7*x^6 + 56*a^3*b^6*x^5 + 70*a^4*b^5*
x^4 + 56*a^5*b^4*x^3 + 28*a^6*b^3*x^2 + 8*a^7*b^2*x + a^8*b)
```

Sympy [B] time = 0.73029, size = 97, normalized size = 6.93

$$-\frac{1}{8a^8b+64a^7b^2x+224a^6b^3x^2+448a^5b^4x^3+560a^4b^5x^4+448a^3b^6x^5+224a^2b^7x^6+64ab^8x^7+8b^9x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**3*x**3+3*a*b**2*x**2+3*a**2*b*x+a**3)**3,x)
```

```
[Out] -1/(8*a**8*b + 64*a**7*b**2*x + 224*a**6*b**3*x**2 + 448*a**5*b**4*x**3 + 5
60*a**4*b**5*x**4 + 448*a**3*b**6*x**5 + 224*a**2*b**7*x**6 + 64*a*b**8*x**
7 + 8*b**9*x**8)
```

Giac [A] time = 1.07468, size = 16, normalized size = 1.14

$$-\frac{1}{8\,(bx+a)^8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^3,x, algorithm="giac")
```

```
[Out] -1/8/((b*x + a)^8*b)
```

3.9 $\int \left(3ab+3b^2x+3bcx^2+c^2x^3\right)^3 \, dx$

Optimal. Leaf size=84

$$-\frac{3b\left(b^2-3ac\right)(b+cx)^7}{7c^4}+\frac{3b^2\left(b^2-3ac\right)^2(b+cx)^4}{4c^4}-\frac{b^3x\left(b^2-3ac\right)^3}{c^3}+\frac{(b+cx)^{10}}{10c^4}$$

```
[Out] -((b^3*(b^2 - 3*a*c)^3*x)/c^3) + (3*b^2*(b^2 - 3*a*c)^2*(b + c*x)^4)/(4*c^4
) - (3*b*(b^2 - 3*a*c)*(b + c*x)^7)/(7*c^4) + (b + c*x)^10/(10*c^4)
```

Rubi [A] time = 0.124573, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2060, 194}

$$-\frac{3b\left(b^2-3ac\right)(b+cx)^7}{7c^4}+\frac{3b^2\left(b^2-3ac\right)^2(b+cx)^4}{4c^4}-\frac{b^3x\left(b^2-3ac\right)^3}{c^3}+\frac{(b+cx)^{10}}{10c^4}$$

Antiderivative was successfully verified.

```
[In] Int[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^3,x]
```

```
[Out] -((b^3*(b^2 - 3*a*c)^3*x)/c^3) + (3*b^2*(b^2 - 3*a*c)^2*(b + c*x)^4)/(4*c^4
) - (3*b*(b^2 - 3*a*c)*(b + c*x)^7)/(7*c^4) + (b + c*x)^10/(10*c^4)
```

Rule 2060

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3)^(p_), x_Symbol] :> D
ist[1/3^p, Subst[Int[Simp[(3*a*c - b^2)/c + (c^2*x^3)/b, x]^p, x], x, c/(3*
d) + x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[p, 0] && EqQ[c^2 - 3*b*d, 0]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \left(3ab+3b^2x+3bcx^2+c^2x^3\right)^3\,dx = \frac{1}{27}\,\mathrm{Subst}\left(\int \left(3b\left(3a-\frac{b^2}{c}\right)+3c^2x^3\right)^3\,dx, x, \frac{b}{c}+x\right)$$

$$= \frac{1}{27}\,\mathrm{Subst}\left(\int \left(\frac{27\left(-b^3+3abc\right)^3}{c^3}+81\left(b^3-3abc\right)^2x^3-81bc^3\left(b^2-3ac\right)x^6+27c^6 \ldots\right.\right.$$

$$= -\frac{b^3\left(b^2-3ac\right)^3x}{c^3}+\frac{3b^2\left(b^2-3ac\right)^2(b+cx)^4}{4c^4}-\frac{3b\left(b^2-3ac\right)(b+cx)^7}{7c^4}+\frac{(b+cx)^{1\ldots}}{10c^4}$$

Mathematica [A] time = 0.0187318, size = 159, normalized size = 1.89

$$\frac{27}{4}b^2x^4\left(a^2c^2+6ab^2c+b^4\right)+\frac{81}{2}a^2b^4x^2+27a^3b^3x+\frac{9}{7}bc^3x^7\left(ac+9b^2\right)+9b^2c^2x^6\left(ac+2b^2\right)+\frac{27}{5}b^3cx^5\left(5ac+3b^2\right)+27 \ldots$$

Antiderivative was successfully verified.

```
[In] Integrate[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^3,x]
```

```
[Out] 27*a^3*b^3*x + (81*a^2*b^4*x^2)/2 + 27*a*b^3*(b^2 + a*c)*x^3 + (27*b^2*(b^4
 + 6*a*b^2*c + a^2*c^2)*x^4)/4 + (27*b^3*c*(3*b^2 + 5*a*c)*x^5)/5 + 9*b^2*c
^2*(2*b^2 + a*c)*x^6 + (9*b*c^3*(9*b^2 + a*c)*x^7)/7 + (9*b^2*c^4*x^8)/2 +
b*c^5*x^9 + (c^6*x^10)/10
```

Maple [B] time = 0.001, size = 295, normalized size = 3.5

$$\frac{c^6x^{10}}{10}+bc^5x^9+\frac{9\,b^2c^4x^8}{2}+\frac{\left(3\,abc^4+63\,b^3c^3+c^2\left(6\,abc^2+18\,b^3c\right)\right)x^7}{7}+\frac{\left(18\,ab^2c^3+45\,b^4c^2+3\,bc\left(6\,abc^2+18\,b^3c\right)+c \ldots\right.}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^3,x)
```

```
[Out] 1/10*c^6*x^10+b*c^5*x^9+9/2*b^2*c^4*x^8+1/7*(3*a*b*c^4+63*b^3*c^3+c^2*(6*a*
b*c^2+18*b^3*c))*x^7+1/6*(18*a*b^2*c^3+45*b^4*c^2+3*b*c*(6*a*b*c^2+18*b^3*c
)+c^2*(18*a*b^2*c+9*b^4))*x^6+1/5*(63*a*b^3*c^2+3*b^2*(6*a*b*c^2+18*b^3*c)+
3*b*c*(18*a*b^2*c+9*b^4))*x^5+1/4*(3*a*b*(6*a*b*c^2+18*b^3*c)+3*b^2*(18*a*b
^2*c+9*b^4)+54*b^4*c*a+9*a^2*b^2*c^2)*x^4+1/3*(3*a*b*(18*a*b^2*c+9*b^4)+54*
b^5*a+27*b^3*c*a^2)*x^3+81/2*a^2*b^4*x^2+27*a^3*b^3*x
```

Maxima [B] time = 1.71501, size = 275, normalized size = 3.27

$$\frac{1}{10}c^6x^{10} + bc^5x^9 + \frac{27}{8}b^2c^4x^8 + \frac{27}{7}b^3c^3x^7 + \frac{27}{4}b^6x^4 + 27a^3b^3x + \frac{27}{4}\left(c^2x^4 + 4bcx^3 + 6b^2x^2\right)a^2b^2 + \frac{9}{10}\left(5c^2x^6 + 18bc\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^3,x, algorithm="maxima")
```

```
[Out] 1/10*c^6*x^10 + b*c^5*x^9 + 27/8*b^2*c^4*x^8 + 27/7*b^3*c^3*x^7 + 27/4*b^6*
x^4 + 27*a^3*b^3*x + 27/4*(c^2*x^4 + 4*b*c*x^3 + 6*b^2*x^2)*a^2*b^2 + 9/10*
(5*c^2*x^6 + 18*b*c*x^5)*b^4 + 9/70*(10*c^4*x^7 + 70*b*c^3*x^6 + 126*b^2*c^
2*x^5 + 210*b^4*x^3 + 21*(4*c^2*x^5 + 15*b*c*x^4)*b^2)*a*b + 9/56*(7*c^4*x^
8 + 48*b*c^3*x^7 + 84*b^2*c^2*x^6)*b^2
```

Fricas [B] time = 1.22675, size = 375, normalized size = 4.46

$$\frac{1}{10}x^{10}c^6 + x^9c^5b + \frac{9}{2}x^8c^4b^2 + \frac{81}{7}x^7c^3b^3 + \frac{9}{7}x^7c^4ba + 18x^6c^2b^4 + 9x^6c^3b^2a + \frac{81}{5}x^5cb^5 + 27x^5c^2b^3a + \frac{27}{4}x^4b^6 + \frac{81}{2}x^4cb^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^3,x, algorithm="fricas")
```

```
[Out] 1/10*x^10*c^6 + x^9*c^5*b + 9/2*x^8*c^4*b^2 + 81/7*x^7*c^3*b^3 + 9/7*x^7*c^
4*b*a + 18*x^6*c^2*b^4 + 9*x^6*c^3*b^2*a + 81/5*x^5*c*b^5 + 27*x^5*c^2*b^3*
a + 27/4*x^4*b^6 + 81/2*x^4*c*b^4*a + 27/4*x^4*c^2*b^2*a^2 + 27*x^3*b^5*a +
 27*x^3*c*b^3*a^2 + 81/2*x^2*b^4*a^2 + 27*x*b^3*a^3
```

Sympy [B] time = 0.092548, size = 175, normalized size = 2.08

$$27a^3b^3x + \frac{81a^2b^4x^2}{2} + \frac{9b^2c^4x^8}{2} + bc^5x^9 + \frac{c^6x^{10}}{10} + x^7\left(\frac{9abc^4}{7} + \frac{81b^3c^3}{7}\right) + x^6\left(9ab^2c^3 + 18b^4c^2\right) + x^5\left(27ab^3c^2 + \frac{81b^5c}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**3+3*b*c*x**2+3*b**2*x+3*a*b)**3,x)
```

```
[Out] 27*a**3*b**3*x + 81*a**2*b**4*x**2/2 + 9*b**2*c**4*x**8/2 + b*c**5*x**9 + c
**6*x**10/10 + x**7*(9*a*b*c**4/7 + 81*b**3*c**3/7) + x**6*(9*a*b**2*c**3 +
 18*b**4*c**2) + x**5*(27*a*b**3*c**2 + 81*b**5*c/5) + x**4*(27*a**2*b**2*c
**2/4 + 81*a*b**4*c/2 + 27*b**6/4) + x**3*(27*a**2*b**3*c + 27*a*b**5)
```

Giac [B] time = 1.07557, size = 224, normalized size = 2.67

$$\frac{1}{10}\,c^6x^{10} + bc^5x^9 + \frac{9}{2}\,b^2c^4x^8 + \frac{81}{7}\,b^3c^3x^7 + \frac{9}{7}\,abc^4x^7 + 18\,b^4c^2x^6 + 9\,ab^2c^3x^6 + \frac{81}{5}\,b^5cx^5 + 27\,ab^3c^2x^5 + \frac{27}{4}\,b^6x^4 + \frac{81}{2}\,ab^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^3,x, algorithm="giac")
```

```
[Out] 1/10*c^6*x^10 + b*c^5*x^9 + 9/2*b^2*c^4*x^8 + 81/7*b^3*c^3*x^7 + 9/7*a*b*c^
4*x^7 + 18*b^4*c^2*x^6 + 9*a*b^2*c^3*x^6 + 81/5*b^5*c*x^5 + 27*a*b^3*c^2*x^
5 + 27/4*b^6*x^4 + 81/2*a*b^4*c*x^4 + 27/4*a^2*b^2*c^2*x^4 + 27*a*b^5*x^3 +
 27*a^2*b^3*c*x^3 + 81/2*a^2*b^4*x^2 + 27*a^3*b^3*x
```

3.10 $\int \left(3ab+3b^2x+3bcx^2+c^2x^3\right)^2 dx$

Optimal. Leaf size=56

$$-\frac{b\left(b^2-3ac\right)(b+cx)^4}{2c^3}+\frac{b^2x\left(b^2-3ac\right)^2}{c^2}+\frac{(b+cx)^7}{7c^3}$$

```
[Out] (b^2*(b^2 - 3*a*c)^2*x)/c^2 - (b*(b^2 - 3*a*c)*(b + c*x)^4)/(2*c^3) + (b +
c*x)^7/(7*c^3)
```

Rubi [A] time = 0.0678793, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2060, 194}

$$-\frac{b\left(b^2-3ac\right)(b+cx)^4}{2c^3}+\frac{b^2x\left(b^2-3ac\right)^2}{c^2}+\frac{(b+cx)^7}{7c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^2,x]
```

```
[Out] (b^2*(b^2 - 3*a*c)^2*x)/c^2 - (b*(b^2 - 3*a*c)*(b + c*x)^4)/(2*c^3) + (b +
c*x)^7/(7*c^3)
```

Rule 2060

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3)^(p_), x_Symbol] :> D
ist[1/3^p, Subst[Int[Simp[(3*a*c - b^2)/c + (c^2*x^3)/b, x]^p, x], x, c/(3*
d) + x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[p, 0] && EqQ[c^2 - 3*b*d, 0]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \left(3ab+3b^2x+3bcx^2+c^2x^3\right)^2\,dx=\frac{1}{9}\,\text{Subst}\left(\int \left(3b\left(3a-\frac{b^2}{c}\right)+3c^2x^3\right)^2\,dx,x,\frac{b}{c}+x\right)$$
$$=\frac{1}{9}\,\text{Subst}\left(\int \left(\frac{9\left(-b^3+3abc\right)^2}{c^2}-18bc\left(b^2-3ac\right)x^3+9c^4x^6\right)dx,x,\frac{b}{c}+x\right)$$
$$=\frac{b^2\left(b^2-3ac\right)^2x}{c^2}-\frac{b\left(b^2-3ac\right)(b+cx)^4}{2c^3}+\frac{(b+cx)^7}{7c^3}$$

Mathematica [A] time = 0.0080767, size = 82, normalized size = 1.46

$$9a^2b^2x+\frac{3}{2}bcx^4\left(ac+3b^2\right)+3b^2x^3\left(2ac+b^2\right)+9ab^3x^2+3b^2c^2x^5+bc^3x^6+\frac{c^4x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^2,x]
```

```
[Out] 9*a^2*b^2*x + 9*a*b^3*x^2 + 3*b^2*(b^2 + 2*a*c)*x^3 + (3*b*c*(3*b^2 + a*c)*
x^4)/2 + 3*b^2*c^2*x^5 + b*c^3*x^6 + (c^4*x^7)/7
```

Maple [A] time = 0.001, size = 84, normalized size = 1.5

$$\frac{c^4x^7}{7}+bc^3x^6+3\,b^2c^2x^5+\frac{\left(6\,abc^2+18\,b^3c\right)x^4}{4}+\frac{\left(18\,ab^2c+9\,b^4\right)x^3}{3}+9\,ab^3x^2+9\,b^2a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^2,x)
```

```
[Out] 1/7*c^4*x^7+b*c^3*x^6+3*b^2*c^2*x^5+1/4*(6*a*b*c^2+18*b^3*c)*x^4+1/3*(18*a*
b^2*c+9*b^4)*x^3+9*a*b^3*x^2+9*b^2*a^2*x
```

Maxima [A] time = 1.25047, size = 126, normalized size = 2.25

$$\frac{1}{7}\,c^4x^7+bc^3x^6+\frac{9}{5}\,b^2c^2x^5+3\,b^4x^3+9\,a^2b^2x+\frac{3}{2}\left(c^2x^4+4\,bcx^3+6\,b^2x^2\right)ab+\frac{3}{10}\left(4\,c^2x^5+15\,bcx^4\right)b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^2,x, algorithm="maxima")
```

```
[Out] 1/7*c^4*x^7 + b*c^3*x^6 + 9/5*b^2*c^2*x^5 + 3*b^4*x^3 + 9*a^2*b^2*x + 3/2*(
c^2*x^4 + 4*b*c*x^3 + 6*b^2*x^2)*a*b + 3/10*(4*c^2*x^5 + 15*b*c*x^4)*b^2
```

Fricas [A] time = 1.14051, size = 177, normalized size = 3.16

$$\frac{1}{7}x^7c^4 + x^6c^3b + 3x^5c^2b^2 + \frac{9}{2}x^4cb^3 + \frac{3}{2}x^4c^2ba + 3x^3b^4 + 6x^3cb^2a + 9x^2b^3a + 9xb^2a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^2,x, algorithm="fricas")
```

```
[Out] 1/7*x^7*c^4 + x^6*c^3*b + 3*x^5*c^2*b^2 + 9/2*x^4*c*b^3 + 3/2*x^4*c^2*b*a +
 3*x^3*b^4 + 6*x^3*c*b^2*a + 9*x^2*b^3*a + 9*x*b^2*a^2
```

Sympy [A] time = 0.078979, size = 87, normalized size = 1.55

$$9a^2b^2x + 9ab^3x^2 + 3b^2c^2x^5 + bc^3x^6 + \frac{c^4x^7}{7} + x^4\left(\frac{3abc^2}{2} + \frac{9b^3c}{2}\right) + x^3\left(6ab^2c + 3b^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**3+3*b*c*x**2+3*b**2*x+3*a*b)**2,x)
```

```
[Out] 9*a**2*b**2*x + 9*a*b**3*x**2 + 3*b**2*c**2*x**5 + b*c**3*x**6 + c**4*x**7/
7 + x**4*(3*a*b*c**2/2 + 9*b**3*c/2) + x**3*(6*a*b**2*c + 3*b**4)
```

Giac [A] time = 1.07039, size = 112, normalized size = 2.

$$\frac{1}{7}c^4x^7 + bc^3x^6 + 3\,b^2c^2x^5 + \frac{9}{2}\,b^3cx^4 + \frac{3}{2}\,abc^2x^4 + 3\,b^4x^3 + 6\,ab^2cx^3 + 9\,ab^3x^2 + 9\,a^2b^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^2,x, algorithm="giac")
```

```
[Out] 1/7*c^4*x^7 + b*c^3*x^6 + 3*b^2*c^2*x^5 + 9/2*b^3*c*x^4 + 3/2*a*b*c^2*x^4 +
 3*b^4*x^3 + 6*a*b^2*c*x^3 + 9*a*b^3*x^2 + 9*a^2*b^2*x
```

3.11 $\int \left(3ab+3b^2x+3bcx^2+c^2x^3\right)\,dx$

Optimal. Leaf size=32

$$3abx+\frac{3b^2x^2}{2}+bcx^3+\frac{c^2x^4}{4}$$

```
[Out] 3*a*b*x + (3*b^2*x^2)/2 + b*c*x^3 + (c^2*x^4)/4
```

Rubi [A] time = 0.0058015, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$3abx+\frac{3b^2x^2}{2}+bcx^3+\frac{c^2x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3,x]
```

```
[Out] 3*a*b*x + (3*b^2*x^2)/2 + b*c*x^3 + (c^2*x^4)/4
```

Rubi steps

$$\int \left(3ab+3b^2x+3bcx^2+c^2x^3\right)\,dx=3abx+\frac{3b^2x^2}{2}+bcx^3+\frac{c^2x^4}{4}$$

Mathematica [A] time = 0.0000507, size = 32, normalized size = 1.

$$3abx+\frac{3b^2x^2}{2}+bcx^3+\frac{c^2x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3,x]
```

```
[Out] 3*a*b*x + (3*b^2*x^2)/2 + b*c*x^3 + (c^2*x^4)/4
```

Maple [A] time = 0., size = 29, normalized size = 0.9

$$3\,abx + \frac{3\,b^2x^2}{2} + bcx^3 + \frac{c^2x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b,x)
```

```
[Out] 3*a*b*x+3/2*b^2*x^2+b*c*x^3+1/4*c^2*x^4
```

Maxima [A] time = 1.14607, size = 38, normalized size = 1.19

$$\frac{1}{4}\,c^2x^4 + bcx^3 + \frac{3}{2}\,b^2x^2 + 3\,abx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b,x, algorithm="maxima")
```

```
[Out] 1/4*c^2*x^4 + b*c*x^3 + 3/2*b^2*x^2 + 3*a*b*x
```

Fricas [A] time = 1.15206, size = 63, normalized size = 1.97

$$\frac{1}{4}x^4c^2 + x^3cb + \frac{3}{2}x^2b^2 + 3xba$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b,x, algorithm="fricas")
```

```
[Out] 1/4*x^4*c^2 + x^3*c*b + 3/2*x^2*b^2 + 3*x*b*a
```

Sympy [A] time = 0.060498, size = 31, normalized size = 0.97

$$3abx + \frac{3b^2x^2}{2} + bcx^3 + \frac{c^2x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c**2*x**3+3*b*c*x**2+3*b**2*x+3*a*b,x)
```

```
[Out] 3*a*b*x + 3*b**2*x**2/2 + b*c*x**3 + c**2*x**4/4
```

Giac [A] time = 1.06694, size = 38, normalized size = 1.19

$$\frac{1}{4}c^2x^4 + bcx^3 + \frac{3}{2}b^2x^2 + 3\,abx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b,x, algorithm="giac")
```

```
[Out] 1/4*c^2*x^4 + b*c*x^3 + 3/2*b^2*x^2 + 3*a*b*x
```

3.12 $\int \frac{1}{3ab+3b^2x+3bcx^2+c^2x^3}\,dx$

Optimal. Leaf size=188

$$-\frac{\log\left(\sqrt[3]{b}c\sqrt[3]{b^2-3ac}\left(\frac{b}{c}+x\right)+b^{2/3}\left(b^2-3ac\right)^{2/3}+c^2\left(\frac{b}{c}+x\right)^2\right)}{6b^{2/3}\left(b^2-3ac\right)^{2/3}}+\frac{\log\left(-\sqrt[3]{b}\sqrt[3]{b^2-3ac}+b+cx\right)}{3b^{2/3}\left(b^2-3ac\right)^{2/3}}-\frac{\tan^{-1}\left(\frac{\frac{2(b+cx)}{\sqrt[3]{b^2-3ac}}+\sqrt[3]{b}}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}b^{2/3}\left(b^2-3ac\right)^{2/3}}$$

```
[Out] -(ArcTan[(b^(1/3) + (2*(b + c*x))/(b^2 - 3*a*c)^(1/3))/(Sqrt[3]*b^(1/3))]/(
Sqrt[3]*b^(2/3)*(b^2 - 3*a*c)^(2/3))) + Log[b - b^(1/3)*(b^2 - 3*a*c)^(1/3)
 + c*x]/(3*b^(2/3)*(b^2 - 3*a*c)^(2/3)) - Log[b^(2/3)*(b^2 - 3*a*c)^(2/3) +
 b^(1/3)*c*(b^2 - 3*a*c)^(1/3)*(b/c + x) + c^2*(b/c + x)^2]/(6*b^(2/3)*(b^2
 - 3*a*c)^(2/3))
```

Rubi [A] time = 0.311988, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2067, 200, 31, 634, 617, 204, 628}

$$-\frac{\log\left(\sqrt[3]{b}c\sqrt[3]{b^2-3ac}\left(\frac{b}{c}+x\right)+b^{2/3}\left(b^2-3ac\right)^{2/3}+c^2\left(\frac{b}{c}+x\right)^2\right)}{6b^{2/3}\left(b^2-3ac\right)^{2/3}}+\frac{\log\left(-\sqrt[3]{b}\sqrt[3]{b^2-3ac}+b+cx\right)}{3b^{2/3}\left(b^2-3ac\right)^{2/3}}-\frac{\tan^{-1}\left(\frac{\frac{2(b+cx)}{\sqrt[3]{b^2-3ac}}+\sqrt[3]{b}}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}b^{2/3}\left(b^2-3ac\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^(-1),x]
```


```
[Out] -(ArcTan[(b^(1/3) + (2*(b + c*x))/(b^2 - 3*a*c)^(1/3))/(Sqrt[3]*b^(1/3))]/(
Sqrt[3]*b^(2/3)*(b^2 - 3*a*c)^(2/3))) + Log[b - b^(1/3)*(b^2 - 3*a*c)^(1/3)
 + c*x]/(3*b^(2/3)*(b^2 - 3*a*c)^(2/3)) - Log[b^(2/3)*(b^2 - 3*a*c)^(2/3) +
 b^(1/3)*c*(b^2 - 3*a*c)^(1/3)*(b/c + x) + c^2*(b/c + x)^2]/(6*b^(2/3)*(b^2
 - 3*a*c)^(2/3))
```


Rule 2067

```
Int[(P3_)^(p_), x_Symbol] :> With[{a = Coeff[P3, x, 0], b = Coeff[P3, x, 1]
, c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Subst[Int[Simp[(2*c^3 - 9*b*c*
d + 27*a*d^2)/(27*d^2) - ((c^2 - 3*b*d)*x)/(3*d) + d*x^3, x]^p, x], x, x +
c/(3*d)] /; NeQ[c, 0]] /; FreeQ[p, x] && PolyQ[P3, x, 3]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{3ab+3b^2x+3bcx^2+c^2x^3}\,dx = \text{Subst}\left(\int \frac{1}{b\left(3a-\frac{b^2}{c}\right)+c^2x^3}\,dx, x, \frac{b}{c}+x\right)$$

$$= \frac{c^{2/3}\,\text{Subst}\left(\int \frac{1}{-\frac{\sqrt[3]{b}\sqrt[3]{b^2-3ac}}{\sqrt[3]{c}}+c^{2/3}x}\,dx, x, \frac{b}{c}+x\right)}{3b^{2/3}\left(b^2-3ac\right)^{2/3}} + \frac{c^{2/3}\,\text{Subst}\left(\int \frac{-\frac{2\sqrt[3]{b}\sqrt[3]{b^2-3ac}}{\sqrt[3]{c}}-c^{2/3}x}{\frac{b^{2/3}\left(b^2-3ac\right)^{2/3}}{c^{2/3}}+\sqrt[3]{b}\sqrt[3]{c}\sqrt[3]{b^2-3ac}x+}\right.}{3b^{2/3}\left(b^2-3ac\right)^{2/3}}$$

$$= \frac{\log\left(\sqrt[3]{b}\left(b^{2/3}-\sqrt[3]{b^2-3ac}\right)+cx\right)}{3b^{2/3}\left(b^2-3ac\right)^{2/3}} - \frac{\text{Subst}\left(\int \frac{\sqrt[3]{b}\sqrt[3]{c}\sqrt[3]{b^2-3ac}+2c^{4/3}x}{\frac{b^{2/3}\left(b^2-3ac\right)^{2/3}}{c^{2/3}}+\sqrt[3]{b}\sqrt[3]{c}\sqrt[3]{b^2-3ac}x+c^{4/3}x^2}\,dx, x, \frac{b}{c}\right.}{6b^{2/3}\left(b^2-3ac\right)^{2/3}}$$

$$= \frac{\log\left(\sqrt[3]{b}\left(b^{2/3}-\sqrt[3]{b^2-3ac}\right)+cx\right)}{3b^{2/3}\left(b^2-3ac\right)^{2/3}} - \frac{\log\left(b^{2/3}\left(b^2-3ac\right)^{2/3}+\sqrt[3]{b}\sqrt[3]{b^2-3ac}(b+cx)+(\right.}{6b^{2/3}\left(b^2-3ac\right)^{2/3}}$$

$$= -\frac{\tan^{-1}\left(\frac{1+\frac{2(b+cx)}{\sqrt[3]{b}\sqrt[3]{b^2-3ac}}}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}\left(b^2-3ac\right)^{2/3}} + \frac{\log\left(\sqrt[3]{b}\left(b^{2/3}-\sqrt[3]{b^2-3ac}\right)+cx\right)}{3b^{2/3}\left(b^2-3ac\right)^{2/3}} - \frac{\log\left(b^{2/3}\left(b^2-3ac\right)^{2/3}+\right.}{6b^{2/}}$$

Mathematica [C] time = 0.016193, size = 63, normalized size = 0.34

$$\frac{1}{3}\text{RootSum}\left[3\#1^2bc+\#1^3c^2+3\#1b^2+3ab\&, \frac{\log(x-\#1)}{\#1^2c^2+2\#1bc+b^2}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^(-1),x]
```

```
[Out] RootSum[3*a*b + 3*b^2*#1 + 3*b*c*#1^2 + c^2*#1^3 & , Log[x - #1]/(b^2 + 2*b
*c*#1 + c^2*#1^2) & ]/3
```

Maple [C] time = 0.003, size = 57, normalized size = 0.3

$$\frac{1}{3}\sum_{\_R=\text{RootOf}\left(c^2\_Z^3+3\,bc\_Z^2+3\,b^2\_Z+3\,ab\right)}\frac{\ln\left(x-\_R\right)}{\_R^2c^2+2\,\_R\,bc+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b),x)
```

```
[Out] 1/3*sum(1/(_R^2*c^2+2*_R*b*c+b^2)*ln(x-_R),_R=RootOf(_Z^3*c^2+3*_Z^2*b*c+3*
_Z*b^2+3*a*b))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{c^2x^3 + 3\,bcx^2 + 3\,b^2x + 3\,ab}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b),x, algorithm="maxima")
```

```
[Out] integrate(1/(c^2*x^3 + 3*b*c*x^2 + 3*b^2*x + 3*a*b), x)
```

Fricas [B] time = 1.41423, size = 878, normalized size = 4.67

$$-\frac{2\,\sqrt{3}\left(b^6 - 6\,ab^4c + 9\,a^2b^2c^2\right)^{\frac{1}{6}}\left(b^3 - 3\,abc\right)\arctan\left(\frac{2\,\sqrt{3}\left(b^6-6\,ab^4c+9\,a^2b^2c^2\right)^{\frac{2}{3}}(cx+b)+\sqrt{3}\left(b^6-6\,ab^4c+9\,a^2b^2c^2\right)^{\frac{1}{3}}\left(b^3-3\,abc\right)}{3\left(b^6-6\,ab^4c+9\,a^2b^2c^2\right)^{\frac{5}{6}}}\right) + \left(b^6 - 6\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(1/6)*(b^3 - 3*a*b*c)*arc
tan(1/3*(2*sqrt(3)*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(2/3)*(c*x + b) + sqrt
(3)*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(1/3)*(b^3 - 3*a*b*c))/(b^6 - 6*a*b^4
*c + 9*a^2*b^2*c^2)^(5/6)) + (b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(2/3)*log(-b
^5 + 3*a*b^3*c - (b^3*c^2 - 3*a*b*c^3)*x^2 - 2*(b^4*c - 3*a*b^2*c^2)*x - (b
^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(2/3)*(c*x + b) - (b^6 - 6*a*b^4*c + 9*a^2*
b^2*c^2)^(1/3)*(b^3 - 3*a*b*c)) - 2*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(2/3)
*log(-b^4 + 3*a*b^2*c - (b^3*c - 3*a*b*c^2)*x + (b^6 - 6*a*b^4*c + 9*a^2*b^
2*c^2)^(2/3)))/(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)
```

Sympy [A] time = 0.375449, size = 53, normalized size = 0.28

$$\mathrm{RootSum}\left(t^3\left(243a^2b^2c^2-162ab^4c+27b^6\right)-1,\left(t\mapsto t\log\left(x+\frac{9tabc-3tb^3+b}{c}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c**2*x**3+3*b*c*x**2+3*b**2*x+3*a*b),x)
```

```
[Out] RootSum(_t**3*(243*a**2*b**2*c**2 - 162*a*b**4*c + 27*b**6) - 1, Lambda(_t,
 _t*log(x + (9*_t*a*b*c - 3*_t*b**3 + b)/c)))
```

Giac [A] time = 1.11204, size = 281, normalized size = 1.49

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}cx+\sqrt{3}b-\sqrt{3}\left(-b^3+3\,abc\right)^{\frac{1}{3}}}{cx+b+\left(-b^3+3\,abc\right)^{\frac{1}{3}}}\right)}{3\left(b^6-6\,ab^4c+9\,a^2b^2c^2\right)^{\frac{1}{3}}}-\frac{\log\left(\left(\sqrt{3}cx+\sqrt{3}b-\sqrt{3}\left(-b^3+3\,abc\right)^{\frac{1}{3}}\right)^2+\left(cx+b+\left(-b^3+3\,abc\right)^{\frac{1}{3}}\right)^2\right)}{6\left(b^6-6\,ab^4c+9\,a^2b^2c^2\right)^{\frac{1}{3}}}+\frac{\log\left(\left|c\right.\right.}{3\left(b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan((sqrt(3)*c*x + sqrt(3)*b - sqrt(3)*(-b^3 + 3*a*b*c)^(1/3
))/(c*x + b + (-b^3 + 3*a*b*c)^(1/3)))/(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(1
/3) - 1/6*log((sqrt(3)*c*x + sqrt(3)*b - sqrt(3)*(-b^3 + 3*a*b*c)^(1/3))^2
+ (c*x + b + (-b^3 + 3*a*b*c)^(1/3))^2)/(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(
1/3) + 1/3*log(abs(c*x + b + (-b^3 + 3*a*b*c)^(1/3)))/(b^6 - 6*a*b^4*c + 9*
a^2*b^2*c^2)^(1/3)
```

3.13 $\int \frac{1}{\left(3ab+3b^2x+3bcx^2+c^2x^3\right)^2}\,dx$

Optimal. Leaf size=245

$$-\frac{c\left(\frac{b}{c}+x\right)}{3b\left(b^2-3ac\right)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)}+\frac{c\log\left(\sqrt[3]{b}c\sqrt[3]{b^2-3ac}\left(\frac{b}{c}+x\right)+b^{2/3}\left(b^2-3ac\right)^{2/3}+c^2\left(\frac{b}{c}+x\right)^2\right)}{9b^{5/3}\left(b^2-3ac\right)^{5/3}}-\frac{2c\log}{}$$

```
[Out] -(c*(b/c + x))/(3*b*(b^2 - 3*a*c)*(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3))
+ (2*c*ArcTan[(b^(1/3) + (2*(b + c*x))/(b^2 - 3*a*c)^(1/3))/(Sqrt[3]*b^(1/3
))])/(3*Sqrt[3]*b^(5/3)*(b^2 - 3*a*c)^(5/3)) - (2*c*Log[b - b^(1/3)*(b^2 -
3*a*c)^(1/3) + c*x])/(9*b^(5/3)*(b^2 - 3*a*c)^(5/3)) + (c*Log[b^(2/3)*(b^2
- 3*a*c)^(2/3) + b^(1/3)*c*(b^2 - 3*a*c)^(1/3)*(b/c + x) + c^2*(b/c + x)^2]
)/(9*b^(5/3)*(b^2 - 3*a*c)^(5/3))
```

Rubi [A] time = 0.248543, antiderivative size = 245, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2067, 199, 200, 31, 634, 617, 204, 628}

$$-\frac{c\left(\frac{b}{c}+x\right)}{3b\left(b^2-3ac\right)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)}+\frac{c\log\left(\sqrt[3]{b}c\sqrt[3]{b^2-3ac}\left(\frac{b}{c}+x\right)+b^{2/3}\left(b^2-3ac\right)^{2/3}+c^2\left(\frac{b}{c}+x\right)^2\right)}{9b^{5/3}\left(b^2-3ac\right)^{5/3}}-\frac{2c\log}{}$$

Antiderivative was successfully verified.

```
[In] Int[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^(-2),x]
```


```
[Out] -(c*(b/c + x))/(3*b*(b^2 - 3*a*c)*(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3))
+ (2*c*ArcTan[(b^(1/3) + (2*(b + c*x))/(b^2 - 3*a*c)^(1/3))/(Sqrt[3]*b^(1/3
))])/(3*Sqrt[3]*b^(5/3)*(b^2 - 3*a*c)^(5/3)) - (2*c*Log[b - b^(1/3)*(b^2 -
3*a*c)^(1/3) + c*x])/(9*b^(5/3)*(b^2 - 3*a*c)^(5/3)) + (c*Log[b^(2/3)*(b^2
- 3*a*c)^(2/3) + b^(1/3)*c*(b^2 - 3*a*c)^(1/3)*(b/c + x) + c^2*(b/c + x)^2]
)/(9*b^(5/3)*(b^2 - 3*a*c)^(5/3))
```


Rule 2067

```
Int[(P3_)^(p_), x_Symbol] :> With[{a = Coeff[P3, x, 0], b = Coeff[P3, x, 1]
, c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Subst[Int[Simp[(2*c^3 - 9*b*c*
```

```
d + 27*a*d^2)/(27*d^2) - ((c^2 - 3*b*d)*x)/(3*d) + d*x^3, x]^p, x], x, x +
c/(3*d)] /; NeQ[c, 0]] /; FreeQ[p, x] && PolyQ[P3, x, 3]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{\left(3ab+3b^2x+3bcx^2+c^2x^3\right)^2}\,dx = \text{Subst}\left(\int \frac{1}{\left(b\left(3a-\frac{b^2}{c}\right)+c^2x^3\right)^2}\,dx, x, \frac{b}{c}+x\right)$$

$$= -\frac{c\left(\frac{b}{c}+x\right)}{3b\left(b^2-3ac\right)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)} - \frac{(2c)\,\text{Subst}\left(\int \frac{1}{b\left(3a-\frac{b^2}{c}\right)+c^2x^3}\,dx, x, \ldots\right.}{3b\left(b^2-3ac\right)}$$

$$= -\frac{c\left(\frac{b}{c}+x\right)}{3b\left(b^2-3ac\right)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)} - \frac{\left(2c^{5/3}\right)\text{Subst}\left(\int \frac{1}{-\frac{\sqrt[3]{b}\sqrt[3]{b^2-3ac}}{\sqrt[3]{c}}+c^{2/3}x}\ldots\right.}{9b^{5/3}\left(b^2-3ac\right)^{5/3}}$$

$$= -\frac{c\left(\frac{b}{c}+x\right)}{3b\left(b^2-3ac\right)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)} - \frac{2c\log\left(\sqrt[3]{b}\left(b^{2/3}-\sqrt[3]{b^2-3ac}\right)+c\ldots\right.}{9b^{5/3}\left(b^2-3ac\right)^{5/3}}$$

$$= -\frac{c\left(\frac{b}{c}+x\right)}{3b\left(b^2-3ac\right)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)} - \frac{2c\log\left(\sqrt[3]{b}\left(b^{2/3}-\sqrt[3]{b^2-3ac}\right)+c\ldots\right.}{9b^{5/3}\left(b^2-3ac\right)^{5/3}}$$

$$= -\frac{c\left(\frac{b}{c}+x\right)}{3b\left(b^2-3ac\right)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)} + \frac{2c\tan^{-1}\left(\frac{1+\frac{2(b+cx)}{\sqrt[3]{b}\sqrt[3]{b^2-3ac}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{5/3}\left(b^2-3ac\right)^{5/3}} - \frac{2c\log\left(\ldots\right.}{\ldots}$$

Mathematica [C] time = 0.054183, size = 112, normalized size = 0.46

$$-\frac{2c\,\text{RootSum}\left[3\#1^2bc+\#1^3c^2+3\#1b^2+3ab\&,\frac{\log(x-\#1)}{\#1^2c^2+2\#1bc+b^2}\&\right]+\frac{3(b+cx)}{3ab+x\left(3b^2+3bcx+c^2x^2\right)}}{9\left(b^3-3abc\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^(-2),x]
```

```
[Out] -((3*(b + c*x))/(3*a*b + x*(3*b^2 + 3*b*c*x + c^2*x^2)) + 2*c*RootSum[3*a*b
 + 3*b^2*#1 + 3*b*c*#1^2 + c^2*#1^3 & , Log[x - #1]/(b^2 + 2*b*c*#1 + c^2*#
1^2) & ])/(9*(b^3 - 3*a*b*c))
```

Maple [C] time = 0.007, size = 136, normalized size = 0.6

$$\frac{1}{c^2x^3+3\,bcx^2+3\,b^2x+3\,ab}\left(\frac{cx}{3\,b\left(3\,ac-b^2\right)}+\frac{1}{9\,ac-3\,b^2}\right)+\frac{2\,c}{9\,b\left(3\,ac-b^2\right)}\sum_{\_R=\mathrm{RootOf}\left(\_Z^3c^2+3\_Z^2bc+3\_Z\,b^2+3\,ab\right)}\frac{\ln(x}{\_R^2c^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^2,x)
```

```
[Out] (1/3*c/b/(3*a*c-b^2)*x+1/3/(3*a*c-b^2))/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)+2
/9*c/b/(3*a*c-b^2)*sum(1/(_R^2*c^2+2*_R*b*c+b^2)*ln(x-_R),_R=RootOf(_Z^3*c^
2+3*_Z^2*b*c+3*_Z*b^2+3*a*b))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\frac{1}{3}\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}cx+\sqrt{3}b-\sqrt{3}\left(-b^3+3\,abc\right)^{\frac{1}{3}}}{cx+b+\left(-b^3+3\,abc\right)^{\frac{1}{3}}}\right)}{\left(b^6-6\,ab^4c+9\,a^2b^2c^2\right)^{\frac{1}{3}}}-\frac{\log\left(\left(\sqrt{3}cx+\sqrt{3}b-\sqrt{3}\left(-b^3+3\,abc\right)^{\frac{1}{3}}\right)^2+\left(cx+b+\left(-b^3+3\,abc\right)^{\frac{1}{3}}\right)^2\right)}{\left(b^6-6\,ab^4c+9\,a^2b^2c^2\right)^{\frac{1}{3}}}+\frac{2\log\left(\left|cx+b+\left(-b^3+3\,abc\right)^{\frac{1}{3}}\right|\right)}{\left(b^6-6\,ab^4c+9\,a^2b^2c^2\right)^{\frac{1}{3}}}\right)c}{3\left(b^3-3\,abc\right)}-\frac{}{3(3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^2,x, algorithm="maxima")
```

```
[Out] -2/3*c*integrate(1/(c^2*x^3 + 3*b*c*x^2 + 3*b^2*x + 3*a*b), x)/(b^3 - 3*a*b
*c) - 1/3*(c*x + b)/(3*a*b^4 - 9*a^2*b^2*c + (b^3*c^2 - 3*a*b*c^3)*x^3 + 3*
(b^4*c - 3*a*b^2*c^2)*x^2 + 3*(b^5 - 3*a*b^3*c)*x)
```

Fricas [B] time = 1.44711, size = 1536, normalized size = 6.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^2,x, algorithm="fricas")
```

```
[Out] -1/9*(3*b^7 - 18*a*b^5*c + 27*a^2*b^3*c^2 - 2*sqrt(3)*(b^6 - 6*a*b^4*c + 9*
a^2*b^2*c^2)^(1/6)*(3*a*b^4*c - 9*a^2*b^2*c^2 + (b^3*c^3 - 3*a*b*c^4)*x^3 +
 3*(b^4*c^2 - 3*a*b^2*c^3)*x^2 + 3*(b^5*c - 3*a*b^3*c^2)*x)*arctan(1/3*(2*s
qrt(3)*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(2/3)*(c*x + b) + sqrt(3)*(b^6 - 6
*a*b^4*c + 9*a^2*b^2*c^2)^(1/3)*(b^3 - 3*a*b*c))/(b^6 - 6*a*b^4*c + 9*a^2*b
^2*c^2)^(5/6)) - (b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(2/3)*(c^3*x^3 + 3*b*c^2
*x^2 + 3*b^2*c*x + 3*a*b*c)*log(-b^5 + 3*a*b^3*c - (b^3*c^2 - 3*a*b*c^3)*x^
2 - 2*(b^4*c - 3*a*b^2*c^2)*x - (b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(2/3)*(c*
x + b) - (b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(1/3)*(b^3 - 3*a*b*c)) + 2*(b^6
- 6*a*b^4*c + 9*a^2*b^2*c^2)^(2/3)*(c^3*x^3 + 3*b*c^2*x^2 + 3*b^2*c*x + 3*a
*b*c)*log(-b^4 + 3*a*b^2*c - (b^3*c - 3*a*b*c^2)*x + (b^6 - 6*a*b^4*c + 9*a
^2*b^2*c^2)^(2/3)) + 3*(b^6*c - 6*a*b^4*c^2 + 9*a^2*b^2*c^3)*x)/(3*a*b^10 -
 27*a^2*b^8*c + 81*a^3*b^6*c^2 - 81*a^4*b^4*c^3 + (b^9*c^2 - 9*a*b^7*c^3 +
27*a^2*b^5*c^4 - 27*a^3*b^3*c^5)*x^3 + 3*(b^10*c - 9*a*b^8*c^2 + 27*a^2*b^6
*c^3 - 27*a^3*b^4*c^4)*x^2 + 3*(b^11 - 9*a*b^9*c + 27*a^2*b^7*c^2 - 27*a^3*
b^5*c^3)*x)
```

Sympy [A] time = 1.49114, size = 192, normalized size = 0.78

$$\frac{b+cx}{27a^2b^2c-9ab^4+x^3\left(9abc^3-3b^3c^2\right)+x^2\left(27ab^2c^2-9b^4c\right)+x\left(27ab^3c-9b^5\right)}+\mathrm{RootSum}\left(t^3\left(177147a^5b^5c^5-29524\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c**2*x**3+3*b*c*x**2+3*b**2*x+3*a*b)**2,x)
```

```
[Out] (b + c*x)/(27*a**2*b**2*c - 9*a*b**4 + x**3*(9*a*b*c**3 - 3*b**3*c**2) + x*
*2*(27*a*b**2*c**2 - 9*b**4*c) + x*(27*a*b**3*c - 9*b**5)) + RootSum(_t**3*
(177147*a**5*b**5*c**5 - 295245*a**4*b**7*c**4 + 196830*a**3*b**9*c**3 - 65
610*a**2*b**11*c**2 + 10935*a*b**13*c - 729*b**15) - 8*c**3, Lambda(_t, _t*
log(x + (81*_t*a**2*b**2*c**2 - 54*_t*a*b**4*c + 9*_t*b**6 + 2*b*c)/(2*c**2
))))
```

Giac [B] time = 1.1683, size = 551, normalized size = 2.25

$$\frac{2}{9}\sqrt{3}\left(-\frac{c^3}{b^{15}-15\,ab^{13}c+90\,a^2b^{11}c^2-270\,a^3b^9c^3+405\,a^4b^7c^4-243\,a^5b^5c^5}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}cx+\sqrt{3}b-\sqrt{3}\left(-b^3+3\,abc\right)^{\frac{1}{3}}}{cx+b+\left(-b^3+3\,abc\right)^{\frac{1}{3}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^2,x, algorithm="giac")
```

```
[Out] 2/9*sqrt(3)*(-c^3/(b^15 - 15*a*b^13*c + 90*a^2*b^11*c^2 - 270*a^3*b^9*c^3 +
 405*a^4*b^7*c^4 - 243*a^5*b^5*c^5))^(1/3)*arctan((sqrt(3)*c*x + sqrt(3)*b
- sqrt(3)*(-b^3 + 3*a*b*c)^(1/3))/(c*x + b + (-b^3 + 3*a*b*c)^(1/3))) - 1/9
*(-c^3/(b^15 - 15*a*b^13*c + 90*a^2*b^11*c^2 - 270*a^3*b^9*c^3 + 405*a^4*b^
7*c^4 - 243*a^5*b^5*c^5))^(1/3)*log((sqrt(3)*c*x + sqrt(3)*b - sqrt(3)*(-b^
3 + 3*a*b*c)^(1/3))^2 + (c*x + b + (-b^3 + 3*a*b*c)^(1/3))^2) + 2/9*(-c^3/(
b^15 - 15*a*b^13*c + 90*a^2*b^11*c^2 - 270*a^3*b^9*c^3 + 405*a^4*b^7*c^4 -
243*a^5*b^5*c^5))^(1/3)*log(abs(3*b^4 - 9*a*b^2*c + 3*(b^3*c - 3*a*b*c^2)*x
 + 3*(b^3 - 3*a*b*c)*(-b^3 + 3*a*b*c)^(1/3))) - 1/3*(c*x + b)/((c^2*x^3 + 3
*b*c*x^2 + 3*b^2*x + 3*a*b)*(b^3 - 3*a*b*c))
```

3.14 $\int \frac{1}{\left(3ab+3b^2x+3bcx^2+c^2x^3\right)^3}\,dx$

Optimal. Leaf size=305

$$\frac{5c^2\left(\frac{b}{c}+x\right)}{18b^2\left(b^2-3ac\right)^2\left(3ab+3b^2x+3bcx^2+c^2x^3\right)}-\frac{c\left(\frac{b}{c}+x\right)}{6b\left(b^2-3ac\right)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)^2}+\frac{5c^2\log\left(-\sqrt[3]{b}\sqrt[3]{b^2-3ac}-\right.}{27b^{8/3}\left(b^2-3ac\right)^{8}}$$

```
[Out] -(c*(b/c + x))/(6*b*(b^2 - 3*a*c)*(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^2
) + (5*c^2*(b/c + x))/(18*b^2*(b^2 - 3*a*c)^2*(3*a*b + 3*b^2*x + 3*b*c*x^2
+ c^2*x^3)) - (5*c^2*ArcTan[(b^(1/3) + (2*(b + c*x))/(b^2 - 3*a*c)^(1/3))/(
Sqrt[3]*b^(1/3))])/(9*Sqrt[3]*b^(8/3)*(b^2 - 3*a*c)^(8/3)) + (5*c^2*Log[b -
 b^(1/3)*(b^2 - 3*a*c)^(1/3) + c*x])/(27*b^(8/3)*(b^2 - 3*a*c)^(8/3)) - (5*
c^2*Log[b^(2/3)*(b^2 - 3*a*c)^(2/3) + b^(1/3)*c*(b^2 - 3*a*c)^(1/3)*(b/c +
x) + c^2*(b/c + x)^2])/(54*b^(8/3)*(b^2 - 3*a*c)^(8/3))
```

Rubi [A] time = 0.301938, antiderivative size = 305, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2067, 199, 200, 31, 634, 617, 204, 628}

$$\frac{5c^2\left(\frac{b}{c}+x\right)}{18b^2\left(b^2-3ac\right)^2\left(3ab+3b^2x+3bcx^2+c^2x^3\right)}-\frac{c\left(\frac{b}{c}+x\right)}{6b\left(b^2-3ac\right)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)^2}+\frac{5c^2\log\left(-\sqrt[3]{b}\sqrt[3]{b^2-3ac}-\right.}{27b^{8/3}\left(b^2-3ac\right)^{8}}$$

Antiderivative was successfully verified.

```
[In] Int[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^(-3),x]
```

```
[Out] -(c*(b/c + x))/(6*b*(b^2 - 3*a*c)*(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^2
) + (5*c^2*(b/c + x))/(18*b^2*(b^2 - 3*a*c)^2*(3*a*b + 3*b^2*x + 3*b*c*x^2
+ c^2*x^3)) - (5*c^2*ArcTan[(b^(1/3) + (2*(b + c*x))/(b^2 - 3*a*c)^(1/3))/(
Sqrt[3]*b^(1/3))])/(9*Sqrt[3]*b^(8/3)*(b^2 - 3*a*c)^(8/3)) + (5*c^2*Log[b -
 b^(1/3)*(b^2 - 3*a*c)^(1/3) + c*x])/(27*b^(8/3)*(b^2 - 3*a*c)^(8/3)) - (5*
c^2*Log[b^(2/3)*(b^2 - 3*a*c)^(2/3) + b^(1/3)*c*(b^2 - 3*a*c)^(1/3)*(b/c +
x) + c^2*(b/c + x)^2])/(54*b^(8/3)*(b^2 - 3*a*c)^(8/3))
```

Rule 2067

```
Int[(P3_)^(p_), x_Symbol] :> With[{a = Coeff[P3, x, 0], b = Coeff[P3, x, 1]
, c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Subst[Int[Simp[(2*c^3 - 9*b*c*
d + 27*a*d^2)/(27*d^2) - ((c^2 - 3*b*d)*x)/(3*d) + d*x^3, x]^p, x], x, x +
c/(3*d)] /; NeQ[c, 0]] /; FreeQ[p, x] && PolyQ[P3, x, 3]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{\left(3ab+3b^2x+3bcx^2+c^2x^3\right)^3}\,dx = \text{Subst}\left(\int \frac{1}{\left(b\left(3a-\frac{b^2}{c}\right)+c^2x^3\right)^3}\,dx, x, \frac{b}{c}+x\right)$$

$$= -\frac{c\left(\frac{b}{c}+x\right)}{6b\left(b^2-3ac\right)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)^2} - \frac{(5c)\,\text{Subst}\left(\int \frac{1}{\left(b\left(3a-\frac{b^2}{c}\right)+c^2x^3\right)^2}\,dx,\right.}{6b\left(b^2-3ac\right)}$$

$$= -\frac{c\left(\frac{b}{c}+x\right)}{6b\left(b^2-3ac\right)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)^2} + \frac{5c(b+cx)}{18b^2\left(b^2-3ac\right)^2\left(3ab+3b^2x+3\right.}$$

$$= -\frac{c\left(\frac{b}{c}+x\right)}{6b\left(b^2-3ac\right)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)^2} + \frac{5c(b+cx)}{18b^2\left(b^2-3ac\right)^2\left(3ab+3b^2x+3\right.}$$

$$= -\frac{c\left(\frac{b}{c}+x\right)}{6b\left(b^2-3ac\right)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)^2} + \frac{5c(b+cx)}{18b^2\left(b^2-3ac\right)^2\left(3ab+3b^2x+3\right.}$$

$$= -\frac{c\left(\frac{b}{c}+x\right)}{6b\left(b^2-3ac\right)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)^2} + \frac{5c(b+cx)}{18b^2\left(b^2-3ac\right)^2\left(3ab+3b^2x+3\right.}$$

$$= -\frac{c\left(\frac{b}{c}+x\right)}{6b\left(b^2-3ac\right)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)^2} + \frac{5c(b+cx)}{18b^2\left(b^2-3ac\right)^2\left(3ab+3b^2x+3\right.}$$

Mathematica [C] time = 0.079559, size = 149, normalized size = 0.49

$$\frac{10c^2\text{RootSum}\left[3\#1^2bc+\#1^3c^2+3\#1b^2+3ab\&,\frac{\log(x-\#1)}{\#1^2c^2+2\#1bc+b^2}\&\right]-\frac{3(b+cx)\left(-3bc\left(8a+5cx^2\right)-15b^2cx+3b^3-5c^3x^3\right)}{\left(3ab+x\left(3b^2+3bcx+c^2x^2\right)\right)^2}}{54\left(b^3-3abc\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^(-3),x]
```

```
[Out] ((-3*(b + c*x)*(3*b^3 - 15*b^2*c*x - 5*c^3*x^3 - 3*b*c*(8*a + 5*c*x^2)))/(3
*a*b + x*(3*b^2 + 3*b*c*x + c^2*x^2))^2 + 10*c^2*RootSum[3*a*b + 3*b^2*#1 +
 3*b*c*#1^2 + c^2*#1^3 & , Log[x - #1]/(b^2 + 2*b*c*#1 + c^2*#1^2) & ])/(54
*(b^3 - 3*a*b*c)^2)
```

Maple [C] time = 0.014, size = 276, normalized size = 0.9

$$\frac{1}{\left(c^2x^3+3\,bcx^2+3\,b^2x+3\,ab\right)^2}\left(\frac{5\,c^4x^4}{18\,b^2\left(9\,a^2c^2-6\,ab^2c+b^4\right)}+\frac{10\,c^3x^3}{9\,b\left(9\,a^2c^2-6\,ab^2c+b^4\right)}+\frac{5\,c^2x^2}{27\,a^2c^2-18\,ab^2c+3\,b^4}+\frac{}{3\,b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^3,x)
```

```
[Out] (5/18*c^4/b^2/(9*a^2*c^2-6*a*b^2*c+b^4)*x^4+10/9/b*c^3/(9*a^2*c^2-6*a*b^2*c
+b^4)*x^3+5/3*c^2/(9*a^2*c^2-6*a*b^2*c+b^4)*x^2+2/3/b*(2*a*c+b^2)*c/(9*a^2*
c^2-6*a*b^2*c+b^4)*x+1/6*(8*a*c-b^2)/(9*a^2*c^2-6*a*b^2*c+b^4))/(c^2*x^3+3*
b*c*x^2+3*b^2*x+3*a*b)^2+5/27*c^2/b^2/(9*a^2*c^2-6*a*b^2*c+b^4)*sum(1/(_R^2
*c^2+2*_R*b*c+b^2)*ln(x-_R),_R=RootOf(_Z^3*c^2+3*_Z^2*b*c+3*_Z*b^2+3*a*b))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\frac{5}{6}\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}cx+\sqrt{3}b-\sqrt{3}\left(-b^3+3\,abc\right)^{\frac{1}{3}}}{cx+b+\left(-b^3+3\,abc\right)^{\frac{1}{3}}}\right)}{\left(b^6-6\,ab^4c+9\,a^2b^2c^2\right)^{\frac{1}{3}}}-\frac{\log\left(\left(\sqrt{3}cx+\sqrt{3}b-\sqrt{3}\left(-b^3+3\,abc\right)^{\frac{1}{3}}\right)^2+\left(cx+b+\left(-b^3+3\,abc\right)^{\frac{1}{3}}\right)^2\right)}{\left(b^6-6\,ab^4c+9\,a^2b^2c^2\right)^{\frac{1}{3}}}+\frac{2\,\log\left(\left|cx+b+\left(-b^3+3\,abc\right)^{\frac{1}{3}}\right|\right)}{\left(b^6-6\,ab^4c+9\,a^2b^2c^2\right)^{\frac{1}{3}}}\right)c^2}{9\left(b^6-6\,ab^4c+9\,a^2b^2c^2\right)}+\frac{}{18\,(9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^3,x, algorithm="maxima")
```

```
[Out] 5/9*c^2*integrate(1/(c^2*x^3 + 3*b*c*x^2 + 3*b^2*x + 3*a*b), x)/(b^6 - 6*a*
b^4*c + 9*a^2*b^2*c^2) + 1/18*(5*c^4*x^4 + 20*b*c^3*x^3 + 30*b^2*c^2*x^2 -
3*b^4 + 24*a*b^2*c + 12*(b^3*c + 2*a*b*c^2)*x)/(9*a^2*b^8 - 54*a^3*b^6*c +
81*a^4*b^4*c^2 + (b^6*c^4 - 6*a*b^4*c^5 + 9*a^2*b^2*c^6)*x^6 + 6*(b^7*c^3 -
 6*a*b^5*c^4 + 9*a^2*b^3*c^5)*x^5 + 15*(b^8*c^2 - 6*a*b^6*c^3 + 9*a^2*b^4*c
^4)*x^4 + 6*(3*b^9*c - 17*a*b^7*c^2 + 21*a^2*b^5*c^3 + 9*a^3*b^3*c^4)*x^3 +
 9*(b^10 - 4*a*b^8*c - 3*a^2*b^6*c^2 + 18*a^3*b^4*c^3)*x^2 + 18*(a*b^9 - 6*
a^2*b^7*c + 9*a^3*b^5*c^2)*x)
```

Fricas [B] time = 1.46634, size = 2731, normalized size = 8.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^3,x, algorithm="fricas")
```

```
[Out] -1/54*(9*b^10 - 126*a*b^8*c + 513*a^2*b^6*c^2 - 648*a^3*b^4*c^3 - 15*(b^6*c
^4 - 6*a*b^4*c^5 + 9*a^2*b^2*c^6)*x^4 - 60*(b^7*c^3 - 6*a*b^5*c^4 + 9*a^2*b
^3*c^5)*x^3 - 90*(b^8*c^2 - 6*a*b^6*c^3 + 9*a^2*b^4*c^4)*x^2 + 10*sqrt(3)*(
9*a^2*b^5*c^2 - 27*a^3*b^3*c^3 + (b^3*c^6 - 3*a*b*c^7)*x^6 + 6*(b^4*c^5 - 3
*a*b^2*c^6)*x^5 + 15*(b^5*c^4 - 3*a*b^3*c^5)*x^4 + 6*(3*b^6*c^3 - 8*a*b^4*c
^4 - 3*a^2*b^2*c^5)*x^3 + 9*(b^7*c^2 - a*b^5*c^3 - 6*a^2*b^3*c^4)*x^2 + 18*
(a*b^6*c^2 - 3*a^2*b^4*c^3)*x)*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(1/6)*arct
an(1/3*(2*sqrt(3)*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(2/3)*(c*x + b) + sqrt(
3)*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(1/3)*(b^3 - 3*a*b*c))/(b^6 - 6*a*b^4*
c + 9*a^2*b^2*c^2)^(5/6)) + 5*(c^6*x^6 + 6*b*c^5*x^5 + 15*b^2*c^4*x^4 + 18*
a*b^3*c^2*x + 9*a^2*b^2*c^2 + 6*(3*b^3*c^3 + a*b*c^4)*x^3 + 9*(b^4*c^2 + 2*
a*b^2*c^3)*x^2)*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(2/3)*log(-b^5 + 3*a*b^3*
c - (b^3*c^2 - 3*a*b*c^3)*x^2 - 2*(b^4*c - 3*a*b^2*c^2)*x - (b^6 - 6*a*b^4*
c + 9*a^2*b^2*c^2)^(2/3)*(c*x + b) - (b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(1/3
)*(b^3 - 3*a*b*c)) - 10*(c^6*x^6 + 6*b*c^5*x^5 + 15*b^2*c^4*x^4 + 18*a*b^3*
c^2*x + 9*a^2*b^2*c^2 + 6*(3*b^3*c^3 + a*b*c^4)*x^3 + 9*(b^4*c^2 + 2*a*b^2*
c^3)*x^2)*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(2/3)*log(-b^4 + 3*a*b^2*c - (b
^3*c - 3*a*b*c^2)*x + (b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(2/3)) - 36*(b^9*c
- 4*a*b^7*c^2 - 3*a^2*b^5*c^3 + 18*a^3*b^3*c^4)*x)/(9*a^2*b^14 - 108*a^3*b^
12*c + 486*a^4*b^10*c^2 - 972*a^5*b^8*c^3 + 729*a^6*b^6*c^4 + (b^12*c^4 - 1
2*a*b^10*c^5 + 54*a^2*b^8*c^6 - 108*a^3*b^6*c^7 + 81*a^4*b^4*c^8)*x^6 + 6*(
```

```
b^13*c^3 - 12*a*b^11*c^4 + 54*a^2*b^9*c^5 - 108*a^3*b^7*c^6 + 81*a^4*b^5*c^
7)*x^5 + 15*(b^14*c^2 - 12*a*b^12*c^3 + 54*a^2*b^10*c^4 - 108*a^3*b^8*c^5 +
 81*a^4*b^6*c^6)*x^4 + 6*(3*b^15*c - 35*a*b^13*c^2 + 150*a^2*b^11*c^3 - 270
*a^3*b^9*c^4 + 135*a^4*b^7*c^5 + 81*a^5*b^5*c^6)*x^3 + 9*(b^16 - 10*a*b^14*
c + 30*a^2*b^12*c^2 - 135*a^4*b^8*c^4 + 162*a^5*b^6*c^5)*x^2 + 18*(a*b^15 -
 12*a^2*b^13*c + 54*a^3*b^11*c^2 - 108*a^4*b^9*c^3 + 81*a^5*b^7*c^4)*x)
```

Sympy [A] time = 5.97428, size = 474, normalized size = 1.55

$$\frac{24a}{1458a^4b^4c^2 - 972a^3b^6c + 162a^2b^8 + x^6\left(162a^2b^2c^6 - 108ab^4c^5 + 18b^6c^4\right) + x^5\left(972a^2b^3c^5 - 648ab^5c^4 + 108b^7c^3\right) + x^4\left(24\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c**2*x**3+3*b*c*x**2+3*b**2*x+3*a*b)**3,x)
```

```
[Out] (24*a*b**2*c - 3*b**4 + 30*b**2*c**2*x**2 + 20*b*c**3*x**3 + 5*c**4*x**4 +
x*(24*a*b*c**2 + 12*b**3*c))/(1458*a**4*b**4*c**2 - 972*a**3*b**6*c + 162*a
**2*b**8 + x**6*(162*a**2*b**2*c**6 - 108*a*b**4*c**5 + 18*b**6*c**4) + x**
5*(972*a**2*b**3*c**5 - 648*a*b**5*c**4 + 108*b**7*c**3) + x**4*(2430*a**2*
b**4*c**4 - 1620*a*b**6*c**3 + 270*b**8*c**2) + x**3*(972*a**3*b**3*c**4 +
2268*a**2*b**5*c**3 - 1836*a*b**7*c**2 + 324*b**9*c) + x**2*(2916*a**3*b**4
*c**3 - 486*a**2*b**6*c**2 - 648*a*b**8*c + 162*b**10) + x*(2916*a**3*b**5*
c**2 - 1944*a**2*b**7*c + 324*a*b**9)) + RootSum(_t**3*(129140163*a**8*b**8
*c**8 - 344373768*a**7*b**10*c**7 + 401769396*a**6*b**12*c**6 - 267846264*a
**5*b**14*c**5 + 111602610*a**4*b**16*c**4 - 29760696*a**3*b**18*c**3 + 496
0116*a**2*b**20*c**2 - 472392*a*b**22*c + 19683*b**24) - 125*c**6, Lambda(_
t, _t*log(x + (729*_t*a**3*b**3*c**3 - 729*_t*a**2*b**5*c**2 + 243*_t*a*b**
7*c - 27*_t*b**9 + 5*b*c**2)/(5*c**3))))
```

Giac [B] time = 1.28468, size = 817, normalized size = 2.68

$$\frac{5}{27}\sqrt{3}\left(\frac{c^6}{b^{24} - 24\,ab^{22}c + 252\,a^2b^{20}c^2 - 1512\,a^3b^{18}c^3 + 5670\,a^4b^{16}c^4 - 13608\,a^5b^{14}c^5 + 20412\,a^6b^{12}c^6 - 17496\,a^7b^{10}c^7 + 65}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^3,x, algorithm="giac")
```

```
[Out] 5/27*sqrt(3)*(c^6/(b^24 - 24*a*b^22*c + 252*a^2*b^20*c^2 - 1512*a^3*b^18*c^
3 + 5670*a^4*b^16*c^4 - 13608*a^5*b^14*c^5 + 20412*a^6*b^12*c^6 - 17496*a^7
*b^10*c^7 + 6561*a^8*b^8*c^8))^(1/3)*arctan((sqrt(3)*c*x + sqrt(3)*b - sqrt
(3)*(-b^3 + 3*a*b*c)^(1/3))/(c*x + b + (-b^3 + 3*a*b*c)^(1/3))) - 5/54*(c^6
/(b^24 - 24*a*b^22*c + 252*a^2*b^20*c^2 - 1512*a^3*b^18*c^3 + 5670*a^4*b^16
*c^4 - 13608*a^5*b^14*c^5 + 20412*a^6*b^12*c^6 - 17496*a^7*b^10*c^7 + 6561*
a^8*b^8*c^8))^(1/3)*log((sqrt(3)*c*x + sqrt(3)*b - sqrt(3)*(-b^3 + 3*a*b*c)
^(1/3))^2 + (c*x + b + (-b^3 + 3*a*b*c)^(1/3))^2) + 5/27*(c^6/(b^24 - 24*a*
b^22*c + 252*a^2*b^20*c^2 - 1512*a^3*b^18*c^3 + 5670*a^4*b^16*c^4 - 13608*a
^5*b^14*c^5 + 20412*a^6*b^12*c^6 - 17496*a^7*b^10*c^7 + 6561*a^8*b^8*c^8))^
(1/3)*log(abs(9*b^7 - 54*a*b^5*c + 81*a^2*b^3*c^2 + 9*(b^6*c - 6*a*b^4*c^2
+ 9*a^2*b^2*c^3)*x + 9*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)*(-b^3 + 3*a*b*c)^(
1/3))) + 1/18*(5*c^4*x^4 + 20*b*c^3*x^3 + 30*b^2*c^2*x^2 + 12*b^3*c*x + 24*
a*b*c^2*x - 3*b^4 + 24*a*b^2*c)/((b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)*(c^2*x^3
 + 3*b*c*x^2 + 3*b^2*x + 3*a*b)^2)
```

3.15 $\int \left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right.$

Optimal. Leaf size=361

$$\frac{3df(a+bx)^8\left(5a^2d^2f^2-5abdf(cf+de)+b^2\left(c^2f^2+3cdef+d^2e^2\right)\right)}{8b^7}+\frac{(a+bx)^7(-2adf+bcf+bde)\left(10a^2d^2f^2-10ab\right.}{7b^7}$$

```
[Out] ((b*c - a*d)^3*(b*e - a*f)^3*(a + b*x)^4)/(4*b^7) + (3*(b*c - a*d)^2*(b*e -
 a*f)^2*(b*d*e + b*c*f - 2*a*d*f)*(a + b*x)^5)/(5*b^7) + ((b*c - a*d)*(b*e
- a*f)*(5*a^2*d^2*f^2 - 5*a*b*d*f*(d*e + c*f) + b^2*(d^2*e^2 + 3*c*d*e*f +
c^2*f^2))*(a + b*x)^6)/(2*b^7) + ((b*d*e + b*c*f - 2*a*d*f)*(10*a^2*d^2*f^2
 - 10*a*b*d*f*(d*e + c*f) + b^2*(d^2*e^2 + 8*c*d*e*f + c^2*f^2))*(a + b*x)^
7)/(7*b^7) + (3*d*f*(5*a^2*d^2*f^2 - 5*a*b*d*f*(d*e + c*f) + b^2*(d^2*e^2 +
 3*c*d*e*f + c^2*f^2))*(a + b*x)^8)/(8*b^7) + (d^2*f^2*(b*d*e + b*c*f - 2*a
*d*f)*(a + b*x)^9)/(3*b^7) + (d^3*f^3*(a + b*x)^10)/(10*b^7)
```

Rubi [A] time = 0.658437, antiderivative size = 361, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {2059, 88}

$$\frac{3df(a+bx)^8\left(5a^2d^2f^2-5abdf(cf+de)+b^2\left(c^2f^2+3cdef+d^2e^2\right)\right)}{8b^7}+\frac{(a+bx)^7(-2adf+bcf+bde)\left(10a^2d^2f^2-10ab\right.}{7b^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2 + b*d*
f*x^3)^3,x]
```


```
[Out] ((b*c - a*d)^3*(b*e - a*f)^3*(a + b*x)^4)/(4*b^7) + (3*(b*c - a*d)^2*(b*e -
 a*f)^2*(b*d*e + b*c*f - 2*a*d*f)*(a + b*x)^5)/(5*b^7) + ((b*c - a*d)*(b*e
- a*f)*(5*a^2*d^2*f^2 - 5*a*b*d*f*(d*e + c*f) + b^2*(d^2*e^2 + 3*c*d*e*f +
c^2*f^2))*(a + b*x)^6)/(2*b^7) + ((b*d*e + b*c*f - 2*a*d*f)*(10*a^2*d^2*f^2
 - 10*a*b*d*f*(d*e + c*f) + b^2*(d^2*e^2 + 8*c*d*e*f + c^2*f^2))*(a + b*x)^
7)/(7*b^7) + (3*d*f*(5*a^2*d^2*f^2 - 5*a*b*d*f*(d*e + c*f) + b^2*(d^2*e^2 +
 3*c*d*e*f + c^2*f^2))*(a + b*x)^8)/(8*b^7) + (d^2*f^2*(b*d*e + b*c*f - 2*a
*d*f)*(a + b*x)^9)/(3*b^7) + (d^3*f^3*(a + b*x)^10)/(10*b^7)
```


Rule 2059

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[u^p, x] /;  !SumQ[Non
freeFactors[u, x]]] /; PolyQ[P, x] && IntegerQ[p]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \left(ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 + bdfx^3\right)^3 dx = \int (a + bx)^3(c + dx)^3(e + fx)^3 dx$$

$$= \int \left(\frac{(bc - ad)^3(be - af)^3(a + bx)^3}{b^6} + \frac{3(bc - ad)^2(be\ldots}{\ldots}\right.$$

$$= \frac{(bc - ad)^3(be - af)^3(a + bx)^4}{4b^7} + \frac{3(bc - ad)^2(be - a\ldots}{\ldots}$$

Mathematica [A] time = 0.204866, size = 653, normalized size = 1.81

$$\frac{3}{8}bdfx^8\left(a^2d^2f^2 + 3abdf(cf + de) + b^2\left(c^2f^2 + 3cdef + d^2e^2\right)\right) + \frac{1}{7}x^7\left(9a^2bd^2f^2(cf + de) + a^3d^3f^3 + 9ab^2df\left(c^2f^2 + 3c\ldots\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2
+ b*d*f*x^3)^3,x]
```

```
[Out] a^3*c^3*e^3*x + (3*a^2*c^2*e^2*(b*c*e + a*d*e + a*c*f)*x^2)/2 + a*c*e*(b^2*
c^2*e^2 + 3*a*b*c*e*(d*e + c*f) + a^2*(d^2*e^2 + 3*c*d*e*f + c^2*f^2))*x^3
+ ((b^3*c^3*e^3 + 9*a*b^2*c^2*e^2*(d*e + c*f) + 9*a^2*b*c*e*(d^2*e^2 + 3*c*
d*e*f + c^2*f^2) + a^3*(d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 + c^3*f^3))
*x^4)/4 + (3*(b^3*c^2*e^2*(d*e + c*f) + 3*a*b^2*c*e*(d^2*e^2 + 3*c*d*e*f +
c^2*f^2) + a^3*d*f*(d^2*e^2 + 3*c*d*e*f + c^2*f^2) + a^2*b*(d^3*e^3 + 9*c*d
^2*e^2*f + 9*c^2*d*e*f^2 + c^3*f^3))*x^5)/5 + ((a^3*d^2*f^2*(d*e + c*f) + b
^3*c*e*(d^2*e^2 + 3*c*d*e*f + c^2*f^2) + 3*a^2*b*d*f*(d^2*e^2 + 3*c*d*e*f +
 c^2*f^2) + a*b^2*(d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 + c^3*f^3))*x^6)
/2 + ((a^3*d^3*f^3 + 9*a^2*b*d^2*f^2*(d*e + c*f) + 9*a*b^2*d*f*(d^2*e^2 + 3
*c*d*e*f + c^2*f^2) + b^3*(d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 + c^3*f^
3))*x^7)/7 + (3*b*d*f*(a^2*d^2*f^2 + 3*a*b*d*f*(d*e + c*f) + b^2*(d^2*e^2 +
 3*c*d*e*f + c^2*f^2))*x^8)/8 + (b^2*d^2*f^2*(b*d*e + b*c*f + a*d*f)*x^9)/3
 + (b^3*d^3*f^3*x^10)/10
```

Maple [B] time = 0.001, size = 861, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^3,x)
```

```
[Out] 1/10*b^3*d^3*f^3*x^10+1/3*(a*d*f+b*c*f+b*d*e)*b^2*d^2*f^2*x^9+1/8*((a*c*f+a
*d*e+b*c*e)*b^2*d^2*f^2+2*(a*d*f+b*c*f+b*d*e)^2*b*d*f+b*d*f*(2*(a*c*f+a*d*e
+b*c*e)*b*d*f+(a*d*f+b*c*f+b*d*e)^2))*x^8+1/7*(a*c*e*b^2*d^2*f^2+2*(a*c*f+a
*d*e+b*c*e)*(a*d*f+b*c*f+b*d*e)*b*d*f+(a*d*f+b*c*f+b*d*e)*(2*(a*c*f+a*d*e+b
*c*e)*b*d*f+(a*d*f+b*c*f+b*d*e)^2)+b*d*f*(2*a*c*e*b*d*f+2*(a*c*f+a*d*e+b*c*
e)*(a*d*f+b*c*f+b*d*e)))*x^7+1/6*(2*a*c*e*(a*d*f+b*c*f+b*d*e)*b*d*f+(a*c*f+
a*d*e+b*c*e)*(2*(a*c*f+a*d*e+b*c*e)*b*d*f+(a*d*f+b*c*f+b*d*e)^2)+(a*d*f+b*c
*f+b*d*e)*(2*a*c*e*b*d*f+2*(a*c*f+a*d*e+b*c*e)*(a*d*f+b*c*f+b*d*e))+b*d*f*(
2*a*c*e*(a*d*f+b*c*f+b*d*e)+(a*c*f+a*d*e+b*c*e)^2))*x^6+1/5*(a*c*e*(2*(a*c*
f+a*d*e+b*c*e)*b*d*f+(a*d*f+b*c*f+b*d*e)^2)+(a*c*f+a*d*e+b*c*e)*(2*a*c*e*b*
d*f+2*(a*c*f+a*d*e+b*c*e)*(a*d*f+b*c*f+b*d*e))+(a*d*f+b*c*f+b*d*e)*(2*a*c*e
*(a*d*f+b*c*f+b*d*e)+(a*c*f+a*d*e+b*c*e)^2)+2*b*d*f*a*c*e*(a*c*f+a*d*e+b*c*
e))*x^5+1/4*(a*c*e*(2*a*c*e*b*d*f+2*(a*c*f+a*d*e+b*c*e)*(a*d*f+b*c*f+b*d*e)
)+(a*c*f+a*d*e+b*c*e)*(2*a*c*e*(a*d*f+b*c*f+b*d*e)+(a*c*f+a*d*e+b*c*e)^2)+2
*(a*d*f+b*c*f+b*d*e)*a*c*e*(a*c*f+a*d*e+b*c*e)+b*d*f*a^2*c^2*e^2)*x^4+1/3*(
a*c*e*(2*a*c*e*(a*d*f+b*c*f+b*d*e)+(a*c*f+a*d*e+b*c*e)^2)+2*(a*c*f+a*d*e+b*
c*e)^2*a*c*e+(a*d*f+b*c*f+b*d*e)*a^2*c^2*e^2)*x^3+3/2*a^2*c^2*e^2*(a*c*f+a*
d*e+b*c*e)*x^2+a^3*c^3*e^3*x
```

Maxima [A] time = 1.20328, size = 622, normalized size = 1.72

$$\frac{1}{10}\,b^3d^3f^3x^{10}+\frac{1}{3}\left(bde+bcf+adf\right)b^2d^2f^2x^9+\frac{3}{8}\left(bde+bcf+adf\right)^2bdfx^8+a^3c^3e^3x+\frac{1}{7}\left(bde+bcf+adf\right)^3x^7+\frac{1}{4}\left(3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^3
,x, algorithm="maxima")
```

```
[Out] 1/10*b^3*d^3*f^3*x^10 + 1/3*(b*d*e + b*c*f + a*d*f)*b^2*d^2*f^2*x^9 + 3/8*(
b*d*e + b*c*f + a*d*f)^2*b*d*f*x^8 + a^3*c^3*e^3*x + 1/7*(b*d*e + b*c*f + a
*d*f)^3*x^7 + 1/4*(3*b*d*f*x^4 + 4*(b*d*e + b*c*f + a*d*f)*x^3 + 6*(b*c*e +
```

```
a*d*e + a*c*f)*x^2)*a^2*c^2*e^2 + 1/4*(b*c*e + a*d*e + a*c*f)^3*x^4 + 1/70
*(30*b^2*d^2*f^2*x^7 + 70*(b*d*e + b*c*f + a*d*f)*b*d*f*x^6 + 42*(b*d*e + b
*c*f + a*d*f)^2*x^5 + 70*(b*c*e + a*d*e + a*c*f)^2*x^3 + 21*(4*b*d*f*x^5 +
5*(b*d*e + (b*c + a*d)*f)*x^4)*(b*c*e + a*d*e + a*c*f))*a*c*e + 1/10*(5*b*d
*f*x^6 + 6*(b*d*e + (b*c + a*d)*f)*x^5)*(b*c*e + a*d*e + a*c*f)^2 + 1/56*(2
1*b^2*d^2*f^2*x^8 + 48*(b^2*d^2*e*f + (b^2*c*d + a*b*d^2)*f^2)*x^7 + 28*(b^
2*d^2*e^2 + 2*(b^2*c*d + a*b*d^2)*e*f + (b^2*c^2 + 2*a*b*c*d + a^2*d^2)*f^2
)*x^6)*(b*c*e + a*d*e + a*c*f)
```

Fricas [B] time = 1.12296, size = 2136, normalized size = 5.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^3
,x, algorithm="fricas")
```

```
[Out] 1/10*x^10*f^3*d^3*b^3 + 1/3*x^9*f^2*e*d^3*b^3 + 1/3*x^9*f^3*d^2*c*b^3 + 1/3
*x^9*f^3*d^3*b^2*a + 3/8*x^8*f*e^2*d^3*b^3 + 9/8*x^8*f^2*e*d^2*c*b^3 + 3/8*
x^8*f^3*d*c^2*b^3 + 9/8*x^8*f^2*e*d^3*b^2*a + 9/8*x^8*f^3*d^2*c*b^2*a + 3/8
*x^8*f^3*d^3*b*a^2 + 1/7*x^7*e^3*d^3*b^3 + 9/7*x^7*f*e^2*d^2*c*b^3 + 9/7*x^
7*f^2*e*d*c^2*b^3 + 1/7*x^7*f^3*c^3*b^3 + 9/7*x^7*f*e^2*d^3*b^2*a + 27/7*x^
7*f^2*e*d^2*c*b^2*a + 9/7*x^7*f^3*d*c^2*b^2*a + 9/7*x^7*f^2*e*d^3*b*a^2 + 9
/7*x^7*f^3*d^2*c*b*a^2 + 1/7*x^7*f^3*d^3*a^3 + 1/2*x^6*e^3*d^2*c*b^3 + 3/2*
x^6*f*e^2*d*c^2*b^3 + 1/2*x^6*f^2*e*c^3*b^3 + 1/2*x^6*e^3*d^3*b^2*a + 9/2*x
^6*f*e^2*d^2*c*b^2*a + 9/2*x^6*f^2*e*d*c^2*b^2*a + 1/2*x^6*f^3*c^3*b^2*a +
3/2*x^6*f*e^2*d^3*b*a^2 + 9/2*x^6*f^2*e*d^2*c*b*a^2 + 3/2*x^6*f^3*d*c^2*b*a
^2 + 1/2*x^6*f^2*e*d^3*a^3 + 1/2*x^6*f^3*d^2*c*a^3 + 3/5*x^5*e^3*d*c^2*b^3
+ 3/5*x^5*f*e^2*c^3*b^3 + 9/5*x^5*e^3*d^2*c*b^2*a + 27/5*x^5*f*e^2*d*c^2*b^
2*a + 9/5*x^5*f^2*e*c^3*b^2*a + 3/5*x^5*e^3*d^3*b*a^2 + 27/5*x^5*f*e^2*d^2*
c*b*a^2 + 27/5*x^5*f^2*e*d*c^2*b*a^2 + 3/5*x^5*f^3*c^3*b*a^2 + 3/5*x^5*f*e^
2*d^3*a^3 + 9/5*x^5*f^2*e*d^2*c*a^3 + 3/5*x^5*f^3*d*c^2*a^3 + 1/4*x^4*e^3*c
^3*b^3 + 9/4*x^4*e^3*d*c^2*b^2*a + 9/4*x^4*f*e^2*c^3*b^2*a + 9/4*x^4*e^3*d^
2*c*b*a^2 + 27/4*x^4*f*e^2*d*c^2*b*a^2 + 9/4*x^4*f^2*e*c^3*b*a^2 + 1/4*x^4*
e^3*d^3*a^3 + 9/4*x^4*f*e^2*d^2*c*a^3 + 9/4*x^4*f^2*e*d*c^2*a^3 + 1/4*x^4*f
^3*c^3*a^3 + x^3*e^3*c^3*b^2*a + 3*x^3*e^3*d*c^2*b*a^2 + 3*x^3*f*e^2*c^3*b*
a^2 + x^3*e^3*d^2*c*a^3 + 3*x^3*f*e^2*d*c^2*a^3 + x^3*f^2*e*c^3*a^3 + 3/2*x
^2*e^3*c^3*b*a^2 + 3/2*x^2*e^3*d*c^2*a^3 + 3/2*x^2*f*e^2*c^3*a^3 + x*e^3*c^
3*a^3
```

Sympy [B] time = 0.216434, size = 1018, normalized size = 2.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x**2+b*d*f*x**3)
**3,x)
```

```
[Out] a**3*c**3*e**3*x + b**3*d**3*f**3*x**10/10 + x**9*(a*b**2*d**3*f**3/3 + b**
3*c*d**2*f**3/3 + b**3*d**3*e*f**2/3) + x**8*(3*a**2*b*d**3*f**3/8 + 9*a*b*
*2*c*d**2*f**3/8 + 9*a*b**2*d**3*e*f**2/8 + 3*b**3*c**2*d*f**3/8 + 9*b**3*c
*d**2*e*f**2/8 + 3*b**3*d**3*e**2*f/8) + x**7*(a**3*d**3*f**3/7 + 9*a**2*b*
c*d**2*f**3/7 + 9*a**2*b*d**3*e*f**2/7 + 9*a*b**2*c**2*d*f**3/7 + 27*a*b**2
*c*d**2*e*f**2/7 + 9*a*b**2*d**3*e**2*f/7 + b**3*c**3*f**3/7 + 9*b**3*c**2*
d*e*f**2/7 + 9*b**3*c*d**2*e**2*f/7 + b**3*d**3*e**3/7) + x**6*(a**3*c*d**2
*f**3/2 + a**3*d**3*e*f**2/2 + 3*a**2*b*c**2*d*f**3/2 + 9*a**2*b*c*d**2*e*f
**2/2 + 3*a**2*b*d**3*e**2*f/2 + a*b**2*c**3*f**3/2 + 9*a*b**2*c**2*d*e*f**
2/2 + 9*a*b**2*c*d**2*e**2*f/2 + a*b**2*d**3*e**3/2 + b**3*c**3*e*f**2/2 +
3*b**3*c**2*d*e**2*f/2 + b**3*c*d**2*e**3/2) + x**5*(3*a**3*c**2*d*f**3/5 +
 9*a**3*c*d**2*e*f**2/5 + 3*a**3*d**3*e**2*f/5 + 3*a**2*b*c**3*f**3/5 + 27*
a**2*b*c**2*d*e*f**2/5 + 27*a**2*b*c*d**2*e**2*f/5 + 3*a**2*b*d**3*e**3/5 +
 9*a*b**2*c**3*e*f**2/5 + 27*a*b**2*c**2*d*e**2*f/5 + 9*a*b**2*c*d**2*e**3/
5 + 3*b**3*c**3*e**2*f/5 + 3*b**3*c**2*d*e**3/5) + x**4*(a**3*c**3*f**3/4 +
 9*a**3*c**2*d*e*f**2/4 + 9*a**3*c*d**2*e**2*f/4 + a**3*d**3*e**3/4 + 9*a**
2*b*c**3*e*f**2/4 + 27*a**2*b*c**2*d*e**2*f/4 + 9*a**2*b*c*d**2*e**3/4 + 9*
a*b**2*c**3*e**2*f/4 + 9*a*b**2*c**2*d*e**3/4 + b**3*c**3*e**3/4) + x**3*(a
**3*c**3*e*f**2 + 3*a**3*c**2*d*e**2*f + a**3*c*d**2*e**3 + 3*a**2*b*c**3*e
**2*f + 3*a**2*b*c**2*d*e**3 + a*b**2*c**3*e**3) + x**2*(3*a**3*c**3*e**2*f
/2 + 3*a**3*c**2*d*e**3/2 + 3*a**2*b*c**3*e**3/2)
```

Giac [B] time = 1.10761, size = 1311, normalized size = 3.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^3
,x, algorithm="giac")
```

```
[Out] 1/10*b^3*d^3*f^3*x^10 + 1/3*b^3*c*d^2*f^3*x^9 + 1/3*a*b^2*d^3*f^3*x^9 + 1/3
*b^3*d^3*f^2*x^9*e + 3/8*b^3*c^2*d*f^3*x^8 + 9/8*a*b^2*c*d^2*f^3*x^8 + 3/8*
```

```
a^2*b*d^3*f^3*x^8 + 9/8*b^3*c*d^2*f^2*x^8*e + 9/8*a*b^2*d^3*f^2*x^8*e + 1/7
*b^3*c^3*f^3*x^7 + 9/7*a*b^2*c^2*d*f^3*x^7 + 9/7*a^2*b*c*d^2*f^3*x^7 + 1/7*
a^3*d^3*f^3*x^7 + 3/8*b^3*d^3*f*x^8*e^2 + 9/7*b^3*c^2*d*f^2*x^7*e + 27/7*a*
b^2*c*d^2*f^2*x^7*e + 9/7*a^2*b*d^3*f^2*x^7*e + 1/2*a*b^2*c^3*f^3*x^6 + 3/2
*a^2*b*c^2*d*f^3*x^6 + 1/2*a^3*c*d^2*f^3*x^6 + 9/7*b^3*c*d^2*f*x^7*e^2 + 9/
7*a*b^2*d^3*f*x^7*e^2 + 1/2*b^3*c^3*f^2*x^6*e + 9/2*a*b^2*c^2*d*f^2*x^6*e +
 9/2*a^2*b*c*d^2*f^2*x^6*e + 1/2*a^3*d^3*f^2*x^6*e + 3/5*a^2*b*c^3*f^3*x^5
+ 3/5*a^3*c^2*d*f^3*x^5 + 1/7*b^3*d^3*x^7*e^3 + 3/2*b^3*c^2*d*f*x^6*e^2 + 9
/2*a*b^2*c*d^2*f*x^6*e^2 + 3/2*a^2*b*d^3*f*x^6*e^2 + 9/5*a*b^2*c^3*f^2*x^5*
e + 27/5*a^2*b*c^2*d*f^2*x^5*e + 9/5*a^3*c*d^2*f^2*x^5*e + 1/4*a^3*c^3*f^3*
x^4 + 1/2*b^3*c*d^2*x^6*e^3 + 1/2*a*b^2*d^3*x^6*e^3 + 3/5*b^3*c^3*f*x^5*e^2
 + 27/5*a*b^2*c^2*d*f*x^5*e^2 + 27/5*a^2*b*c*d^2*f*x^5*e^2 + 3/5*a^3*d^3*f*
x^5*e^2 + 9/4*a^2*b*c^3*f^2*x^4*e + 9/4*a^3*c^2*d*f^2*x^4*e + 3/5*b^3*c^2*d
*x^5*e^3 + 9/5*a*b^2*c*d^2*x^5*e^3 + 3/5*a^2*b*d^3*x^5*e^3 + 9/4*a*b^2*c^3*
f*x^4*e^2 + 27/4*a^2*b*c^2*d*f*x^4*e^2 + 9/4*a^3*c*d^2*f*x^4*e^2 + a^3*c^3*
f^2*x^3*e + 1/4*b^3*c^3*x^4*e^3 + 9/4*a*b^2*c^2*d*x^4*e^3 + 9/4*a^2*b*c*d^2
*x^4*e^3 + 1/4*a^3*d^3*x^4*e^3 + 3*a^2*b*c^3*f*x^3*e^2 + 3*a^3*c^2*d*f*x^3*
e^2 + a*b^2*c^3*x^3*e^3 + 3*a^2*b*c^2*d*x^3*e^3 + a^3*c*d^2*x^3*e^3 + 3/2*a
^3*c^3*f*x^2*e^2 + 3/2*a^2*b*c^3*x^2*e^3 + 3/2*a^3*c^2*d*x^2*e^3 + a^3*c^3*
x*e^3
```

3.16 $\int \left(ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 + bdfx^3\right.$

Optimal. Leaf size=193

$$\frac{(a+bx)^5\left(6a^2d^2f^2-6abdf(cf+de)+b^2\left(c^2f^2+4cdef+d^2e^2\right)\right)}{5b^5}+\frac{df(a+bx)^6(-2adf+bcf+bde)}{3b^5}+\frac{(a+bx)^4(bc-a}{}$$

```
[Out] ((b*c - a*d)^2*(b*e - a*f)^2*(a + b*x)^3)/(3*b^5) + ((b*c - a*d)*(b*e - a*f
)*(b*d*e + b*c*f - 2*a*d*f)*(a + b*x)^4)/(2*b^5) + ((6*a^2*d^2*f^2 - 6*a*b*
d*f*(d*e + c*f) + b^2*(d^2*e^2 + 4*c*d*e*f + c^2*f^2))*(a + b*x)^5)/(5*b^5)
 + (d*f*(b*d*e + b*c*f - 2*a*d*f)*(a + b*x)^6)/(3*b^5) + (d^2*f^2*(a + b*x)
^7)/(7*b^5)
```

Rubi [A] time = 0.231449, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {2059, 88}

$$\frac{(a+bx)^5\left(6a^2d^2f^2-6abdf(cf+de)+b^2\left(c^2f^2+4cdef+d^2e^2\right)\right)}{5b^5}+\frac{df(a+bx)^6(-2adf+bcf+bde)}{3b^5}+\frac{(a+bx)^4(bc-a}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2 + b*d*
f*x^3)^2,x]
```

```
[Out] ((b*c - a*d)^2*(b*e - a*f)^2*(a + b*x)^3)/(3*b^5) + ((b*c - a*d)*(b*e - a*f
)*(b*d*e + b*c*f - 2*a*d*f)*(a + b*x)^4)/(2*b^5) + ((6*a^2*d^2*f^2 - 6*a*b*
d*f*(d*e + c*f) + b^2*(d^2*e^2 + 4*c*d*e*f + c^2*f^2))*(a + b*x)^5)/(5*b^5)
 + (d*f*(b*d*e + b*c*f - 2*a*d*f)*(a + b*x)^6)/(3*b^5) + (d^2*f^2*(a + b*x)
^7)/(7*b^5)
```

Rule 2059

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[u^p, x] /;  !SumQ[Non
freeFactors[u, x]]] /; PolyQ[P, x] && IntegerQ[p]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
```

```
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \left(ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 + bdfx^3\right)^2 dx = \int (a+bx)^2(c+dx)^2(e+fx)^2 dx$$
$$= \int \left(\frac{(bc-ad)^2(be-af)^2(a+bx)^2}{b^4} + \frac{2(bc-ad)(be\cdot\right.$$
$$= \frac{(bc-ad)^2(be-af)^2(a+bx)^3}{3b^5} + \frac{(bc-ad)(be-af)}{}$$

Mathematica [A] time = 0.0775239, size = 241, normalized size = 1.25

$$\frac{1}{5}x^5\left(a^2d^2f^2 + 4abdf(cf+de) + b^2\left(c^2f^2 + 4cdef + d^2e^2\right)\right) + \frac{1}{2}x^4\left(a^2df(cf+de) + ab\left(c^2f^2 + 4cdef + d^2e^2\right) + b^2ce(cf\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2
+ b*d*f*x^3)^2,x]
```

```
[Out] a^2*c^2*e^2*x + a*c*e*(b*c*e + a*d*e + a*c*f)*x^2 + ((b^2*c^2*e^2 + 4*a*b*c
*e*(d*e + c*f) + a^2*(d^2*e^2 + 4*c*d*e*f + c^2*f^2))*x^3)/3 + ((b^2*c*e*(d
*e + c*f) + a^2*d*f*(d*e + c*f) + a*b*(d^2*e^2 + 4*c*d*e*f + c^2*f^2))*x^4)
/2 + ((a^2*d^2*f^2 + 4*a*b*d*f*(d*e + c*f) + b^2*(d^2*e^2 + 4*c*d*e*f + c^2
*f^2))*x^5)/5 + (b*d*f*(b*d*e + b*c*f + a*d*f)*x^6)/3 + (b^2*d^2*f^2*x^7)/7
```

Maple [A] time = 0.002, size = 188, normalized size = 1.

$$\frac{b^2d^2f^2x^7}{7} + \frac{\left(adf + bcf + bde\right)bdfx^6}{3} + \frac{\left(2\left(acf + ade + bce\right)bdf + \left(adf + bcf + bde\right)^2\right)x^5}{5} + \frac{\left(2\,acebdf + 2\left(acf + ad\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^2,x)
```

```
[Out] 1/7*b^2*d^2*f^2*x^7+1/3*(a*d*f+b*c*f+b*d*e)*b*d*f*x^6+1/5*(2*(a*c*f+a*d*e+b
*c*e)*b*d*f+(a*d*f+b*c*f+b*d*e)^2)*x^5+1/4*(2*a*c*e*b*d*f+2*(a*c*f+a*d*e+b*
```

```
c*e)*(a*d*f+b*c*f+b*d*e))*x^4+1/3*(2*a*c*e*(a*d*f+b*c*f+b*d*e)+(a*c*f+a*d*e
+b*c*e)^2)*x^3+a*c*e*(a*c*f+a*d*e+b*c*e)*x^2+a^2*c^2*e^2*x
```

Maxima [A] time = 1.09127, size = 243, normalized size = 1.26

$$\frac{1}{7}b^2d^2f^2x^7+\frac{1}{3}\left(bde+bcf+adf\right)bdfx^6+a^2c^2e^2x+\frac{1}{5}\left(bde+bcf+adf\right)^2x^5+\frac{1}{3}\left(bce+ade+acf\right)^2x^3+\frac{1}{6}\left(3\,bdfx^4+4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^2
,x, algorithm="maxima")
```

```
[Out] 1/7*b^2*d^2*f^2*x^7 + 1/3*(b*d*e + b*c*f + a*d*f)*b*d*f*x^6 + a^2*c^2*e^2*x
 + 1/5*(b*d*e + b*c*f + a*d*f)^2*x^5 + 1/3*(b*c*e + a*d*e + a*c*f)^2*x^3 +
1/6*(3*b*d*f*x^4 + 4*(b*d*e + b*c*f + a*d*f)*x^3 + 6*(b*c*e + a*d*e + a*c*f
)*x^2)*a*c*e + 1/10*(4*b*d*f*x^5 + 5*(b*d*e + (b*c + a*d)*f)*x^4)*(b*c*e +
a*d*e + a*c*f)
```

Fricas [A] time = 1.09973, size = 774, normalized size = 4.01

$$\frac{1}{7}x^7f^2d^2b^2+\frac{1}{3}x^6fed^2b^2+\frac{1}{3}x^6f^2dcb^2+\frac{1}{3}x^6f^2d^2ba+\frac{1}{5}x^5e^2d^2b^2+\frac{4}{5}x^5fedcb^2+\frac{1}{5}x^5f^2c^2b^2+\frac{4}{5}x^5fed^2ba+\frac{4}{5}x^5f^2dcba$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^2
,x, algorithm="fricas")
```

```
[Out] 1/7*x^7*f^2*d^2*b^2 + 1/3*x^6*f*e*d^2*b^2 + 1/3*x^6*f^2*d*c*b^2 + 1/3*x^6*f
^2*d^2*b*a + 1/5*x^5*e^2*d^2*b^2 + 4/5*x^5*f*e*d*c*b^2 + 1/5*x^5*f^2*c^2*b^
2 + 4/5*x^5*f*e*d^2*b*a + 4/5*x^5*f^2*d*c*b*a + 1/5*x^5*f^2*d^2*a^2 + 1/2*x
^4*e^2*d*c*b^2 + 1/2*x^4*f*e*c^2*b^2 + 1/2*x^4*e^2*d^2*b*a + 2*x^4*f*e*d*c*
b*a + 1/2*x^4*f^2*c^2*b*a + 1/2*x^4*f*e*d^2*a^2 + 1/2*x^4*f^2*d*c*a^2 + 1/3
*x^3*e^2*c^2*b^2 + 4/3*x^3*e^2*d*c*b*a + 4/3*x^3*f*e*c^2*b*a + 1/3*x^3*e^2*
d^2*a^2 + 4/3*x^3*f*e*d*c*a^2 + 1/3*x^3*f^2*c^2*a^2 + x^2*e^2*c^2*b*a + x^2
*e^2*d*c*a^2 + x^2*f*e*c^2*a^2 + x*e^2*c^2*a^2
```

Sympy [A] time = 0.123666, size = 345, normalized size = 1.79

$$a^2c^2e^2x + \frac{b^2d^2f^2x^7}{7} + x^6\left(\frac{abd^2f^2}{3} + \frac{b^2cdf^2}{3} + \frac{b^2d^2ef}{3}\right) + x^5\left(\frac{a^2d^2f^2}{5} + \frac{4abcdf^2}{5} + \frac{4abd^2ef}{5} + \frac{b^2c^2f^2}{5} + \frac{4b^2cdef}{5} + \frac{b^2}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x**2+b*d*f*x**3)
**2,x)
```

```
[Out] a**2*c**2*e**2*x + b**2*d**2*f**2*x**7/7 + x**6*(a*b*d**2*f**2/3 + b**2*c*d
*f**2/3 + b**2*d**2*e*f/3) + x**5*(a**2*d**2*f**2/5 + 4*a*b*c*d*f**2/5 + 4*
a*b*d**2*e*f/5 + b**2*c**2*f**2/5 + 4*b**2*c*d*e*f/5 + b**2*d**2*e**2/5) +
x**4*(a**2*c*d*f**2/2 + a**2*d**2*e*f/2 + a*b*c**2*f**2/2 + 2*a*b*c*d*e*f +
 a*b*d**2*e**2/2 + b**2*c**2*e*f/2 + b**2*c*d*e**2/2) + x**3*(a**2*c**2*f**
2/3 + 4*a**2*c*d*e*f/3 + a**2*d**2*e**2/3 + 4*a*b*c**2*e*f/3 + 4*a*b*c*d*e*
*2/3 + b**2*c**2*e**2/3) + x**2*(a**2*c**2*e*f + a**2*c*d*e**2 + a*b*c**2*e
**2)
```

Giac [A] time = 1.08288, size = 467, normalized size = 2.42

$$\frac{1}{7}b^2d^2f^2x^7 + \frac{1}{3}b^2cdf^2x^6 + \frac{1}{3}abd^2f^2x^6 + \frac{1}{3}b^2d^2fx^6e + \frac{1}{5}b^2c^2f^2x^5 + \frac{4}{5}abcdf^2x^5 + \frac{1}{5}a^2d^2f^2x^5 + \frac{4}{5}b^2cdfx^5e + \frac{4}{5}abd^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^2
,x, algorithm="giac")
```

```
[Out] 1/7*b^2*d^2*f^2*x^7 + 1/3*b^2*c*d*f^2*x^6 + 1/3*a*b*d^2*f^2*x^6 + 1/3*b^2*d
^2*f*x^6*e + 1/5*b^2*c^2*f^2*x^5 + 4/5*a*b*c*d*f^2*x^5 + 1/5*a^2*d^2*f^2*x^
5 + 4/5*b^2*c*d*f*x^5*e + 4/5*a*b*d^2*f*x^5*e + 1/2*a*b*c^2*f^2*x^4 + 1/2*a
^2*c*d*f^2*x^4 + 1/5*b^2*d^2*x^5*e^2 + 1/2*b^2*c^2*f*x^4*e + 2*a*b*c*d*f*x^
4*e + 1/2*a^2*d^2*f*x^4*e + 1/3*a^2*c^2*f^2*x^3 + 1/2*b^2*c*d*x^4*e^2 + 1/2
*a*b*d^2*x^4*e^2 + 4/3*a*b*c^2*f*x^3*e + 4/3*a^2*c*d*f*x^3*e + 1/3*b^2*c^2*
x^3*e^2 + 4/3*a*b*c*d*x^3*e^2 + 1/3*a^2*d^2*x^3*e^2 + a^2*c^2*f*x^2*e + a*b
*c^2*x^2*e^2 + a^2*c*d*x^2*e^2 + a^2*c^2*x*e^2
```

3.17 $\int \left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right.$

Optimal. Leaf size=56

$$\frac{1}{3}x^3(adf+bcf+bde)+\frac{1}{2}x^2(acf+ade+bce)+acex+\frac{1}{4}bdfx^4$$

```
[Out] a*c*e*x + ((b*c*e + a*d*e + a*c*f)*x^2)/2 + ((b*d*e + b*c*f + a*d*f)*x^3)/3
 + (b*d*f*x^4)/4
```

Rubi [A] time = 0.0164289, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\frac{1}{3}x^3(adf+bcf+bde)+\frac{1}{2}x^2(acf+ade+bce)+acex+\frac{1}{4}bdfx^4$$

Antiderivative was successfully verified.

```
[In] Int[a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2 + b*d*f
*x^3,x]
```

```
[Out] a*c*e*x + ((b*c*e + a*d*e + a*c*f)*x^2)/2 + ((b*d*e + b*c*f + a*d*f)*x^3)/3
 + (b*d*f*x^4)/4
```

Rubi steps

$$\int \left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right)\,dx=acex+\frac{1}{2}(bce+ade+acf)x^2+\frac{1}{3}(bde+bcf+adf)x^3$$

Mathematica [A] time = 0.0000704, size = 76, normalized size = 1.36

$$acex+\frac{1}{2}acfx^2+\frac{1}{2}adex^2+\frac{1}{3}adfx^3+\frac{1}{2}bcex^2+\frac{1}{3}bcfx^3+\frac{1}{3}bdex^3+\frac{1}{4}bdfx^4$$

Antiderivative was successfully verified.

```
[In] Integrate[a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2 +
 b*d*f*x^3,x]
```

```
[Out] a*c*e*x + (b*c*e*x^2)/2 + (a*d*e*x^2)/2 + (a*c*f*x^2)/2 + (b*d*e*x^3)/3 + (
b*c*f*x^3)/3 + (a*d*f*x^3)/3 + (b*d*f*x^4)/4
```

Maple [A] time = 0., size = 51, normalized size = 0.9

$$acex + \frac{\left(acf + ade + bce\right)x^2}{2} + \frac{\left(adf + bcf + bde\right)x^3}{3} + \frac{bdfx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3,x)
```

```
[Out] a*c*e*x+1/2*(a*c*f+a*d*e+b*c*e)*x^2+1/3*(a*d*f+b*c*f+b*d*e)*x^3+1/4*b*d*f*x
^4
```

Maxima [A] time = 1.22255, size = 68, normalized size = 1.21

$$\frac{1}{4}bdfx^4 + acex + \frac{1}{3}\left(bde + bcf + adf\right)x^3 + \frac{1}{2}\left(bce + ade + acf\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3,x,
algorithm="maxima")
```

```
[Out] 1/4*b*d*f*x^4 + a*c*e*x + 1/3*(b*d*e + b*c*f + a*d*f)*x^3 + 1/2*(b*c*e + a*
d*e + a*c*f)*x^2
```

Fricas [A] time = 1.08366, size = 163, normalized size = 2.91

$$\frac{1}{4}x^4fdb + \frac{1}{3}x^3edb + \frac{1}{3}x^3fcb + \frac{1}{3}x^3fda + \frac{1}{2}x^2ecb + \frac{1}{2}x^2eda + \frac{1}{2}x^2fca + xeca$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3,x,
algorithm="fricas")
```

```
[Out] 1/4*x^4*f*d*b + 1/3*x^3*e*d*b + 1/3*x^3*f*c*b + 1/3*x^3*f*d*a + 1/2*x^2*e*c
*b + 1/2*x^2*e*d*a + 1/2*x^2*f*c*a + x*e*c*a
```

Sympy [A] time = 0.067543, size = 63, normalized size = 1.12

$$acex + \frac{bdfx^4}{4} + x^3\left(\frac{adf}{3} + \frac{bcf}{3} + \frac{bde}{3}\right) + x^2\left(\frac{acf}{2} + \frac{ade}{2} + \frac{bce}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x**2+b*d*f*x**3,x
)
```

```
[Out] a*c*e*x + b*d*f*x**4/4 + x**3*(a*d*f/3 + b*c*f/3 + b*d*e/3) + x**2*(a*c*f/2
 + a*d*e/2 + b*c*e/2)
```

Giac [A] time = 1.06873, size = 73, normalized size = 1.3

$$\frac{1}{4}bdfx^4 + \frac{1}{3}\left(bcf + adf + bde\right)x^3 + acxe + \frac{1}{2}\left(acf + bce + ade\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3,x,
algorithm="giac")
```

```
[Out] 1/4*b*d*f*x^4 + 1/3*(b*c*f + a*d*f + b*d*e)*x^3 + a*c*x*e + 1/2*(a*c*f + b*
c*e + a*d*e)*x^2
```

3.18 $\int \frac{1}{ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3}\,dx$

Optimal. Leaf size=86

$$\frac{b\log(a+bx)}{(bc-ad)(be-af)}-\frac{d\log(c+dx)}{(bc-ad)(de-cf)}+\frac{f\log(e+fx)}{(be-af)(de-cf)}$$

```
[Out] (b*Log[a + b*x])/((b*c - a*d)*(b*e - a*f)) - (d*Log[c + d*x])/((b*c - a*d)*
(d*e - c*f)) + (f*Log[e + f*x])/((b*e - a*f)*(d*e - c*f))
```

Rubi [A] time = 0.0733537, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.022, Rules used = {2058}

$$\frac{b\log(a+bx)}{(bc-ad)(be-af)}-\frac{d\log(c+dx)}{(bc-ad)(de-cf)}+\frac{f\log(e+fx)}{(be-af)(de-cf)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2 + b*d*
f*x^3)^(-1),x]
```

```
[Out] (b*Log[a + b*x])/((b*c - a*d)*(b*e - a*f)) - (d*Log[c + d*x])/((b*c - a*d)*
(d*e - c*f)) + (f*Log[e + f*x])/((b*e - a*f)*(d*e - c*f))
```

Rule 2058

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rubi steps

$$\int \frac{1}{ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3}\,dx = \int \left(\frac{b^2}{(bc-ad)(be-af)(a+bx)}+\frac{d^2}{(bc-ad)(-de+cf)(\ldots}\right.$$
$$= \frac{b\log(a+bx)}{(bc-ad)(be-af)}-\frac{d\log(c+dx)}{(bc-ad)(de-cf)}+\frac{f\log(e+\ldots}{(be-af)(d\ldots}$$

Mathematica [A] time = 0.0476963, size = 80, normalized size = 0.93

$$\frac{b\log(a+bx)(cf-de)+d(be-af)\log(c+dx)+f(ad-bc)\log(e+fx)}{(bc-ad)(be-af)(cf-de)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2
+ b*d*f*x^3)^(-1),x]
```

```
[Out] (b*(-(d*e) + c*f)*Log[a + b*x] + d*(b*e - a*f)*Log[c + d*x] + (-(b*c) + a*d
)*f*Log[e + f*x])/((b*c - a*d)*(b*e - a*f)*(-(d*e) + c*f))
```

Maple [A] time = 0.008, size = 87, normalized size = 1.

$$\frac{b\ln\left(bx+a\right)}{\left(af-be\right)\left(ad-bc\right)}-\frac{d\ln\left(dx+c\right)}{\left(cf-de\right)\left(ad-bc\right)}+\frac{f\ln\left(fx+e\right)}{\left(cf-de\right)\left(af-be\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3),x)
```

```
[Out] b/(a*f-b*e)/(a*d-b*c)*ln(b*x+a)-d/(c*f-d*e)/(a*d-b*c)*ln(d*x+c)+f/(c*f-d*e)
/(a*f-b*e)*ln(f*x+e)
```

Maxima [A] time = 1.04476, size = 151, normalized size = 1.76

$$\frac{b\log\left(bx+a\right)}{\left(b^2c-abd\right)e-\left(abc-a^2d\right)f}-\frac{d\log\left(dx+c\right)}{\left(bcd-ad^2\right)e-\left(bc^2-acd\right)f}+\frac{f\log\left(fx+e\right)}{bde^2+acf^2-(bc+ad)ef}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)
,x, algorithm="maxima")
```

```
[Out] b*log(b*x + a)/((b^2*c - a*b*d)*e - (a*b*c - a^2*d)*f) - d*log(d*x + c)/((b
*c*d - a*d^2)*e - (b*c^2 - a*c*d)*f) + f*log(f*x + e)/(b*d*e^2 + a*c*f^2 -
(b*c + a*d)*e*f)
```

Fricas [A] time = 28.6856, size = 230, normalized size = 2.67

$$\frac{(bc-ad)f\log\left(fx+e\right)+\left(bde-bcf\right)\log\left(bx+a\right)-\left(bde-adf\right)\log\left(dx+c\right)}{\left(b^2cd-abd^2\right)e^2-\left(b^2c^2-a^2d^2\right)ef+\left(abc^2-a^2cd\right)f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)
,x, algorithm="fricas")
```

```
[Out] ((b*c - a*d)*f*log(f*x + e) + (b*d*e - b*c*f)*log(b*x + a) - (b*d*e - a*d*f
)*log(d*x + c))/((b^2*c*d - a*b*d^2)*e^2 - (b^2*c^2 - a^2*d^2)*e*f + (a*b*c
^2 - a^2*c*d)*f^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x**2+b*d*f*x**
3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{bdfx^3+ace+\left(bde+bcf+adf\right)x^2+\left(bce+ade+acf\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)
,x, algorithm="giac")
```

```
[Out] integrate(1/(b*d*f*x^3 + a*c*e + (b*d*e + b*c*f + a*d*f)*x^2 + (b*c*e + a*d
*e + a*c*f)*x), x)
```

3.19 $\int \frac{1}{\left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right)^2}\,dx$

Optimal. Leaf size=234

$$-\frac{b^3}{(a+bx)(bc-ad)^2(be-af)^2}-\frac{2b^3\log(a+bx)(-2adf+bcf+bde)}{(bc-ad)^3(be-af)^3}-\frac{d^3}{(c+dx)(bc-ad)^2(de-cf)^2}+\frac{2d^3\log(c+dx)(a\ldots}{(bc-ad)^3(\ldots}$$

```
[Out] -(b^3/((b*c - a*d)^2*(b*e - a*f)^2*(a + b*x))) - d^3/((b*c - a*d)^2*(d*e -
c*f)^2*(c + d*x)) - f^3/((b*e - a*f)^2*(d*e - c*f)^2*(e + f*x)) - (2*b^3*(b
*d*e + b*c*f - 2*a*d*f)*Log[a + b*x])/((b*c - a*d)^3*(b*e - a*f)^3) + (2*d^
3*(b*d*e - 2*b*c*f + a*d*f)*Log[c + d*x])/((b*c - a*d)^3*(d*e - c*f)^3) + (
2*f^3*(2*b*d*e - b*c*f - a*d*f)*Log[e + f*x])/((b*e - a*f)^3*(d*e - c*f)^3)
```

Rubi [A] time = 0.405178, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.022, Rules used = {2058}

$$-\frac{b^3}{(a+bx)(bc-ad)^2(be-af)^2}-\frac{2b^3\log(a+bx)(-2adf+bcf+bde)}{(bc-ad)^3(be-af)^3}-\frac{d^3}{(c+dx)(bc-ad)^2(de-cf)^2}+\frac{2d^3\log(c+dx)(a\ldots}{(bc-ad)^3(\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2 + b*d*
f*x^3)^(-2),x]
```

```
[Out] -(b^3/((b*c - a*d)^2*(b*e - a*f)^2*(a + b*x))) - d^3/((b*c - a*d)^2*(d*e -
c*f)^2*(c + d*x)) - f^3/((b*e - a*f)^2*(d*e - c*f)^2*(e + f*x)) - (2*b^3*(b
*d*e + b*c*f - 2*a*d*f)*Log[a + b*x])/((b*c - a*d)^3*(b*e - a*f)^3) + (2*d^
3*(b*d*e - 2*b*c*f + a*d*f)*Log[c + d*x])/((b*c - a*d)^3*(d*e - c*f)^3) + (
2*f^3*(2*b*d*e - b*c*f - a*d*f)*Log[e + f*x])/((b*e - a*f)^3*(d*e - c*f)^3)
```

Rule 2058

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rubi steps

$$\int \frac{1}{\left(ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 + bdfx^3\right)^2} dx = \int \left(\frac{b^4}{(bc-ad)^2(be-af)^2(a+bx)^2} - \frac{2b^4(bde+b}{(bc-ad)^3(be-}\right.$$
$$= -\frac{b^3}{(bc-ad)^2(be-af)^2(a+bx)} - \frac{d^3}{(bc-ad)^2(de-cf}$$

Mathematica [A] time = 0.652643, size = 232, normalized size = 0.99

$$-\frac{b^3}{(a+bx)(bc-ad)^2(be-af)^2} - \frac{2b^3\log(a+bx)(-2adf+bcf+bde)}{(bc-ad)^3(be-af)^3} - \frac{d^3}{(c+dx)(bc-ad)^2(de-cf)^2} - \frac{2d^3\log(c+dx)(}{(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2
+ b*d*f*x^3)^(-2),x]
```

```
[Out] -(b^3/((b*c - a*d)^2*(b*e - a*f)^2*(a + b*x))) - d^3/((b*c - a*d)^2*(d*e -
c*f)^2*(c + d*x)) - f^3/((b*e - a*f)^2*(d*e - c*f)^2*(e + f*x)) - (2*b^3*(b
*d*e + b*c*f - 2*a*d*f)*Log[a + b*x])/((b*c - a*d)^3*(b*e - a*f)^3) - (2*d^
3*(b*d*e - 2*b*c*f + a*d*f)*Log[c + d*x])/((b*c - a*d)^3*(-(d*e) + c*f)^3)
- (2*f^3*(-2*b*d*e + b*c*f + a*d*f)*Log[e + f*x])/((b*e - a*f)^3*(d*e - c*f
)^3)
```

Maple [A] time = 0.05, size = 398, normalized size = 1.7

$$-\frac{b^3}{\left(af-be\right)^2(ad-bc)^2(bx+a)} + 4\frac{b^3\ln(bx+a)adf}{\left(af-be\right)^3(ad-bc)^3} - 2\frac{b^4\ln(bx+a)cf}{\left(af-be\right)^3(ad-bc)^3} - 2\frac{b^4\ln(bx+a)de}{\left(af-be\right)^3(ad-bc)^3} - \frac{}{\left(cf-de\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^2,x)
```

```
[Out] -b^3/(a*f-b*e)^2/(a*d-b*c)^2/(b*x+a)+4*b^3/(a*f-b*e)^3/(a*d-b*c)^3*ln(b*x+a
)*a*d*f-2*b^4/(a*f-b*e)^3/(a*d-b*c)^3*ln(b*x+a)*c*f-2*b^4/(a*f-b*e)^3/(a*d-
b*c)^3*ln(b*x+a)*d*e-d^3/(c*f-d*e)^2/(a*d-b*c)^2/(d*x+c)+2*d^4/(c*f-d*e)^3/
(a*d-b*c)^3*ln(d*x+c)*a*f-4*d^3/(c*f-d*e)^3/(a*d-b*c)^3*ln(d*x+c)*b*c*f+2*d
^4/(c*f-d*e)^3/(a*d-b*c)^3*ln(d*x+c)*b*e-f^3/(c*f-d*e)^2/(a*f-b*e)^2/(f*x+e
)-2*f^4/(c*f-d*e)^3/(a*f-b*e)^3*ln(f*x+e)*a*d-2*f^4/(c*f-d*e)^3/(a*f-b*e)^3
```

```
*ln(f*x+e)*b*c+4*f^3/(c*f-d*e)^3/(a*f-b*e)^3*ln(f*x+e)*b*d*e
```

Maxima [B] time = 1.85039, size = 2830, normalized size = 12.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)
^2,x, algorithm="maxima")
```

```
[Out] -2*(b^4*d*e + (b^4*c - 2*a*b^3*d)*f)*log(b*x + a)/((b^6*c^3 - 3*a*b^5*c^2*d
 + 3*a^2*b^4*c*d^2 - a^3*b^3*d^3)*e^3 - 3*(a*b^5*c^3 - 3*a^2*b^4*c^2*d + 3*
a^3*b^3*c*d^2 - a^4*b^2*d^3)*e^2*f + 3*(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a
^4*b^2*c*d^2 - a^5*b*d^3)*e*f^2 - (a^3*b^3*c^3 - 3*a^4*b^2*c^2*d + 3*a^5*b*
c*d^2 - a^6*d^3)*f^3) + 2*(b*d^4*e - (2*b*c*d^3 - a*d^4)*f)*log(d*x + c)/((
b^3*c^3*d^3 - 3*a*b^2*c^2*d^4 + 3*a^2*b*c*d^5 - a^3*d^6)*e^3 - 3*(b^3*c^4*d
^2 - 3*a*b^2*c^3*d^3 + 3*a^2*b*c^2*d^4 - a^3*c*d^5)*e^2*f + 3*(b^3*c^5*d -
3*a*b^2*c^4*d^2 + 3*a^2*b*c^3*d^3 - a^3*c^2*d^4)*e*f^2 - (b^3*c^6 - 3*a*b^2
*c^5*d + 3*a^2*b*c^4*d^2 - a^3*c^3*d^3)*f^3) + 2*(2*b*d*e*f^3 - (b*c + a*d)
*f^4)*log(f*x + e)/(b^3*d^3*e^6 + a^3*c^3*f^6 - 3*(b^3*c*d^2 + a*b^2*d^3)*e
^5*f + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*e^4*f^2 - (b^3*c^3 + 9*a*b
^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*e^3*f^3 + 3*(a*b^2*c^3 + 3*a^2*b*c^2*d
+ a^3*c*d^2)*e^2*f^4 - 3*(a^2*b*c^3 + a^3*c^2*d)*e*f^5) - ((b^3*c*d^2 + a*b
^2*d^3)*e^3 - 2*(b^3*c^2*d + a^2*b*d^3)*e^2*f + (b^3*c^3 + a^3*d^3)*e*f^2 +
 (a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2)*f^3 + 2*(b^3*d^3*e^2*f - (b^3*c*d^
2 + a*b^2*d^3)*e*f^2 + (b^3*c^2*d - a*b^2*c*d^2 + a^2*b*d^3)*f^3)*x^2 + (2*
b^3*d^3*e^3 - (b^3*c*d^2 + a*b^2*d^3)*e^2*f - (b^3*c^2*d + a^2*b*d^3)*e*f^2
 + (2*b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + 2*a^3*d^3)*f^3)*x)/((a*b^4*c^3*
d^2 - 2*a^2*b^3*c^2*d^3 + a^3*b^2*c*d^4)*e^5 - 2*(a*b^4*c^4*d - a^2*b^3*c^3
*d^2 - a^3*b^2*c^2*d^3 + a^4*b*c*d^4)*e^4*f + (a*b^4*c^5 + 2*a^2*b^3*c^4*d
- 6*a^3*b^2*c^3*d^2 + 2*a^4*b*c^2*d^3 + a^5*c*d^4)*e^3*f^2 - 2*(a^2*b^3*c^5
 - a^3*b^2*c^4*d - a^4*b*c^3*d^2 + a^5*c^2*d^3)*e^2*f^3 + (a^3*b^2*c^5 - 2*
a^4*b*c^4*d + a^5*c^3*d^2)*e*f^4 + ((b^5*c^2*d^3 - 2*a*b^4*c*d^4 + a^2*b^3*
d^5)*e^4*f - 2*(b^5*c^3*d^2 - a*b^4*c^2*d^3 - a^2*b^3*c*d^4 + a^3*b^2*d^5)*
e^3*f^2 + (b^5*c^4*d + 2*a*b^4*c^3*d^2 - 6*a^2*b^3*c^2*d^3 + 2*a^3*b^2*c*d^
4 + a^4*b*d^5)*e^2*f^3 - 2*(a*b^4*c^4*d - a^2*b^3*c^3*d^2 - a^3*b^2*c^2*d^3
 + a^4*b*c*d^4)*e*f^4 + (a^2*b^3*c^4*d - 2*a^3*b^2*c^3*d^2 + a^4*b*c^2*d^3)
*f^5)*x^3 + ((b^5*c^2*d^3 - 2*a*b^4*c*d^4 + a^2*b^3*d^5)*e^5 - (b^5*c^3*d^2
 - a*b^4*c^2*d^3 - a^2*b^3*c*d^4 + a^3*b^2*d^5)*e^4*f - (b^5*c^4*d - 2*a*b^
4*c^3*d^2 + 2*a^2*b^3*c^2*d^3 - 2*a^3*b^2*c*d^4 + a^4*b*d^5)*e^3*f^2 + (b^5
*c^5 + a*b^4*c^4*d - 2*a^2*b^3*c^3*d^2 - 2*a^3*b^2*c^2*d^3 + a^4*b*c*d^4 +
```

```
a^5*d^5)*e^2*f^3 - (2*a*b^4*c^5 - a^2*b^3*c^4*d - 2*a^3*b^2*c^3*d^2 - a^4*b
*c^2*d^3 + 2*a^5*c*d^4)*e*f^4 + (a^2*b^3*c^5 - a^3*b^2*c^4*d - a^4*b*c^3*d^
2 + a^5*c^2*d^3)*f^5)*x^2 + ((b^5*c^3*d^2 - a*b^4*c^2*d^3 - a^2*b^3*c*d^4 +
 a^3*b^2*d^5)*e^5 - (2*b^5*c^4*d - a*b^4*c^3*d^2 - 2*a^2*b^3*c^2*d^3 - a^3*
b^2*c*d^4 + 2*a^4*b*d^5)*e^4*f + (b^5*c^5 + a*b^4*c^4*d - 2*a^2*b^3*c^3*d^2
 - 2*a^3*b^2*c^2*d^3 + a^4*b*c*d^4 + a^5*d^5)*e^3*f^2 - (a*b^4*c^5 - 2*a^2*
b^3*c^4*d + 2*a^3*b^2*c^3*d^2 - 2*a^4*b*c^2*d^3 + a^5*c*d^4)*e^2*f^3 - (a^2
*b^3*c^5 - a^3*b^2*c^4*d - a^4*b*c^3*d^2 + a^5*c^2*d^3)*e*f^4 + (a^3*b^2*c^
5 - 2*a^4*b*c^4*d + a^5*c^3*d^2)*f^5)*x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)
^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x**2+b*d*f*x**
3)**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)
^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.20 $\int \frac{1}{\left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right)^3} dx$

Optimal. Leaf size=495

$$\frac{3b^5 \log(a+bx)\left(7a^2d^2f^2-7abdf(cf+de)+b^2\left(2c^2f^2+3cdef+2d^2e^2\right)\right)}{(bc-ad)^5(be-af)^5} - \frac{3d^5 \log(c+dx)\left(2a^2d^2f^2+abdf(3de-7c\right.}{(bc-ad)^5(de-}$$

```
[Out] -b^5/(2*(b*c - a*d)^3*(b*e - a*f)^3*(a + b*x)^2) + (3*b^5*(b*d*e + b*c*f -
2*a*d*f))/((b*c - a*d)^4*(b*e - a*f)^4*(a + b*x)) + d^5/(2*(b*c - a*d)^3*(d
*e - c*f)^3*(c + d*x)^2) + (3*d^5*(b*d*e - 2*b*c*f + a*d*f))/((b*c - a*d)^4
*(d*e - c*f)^4*(c + d*x)) - f^5/(2*(b*e - a*f)^3*(d*e - c*f)^3*(e + f*x)^2)
 - (3*f^5*(2*b*d*e - b*c*f - a*d*f))/((b*e - a*f)^4*(d*e - c*f)^4*(e + f*x)
) + (3*b^5*(7*a^2*d^2*f^2 - 7*a*b*d*f*(d*e + c*f) + b^2*(2*d^2*e^2 + 3*c*d*
e*f + 2*c^2*f^2))*Log[a + b*x])/((b*c - a*d)^5*(b*e - a*f)^5) - (3*d^5*(2*a
^2*d^2*f^2 + a*b*d*f*(3*d*e - 7*c*f) + b^2*(2*d^2*e^2 - 7*c*d*e*f + 7*c^2*f
^2))*Log[c + d*x])/((b*c - a*d)^5*(d*e - c*f)^5) + (3*f^5*(2*a^2*d^2*f^2 -
a*b*d*f*(7*d*e - 3*c*f) + b^2*(7*d^2*e^2 - 7*c*d*e*f + 2*c^2*f^2))*Log[e +
f*x])/((b*e - a*f)^5*(d*e - c*f)^5)
```

Rubi [A] time = 1.46179, antiderivative size = 495, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.022, Rules used = {2058}

$$\frac{3b^5 \log(a+bx)\left(7a^2d^2f^2-7abdf(cf+de)+b^2\left(2c^2f^2+3cdef+2d^2e^2\right)\right)}{(bc-ad)^5(be-af)^5} - \frac{3d^5 \log(c+dx)\left(2a^2d^2f^2+abdf(3de-7c\right.}{(bc-ad)^5(de-}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2 + b*d*
f*x^3)^(-3),x]
```

```
[Out] -b^5/(2*(b*c - a*d)^3*(b*e - a*f)^3*(a + b*x)^2) + (3*b^5*(b*d*e + b*c*f -
2*a*d*f))/((b*c - a*d)^4*(b*e - a*f)^4*(a + b*x)) + d^5/(2*(b*c - a*d)^3*(d
*e - c*f)^3*(c + d*x)^2) + (3*d^5*(b*d*e - 2*b*c*f + a*d*f))/((b*c - a*d)^4
*(d*e - c*f)^4*(c + d*x)) - f^5/(2*(b*e - a*f)^3*(d*e - c*f)^3*(e + f*x)^2)
 - (3*f^5*(2*b*d*e - b*c*f - a*d*f))/((b*e - a*f)^4*(d*e - c*f)^4*(e + f*x)
) + (3*b^5*(7*a^2*d^2*f^2 - 7*a*b*d*f*(d*e + c*f) + b^2*(2*d^2*e^2 + 3*c*d*
e*f + 2*c^2*f^2))*Log[a + b*x])/((b*c - a*d)^5*(b*e - a*f)^5) - (3*d^5*(2*a
^2*d^2*f^2 + a*b*d*f*(3*d*e - 7*c*f) + b^2*(2*d^2*e^2 - 7*c*d*e*f + 7*c^2*f
^2))*Log[c + d*x])/((b*c - a*d)^5*(d*e - c*f)^5) + (3*f^5*(2*a^2*d^2*f^2 -
```

```
a*b*d*f*(7*d*e - 3*c*f) + b^2*(7*d^2*e^2 - 7*c*d*e*f + 2*c^2*f^2))*Log[e +
f*x])/((b*e - a*f)^5*(d*e - c*f)^5)
```

Rule 2058

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rubi steps

$$\int \frac{1}{\left(ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 + bdfx^3\right)^3}\,dx = \int \left(\frac{b^6}{(bc-ad)^3(be-af)^3(a+bx)^3} - \frac{3b^6(bde+bc}{(bc-ad)^4(be-}\right.$$

$$= -\frac{b^5}{2(bc-ad)^3(be-af)^3(a+bx)^2} + \frac{3b^5(bde+bcf}{(bc-ad)^4(be-a}$$

Mathematica [A] time = 1.2813, size = 490, normalized size = 0.99

$$\frac{1}{2}\left(\frac{6b^5\log(a+bx)\left(7a^2d^2f^2 - 7abdf(cf+de) + b^2\left(2c^2f^2 + 3cdef + 2d^2e^2\right)\right)}{(bc-ad)^5(be-af)^5} + \frac{6d^5\log(c+dx)\left(2a^2d^2f^2 + abdf(3de-7\right.}{(bc-ad)^5(cf}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2
+ b*d*f*x^3)^(-3),x]
```

```
[Out] (-(b^5/((b*c - a*d)^3*(b*e - a*f)^3*(a + b*x)^2)) + (6*b^5*(b*d*e + b*c*f -
 2*a*d*f))/((b*c - a*d)^4*(b*e - a*f)^4*(a + b*x)) - d^5/((b*c - a*d)^3*(-(
d*e) + c*f)^3*(c + d*x)^2) + (6*d^5*(b*d*e - 2*b*c*f + a*d*f))/((b*c - a*d)
^4*(d*e - c*f)^4*(c + d*x)) - f^5/((b*e - a*f)^3*(d*e - c*f)^3*(e + f*x)^2)
 + (6*f^5*(-2*b*d*e + b*c*f + a*d*f))/((b*e - a*f)^4*(d*e - c*f)^4*(e + f*x
)) + (6*b^5*(7*a^2*d^2*f^2 - 7*a*b*d*f*(d*e + c*f) + b^2*(2*d^2*e^2 + 3*c*d
*e*f + 2*c^2*f^2))*Log[a + b*x])/((b*c - a*d)^5*(b*e - a*f)^5) + (6*d^5*(2*
a^2*d^2*f^2 + a*b*d*f*(3*d*e - 7*c*f) + b^2*(2*d^2*e^2 - 7*c*d*e*f + 7*c^2*
f^2))*Log[c + d*x])/((b*c - a*d)^5*(-(d*e) + c*f)^5) + (6*f^5*(2*a^2*d^2*f^
2 + a*b*d*f*(-7*d*e + 3*c*f) + b^2*(7*d^2*e^2 - 7*c*d*e*f + 2*c^2*f^2))*Log
[e + f*x])/((b*e - a*f)^5*(d*e - c*f)^5))/2
```

Maple [B] time = 0.032, size = 1076, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^3,x)
```

```
[Out] 6*f^7/(c*f-d*e)^5/(a*f-b*e)^5*ln(f*x+e)*a^2*d^2+6*f^7/(c*f-d*e)^5/(a*f-b*e)
^5*ln(f*x+e)*c^2*b^2-6*d^7/(c*f-d*e)^5/(a*d-b*c)^5*ln(d*x+c)*a^2*f^2-6*d^7/
(c*f-d*e)^5/(a*d-b*c)^5*ln(d*x+c)*e^2*b^2+3*f^6/(c*f-d*e)^4/(a*f-b*e)^4/(f*
x+e)*a*d+3*f^6/(c*f-d*e)^4/(a*f-b*e)^4/(f*x+e)*b*c+3*b^6/(a*f-b*e)^4/(a*d-b
*c)^4/(b*x+a)*d*e+6*b^7/(a*f-b*e)^5/(a*d-b*c)^5*ln(b*x+a)*c^2*f^2+6*b^7/(a*
f-b*e)^5/(a*d-b*c)^5*ln(b*x+a)*e^2*d^2+3*d^6/(c*f-d*e)^4/(a*d-b*c)^4/(d*x+c
)*a*f+3*d^6/(c*f-d*e)^4/(a*d-b*c)^4/(d*x+c)*b*e+3*b^6/(a*f-b*e)^4/(a*d-b*c)
^4/(b*x+a)*c*f+1/2*d^5/(c*f-d*e)^3/(a*d-b*c)^3/(d*x+c)^2-1/2*f^5/(c*f-d*e)^
3/(a*f-b*e)^3/(f*x+e)^2-1/2*b^5/(a*f-b*e)^3/(a*d-b*c)^3/(b*x+a)^2-6*b^5/(a*
f-b*e)^4/(a*d-b*c)^4/(b*x+a)*a*d*f+21*b^5/(a*f-b*e)^5/(a*d-b*c)^5*ln(b*x+a)
*a^2*d^2*f^2-6*d^5/(c*f-d*e)^4/(a*d-b*c)^4/(d*x+c)*b*c*f-21*d^5/(c*f-d*e)^5
/(a*d-b*c)^5*ln(d*x+c)*c^2*f^2*b^2-6*f^5/(c*f-d*e)^4/(a*f-b*e)^4/(f*x+e)*b*
d*e+21*f^5/(c*f-d*e)^5/(a*f-b*e)^5*ln(f*x+e)*e^2*d^2*b^2+9*f^7/(c*f-d*e)^5/
(a*f-b*e)^5*ln(f*x+e)*a*b*c*d-21*f^6/(c*f-d*e)^5/(a*f-b*e)^5*ln(f*x+e)*a*b*
d^2*e+21*d^6/(c*f-d*e)^5/(a*d-b*c)^5*ln(d*x+c)*c*e*f*b^2-21*b^6/(a*f-b*e)^5
/(a*d-b*c)^5*ln(b*x+a)*a*d^2*e*f+9*b^7/(a*f-b*e)^5/(a*d-b*c)^5*ln(b*x+a)*c*
d*e*f+21*d^6/(c*f-d*e)^5/(a*d-b*c)^5*ln(d*x+c)*a*b*c*f^2-9*d^7/(c*f-d*e)^5/
(a*d-b*c)^5*ln(d*x+c)*a*b*e*f-21*b^6/(a*f-b*e)^5/(a*d-b*c)^5*ln(b*x+a)*a*c*
d*f^2-21*f^6/(c*f-d*e)^5/(a*f-b*e)^5*ln(f*x+e)*c*d*e*b^2
```

Maxima [B] time = 4.52537, size = 14857, normalized size = 30.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)
^3,x, algorithm="maxima")
```

```
[Out] 3*(2*b^7*d^2*e^2 + (3*b^7*c*d - 7*a*b^6*d^2)*e*f + (2*b^7*c^2 - 7*a*b^6*c*d
 + 7*a^2*b^5*d^2)*f^2)*log(b*x + a)/((b^10*c^5 - 5*a*b^9*c^4*d + 10*a^2*b^8
*c^3*d^2 - 10*a^3*b^7*c^2*d^3 + 5*a^4*b^6*c*d^4 - a^5*b^5*d^5)*e^5 - 5*(a*b
^9*c^5 - 5*a^2*b^8*c^4*d + 10*a^3*b^7*c^3*d^2 - 10*a^4*b^6*c^2*d^3 + 5*a^5*
b^5*c*d^4 - a^6*b^4*d^5)*e^4*f + 10*(a^2*b^8*c^5 - 5*a^3*b^7*c^4*d + 10*a^4
```

```
*b^6*c^3*d^2 - 10*a^5*b^5*c^2*d^3 + 5*a^6*b^4*c*d^4 - a^7*b^3*d^5)*e^3*f^2
- 10*(a^3*b^7*c^5 - 5*a^4*b^6*c^4*d + 10*a^5*b^5*c^3*d^2 - 10*a^6*b^4*c^2*d
^3 + 5*a^7*b^3*c*d^4 - a^8*b^2*d^5)*e^2*f^3 + 5*(a^4*b^6*c^5 - 5*a^5*b^5*c^
4*d + 10*a^6*b^4*c^3*d^2 - 10*a^7*b^3*c^2*d^3 + 5*a^8*b^2*c*d^4 - a^9*b*d^5
)*e*f^4 - (a^5*b^5*c^5 - 5*a^6*b^4*c^4*d + 10*a^7*b^3*c^3*d^2 - 10*a^8*b^2*
c^2*d^3 + 5*a^9*b*c*d^4 - a^10*d^5)*f^5) - 3*(2*b^2*d^7*e^2 - (7*b^2*c*d^6
- 3*a*b*d^7)*e*f + (7*b^2*c^2*d^5 - 7*a*b*c*d^6 + 2*a^2*d^7)*f^2)*log(d*x +
 c)/((b^5*c^5*d^5 - 5*a*b^4*c^4*d^6 + 10*a^2*b^3*c^3*d^7 - 10*a^3*b^2*c^2*d
^8 + 5*a^4*b*c*d^9 - a^5*d^10)*e^5 - 5*(b^5*c^6*d^4 - 5*a*b^4*c^5*d^5 + 10*
a^2*b^3*c^4*d^6 - 10*a^3*b^2*c^3*d^7 + 5*a^4*b*c^2*d^8 - a^5*c*d^9)*e^4*f +
 10*(b^5*c^7*d^3 - 5*a*b^4*c^6*d^4 + 10*a^2*b^3*c^5*d^5 - 10*a^3*b^2*c^4*d^
6 + 5*a^4*b*c^3*d^7 - a^5*c^2*d^8)*e^3*f^2 - 10*(b^5*c^8*d^2 - 5*a*b^4*c^7*
d^3 + 10*a^2*b^3*c^6*d^4 - 10*a^3*b^2*c^5*d^5 + 5*a^4*b*c^4*d^6 - a^5*c^3*d
^7)*e^2*f^3 + 5*(b^5*c^9*d - 5*a*b^4*c^8*d^2 + 10*a^2*b^3*c^7*d^3 - 10*a^3*
b^2*c^6*d^4 + 5*a^4*b*c^5*d^5 - a^5*c^4*d^6)*e*f^4 - (b^5*c^10 - 5*a*b^4*c^
9*d + 10*a^2*b^3*c^8*d^2 - 10*a^3*b^2*c^7*d^3 + 5*a^4*b*c^6*d^4 - a^5*c^5*d
^5)*f^5) + 3*(7*b^2*d^2*e^2*f^5 - 7*(b^2*c*d + a*b*d^2)*e*f^6 + (2*b^2*c^2
+ 3*a*b*c*d + 2*a^2*d^2)*f^7)*log(f*x + e)/(b^5*d^5*e^10 + a^5*c^5*f^10 - 5
*(b^5*c*d^4 + a*b^4*d^5)*e^9*f + 5*(2*b^5*c^2*d^3 + 5*a*b^4*c*d^4 + 2*a^2*b
^3*d^5)*e^8*f^2 - 10*(b^5*c^3*d^2 + 5*a*b^4*c^2*d^3 + 5*a^2*b^3*c*d^4 + a^3
*b^2*d^5)*e^7*f^3 + 5*(b^5*c^4*d + 10*a*b^4*c^3*d^2 + 20*a^2*b^3*c^2*d^3 +
10*a^3*b^2*c*d^4 + a^4*b*d^5)*e^6*f^4 - (b^5*c^5 + 25*a*b^4*c^4*d + 100*a^2
*b^3*c^3*d^2 + 100*a^3*b^2*c^2*d^3 + 25*a^4*b*c*d^4 + a^5*d^5)*e^5*f^5 + 5*
(a*b^4*c^5 + 10*a^2*b^3*c^4*d + 20*a^3*b^2*c^3*d^2 + 10*a^4*b*c^2*d^3 + a^5
*c*d^4)*e^4*f^6 - 10*(a^2*b^3*c^5 + 5*a^3*b^2*c^4*d + 5*a^4*b*c^3*d^2 + a^5
*c^2*d^3)*e^3*f^7 + 5*(2*a^3*b^2*c^5 + 5*a^4*b*c^4*d + 2*a^5*c^3*d^2)*e^2*f
^8 - 5*(a^4*b*c^5 + a^5*c^4*d)*e*f^9) - 1/2*((b^7*c^3*d^4 - 7*a*b^6*c^2*d^5
 - 7*a^2*b^5*c*d^6 + a^3*b^4*d^7)*e^7 - (4*b^7*c^4*d^3 - 21*a*b^6*c^3*d^4 -
 26*a^2*b^5*c^2*d^5 - 21*a^3*b^4*c*d^6 + 4*a^4*b^3*d^7)*e^6*f + 2*(3*b^7*c^
5*d^2 - 7*a*b^6*c^4*d^3 - 26*a^2*b^5*c^3*d^4 - 26*a^3*b^4*c^2*d^5 - 7*a^4*b
^3*c*d^6 + 3*a^5*b^2*d^7)*e^5*f^2 - 2*(2*b^7*c^6*d + 7*a*b^6*c^5*d^2 - 39*a
^2*b^5*c^4*d^3 - 39*a^4*b^3*c^2*d^5 + 7*a^5*b^2*c*d^6 + 2*a^6*b*d^7)*e^4*f^
3 + (b^7*c^7 + 21*a*b^6*c^6*d - 52*a^2*b^5*c^5*d^2 - 52*a^5*b^2*c^2*d^5 + 2
1*a^6*b*c*d^6 + a^7*d^7)*e^3*f^4 - (7*a*b^6*c^7 - 26*a^2*b^5*c^6*d + 52*a^3
*b^4*c^5*d^2 - 78*a^4*b^3*c^4*d^3 + 52*a^5*b^2*c^3*d^4 - 26*a^6*b*c^2*d^5 +
 7*a^7*c*d^6)*e^2*f^5 - 7*(a^2*b^5*c^7 - 3*a^3*b^4*c^6*d + 2*a^4*b^3*c^5*d^
2 + 2*a^5*b^2*c^4*d^3 - 3*a^6*b*c^3*d^4 + a^7*c^2*d^5)*e*f^6 + (a^3*b^4*c^7
 - 4*a^4*b^3*c^6*d + 6*a^5*b^2*c^5*d^2 - 4*a^6*b*c^4*d^3 + a^7*c^3*d^4)*f^7
 - 6*(2*b^7*d^7*e^5*f^2 - 5*(b^7*c*d^6 + a*b^6*d^7)*e^4*f^3 + 2*(b^7*c^2*d^
5 + 8*a*b^6*c*d^6 + a^2*b^5*d^7)*e^3*f^4 + 2*(b^7*c^3*d^4 - 6*a*b^6*c^2*d^5
 - 6*a^2*b^5*c*d^6 + a^3*b^4*d^7)*e^2*f^5 - (5*b^7*c^4*d^3 - 16*a*b^6*c^3*d
^4 + 12*a^2*b^5*c^2*d^5 - 16*a^3*b^4*c*d^6 + 5*a^4*b^3*d^7)*e*f^6 + (2*b^7*
c^5*d^2 - 5*a*b^6*c^4*d^3 + 2*a^2*b^5*c^3*d^4 + 2*a^3*b^4*c^2*d^5 - 5*a^4*b
^3*c*d^6 + 2*a^5*b^2*d^7)*f^7)*x^5 - 3*(8*b^7*d^7*e^6*f - 14*(b^7*c*d^6 + a
*b^6*d^7)*e^5*f^2 - (7*b^7*c^2*d^5 - 34*a*b^6*c*d^6 + 7*a^2*b^5*d^7)*e^4*f^
```

3 + 2*(7*b^7*c^3*d^4 + 3*a*b^6*c^2*d^5 + 3*a^2*b^5*c*d^6 + 7*a^3*b^4*d^7)*e
^3*f^4 - (7*b^7*c^4*d^3 - 6*a*b^6*c^3*d^4 + 78*a^2*b^5*c^2*d^5 - 6*a^3*b^4*
c*d^6 + 7*a^4*b^3*d^7)*e^2*f^5 - 2*(7*b^7*c^5*d^2 - 17*a*b^6*c^4*d^3 - 3*a^
2*b^5*c^3*d^4 - 3*a^3*b^4*c^2*d^5 - 17*a^4*b^3*c*d^6 + 7*a^5*b^2*d^7)*e*f^6
 + (8*b^7*c^6*d - 14*a*b^6*c^5*d^2 - 7*a^2*b^5*c^4*d^3 + 14*a^3*b^4*c^3*d^4
 - 7*a^4*b^3*c^2*d^5 - 14*a^5*b^2*c*d^6 + 8*a^6*b*d^7)*f^7)*x^4 - 2*(6*b^7*
d^7*e^7 + 3*(b^7*c*d^6 + a*b^6*d^7)*e^6*f - (37*b^7*c^2*d^5 + 28*a*b^6*c*d^
6 + 37*a^2*b^5*d^7)*e^5*f^2 + (19*b^7*c^3*d^4 + 86*a*b^6*c^2*d^5 + 86*a^2*b
^5*c*d^6 + 19*a^3*b^4*d^7)*e^4*f^3 + (19*b^7*c^4*d^3 - 68*a*b^6*c^3*d^4 - 5
2*a^2*b^5*c^2*d^5 - 68*a^3*b^4*c*d^6 + 19*a^4*b^3*d^7)*e^3*f^4 - (37*b^7*c^
5*d^2 - 86*a*b^6*c^4*d^3 + 52*a^2*b^5*c^3*d^4 + 52*a^3*b^4*c^2*d^5 - 86*a^4
*b^3*c*d^6 + 37*a^5*b^2*d^7)*e^2*f^5 + (3*b^7*c^6*d - 28*a*b^6*c^5*d^2 + 86
*a^2*b^5*c^4*d^3 - 68*a^3*b^4*c^3*d^4 + 86*a^4*b^3*c^2*d^5 - 28*a^5*b^2*c*d
^6 + 3*a^6*b*d^7)*e*f^6 + (6*b^7*c^7 + 3*a*b^6*c^6*d - 37*a^2*b^5*c^5*d^2 +
 19*a^3*b^4*c^4*d^3 + 19*a^4*b^3*c^3*d^4 - 37*a^5*b^2*c^2*d^5 + 3*a^6*b*c*d
^6 + 6*a^7*d^7)*f^7)*x^3 - (18*(b^7*c*d^6 + a*b^6*d^7)*e^7 - (37*b^7*c^2*d^
5 + 34*a*b^6*c*d^6 + 37*a^2*b^5*d^7)*e^6*f - 3*(b^7*c^3*d^4 - 3*a*b^6*c^2*d
^5 - 3*a^2*b^5*c*d^6 + a^3*b^4*d^7)*e^5*f^2 + (32*b^7*c^4*d^3 + a*b^6*c^3*d
^4 + 234*a^2*b^5*c^2*d^5 + a^3*b^4*c*d^6 + 32*a^4*b^3*d^7)*e^4*f^3 - (3*b^7
*c^5*d^2 - a*b^6*c^4*d^3 + 208*a^2*b^5*c^3*d^4 + 208*a^3*b^4*c^2*d^5 - a^4*
b^3*c*d^6 + 3*a^5*b^2*d^7)*e^3*f^4 - (37*b^7*c^6*d - 9*a*b^6*c^5*d^2 - 234*
a^2*b^5*c^4*d^3 + 208*a^3*b^4*c^3*d^4 - 234*a^4*b^3*c^2*d^5 - 9*a^5*b^2*c*d
^6 + 37*a^6*b*d^7)*e^2*f^5 + (18*b^7*c^7 - 34*a*b^6*c^6*d + 9*a^2*b^5*c^5*d
^2 + a^3*b^4*c^4*d^3 + a^4*b^3*c^3*d^4 + 9*a^5*b^2*c^2*d^5 - 34*a^6*b*c*d^6
 + 18*a^7*d^7)*e*f^6 + (18*a*b^6*c^7 - 37*a^2*b^5*c^6*d - 3*a^3*b^4*c^5*d^2
 + 32*a^4*b^3*c^4*d^3 - 3*a^5*b^2*c^3*d^4 - 37*a^6*b*c^2*d^5 + 18*a^7*c*d^6
)*f^7)*x^2 - 2*(2*(b^7*c^2*d^5 + 7*a*b^6*c*d^6 + a^2*b^5*d^7)*e^7 - 3*(2*b^
7*c^3*d^4 + 11*a*b^6*c^2*d^5 + 11*a^2*b^5*c*d^6 + 2*a^3*b^4*d^7)*e^6*f + (4
*b^7*c^4*d^3 + 17*a*b^6*c^3*d^4 + 78*a^2*b^5*c^2*d^5 + 17*a^3*b^4*c*d^6 + 4
*a^4*b^3*d^7)*e^5*f^2 + 2*(2*b^7*c^5*d^2 - 4*a*b^6*c^4*d^3 - 13*a^2*b^5*c^3
*d^4 - 13*a^3*b^4*c^2*d^5 - 4*a^4*b^3*c*d^6 + 2*a^5*b^2*d^7)*e^4*f^3 - (6*b
^7*c^6*d - 17*a*b^6*c^5*d^2 + 26*a^2*b^5*c^4*d^3 + 26*a^4*b^3*c^2*d^5 - 17*
a^5*b^2*c*d^6 + 6*a^6*b*d^7)*e^3*f^4 + (2*b^7*c^7 - 33*a*b^6*c^6*d + 78*a^2
*b^5*c^5*d^2 - 26*a^3*b^4*c^4*d^3 - 26*a^4*b^3*c^3*d^4 + 78*a^5*b^2*c^2*d^5
 - 33*a^6*b*c*d^6 + 2*a^7*d^7)*e^2*f^5 + (14*a*b^6*c^7 - 33*a^2*b^5*c^6*d +
 17*a^3*b^4*c^5*d^2 - 8*a^4*b^3*c^4*d^3 + 17*a^5*b^2*c^3*d^4 - 33*a^6*b*c^2
*d^5 + 14*a^7*c*d^6)*e*f^6 + 2*(a^2*b^5*c^7 - 3*a^3*b^4*c^6*d + 2*a^4*b^3*c
^5*d^2 + 2*a^5*b^2*c^4*d^3 - 3*a^6*b*c^3*d^4 + a^7*c^2*d^5)*f^7)*x)/((a^2*b
^8*c^6*d^4 - 4*a^3*b^7*c^5*d^5 + 6*a^4*b^6*c^4*d^6 - 4*a^5*b^5*c^3*d^7 + a^
6*b^4*c^2*d^8)*e^10 - 4*(a^2*b^8*c^7*d^3 - 3*a^3*b^7*c^6*d^4 + 2*a^4*b^6*c^
5*d^5 + 2*a^5*b^5*c^4*d^6 - 3*a^6*b^4*c^3*d^7 + a^7*b^3*c^2*d^8)*e^9*f + 2*
(3*a^2*b^8*c^8*d^2 - 4*a^3*b^7*c^7*d^3 - 11*a^4*b^6*c^6*d^4 + 24*a^5*b^5*c^
5*d^5 - 11*a^6*b^4*c^4*d^6 - 4*a^7*b^3*c^3*d^7 + 3*a^8*b^2*c^2*d^8)*e^8*f^2
 - 4*(a^2*b^8*c^9*d + 2*a^3*b^7*c^8*d^2 - 12*a^4*b^6*c^7*d^3 + 9*a^5*b^5*c^
6*d^4 + 9*a^6*b^4*c^5*d^5 - 12*a^7*b^3*c^4*d^6 + 2*a^8*b^2*c^3*d^7 + a^9*b*

c^2*d^8)*e^7*f^3 + (a^2*b^8*c^10 + 12*a^3*b^7*c^9*d - 22*a^4*b^6*c^8*d^2 -
36*a^5*b^5*c^7*d^3 + 90*a^6*b^4*c^6*d^4 - 36*a^7*b^3*c^5*d^5 - 22*a^8*b^2*c
^4*d^6 + 12*a^9*b*c^3*d^7 + a^10*c^2*d^8)*e^6*f^4 - 4*(a^3*b^7*c^10 + 2*a^4
*b^6*c^9*d - 12*a^5*b^5*c^8*d^2 + 9*a^6*b^4*c^7*d^3 + 9*a^7*b^3*c^6*d^4 - 1
2*a^8*b^2*c^5*d^5 + 2*a^9*b*c^4*d^6 + a^10*c^3*d^7)*e^5*f^5 + 2*(3*a^4*b^6*
c^10 - 4*a^5*b^5*c^9*d - 11*a^6*b^4*c^8*d^2 + 24*a^7*b^3*c^7*d^3 - 11*a^8*b
^2*c^6*d^4 - 4*a^9*b*c^5*d^5 + 3*a^10*c^4*d^6)*e^4*f^6 - 4*(a^5*b^5*c^10 -
3*a^6*b^4*c^9*d + 2*a^7*b^3*c^8*d^2 + 2*a^8*b^2*c^7*d^3 - 3*a^9*b*c^6*d^4 +
a^10*c^5*d^5)*e^3*f^7 + (a^6*b^4*c^10 - 4*a^7*b^3*c^9*d + 6*a^8*b^2*c^8*d^
2 - 4*a^9*b*c^7*d^3 + a^10*c^6*d^4)*e^2*f^8 + ((b^10*c^4*d^6 - 4*a*b^9*c^3*
d^7 + 6*a^2*b^8*c^2*d^8 - 4*a^3*b^7*c*d^9 + a^4*b^6*d^10)*e^8*f^2 - 4*(b^10
*c^5*d^5 - 3*a*b^9*c^4*d^6 + 2*a^2*b^8*c^3*d^7 + 2*a^3*b^7*c^2*d^8 - 3*a^4*
b^6*c*d^9 + a^5*b^5*d^10)*e^7*f^3 + 2*(3*b^10*c^6*d^4 - 4*a*b^9*c^5*d^5 - 1
1*a^2*b^8*c^4*d^6 + 24*a^3*b^7*c^3*d^7 - 11*a^4*b^6*c^2*d^8 - 4*a^5*b^5*c*d
^9 + 3*a^6*b^4*d^10)*e^6*f^4 - 4*(b^10*c^7*d^3 + 2*a*b^9*c^6*d^4 - 12*a^2*b
^8*c^5*d^5 + 9*a^3*b^7*c^4*d^6 + 9*a^4*b^6*c^3*d^7 - 12*a^5*b^5*c^2*d^8 + 2
*a^6*b^4*c*d^9 + a^7*b^3*d^10)*e^5*f^5 + (b^10*c^8*d^2 + 12*a*b^9*c^7*d^3 -
22*a^2*b^8*c^6*d^4 - 36*a^3*b^7*c^5*d^5 + 90*a^4*b^6*c^4*d^6 - 36*a^5*b^5*
c^3*d^7 - 22*a^6*b^4*c^2*d^8 + 12*a^7*b^3*c*d^9 + a^8*b^2*d^10)*e^4*f^6 - 4
*(a*b^9*c^8*d^2 + 2*a^2*b^8*c^7*d^3 - 12*a^3*b^7*c^6*d^4 + 9*a^4*b^6*c^5*d^
5 + 9*a^5*b^5*c^4*d^6 - 12*a^6*b^4*c^3*d^7 + 2*a^7*b^3*c^2*d^8 + a^8*b^2*c*
d^9)*e^3*f^7 + 2*(3*a^2*b^8*c^8*d^2 - 4*a^3*b^7*c^7*d^3 - 11*a^4*b^6*c^6*d^
4 + 24*a^5*b^5*c^5*d^5 - 11*a^6*b^4*c^4*d^6 - 4*a^7*b^3*c^3*d^7 + 3*a^8*b^2
*c^2*d^8)*e^2*f^8 - 4*(a^3*b^7*c^8*d^2 - 3*a^4*b^6*c^7*d^3 + 2*a^5*b^5*c^6*
d^4 + 2*a^6*b^4*c^5*d^5 - 3*a^7*b^3*c^4*d^6 + a^8*b^2*c^3*d^7)*e*f^9 + (a^4
*b^6*c^8*d^2 - 4*a^5*b^5*c^7*d^3 + 6*a^6*b^4*c^6*d^4 - 4*a^7*b^3*c^5*d^5 +
a^8*b^2*c^4*d^6)*f^10)*x^6 + 2*((b^10*c^4*d^6 - 4*a*b^9*c^3*d^7 + 6*a^2*b^8
*c^2*d^8 - 4*a^3*b^7*c*d^9 + a^4*b^6*d^10)*e^9*f - 3*(b^10*c^5*d^5 - 3*a*b^
9*c^4*d^6 + 2*a^2*b^8*c^3*d^7 + 2*a^3*b^7*c^2*d^8 - 3*a^4*b^6*c*d^9 + a^5*b
^5*d^10)*e^8*f^2 + 2*(b^10*c^6*d^4 - 9*a^2*b^8*c^4*d^6 + 16*a^3*b^7*c^3*d^7
- 9*a^4*b^6*c^2*d^8 + a^6*b^4*d^10)*e^7*f^3 + 2*(b^10*c^7*d^3 - 5*a*b^9*c^
6*d^4 + 9*a^2*b^8*c^5*d^5 - 5*a^3*b^7*c^4*d^6 - 5*a^4*b^6*c^3*d^7 + 9*a^5*b
^5*c^2*d^8 - 5*a^6*b^4*c*d^9 + a^7*b^3*d^10)*e^6*f^4 - 3*(b^10*c^8*d^2 - 6*
a^2*b^8*c^6*d^4 + 8*a^3*b^7*c^5*d^5 - 6*a^4*b^6*c^4*d^6 + 8*a^5*b^5*c^3*d^7
- 6*a^6*b^4*c^2*d^8 + a^8*b^2*d^10)*e^5*f^5 + (b^10*c^9*d + 9*a*b^9*c^8*d^
2 - 18*a^2*b^8*c^7*d^3 - 10*a^3*b^7*c^6*d^4 + 18*a^4*b^6*c^5*d^5 + 18*a^5*b
^5*c^4*d^6 - 10*a^6*b^4*c^3*d^7 - 18*a^7*b^3*c^2*d^8 + 9*a^8*b^2*c*d^9 + a^
9*b*d^10)*e^4*f^6 - 2*(2*a*b^9*c^9*d + 3*a^2*b^8*c^8*d^2 - 16*a^3*b^7*c^7*d
^3 + 5*a^4*b^6*c^6*d^4 + 12*a^5*b^5*c^5*d^5 + 5*a^6*b^4*c^4*d^6 - 16*a^7*b^
3*c^3*d^7 + 3*a^8*b^2*c^2*d^8 + 2*a^9*b*c*d^9)*e^3*f^7 + 6*(a^2*b^8*c^9*d -
a^3*b^7*c^8*d^2 - 3*a^4*b^6*c^7*d^3 + 3*a^5*b^5*c^6*d^4 + 3*a^6*b^4*c^5*d^
5 - 3*a^7*b^3*c^4*d^6 - a^8*b^2*c^3*d^7 + a^9*b*c^2*d^8)*e^2*f^8 - (4*a^3*b
^7*c^9*d - 9*a^4*b^6*c^8*d^2 + 10*a^6*b^4*c^6*d^4 - 9*a^8*b^2*c^4*d^6 + 4*a
^9*b*c^3*d^7)*e*f^9 + (a^4*b^6*c^9*d - 3*a^5*b^5*c^8*d^2 + 2*a^6*b^4*c^7*d^
3 + 2*a^7*b^3*c^6*d^4 - 3*a^8*b^2*c^5*d^5 + a^9*b*c^4*d^6)*f^10)*x^5 + ((b^

10*c^4*d^6 - 4*a*b^9*c^3*d^7 + 6*a^2*b^8*c^2*d^8 - 4*a^3*b^7*c*d^9 + a^4*b^
6*d^10)*e^10 - 3*(3*b^10*c^6*d^4 - 8*a*b^9*c^5*d^5 + 5*a^2*b^8*c^4*d^6 + 5*
a^4*b^6*c^2*d^8 - 8*a^5*b^5*c*d^9 + 3*a^6*b^4*d^10)*e^8*f^2 + 4*(4*b^10*c^7
*d^3 - 5*a*b^9*c^6*d^4 - 9*a^2*b^8*c^5*d^5 + 10*a^3*b^7*c^4*d^6 + 10*a^4*b^
6*c^3*d^7 - 9*a^5*b^5*c^2*d^8 - 5*a^6*b^4*c*d^9 + 4*a^7*b^3*d^10)*e^7*f^3 -
(9*b^10*c^8*d^2 + 20*a*b^9*c^7*d^3 - 90*a^2*b^8*c^6*d^4 + 36*a^3*b^7*c^5*d
^5 + 50*a^4*b^6*c^4*d^6 + 36*a^5*b^5*c^3*d^7 - 90*a^6*b^4*c^2*d^8 + 20*a^7*
b^3*c*d^9 + 9*a^8*b^2*d^10)*e^6*f^4 + 12*(2*a*b^9*c^8*d^2 - 3*a^2*b^8*c^7*d
^3 - 3*a^3*b^7*c^6*d^4 + 4*a^4*b^6*c^5*d^5 + 4*a^5*b^5*c^4*d^6 - 3*a^6*b^4*
c^3*d^7 - 3*a^7*b^3*c^2*d^8 + 2*a^8*b^2*c*d^9)*e^5*f^5 + (b^10*c^10 - 15*a^
2*b^8*c^8*d^2 + 40*a^3*b^7*c^7*d^3 - 50*a^4*b^6*c^6*d^4 + 48*a^5*b^5*c^5*d^
5 - 50*a^6*b^4*c^4*d^6 + 40*a^7*b^3*c^3*d^7 - 15*a^8*b^2*c^2*d^8 + a^10*d^1
0)*e^4*f^6 - 4*(a*b^9*c^10 - 10*a^4*b^6*c^7*d^3 + 9*a^5*b^5*c^6*d^4 + 9*a^6
*b^4*c^5*d^5 - 10*a^7*b^3*c^4*d^6 + a^10*c*d^9)*e^3*f^7 + 3*(2*a^2*b^8*c^10
- 5*a^4*b^6*c^8*d^2 - 12*a^5*b^5*c^7*d^3 + 30*a^6*b^4*c^6*d^4 - 12*a^7*b^3
*c^5*d^5 - 5*a^8*b^2*c^4*d^6 + 2*a^10*c^2*d^8)*e^2*f^8 - 4*(a^3*b^7*c^10 -
6*a^5*b^5*c^8*d^2 + 5*a^6*b^4*c^7*d^3 + 5*a^7*b^3*c^6*d^4 - 6*a^8*b^2*c^5*d
^5 + a^10*c^3*d^7)*e*f^9 + (a^4*b^6*c^10 - 9*a^6*b^4*c^8*d^2 + 16*a^7*b^3*c
^7*d^3 - 9*a^8*b^2*c^6*d^4 + a^10*c^4*d^6)*f^10)*x^4 + 2*((b^10*c^5*d^5 - 3
*a*b^9*c^4*d^6 + 2*a^2*b^8*c^3*d^7 + 2*a^3*b^7*c^2*d^8 - 3*a^4*b^6*c*d^9 +
a^5*b^5*d^10)*e^10 - (3*b^10*c^6*d^4 - 8*a*b^9*c^5*d^5 + 5*a^2*b^8*c^4*d^6
+ 5*a^4*b^6*c^2*d^8 - 8*a^5*b^5*c*d^9 + 3*a^6*b^4*d^10)*e^9*f + (2*b^10*c^7
*d^3 - 5*a*b^9*c^6*d^4 + 3*a^2*b^8*c^5*d^5 + 3*a^5*b^5*c^2*d^8 - 5*a^6*b^4*
c*d^9 + 2*a^7*b^3*d^10)*e^8*f^2 + 2*(b^10*c^8*d^2 - 16*a^3*b^7*c^5*d^5 + 30
*a^4*b^6*c^4*d^6 - 16*a^5*b^5*c^3*d^7 + a^8*b^2*d^10)*e^7*f^3 - (3*b^10*c^9
*d + 5*a*b^9*c^8*d^2 - 60*a^3*b^7*c^6*d^4 + 52*a^4*b^6*c^5*d^5 + 52*a^5*b^5
*c^4*d^6 - 60*a^6*b^4*c^3*d^7 + 5*a^8*b^2*c*d^9 + 3*a^9*b*d^10)*e^6*f^4 + (
b^10*c^10 + 8*a*b^9*c^9*d + 3*a^2*b^8*c^8*d^2 - 32*a^3*b^7*c^7*d^3 - 52*a^4
*b^6*c^6*d^4 + 144*a^5*b^5*c^5*d^5 - 52*a^6*b^4*c^4*d^6 - 32*a^7*b^3*c^3*d^
7 + 3*a^8*b^2*c^2*d^8 + 8*a^9*b*c*d^9 + a^10*d^10)*e^5*f^5 - (3*a*b^9*c^10
+ 5*a^2*b^8*c^9*d - 60*a^4*b^6*c^7*d^3 + 52*a^5*b^5*c^6*d^4 + 52*a^6*b^4*c^
5*d^5 - 60*a^7*b^3*c^4*d^6 + 5*a^9*b*c^2*d^8 + 3*a^10*c*d^9)*e^4*f^6 + 2*(a
^2*b^8*c^10 - 16*a^5*b^5*c^7*d^3 + 30*a^6*b^4*c^6*d^4 - 16*a^7*b^3*c^5*d^5
+ a^10*c^2*d^8)*e^3*f^7 + (2*a^3*b^7*c^10 - 5*a^4*b^6*c^9*d + 3*a^5*b^5*c^8
*d^2 + 3*a^8*b^2*c^5*d^5 - 5*a^9*b*c^4*d^6 + 2*a^10*c^3*d^7)*e^2*f^8 - (3*a
^4*b^6*c^10 - 8*a^5*b^5*c^9*d + 5*a^6*b^4*c^8*d^2 + 5*a^8*b^2*c^6*d^4 - 8*a
^9*b*c^5*d^5 + 3*a^10*c^4*d^6)*e*f^9 + (a^5*b^5*c^10 - 3*a^6*b^4*c^9*d + 2*
a^7*b^3*c^8*d^2 + 2*a^8*b^2*c^7*d^3 - 3*a^9*b*c^6*d^4 + a^10*c^5*d^5)*f^10)
*x^3 + ((b^10*c^6*d^4 - 9*a^2*b^8*c^4*d^6 + 16*a^3*b^7*c^3*d^7 - 9*a^4*b^6*
c^2*d^8 + a^6*b^4*d^10)*e^10 - 4*(b^10*c^7*d^3 - 6*a^2*b^8*c^5*d^5 + 5*a^3*
b^7*c^4*d^6 + 5*a^4*b^6*c^3*d^7 - 6*a^5*b^5*c^2*d^8 + a^7*b^3*d^10)*e^9*f +
3*(2*b^10*c^8*d^2 - 5*a^2*b^8*c^6*d^4 - 12*a^3*b^7*c^5*d^5 + 30*a^4*b^6*c^
4*d^6 - 12*a^5*b^5*c^3*d^7 - 5*a^6*b^4*c^2*d^8 + 2*a^8*b^2*d^10)*e^8*f^2 -
4*(b^10*c^9*d - 10*a^3*b^7*c^6*d^4 + 9*a^4*b^6*c^5*d^5 + 9*a^5*b^5*c^4*d^6
- 10*a^6*b^4*c^3*d^7 + a^9*b*d^10)*e^7*f^3 + (b^10*c^10 - 15*a^2*b^8*c^8*d^

```
2 + 40*a^3*b^7*c^7*d^3 - 50*a^4*b^6*c^6*d^4 + 48*a^5*b^5*c^5*d^5 - 50*a^6*b
^4*c^4*d^6 + 40*a^7*b^3*c^3*d^7 - 15*a^8*b^2*c^2*d^8 + a^10*d^10)*e^6*f^4 +
 12*(2*a^2*b^8*c^9*d - 3*a^3*b^7*c^8*d^2 - 3*a^4*b^6*c^7*d^3 + 4*a^5*b^5*c^
6*d^4 + 4*a^6*b^4*c^5*d^5 - 3*a^7*b^3*c^4*d^6 - 3*a^8*b^2*c^3*d^7 + 2*a^9*b
*c^2*d^8)*e^5*f^5 - (9*a^2*b^8*c^10 + 20*a^3*b^7*c^9*d - 90*a^4*b^6*c^8*d^2
 + 36*a^5*b^5*c^7*d^3 + 50*a^6*b^4*c^6*d^4 + 36*a^7*b^3*c^5*d^5 - 90*a^8*b^
2*c^4*d^6 + 20*a^9*b*c^3*d^7 + 9*a^10*c^2*d^8)*e^4*f^6 + 4*(4*a^3*b^7*c^10
- 5*a^4*b^6*c^9*d - 9*a^5*b^5*c^8*d^2 + 10*a^6*b^4*c^7*d^3 + 10*a^7*b^3*c^6
*d^4 - 9*a^8*b^2*c^5*d^5 - 5*a^9*b*c^4*d^6 + 4*a^10*c^3*d^7)*e^3*f^7 - 3*(3
*a^4*b^6*c^10 - 8*a^5*b^5*c^9*d + 5*a^6*b^4*c^8*d^2 + 5*a^8*b^2*c^6*d^4 - 8
*a^9*b*c^5*d^5 + 3*a^10*c^4*d^6)*e^2*f^8 + (a^6*b^4*c^10 - 4*a^7*b^3*c^9*d
+ 6*a^8*b^2*c^8*d^2 - 4*a^9*b*c^7*d^3 + a^10*c^6*d^4)*f^10)*x^2 + 2*((a*b^9
*c^6*d^4 - 3*a^2*b^8*c^5*d^5 + 2*a^3*b^7*c^4*d^6 + 2*a^4*b^6*c^3*d^7 - 3*a^
5*b^5*c^2*d^8 + a^6*b^4*c*d^9)*e^10 - (4*a*b^9*c^7*d^3 - 9*a^2*b^8*c^6*d^4
+ 10*a^4*b^6*c^4*d^6 - 9*a^6*b^4*c^2*d^8 + 4*a^7*b^3*c*d^9)*e^9*f + 6*(a*b^
9*c^8*d^2 - a^2*b^8*c^7*d^3 - 3*a^3*b^7*c^6*d^4 + 3*a^4*b^6*c^5*d^5 + 3*a^5
*b^5*c^4*d^6 - 3*a^6*b^4*c^3*d^7 - a^7*b^3*c^2*d^8 + a^8*b^2*c*d^9)*e^8*f^2
 - 2*(2*a*b^9*c^9*d + 3*a^2*b^8*c^8*d^2 - 16*a^3*b^7*c^7*d^3 + 5*a^4*b^6*c^
6*d^4 + 12*a^5*b^5*c^5*d^5 + 5*a^6*b^4*c^4*d^6 - 16*a^7*b^3*c^3*d^7 + 3*a^8
*b^2*c^2*d^8 + 2*a^9*b*c*d^9)*e^7*f^3 + (a*b^9*c^10 + 9*a^2*b^8*c^9*d - 18*
a^3*b^7*c^8*d^2 - 10*a^4*b^6*c^7*d^3 + 18*a^5*b^5*c^6*d^4 + 18*a^6*b^4*c^5*
d^5 - 10*a^7*b^3*c^4*d^6 - 18*a^8*b^2*c^3*d^7 + 9*a^9*b*c^2*d^8 + a^10*c*d^
9)*e^6*f^4 - 3*(a^2*b^8*c^10 - 6*a^4*b^6*c^8*d^2 + 8*a^5*b^5*c^7*d^3 - 6*a^
6*b^4*c^6*d^4 + 8*a^7*b^3*c^5*d^5 - 6*a^8*b^2*c^4*d^6 + a^10*c^2*d^8)*e^5*f
^5 + 2*(a^3*b^7*c^10 - 5*a^4*b^6*c^9*d + 9*a^5*b^5*c^8*d^2 - 5*a^6*b^4*c^7*
d^3 - 5*a^7*b^3*c^6*d^4 + 9*a^8*b^2*c^5*d^5 - 5*a^9*b*c^4*d^6 + a^10*c^3*d^
7)*e^4*f^6 + 2*(a^4*b^6*c^10 - 9*a^6*b^4*c^8*d^2 + 16*a^7*b^3*c^7*d^3 - 9*a
^8*b^2*c^6*d^4 + a^10*c^4*d^6)*e^3*f^7 - 3*(a^5*b^5*c^10 - 3*a^6*b^4*c^9*d
+ 2*a^7*b^3*c^8*d^2 + 2*a^8*b^2*c^7*d^3 - 3*a^9*b*c^6*d^4 + a^10*c^5*d^5)*e
^2*f^8 + (a^6*b^4*c^10 - 4*a^7*b^3*c^9*d + 6*a^8*b^2*c^8*d^2 - 4*a^9*b*c^7*
d^3 + a^10*c^6*d^4)*e*f^9)*x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)
^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x**2+b*d*f*x**
3)**3,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)
^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.21 $\int \frac{1}{1+x+x^2+x^3} dx$

Optimal. Leaf size=25

$$-\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{2}\log(x+1)+\frac{1}{2}\tan^{-1}(x)$$

```
[Out] ArcTan[x]/2 + Log[1 + x]/2 - Log[1 + x^2]/4
```

Rubi [A] time = 0.0146004, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2058, 635, 203, 260}

$$-\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{2}\log(x+1)+\frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + x^2 + x^3)^(-1),x]
```

```
[Out] ArcTan[x]/2 + Log[1 + x]/2 - Log[1 + x^2]/4
```

Rule 2058

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{1+x+x^2+x^3}\,dx &= \int \left(\frac{1}{2(1+x)}+\frac{1-x}{2\left(1+x^2\right)}\right)dx \\
&= \frac{1}{2}\log(1+x)+\frac{1}{2}\int \frac{1-x}{1+x^2}\,dx \\
&= \frac{1}{2}\log(1+x)+\frac{1}{2}\int \frac{1}{1+x^2}\,dx-\frac{1}{2}\int \frac{x}{1+x^2}\,dx \\
&= \frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\log(1+x)-\frac{1}{4}\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.007587, size = 25, normalized size = 1.

$$-\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{2}\log(x+1)+\frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + x^2 + x^3)^(-1),x]
```

```
[Out] ArcTan[x]/2 + Log[1 + x]/2 - Log[1 + x^2]/4
```

Maple [A] time = 0.004, size = 20, normalized size = 0.8

$$\frac{\arctan(x)}{2}+\frac{\ln(1+x)}{2}-\frac{\ln\left(x^2+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3+x^2+x+1),x)
```

```
[Out] 1/2*arctan(x)+1/2*ln(1+x)-1/4*ln(x^2+1)
```

Maxima [A] time = 1.59432, size = 26, normalized size = 1.04

$$\frac{1}{2}\arctan\left(x\right) - \frac{1}{4}\log\left(x^{2} + 1\right) + \frac{1}{2}\log\left(x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3+x^2+x+1),x, algorithm="maxima")
```

```
[Out] 1/2*arctan(x) - 1/4*log(x^2 + 1) + 1/2*log(x + 1)
```

Fricas [A] time = 1.80641, size = 69, normalized size = 2.76

$$\frac{1}{2}\arctan\left(x\right) - \frac{1}{4}\log\left(x^{2} + 1\right) + \frac{1}{2}\log\left(x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3+x^2+x+1),x, algorithm="fricas")
```

```
[Out] 1/2*arctan(x) - 1/4*log(x^2 + 1) + 1/2*log(x + 1)
```

Sympy [A] time = 0.119343, size = 19, normalized size = 0.76

$$\frac{\log\left(x + 1\right)}{2} - \frac{\log\left(x^{2} + 1\right)}{4} + \frac{\operatorname{atan}\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**3+x**2+x+1),x)
```

```
[Out] log(x + 1)/2 - log(x**2 + 1)/4 + atan(x)/2
```

Giac [A] time = 1.24522, size = 27, normalized size = 1.08

$$\frac{1}{2}\arctan\left(x\right) - \frac{1}{4}\log\left(x^{2} + 1\right) + \frac{1}{2}\log\left(|x + 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3+x^2+x+1),x, algorithm="giac")
```

```
[Out] 1/2*arctan(x) - 1/4*log(x^2 + 1) + 1/2*log(abs(x + 1))
```

3.22 $\int \frac{1}{-1+4x-4x^2+16x^3}\,dx$

Optimal. Leaf size=31

$$-\frac{1}{10}\log\left(4x^2+1\right)+\frac{1}{5}\log(1-4x)-\frac{1}{10}\tan^{-1}(2x)$$

```
[Out] -ArcTan[2*x]/10 + Log[1 - 4*x]/5 - Log[1 + 4*x^2]/10
```

Rubi [A] time = 0.0198343, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2058, 635, 203, 260}

$$-\frac{1}{10}\log\left(4x^2+1\right)+\frac{1}{5}\log(1-4x)-\frac{1}{10}\tan^{-1}(2x)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 4*x - 4*x^2 + 16*x^3)^(-1),x]
```

```
[Out] -ArcTan[2*x]/10 + Log[1 - 4*x]/5 - Log[1 + 4*x^2]/10
```

Rule 2058

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{-1+4x-4x^2+16x^3}\,dx &= \int \left(\frac{4}{5(-1+4x)}+\frac{-1-4x}{5\left(1+4x^2\right)}\right)dx\\ &= \frac{1}{5}\log(1-4x)+\frac{1}{5}\int \frac{-1-4x}{1+4x^2}\,dx\\ &= \frac{1}{5}\log(1-4x)-\frac{1}{5}\int \frac{1}{1+4x^2}\,dx-\frac{4}{5}\int \frac{x}{1+4x^2}\,dx\\ &= -\frac{1}{10}\tan^{-1}(2x)+\frac{1}{5}\log(1-4x)-\frac{1}{10}\log\left(1+4x^2\right)\end{aligned}$$

Mathematica [A] time = 0.0075836, size = 31, normalized size = 1.

$$-\frac{1}{10}\log\left(4x^2+1\right)+\frac{1}{5}\log(1-4x)-\frac{1}{10}\tan^{-1}(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + 4*x - 4*x^2 + 16*x^3)^(-1),x]
```

```
[Out] -ArcTan[2*x]/10 + Log[1 - 4*x]/5 - Log[1 + 4*x^2]/10
```

Maple [A] time = 0.006, size = 26, normalized size = 0.8

$$\frac{\ln(-1+4x)}{5}-\frac{\ln\left(4x^2+1\right)}{10}-\frac{\arctan(2x)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(16*x^3-4*x^2+4*x-1),x)
```

```
[Out] 1/5*ln(-1+4*x)-1/10*ln(4*x^2+1)-1/10*arctan(2*x)
```

Maxima [A] time = 1.55369, size = 34, normalized size = 1.1

$$-\frac{1}{10}\arctan(2x)-\frac{1}{10}\log\left(4x^2+1\right)+\frac{1}{5}\log(4x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(16*x^3-4*x^2+4*x-1),x, algorithm="maxima")
```

```
[Out] -1/10*arctan(2*x) - 1/10*log(4*x^2 + 1) + 1/5*log(4*x - 1)
```

Fricas [A] time = 1.83125, size = 81, normalized size = 2.61

$$-\frac{1}{10}\arctan(2x)-\frac{1}{10}\log\left(4x^2+1\right)+\frac{1}{5}\log(4x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(16*x^3-4*x^2+4*x-1),x, algorithm="fricas")
```

```
[Out] -1/10*arctan(2*x) - 1/10*log(4*x^2 + 1) + 1/5*log(4*x - 1)
```

Sympy [A] time = 0.131789, size = 24, normalized size = 0.77

$$\frac{\log\left(x-\frac{1}{4}\right)}{5}-\frac{\log\left(x^2+\frac{1}{4}\right)}{10}-\frac{\operatorname{atan}(2x)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(16*x**3-4*x**2+4*x-1),x)
```

```
[Out] log(x - 1/4)/5 - log(x**2 + 1/4)/10 - atan(2*x)/10
```

Giac [A] time = 1.23862, size = 35, normalized size = 1.13

$$-\frac{1}{10}\arctan(2x)-\frac{1}{10}\log\left(4x^2+1\right)+\frac{1}{5}\log(|4x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(16*x^3-4*x^2+4*x-1),x, algorithm="giac")
```

```
[Out] -1/10*arctan(2*x) - 1/10*log(4*x^2 + 1) + 1/5*log(abs(4*x - 1))
```

3.23 $\int \frac{1}{dx^3}\,dx$

Optimal. Leaf size=10

$$-\frac{1}{2dx^2}$$

[Out] -1/(2*d*x^2)

Rubi [A] time = 0.0014073, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {12, 30}

$$-\frac{1}{2dx^2}$$

Antiderivative was successfully verified.

[In] Int[1/(d*x^3),x]

[Out] -1/(2*d*x^2)

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{dx^3}\,dx &= \frac{\int \frac{1}{x^3}\,dx}{d}\\ &= -\frac{1}{2dx^2}\end{aligned}$$

Mathematica [A] time = 0.0003004, size = 10, normalized size = 1.

$$-\frac{1}{2dx^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(d*x^3),x]
```

```
[Out] -1/(2*d*x^2)
```

Maple [A] time = 0.002, size = 9, normalized size = 0.9

$$-\frac{1}{2\,d x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/d/x^3,x)
```

```
[Out] -1/2/d/x^2
```

Maxima [A] time = 1.00634, size = 11, normalized size = 1.1

$$-\frac{1}{2\,d x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/d/x^3,x, algorithm="maxima")
```

```
[Out] -1/2/(d*x^2)
```

Fricas [A] time = 1.49792, size = 19, normalized size = 1.9

$$-\frac{1}{2\,d x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/d/x^3,x, algorithm="fricas")
```

```
[Out] -1/2/(d*x^2)
```

Sympy [A] time = 0.062868, size = 8, normalized size = 0.8

$$-\frac{1}{2dx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/d/x**3,x)
```

```
[Out] -1/(2*d*x**2)
```

Giac [A] time = 1.2243, size = 11, normalized size = 1.1

$$-\frac{1}{2\,dx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/d/x^3,x, algorithm="giac")
```

```
[Out] -1/2/(d*x^2)
```

3.24 $\int \frac{1}{cx^2+dx^3}\,dx$

Optimal. Leaf size=28

$$-\frac{d\log(x)}{c^2}+\frac{d\log(c+dx)}{c^2}-\frac{1}{cx}$$

```
[Out] -(1/(c*x)) - (d*Log[x])/c^2 + (d*Log[c + d*x])/c^2
```

Rubi [A] time = 0.0151371, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1593, 44}

$$-\frac{d\log(x)}{c^2}+\frac{d\log(c+dx)}{c^2}-\frac{1}{cx}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2 + d*x^3)^(-1),x]
```

```
[Out] -(1/(c*x)) - (d*Log[x])/c^2 + (d*Log[c + d*x])/c^2
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{1}{cx^2 + dx^3} dx = \int \frac{1}{x^2(c + dx)} dx$$
$$= \int \left(\frac{1}{cx^2} - \frac{d}{c^2 x} + \frac{d^2}{c^2(c + dx)}\right) dx$$
$$= -\frac{1}{cx} - \frac{d\log(x)}{c^2} + \frac{d\log(c + dx)}{c^2}$$

Mathematica [A] time = 0.004111, size = 28, normalized size = 1.

$$-\frac{d\log(x)}{c^2} + \frac{d\log(c + dx)}{c^2} - \frac{1}{cx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2 + d*x^3)^(-1),x]
```

```
[Out] -(1/(c*x)) - (d*Log[x])/c^2 + (d*Log[c + d*x])/c^2
```

Maple [A] time = 0.007, size = 29, normalized size = 1.

$$-\frac{1}{cx} - \frac{d\ln(x)}{c^2} + \frac{d\ln(dx + c)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x^3+c*x^2),x)
```

```
[Out] -1/c/x-d*ln(x)/c^2+d*ln(d*x+c)/c^2
```

Maxima [A] time = 1.14695, size = 38, normalized size = 1.36

$$\frac{d\log(dx + c)}{c^2} - \frac{d\log(x)}{c^2} - \frac{1}{cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x^3+c*x^2),x, algorithm="maxima")
```

```
[Out] d*log(d*x + c)/c^2 - d*log(x)/c^2 - 1/(c*x)
```

Fricas [A] time = 1.22864, size = 61, normalized size = 2.18

$$\frac{dx\log(dx+c)-dx\log(x)-c}{c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x^3+c*x^2),x, algorithm="fricas")
```

```
[Out] (d*x*log(d*x + c) - d*x*log(x) - c)/(c^2*x)
```

Sympy [A] time = 0.435983, size = 19, normalized size = 0.68

$$-\frac{1}{cx}+\frac{d\left(-\log(x)+\log\left(\frac{c}{d}+x\right)\right)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x**3+c*x**2),x)
```

```
[Out] -1/(c*x) + d*(-log(x) + log(c/d + x))/c**2
```

Giac [A] time = 1.26542, size = 41, normalized size = 1.46

$$\frac{d\log(|dx+c|)}{c^2}-\frac{d\log(|x|)}{c^2}-\frac{1}{cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x^3+c*x^2),x, algorithm="giac")
```

```
[Out] d*log(abs(d*x + c))/c^2 - d*log(abs(x))/c^2 - 1/(c*x)
```

3.25 $\int \frac{1}{bx+dx^3} \, dx$

Optimal. Leaf size=22

$$\frac{\log(x)}{b} - \frac{\log\left(b + dx^2\right)}{2b}$$

```
[Out] Log[x]/b - Log[b + d*x^2]/(2*b)
```

Rubi [A] time = 0.0115629, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {1593, 266, 36, 29, 31}

$$\frac{\log(x)}{b} - \frac{\log\left(b + dx^2\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + d*x^3)^(-1),x]
```

```
[Out] Log[x]/b - Log[b + d*x^2]/(2*b)
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{bx+dx^3}\,dx &= \int \frac{1}{x\left(b+dx^2\right)}\,dx \\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x(b+dx)}\,dx, x, x^2\right) \\
&= \frac{\operatorname{Subst}\left(\int \frac{1}{x}\,dx, x, x^2\right)}{2b} - \frac{d\operatorname{Subst}\left(\int \frac{1}{b+dx}\,dx, x, x^2\right)}{2b} \\
&= \frac{\log(x)}{b} - \frac{\log\left(b+dx^2\right)}{2b}
\end{aligned}
$$

Mathematica [A] time = 0.0045165, size = 22, normalized size = 1.

$$\frac{\log(x)}{b} - \frac{\log\left(b+dx^2\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + d*x^3)^(-1),x]
```

```
[Out] Log[x]/b - Log[b + d*x^2]/(2*b)
```

Maple [A] time = 0.006, size = 21, normalized size = 1.

$$\frac{\ln(x)}{b} - \frac{\ln\left(dx^2+b\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x^3+b*x),x)
```

```
[Out] ln(x)/b-1/2*ln(d*x^2+b)/b
```

Maxima [A] time = 1.0347, size = 27, normalized size = 1.23

$$-\frac{\log\left(dx^2+b\right)}{2\,b}+\frac{\log\left(x\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x^3+b*x),x, algorithm="maxima")
```

```
[Out] -1/2*log(d*x^2 + b)/b + log(x)/b
```

Fricas [A] time = 1.28758, size = 49, normalized size = 2.23

$$-\frac{\log\left(dx^2+b\right)-2\,\log\left(x\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x^3+b*x),x, algorithm="fricas")
```

```
[Out] -1/2*(log(d*x^2 + b) - 2*log(x))/b
```

Sympy [A] time = 0.188181, size = 15, normalized size = 0.68

$$\frac{\log\left(x\right)}{b}-\frac{\log\left(\frac{b}{d}+x^2\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x**3+b*x),x)
```

```
[Out] log(x)/b - log(b/d + x**2)/(2*b)
```

Giac [A] time = 1.18816, size = 32, normalized size = 1.45

$$\frac{\log\left(x^2\right)}{2\,b} - \frac{\log\left(\left|dx^2 + b\right|\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x^3+b*x),x, algorithm="giac")
```

```
[Out] 1/2*log(x^2)/b - 1/2*log(abs(d*x^2 + b))/b
```

3.26 $\int \frac{1}{bx+cx^2+dx^3}\,dx$

Optimal. Leaf size=62

$$\frac{c\tanh^{-1}\left(\frac{c+2dx}{\sqrt{c^2-4bd}}\right)}{b\sqrt{c^2-4bd}}-\frac{\log\left(b+cx+dx^2\right)}{2b}+\frac{\log(x)}{b}$$

```
[Out] (c*ArcTanh[(c + 2*d*x)/Sqrt[c^2 - 4*b*d]])/(b*Sqrt[c^2 - 4*b*d]) + Log[x]/b
 - Log[b + c*x + d*x^2]/(2*b)
```

Rubi [A] time = 0.0545669, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1594, 705, 29, 634, 618, 206, 628}

$$\frac{c\tanh^{-1}\left(\frac{c+2dx}{\sqrt{c^2-4bd}}\right)}{b\sqrt{c^2-4bd}}-\frac{\log\left(b+cx+dx^2\right)}{2b}+\frac{\log(x)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2 + d*x^3)^(-1),x]
```

```
[Out] (c*ArcTanh[(c + 2*d*x)/Sqrt[c^2 - 4*b*d]])/(b*Sqrt[c^2 - 4*b*d]) + Log[x]/b
 - Log[b + c*x + d*x^2]/(2*b)
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{bx+cx^2+dx^3}\,dx &= \int \frac{1}{x\left(b+cx+dx^2\right)}\,dx\\
&= \frac{\int \frac{1}{x}\,dx}{b} + \frac{\int \frac{-c-dx}{b+cx+dx^2}\,dx}{b}\\
&= \frac{\log(x)}{b} - \frac{\int \frac{c+2dx}{b+cx+dx^2}\,dx}{2b} - \frac{c\int \frac{1}{b+cx+dx^2}\,dx}{2b}\\
&= \frac{\log(x)}{b} - \frac{\log\left(b+cx+dx^2\right)}{2b} + \frac{c\,\mathrm{Subst}\left(\int \frac{1}{c^2-4bd-x^2}\,dx, x, c+2dx\right)}{b}\\
&= \frac{c\tanh^{-1}\left(\frac{c+2dx}{\sqrt{c^2-4bd}}\right)}{b\sqrt{c^2-4bd}} + \frac{\log(x)}{b} - \frac{\log\left(b+cx+dx^2\right)}{2b}
\end{aligned}
$$

Mathematica [A] time = 0.0690686, size = 61, normalized size = 0.98

$$-\frac{\frac{2c\tan^{-1}\left(\frac{c+2dx}{\sqrt{4bd-c^2}}\right)}{\sqrt{4bd-c^2}}+\log(b+x(c+dx))-2\log(x)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2 + d*x^3)^(-1),x]
```

```
[Out] -((2*c*ArcTan[(c + 2*d*x)/Sqrt[-c^2 + 4*b*d]])/Sqrt[-c^2 + 4*b*d] - 2*Log[x
] + Log[b + x*(c + d*x)])/(2*b)
```

Maple [A] time = 0.007, size = 62, normalized size = 1.

$$\frac{\ln(x)}{b}-\frac{\ln\left(dx^2+cx+b\right)}{2b}-\frac{c}{b}\arctan\left((2dx+c)\frac{1}{\sqrt{4bd-c^2}}\right)\frac{1}{\sqrt{4bd-c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x^3+c*x^2+b*x),x)
```

```
[Out] ln(x)/b-1/2*ln(d*x^2+c*x+b)/b-1/b*c/(4*b*d-c^2)^(1/2)*arctan((2*d*x+c)/(4*b
*d-c^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x^3+c*x^2+b*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.31094, size = 494, normalized size = 7.97

$$\left[\frac{\sqrt{c^2-4\,bd}c\log\left(\frac{2\,d^2x^2+2\,cdx+c^2-2\,bd+\sqrt{c^2-4\,bd}(2\,dx+c)}{dx^2+cx+b}\right)-\left(c^2-4\,bd\right)\log\left(dx^2+cx+b\right)+2\left(c^2-4\,bd\right)\log\left(x\right)}{2\left(bc^2-4\,b^2d\right)},\ \frac{2\sqrt{-c^2+4\,b\ldots}}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x^3+c*x^2+b*x),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(c^2 - 4*b*d)*c*log((2*d^2*x^2 + 2*c*d*x + c^2 - 2*b*d + sqrt(c^2
 - 4*b*d)*(2*d*x + c))/(d*x^2 + c*x + b)) - (c^2 - 4*b*d)*log(d*x^2 + c*x +
 b) + 2*(c^2 - 4*b*d)*log(x))/(b*c^2 - 4*b^2*d), 1/2*(2*sqrt(-c^2 + 4*b*d)*
c*arctan(-sqrt(-c^2 + 4*b*d)*(2*d*x + c)/(c^2 - 4*b*d)) - (c^2 - 4*b*d)*log
(d*x^2 + c*x + b) + 2*(c^2 - 4*b*d)*log(x))/(b*c^2 - 4*b^2*d)]
```

Sympy [B] time = 1.93077, size = 564, normalized size = 9.1

$$\left(-\frac{c\sqrt{-4bd+c^2}}{2b\left(4bd-c^2\right)}-\frac{1}{2b}\right)\log\left(x+\frac{24b^4d^2\left(-\frac{c\sqrt{-4bd+c^2}}{2b\left(4bd-c^2\right)}-\frac{1}{2b}\right)^2-14b^3c^2d\left(-\frac{c\sqrt{-4bd+c^2}}{2b\left(4bd-c^2\right)}-\frac{1}{2b}\right)^2-12b^3d^2\left(-\frac{c\sqrt{-4bd+c^2}}{2b\left(4bd-c^2\right)}-\frac{1}{2b}\right)+2\ldots}{9bcd^2-2c^3\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x**3+c*x**2+b*x),x)
```

```
[Out] (-c*sqrt(-4*b*d + c**2)/(2*b*(4*b*d - c**2)) - 1/(2*b))*log(x + (24*b**4*d*
*2*(-c*sqrt(-4*b*d + c**2)/(2*b*(4*b*d - c**2)) - 1/(2*b))**2 - 14*b**3*c**
2*d*(-c*sqrt(-4*b*d + c**2)/(2*b*(4*b*d - c**2)) - 1/(2*b))**2 - 12*b**3*d*
*2*(-c*sqrt(-4*b*d + c**2)/(2*b*(4*b*d - c**2)) - 1/(2*b)) + 2*b**2*c**4*(-
c*sqrt(-4*b*d + c**2)/(2*b*(4*b*d - c**2)) - 1/(2*b))**2 + 3*b**2*c**2*d*(-
c*sqrt(-4*b*d + c**2)/(2*b*(4*b*d - c**2)) - 1/(2*b)) - 12*b**2*d**2 + 11*b
*c**2*d - 2*c**4)/(9*b*c*d**2 - 2*c**3*d)) + (c*sqrt(-4*b*d + c**2)/(2*b*(4
*b*d - c**2)) - 1/(2*b))*log(x + (24*b**4*d**2*(c*sqrt(-4*b*d + c**2)/(2*b*
(4*b*d - c**2)) - 1/(2*b))**2 - 14*b**3*c**2*d*(c*sqrt(-4*b*d + c**2)/(2*b*
(4*b*d - c**2)) - 1/(2*b))**2 - 12*b**3*d**2*(c*sqrt(-4*b*d + c**2)/(2*b*(4
*b*d - c**2)) - 1/(2*b)) + 2*b**2*c**4*(c*sqrt(-4*b*d + c**2)/(2*b*(4*b*d -
 c**2)) - 1/(2*b))**2 + 3*b**2*c**2*d*(c*sqrt(-4*b*d + c**2)/(2*b*(4*b*d -
c**2)) - 1/(2*b)) - 12*b**2*d**2 + 11*b*c**2*d - 2*c**4)/(9*b*c*d**2 - 2*c*
```

```
*3*d)) + log(x)/b
```

Giac [A] time = 1.12708, size = 84, normalized size = 1.35

$$-\frac{c \arctan\left(\frac{2\,dx+c}{\sqrt{-c^2+4\,bd}}\right)}{\sqrt{-c^2+4\,bd}b}-\frac{\log\left(dx^2+cx+b\right)}{2\,b}+\frac{\log\left(|x|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x^3+c*x^2+b*x),x, algorithm="giac")
```

```
[Out] -c*arctan((2*d*x + c)/sqrt(-c^2 + 4*b*d))/(sqrt(-c^2 + 4*b*d)*b) - 1/2*log(
d*x^2 + c*x + b)/b + log(abs(x))/b
```

3.27 $\int \frac{1}{a+dx^3}\,dx$

Optimal. Leaf size=115

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{d}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{d}x\right)}{3a^{2/3}\sqrt[3]{d}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\sqrt[3]{d}}$$

```
[Out] -(ArcTan[(a^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(2/3)*d^(1/3
))) + Log[a^(1/3) + d^(1/3)*x]/(3*a^(2/3)*d^(1/3)) - Log[a^(2/3) - a^(1/3)*
d^(1/3)*x + d^(2/3)*x^2]/(6*a^(2/3)*d^(1/3))
```

Rubi [A] time = 0.0630292, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {200, 31, 634, 617, 204, 628}

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{d}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{d}x\right)}{3a^{2/3}\sqrt[3]{d}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\sqrt[3]{d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + d*x^3)^(-1),x]
```

```
[Out] -(ArcTan[(a^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(2/3)*d^(1/3
))) + Log[a^(1/3) + d^(1/3)*x]/(3*a^(2/3)*d^(1/3)) - Log[a^(2/3) - a^(1/3)*
d^(1/3)*x + d^(2/3)*x^2]/(6*a^(2/3)*d^(1/3))
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{a+dx^3}\,dx &= \frac{\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{d}x}\,dx}{3a^{2/3}}+\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{d}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{d}x+d^{2/3}x^2}\,dx}{3a^{2/3}}\\
&= \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{d}x\right)}{3a^{2/3}\sqrt[3]{d}}+\frac{\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{d}x+d^{2/3}x^2}\,dx}{2\sqrt[3]{a}}-\frac{\int \frac{-\sqrt[3]{a}\sqrt[3]{d}+2d^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{d}x+d^{2/3}x^2}\,dx}{6a^{2/3}\sqrt[3]{d}}\\
&= \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{d}x\right)}{3a^{2/3}\sqrt[3]{d}}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{d}}+\frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{a}}\right)}{a^{2/3}\sqrt[3]{d}}\\
&= -\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\sqrt[3]{d}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{d}x\right)}{3a^{2/3}\sqrt[3]{d}}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{d}}
\end{aligned}
$$

Mathematica [A] time = 0.0266957, size = 89, normalized size = 0.77

$$-\frac{\log \left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{d}x+d^{2/3}x^2\right)-2\log \left(\sqrt[3]{a}+\sqrt[3]{d}x\right)+2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{6a^{2/3}\sqrt[3]{d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + d*x^3)^(-1),x]
```

```
[Out] -(2*Sqrt[3]*ArcTan[(1 - (2*d^(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*Log[a^(1/3) + d
^(1/3)*x] + Log[a^(2/3) - a^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(6*a^(2/3)*d^(1
/3))
```

Maple [A] time = 0.004, size = 91, normalized size = 0.8

$$\frac{1}{3d}\ln\left(x+\sqrt[3]{\frac{a}{d}}\right)\left(\frac{a}{d}\right)^{-\frac{2}{3}}-\frac{1}{6d}\ln\left(x^2-\sqrt[3]{\frac{a}{d}}x+\left(\frac{a}{d}\right)^{\frac{2}{3}}\right)\left(\frac{a}{d}\right)^{-\frac{2}{3}}+\frac{\sqrt{3}}{3d}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{d}}}-1\right)\right)\left(\frac{a}{d}\right)^{-\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x^3+a),x)
```

```
[Out] 1/3/d/(a/d)^(2/3)*ln(x+(a/d)^(1/3))-1/6/d/(a/d)^(2/3)*ln(x^2-(a/d)^(1/3)*x+
(a/d)^(2/3))+1/3/d/(a/d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/d)^(1/3)*x-
1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.22155, size = 749, normalized size = 6.51

$$\left[\frac{3\sqrt{\frac{1}{3}}ad\sqrt{-\frac{\left(a^2d\right)^{\frac{1}{3}}}{d}}\log\left(\frac{2\,adx^3-3\left(a^2d\right)^{\frac{1}{3}}ax-a^2+3\sqrt{\frac{1}{3}}\left(2\,adx^2+\left(a^2d\right)^{\frac{2}{3}}x-\left(a^2d\right)^{\frac{1}{3}}a\right)\sqrt{-\frac{\left(a^2d\right)^{\frac{1}{3}}}{d}}}{dx^3+a}\right)-\left(a^2d\right)^{\frac{2}{3}}\log\left(adx^2-\left(a^2d\right)^{\frac{2}{3}}x+\left(a^2d\right)^{\frac{1}{3}}a\right)}{6\,a^2d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*a*d*sqrt(-(a^2*d)^(1/3)/d)*log((2*a*d*x^3 - 3*(a^2*d)^(1/
3)*a*x - a^2 + 3*sqrt(1/3)*(2*a*d*x^2 + (a^2*d)^(2/3)*x - (a^2*d)^(1/3)*a)*
sqrt(-(a^2*d)^(1/3)/d))/(d*x^3 + a)) - (a^2*d)^(2/3)*log(a*d*x^2 - (a^2*d)^
(2/3)*x + (a^2*d)^(1/3)*a) + 2*(a^2*d)^(2/3)*log(a*d*x + (a^2*d)^(2/3)))/(a
^2*d), 1/6*(6*sqrt(1/3)*a*d*sqrt((a^2*d)^(1/3)/d)*arctan(sqrt(1/3)*(2*(a^2*
d)^(2/3)*x - (a^2*d)^(1/3)*a)*sqrt((a^2*d)^(1/3)/d)/a^2) - (a^2*d)^(2/3)*lo
g(a*d*x^2 - (a^2*d)^(2/3)*x + (a^2*d)^(1/3)*a) + 2*(a^2*d)^(2/3)*log(a*d*x
+ (a^2*d)^(2/3)))/(a^2*d)]
```

Sympy [A] time = 0.144555, size = 20, normalized size = 0.17

$$\operatorname{RootSum}\left(27t^3a^2d-1,\left(t\mapsto t\log\left(3ta+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a**2*d - 1, Lambda(_t, _t*log(3*_t*a + x)))
```

<u>Giac</u> [A] time = 1.15588, size = 151, normalized size = 1.31

$$-\frac{\left(-\frac{a}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,a}+\frac{\sqrt{3}\left(-ad^{2}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{d}\right)^{\frac{1}{3}}}\right)}{3\,ad}+\frac{\left(-ad^{2}\right)^{\frac{1}{3}}\log\left(x^{2}+x\left(-\frac{a}{d}\right)^{\frac{1}{3}}+\left(-\frac{a}{d}\right)^{\frac{2}{3}}\right)}{6\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*(-a/d)^(1/3)*log(abs(x - (-a/d)^(1/3)))/a + 1/3*sqrt(3)*(-a*d^2)^(1/3)
*arctan(1/3*sqrt(3)*(2*x + (-a/d)^(1/3))/(-a/d)^(1/3))/(a*d) + 1/6*(-a*d^2)
^(1/3)*log(x^2 + x*(-a/d)^(1/3) + (-a/d)^(2/3))/(a*d)
```

3.28 $\int (dx^3)^n \, dx$

Optimal. Leaf size=16

$$\frac{x\left(dx^3\right)^n}{3n+1}$$

```
[Out] (x*(d*x^3)^n)/(1 + 3*n)
```

Rubi [A] time = 0.0040014, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {15, 30}

$$\frac{x\left(dx^3\right)^n}{3n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x^3)^n,x]
```

```
[Out] (x*(d*x^3)^n)/(1 + 3*n)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \left(dx^3\right)^n \, dx &= \left(x^{-3n}\left(dx^3\right)^n\right)\int x^{3n}\,dx\\ &= \frac{x\left(dx^3\right)^n}{1+3n}\end{aligned}$$

Mathematica [A] time = 0.0018816, size = 16, normalized size = 1.

$$\frac{x\left(dx^3\right)^n}{3n+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x^3)^n,x]
```

```
[Out] (x*(d*x^3)^n)/(1 + 3*n)
```

Maple [A] time = 0.002, size = 17, normalized size = 1.1

$$\frac{x\left(dx^3\right)^n}{1+3\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3)^n,x)
```

```
[Out] x*(d*x^3)^n/(1+3*n)
```

Maxima [A] time = 1.13058, size = 23, normalized size = 1.44

$$\frac{d^n x x^{3\,n}}{3\,n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3)^n,x, algorithm="maxima")
```

```
[Out] d^n*x*x^(3*n)/(3*n + 1)
```

Fricas [A] time = 1.37066, size = 31, normalized size = 1.94

$$\frac{\left(dx^3\right)^n x}{3\,n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3)^n,x, algorithm="fricas")
```

```
[Out] (d*x^3)^n*x/(3*n + 1)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3)**n,x)
```

```
[Out] Exception raised: TypeError
```

Giac [A] time = 1.1404, size = 22, normalized size = 1.38

$$\frac{\left(dx^3\right)^n x}{3\,n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3)^n,x, algorithm="giac")
```

```
[Out] (d*x^3)^n*x/(3*n + 1)
```

3.29 $\int (cx^2 + dx^3)^n \, dx$

Optimal. Leaf size=55

$$\frac{x\left(\frac{dx}{c}+1\right)^{-n}\left(cx^2+dx^3\right)^n \text{Hypergeometric2F1}\left(-n,2n+1,2(n+1),-\frac{dx}{c}\right)}{2n+1}$$

```
[Out] (x*(c*x^2 + d*x^3)^n*Hypergeometric2F1[-n, 1 + 2*n, 2*(1 + n), -((d*x)/c)])
/((1 + 2*n)*(1 + (d*x)/c)^n)
```

Rubi [A] time = 0.0192491, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2011, 66, 64}

$$\frac{x\left(\frac{dx}{c}+1\right)^{-n}\left(cx^2+dx^3\right)^n \text{Hypergeometric2F1}\left(-n,2n+1,2(n+1),-\frac{dx}{c}\right)}{2n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2 + d*x^3)^n,x]
```

```
[Out] (x*(c*x^2 + d*x^3)^n*Hypergeometric2F1[-n, 1 + 2*n, 2*(1 + n), -((d*x)/c)])
/((1 + 2*n)*(1 + (d*x)/c)^n)
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c^IntPar
t[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart[n], Int[(b*x)^m*(1 + (d*
x)/c)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ
[n] && !GtQ[c, 0] && !GtQ[-(d/(b*c)), 0] && ((RationalQ[m] && !(EqQ[n, -
2^(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\begin{aligned}
\int \left(cx^2+dx^3\right)^n dx &= \left(x^{-2n}(c+dx)^{-n}\left(cx^2+dx^3\right)^n\right)\int x^{2n}(c+dx)^n\,dx \\
&= \left(x^{-2n}\left(1+\frac{dx}{c}\right)^{-n}\left(cx^2+dx^3\right)^n\right)\int x^{2n}\left(1+\frac{dx}{c}\right)^n dx \\
&= \frac{x\left(1+\frac{dx}{c}\right)^{-n}\left(cx^2+dx^3\right)^n {}_2F_1\left(-n,1+2n;2(1+n);-\frac{dx}{c}\right)}{1+2n}
\end{aligned}$$

Mathematica [A] time = 0.0121895, size = 53, normalized size = 0.96

$$\frac{x\left(x^2(c+dx)\right)^n\left(\frac{dx}{c}+1\right)^{-n}\text{Hypergeometric2F1}\left(-n,2n+1,2n+2,-\frac{dx}{c}\right)}{2n+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2 + d*x^3)^n,x]
```

```
[Out] (x*(x^2*(c + d*x))^n*Hypergeometric2F1[-n, 1 + 2*n, 2 + 2*n, -((d*x)/c)])/(
(1 + 2*n)*(1 + (d*x)/c)^n)
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$\int \left(dx^3+cx^2\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c*x^2)^n,x)
```

```
[Out] int((d*x^3+c*x^2)^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(dx^3 + cx^2\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2)^n,x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c*x^2)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(dx^3 + cx^2\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2)^n,x, algorithm="fricas")
```

```
[Out] integral((d*x^3 + c*x^2)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2 + dx^3\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c*x**2)**n,x)
```

```
[Out] Integral((c*x**2 + d*x**3)**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(dx^3 + cx^2\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2)^n,x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c*x^2)^n, x)
```

3.30 $\int \left(bx + dx^3\right)^n \, dx$

Optimal. Leaf size=53

$$\frac{x\left(b+dx^2\right)\left(bx+dx^3\right)^n \text{Hypergeometric2F1}\left(1,\frac{3(n+1)}{2},\frac{n+3}{2},-\frac{dx^2}{b}\right)}{b(n+1)}$$

```
[Out] (x*(b + d*x^2)*(b*x + d*x^3)^n*Hypergeometric2F1[1, (3*(1 + n))/2, (3 + n)/
2, -((d*x^2)/b)])/(b*(1 + n))
```

Rubi [A] time = 0.0227277, antiderivative size = 59, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2011, 365, 364}

$$\frac{x\left(\frac{dx^2}{b}+1\right)^{-n}\left(bx+dx^3\right)^n \text{Hypergeometric2F1}\left(-n,\frac{n+1}{2},\frac{n+3}{2},-\frac{dx^2}{b}\right)}{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + d*x^3)^n,x]
```

```
[Out] (x*(b*x + d*x^3)^n*Hypergeometric2F1[-n, (1 + n)/2, (3 + n)/2, -((d*x^2)/b)
])/((1 + n)*(1 + (d*x^2)/b)^n)
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \left(bx+dx^3\right)^n\,dx &= \left(x^{-n}\left(b+dx^2\right)^{-n}\left(bx+dx^3\right)^n\right)\int x^n\left(b+dx^2\right)^n\,dx\\ &= \left(x^{-n}\left(1+\frac{dx^2}{b}\right)^{-n}\left(bx+dx^3\right)^n\right)\int x^n\left(1+\frac{dx^2}{b}\right)^n\,dx\\ &= \frac{x\left(1+\frac{dx^2}{b}\right)^{-n}\left(bx+dx^3\right)^n\,{}_2F_1\left(-n,\frac{1+n}{2};\frac{3+n}{2};-\frac{dx^2}{b}\right)}{1+n}\end{aligned}$$

Mathematica [A] time = 0.0123611, size = 61, normalized size = 1.15

$$\frac{x\left(x\left(b+dx^2\right)\right)^n\left(\frac{dx^2}{b}+1\right)^{-n}\text{Hypergeometric2F1}\left(-n,\frac{n+1}{2},\frac{n+1}{2}+1,-\frac{dx^2}{b}\right)}{n+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + d*x^3)^n,x]
```

```
[Out] (x*(x*(b + d*x^2))^n*Hypergeometric2F1[-n, (1 + n)/2, 1 + (1 + n)/2, -((d*x
^2)/b)])/((1 + n)*(1 + (d*x^2)/b)^n)
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$\int \left(dx^3+bx\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+b*x)^n,x)
```

```
[Out] int((d*x^3+b*x)^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dx^3 + bx)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+b*x)^n,x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + b*x)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((dx^3 + bx)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+b*x)^n,x, algorithm="fricas")
```

```
[Out] integral((d*x^3 + b*x)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (bx + dx^3)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+b*x)**n,x)
```

```
[Out] Integral((b*x + d*x**3)**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx^3 + bx)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+b*x)^n,x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + b*x)^n, x)
```

3.31 $\int \left(bx+cx^2+dx^3\right)^n\,dx$

Optimal. Leaf size=132

$$\frac{x\left(\frac{2dx}{c-\sqrt{c^2-4bd}}+1\right)^{-n}\left(\frac{2dx}{\sqrt{c^2-4bd}+c}+1\right)^{-n}\left(bx+cx^2+dx^3\right)^nF_1\left(n+1;-n,-n;n+2;-\frac{2dx}{c-\sqrt{c^2-4bd}},-\frac{2dx}{c+\sqrt{c^2-4bd}}\right)}{n+1}$$

```
[Out] (x*(b*x + c*x^2 + d*x^3)^n*AppellF1[1 + n, -n, -n, 2 + n, (-2*d*x)/(c - Sqr
t[c^2 - 4*b*d]), (-2*d*x)/(c + Sqrt[c^2 - 4*b*d])])/((1 + n)*(1 + (2*d*x)/(
c - Sqrt[c^2 - 4*b*d]))^n*(1 + (2*d*x)/(c + Sqrt[c^2 - 4*b*d]))^n)
```

Rubi [A] time = 0.158354, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1908, 759, 133}

$$\frac{x\left(\frac{2dx}{c-\sqrt{c^2-4bd}}+1\right)^{-n}\left(\frac{2dx}{\sqrt{c^2-4bd}+c}+1\right)^{-n}\left(bx+cx^2+dx^3\right)^nF_1\left(n+1;-n,-n;n+2;-\frac{2dx}{c-\sqrt{c^2-4bd}},-\frac{2dx}{c+\sqrt{c^2-4bd}}\right)}{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2 + d*x^3)^n,x]
```

```
[Out] (x*(b*x + c*x^2 + d*x^3)^n*AppellF1[1 + n, -n, -n, 2 + n, (-2*d*x)/(c - Sqr
t[c^2 - 4*b*d]), (-2*d*x)/(c + Sqrt[c^2 - 4*b*d])])/((1 + n)*(1 + (2*d*x)/(
c - Sqrt[c^2 - 4*b*d]))^n*(1 + (2*d*x)/(c + Sqrt[c^2 - 4*b*d]))^n)
```

Rule 1908

```
Int[((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_), x_Symbol
] :> Dist[(a*x^q + b*x^n + c*x^(2*n - q))^p/(x^(p*q)*(a + b*x^(n - q) + c*x
^(2*(n - q)))^p), Int[x^(p*q)*(a + b*x^(n - q) + c*x^(2*(n - q)))^p, x], x]
 /; FreeQ[{a, b, c, n, p, q}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !Int
egerQ[p]
```

Rule 759

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - (e*(b - q))/(2*c)))^p*(1 - (d + e*x)/(d - (e*(b + q))/(2*c)))
^p), Subst[Int[x^m*Simp[1 - x/(d - (e*(b - q))/(2*c)), x]^p*Simp[1 - x/(d -
```

```
 (e*(b + q))/(2*c)), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*
d - b*e, 0] &&  !IntegerQ[p]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\begin{aligned}\int \left(bx+cx^2+dx^3\right)^n\,dx &= \left(x^{-n}\left(b+cx+dx^2\right)^{-n}\left(bx+cx^2+dx^3\right)^n\right)\int x^n\left(b+cx+dx^2\right)^n\,dx\\ &= \left(x^{-n}\left(1+\frac{2dx}{c-\sqrt{c^2-4bd}}\right)^{-n}\left(1+\frac{2dx}{c+\sqrt{c^2-4bd}}\right)^{-n}\left(bx+cx^2+dx^3\right)^n\right)\text{Subst}\left(\int x^n\left(1+\frac{}{c-}\right.\right.\\ &= \frac{x\left(1+\frac{2dx}{c-\sqrt{c^2-4bd}}\right)^{-n}\left(1+\frac{2dx}{c+\sqrt{c^2-4bd}}\right)^{-n}\left(bx+cx^2+dx^3\right)^n F_1\left(1+n;-n,-n;2+n;-\frac{2dx}{c-\sqrt{c^2-4bd}},-\right.}{1+n}\end{aligned}$$

Mathematica [A] time = 0.283606, size = 157, normalized size = 1.19

$$\frac{x\left(\frac{-\sqrt{c^2-4bd}+c+2dx}{c-\sqrt{c^2-4bd}}\right)^{-n}\left(\frac{\sqrt{c^2-4bd}+c+2dx}{\sqrt{c^2-4bd}+c}\right)^{-n}(x(b+x(c+dx)))^nF_1\left(n+1;-n,-n;n+2;-\frac{2dx}{c+\sqrt{c^2-4bd}},\frac{2dx}{\sqrt{c^2-4bd}-c}\right)}{n+1}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(b*x + c*x^2 + d*x^3)^n,x]
```

```
[Out] (x*(x*(b + x*(c + d*x)))^n*AppellF1[1 + n, -n, -n, 2 + n, (-2*d*x)/(c + Sqr
t[c^2 - 4*b*d]), (2*d*x)/(-c + Sqrt[c^2 - 4*b*d])])/((1 + n)*((c - Sqrt[c^2
 - 4*b*d] + 2*d*x)/(c - Sqrt[c^2 - 4*b*d]))^n*((c + Sqrt[c^2 - 4*b*d] + 2*d
*x)/(c + Sqrt[c^2 - 4*b*d]))^n)
```

Maple [F] time = 0.02, size = 0, normalized size = 0.

$$\int \left(dx^3+cx^2+bx\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c*x^2+b*x)^n,x)
```

```
[Out] int((d*x^3+c*x^2+b*x)^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(dx^3 + cx^2 + bx\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x)^n,x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c*x^2 + b*x)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(dx^3 + cx^2 + bx\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x)^n,x, algorithm="fricas")
```

```
[Out] integral((d*x^3 + c*x^2 + b*x)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx + cx^2 + dx^3\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c*x**2+b*x)**n,x)
```

```
[Out] Integral((b*x + c*x**2 + d*x**3)**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(dx^3 + cx^2 + bx\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x)^n,x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c*x^2 + b*x)^n, x)
```

3.32 $\int \left(a+dx^3\right)^n \, dx$

Optimal. Leaf size=35

$$\frac{x\left(a+dx^3\right)^{n+1} {}_2F_1\left(1,n+\frac{4}{3};\frac{4}{3};-\frac{dx^3}{a}\right)}{a}$$

```
[Out] (x*(a + d*x^3)^(1 + n)*Hypergeometric2F1[1, 4/3 + n, 4/3, -((d*x^3)/a)])/a
```

Rubi [A] time = 0.0088327, antiderivative size = 44, normalized size of antiderivative = 1.26, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {246, 245}

$$x\left(a+dx^3\right)^n\left(\frac{dx^3}{a}+1\right)^{-n} {}_2F_1\left(\frac{1}{3},-n;\frac{4}{3};-\frac{dx^3}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + d*x^3)^n,x]
```

```
[Out] (x*(a + d*x^3)^n*Hypergeometric2F1[1/3, -n, 4/3, -((d*x^3)/a)])/(1 + (d*x^3
)/a)^n
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\int \left(a+dx^3\right)^n\,dx = \left(\left(a+dx^3\right)^n\left(1+\frac{dx^3}{a}\right)^{-n}\right)\int \left(1+\frac{dx^3}{a}\right)^n\,dx$$
$$= x\left(a+dx^3\right)^n\left(1+\frac{dx^3}{a}\right)^{-n}\,{}_2F_1\left(\frac{1}{3},-n;\frac{4}{3};-\frac{dx^3}{a}\right)$$

Mathematica [C] time = 0.150824, size = 196, normalized size = 5.6

$$\frac{2^{-n}\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{d}x\right)\left(\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{d}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}\right)^{-n}\left(\frac{i\left(\frac{\sqrt[3]{d}x}{\sqrt[3]{a}}+1\right)}{\sqrt{3}+3i}\right)^{-n}\left(a+dx^3\right)^n F_1\left(n+1;-n,-n;n+2;-\frac{i\left(\sqrt[3]{d}x+(-1)^{2/3}\sqrt[3]{a}\right)}{\sqrt{3}\sqrt[3]{a}},\frac{-\frac{2i\sqrt[3]{d}x}{\sqrt[3]{a}}+\sqrt{3}+i}{3i+\sqrt{3}}\right)}{\sqrt[3]{d}(n+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + d*x^3)^n,x]
```

```
[Out] (((-1)^(2/3)*a^(1/3) + d^(1/3)*x)*(a + d*x^3)^n*AppellF1[1 + n, -n, -n, 2 +
 n, ((-I)*((-1)^(2/3)*a^(1/3) + d^(1/3)*x))/(Sqrt[3]*a^(1/3)), (I + Sqrt[3]
 - ((2*I)*d^(1/3)*x)/a^(1/3))/(3*I + Sqrt[3])])/(2^n*d^(1/3)*(1 + n)*((a^(1
/3) + (-1)^(2/3)*d^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3)))^n*((I*(1 + (d^(1/3)
*x)/a^(1/3)))/(3*I + Sqrt[3]))^n)
```

Maple [F] time = 0.029, size = 0, normalized size = 0.

$$\int \left(dx^3+a\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+a)^n,x)
```

```
[Out] int((d*x^3+a)^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(dx^3+a\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+a)^n,x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + a)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(dx^3+a\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+a)^n,x, algorithm="fricas")
```

```
[Out] integral((d*x^3 + a)^n, x)
```

Sympy [C] time = 16.0086, size = 34, normalized size = 0.97

$$\frac{a^n x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},-n\\\frac{4}{3}\end{matrix}\right|\frac{dx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+a)**n,x)
```

```
[Out] a**n*x*gamma(1/3)*hyper((1/3, -n), (4/3,), d*x**3*exp_polar(I*pi)/a)/(3*gam
ma(4/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(dx^3+a\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+a)^n,x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + a)^n, x)
```

3.33 $\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^4\,dx$

Optimal. Leaf size=270

$$\frac{2}{9}c^2\left(48a^2d^4+120ac^3d^2+35c^6\right)\left(\frac{c}{d}+x\right)^9+\frac{4}{13}cd^4\left(4ad^2+7c^3\right)\left(\frac{c}{d}+x\right)^{13}-\frac{8}{11}c^3d^2\left(12ad^2+7c^3\right)\left(\frac{c}{d}+x\right)^{11}-\frac{8c^4\left(4ad\right.}{}$$

```
[Out] (c^4*(c^3 + 4*a*d^2)^4*x)/d^8 - (8*c^5*(c^3 + 4*a*d^2)^3*(c/d + x)^3)/(3*d^
6) + (4*c^3*(c^3 + 4*a*d^2)^2*(7*c^3 + 4*a*d^2)*(c/d + x)^5)/(5*d^4) - (8*c
^4*(c^3 + 4*a*d^2)*(7*c^3 + 12*a*d^2)*(c/d + x)^7)/(7*d^2) + (2*c^2*(35*c^6
 + 120*a*c^3*d^2 + 48*a^2*d^4)*(c/d + x)^9)/9 - (8*c^3*d^2*(7*c^3 + 12*a*d^
2)*(c/d + x)^11)/11 + (4*c*d^4*(7*c^3 + 4*a*d^2)*(c/d + x)^13)/13 - (8*c^2*
d^6*(c/d + x)^15)/15 + (d^8*(c/d + x)^17)/17
```

Rubi [A] time = 0.538616, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {1106, 1090}

$$\frac{2}{9}c^2\left(48a^2d^4+120ac^3d^2+35c^6\right)\left(\frac{c}{d}+x\right)^9+\frac{4}{13}cd^4\left(4ad^2+7c^3\right)\left(\frac{c}{d}+x\right)^{13}-\frac{8}{11}c^3d^2\left(12ad^2+7c^3\right)\left(\frac{c}{d}+x\right)^{11}-\frac{8c^4\left(4ad\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^4,x]
```

```
[Out] (c^4*(c^3 + 4*a*d^2)^4*x)/d^8 - (8*c^5*(c^3 + 4*a*d^2)^3*(c/d + x)^3)/(3*d^
6) + (4*c^3*(c^3 + 4*a*d^2)^2*(7*c^3 + 4*a*d^2)*(c/d + x)^5)/(5*d^4) - (8*c
^4*(c^3 + 4*a*d^2)*(7*c^3 + 12*a*d^2)*(c/d + x)^7)/(7*d^2) + (2*c^2*(35*c^6
 + 120*a*c^3*d^2 + 48*a^2*d^4)*(c/d + x)^9)/9 - (8*c^3*d^2*(7*c^3 + 12*a*d^
2)*(c/d + x)^11)/11 + (4*c*d^4*(7*c^3 + 4*a*d^2)*(c/d + x)^13)/13 - (8*c^2*
d^6*(c/d + x)^15)/15 + (d^8*(c/d + x)^17)/17
```

Rule 1106

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1090

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandInteg
rand[(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^4\,dx = \text{Subst}\left(\int \left(c\left(4a+\frac{c^3}{d^2}\right)-2c^2x^2+d^2x^4\right)^4\,dx, x, \frac{c}{d}+x\right)$$

$$= \text{Subst}\left(\int \left(\frac{\left(c^4+4acd^2\right)^4}{d^8}-\frac{8c^5\left(c^3+4ad^2\right)^3x^2}{d^6}+\frac{24c^6\left(c^3+4ad^2\right)^2\left(\frac{7}{6}+\frac{2ad^2}{3c^3}\right)x}{d^4}\right.\right.$$

$$= \frac{c^4\left(c^3+4ad^2\right)^4x}{d^8}-\frac{8c^5\left(c^3+4ad^2\right)^3\left(\frac{c}{d}+x\right)^3}{3d^6}+\frac{4c^3\left(c^3+4ad^2\right)^2\left(7c^3+4ad^2\right)\left(\frac{c}{d}\right.}{5d^4}$$

Mathematica [A] time = 0.0355397, size = 285, normalized size = 1.06

$$\frac{32}{9}c^2x^9\left(3a^2d^4+120ac^3d^2+8c^6\right)+\frac{256}{5}a^2c^3x^5\left(ad^2+6c^3\right)+512a^2c^5dx^6+256a^3c^4dx^4+\frac{1024}{3}a^3c^5x^3+256a^4c^4x+\frac{16}{13}c$$

Antiderivative was successfully verified.

```
[In] Integrate[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^4,x]
```

```
[Out] 256*a^4*c^4*x + (1024*a^3*c^5*x^3)/3 + 256*a^3*c^4*d*x^4 + (256*a^2*c^3*(6*
c^3 + a*d^2)*x^5)/5 + 512*a^2*c^5*d*x^6 + (256*a*c^4*(4*c^3 + 9*a*d^2)*x^7)
/7 + 96*a*c^3*d*(4*c^3 + a*d^2)*x^8 + (32*c^2*(8*c^6 + 120*a*c^3*d^2 + 3*a^
2*d^4)*x^9)/9 + (256*c^4*d*(2*c^3 + 5*a*d^2)*x^10)/5 + (64*c^3*d^2*(28*c^3
+ 15*a*d^2)*x^11)/11 + (16*c^2*d^3*(28*c^3 + 3*a*d^2)*x^12)/3 + (16*c*d^4*(
70*c^3 + a*d^2)*x^13)/13 + 32*c^3*d^5*x^14 + (112*c^2*d^6*x^15)/15 + c*d^7*
x^16 + (d^8*x^17)/17
```

Maple [A] time = 0.003, size = 392, normalized size = 1.5

$$\frac{d^8x^{17}}{17}+cd^7x^{16}+\frac{112\,c^2d^6x^{15}}{15}+32\,c^3d^5x^{14}+\frac{\left(2\left(8\,acd^2+16\,c^4\right)d^4+1088\,c^4d^4\right)x^{13}}{13}+\frac{\left(64\,ac^2d^5+16\left(8\,acd^2+16\,c^4\right)c\right.}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^4,x)
```

```
[Out] 1/17*d^8*x^17+c*d^7*x^16+112/15*c^2*d^6*x^15+32*c^3*d^5*x^14+1/13*(2*(8*a*c
*d^2+16*c^4)*d^4+1088*c^4*d^4)*x^13+1/12*(64*a*c^2*d^5+16*(8*a*c*d^2+16*c^4
)*c*d^3+1536*c^5*d^3)*x^12+1/11*(576*a*c^3*d^4+48*(8*a*c*d^2+16*c^4)*c^2*d^
2+1024*c^6*d^2)*x^11+1/10*(2048*a*c^4*d^3+64*(8*a*c*d^2+16*c^4)*c^3*d)*x^10
+1/9*(32*a^2*c^2*d^4+3584*a*c^5*d^2+(8*a*c*d^2+16*c^4)^2)*x^9+1/8*(256*a^2*
c^3*d^3+2048*a*c^6*d+64*a*c^2*d*(8*a*c*d^2+16*c^4))*x^8+1/7*(1792*a^2*c^4*d
^2+64*a*c^3*(8*a*c*d^2+16*c^4))*x^7+512*a^2*c^5*d*x^6+1/5*(32*a^2*c^2*(8*a*
c*d^2+16*c^4)+1024*a^2*c^6)*x^5+256*a^3*c^4*d*x^4+1024/3*a^3*c^5*x^3+256*a^
4*c^4*x
```

Maxima [A] time = 1.18094, size = 502, normalized size = 1.86

$$\frac{1}{17}\,d^8x^{17}+cd^7x^{16}+\frac{32}{5}\,c^2d^6x^{15}+\frac{128}{7}\,c^3d^5x^{14}+\frac{256}{13}\,c^4d^4x^{13}+\frac{256}{9}\,c^8x^9+256\,a^4c^4x+\frac{256}{15}\,\big(3\,d^2x^5+15\,cdx^4+20\,c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^4,x, algorithm="maxima")
```

```
[Out] 1/17*d^8*x^17 + c*d^7*x^16 + 32/5*c^2*d^6*x^15 + 128/7*c^3*d^5*x^14 + 256/1
3*c^4*d^4*x^13 + 256/9*c^8*x^9 + 256*a^4*c^4*x + 256/15*(3*d^2*x^5 + 15*c*d
*x^4 + 20*c^2*x^3)*a^3*c^3 + 256/55*(5*d^2*x^11 + 22*c*d*x^10)*c^6 + 32/105
*(35*d^4*x^9 + 315*c*d^3*x^8 + 720*c^2*d^2*x^7 + 1008*c^4*x^5 + 120*(3*d^2*
x^7 + 14*c*d*x^6)*c^2)*a^2*c^2 + 32/143*(33*d^4*x^13 + 286*c*d^3*x^12 + 624
*c^2*d^2*x^11)*c^4 + 16/15015*(1155*d^6*x^13 + 15015*c*d^5*x^12 + 65520*c^2
*d^4*x^11 + 96096*c^3*d^3*x^10 + 137280*c^6*x^7 + 40040*(2*d^2*x^9 + 9*c*d*
x^8)*c^4 + 364*(45*d^4*x^11 + 396*c*d^3*x^10 + 880*c^2*d^2*x^9)*c^2)*a*c +
16/1365*(91*d^6*x^15 + 1170*c*d^5*x^14 + 5040*c^2*d^4*x^13 + 7280*c^3*d^3*x
^12)*c^2
```

Fricas [A] time = 1.03737, size = 684, normalized size = 2.53

$$\frac{1}{17}x^{17}d^8+x^{16}d^7c+\frac{112}{15}x^{15}d^6c^2+32x^{14}d^5c^3+\frac{1120}{13}x^{13}d^4c^4+\frac{16}{13}x^{13}d^6ca+\frac{448}{3}x^{12}d^3c^5+16x^{12}d^5c^2a+\frac{1792}{11}x^{11}d^2c^6+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^4,x, algorithm="fricas")
```

```
[Out] 1/17*x^17*d^8 + x^16*d^7*c + 112/15*x^15*d^6*c^2 + 32*x^14*d^5*c^3 + 1120/1
3*x^13*d^4*c^4 + 16/13*x^13*d^6*c*a + 448/3*x^12*d^3*c^5 + 16*x^12*d^5*c^2*
a + 1792/11*x^11*d^2*c^6 + 960/11*x^11*d^4*c^3*a + 512/5*x^10*d*c^7 + 256*x
^10*d^3*c^4*a + 256/9*x^9*c^8 + 1280/3*x^9*d^2*c^5*a + 32/3*x^9*d^4*c^2*a^2
 + 384*x^8*d*c^6*a + 96*x^8*d^3*c^3*a^2 + 1024/7*x^7*c^7*a + 2304/7*x^7*d^2
*c^4*a^2 + 512*x^6*d*c^5*a^2 + 1536/5*x^5*c^6*a^2 + 256/5*x^5*d^2*c^3*a^3 +
 256*x^4*d*c^4*a^3 + 1024/3*x^3*c^5*a^3 + 256*x*c^4*a^4
```

Sympy [A] time = 0.118807, size = 299, normalized size = 1.11

$$256a^4c^4x + \frac{1024a^3c^5x^3}{3} + 256a^3c^4dx^4 + 512a^2c^5dx^6 + 32c^3d^5x^{14} + \frac{112c^2d^6x^{15}}{15} + cd^7x^{16} + \frac{d^8x^{17}}{17} + x^{13}\left(\frac{16acd^6}{13} + \frac{1120c^4}{13}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c)**4,x)
```

```
[Out] 256*a**4*c**4*x + 1024*a**3*c**5*x**3/3 + 256*a**3*c**4*d*x**4 + 512*a**2*c
**5*d*x**6 + 32*c**3*d**5*x**14 + 112*c**2*d**6*x**15/15 + c*d**7*x**16 + d
**8*x**17/17 + x**13*(16*a*c*d**6/13 + 1120*c**4*d**4/13) + x**12*(16*a*c**
2*d**5 + 448*c**5*d**3/3) + x**11*(960*a*c**3*d**4/11 + 1792*c**6*d**2/11)
+ x**10*(256*a*c**4*d**3 + 512*c**7*d/5) + x**9*(32*a**2*c**2*d**4/3 + 1280
*a*c**5*d**2/3 + 256*c**8/9) + x**8*(96*a**2*c**3*d**3 + 384*a*c**6*d) + x*
*7*(2304*a**2*c**4*d**2/7 + 1024*a*c**7/7) + x**5*(256*a**3*c**3*d**2/5 + 1
536*a**2*c**6/5)
```

Giac [A] time = 1.14298, size = 374, normalized size = 1.39

$$\frac{1}{17}d^8x^{17} + cd^7x^{16} + \frac{112}{15}c^2d^6x^{15} + 32\,c^3d^5x^{14} + \frac{1120}{13}c^4d^4x^{13} + \frac{16}{13}acd^6x^{13} + \frac{448}{3}c^5d^3x^{12} + 16\,ac^2d^5x^{12} + \frac{1792}{11}c^6d^2x^{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^4,x, algorithm="giac")
```

```
[Out] 1/17*d^8*x^17 + c*d^7*x^16 + 112/15*c^2*d^6*x^15 + 32*c^3*d^5*x^14 + 1120/1
3*c^4*d^4*x^13 + 16/13*a*c*d^6*x^13 + 448/3*c^5*d^3*x^12 + 16*a*c^2*d^5*x^1
```

```
2 + 1792/11*c^6*d^2*x^11 + 960/11*a*c^3*d^4*x^11 + 512/5*c^7*d*x^10 + 256*a
*c^4*d^3*x^10 + 256/9*c^8*x^9 + 1280/3*a*c^5*d^2*x^9 + 32/3*a^2*c^2*d^4*x^9
 + 384*a*c^6*d*x^8 + 96*a^2*c^3*d^3*x^8 + 1024/7*a*c^7*x^7 + 2304/7*a^2*c^4
*d^2*x^7 + 512*a^2*c^5*d*x^6 + 1536/5*a^2*c^6*x^5 + 256/5*a^3*c^3*d^2*x^5 +
 256*a^3*c^4*d*x^4 + 1024/3*a^3*c^5*x^3 + 256*a^4*c^4*x
```

3.34 $\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^3 \, dx$

Optimal. Leaf size=171

$$48a^2c^3dx^4+64a^2c^4x^3+64a^3c^3x+\frac{4}{3}cd^2x^9\left(ad^2+20c^3\right)+12c^2dx^8\left(ad^2+2c^3\right)+\frac{32}{7}c^3x^7\left(9ad^2+2c^3\right)+\frac{48}{5}ac^2x^5\left(ad^2+\right.$$

```
[Out] 64*a^3*c^3*x + 64*a^2*c^4*x^3 + 48*a^2*c^3*d*x^4 + (48*a*c^2*(4*c^3 + a*d^2
)*x^5)/5 + 64*a*c^4*d*x^6 + (32*c^3*(2*c^3 + 9*a*d^2)*x^7)/7 + 12*c^2*d*(2*
c^3 + a*d^2)*x^8 + (4*c*d^2*(20*c^3 + a*d^2)*x^9)/3 + 16*c^3*d^3*x^10 + (60
*c^2*d^4*x^11)/11 + c*d^5*x^12 + (d^6*x^13)/13
```

Rubi [A] time = 0.0919228, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {2061}

$$48a^2c^3dx^4+64a^2c^4x^3+64a^3c^3x+\frac{4}{3}cd^2x^9\left(ad^2+20c^3\right)+12c^2dx^8\left(ad^2+2c^3\right)+\frac{32}{7}c^3x^7\left(9ad^2+2c^3\right)+\frac{48}{5}ac^2x^5\left(ad^2+\right.$$

Antiderivative was successfully verified.

```
[In] Int[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^3,x]
```


```
[Out] 64*a^3*c^3*x + 64*a^2*c^4*x^3 + 48*a^2*c^3*d*x^4 + (48*a*c^2*(4*c^3 + a*d^2
)*x^5)/5 + 64*a*c^4*d*x^6 + (32*c^3*(2*c^3 + 9*a*d^2)*x^7)/7 + 12*c^2*d*(2*
c^3 + a*d^2)*x^8 + (4*c*d^2*(20*c^3 + a*d^2)*x^9)/3 + 16*c^3*d^3*x^10 + (60
*c^2*d^4*x^11)/11 + c*d^5*x^12 + (d^6*x^13)/13
```


Rule 2061

```
Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I
GtQ[p, 0]
```

Rubi steps

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^3 \, dx=\int \left(64a^3c^3+192a^2c^4x^2+192a^2c^3dx^3+48ac^2\left(4c^3+ad^2\right)x^4+384ac^4dx^5+32c^3\right.$$

$$=64a^3c^3x+64a^2c^4x^3+48a^2c^3dx^4+\frac{48}{5}ac^2\left(4c^3+ad^2\right)x^5+64ac^4dx^6+\frac{32}{7}c^3\left(2c^3\right.$$

Mathematica [A] time = 0.0171551, size = 171, normalized size = 1.

$$48a^2c^3dx^4 + 64a^2c^4x^3 + 64a^3c^3x + \frac{4}{3}cd^2x^9\left(ad^2 + 20c^3\right) + 12c^2dx^8\left(ad^2 + 2c^3\right) + \frac{32}{7}c^3x^7\left(9ad^2 + 2c^3\right) + \frac{48}{5}ac^2x^5\left(ad^2 +\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^3,x]
```

```
[Out] 64*a^3*c^3*x + 64*a^2*c^4*x^3 + 48*a^2*c^3*d*x^4 + (48*a*c^2*(4*c^3 + a*d^2
)*x^5)/5 + 64*a*c^4*d*x^6 + (32*c^3*(2*c^3 + 9*a*d^2)*x^7)/7 + 12*c^2*d*(2*
c^3 + a*d^2)*x^8 + (4*c*d^2*(20*c^3 + a*d^2)*x^9)/3 + 16*c^3*d^3*x^10 + (60
*c^2*d^4*x^11)/11 + c*d^5*x^12 + (d^6*x^13)/13
```

Maple [A] time = 0.001, size = 231, normalized size = 1.4

$$\frac{d^6x^{13}}{13} + cd^5x^{12} + \frac{60\,c^2d^4x^{11}}{11} + 16\,c^3d^3x^{10} + \frac{\left(4\,acd^4 + 224\,c^4d^2 + d^2\left(8\,acd^2 + 16\,c^4\right)\right)x^9}{9} + \frac{\left(64\,ac^2d^3 + 128\,c^5d + 4\,cd\left(\right.\right.}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^3,x)
```

```
[Out] 1/13*d^6*x^13+c*d^5*x^12+60/11*c^2*d^4*x^11+16*c^3*d^3*x^10+1/9*(4*a*c*d^4+
224*c^4*d^2+d^2*(8*a*c*d^2+16*c^4))*x^9+1/8*(64*a*c^2*d^3+128*c^5*d+4*c*d*(
8*a*c*d^2+16*c^4))*x^8+1/7*(256*a*c^3*d^2+4*c^2*(8*a*c*d^2+16*c^4))*x^7+64*
a*c^4*d*x^6+1/5*(4*a*c*(8*a*c*d^2+16*c^4)+128*c^5*a+16*a^2*c^2*d^2)*x^5+48*
a^2*c^3*d*x^4+64*a^2*c^4*x^3+64*a^3*c^3*x
```

Maxima [A] time = 1.11101, size = 277, normalized size = 1.62

$$\frac{1}{13}\,d^6x^{13} + cd^5x^{12} + \frac{48}{11}\,c^2d^4x^{11} + \frac{32}{5}\,c^3d^3x^{10} + \frac{64}{7}\,c^6x^7 + 64\,a^3c^3x + \frac{16}{5}\left(3\,d^2x^5 + 15\,cdx^4 + 20\,c^2x^3\right)a^2c^2 + \frac{8}{3}\left(2\,d^2x^9 +\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^3,x, algorithm="maxima")
```

```
[Out] 1/13*d^6*x^13 + c*d^5*x^12 + 48/11*c^2*d^4*x^11 + 32/5*c^3*d^3*x^10 + 64/7*
c^6*x^7 + 64*a^3*c^3*x + 16/5*(3*d^2*x^5 + 15*c*d*x^4 + 20*c^2*x^3)*a^2*c^2
```

```
 + 8/3*(2*d^2*x^9 + 9*c*d*x^8)*c^4 + 4/105*(35*d^4*x^9 + 315*c*d^3*x^8 + 72
0*c^2*d^2*x^7 + 1008*c^4*x^5 + 120*(3*d^2*x^7 + 14*c*d*x^6)*c^2)*a*c + 4/16
5*(45*d^4*x^11 + 396*c*d^3*x^10 + 880*c^2*d^2*x^9)*c^2
```

Fricas [A] time = 1.17696, size = 383, normalized size = 2.24

$$\frac{1}{13}x^{13}d^6 + x^{12}d^5c + \frac{60}{11}x^{11}d^4c^2 + 16x^{10}d^3c^3 + \frac{80}{3}x^9d^2c^4 + \frac{4}{3}x^9d^4ca + 24x^8dc^5 + 12x^8d^3c^2a + \frac{64}{7}x^7c^6 + \frac{288}{7}x^7d^2c^3a + 64$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^3,x, algorithm="fricas")
```

```
[Out] 1/13*x^13*d^6 + x^12*d^5*c + 60/11*x^11*d^4*c^2 + 16*x^10*d^3*c^3 + 80/3*x^
9*d^2*c^4 + 4/3*x^9*d^4*c*a + 24*x^8*d*c^5 + 12*x^8*d^3*c^2*a + 64/7*x^7*c^
6 + 288/7*x^7*d^2*c^3*a + 64*x^6*d*c^4*a + 192/5*x^5*c^5*a + 48/5*x^5*d^2*c
^2*a^2 + 48*x^4*d*c^3*a^2 + 64*x^3*c^4*a^2 + 64*x*c^3*a^3
```

Sympy [A] time = 0.092107, size = 180, normalized size = 1.05

$$64a^3c^3x + 64a^2c^4x^3 + 48a^2c^3dx^4 + 64ac^4dx^6 + 16c^3d^3x^{10} + \frac{60c^2d^4x^{11}}{11} + cd^5x^{12} + \frac{d^6x^{13}}{13} + x^9\left(\frac{4acd^4}{3} + \frac{80c^4d^2}{3}\right) + x^8\left(12\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c)**3,x)
```

```
[Out] 64*a**3*c**3*x + 64*a**2*c**4*x**3 + 48*a**2*c**3*d*x**4 + 64*a*c**4*d*x**6
 + 16*c**3*d**3*x**10 + 60*c**2*d**4*x**11/11 + c*d**5*x**12 + d**6*x**13/1
3 + x**9*(4*a*c*d**4/3 + 80*c**4*d**2/3) + x**8*(12*a*c**2*d**3 + 24*c**5*d
) + x**7*(288*a*c**3*d**2/7 + 64*c**6/7) + x**5*(48*a**2*c**2*d**2/5 + 192*
a*c**5/5)
```

Giac [A] time = 1.1121, size = 224, normalized size = 1.31

$$\frac{1}{13}d^6x^{13} + cd^5x^{12} + \frac{60}{11}c^2d^4x^{11} + 16\,c^3d^3x^{10} + \frac{80}{3}c^4d^2x^9 + \frac{4}{3}acd^4x^9 + 24\,c^5dx^8 + 12\,ac^2d^3x^8 + \frac{64}{7}c^6x^7 + \frac{288}{7}ac^3d^2x^7 +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^3,x, algorithm="giac")
```

```
[Out] 1/13*d^6*x^13 + c*d^5*x^12 + 60/11*c^2*d^4*x^11 + 16*c^3*d^3*x^10 + 80/3*c^
4*d^2*x^9 + 4/3*a*c*d^4*x^9 + 24*c^5*d*x^8 + 12*a*c^2*d^3*x^8 + 64/7*c^6*x^
7 + 288/7*a*c^3*d^2*x^7 + 64*a*c^4*d*x^6 + 192/5*a*c^5*x^5 + 48/5*a^2*c^2*d
^2*x^5 + 48*a^2*c^3*d*x^4 + 64*a^2*c^4*x^3 + 64*a^3*c^3*x
```

3.35 $\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^2 \, dx$

Optimal. Leaf size=92

$$16a^2c^2x+\frac{8}{5}cx^5\left(ad^2+2c^3\right)+8ac^2dx^4+\frac{32}{3}ac^3x^3+\frac{24}{7}c^2d^2x^7+\frac{16}{3}c^3dx^6+cd^3x^8+\frac{d^4x^9}{9}$$

```
[Out] 16*a^2*c^2*x + (32*a*c^3*x^3)/3 + 8*a*c^2*d*x^4 + (8*c*(2*c^3 + a*d^2)*x^5)
/5 + (16*c^3*d*x^6)/3 + (24*c^2*d^2*x^7)/7 + c*d^3*x^8 + (d^4*x^9)/9
```

Rubi [A] time = 0.0430304, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {2061}

$$16a^2c^2x+\frac{8}{5}cx^5\left(ad^2+2c^3\right)+8ac^2dx^4+\frac{32}{3}ac^3x^3+\frac{24}{7}c^2d^2x^7+\frac{16}{3}c^3dx^6+cd^3x^8+\frac{d^4x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Int[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^2,x]
```

```
[Out] 16*a^2*c^2*x + (32*a*c^3*x^3)/3 + 8*a*c^2*d*x^4 + (8*c*(2*c^3 + a*d^2)*x^5)
/5 + (16*c^3*d*x^6)/3 + (24*c^2*d^2*x^7)/7 + c*d^3*x^8 + (d^4*x^9)/9
```

Rule 2061

```
Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I
GtQ[p, 0]
```

Rubi steps

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^2 \, dx = \int \left(16a^2c^2+32ac^3x^2+32ac^2dx^3+8c\left(2c^3+ad^2\right)x^4+32c^3dx^5+24c^2d^2x^6+8c\right.$$

$$=16a^2c^2x+\frac{32}{3}ac^3x^3+8ac^2dx^4+\frac{8}{5}c\left(2c^3+ad^2\right)x^5+\frac{16}{3}c^3dx^6+\frac{24}{7}c^2d^2x^7+cd^3x$$

Mathematica [A] time = 0.0082144, size = 92, normalized size = 1.

$$16a^2c^2x+\frac{8}{5}cx^5\left(ad^2+2c^3\right)+8ac^2dx^4+\frac{32}{3}ac^3x^3+\frac{24}{7}c^2d^2x^7+\frac{16}{3}c^3dx^6+cd^3x^8+\frac{d^4x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^2,x]
```

```
[Out] 16*a^2*c^2*x + (32*a*c^3*x^3)/3 + 8*a*c^2*d*x^4 + (8*c*(2*c^3 + a*d^2)*x^5)
/5 + (16*c^3*d*x^6)/3 + (24*c^2*d^2*x^7)/7 + c*d^3*x^8 + (d^4*x^9)/9
```

Maple [A] time = 0., size = 84, normalized size = 0.9

$$\frac{d^4x^9}{9} + cd^3x^8 + \frac{24\,c^2d^2x^7}{7} + \frac{16\,c^3dx^6}{3} + \frac{\left(8\,acd^2 + 16\,c^4\right)x^5}{5} + 8\,ac^2dx^4 + \frac{32\,ac^3x^3}{3} + 16\,a^2c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^2,x)
```

```
[Out] 1/9*d^4*x^9+c*d^3*x^8+24/7*c^2*d^2*x^7+16/3*c^3*d*x^6+1/5*(8*a*c*d^2+16*c^4
)*x^5+8*a*c^2*d*x^4+32/3*a*c^3*x^3+16*a^2*c^2*x
```

Maxima [A] time = 1.09146, size = 127, normalized size = 1.38

$$\frac{1}{9}\,d^4x^9 + cd^3x^8 + \frac{16}{7}\,c^2d^2x^7 + \frac{16}{5}\,c^4x^5 + 16\,a^2c^2x + \frac{8}{15}\left(3\,d^2x^5 + 15\,cdx^4 + 20\,c^2x^3\right)ac + \frac{8}{21}\left(3\,d^2x^7 + 14\,cdx^6\right)c^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^2,x, algorithm="maxima")
```

```
[Out] 1/9*d^4*x^9 + c*d^3*x^8 + 16/7*c^2*d^2*x^7 + 16/5*c^4*x^5 + 16*a^2*c^2*x +
8/15*(3*d^2*x^5 + 15*c*d*x^4 + 20*c^2*x^3)*a*c + 8/21*(3*d^2*x^7 + 14*c*d*x
^6)*c^2
```

Fricas [A] time = 1.12068, size = 192, normalized size = 2.09

$$\frac{1}{9}x^9d^4 + x^8d^3c + \frac{24}{7}x^7d^2c^2 + \frac{16}{3}x^6dc^3 + \frac{16}{5}x^5c^4 + \frac{8}{5}x^5d^2ca + 8x^4dc^2a + \frac{32}{3}x^3c^3a + 16xc^2a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^2,x, algorithm="fricas")
```

```
[Out] 1/9*x^9*d^4 + x^8*d^3*c + 24/7*x^7*d^2*c^2 + 16/3*x^6*d*c^3 + 16/5*x^5*c^4
+ 8/5*x^5*d^2*c*a + 8*x^4*d*c^2*a + 32/3*x^3*c^3*a + 16*x*c^2*a^2
```

Sympy [A] time = 0.078466, size = 95, normalized size = 1.03

$$16a^2c^2x + \frac{32ac^3x^3}{3} + 8ac^2dx^4 + \frac{16c^3dx^6}{3} + \frac{24c^2d^2x^7}{7} + cd^3x^8 + \frac{d^4x^9}{9} + x^5\left(\frac{8acd^2}{5} + \frac{16c^4}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c)**2,x)
```

```
[Out] 16*a**2*c**2*x + 32*a*c**3*x**3/3 + 8*a*c**2*d*x**4 + 16*c**3*d*x**6/3 + 24
*c**2*d**2*x**7/7 + c*d**3*x**8 + d**4*x**9/9 + x**5*(8*a*c*d**2/5 + 16*c**
4/5)
```

Giac [A] time = 1.14216, size = 112, normalized size = 1.22

$$\frac{1}{9}\,d^4x^9 + cd^3x^8 + \frac{24}{7}\,c^2d^2x^7 + \frac{16}{3}\,c^3dx^6 + \frac{16}{5}\,c^4x^5 + \frac{8}{5}\,acd^2x^5 + 8\,ac^2dx^4 + \frac{32}{3}\,ac^3x^3 + 16\,a^2c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^2,x, algorithm="giac")
```

```
[Out] 1/9*d^4*x^9 + c*d^3*x^8 + 24/7*c^2*d^2*x^7 + 16/3*c^3*d*x^6 + 16/5*c^4*x^5
+ 8/5*a*c*d^2*x^5 + 8*a*c^2*d*x^4 + 32/3*a*c^3*x^3 + 16*a^2*c^2*x
```

3.36 $\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right) dx$

Optimal. Leaf size=32

$$4acx+\frac{4c^2x^3}{3}+cdx^4+\frac{d^2x^5}{5}$$

```
[Out] 4*a*c*x + (4*c^2*x^3)/3 + c*d*x^4 + (d^2*x^5)/5
```

Rubi [A] time = 0.0058112, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$4acx+\frac{4c^2x^3}{3}+cdx^4+\frac{d^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4,x]
```

```
[Out] 4*a*c*x + (4*c^2*x^3)/3 + c*d*x^4 + (d^2*x^5)/5
```

Rubi steps

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right) dx=4acx+\frac{4c^2x^3}{3}+cdx^4+\frac{d^2x^5}{5}$$

Mathematica [A] time = 0.0000498, size = 32, normalized size = 1.

$$4acx+\frac{4c^2x^3}{3}+cdx^4+\frac{d^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4,x]
```

```
[Out] 4*a*c*x + (4*c^2*x^3)/3 + c*d*x^4 + (d^2*x^5)/5
```

Maple [A] time = 0., size = 29, normalized size = 0.9

$$4\,acx+\frac{4\,c^{2}x^{3}}{3}+cd x^{4}+\frac{d^{2}x^{5}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c,x)
```

```
[Out] 4*a*c*x+4/3*c^2*x^3+c*d*x^4+1/5*d^2*x^5
```

Maxima [A] time = 1.04914, size = 38, normalized size = 1.19

$$\frac{1}{5}\,d^{2}x^{5}+cdx^{4}+\frac{4}{3}\,c^{2}x^{3}+4\,acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c,x, algorithm="maxima")
```

```
[Out] 1/5*d^2*x^5 + c*d*x^4 + 4/3*c^2*x^3 + 4*a*c*x
```

Fricas [A] time = 1.10546, size = 63, normalized size = 1.97

$$\frac{1}{5}x^{5}d^{2}+x^{4}dc+\frac{4}{3}x^{3}c^{2}+4xca$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c,x, algorithm="fricas")
```

```
[Out] 1/5*x^5*d^2 + x^4*d*c + 4/3*x^3*c^2 + 4*x*c*a
```

Sympy [A] time = 0.066172, size = 31, normalized size = 0.97

$$4acx+\frac{4c^{2}x^{3}}{3}+cdx^{4}+\frac{d^{2}x^{5}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c,x)
```

```
[Out] 4*a*c*x + 4*c**2*x**3/3 + c*d*x**4 + d**2*x**5/5
```

Giac [A] time = 1.11857, size = 38, normalized size = 1.19

$$\frac{1}{5}\,d^2x^5 + cdx^4 + \frac{4}{3}\,c^2x^3 + 4\,acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c,x, algorithm="giac")
```

```
[Out] 1/5*d^2*x^5 + c*d*x^4 + 4/3*c^2*x^3 + 4*a*c*x
```

3.37 $\int \frac{1}{4ac+4c^2x^2+4cdx^3+d^2x^4}\,dx$

Optimal. Leaf size=529

$$-\frac{d\log\left(-\sqrt{2}\sqrt[4]{c}d\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}\left(\frac{c}{d}+x\right)+\sqrt{c}\sqrt{4ad^2+c^3}+d^2\left(\frac{c}{d}+x\right)^2\right)}{4\sqrt{2}c^{3/4}\sqrt{4ad^2+c^3}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}}+\frac{d\log\left(\sqrt{2}\sqrt[4]{c}d\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}\left(\frac{c}{d}+x\right.\right.}{4\sqrt{2}c^{3/4}\sqrt{4ad^2+c^3}\sqrt{\sqrt{}}}$$

```
[Out] -(d*ArcTanh[(Sqrt[2]*c + c^(1/4)*Sqrt[c^(3/2) + Sqrt[c^3 + 4*a*d^2]] + Sqrt
[2]*d*x)/(c^(1/4)*Sqrt[c^(3/2) - Sqrt[c^3 + 4*a*d^2]])])/(2*Sqrt[2]*c^(3/4)
*Sqrt[c^3 + 4*a*d^2]*Sqrt[c^(3/2) - Sqrt[c^3 + 4*a*d^2]]) + (d*ArcTanh[(c^(
1/4)*Sqrt[c^(3/2) + Sqrt[c^3 + 4*a*d^2]] - Sqrt[2]*(c + d*x))/(c^(1/4)*Sqrt
[c^(3/2) - Sqrt[c^3 + 4*a*d^2]])])/(2*Sqrt[2]*c^(3/4)*Sqrt[c^3 + 4*a*d^2]*S
qrt[c^(3/2) - Sqrt[c^3 + 4*a*d^2]]) - (d*Log[Sqrt[c]*Sqrt[c^3 + 4*a*d^2] -
Sqrt[2]*c^(1/4)*d*Sqrt[c^(3/2) + Sqrt[c^3 + 4*a*d^2]]*(c/d + x) + d^2*(c/d
+ x)^2])/(4*Sqrt[2]*c^(3/4)*Sqrt[c^3 + 4*a*d^2]*Sqrt[c^(3/2) + Sqrt[c^3 + 4
*a*d^2]]) + (d*Log[Sqrt[c]*Sqrt[c^3 + 4*a*d^2] + Sqrt[2]*c^(1/4)*d*Sqrt[c^(
3/2) + Sqrt[c^3 + 4*a*d^2]]*(c/d + x) + d^2*(c/d + x)^2])/(4*Sqrt[2]*c^(3/4
)*Sqrt[c^3 + 4*a*d^2]*Sqrt[c^(3/2) + Sqrt[c^3 + 4*a*d^2]])
```

Rubi [A] time = 0.895749, antiderivative size = 529, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1106, 1094, 634, 618, 206, 628}

$$-\frac{d\log\left(-\sqrt{2}\sqrt[4]{c}d\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}\left(\frac{c}{d}+x\right)+\sqrt{c}\sqrt{4ad^2+c^3}+d^2\left(\frac{c}{d}+x\right)^2\right)}{4\sqrt{2}c^{3/4}\sqrt{4ad^2+c^3}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}}+\frac{d\log\left(\sqrt{2}\sqrt[4]{c}d\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}\left(\frac{c}{d}+x\right.\right.}{4\sqrt{2}c^{3/4}\sqrt{4ad^2+c^3}\sqrt{\sqrt{}}}$$

Antiderivative was successfully verified.

```
[In] Int[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^(-1),x]
```


```
[Out] -(d*ArcTanh[(Sqrt[2]*c + c^(1/4)*Sqrt[c^(3/2) + Sqrt[c^3 + 4*a*d^2]] + Sqrt
[2]*d*x)/(c^(1/4)*Sqrt[c^(3/2) - Sqrt[c^3 + 4*a*d^2]])])/(2*Sqrt[2]*c^(3/4)
*Sqrt[c^3 + 4*a*d^2]*Sqrt[c^(3/2) - Sqrt[c^3 + 4*a*d^2]]) + (d*ArcTanh[(c^(
1/4)*Sqrt[c^(3/2) + Sqrt[c^3 + 4*a*d^2]] - Sqrt[2]*(c + d*x))/(c^(1/4)*Sqrt
[c^(3/2) - Sqrt[c^3 + 4*a*d^2]])])/(2*Sqrt[2]*c^(3/4)*Sqrt[c^3 + 4*a*d^2]*S
qrt[c^(3/2) - Sqrt[c^3 + 4*a*d^2]]) - (d*Log[Sqrt[c]*Sqrt[c^3 + 4*a*d^2] -
Sqrt[2]*c^(1/4)*d*Sqrt[c^(3/2) + Sqrt[c^3 + 4*a*d^2]]*(c/d + x) + d^2*(c/d
```

```
+ x)^2])/(4*Sqrt[2]*c^(3/4)*Sqrt[c^3 + 4*a*d^2]*Sqrt[c^(3/2) + Sqrt[c^3 + 4
*a*d^2]]) + (d*Log[Sqrt[c]*Sqrt[c^3 + 4*a*d^2] + Sqrt[2]*c^(1/4)*d*Sqrt[c^(
3/2) + Sqrt[c^3 + 4*a*d^2]]*(c/d + x) + d^2*(c/d + x)^2])/(4*Sqrt[2]*c^(3/4
)*Sqrt[c^3 + 4*a*d^2]*Sqrt[c^(3/2) + Sqrt[c^3 + 4*a*d^2]])
```

Rule 1106

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1094

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{4ac+4c^2x^2+4cdx^3+d^2x^4}\,dx = \text{Subst}\left(\int \frac{1}{c\left(4a+\frac{c^3}{d^2}\right)-2c^2x^2+d^2x^4}\,dx, x, \frac{c}{d}+x\right)$$

$$= \frac{d\,\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}}{d}-x}{\frac{\sqrt{c}\sqrt{c^3+4ad^2}}{d^2}-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}x}{d}+x^2}\,dx, x, \frac{c}{d}+x\right)}{2\sqrt{2}c^{3/4}\sqrt{c^3+4ad^2}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}} + \frac{d\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2} \cdots}}{\frac{\sqrt{c}\sqrt{c^3+4ad^2}}{d^2}+\frac{\sqrt{2}\sqrt[4]{\cdots}}{\cdots}}\cdots\right)}{2\sqrt{2}c^{3/4}\sqrt{c^3+4a \cdots}}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{c}\sqrt{c^3+4ad^2}}{d^2}-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}x}{d}+x^2}\,dx, x, \frac{c}{d}+x\right)}{4\sqrt{c}\sqrt{c^3+4ad^2}} + \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{c}\sqrt{c^3+4ad^2}}{d^2}+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3 \cdots}}}{\cdots}}\cdots\right)}{4\sqrt{c}\sqrt{c^3 \cdots}}$$

$$= -\frac{d\log\left(\sqrt{c}\sqrt{c^3+4ad^2}-\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}(c+dx)+(c+dx)^2\right)}{4\sqrt{2}c^{3/4}\sqrt{c^3+4ad^2}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}} + \frac{d\log\left(\sqrt{\cdots}\right.}{\cdots}$$

$$= -\frac{d\tanh^{-1}\left(\frac{\sqrt[4]{c}\left(\sqrt{2}c^{3/4}-\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\right)+\sqrt{2}dx}{\sqrt[4]{c}\sqrt{c^{3/2}-\sqrt{c^3+4ad^2}}}\right)}{2\sqrt{2}c^{3/4}\sqrt{c^3+4ad^2}\sqrt{c^{3/2}-\sqrt{c^3+4ad^2}}} - \frac{d\tanh^{-1}\left(\frac{\sqrt[4]{c}\left(\sqrt{2}c^{3/4}+\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\right)+\sqrt{2}d \cdots}{\sqrt[4]{c}\sqrt{c^{3/2}-\sqrt{c^3+4ad^2}}}\right.}{2\sqrt{2}c^{3/4}\sqrt{c^3+4ad^2}\sqrt{c^{3/2}-\sqrt{c^3+4a \cdots}}}$$

Mathematica [C] time = 0.0263068, size = 71, normalized size = 0.13

$$\frac{1}{4}\text{RootSum}\left[4\#1^2c^2+4\#1^3cd+\#1^4d^2+4ac\&, \frac{\log(x-\#1)}{3\#1^2cd+\#1^3d^2+2\#1c^2}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^(-1),x]
```

```
[Out] RootSum[4*a*c + 4*c^2*#1^2 + 4*c*d*#1^3 + d^2*#1^4 & , Log[x - #1]/(2*c^2*#
1 + 3*c*d*#1^2 + d^2*#1^3) & ]/4
```

Maple [C] time = 0.059, size = 64, normalized size = 0.1

$$\frac{1}{4}\sum_{\_R=\mathrm{RootOf}\left(d^2\_Z^4+4cd\_Z^3+4c^2\_Z^2+4ac\right)}\frac{\ln\left(x-\_R\right)}{\_R^3d^2+3\_R^2cd+2\_R\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c),x)
```

```
[Out] 1/4*sum(1/(_R^3*d^2+3*_R^2*c*d+2*_R*c^2)*ln(x-_R),_R=RootOf(_Z^4*d^2+4*_Z^3
*c*d+4*_Z^2*c^2+4*a*c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{d^2x^4+4\,cdx^3+4\,c^2x^2+4\,ac}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c),x, algorithm="maxima")
```

```
[Out] integrate(1/(d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c), x)
```

Fricas [B] time = 1.38816, size = 1770, normalized size = 3.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(-(2*(a*c^3 + 4*a^2*d^2)*sqrt(-d^2/(a*c^9 + 8*a^2*c^6*d^2 + 16*a^3*
c^3*d^4)) + 1)/(a*c^3 + 4*a^2*d^2))*log(d^2*x + c*d + (2*a*c*d^2 + (a*c^7 +
 4*a^2*c^4*d^2)*sqrt(-d^2/(a*c^9 + 8*a^2*c^6*d^2 + 16*a^3*c^3*d^4)))*sqrt(-
(2*(a*c^3 + 4*a^2*d^2)*sqrt(-d^2/(a*c^9 + 8*a^2*c^6*d^2 + 16*a^3*c^3*d^4))
+ 1)/(a*c^3 + 4*a^2*d^2))) - 1/8*sqrt(-(2*(a*c^3 + 4*a^2*d^2)*sqrt(-d^2/(a*
c^9 + 8*a^2*c^6*d^2 + 16*a^3*c^3*d^4)) + 1)/(a*c^3 + 4*a^2*d^2))*log(d^2*x
+ c*d - (2*a*c*d^2 + (a*c^7 + 4*a^2*c^4*d^2)*sqrt(-d^2/(a*c^9 + 8*a^2*c^6*d
^2 + 16*a^3*c^3*d^4)))*sqrt(-(2*(a*c^3 + 4*a^2*d^2)*sqrt(-d^2/(a*c^9 + 8*a^
```

```
2*c^6*d^2 + 16*a^3*c^3*d^4)) + 1)/(a*c^3 + 4*a^2*d^2))) + 1/8*sqrt((2*(a*c^
3 + 4*a^2*d^2)*sqrt(-d^2/(a*c^9 + 8*a^2*c^6*d^2 + 16*a^3*c^3*d^4)) - 1)/(a*
c^3 + 4*a^2*d^2))*log(d^2*x + c*d + (2*a*c*d^2 - (a*c^7 + 4*a^2*c^4*d^2)*sq
rt(-d^2/(a*c^9 + 8*a^2*c^6*d^2 + 16*a^3*c^3*d^4)))*sqrt((2*(a*c^3 + 4*a^2*d
^2)*sqrt(-d^2/(a*c^9 + 8*a^2*c^6*d^2 + 16*a^3*c^3*d^4)) - 1)/(a*c^3 + 4*a^2
*d^2))) - 1/8*sqrt((2*(a*c^3 + 4*a^2*d^2)*sqrt(-d^2/(a*c^9 + 8*a^2*c^6*d^2
+ 16*a^3*c^3*d^4)) - 1)/(a*c^3 + 4*a^2*d^2))*log(d^2*x + c*d - (2*a*c*d^2 -
 (a*c^7 + 4*a^2*c^4*d^2)*sqrt(-d^2/(a*c^9 + 8*a^2*c^6*d^2 + 16*a^3*c^3*d^4)
))*sqrt((2*(a*c^3 + 4*a^2*d^2)*sqrt(-d^2/(a*c^9 + 8*a^2*c^6*d^2 + 16*a^3*c^
3*d^4)) - 1)/(a*c^3 + 4*a^2*d^2)))
```

Sympy [A] time = 0.935703, size = 88, normalized size = 0.17

$$\mathrm{RootSum}\left(t^4\left(16384a^3c^3d^2+4096a^2c^6\right)+128t^2ac^3+1,\left(t\mapsto t\log\left(x+\frac{-1024t^3a^2c^4d^2-256t^3ac^7+16tacd^2-4tc^4+c\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c),x)
```

```
[Out] RootSum(_t**4*(16384*a**3*c**3*d**2 + 4096*a**2*c**6) + 128*_t**2*a*c**3 +
1, Lambda(_t, _t*log(x + (-1024*_t**3*a**2*c**4*d**2 - 256*_t**3*a*c**7 + 1
6*_t*a*c*d**2 - 4*_t*c**4 + c*d)/d**2)))
```

Giac [A] time = 1.32178, size = 471, normalized size = 0.89

$$-\frac{1}{8}\sqrt{-\frac{ac^3+2\sqrt{-ac}acd}{a^2c^6+4a^3c^3d^2}}\log\left(\left|\sqrt{-ac}dx+\sqrt{-ac}c-\sqrt{-ac^3+2\sqrt{-ac}acd}\right|\right)+\frac{1}{8}\sqrt{-\frac{ac^3-2\sqrt{-ac}acd}{a^2c^6+4a^3c^3d^2}}\log\left(\left|\sqrt{-ac}dx+\sqrt{-ac}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(-(a*c^3 + 2*sqrt(-a*c)*a*c*d)/(a^2*c^6 + 4*a^3*c^3*d^2))*log(abs(
sqrt(-a*c)*d*x + sqrt(-a*c)*c - sqrt(-a*c^3 + 2*sqrt(-a*c)*a*c*d))) + 1/8*s
qrt(-(a*c^3 - 2*sqrt(-a*c)*a*c*d)/(a^2*c^6 + 4*a^3*c^3*d^2))*log(abs(sqrt(-
a*c)*d*x + sqrt(-a*c)*c - sqrt(-a*c^3 - 2*sqrt(-a*c)*a*c*d))) + 1/8*sqrt(-(
a*c^3 + 2*sqrt(-a*c)*a*c*d)/(a^2*c^6 + 4*a^3*c^3*d^2))*log(abs(-sqrt(-a*c)*
d*x - sqrt(-a*c)*c - sqrt(-a*c^3 + 2*sqrt(-a*c)*a*c*d))) - 1/8*sqrt(-(a*c^3
```

```
- 2*sqrt(-a*c)*a*c*d)/(a^2*c^6 + 4*a^3*c^3*d^2))*log(abs(-sqrt(-a*c)*d*x -
sqrt(-a*c)*c - sqrt(-a*c^3 - 2*sqrt(-a*c)*a*c*d)))
```

3.38 $\int \frac{1}{\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^2}\,dx$

Optimal. Leaf size=746

$$-\frac{\left(\frac{c}{d}+x\right)\left(-4ad^2+c^3-cd^2\left(\frac{c}{d}+x\right)^2\right)}{16ac\left(4ad^2+c^3\right)\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)}-\frac{d\left(-c^{3/2}\sqrt{4ad^2+c^3}+12ad^2+c^3\right)\log\left(-\sqrt{2}\sqrt[4]{c}d\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}\left(\frac{c}{d}\right.\right.}{64\sqrt{2}ac^{7/4}\left(4ad^2+c^3\right)^{3/2}\sqrt{\sqrt{4ad^2+c^3}}}$$

```
[Out] -((c/d + x)*(c^3 - 4*a*d^2 - c*d^2*(c/d + x)^2))/(16*a*c*(c^3 + 4*a*d^2)*(4
*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)) - (d*(c^3 + 12*a*d^2 + c^(3/2)*Sqr
t[c^3 + 4*a*d^2])*ArcTanh[(Sqrt[2]*c + c^(1/4)*Sqrt[c^(3/2) + Sqrt[c^3 + 4*
a*d^2]] + Sqrt[2]*d*x)/(c^(1/4)*Sqrt[c^(3/2) - Sqrt[c^3 + 4*a*d^2]])])/(32*
Sqrt[2]*a*c^(7/4)*(c^3 + 4*a*d^2)^(3/2)*Sqrt[c^(3/2) - Sqrt[c^3 + 4*a*d^2]]
) + (d*(c^3 + 12*a*d^2 + c^(3/2)*Sqrt[c^3 + 4*a*d^2])*ArcTanh[(c^(1/4)*Sqrt
[c^(3/2) + Sqrt[c^3 + 4*a*d^2]] - Sqrt[2]*(c + d*x))/(c^(1/4)*Sqrt[c^(3/2)
- Sqrt[c^3 + 4*a*d^2]])])/(32*Sqrt[2]*a*c^(7/4)*(c^3 + 4*a*d^2)^(3/2)*Sqrt[
c^(3/2) - Sqrt[c^3 + 4*a*d^2]]) - (d*(c^3 + 12*a*d^2 - c^(3/2)*Sqrt[c^3 + 4
*a*d^2])*Log[Sqrt[c]*Sqrt[c^3 + 4*a*d^2] - Sqrt[2]*c^(1/4)*d*Sqrt[c^(3/2) +
 Sqrt[c^3 + 4*a*d^2]]*(c/d + x) + d^2*(c/d + x)^2])/(64*Sqrt[2]*a*c^(7/4)*(
c^3 + 4*a*d^2)^(3/2)*Sqrt[c^(3/2) + Sqrt[c^3 + 4*a*d^2]]) + (d*(c^3 + 12*a*
d^2 - c^(3/2)*Sqrt[c^3 + 4*a*d^2])*Log[Sqrt[c]*Sqrt[c^3 + 4*a*d^2] + Sqrt[2
]*c^(1/4)*d*Sqrt[c^(3/2) + Sqrt[c^3 + 4*a*d^2]]*(c/d + x) + d^2*(c/d + x)^2
])/(64*Sqrt[2]*a*c^(7/4)*(c^3 + 4*a*d^2)^(3/2)*Sqrt[c^(3/2) + Sqrt[c^3 + 4*
a*d^2]])
```

Rubi [A] time = 1.32817, antiderivative size = 746, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {1106, 1092, 1169, 634, 618, 206, 628}

$$-\frac{\left(\frac{c}{d}+x\right)\left(-4ad^2+c^3-cd^2\left(\frac{c}{d}+x\right)^2\right)}{16ac\left(4ad^2+c^3\right)\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)}-\frac{d\left(-c^{3/2}\sqrt{4ad^2+c^3}+12ad^2+c^3\right)\log\left(-\sqrt{2}\sqrt[4]{c}d\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}\left(\frac{c}{d}\right.\right.}{64\sqrt{2}ac^{7/4}\left(4ad^2+c^3\right)^{3/2}\sqrt{\sqrt{4ad^2+c^3}}}$$

Antiderivative was successfully verified.

```
[In] Int[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^(-2),x]
```

```
[Out] -((c/d + x)*(c^3 - 4*a*d^2 - c*d^2*(c/d + x)^2))/(16*a*c*(c^3 + 4*a*d^2)*(4
*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)) - (d*(c^3 + 12*a*d^2 + c^(3/2)*Sqr
```

```
t[c^3 + 4*a*d^2])*ArcTanh[(Sqrt[2]*c + c^(1/4)*Sqrt[c^(3/2) + Sqrt[c^3 + 4*
a*d^2]] + Sqrt[2]*d*x)/(c^(1/4)*Sqrt[c^(3/2) - Sqrt[c^3 + 4*a*d^2]])])/(32*
Sqrt[2]*a*c^(7/4)*(c^3 + 4*a*d^2)^(3/2)*Sqrt[c^(3/2) - Sqrt[c^3 + 4*a*d^2]]
) + (d*(c^3 + 12*a*d^2 + c^(3/2)*Sqrt[c^3 + 4*a*d^2])*ArcTanh[(c^(1/4)*Sqrt
[c^(3/2) + Sqrt[c^3 + 4*a*d^2]] - Sqrt[2]*(c + d*x))/(c^(1/4)*Sqrt[c^(3/2)
- Sqrt[c^3 + 4*a*d^2]])])/(32*Sqrt[2]*a*c^(7/4)*(c^3 + 4*a*d^2)^(3/2)*Sqrt[
c^(3/2) - Sqrt[c^3 + 4*a*d^2]]) - (d*(c^3 + 12*a*d^2 - c^(3/2)*Sqrt[c^3 + 4
*a*d^2])*Log[Sqrt[c]*Sqrt[c^3 + 4*a*d^2] - Sqrt[2]*c^(1/4)*d*Sqrt[c^(3/2) +
 Sqrt[c^3 + 4*a*d^2]]*(c/d + x) + d^2*(c/d + x)^2])/(64*Sqrt[2]*a*c^(7/4)*(
c^3 + 4*a*d^2)^(3/2)*Sqrt[c^(3/2) + Sqrt[c^3 + 4*a*d^2]]) + (d*(c^3 + 12*a*
d^2 - c^(3/2)*Sqrt[c^3 + 4*a*d^2])*Log[Sqrt[c]*Sqrt[c^3 + 4*a*d^2] + Sqrt[2
]*c^(1/4)*d*Sqrt[c^(3/2) + Sqrt[c^3 + 4*a*d^2]]*(c/d + x) + d^2*(c/d + x)^2
])/(64*Sqrt[2]*a*c^(7/4)*(c^3 + 4*a*d^2)^(3/2)*Sqrt[c^(3/2) + Sqrt[c^3 + 4*
a*d^2]])
```

Rule 1106

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1092

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x*(b^2 -
 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2
 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^2}\,dx = \text{Subst}\left(\int \frac{1}{\left(c\left(4a+\frac{c^3}{d^2}\right)-2c^2x^2+d^2x^4\right)^2}\,dx, x, \frac{c}{d}+x\right)$$

$$= -\frac{\left(\frac{c}{d}+x\right)\left(c^3-4ad^2-cd^2\left(\frac{c}{d}+x\right)^2\right)}{16ac\left(c^3+4ad^2\right)\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)} + \frac{\text{Subst}\left(\int \frac{4c^4-2c\left(4a+\frac{c^3}{d^2}\right)d^2-2\left(4 \cdots\right.}{c\left(4a+\frac{c^3}{d^2}\right) \cdots} \cdots\right.}{32ac^2 \cdots}$$

$$= -\frac{\left(\frac{c}{d}+x\right)\left(c^3-4ad^2-cd^2\left(\frac{c}{d}+x\right)^2\right)}{16ac\left(c^3+4ad^2\right)\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)} + \frac{d\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}} \cdots}{\cdots} \cdots\right.}{\cdots}$$

$$= -\frac{\left(\frac{c}{d}+x\right)\left(c^3-4ad^2-cd^2\left(\frac{c}{d}+x\right)^2\right)}{16ac\left(c^3+4ad^2\right)\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)} - \frac{\left(d\left(c^3+12ad^2-c^{3/2}\sqrt{c^3+4 \cdots}\right.\right.}{64\sqrt{2}a \cdots}$$

$$= -\frac{\left(\frac{c}{d}+x\right)\left(c^3-4ad^2-cd^2\left(\frac{c}{d}+x\right)^2\right)}{16ac\left(c^3+4ad^2\right)\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)} - \frac{d\left(c^3+12ad^2-c^{3/2}\sqrt{c^3+4a \cdots}\right.}{\cdots}$$

$$= -\frac{\left(\frac{c}{d}+x\right)\left(c^3-4ad^2-cd^2\left(\frac{c}{d}+x\right)^2\right)}{16ac\left(c^3+4ad^2\right)\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)} - \frac{d\left(c^3+12ad^2+c^{3/2}\sqrt{c^3+4a \cdots}\right.}{32\sqrt{2}ac^{7/4}\left(c^3 \cdots\right.}$$

Mathematica [C] time = 0.102979, size = 182, normalized size = 0.24

$$\frac{\text{RootSum}\left[4\#1^2c^2+4\#1^3cd+\#1^4d^2+4ac\&, \frac{\#1^2cd^2\log(x-\#1)+12ad^2\log(x-\#1)+2\#1c^2d\log(x-\#1)+2c^3\log(x-\#1)}{3\#1^2cd+\#1^3d^2+2\#1c^2}\&\right]+\frac{4(c+dx)(4ad+cx(2 \cdots}{4ac+x^2(2c+d \cdots}}{64ac\left(4ad^2+c^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^(-2),x]
```

```
[Out] ((4*(c + d*x)*(4*a*d + c*x*(2*c + d*x)))/(4*a*c + x^2*(2*c + d*x)^2) + Root
Sum[4*a*c + 4*c^2*#1^2 + 4*c*d*#1^3 + d^2*#1^4 & , (2*c^3*Log[x - #1] + 12*
a*d^2*Log[x - #1] + 2*c^2*d*Log[x - #1]*#1 + c*d^2*Log[x - #1]*#1^2)/(2*c^2
*#1 + 3*c*d*#1^2 + d^2*#1^3) & ])/(64*a*c*(c^3 + 4*a*d^2))
```

Maple [C] time = 0.014, size = 232, normalized size = 0.3

$$\frac{1}{d^2x^4+4\,cdx^3+4\,c^2x^2+4\,ac}\left(\frac{d^2x^3}{16\,a\left(4\,ad^2+c^3\right)}+\frac{3\,cdx^2}{16\,a\left(4\,ad^2+c^3\right)}+\frac{\left(2\,ad^2+c^3\right)x}{8\,\left(4\,ad^2+c^3\right)ac}+\frac{d}{16\,ad^2+4\,c^3}\right)+\frac{1}{\left(256\,ad^2+64\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^2,x)
```

```
[Out] (1/16*d^2/a/(4*a*d^2+c^3)*x^3+3/16/a*c*d/(4*a*d^2+c^3)*x^2+1/8/c*(2*a*d^2+c
^3)/(4*a*d^2+c^3)/a*x+1/4*d/(4*a*d^2+c^3))/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a
*c)+1/64/(4*a*d^2+c^3)/a/c*sum((_R^2*c*d^2+2*_R*c^2*d+12*a*d^2+2*c^3)/(_R^3
*d^2+3*_R^2*c*d+2*_R*c^2)*ln(x-_R),_R=RootOf(_Z^4*d^2+4*_Z^3*c*d+4*_Z^2*c^2
+4*a*c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{cd^2x^3+3\,c^2dx^2+4\,acd+2\left(c^3+2\,ad^2\right)x}{16\left(4\,a^2c^5+16\,a^3c^2d^2+\left(ac^4d^2+4\,a^2cd^4\right)x^4+4\left(ac^5d+4\,a^2c^2d^3\right)x^3+4\left(ac^6+4\,a^2c^3d^2\right)x^2\right)}+\frac{\text{sage}_2}{16\left(ac^4+4\,a^2cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^2,x, algorithm="maxima")
```

```
[Out] 1/16*(c*d^2*x^3 + 3*c^2*d*x^2 + 4*a*c*d + 2*(c^3 + 2*a*d^2)*x)/(4*a^2*c^5 +
 16*a^3*c^2*d^2 + (a*c^4*d^2 + 4*a^2*c*d^4)*x^4 + 4*(a*c^5*d + 4*a^2*c^2*d^
3)*x^3 + 4*(a*c^6 + 4*a^2*c^3*d^2)*x^2) + 1/16*integrate((c*d^2*x^2 + 2*c^2
*d*x + 2*c^3 + 12*a*d^2)/(d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c), x)/(a*c
^4 + 4*a^2*c*d^2)
```

Fricas [B] time = 1.82838, size = 6965, normalized size = 9.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^2,x, algorithm="fricas")
```

```
[Out] 1/64*(4*c*d^2*x^3 + 12*c^2*d*x^2 + 16*a*c*d + (4*a^2*c^5 + 16*a^3*c^2*d^2 +
 (a*c^4*d^2 + 4*a^2*c*d^4)*x^4 + 4*(a*c^5*d + 4*a^2*c^2*d^3)*x^3 + 4*(a*c^6
 + 4*a^2*c^3*d^2)*x^2)*sqrt(-(c^6 + 15*a*c^3*d^2 + 60*a^2*d^4 + 2*(a^3*c^11
 + 12*a^4*c^8*d^2 + 48*a^5*c^5*d^4 + 64*a^6*c^2*d^6)*sqrt(-(25*c^6*d^6 + 36
0*a*c^3*d^8 + 1296*a^2*d^10)/(a^3*c^25 + 24*a^4*c^22*d^2 + 240*a^5*c^19*d^4
 + 1280*a^6*c^16*d^6 + 3840*a^7*c^13*d^8 + 6144*a^8*c^10*d^10 + 4096*a^9*c^
7*d^12)))/(a^3*c^11 + 12*a^4*c^8*d^2 + 48*a^5*c^5*d^4 + 64*a^6*c^2*d^6))*lo
g(5*c^7*d^3 + 81*a*c^4*d^5 + 324*a^2*c*d^7 + (5*c^6*d^4 + 81*a*c^3*d^6 + 32
4*a^2*d^8)*x + (5*a^2*c^8*d^4 + 96*a^3*c^5*d^6 + 432*a^4*c^2*d^8 + (a^3*c^1
9 + 20*a^4*c^16*d^2 + 144*a^5*c^13*d^4 + 448*a^6*c^10*d^6 + 512*a^7*c^7*d^8
)*sqrt(-(25*c^6*d^6 + 360*a*c^3*d^8 + 1296*a^2*d^10)/(a^3*c^25 + 24*a^4*c^2
2*d^2 + 240*a^5*c^19*d^4 + 1280*a^6*c^16*d^6 + 3840*a^7*c^13*d^8 + 6144*a^8
*c^10*d^10 + 4096*a^9*c^7*d^12)))*sqrt(-(c^6 + 15*a*c^3*d^2 + 60*a^2*d^4 +
2*(a^3*c^11 + 12*a^4*c^8*d^2 + 48*a^5*c^5*d^4 + 64*a^6*c^2*d^6)*sqrt(-(25*c
^6*d^6 + 360*a*c^3*d^8 + 1296*a^2*d^10)/(a^3*c^25 + 24*a^4*c^22*d^2 + 240*a
^5*c^19*d^4 + 1280*a^6*c^16*d^6 + 3840*a^7*c^13*d^8 + 6144*a^8*c^10*d^10 +
4096*a^9*c^7*d^12)))/(a^3*c^11 + 12*a^4*c^8*d^2 + 48*a^5*c^5*d^4 + 64*a^6*c
^2*d^6))) - (4*a^2*c^5 + 16*a^3*c^2*d^2 + (a*c^4*d^2 + 4*a^2*c*d^4)*x^4 + 4
*(a*c^5*d + 4*a^2*c^2*d^3)*x^3 + 4*(a*c^6 + 4*a^2*c^3*d^2)*x^2)*sqrt(-(c^6
+ 15*a*c^3*d^2 + 60*a^2*d^4 + 2*(a^3*c^11 + 12*a^4*c^8*d^2 + 48*a^5*c^5*d^4
 + 64*a^6*c^2*d^6)*sqrt(-(25*c^6*d^6 + 360*a*c^3*d^8 + 1296*a^2*d^10)/(a^3*
c^25 + 24*a^4*c^22*d^2 + 240*a^5*c^19*d^4 + 1280*a^6*c^16*d^6 + 3840*a^7*c^
13*d^8 + 6144*a^8*c^10*d^10 + 4096*a^9*c^7*d^12)))/(a^3*c^11 + 12*a^4*c^8*d
^2 + 48*a^5*c^5*d^4 + 64*a^6*c^2*d^6))*log(5*c^7*d^3 + 81*a*c^4*d^5 + 324*a
^2*c*d^7 + (5*c^6*d^4 + 81*a*c^3*d^6 + 324*a^2*d^8)*x - (5*a^2*c^8*d^4 + 96
*a^3*c^5*d^6 + 432*a^4*c^2*d^8 + (a^3*c^19 + 20*a^4*c^16*d^2 + 144*a^5*c^13
*d^4 + 448*a^6*c^10*d^6 + 512*a^7*c^7*d^8)*sqrt(-(25*c^6*d^6 + 360*a*c^3*d^
8 + 1296*a^2*d^10)/(a^3*c^25 + 24*a^4*c^22*d^2 + 240*a^5*c^19*d^4 + 1280*a^
6*c^16*d^6 + 3840*a^7*c^13*d^8 + 6144*a^8*c^10*d^10 + 4096*a^9*c^7*d^12)))*
sqrt(-(c^6 + 15*a*c^3*d^2 + 60*a^2*d^4 + 2*(a^3*c^11 + 12*a^4*c^8*d^2 + 48*
a^5*c^5*d^4 + 64*a^6*c^2*d^6)*sqrt(-(25*c^6*d^6 + 360*a*c^3*d^8 + 1296*a^2*
d^10)/(a^3*c^25 + 24*a^4*c^22*d^2 + 240*a^5*c^19*d^4 + 1280*a^6*c^16*d^6 +
3840*a^7*c^13*d^8 + 6144*a^8*c^10*d^10 + 4096*a^9*c^7*d^12)))/(a^3*c^11 + 1
2*a^4*c^8*d^2 + 48*a^5*c^5*d^4 + 64*a^6*c^2*d^6))) + (4*a^2*c^5 + 16*a^3*c^
2*d^2 + (a*c^4*d^2 + 4*a^2*c*d^4)*x^4 + 4*(a*c^5*d + 4*a^2*c^2*d^3)*x^3 + 4
*(a*c^6 + 4*a^2*c^3*d^2)*x^2)*sqrt(-(c^6 + 15*a*c^3*d^2 + 60*a^2*d^4 - 2*(a
^3*c^11 + 12*a^4*c^8*d^2 + 48*a^5*c^5*d^4 + 64*a^6*c^2*d^6)*sqrt(-(25*c^6*d
^6 + 360*a*c^3*d^8 + 1296*a^2*d^10)/(a^3*c^25 + 24*a^4*c^22*d^2 + 240*a^5*c
^19*d^4 + 1280*a^6*c^16*d^6 + 3840*a^7*c^13*d^8 + 6144*a^8*c^10*d^10 + 4096
*a^9*c^7*d^12)))/(a^3*c^11 + 12*a^4*c^8*d^2 + 48*a^5*c^5*d^4 + 64*a^6*c^2*d
```

```
^6))*log(5*c^7*d^3 + 81*a*c^4*d^5 + 324*a^2*c*d^7 + (5*c^6*d^4 + 81*a*c^3*d
^6 + 324*a^2*d^8)*x + (5*a^2*c^8*d^4 + 96*a^3*c^5*d^6 + 432*a^4*c^2*d^8 - (
a^3*c^19 + 20*a^4*c^16*d^2 + 144*a^5*c^13*d^4 + 448*a^6*c^10*d^6 + 512*a^7*
c^7*d^8)*sqrt(-(25*c^6*d^6 + 360*a*c^3*d^8 + 1296*a^2*d^10)/(a^3*c^25 + 24*
a^4*c^22*d^2 + 240*a^5*c^19*d^4 + 1280*a^6*c^16*d^6 + 3840*a^7*c^13*d^8 + 6
144*a^8*c^10*d^10 + 4096*a^9*c^7*d^12)))*sqrt(-(c^6 + 15*a*c^3*d^2 + 60*a^2
*d^4 - 2*(a^3*c^11 + 12*a^4*c^8*d^2 + 48*a^5*c^5*d^4 + 64*a^6*c^2*d^6)*sqrt
(-(25*c^6*d^6 + 360*a*c^3*d^8 + 1296*a^2*d^10)/(a^3*c^25 + 24*a^4*c^22*d^2
+ 240*a^5*c^19*d^4 + 1280*a^6*c^16*d^6 + 3840*a^7*c^13*d^8 + 6144*a^8*c^10*
d^10 + 4096*a^9*c^7*d^12)))/(a^3*c^11 + 12*a^4*c^8*d^2 + 48*a^5*c^5*d^4 + 6
4*a^6*c^2*d^6))) - (4*a^2*c^5 + 16*a^3*c^2*d^2 + (a*c^4*d^2 + 4*a^2*c*d^4)*
x^4 + 4*(a*c^5*d + 4*a^2*c^2*d^3)*x^3 + 4*(a*c^6 + 4*a^2*c^3*d^2)*x^2)*sqrt
(-(c^6 + 15*a*c^3*d^2 + 60*a^2*d^4 - 2*(a^3*c^11 + 12*a^4*c^8*d^2 + 48*a^5*
c^5*d^4 + 64*a^6*c^2*d^6)*sqrt(-(25*c^6*d^6 + 360*a*c^3*d^8 + 1296*a^2*d^10
)/(a^3*c^25 + 24*a^4*c^22*d^2 + 240*a^5*c^19*d^4 + 1280*a^6*c^16*d^6 + 3840
*a^7*c^13*d^8 + 6144*a^8*c^10*d^10 + 4096*a^9*c^7*d^12)))/(a^3*c^11 + 12*a^
4*c^8*d^2 + 48*a^5*c^5*d^4 + 64*a^6*c^2*d^6))*log(5*c^7*d^3 + 81*a*c^4*d^5
+ 324*a^2*c*d^7 + (5*c^6*d^4 + 81*a*c^3*d^6 + 324*a^2*d^8)*x - (5*a^2*c^8*d
^4 + 96*a^3*c^5*d^6 + 432*a^4*c^2*d^8 - (a^3*c^19 + 20*a^4*c^16*d^2 + 144*a
^5*c^13*d^4 + 448*a^6*c^10*d^6 + 512*a^7*c^7*d^8)*sqrt(-(25*c^6*d^6 + 360*a
*c^3*d^8 + 1296*a^2*d^10)/(a^3*c^25 + 24*a^4*c^22*d^2 + 240*a^5*c^19*d^4 +
1280*a^6*c^16*d^6 + 3840*a^7*c^13*d^8 + 6144*a^8*c^10*d^10 + 4096*a^9*c^7*d
^12)))*sqrt(-(c^6 + 15*a*c^3*d^2 + 60*a^2*d^4 - 2*(a^3*c^11 + 12*a^4*c^8*d^
2 + 48*a^5*c^5*d^4 + 64*a^6*c^2*d^6)*sqrt(-(25*c^6*d^6 + 360*a*c^3*d^8 + 12
96*a^2*d^10)/(a^3*c^25 + 24*a^4*c^22*d^2 + 240*a^5*c^19*d^4 + 1280*a^6*c^16
*d^6 + 3840*a^7*c^13*d^8 + 6144*a^8*c^10*d^10 + 4096*a^9*c^7*d^12)))/(a^3*c
^11 + 12*a^4*c^8*d^2 + 48*a^5*c^5*d^4 + 64*a^6*c^2*d^6))) + 8*(c^3 + 2*a*d^
2)*x)/(4*a^2*c^5 + 16*a^3*c^2*d^2 + (a*c^4*d^2 + 4*a^2*c*d^4)*x^4 + 4*(a*c^
5*d + 4*a^2*c^2*d^3)*x^3 + 4*(a*c^6 + 4*a^2*c^3*d^2)*x^2)
```

Sympy [A] time = 5.8148, size = 427, normalized size = 0.57

$$\frac{4acd + 3c^2dx^2 + cd^2x^3 + x\left(4ad^2 + 2c^3\right)}{256a^3c^2d^2 + 64a^2c^5 + x^4\left(64a^2cd^4 + 16ac^4d^2\right) + x^3\left(256a^2c^2d^3 + 64ac^5d\right) + x^2\left(256a^2c^3d^2 + 64ac^6\right)} + \text{RootSum}\left(t^4\left(10\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c)**2,x)
```

```
[Out] (4*a*c*d + 3*c**2*d*x**2 + c*d**2*x**3 + x*(4*a*d**2 + 2*c**3))/(256*a**3*c
**2*d**2 + 64*a**2*c**5 + x**4*(64*a**2*c*d**4 + 16*a*c**4*d**2) + x**3*(25
6*a**2*c**2*d**3 + 64*a*c**5*d) + x**2*(256*a**2*c**3*d**2 + 64*a*c**6)) +
```

```
RootSum(_t**4*(1073741824*a**9*c**7*d**6 + 805306368*a**8*c**10*d**4 + 2013
26592*a**7*c**13*d**2 + 16777216*a**6*c**16) + _t**2*(491520*a**5*c**5*d**4
 + 122880*a**4*c**8*d**2 + 8192*a**3*c**11) + 81*a**2*d**4 + 18*a*c**3*d**2
 + c**6, Lambda(_t, _t*log(x + (-67108864*_t**3*a**7*c**7*d**8 - 58720256*_
t**3*a**6*c**10*d**6 - 18874368*_t**3*a**5*c**13*d**4 - 2621440*_t**3*a**4*
c**16*d**2 - 131072*_t**3*a**3*c**19 + 27648*_t*a**4*c**2*d**8 - 9216*_t*a*
*3*c**5*d**6 - 5440*_t*a**2*c**8*d**4 - 736*_t*a*c**11*d**2 - 32*_t*c**14 +
 324*a**2*c*d**7 + 81*a*c**4*d**5 + 5*c**7*d**3)/(324*a**2*d**8 + 81*a*c**3
*d**6 + 5*c**6*d**4))))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.39 $\int \left(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4\right)^4 dx$

Optimal. Leaf size=295

$$\frac{1}{24}e^4\left(65536a^2e^6+20992ad^4e^3+601d^8\right)\left(\frac{d}{4e}+x\right)^9+\frac{64}{13}e^8\left(256ae^3+59d^4\right)\left(\frac{d}{4e}+x\right)^{13}-\frac{72}{11}d^2e^6\left(256ae^3+17d^4\right)\left(\frac{d}{4e}+\right.$$

```
[Out] ((5*d^4 + 256*a*e^3)^4*x)/(1048576*e^4) - (d^2*(5*d^4 + 256*a*e^3)^3*(d/(4*
e) + x)^3)/(8192*e^2) + ((5*d^4 + 256*a*e^3)^2*(59*d^4 + 256*a*e^3)*(d/(4*e
) + x)^5)/5120 - (9*d^2*e^2*(5*d^4 + 256*a*e^3)*(17*d^4 + 256*a*e^3)*(d/(4*
e) + x)^7)/224 + (e^4*(601*d^8 + 20992*a*d^4*e^3 + 65536*a^2*e^6)*(d/(4*e)
+ x)^9)/24 - (72*d^2*e^6*(17*d^4 + 256*a*e^3)*(d/(4*e) + x)^11)/11 + (64*e^
8*(59*d^4 + 256*a*e^3)*(d/(4*e) + x)^13)/13 - (2048*d^2*e^10*(d/(4*e) + x)^
15)/5 + (4096*e^12*(d/(4*e) + x)^17)/17
```

Rubi [A] time = 0.532395, antiderivative size = 295, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1106, 1090}

$$\frac{1}{24}e^4\left(65536a^2e^6+20992ad^4e^3+601d^8\right)\left(\frac{d}{4e}+x\right)^9+\frac{64}{13}e^8\left(256ae^3+59d^4\right)\left(\frac{d}{4e}+x\right)^{13}-\frac{72}{11}d^2e^6\left(256ae^3+17d^4\right)\left(\frac{d}{4e}+\right.$$

Antiderivative was successfully verified.

```
[In] Int[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^4,x]
```


```
[Out] ((5*d^4 + 256*a*e^3)^4*x)/(1048576*e^4) - (d^2*(5*d^4 + 256*a*e^3)^3*(d/(4*
e) + x)^3)/(8192*e^2) + ((5*d^4 + 256*a*e^3)^2*(59*d^4 + 256*a*e^3)*(d/(4*e
) + x)^5)/5120 - (9*d^2*e^2*(5*d^4 + 256*a*e^3)*(17*d^4 + 256*a*e^3)*(d/(4*
e) + x)^7)/224 + (e^4*(601*d^8 + 20992*a*d^4*e^3 + 65536*a^2*e^6)*(d/(4*e)
+ x)^9)/24 - (72*d^2*e^6*(17*d^4 + 256*a*e^3)*(d/(4*e) + x)^11)/11 + (64*e^
8*(59*d^4 + 256*a*e^3)*(d/(4*e) + x)^13)/13 - (2048*d^2*e^10*(d/(4*e) + x)^
15)/5 + (4096*e^12*(d/(4*e) + x)^17)/17
```


Rule 1106

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x
```

```
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1090

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandInteg
rand[(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \left(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4\right)^4 dx = \text{Subst}\left(\int \left(\frac{1}{32}\left(\frac{5d^4}{e} + 256ae^2\right) - 3d^2ex^2 + 8e^3x^4\right)^4 dx, x, \frac{d}{4e} + x\right)$$

$$= \text{Subst}\left(\int \left(\frac{\left(5d^4 + 256ae^3\right)^4}{1048576e^4} - \frac{3d^2\left(5d^4 + 256ae^3\right)^3 x^2}{8192e^2} + \frac{27}{512}d^4\left(5d^4 + 256ae^3\right.\right.\right.$$

$$= \frac{\left(5d^4 + 256ae^3\right)^4 x}{1048576e^4} - \frac{d^2\left(5d^4 + 256ae^3\right)^3\left(\frac{d}{4e} + x\right)^3}{8192e^2} + \frac{\left(5d^4 + 256ae^3\right)^2\left(59d^4 + \right.}{5120}$$

Mathematica [A] time = 0.0489849, size = 345, normalized size = 1.17

$$\frac{128}{3}e^4x^9\left(64a^2e^6 - 32ad^4e^3 + d^8\right) - 4de^3x^8\left(-1536a^2e^6 + 192ad^4e^3 + d^8\right) - \frac{32}{7}d^2e^2x^7\left(-768a^2e^6 - 24ad^4e^3 + d^8\right) + \frac{1}{5}x^5$$

Antiderivative was successfully verified.

```
[In] Integrate[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^4,x]
```

```
[Out] 4096*a^4*e^8*x - 1024*a^3*d^3*e^6*x^2 + 128*a^2*d^6*e^4*x^3 + 8*a*d*e^2*(-d
^8 + 512*a^2*e^6)*x^4 + ((d^12 - 6144*a^2*d^4*e^6 + 16384*a^3*e^9)*x^5)/5 -
 128*a*d^3*e^4*(-d^4 + 8*a*e^3)*x^6 - (32*d^2*e^2*(d^8 - 24*a*d^4*e^3 - 768
*a^2*e^6)*x^7)/7 - 4*d*e^3*(d^8 + 192*a*d^4*e^3 - 1536*a^2*e^6)*x^8 + (128*
e^4*(d^8 - 32*a*d^4*e^3 + 64*a^2*e^6)*x^9)/3 + (128*d^3*e^5*(3*d^4 + 40*a*e
^3)*x^10)/5 + (128*d^2*e^6*(-13*d^4 + 384*a*e^3)*x^11)/11 - 512*d*e^7*(d^4
- 8*a*e^3)*x^12 + (2048*e^8*(-d^4 + 8*a*e^3)*x^13)/13 + 1024*d^3*e^9*x^14 +
 (8192*d^2*e^10*x^15)/5 + 1024*d*e^11*x^16 + (4096*e^12*x^17)/17
```

Maple [A] time = 0.003, size = 500, normalized size = 1.7

$$\frac{4096\,e^{12}x^{17}}{17}+1024\,d e^{11}x^{16}+\frac{8192\,d^{2}e^{10}x^{15}}{5}+1024\,d^{3}e^{9}x^{14}+\frac{\left(16384\,a e^{5}-2048\,d^{4}e^{2}\right)e^{6}x^{13}}{13}+\frac{\left(16384\,a e^{10}d+256\,\left(128\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^4,x)
```

```
[Out] 4096/17*e^12*x^17+1024*d*e^11*x^16+8192/5*d^2*e^10*x^15+1024*d^3*e^9*x^14+1
28/13*(128*a*e^5-16*d^4*e^2)*e^6*x^13+1/12*(16384*a*e^10*d+256*(128*a*e^5-1
6*d^4*e^2)*d*e^5-2048*d^5*e^7)*x^12+1/11*(384*d^6*e^6+32768*a*e^9*d^2+128*(
128*a*e^5-16*d^4*e^2)*d^2*e^4)*x^11+1/10*(14336*a*e^8*d^3+256*d^7*e^5-32*(1
28*a*e^5-16*d^4*e^2)*d^3*e^3)*x^10+1/9*(8192*a^2*e^10-8192*a*e^7*d^4+128*d^
8*e^4+(128*a*e^5-16*d^4*e^2)^2)*x^9+1/8*(16384*a^2*e^9*d-2048*a*e^6*d^5-32*
d^9*e^3+256*a*e^4*d*(128*a*e^5-16*d^4*e^2))*x^8+1/7*(24576*a^2*e^8*d^2+512*
a*e^5*d^6+2*d^6*(128*a*e^5-16*d^4*e^2))*x^7+1/6*(-2048*a^2*e^7*d^3-32*a*e^2
*d^3*(128*a*e^5-16*d^4*e^2)+256*d^7*a*e^4)*x^6+1/5*(128*a^2*e^4*(128*a*e^5-
16*d^4*e^2)-4096*a^2*e^6*d^4+d^12)*x^5+1/4*(16384*a^3*d*e^8-32*a*d^9*e^2)*x
^4+128*a^2*e^4*d^6*x^3-1024*a^3*e^6*d^3*x^2+4096*a^4*e^8*x
```

Maxima [A] time = 1.02578, size = 517, normalized size = 1.75

$$\frac{4096}{17}\,e^{12}x^{17}+1024\,d e^{11}x^{16}+\frac{8192}{5}\,d^{2}e^{10}x^{15}+\frac{8192}{7}\,d^{3}e^{9}x^{14}+\frac{4096}{13}\,d^{4}e^{8}x^{13}+\frac{1}{5}\,d^{12}x^{5}+4096\,a^{4}e^{8}x-\frac{4}{7}\left(7\,e^{3}x^{8}+8\,d e^{2}x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^4,x, algorithm="maxima")
```

```
[Out] 4096/17*e^12*x^17 + 1024*d*e^11*x^16 + 8192/5*d^2*e^10*x^15 + 8192/7*d^3*e^
9*x^14 + 4096/13*d^4*e^8*x^13 + 1/5*d^12*x^5 + 4096*a^4*e^8*x - 4/7*(7*e^3*
x^8 + 8*d*e^2*x^7)*d^9 + 1024/5*(16*e^3*x^5 + 20*d*e^2*x^4 - 5*d^3*x^2)*a^3
*e^6 + 128/165*(45*e^6*x^11 + 99*d*e^5*x^10 + 55*d^2*e^4*x^9)*d^6 + 128/105
*(2240*e^6*x^9 + 5040*d*e^5*x^8 + 2880*d^2*e^4*x^7 + 105*d^6*x^3 - 168*(5*e
^3*x^6 + 6*d*e^2*x^5)*d^3)*a^2*e^4 - 512/1001*(286*e^9*x^14 + 924*d*e^8*x^1
3 + 1001*d^2*e^7*x^12 + 364*d^3*e^6*x^11)*d^3 + 8/15015*(2365440*e^9*x^13 +
 7687680*d*e^8*x^12 + 8386560*d^2*e^7*x^11 + 3075072*d^3*e^6*x^10 - 15015*d
^9*x^4 + 34320*(6*e^3*x^7 + 7*d*e^2*x^6)*d^6 - 32032*(36*e^6*x^10 + 80*d*e^
5*x^9 + 45*d^2*e^4*x^8)*d^3)*a*e^2
```

Fricas [A] time = 1.13587, size = 884, normalized size = 3.

$$\frac{4096}{17}x^{17}e^{12} + 1024x^{16}e^{11}d + \frac{8192}{5}x^{15}e^{10}d^2 + 1024x^{14}e^9d^3 - \frac{2048}{13}x^{13}e^8d^4 + \frac{16384}{13}x^{13}e^{11}a - 512x^{12}e^7d^5 + 4096x^{12}e^{10}da$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^4,x, algorithm="fricas")
```

```
[Out] 4096/17*x^17*e^12 + 1024*x^16*e^11*d + 8192/5*x^15*e^10*d^2 + 1024*x^14*e^9
*d^3 - 2048/13*x^13*e^8*d^4 + 16384/13*x^13*e^11*a - 512*x^12*e^7*d^5 + 409
6*x^12*e^10*d*a - 1664/11*x^11*e^6*d^6 + 49152/11*x^11*e^9*d^2*a + 384/5*x^
10*e^5*d^7 + 1024*x^10*e^8*d^3*a + 128/3*x^9*e^4*d^8 - 4096/3*x^9*e^7*d^4*a
 + 8192/3*x^9*e^10*a^2 - 4*x^8*e^3*d^9 - 768*x^8*e^6*d^5*a + 6144*x^8*e^9*d
*a^2 - 32/7*x^7*e^2*d^10 + 768/7*x^7*e^5*d^6*a + 24576/7*x^7*e^8*d^2*a^2 +
128*x^6*e^4*d^7*a - 1024*x^6*e^7*d^3*a^2 + 1/5*x^5*d^12 - 6144/5*x^5*e^6*d^
4*a^2 + 16384/5*x^5*e^9*a^3 - 8*x^4*e^2*d^9*a + 4096*x^4*e^8*d*a^3 + 128*x^
3*e^4*d^6*a^2 - 1024*x^2*e^6*d^3*a^3 + 4096*x*e^8*a^4
```

Sympy [A] time = 0.124367, size = 366, normalized size = 1.24

$$4096a^4e^8x - 1024a^3d^3e^6x^2 + 128a^2d^6e^4x^3 + 1024d^3e^9x^{14} + \frac{8192d^2e^{10}x^{15}}{5} + 1024de^{11}x^{16} + \frac{4096e^{12}x^{17}}{17} + x^{13}\left(\frac{16384ae^{1}}{13}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e**3*x**4+8*d*e**2*x**3-d**3*x+8*a*e**2)**4,x)
```

```
[Out] 4096*a**4*e**8*x - 1024*a**3*d**3*e**6*x**2 + 128*a**2*d**6*e**4*x**3 + 102
4*d**3*e**9*x**14 + 8192*d**2*e**10*x**15/5 + 1024*d*e**11*x**16 + 4096*e**
12*x**17/17 + x**13*(16384*a*e**11/13 - 2048*d**4*e**8/13) + x**12*(4096*a*
d*e**10 - 512*d**5*e**7) + x**11*(49152*a*d**2*e**9/11 - 1664*d**6*e**6/11)
 + x**10*(1024*a*d**3*e**8 + 384*d**7*e**5/5) + x**9*(8192*a**2*e**10/3 - 4
096*a*d**4*e**7/3 + 128*d**8*e**4/3) + x**8*(6144*a**2*d*e**9 - 768*a*d**5*
e**6 - 4*d**9*e**3) + x**7*(24576*a**2*d**2*e**8/7 + 768*a*d**6*e**5/7 - 32
*d**10*e**2/7) + x**6*(-1024*a**2*d**3*e**7 + 128*a*d**7*e**4) + x**5*(1638
4*a**3*e**9/5 - 6144*a**2*d**4*e**6/5 + d**12/5) + x**4*(4096*a**3*d*e**8 -
 8*a*d**9*e**2)
```

Giac [A] time = 1.11671, size = 436, normalized size = 1.48

$$\frac{4096}{17}x^{17}e^{12}+1024\,dx^{16}e^{11}+\frac{8192}{5}\,d^2x^{15}e^{10}+1024\,d^3x^{14}e^9-\frac{2048}{13}\,d^4x^{13}e^8-512\,d^5x^{12}e^7-\frac{1664}{11}\,d^6x^{11}e^6+\frac{384}{5}\,d^7x^{10}e^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^4,x, algorithm="giac")
```

```
[Out] 4096/17*x^17*e^12 + 1024*d*x^16*e^11 + 8192/5*d^2*x^15*e^10 + 1024*d^3*x^14
*e^9 - 2048/13*d^4*x^13*e^8 - 512*d^5*x^12*e^7 - 1664/11*d^6*x^11*e^6 + 384
/5*d^7*x^10*e^5 + 128/3*d^8*x^9*e^4 - 4*d^9*x^8*e^3 - 32/7*d^10*x^7*e^2 + 1
/5*d^12*x^5 + 16384/13*a*x^13*e^11 + 4096*a*d*x^12*e^10 + 49152/11*a*d^2*x^
11*e^9 + 1024*a*d^3*x^10*e^8 - 4096/3*a*d^4*x^9*e^7 - 768*a*d^5*x^8*e^6 + 7
68/7*a*d^6*x^7*e^5 + 128*a*d^7*x^6*e^4 - 8*a*d^9*x^4*e^2 + 8192/3*a^2*x^9*e
^10 + 6144*a^2*d*x^8*e^9 + 24576/7*a^2*d^2*x^7*e^8 - 1024*a^2*d^3*x^6*e^7 -
 6144/5*a^2*d^4*x^5*e^6 + 128*a^2*d^6*x^3*e^4 + 16384/5*a^3*x^5*e^9 + 4096*
a^3*d*x^4*e^8 - 1024*a^3*d^3*x^2*e^6 + 4096*a^4*x*e^8
```

3.40 $\int \left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^3 \, dx$

Optimal. Leaf size=203

$$-\frac{1}{4}dx^4\left(d^8-1536a^2e^6\right)-96a^2d^3e^4x^2+512a^3e^6x-\frac{128}{3}e^5x^9\left(d^4-4ae^3\right)-24de^4x^8\left(d^4-16ae^3\right)+\frac{24}{7}d^2e^3x^7\left(64ae^3+d^4\right.$$

```
[Out] 512*a^3*e^6*x - 96*a^2*d^3*e^4*x^2 + 8*a*d^6*e^2*x^3 - (d*(d^8 - 1536*a^2*e
^6)*x^4)/4 - (384*a*e^4*(d^4 - 4*a*e^3)*x^5)/5 + 4*d^3*e^2*(d^4 - 16*a*e^3)
*x^6 + (24*d^2*e^3*(d^4 + 64*a*e^3)*x^7)/7 - 24*d*e^4*(d^4 - 16*a*e^3)*x^8
- (128*e^5*(d^4 - 4*a*e^3)*x^9)/3 + 32*d^3*e^6*x^10 + (1536*d^2*e^7*x^11)/1
1 + 128*d*e^8*x^12 + (512*e^9*x^13)/13
```

Rubi [A] time = 0.122958, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.031, Rules used = {2061}

$$-\frac{1}{4}dx^4\left(d^8-1536a^2e^6\right)-96a^2d^3e^4x^2+512a^3e^6x-\frac{128}{3}e^5x^9\left(d^4-4ae^3\right)-24de^4x^8\left(d^4-16ae^3\right)+\frac{24}{7}d^2e^3x^7\left(64ae^3+d^4\right.$$

Antiderivative was successfully verified.

```
[In] Int[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^3,x]
```

```
[Out] 512*a^3*e^6*x - 96*a^2*d^3*e^4*x^2 + 8*a*d^6*e^2*x^3 - (d*(d^8 - 1536*a^2*e
^6)*x^4)/4 - (384*a*e^4*(d^4 - 4*a*e^3)*x^5)/5 + 4*d^3*e^2*(d^4 - 16*a*e^3)
*x^6 + (24*d^2*e^3*(d^4 + 64*a*e^3)*x^7)/7 - 24*d*e^4*(d^4 - 16*a*e^3)*x^8
- (128*e^5*(d^4 - 4*a*e^3)*x^9)/3 + 32*d^3*e^6*x^10 + (1536*d^2*e^7*x^11)/1
1 + 128*d*e^8*x^12 + (512*e^9*x^13)/13
```

Rule 2061

```
Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I
GtQ[p, 0]
```

Rubi steps

$$\int \left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^3 \, dx = \int \left(512a^3e^6-192a^2d^3e^4x+24ad^6e^2x^2-d\left(d^8-1536a^2e^6\right)x^3-384ae^4\left(d^4-4a\right.\right.$$

$$= 512a^3e^6x-96a^2d^3e^4x^2+8ad^6e^2x^3-\frac{1}{4}d\left(d^8-1536a^2e^6\right)x^4-\frac{384}{5}ae^4\left(d^4-4ae\right.$$

Mathematica [A] time = 0.0248907, size = 207, normalized size = 1.02

$$-\frac{1}{4}dx^4\left(d^8-1536a^2e^6\right)-96a^2d^3e^4x^2+512a^3e^6x+\frac{128}{3}e^5x^9\left(4ae^3-d^4\right)-24de^4x^8\left(d^4-16ae^3\right)+\frac{24}{7}d^2e^3x^7\left(64ae^3+d^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^3,x]
```

```
[Out] 512*a^3*e^6*x - 96*a^2*d^3*e^4*x^2 + 8*a*d^6*e^2*x^3 - (d*(d^8 - 1536*a^2*e
^6)*x^4)/4 + (384*a*e^4*(-d^4 + 4*a*e^3)*x^5)/5 + 4*d^3*e^2*(d^4 - 16*a*e^3
)*x^6 + (24*d^2*e^3*(d^4 + 64*a*e^3)*x^7)/7 - 24*d*e^4*(d^4 - 16*a*e^3)*x^8
 + (128*e^5*(-d^4 + 4*a*e^3)*x^9)/3 + 32*d^3*e^6*x^10 + (1536*d^2*e^7*x^11)
/11 + 128*d*e^8*x^12 + (512*e^9*x^13)/13
```

Maple [A] time = 0.001, size = 288, normalized size = 1.4

$$\frac{512\,e^9x^{13}}{13}+128\,de^8x^{12}+\frac{1536\,d^2e^7x^{11}}{11}+32\,d^3e^6x^{10}+\frac{\left(512\,ae^8-256\,d^4e^5+8\,e^3\left(128\,ae^5-16\,d^4e^2\right)\right)x^9}{9}+\frac{\left(2048\,ae^7d-6\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^3,x)
```

```
[Out] 512/13*e^9*x^13+128*d*e^8*x^12+1536/11*d^2*e^7*x^11+32*d^3*e^6*x^10+1/9*(51
2*a*e^8-256*d^4*e^5+8*e^3*(128*a*e^5-16*d^4*e^2))*x^9+1/8*(2048*a*e^7*d-64*
d^5*e^4+8*d*e^2*(128*a*e^5-16*d^4*e^2))*x^8+1/7*(1536*a*d^2*e^6+24*d^6*e^3)
*x^7+1/6*(-256*a*e^5*d^3-d^3*(128*a*e^5-16*d^4*e^2)+8*d^7*e^2)*x^6+1/5*(8*a
*e^2*(128*a*e^5-16*d^4*e^2)-256*d^4*a*e^4+512*e^7*a^2)*x^5+1/4*(1536*a^2*d*
e^6-d^9)*x^4+8*a*d^6*e^2*x^3-96*a^2*d^3*e^4*x^2+512*a^3*e^6*x
```

Maxima [A] time = 1.16221, size = 289, normalized size = 1.42

$$\frac{512}{13}\,e^9x^{13}+128\,de^8x^{12}+\frac{1536}{11}\,d^2e^7x^{11}+\frac{256}{5}\,d^3e^6x^{10}-\frac{1}{4}\,d^9x^4+512\,a^3e^6x+\frac{4}{7}\left(6\,e^3x^7+7\,de^2x^6\right)d^6+\frac{96}{5}\left(16\,e^3x^5+20\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^3,x, algorithm="maxima")
```

```
[Out] 512/13*e^9*x^13 + 128*d*e^8*x^12 + 1536/11*d^2*e^7*x^11 + 256/5*d^3*e^6*x^1
0 - 1/4*d^9*x^4 + 512*a^3*e^6*x + 4/7*(6*e^3*x^7 + 7*d*e^2*x^6)*d^6 + 96/5*
(16*e^3*x^5 + 20*d*e^2*x^4 - 5*d^3*x^2)*a^2*e^4 - 8/15*(36*e^6*x^10 + 80*d*
e^5*x^9 + 45*d^2*e^4*x^8)*d^3 + 8/105*(2240*e^6*x^9 + 5040*d*e^5*x^8 + 2880
*d^2*e^4*x^7 + 105*d^6*x^3 - 168*(5*e^3*x^6 + 6*d*e^2*x^5)*d^3)*a*e^2
```

Fricas [A] time = 1.1556, size = 482, normalized size = 2.37

$$\frac{512}{13}x^{13}e^9 + 128x^{12}e^8d + \frac{1536}{11}x^{11}e^7d^2 + 32x^{10}e^6d^3 - \frac{128}{3}x^9e^5d^4 + \frac{512}{3}x^9e^8a - 24x^8e^4d^5 + 384x^8e^7da + \frac{24}{7}x^7e^3d^6 + \frac{153}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^3,x, algorithm="fricas")
```

```
[Out] 512/13*x^13*e^9 + 128*x^12*e^8*d + 1536/11*x^11*e^7*d^2 + 32*x^10*e^6*d^3 -
 128/3*x^9*e^5*d^4 + 512/3*x^9*e^8*a - 24*x^8*e^4*d^5 + 384*x^8*e^7*d*a + 2
4/7*x^7*e^3*d^6 + 1536/7*x^7*e^6*d^2*a + 4*x^6*e^2*d^7 - 64*x^6*e^5*d^3*a -
 384/5*x^5*e^4*d^4*a + 1536/5*x^5*e^7*a^2 - 1/4*x^4*d^9 + 384*x^4*e^6*d*a^2
 + 8*x^3*e^2*d^6*a - 96*x^2*e^4*d^3*a^2 + 512*x*e^6*a^3
```

Sympy [A] time = 0.099933, size = 218, normalized size = 1.07

$$512a^3e^6x - 96a^2d^3e^4x^2 + 8ad^6e^2x^3 + 32d^3e^6x^{10} + \frac{1536d^2e^7x^{11}}{11} + 128de^8x^{12} + \frac{512e^9x^{13}}{13} + x^9\left(\frac{512ae^8}{3} - \frac{128d^4e^5}{3}\right) + x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e**3*x**4+8*d*e**2*x**3-d**3*x+8*a*e**2)**3,x)
```

```
[Out] 512*a**3*e**6*x - 96*a**2*d**3*e**4*x**2 + 8*a*d**6*e**2*x**3 + 32*d**3*e**
6*x**10 + 1536*d**2*e**7*x**11/11 + 128*d*e**8*x**12 + 512*e**9*x**13/13 +
x**9*(512*a*e**8/3 - 128*d**4*e**5/3) + x**8*(384*a*d*e**7 - 24*d**5*e**4)
+ x**7*(1536*a*d**2*e**6/7 + 24*d**6*e**3/7) + x**6*(-64*a*d**3*e**5 + 4*d*
*7*e**2) + x**5*(1536*a**2*e**7/5 - 384*a*d**4*e**4/5) + x**4*(384*a**2*d*e
**6 - d**9/4)
```

Giac [A] time = 1.14532, size = 252, normalized size = 1.24

$$\frac{512}{13}\,x^{13}e^9 + 128\,dx^{12}e^8 + \frac{1536}{11}\,d^2x^{11}e^7 + 32\,d^3x^{10}e^6 - \frac{128}{3}\,d^4x^9e^5 - 24\,d^5x^8e^4 + \frac{24}{7}\,d^6x^7e^3 + 4\,d^7x^6e^2 - \frac{1}{4}\,d^9x^4 + \frac{512}{3}\,ax^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^3,x, algorithm="giac")
```

```
[Out] 512/13*x^13*e^9 + 128*d*x^12*e^8 + 1536/11*d^2*x^11*e^7 + 32*d^3*x^10*e^6 -
 128/3*d^4*x^9*e^5 - 24*d^5*x^8*e^4 + 24/7*d^6*x^7*e^3 + 4*d^7*x^6*e^2 - 1/
4*d^9*x^4 + 512/3*a*x^9*e^8 + 384*a*d*x^8*e^7 + 1536/7*a*d^2*x^7*e^6 - 64*a
*d^3*x^6*e^5 - 384/5*a*d^4*x^5*e^4 + 8*a*d^6*x^3*e^2 + 1536/5*a^2*x^5*e^7 +
 384*a^2*d*x^4*e^6 - 96*a^2*d^3*x^2*e^4 + 512*a^3*x*e^6
```

3.41 $\int \left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^2 \, dx$

Optimal. Leaf size=107

$$64a^2e^4x-\frac{16}{5}e^2x^5\left(d^4-8ae^3\right)-8ad^3e^2x^2+32ade^4x^4+\frac{64}{7}d^2e^4x^7-\frac{8}{3}d^3e^3x^6+\frac{d^6x^3}{3}+16de^5x^8+\frac{64e^6x^9}{9}$$

```
[Out] 64*a^2*e^4*x - 8*a*d^3*e^2*x^2 + (d^6*x^3)/3 + 32*a*d*e^4*x^4 - (16*e^2*(d^
4 - 8*a*e^3)*x^5)/5 - (8*d^3*e^3*x^6)/3 + (64*d^2*e^4*x^7)/7 + 16*d*e^5*x^8
 + (64*e^6*x^9)/9
```

Rubi [A] time = 0.0509105, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.031, Rules used = {2061}

$$64a^2e^4x-\frac{16}{5}e^2x^5\left(d^4-8ae^3\right)-8ad^3e^2x^2+32ade^4x^4+\frac{64}{7}d^2e^4x^7-\frac{8}{3}d^3e^3x^6+\frac{d^6x^3}{3}+16de^5x^8+\frac{64e^6x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Int[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^2,x]
```

```
[Out] 64*a^2*e^4*x - 8*a*d^3*e^2*x^2 + (d^6*x^3)/3 + 32*a*d*e^4*x^4 - (16*e^2*(d^
4 - 8*a*e^3)*x^5)/5 - (8*d^3*e^3*x^6)/3 + (64*d^2*e^4*x^7)/7 + 16*d*e^5*x^8
 + (64*e^6*x^9)/9
```

Rule 2061

```
Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I
GtQ[p, 0]
```

Rubi steps

$$\int \left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^2 \, dx = \int \left(64a^2e^4-16ad^3e^2x+d^6x^2+128ade^4x^3-16e^2\left(d^4-8ae^3\right)x^4-16d^3e^3x^5+6\right.$$

$$= 64a^2e^4x-8ad^3e^2x^2+\frac{d^6x^3}{3}+32ade^4x^4-\frac{16}{5}e^2\left(d^4-8ae^3\right)x^5-\frac{8}{3}d^3e^3x^6+\frac{64}{7}d$$

Mathematica [A] time = 0.0125221, size = 109, normalized size = 1.02

$$64a^2e^4x+\frac{16}{5}e^2x^5\left(8ae^3-d^4\right)-8ad^3e^2x^2+32ade^4x^4+\frac{64}{7}d^2e^4x^7-\frac{8}{3}d^3e^3x^6+\frac{d^6x^3}{3}+16de^5x^8+\frac{64e^6x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^2,x]
```

```
[Out] 64*a^2*e^4*x - 8*a*d^3*e^2*x^2 + (d^6*x^3)/3 + 32*a*d*e^4*x^4 + (16*e^2*(-d
^4 + 8*a*e^3)*x^5)/5 - (8*d^3*e^3*x^6)/3 + (64*d^2*e^4*x^7)/7 + 16*d*e^5*x^
8 + (64*e^6*x^9)/9
```

Maple [A] time = 0., size = 100, normalized size = 0.9

$$\frac{64\,e^{6}x^{9}}{9}+16\,de^{5}x^{8}+\frac{64\,d^{2}e^{4}x^{7}}{7}-\frac{8\,d^{3}e^{3}x^{6}}{3}+\frac{\left(128\,ae^{5}-16\,d^{4}e^{2}\right)x^{5}}{5}+32\,ade^{4}x^{4}+\frac{d^{6}x^{3}}{3}-8\,ad^{3}e^{2}x^{2}+64\,a^{2}e^{4}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^2,x)
```

```
[Out] 64/9*e^6*x^9+16*d*e^5*x^8+64/7*d^2*e^4*x^7-8/3*d^3*e^3*x^6+1/5*(128*a*e^5-1
6*d^4*e^2)*x^5+32*a*d*e^4*x^4+1/3*d^6*x^3-8*a*d^3*e^2*x^2+64*a^2*e^4*x
```

Maxima [A] time = 1.1223, size = 136, normalized size = 1.27

$$\frac{64}{9}\,e^{6}x^{9}+16\,de^{5}x^{8}+\frac{64}{7}\,d^{2}e^{4}x^{7}+\frac{1}{3}\,d^{6}x^{3}+64\,a^{2}e^{4}x-\frac{8}{15}\left(5\,e^{3}x^{6}+6\,de^{2}x^{5}\right)d^{3}+\frac{8}{5}\left(16\,e^{3}x^{5}+20\,de^{2}x^{4}-5\,d^{3}x^{2}\right)ae^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^2,x, algorithm="maxima")
```

```
[Out] 64/9*e^6*x^9 + 16*d*e^5*x^8 + 64/7*d^2*e^4*x^7 + 1/3*d^6*x^3 + 64*a^2*e^4*x
 - 8/15*(5*e^3*x^6 + 6*d*e^2*x^5)*d^3 + 8/5*(16*e^3*x^5 + 20*d*e^2*x^4 - 5*
d^3*x^2)*a*e^2
```

Fricas [A] time = 1.3873, size = 225, normalized size = 2.1

$$\frac{64}{9}x^{9}e^{6}+16x^{8}e^{5}d+\frac{64}{7}x^{7}e^{4}d^{2}-\frac{8}{3}x^{6}e^{3}d^{3}-\frac{16}{5}x^{5}e^{2}d^{4}+\frac{128}{5}x^{5}e^{5}a+32x^{4}e^{4}da+\frac{1}{3}x^{3}d^{6}-8x^{2}e^{2}d^{3}a+64xe^{4}a^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^2,x, algorithm="fricas")
```

```
[Out] 64/9*x^9*e^6 + 16*x^8*e^5*d + 64/7*x^7*e^4*d^2 - 8/3*x^6*e^3*d^3 - 16/5*x^5
*e^2*d^4 + 128/5*x^5*e^5*a + 32*x^4*e^4*d*a + 1/3*x^3*d^6 - 8*x^2*e^2*d^3*a
 + 64*x*e^4*a^2
```

Sympy [A] time = 0.080088, size = 112, normalized size = 1.05

$$64a^2e^4x - 8ad^3e^2x^2 + 32ade^4x^4 + \frac{d^6x^3}{3} - \frac{8d^3e^3x^6}{3} + \frac{64d^2e^4x^7}{7} + 16de^5x^8 + \frac{64e^6x^9}{9} + x^5\left(\frac{128ae^5}{5} - \frac{16d^4e^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e**3*x**4+8*d*e**2*x**3-d**3*x+8*a*e**2)**2,x)
```

```
[Out] 64*a**2*e**4*x - 8*a*d**3*e**2*x**2 + 32*a*d*e**4*x**4 + d**6*x**3/3 - 8*d*
*3*e**3*x**6/3 + 64*d**2*e**4*x**7/7 + 16*d*e**5*x**8 + 64*e**6*x**9/9 + x*
*5*(128*a*e**5/5 - 16*d**4*e**2/5)
```

Giac [A] time = 1.12963, size = 122, normalized size = 1.14

$$\frac{64}{9}x^9e^6 + 16\,dx^8e^5 + \frac{64}{7}d^2x^7e^4 - \frac{8}{3}d^3x^6e^3 - \frac{16}{5}d^4x^5e^2 + \frac{1}{3}d^6x^3 + \frac{128}{5}ax^5e^5 + 32\,adx^4e^4 - 8\,ad^3x^2e^2 + 64\,a^2xe^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^2,x, algorithm="giac")
```

```
[Out] 64/9*x^9*e^6 + 16*d*x^8*e^5 + 64/7*d^2*x^7*e^4 - 8/3*d^3*x^6*e^3 - 16/5*d^4
*x^5*e^2 + 1/3*d^6*x^3 + 128/5*a*x^5*e^5 + 32*a*d*x^4*e^4 - 8*a*d^3*x^2*e^2
 + 64*a^2*x*e^4
```

3.42 $\int \left(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4\right) dx$

Optimal. Leaf size=37

$$8ae^2x - \frac{d^3x^2}{2} + 2de^2x^4 + \frac{8e^3x^5}{5}$$

```
[Out] 8*a*e^2*x - (d^3*x^2)/2 + 2*d*e^2*x^4 + (8*e^3*x^5)/5
```

Rubi [A] time = 0.0073737, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$8ae^2x - \frac{d^3x^2}{2} + 2de^2x^4 + \frac{8e^3x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4,x]
```

```
[Out] 8*a*e^2*x - (d^3*x^2)/2 + 2*d*e^2*x^4 + (8*e^3*x^5)/5
```

Rubi steps

$$\int \left(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4\right) dx = 8ae^2x - \frac{d^3x^2}{2} + 2de^2x^4 + \frac{8e^3x^5}{5}$$

Mathematica [A] time = 0.0000512, size = 37, normalized size = 1.

$$8ae^2x - \frac{d^3x^2}{2} + 2de^2x^4 + \frac{8e^3x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4,x]
```

```
[Out] 8*a*e^2*x - (d^3*x^2)/2 + 2*d*e^2*x^4 + (8*e^3*x^5)/5
```

Maple [A] time = 0., size = 34, normalized size = 0.9

$$8\,a e^{2} x -\frac{d^{3} x^{2}}{2}+2\,d e^{2} x^{4}+\frac{8\,e^{3} x^{5}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2,x)
```

```
[Out] 8*a*e^2*x-1/2*d^3*x^2+2*d*e^2*x^4+8/5*e^3*x^5
```

Maxima [A] time = 1.15237, size = 45, normalized size = 1.22

$$\frac{8}{5}\,e^{3} x^{5}+2\,d e^{2} x^{4}-\frac{1}{2}\,d^{3} x^{2}+8\,a e^{2} x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2,x, algorithm="maxima")
```

```
[Out] 8/5*e^3*x^5 + 2*d*e^2*x^4 - 1/2*d^3*x^2 + 8*a*e^2*x
```

Fricas [A] time = 1.26313, size = 72, normalized size = 1.95

$$\frac{8}{5}x^{5}e^{3}+2x^{4}e^{2}d-\frac{1}{2}x^{2}d^{3}+8xe^{2}a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2,x, algorithm="fricas")
```

```
[Out] 8/5*x^5*e^3 + 2*x^4*e^2*d - 1/2*x^2*d^3 + 8*x*e^2*a
```

Sympy [A] time = 0.06298, size = 36, normalized size = 0.97

$$8ae^{2}x-\frac{d^{3}x^{2}}{2}+2de^{2}x^{4}+\frac{8e^{3}x^{5}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(8*e**3*x**4+8*d*e**2*x**3-d**3*x+8*a*e**2,x)
```

```
[Out] 8*a*e**2*x - d**3*x**2/2 + 2*d*e**2*x**4 + 8*e**3*x**5/5
```

Giac [A] time = 1.14496, size = 41, normalized size = 1.11

$$\frac{8}{5}\,x^5e^3 + 2\,dx^4e^2 - \frac{1}{2}\,d^3x^2 + 8\,axe^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2,x, algorithm="giac")
```

```
[Out] 8/5*x^5*e^3 + 2*d*x^4*e^2 - 1/2*d^3*x^2 + 8*a*x*e^2
```

3.43 $\int \frac{1}{8ae^2-d^3x+8de^2x^3+8e^3x^4}\,dx$

Optimal. Leaf size=153

$$\frac{2\tanh^{-1}\left(\frac{d+4ex}{\sqrt{3d^2-2\sqrt{d^4-64ae^3}}}\right)}{\sqrt{d^4-64ae^3}\sqrt{3d^2-2\sqrt{d^4-64ae^3}}}-\frac{2\tanh^{-1}\left(\frac{d+4ex}{\sqrt{2\sqrt{d^4-64ae^3}+3d^2}}\right)}{\sqrt{d^4-64ae^3}\sqrt{2\sqrt{d^4-64ae^3}+3d^2}}$$

```
[Out] (2*ArcTanh[(d + 4*e*x)/Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]]])/(Sqrt[d^4 - 6
4*a*e^3]*Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]]) - (2*ArcTanh[(d + 4*e*x)/Sqr
t[3*d^2 + 2*Sqrt[d^4 - 64*a*e^3]]])/(Sqrt[d^4 - 64*a*e^3]*Sqrt[3*d^2 + 2*Sq
rt[d^4 - 64*a*e^3]])
```

Rubi [A] time = 0.254435, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {1106, 1093, 208}

$$\frac{2\tanh^{-1}\left(\frac{d+4ex}{\sqrt{3d^2-2\sqrt{d^4-64ae^3}}}\right)}{\sqrt{d^4-64ae^3}\sqrt{3d^2-2\sqrt{d^4-64ae^3}}}-\frac{2\tanh^{-1}\left(\frac{d+4ex}{\sqrt{2\sqrt{d^4-64ae^3}+3d^2}}\right)}{\sqrt{d^4-64ae^3}\sqrt{2\sqrt{d^4-64ae^3}+3d^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^(-1),x]
```


```
[Out] (2*ArcTanh[(d + 4*e*x)/Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]]])/(Sqrt[d^4 - 6
4*a*e^3]*Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]]) - (2*ArcTanh[(d + 4*e*x)/Sqr
t[3*d^2 + 2*Sqrt[d^4 - 64*a*e^3]]])/(Sqrt[d^4 - 64*a*e^3]*Sqrt[3*d^2 + 2*Sq
rt[d^4 - 64*a*e^3]])
```


Rule 1106

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{8ae^2-d^3x+8de^2x^3+8e^3x^4}\,dx = \text{Subst}\left(\int \frac{1}{\frac{1}{32}\left(\frac{5d^4}{e}+256ae^2\right)-3d^2ex^2+8e^3x^4}\,dx, x, \frac{d}{4e}+x\right)$$

$$= \frac{\left(4e^2\right)\text{Subst}\left(\int \frac{1}{-\frac{3d^2e}{2}-e\sqrt{d^4-64ae^3}+8e^3x^2}\,dx, x, \frac{d}{4e}+x\right)}{\sqrt{d^4-64ae^3}} - \frac{\left(4e^2\right)\text{Subst}\left(\int \frac{1}{-\frac{3d^2e}{2}+e\sqrt{d^4-64ae}}\right.}{\sqrt{d^4-64ae}}$$

$$= \frac{2\tanh^{-1}\left(\frac{d+4ex}{\sqrt{3d^2-2\sqrt{d^4-64ae^3}}}\right)}{\sqrt{d^4-64ae^3}\sqrt{3d^2-2\sqrt{d^4-64ae^3}}} - \frac{2\tanh^{-1}\left(\frac{d+4ex}{\sqrt{3d^2+2\sqrt{d^4-64ae^3}}}\right)}{\sqrt{d^4-64ae^3}\sqrt{3d^2+2\sqrt{d^4-64ae^3}}}$$

Mathematica [C] time = 0.0224046, size = 71, normalized size = 0.46

$$-\text{RootSum}\left[8\#1^3de^2+8\#1^4e^3-\#1d^3+8ae^2\&, \frac{\log(x-\#1)}{-24\#1^2de^2-32\#1^3e^3+d^3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^(-1),x]
```

```
[Out] -RootSum[8*a*e^2 - d^3*#1 + 8*d*e^2*#1^3 + 8*e^3*#1^4 & , Log[x - #1]/(d^3
- 24*d*e^2*#1^2 - 32*e^3*#1^3) & ]
```

Maple [C] time = 0.067, size = 67, normalized size = 0.4

$$\sum_{\_R=\text{RootOf}\left(8e^3\_Z^4+8de^2\_Z^3-d^3\_Z+8ae^2\right)} \frac{\ln(x-\_R)}{32\_R^3e^3+24\_R^2de^2-d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2),x)
```

```
[Out] sum(1/(32*_R^3*e^3+24*_R^2*d*e^2-d^3)*ln(x-_R),_R=RootOf(8*_Z^4*e^3+8*_Z^3*
d*e^2-_Z*d^3+8*a*e^2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{8\,e^3x^4 + 8\,de^2x^3 - d^3x + 8\,ae^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2),x, algorithm="maxima")
```

```
[Out] integrate(1/(8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2), x)
```

Fricas [B] time = 1.6947, size = 2630, normalized size = 17.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2),x, algorithm="fricas")
```

```
[Out] -sqrt((3*d^2 + 2*(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6)/sqrt(25*d^12 + 960*
a*d^8*e^3 - 98304*a^2*d^4*e^6 - 4194304*a^3*e^9))/(5*d^8 - 64*a*d^4*e^3 - 1
6384*a^2*e^6))*log(8*e*x + 2*(2*d^4 - 128*a*e^3 - 3*(5*d^10 - 64*a*d^6*e^3
- 16384*a^2*d^2*e^6)/sqrt(25*d^12 + 960*a*d^8*e^3 - 98304*a^2*d^4*e^6 - 419
4304*a^3*e^9))*sqrt((3*d^2 + 2*(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6)/sqrt(
25*d^12 + 960*a*d^8*e^3 - 98304*a^2*d^4*e^6 - 4194304*a^3*e^9))/(5*d^8 - 64
*a*d^4*e^3 - 16384*a^2*e^6)) + 2*d) + sqrt((3*d^2 + 2*(5*d^8 - 64*a*d^4*e^3
 - 16384*a^2*e^6)/sqrt(25*d^12 + 960*a*d^8*e^3 - 98304*a^2*d^4*e^6 - 419430
4*a^3*e^9))/(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6))*log(8*e*x - 2*(2*d^4 -
128*a*e^3 - 3*(5*d^10 - 64*a*d^6*e^3 - 16384*a^2*d^2*e^6)/sqrt(25*d^12 + 96
0*a*d^8*e^3 - 98304*a^2*d^4*e^6 - 4194304*a^3*e^9))*sqrt((3*d^2 + 2*(5*d^8
- 64*a*d^4*e^3 - 16384*a^2*e^6)/sqrt(25*d^12 + 960*a*d^8*e^3 - 98304*a^2*d^
4*e^6 - 4194304*a^3*e^9))/(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6)) + 2*d) -
sqrt((3*d^2 - 2*(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6)/sqrt(25*d^12 + 960*a
```

```
*d^8*e^3 - 98304*a^2*d^4*e^6 - 4194304*a^3*e^9))/(5*d^8 - 64*a*d^4*e^3 - 16
384*a^2*e^6))*log(8*e*x + 2*(2*d^4 - 128*a*e^3 + 3*(5*d^10 - 64*a*d^6*e^3 -
 16384*a^2*d^2*e^6)/sqrt(25*d^12 + 960*a*d^8*e^3 - 98304*a^2*d^4*e^6 - 4194
304*a^3*e^9))*sqrt((3*d^2 - 2*(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6)/sqrt(2
5*d^12 + 960*a*d^8*e^3 - 98304*a^2*d^4*e^6 - 4194304*a^3*e^9))/(5*d^8 - 64*
a*d^4*e^3 - 16384*a^2*e^6)) + 2*d) + sqrt((3*d^2 - 2*(5*d^8 - 64*a*d^4*e^3
- 16384*a^2*e^6)/sqrt(25*d^12 + 960*a*d^8*e^3 - 98304*a^2*d^4*e^6 - 4194304
*a^3*e^9))/(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6))*log(8*e*x - 2*(2*d^4 - 1
28*a*e^3 + 3*(5*d^10 - 64*a*d^6*e^3 - 16384*a^2*d^2*e^6)/sqrt(25*d^12 + 960
*a*d^8*e^3 - 98304*a^2*d^4*e^6 - 4194304*a^3*e^9))*sqrt((3*d^2 - 2*(5*d^8 -
 64*a*d^4*e^3 - 16384*a^2*e^6)/sqrt(25*d^12 + 960*a*d^8*e^3 - 98304*a^2*d^4
*e^6 - 4194304*a^3*e^9))/(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6)) + 2*d)
```

Sympy [A] time = 1.45378, size = 122, normalized size = 0.8

$$\operatorname{RootSum}\left(t^4\left(1048576a^3e^9-12288a^2d^4e^6-384ad^8e^3+5d^{12}\right)+t^2\left(384ad^2e^3-6d^6\right)+1,\left(t\mapsto t\log\left(x+\frac{-49152t^3a^2d^2\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*e**3*x**4+8*d*e**2*x**3-d**3*x+8*a*e**2),x)
```

```
[Out] RootSum(_t**4*(1048576*a**3*e**9 - 12288*a**2*d**4*e**6 - 384*a*d**8*e**3 +
 5*d**12) + _t**2*(384*a*d**2*e**3 - 6*d**6) + 1, Lambda(_t, _t*log(x + (-4
9152*_t**3*a**2*d**2*e**6 - 192*_t**3*a*d**6*e**3 + 15*_t**3*d**10 + 256*_t
*a*e**3 - 13*_t*d**4 + 2*d)/(8*e))))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{8\,e^3x^4+8\,de^2x^3-d^3x+8\,ae^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2),x, algorithm="giac")
```

```
[Out] integrate(1/(8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2), x)
```

3.44 $\int \frac{1}{(8ae^2-d^3x+8de^2x^3+8e^3x^4)^2}\,dx$

Optimal. Leaf size=342

$$\frac{2e\left(\frac{d}{4e}+x\right)\left(-256ae^3-48d^2e^2\left(\frac{d}{4e}+x\right)^2+13d^4\right)}{\left(-16384a^2e^6-64ad^4e^3+5d^8\right)\left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)}-\frac{24e\left(-d^2\sqrt{d^4-64ae^3}+128ae^3+d^4\right)\tanh^{-1}\left(\frac{d+4e}{\sqrt{3d^2-2\sqrt{d}}}\right.}{\left(d^4-64ae^3\right)^{3/2}\left(256ae^3+5d^4\right)\sqrt{3d^2-2\sqrt{d^4-64a}}}$$

```
[Out] (2*e*(d/(4*e) + x)*(13*d^4 - 256*a*e^3 - 48*d^2*e^2*(d/(4*e) + x)^2))/((5*d
^8 - 64*a*d^4*e^3 - 16384*a^2*e^6)*(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x
^4)) - (24*e*(d^4 + 128*a*e^3 - d^2*Sqrt[d^4 - 64*a*e^3])*ArcTanh[(d + 4*e*
x)/Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]]])/((d^4 - 64*a*e^3)^(3/2)*(5*d^4 +
256*a*e^3)*Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]]) + (24*e*(d^4 + 128*a*e^3 +
 d^2*Sqrt[d^4 - 64*a*e^3])*ArcTanh[(d + 4*e*x)/Sqrt[3*d^2 + 2*Sqrt[d^4 - 64
*a*e^3]]])/((d^4 - 64*a*e^3)^(3/2)*(5*d^4 + 256*a*e^3)*Sqrt[3*d^2 + 2*Sqrt[
d^4 - 64*a*e^3]])
```

Rubi [A] time = 0.532286, antiderivative size = 342, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1106, 1092, 1166, 208}

$$\frac{2e\left(\frac{d}{4e}+x\right)\left(-256ae^3-48d^2e^2\left(\frac{d}{4e}+x\right)^2+13d^4\right)}{\left(-16384a^2e^6-64ad^4e^3+5d^8\right)\left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)}-\frac{24e\left(-d^2\sqrt{d^4-64ae^3}+128ae^3+d^4\right)\tanh^{-1}\left(\frac{d+4e}{\sqrt{3d^2-2\sqrt{d}}}\right.}{\left(d^4-64ae^3\right)^{3/2}\left(256ae^3+5d^4\right)\sqrt{3d^2-2\sqrt{d^4-64a}}}$$

Antiderivative was successfully verified.

```
[In] Int[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^(-2),x]
```

```
[Out] (2*e*(d/(4*e) + x)*(13*d^4 - 256*a*e^3 - 48*d^2*e^2*(d/(4*e) + x)^2))/((5*d
^8 - 64*a*d^4*e^3 - 16384*a^2*e^6)*(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x
^4)) - (24*e*(d^4 + 128*a*e^3 - d^2*Sqrt[d^4 - 64*a*e^3])*ArcTanh[(d + 4*e*
x)/Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]]])/((d^4 - 64*a*e^3)^(3/2)*(5*d^4 +
256*a*e^3)*Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]]) + (24*e*(d^4 + 128*a*e^3 +
 d^2*Sqrt[d^4 - 64*a*e^3])*ArcTanh[(d + 4*e*x)/Sqrt[3*d^2 + 2*Sqrt[d^4 - 64
*a*e^3]]])/((d^4 - 64*a*e^3)^(3/2)*(5*d^4 + 256*a*e^3)*Sqrt[3*d^2 + 2*Sqrt[
d^4 - 64*a*e^3]])
```

Rule 1106

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1092

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x*(b^2 -
 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2
 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4)^2} dx = \text{Subst}\left(\int \frac{1}{\left(\frac{1}{32}\left(\frac{5d^4}{e} + 256ae^2\right) - 3d^2ex^2 + 8e^3x^4\right)^2} dx, x, \frac{d}{4e} + x\right)$$

$$= \frac{2e\left(\frac{d}{4e} + x\right)\left(13d^4 - 256ae^3 - 48d^2e^2\left(\frac{d}{4e} + x\right)^2\right)}{(5d^8 - 64ad^4e^3 - 16384a^2e^6)(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4)} - \frac{4\,\text{Subst}\left(\int \frac{9d^4e^{\cdots}}{\cdots}\right.}{\cdots}$$

$$= \frac{2e\left(\frac{d}{4e} + x\right)\left(13d^4 - 256ae^3 - 48d^2e^2\left(\frac{d}{4e} + x\right)^2\right)}{(5d^8 - 64ad^4e^3 - 16384a^2e^6)(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4)} + \frac{\left(48e^3\left(d^4 + 128\cdots\right.\right.}{\cdots}$$

$$= \frac{2e\left(\frac{d}{4e} + x\right)\left(13d^4 - 256ae^3 - 48d^2e^2\left(\frac{d}{4e} + x\right)^2\right)}{(5d^8 - 64ad^4e^3 - 16384a^2e^6)(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4)} - \frac{24e\left(d^4 + 128ae\cdots\right.}{\left(d^4 - 64ae^3\cdots\right.}$$

Mathematica [C] time = 0.17972, size = 234, normalized size = 0.68

$$\frac{48e^2\text{RootSum}\left[8\#1^3de^2 + 8\#1^4e^3 - \#1d^3 + 8ae^2\&, \frac{2\#1^2d^2e\log(x-\#1) + 32ae^2\log(x-\#1) + \#1d^3\log(x-\#1)}{24\#1^2de^2 + 32\#1^3e^3 - d^3}\&\right]}{16384a^2e^6 + 64ad^4e^3 - 5d^8} + \frac{(d + 4ex)(-12\cdots}{(d^4 - 64ae^3)(256ae^{\cdots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^(-2),x]
```

```
[Out] ((d + 4*e*x)*(5*d^4 - 128*a*e^3 - 12*d^3*e*x - 24*d^2*e^2*x^2))/((d^4 - 64*
a*e^3)*(5*d^4 + 256*a*e^3)*(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)) + (
48*e^2*RootSum[8*a*e^2 - d^3*#1 + 8*d*e^2*#1^3 + 8*e^3*#1^4 & , (32*a*e^2*L
og[x - #1] + d^3*Log[x - #1]*#1 + 2*d^2*e*Log[x - #1]*#1^2)/(-d^3 + 24*d*e^
2*#1^2 + 32*e^3*#1^3) & ])/(-5*d^8 + 64*a*d^4*e^3 + 16384*a^2*e^6)
```

Maple [C] time = 0.015, size = 288, normalized size = 0.8

$$\left(12\frac{d^2e^3x^3}{(256\,ae^3 + 5\,d^4)(64\,ae^3 - d^4)} + 9\frac{d^3e^2x^2}{(256\,ae^3 + 5\,d^4)(64\,ae^3 - d^4)} + \frac{ex}{256\,ae^3 + 5\,d^4} + \frac{d(128\,ae^3 - 5\,d^4)}{131072\,a^2e^6 + 512\,ad^4e^3 - 40\,d\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^2,x)
```

```
[Out] (12*d^2*e^3/(256*a*e^3+5*d^4)/(64*a*e^3-d^4)*x^3+9*d^3*e^2/(256*a*e^3+5*d^4
)/(64*a*e^3-d^4)*x^2+e/(256*a*e^3+5*d^4)*x+1/8*d*(128*a*e^3-5*d^4)/(16384*a
^2*e^6+64*a*d^4*e^3-5*d^8))/(e^3*x^4+d*e^2*x^3-1/8*d^3*x+a*e^2)+384*e^2/(20
48*a*e^3+40*d^4)/(64*a*e^3-d^4)*sum((2*_R^2*d^2*e+_R*d^3+32*a*e^2)/(32*_R^3
*e^3+24*_R^2*d*e^2-d^3)*ln(x-_R),_R=RootOf(8*_Z^4*e^3+8*_Z^3*d*e^2-_Z*d^3+8
*a*e^2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: AttributeError
```

Fricas [B] time = 2.36008, size = 12891, normalized size = 37.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^2,x, algorithm="fricas")
```

```
[Out] -(96*d^2*e^3*x^3 + 72*d^3*e^2*x^2 - 5*d^5 + 128*a*d*e^3 + 12*sqrt(2)*(40*a*
d^8*e^2 - 512*a^2*d^4*e^5 - 131072*a^3*e^8 + 8*(5*d^8*e^3 - 64*a*d^4*e^6 -
16384*a^2*e^9)*x^4 + 8*(5*d^9*e^2 - 64*a*d^5*e^5 - 16384*a^2*d*e^8)*x^3 - (
5*d^11 - 64*a*d^7*e^3 - 16384*a^2*d^3*e^6)*x)*sqrt((d^10*e^2 + 160*a*d^6*e^
5 + 40960*a^2*d^2*e^8 + (125*d^24 - 4800*a*d^20*e^3 - 1167360*a^2*d^16*e^6
+ 31195136*a^3*d^12*e^9 + 3825205248*a^4*d^8*e^12 - 51539607552*a^5*d^4*e^1
5 - 4398046511104*a^6*e^18)*sqrt((d^8*e^4 + 512*a*d^4*e^7 + 65536*a^2*e^10)
/(15625*d^36 + 1800000*a*d^32*e^3 - 115200000*a^2*d^28*e^6 - 21135360000*a^
3*d^24*e^9 - 150994944000*a^4*d^20*e^12 + 78082505441280*a^5*d^16*e^15 + 27
44381022928896*a^6*d^12*e^18 - 70931694131085312*a^7*d^8*e^21 - 51881467707
```

30811392*a^8*d^4*e^24 - 73786976294838206464*a^9*e^27)))/(125*d^24 - 4800*a
*d^20*e^3 - 1167360*a^2*d^16*e^6 + 31195136*a^3*d^12*e^9 + 3825205248*a^4*d
^8*e^12 - 51539607552*a^5*d^4*e^15 - 4398046511104*a^6*e^18))*log(884736*a*
d^5*e^6 + 226492416*a^2*d*e^9 + 3538944*(a*d^4*e^7 + 256*a^2*e^10)*x + 1382
4*sqrt(2)*(d^16*e^2 - 128*a*d^12*e^5 - 61440*a^2*d^8*e^8 + 8388608*a^3*d^4*
e^11 - 268435456*a^4*e^14 - (125*d^30 + 59200*a*d^26*e^3 - 3624960*a^2*d^22
*e^6 - 566493184*a^3*d^18*e^9 + 19797114880*a^4*d^14*e^12 + 1906965479424*a
^5*d^10*e^15 - 30786325577728*a^6*d^6*e^18 - 2251799813685248*a^7*d^2*e^21)
*sqrt((d^8*e^4 + 512*a*d^4*e^7 + 65536*a^2*e^10)/(15625*d^36 + 1800000*a*d^
32*e^3 - 115200000*a^2*d^28*e^6 - 21135360000*a^3*d^24*e^9 - 150994944000*a
^4*d^20*e^12 + 78082505441280*a^5*d^16*e^15 + 2744381022928896*a^6*d^12*e^1
8 - 70931694131085312*a^7*d^8*e^21 - 5188146770730811392*a^8*d^4*e^24 - 737
86976294838206464*a^9*e^27)))*sqrt((d^10*e^2 + 160*a*d^6*e^5 + 40960*a^2*d^
2*e^8 + (125*d^24 - 4800*a*d^20*e^3 - 1167360*a^2*d^16*e^6 + 31195136*a^3*d
^12*e^9 + 3825205248*a^4*d^8*e^12 - 51539607552*a^5*d^4*e^15 - 439804651110
4*a^6*e^18)*sqrt((d^8*e^4 + 512*a*d^4*e^7 + 65536*a^2*e^10)/(15625*d^36 + 1
800000*a*d^32*e^3 - 115200000*a^2*d^28*e^6 - 21135360000*a^3*d^24*e^9 - 150
994944000*a^4*d^20*e^12 + 78082505441280*a^5*d^16*e^15 + 2744381022928896*a
^6*d^12*e^18 - 70931694131085312*a^7*d^8*e^21 - 5188146770730811392*a^8*d^4
*e^24 - 73786976294838206464*a^9*e^27)))/(125*d^24 - 4800*a*d^20*e^3 - 1167
360*a^2*d^16*e^6 + 31195136*a^3*d^12*e^9 + 3825205248*a^4*d^8*e^12 - 515396
07552*a^5*d^4*e^15 - 4398046511104*a^6*e^18))) - 12*sqrt(2)*(40*a*d^8*e^2 -
512*a^2*d^4*e^5 - 131072*a^3*e^8 + 8*(5*d^8*e^3 - 64*a*d^4*e^6 - 16384*a^2
*e^9)*x^4 + 8*(5*d^9*e^2 - 64*a*d^5*e^5 - 16384*a^2*d*e^8)*x^3 - (5*d^11 -
64*a*d^7*e^3 - 16384*a^2*d^3*e^6)*x)*sqrt((d^10*e^2 + 160*a*d^6*e^5 + 40960
*a^2*d^2*e^8 + (125*d^24 - 4800*a*d^20*e^3 - 1167360*a^2*d^16*e^6 + 3119513
6*a^3*d^12*e^9 + 3825205248*a^4*d^8*e^12 - 51539607552*a^5*d^4*e^15 - 43980
46511104*a^6*e^18)*sqrt((d^8*e^4 + 512*a*d^4*e^7 + 65536*a^2*e^10)/(15625*d
^36 + 1800000*a*d^32*e^3 - 115200000*a^2*d^28*e^6 - 21135360000*a^3*d^24*e^
9 - 150994944000*a^4*d^20*e^12 + 78082505441280*a^5*d^16*e^15 + 27443810229
28896*a^6*d^12*e^18 - 70931694131085312*a^7*d^8*e^21 - 5188146770730811392*
a^8*d^4*e^24 - 73786976294838206464*a^9*e^27)))/(125*d^24 - 4800*a*d^20*e^3
- 1167360*a^2*d^16*e^6 + 31195136*a^3*d^12*e^9 + 3825205248*a^4*d^8*e^12 -
51539607552*a^5*d^4*e^15 - 4398046511104*a^6*e^18))*log(884736*a*d^5*e^6 +
226492416*a^2*d*e^9 + 3538944*(a*d^4*e^7 + 256*a^2*e^10)*x - 13824*sqrt(2)
*(d^16*e^2 - 128*a*d^12*e^5 - 61440*a^2*d^8*e^8 + 8388608*a^3*d^4*e^11 - 26
8435456*a^4*e^14 - (125*d^30 + 59200*a*d^26*e^3 - 3624960*a^2*d^22*e^6 - 56
6493184*a^3*d^18*e^9 + 19797114880*a^4*d^14*e^12 + 1906965479424*a^5*d^10*e
^15 - 30786325577728*a^6*d^6*e^18 - 2251799813685248*a^7*d^2*e^21)*sqrt((d^
8*e^4 + 512*a*d^4*e^7 + 65536*a^2*e^10)/(15625*d^36 + 1800000*a*d^32*e^3 -
115200000*a^2*d^28*e^6 - 21135360000*a^3*d^24*e^9 - 150994944000*a^4*d^20*e
^12 + 78082505441280*a^5*d^16*e^15 + 2744381022928896*a^6*d^12*e^18 - 70931
694131085312*a^7*d^8*e^21 - 5188146770730811392*a^8*d^4*e^24 - 737869762948
38206464*a^9*e^27)))*sqrt((d^10*e^2 + 160*a*d^6*e^5 + 40960*a^2*d^2*e^8 + (
125*d^24 - 4800*a*d^20*e^3 - 1167360*a^2*d^16*e^6 + 31195136*a^3*d^12*e^9 +

3825205248*a^4*d^8*e^12 - 51539607552*a^5*d^4*e^15 - 4398046511104*a^6*e^1
8)*sqrt((d^8*e^4 + 512*a*d^4*e^7 + 65536*a^2*e^10)/(15625*d^36 + 1800000*a*
d^32*e^3 - 115200000*a^2*d^28*e^6 - 21135360000*a^3*d^24*e^9 - 150994944000
*a^4*d^20*e^12 + 78082505441280*a^5*d^16*e^15 + 2744381022928896*a^6*d^12*e
^18 - 70931694131085312*a^7*d^8*e^21 - 5188146770730811392*a^8*d^4*e^24 - 7
378697629483820646 4*a^9*e^27)))/(125*d^24 - 4800*a*d^20*e^3 - 1167360*a^2*d
^16*e^6 + 31195136*a^3*d^12*e^9 + 3825205248*a^4*d^8*e^12 - 51539607552*a^5
*d^4*e^15 - 4398046511104*a^6*e^18))) + 12*sqrt(2)*(40*a*d^8*e^2 - 512*a^2*
d^4*e^5 - 131072*a^3*e^8 + 8*(5*d^8*e^3 - 64*a*d^4*e^6 - 16384*a^2*e^9)*x^4
 + 8*(5*d^9*e^2 - 64*a*d^5*e^5 - 16384*a^2*d*e^8)*x^3 - (5*d^11 - 64*a*d^7*
e^3 - 16384*a^2*d^3*e^6)*x)*sqrt((d^10*e^2 + 160*a*d^6*e^5 + 40960*a^2*d^2*
e^8 - (125*d^24 - 4800*a*d^20*e^3 - 1167360*a^2*d^16*e^6 + 31195136*a^3*d^1
2*e^9 + 3825205248*a^4*d^8*e^12 - 51539607552*a^5*d^4*e^15 - 4398046511104*
a^6*e^18)*sqrt((d^8*e^4 + 512*a*d^4*e^7 + 65536*a^2*e^10)/(15625*d^36 + 180
0000*a*d^32*e^3 - 115200000*a^2*d^28*e^6 - 21135360000*a^3*d^24*e^9 - 15099
4944000*a^4*d^20*e^12 + 78082505441280*a^5*d^16*e^15 + 2744381022928896*a^6
*d^12*e^18 - 70931694131085312*a^7*d^8*e^21 - 5188146770730811392*a^8*d^4*e
^24 - 73786976294838206464*a^9*e^27)))/(125*d^24 - 4800*a*d^20*e^3 - 116736
0*a^2*d^16*e^6 + 31195136*a^3*d^12*e^9 + 3825205248*a^4*d^8*e^12 - 51539607
552*a^5*d^4*e^15 - 4398046511104*a^6*e^18))*log(884736*a*d^5*e^6 + 22649241
6*a^2*d*e^9 + 3538944*(a*d^4*e^7 + 256*a^2*e^10)*x + 13824*sqrt(2)*(d^16*e^
2 - 128*a*d^12*e^5 - 61440*a^2*d^8*e^8 + 8388608*a^3*d^4*e^11 - 268435456*a
^4*e^14 + (125*d^30 + 59200*a*d^26*e^3 - 3624960*a^2*d^22*e^6 - 566493184*a
^3*d^18*e^9 + 19797114880*a^4*d^14*e^12 + 1906965479424*a^5*d^10*e^15 - 307
86325577728*a^6*d^6*e^18 - 2251799813685248*a^7*d^2*e^21)*sqrt((d^8*e^4 + 5
12*a*d^4*e^7 + 65536*a^2*e^10)/(15625*d^36 + 1800000*a*d^32*e^3 - 115200000
*a^2*d^28*e^6 - 21135360000*a^3*d^24*e^9 - 150994944000*a^4*d^20*e^12 + 780
82505441280*a^5*d^16*e^15 + 2744381022928896*a^6*d^12*e^18 - 70931694131085
312*a^7*d^8*e^21 - 5188146770730811392*a^8*d^4*e^24 - 73786976294838206464*
a^9*e^27)))*sqrt((d^10*e^2 + 160*a*d^6*e^5 + 40960*a^2*d^2*e^8 - (125*d^24
 - 4800*a*d^20*e^3 - 1167360*a^2*d^16*e^6 + 31195136*a^3*d^12*e^9 + 38252052
48*a^4*d^8*e^12 - 51539607552*a^5*d^4*e^15 - 4398046511104*a^6*e^18)*sqrt((
d^8*e^4 + 512*a*d^4*e^7 + 65536*a^2*e^10)/(15625*d^36 + 1800000*a*d^32*e^3
 - 115200000*a^2*d^28*e^6 - 21135360000*a^3*d^24*e^9 - 150994944000*a^4*d^20
*e^12 + 78082505441280*a^5*d^16*e^15 + 2744381022928896*a^6*d^12*e^18 - 709
31694131085312*a^7*d^8*e^21 - 5188146770730811392*a^8*d^4*e^24 - 7378697629
4838206464*a^9*e^27)))/(125*d^24 - 4800*a*d^20*e^3 - 1167360*a^2*d^16*e^6 +
 31195136*a^3*d^12*e^9 + 3825205248*a^4*d^8*e^12 - 51539607552*a^5*d^4*e^15
 - 4398046511104*a^6*e^18))) - 12*sqrt(2)*(40*a*d^8*e^2 - 512*a^2*d^4*e^5 -
 131072*a^3*e^8 + 8*(5*d^8*e^3 - 64*a*d^4*e^6 - 16384*a^2*e^9)*x^4 + 8*(5*d
^9*e^2 - 64*a*d^5*e^5 - 16384*a^2*d*e^8)*x^3 - (5*d^11 - 64*a*d^7*e^3 - 163
84*a^2*d^3*e^6)*x)*sqrt((d^10*e^2 + 160*a*d^6*e^5 + 40960*a^2*d^2*e^8 - (12
5*d^24 - 4800*a*d^20*e^3 - 1167360*a^2*d^16*e^6 + 31195136*a^3*d^12*e^9 + 3
825205248*a^4*d^8*e^12 - 51539607552*a^5*d^4*e^15 - 4398046511104*a^6*e^18)
*sqrt((d^8*e^4 + 512*a*d^4*e^7 + 65536*a^2*e^10)/(15625*d^36 + 1800000*a*d^

```
32*e^3 - 115200000*a^2*d^28*e^6 - 21135360000*a^3*d^24*e^9 - 150994944000*a
^4*d^20*e^12 + 78082505441280*a^5*d^16*e^15 + 2744381022928896*a^6*d^12*e^1
8 - 70931694131085312*a^7*d^8*e^21 - 518814677073081l392*a^8*d^4*e^24 - 737
86976294838206464*a^9*e^27)))/(125*d^24 - 4800*a*d^20*e^3 - 1167360*a^2*d^1
6*e^6 + 31195136*a^3*d^12*e^9 + 3825205248*a^4*d^8*e^12 - 51539607552*a^5*d
^4*e^15 - 4398046511104*a^6*e^18))*log(884736*a*d^5*e^6 + 226492416*a^2*d*e
^9 + 3538944*(a*d^4*e^7 + 256*a^2*e^10)*x - 13824*sqrt(2)*(d^16*e^2 - 128*a
*d^12*e^5 - 61440*a^2*d^8*e^8 + 8388608*a^3*d^4*e^11 - 268435456*a^4*e^14 +
 (125*d^30 + 59200*a*d^26*e^3 - 3624960*a^2*d^22*e^6 - 566493184*a^3*d^18*e
^9 + 19797114880*a^4*d^14*e^12 + 1906965479424*a^5*d^10*e^15 - 307863255777
28*a^6*d^6*e^18 - 2251799813685248*a^7*d^2*e^21)*sqrt((d^8*e^4 + 512*a*d^4*
e^7 + 65536*a^2*e^10)/(15625*d^36 + 1800000*a*d^32*e^3 - 115200000*a^2*d^28
*e^6 - 21135360000*a^3*d^24*e^9 - 150994944000*a^4*d^20*e^12 + 780825054412
80*a^5*d^16*e^15 + 2744381022928896*a^6*d^12*e^18 - 70931694131085312*a^7*d
^8*e^21 - 5188146770730811392*a^8*d^4*e^24 - 73786976294838206464*a^9*e^27)
))*sqrt((d^10*e^2 + 160*a*d^6*e^5 + 40960*a^2*d^2*e^8 - (125*d^24 - 4800*a*
d^20*e^3 - 1167360*a^2*d^16*e^6 + 31195136*a^3*d^12*e^9 + 3825205248*a^4*d^
8*e^12 - 51539607552*a^5*d^4*e^15 - 4398046511104*a^6*e^18)*sqrt((d^8*e^4 +
 512*a*d^4*e^7 + 65536*a^2*e^10)/(15625*d^36 + 1800000*a*d^32*e^3 - 1152000
00*a^2*d^28*e^6 - 21135360000*a^3*d^24*e^9 - 150994944000*a^4*d^20*e^12 + 7
8082505441280*a^5*d^16*e^15 + 2744381022928896*a^6*d^12*e^18 - 709316941310
85312*a^7*d^8*e^21 - 5188146770730811392*a^8*d^4*e^24 - 7378697629483820646
4*a^9*e^27)))/(125*d^24 - 4800*a*d^20*e^3 - 1167360*a^2*d^16*e^6 + 31195136
*a^3*d^12*e^9 + 3825205248*a^4*d^8*e^12 - 51539607552*a^5*d^4*e^15 - 439804
6511104*a^6*e^18))) - 8*(d^4*e - 64*a*e^4)*x)/(40*a*d^8*e^2 - 512*a^2*d^4*e
^5 - 131072*a^3*e^8 + 8*(5*d^8*e^3 - 64*a*d^4*e^6 - 16384*a^2*e^9)*x^4 + 8*
(5*d^9*e^2 - 64*a*d^5*e^5 - 16384*a^2*d*e^8)*x^3 - (5*d^11 - 64*a*d^7*e^3 -
 16384*a^2*d^3*e^6)*x)
```

Sympy [A] time = 14.0486, size = 580, normalized size = 1.7

$$\frac{128ade^3 - 5d^5 + 72d^3e^2x^2 + 96d^2e^3x^3 + x\left(512ae^4 - 8d^4e\right)}{131072a^3e^8 + 512a^2d^4e^5 - 40ad^8e^2 + x^4\left(131072a^2e^9 + 512ad^4e^6 - 40d^8e^3\right) + x^3\left(131072a^2de^8 + 512ad^5e^5 - 40d^9e^2\right) + \cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*e**3*x**4+8*d*e**2*x**3-d**3*x+8*a*e**2)**2,x)
```

```
[Out] (128*a*d*e**3 - 5*d**5 + 72*d**3*e**2*x**2 + 96*d**2*e**3*x**3 + x*(512*a*e
**4 - 8*d**4*e))/(131072*a**3*e**8 + 512*a**2*d**4*e**5 - 40*a*d**8*e**2 +
x**4*(131072*a**2*e**9 + 512*a*d**4*e**6 - 40*d**8*e**3) + x**3*(131072*a**
2*d*e**8 + 512*a*d**5*e**5 - 40*d**9*e**2) + x*(-16384*a**2*d**3*e**6 - 64*
```

```
a*d**7*e**3 + 5*d**11)) + RootSum(_t**4*(1152921504606846976*a**9*e**27 - 4
0532396646334464*a**8*d**4*e**24 - 791648371998720*a**7*d**8*e**21 + 443240
62494720*a**6*d**12*e**18 - 96636764160*a**5*d**16*e**15 - 15250489344*a**4
*d**20*e**12 + 163577856*a**3*d**24*e**9 + 1290240*a**2*d**28*e**6 - 28800*
a*d**32*e**3 + 125*d**36) + _t**2*(6184752906240*a**5*d**2*e**17 - 26575110
1440*a**4*d**6*e**14 + 3548381184*a**3*d**10*e**11 - 12976128*a**2*d**14*e*
*8 + 18432*a*d**18*e**5 - 576*d**22*e**2) + 84934656*a**2*e**10, Lambda(_t,
 _t*log(x + (-2251799813685248*_t**3*a**7*d**2*e**21 - 30786325577728*_t**3
*a**6*d**6*e**18 + 1906965479424*_t**3*a**5*d**10*e**15 + 19797114880*_t**3
*a**4*d**14*e**12 - 566493184*_t**3*a**3*d**18*e**9 - 3624960*_t**3*a**2*d*
*22*e**6 + 59200*_t**3*a*d**26*e**3 + 125*_t**3*d**30 + 77309411328*_t*a**4
*e**14 - 8455716864*_t*a**3*d**4*e**11 - 17694720*_t*a**2*d**8*e**8 - 15667
2*_t*a*d**12*e**5 - 576*_t*d**16*e**2 + 56623104*a**2*d*e**9 + 221184*a*d**
5*e**6)/(226492416*a**2*e**10 + 884736*a*d**4*e**7))))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.45 $\int \left(8+8x-x^3+8x^4\right)^4 \, dx$

Optimal. Leaf size=96

$$\frac{4096x^{17}}{17}-128x^{16}+\frac{128x^{15}}{5}+1168x^{14}+\frac{10241x^{13}}{13}-448x^{12}+\frac{25312x^{11}}{11}+\frac{21488x^{10}}{5}+1408x^9+1376x^8+6784x^7+7$$

```
[Out] 4096*x + 8192*x^2 + 8192*x^3 + 3584*x^4 + (14336*x^5)/5 + 7168*x^6 + 6784*x
^7 + 1376*x^8 + 1408*x^9 + (21488*x^10)/5 + (25312*x^11)/11 - 448*x^12 + (1
0241*x^13)/13 + 1168*x^14 + (128*x^15)/5 - 128*x^16 + (4096*x^17)/17
```

Rubi [A] time = 0.0301725, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2061}

$$\frac{4096x^{17}}{17}-128x^{16}+\frac{128x^{15}}{5}+1168x^{14}+\frac{10241x^{13}}{13}-448x^{12}+\frac{25312x^{11}}{11}+\frac{21488x^{10}}{5}+1408x^9+1376x^8+6784x^7+7$$

Antiderivative was successfully verified.

```
[In] Int[(8 + 8*x - x^3 + 8*x^4)^4,x]
```

```
[Out] 4096*x + 8192*x^2 + 8192*x^3 + 3584*x^4 + (14336*x^5)/5 + 7168*x^6 + 6784*x
^7 + 1376*x^8 + 1408*x^9 + (21488*x^10)/5 + (25312*x^11)/11 - 448*x^12 + (1
0241*x^13)/13 + 1168*x^14 + (128*x^15)/5 - 128*x^16 + (4096*x^17)/17
```

Rule 2061

```
Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I
GtQ[p, 0]
```

Rubi steps

$$\begin{aligned} \int \left(8+8x-x^3+8x^4\right)^4 \, dx &= \int \left(4096+16384x+24576x^2+14336x^3+14336x^4+43008x^5+47488x^6+11008x^7+1\right. \\ &= 4096x+8192x^2+8192x^3+3584x^4+\frac{14336x^5}{5}+7168x^6+6784x^7+1376x^8+1408x^9+ \end{aligned}$$

Mathematica [A] time = 0.0015055, size = 96, normalized size = 1.

$$\frac{4096x^{17}}{17}-128x^{16}+\frac{128x^{15}}{5}+1168x^{14}+\frac{10241x^{13}}{13}-448x^{12}+\frac{25312x^{11}}{11}+\frac{21488x^{10}}{5}+1408x^9+1376x^8+6784x^7+7$$

Antiderivative was successfully verified.

```
[In] Integrate[(8 + 8*x - x^3 + 8*x^4)^4,x]
```

```
[Out] 4096*x + 8192*x^2 + 8192*x^3 + 3584*x^4 + (14336*x^5)/5 + 7168*x^6 + 6784*x
^7 + 1376*x^8 + 1408*x^9 + (21488*x^10)/5 + (25312*x^11)/11 - 448*x^12 + (1
0241*x^13)/13 + 1168*x^14 + (128*x^15)/5 - 128*x^16 + (4096*x^17)/17
```

Maple [A] time = 0., size = 85, normalized size = 0.9

$$4096\,x+8192\,x^2+8192\,x^3+3584\,x^4+\frac{14336\,x^5}{5}+7168\,x^6+6784\,x^7+1376\,x^8+1408\,x^9+\frac{21488\,x^{10}}{5}+\frac{25312\,x^{11}}{11}-4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*x^4-x^3+8*x+8)^4,x)
```

```
[Out] 4096*x+8192*x^2+8192*x^3+3584*x^4+14336/5*x^5+7168*x^6+6784*x^7+1376*x^8+14
08*x^9+21488/5*x^10+25312/11*x^11-448*x^12+10241/13*x^13+1168*x^14+128/5*x^
15-128*x^16+4096/17*x^17
```

Maxima [A] time = 1.04722, size = 113, normalized size = 1.18

$$\frac{4096}{17}\,x^{17}-128\,x^{16}+\frac{128}{5}\,x^{15}+1168\,x^{14}+\frac{10241}{13}\,x^{13}-448\,x^{12}+\frac{25312}{11}\,x^{11}+\frac{21488}{5}\,x^{10}+1408\,x^9+1376\,x^8+6784\,x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^4-x^3+8*x+8)^4,x, algorithm="maxima")
```

```
[Out] 4096/17*x^17 - 128*x^16 + 128/5*x^15 + 1168*x^14 + 10241/13*x^13 - 448*x^12
 + 25312/11*x^11 + 21488/5*x^10 + 1408*x^9 + 1376*x^8 + 6784*x^7 + 7168*x^6
 + 14336/5*x^5 + 3584*x^4 + 8192*x^3 + 8192*x^2 + 4096*x
```

Fricas [A] time = 1.11129, size = 281, normalized size = 2.93

$$\frac{4096}{17}x^{17}-128x^{16}+\frac{128}{5}x^{15}+1168x^{14}+\frac{10241}{13}x^{13}-448x^{12}+\frac{25312}{11}x^{11}+\frac{21488}{5}x^{10}+1408x^9+1376x^8+6784x^7+71$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^4-x^3+8*x+8)^4,x, algorithm="fricas")
```

```
[Out] 4096/17*x^17 - 128*x^16 + 128/5*x^15 + 1168*x^14 + 10241/13*x^13 - 448*x^12
 + 25312/11*x^11 + 21488/5*x^10 + 1408*x^9 + 1376*x^8 + 6784*x^7 + 7168*x^6
 + 14336/5*x^5 + 3584*x^4 + 8192*x^3 + 8192*x^2 + 4096*x
```

Sympy [A] time = 0.073529, size = 94, normalized size = 0.98

$$\frac{4096x^{17}}{17} - 128x^{16} + \frac{128x^{15}}{5} + 1168x^{14} + \frac{10241x^{13}}{13} - 448x^{12} + \frac{25312x^{11}}{11} + \frac{21488x^{10}}{5} + 1408x^9 + 1376x^8 + 6784x^7 + 7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x**4-x**3+8*x+8)**4,x)
```

```
[Out] 4096*x**17/17 - 128*x**16 + 128*x**15/5 + 1168*x**14 + 10241*x**13/13 - 448
*x**12 + 25312*x**11/11 + 21488*x**10/5 + 1408*x**9 + 1376*x**8 + 6784*x**7
 + 7168*x**6 + 14336*x**5/5 + 3584*x**4 + 8192*x**3 + 8192*x**2 + 4096*x
```

Giac [A] time = 1.16385, size = 113, normalized size = 1.18

$$\frac{4096}{17}x^{17} - 128x^{16} + \frac{128}{5}x^{15} + 1168x^{14} + \frac{10241}{13}x^{13} - 448x^{12} + \frac{25312}{11}x^{11} + \frac{21488}{5}x^{10} + 1408x^9 + 1376x^8 + 6784x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^4-x^3+8*x+8)^4,x, algorithm="giac")
```

```
[Out] 4096/17*x^17 - 128*x^16 + 128/5*x^15 + 1168*x^14 + 10241/13*x^13 - 448*x^12
 + 25312/11*x^11 + 21488/5*x^10 + 1408*x^9 + 1376*x^8 + 6784*x^7 + 7168*x^6
 + 14336/5*x^5 + 3584*x^4 + 8192*x^3 + 8192*x^2 + 4096*x
```

3.46 $\int \left(8+8x-x^3+8x^4\right)^3\,dx$

Optimal. Leaf size=74

$$\frac{512x^{13}}{13}-16x^{12}+\frac{24x^{11}}{11}+\frac{307x^{10}}{2}+128x^9-45x^8+\frac{1560x^7}{7}+480x^6+\frac{1152x^5}{5}+80x^4+512x^3+768x^2+512x$$

```
[Out] 512*x + 768*x^2 + 512*x^3 + 80*x^4 + (1152*x^5)/5 + 480*x^6 + (1560*x^7)/7
- 45*x^8 + 128*x^9 + (307*x^10)/2 + (24*x^11)/11 - 16*x^12 + (512*x^13)/13
```

Rubi [A] time = 0.0225196, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2061}

$$\frac{512x^{13}}{13}-16x^{12}+\frac{24x^{11}}{11}+\frac{307x^{10}}{2}+128x^9-45x^8+\frac{1560x^7}{7}+480x^6+\frac{1152x^5}{5}+80x^4+512x^3+768x^2+512x$$

Antiderivative was successfully verified.

```
[In] Int[(8 + 8*x - x^3 + 8*x^4)^3,x]
```

```
[Out] 512*x + 768*x^2 + 512*x^3 + 80*x^4 + (1152*x^5)/5 + 480*x^6 + (1560*x^7)/7
- 45*x^8 + 128*x^9 + (307*x^10)/2 + (24*x^11)/11 - 16*x^12 + (512*x^13)/13
```

Rule 2061

```
Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I
GtQ[p, 0]
```

Rubi steps

$$\int \left(8+8x-x^3+8x^4\right)^3\,dx=\int \left(512+1536x+1536x^2+320x^3+1152x^4+2880x^5+1560x^6-360x^7+1152x^8+1535\right.$$

$$=512x+768x^2+512x^3+80x^4+\frac{1152x^5}{5}+480x^6+\frac{1560x^7}{7}-45x^8+128x^9+\frac{307x^{10}}{2}+24$$

Mathematica [A] time = 0.0012485, size = 74, normalized size = 1.

$$\frac{512x^{13}}{13}-16x^{12}+\frac{24x^{11}}{11}+\frac{307x^{10}}{2}+128x^9-45x^8+\frac{1560x^7}{7}+480x^6+\frac{1152x^5}{5}+80x^4+512x^3+768x^2+512x$$

Antiderivative was successfully verified.

```
[In] Integrate[(8 + 8*x - x^3 + 8*x^4)^3,x]
```

```
[Out] 512*x + 768*x^2 + 512*x^3 + 80*x^4 + (1152*x^5)/5 + 480*x^6 + (1560*x^7)/7
- 45*x^8 + 128*x^9 + (307*x^10)/2 + (24*x^11)/11 - 16*x^12 + (512*x^13)/13
```

Maple [A] time = 0.002, size = 65, normalized size = 0.9

$$512x+768x^2+512x^3+80x^4+\frac{1152x^5}{5}+480x^6+\frac{1560x^7}{7}-45x^8+128x^9+\frac{307x^{10}}{2}+\frac{24x^{11}}{11}-16x^{12}+\frac{512x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*x^4-x^3+8*x+8)^3,x)
```

```
[Out] 512*x+768*x^2+512*x^3+80*x^4+1152/5*x^5+480*x^6+1560/7*x^7-45*x^8+128*x^9+3
07/2*x^10+24/11*x^11-16*x^12+512/13*x^13
```

Maxima [A] time = 1.16596, size = 86, normalized size = 1.16

$$\frac{512}{13}x^{13}-16x^{12}+\frac{24}{11}x^{11}+\frac{307}{2}x^{10}+128x^9-45x^8+\frac{1560}{7}x^7+480x^6+\frac{1152}{5}x^5+80x^4+512x^3+768x^2+512x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^4-x^3+8*x+8)^3,x, algorithm="maxima")
```

```
[Out] 512/13*x^13 - 16*x^12 + 24/11*x^11 + 307/2*x^10 + 128*x^9 - 45*x^8 + 1560/7
*x^7 + 480*x^6 + 1152/5*x^5 + 80*x^4 + 512*x^3 + 768*x^2 + 512*x
```

Fricas [A] time = 1.12793, size = 190, normalized size = 2.57

$$\frac{512}{13}x^{13}-16x^{12}+\frac{24}{11}x^{11}+\frac{307}{2}x^{10}+128x^9-45x^8+\frac{1560}{7}x^7+480x^6+\frac{1152}{5}x^5+80x^4+512x^3+768x^2+512x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^4-x^3+8*x+8)^3,x, algorithm="fricas")
```

```
[Out] 512/13*x^13 - 16*x^12 + 24/11*x^11 + 307/2*x^10 + 128*x^9 - 45*x^8 + 1560/7
*x^7 + 480*x^6 + 1152/5*x^5 + 80*x^4 + 512*x^3 + 768*x^2 + 512*x
```

Sympy [A] time = 0.064054, size = 71, normalized size = 0.96

$$\frac{512x^{13}}{13} - 16x^{12} + \frac{24x^{11}}{11} + \frac{307x^{10}}{2} + 128x^9 - 45x^8 + \frac{1560x^7}{7} + 480x^6 + \frac{1152x^5}{5} + 80x^4 + 512x^3 + 768x^2 + 512x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x**4-x**3+8*x+8)**3,x)
```

```
[Out] 512*x**13/13 - 16*x**12 + 24*x**11/11 + 307*x**10/2 + 128*x**9 - 45*x**8 +
1560*x**7/7 + 480*x**6 + 1152*x**5/5 + 80*x**4 + 512*x**3 + 768*x**2 + 512*
x
```

Giac [A] time = 1.11133, size = 86, normalized size = 1.16

$$\frac{512}{13}\,x^{13} - 16\,x^{12} + \frac{24}{11}\,x^{11} + \frac{307}{2}\,x^{10} + 128\,x^9 - 45\,x^8 + \frac{1560}{7}\,x^7 + 480\,x^6 + \frac{1152}{5}\,x^5 + 80\,x^4 + 512\,x^3 + 768\,x^2 + 512\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^4-x^3+8*x+8)^3,x, algorithm="giac")
```

```
[Out] 512/13*x^13 - 16*x^12 + 24/11*x^11 + 307/2*x^10 + 128*x^9 - 45*x^8 + 1560/7
*x^7 + 480*x^6 + 1152/5*x^5 + 80*x^4 + 512*x^3 + 768*x^2 + 512*x
```

3.47 $\int \left(8+8x-x^3+8x^4\right)^2 \, dx$

Optimal. Leaf size=54

$$\frac{64x^9}{9}-2x^8+\frac{x^7}{7}+\frac{64x^6}{3}+\frac{112x^5}{5}-4x^4+\frac{64x^3}{3}+64x^2+64x$$

```
[Out] 64*x + 64*x^2 + (64*x^3)/3 - 4*x^4 + (112*x^5)/5 + (64*x^6)/3 + x^7/7 - 2*x
^8 + (64*x^9)/9
```

Rubi [A] time = 0.0165229, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2061}

$$\frac{64x^9}{9}-2x^8+\frac{x^7}{7}+\frac{64x^6}{3}+\frac{112x^5}{5}-4x^4+\frac{64x^3}{3}+64x^2+64x$$

Antiderivative was successfully verified.

```
[In] Int[(8 + 8*x - x^3 + 8*x^4)^2,x]
```

```
[Out] 64*x + 64*x^2 + (64*x^3)/3 - 4*x^4 + (112*x^5)/5 + (64*x^6)/3 + x^7/7 - 2*x
^8 + (64*x^9)/9
```

Rule 2061

```
Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I
GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \left(8+8x-x^3+8x^4\right)^2 \, dx &= \int \left(64+128x+64x^2-16x^3+112x^4+128x^5+x^6-16x^7+64x^8\right) dx\\ &= 64x+64x^2+\frac{64x^3}{3}-4x^4+\frac{112x^5}{5}+\frac{64x^6}{3}+\frac{x^7}{7}-2x^8+\frac{64x^9}{9}\end{aligned}$$

Mathematica [A] time = 0.0012604, size = 54, normalized size = 1.

$$\frac{64x^9}{9}-2x^8+\frac{x^7}{7}+\frac{64x^6}{3}+\frac{112x^5}{5}-4x^4+\frac{64x^3}{3}+64x^2+64x$$

Antiderivative was successfully verified.

```
[In] Integrate[(8 + 8*x - x^3 + 8*x^4)^2,x]
```

```
[Out] 64*x + 64*x^2 + (64*x^3)/3 - 4*x^4 + (112*x^5)/5 + (64*x^6)/3 + x^7/7 - 2*x
^8 + (64*x^9)/9
```

Maple [A] time = 0.001, size = 45, normalized size = 0.8

$$64\,x+64\,x^{2}+\frac{64\,x^{3}}{3}-4\,x^{4}+\frac{112\,x^{5}}{5}+\frac{64\,x^{6}}{3}+\frac{x^{7}}{7}-2\,x^{8}+\frac{64\,x^{9}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*x^4-x^3+8*x+8)^2,x)
```

```
[Out] 64*x+64*x^2+64/3*x^3-4*x^4+112/5*x^5+64/3*x^6+1/7*x^7-2*x^8+64/9*x^9
```

Maxima [A] time = 1.13361, size = 59, normalized size = 1.09

$$\frac{64}{9}\,x^{9}-2\,x^{8}+\frac{1}{7}\,x^{7}+\frac{64}{3}\,x^{6}+\frac{112}{5}\,x^{5}-4\,x^{4}+\frac{64}{3}\,x^{3}+64\,x^{2}+64\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^4-x^3+8*x+8)^2,x, algorithm="maxima")
```

```
[Out] 64/9*x^9 - 2*x^8 + 1/7*x^7 + 64/3*x^6 + 112/5*x^5 - 4*x^4 + 64/3*x^3 + 64*x
^2 + 64*x
```

Fricas [A] time = 1.06795, size = 116, normalized size = 2.15

$$\frac{64}{9}x^{9}-2x^{8}+\frac{1}{7}x^{7}+\frac{64}{3}x^{6}+\frac{112}{5}x^{5}-4x^{4}+\frac{64}{3}x^{3}+64x^{2}+64x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^4-x^3+8*x+8)^2,x, algorithm="fricas")
```

```
[Out] 64/9*x^9 - 2*x^8 + 1/7*x^7 + 64/3*x^6 + 112/5*x^5 - 4*x^4 + 64/3*x^3 + 64*x
^2 + 64*x
```

Sympy [A] time = 0.058537, size = 49, normalized size = 0.91

$$\frac{64x^9}{9} - 2x^8 + \frac{x^7}{7} + \frac{64x^6}{3} + \frac{112x^5}{5} - 4x^4 + \frac{64x^3}{3} + 64x^2 + 64x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x**4-x**3+8*x+8)**2,x)
```

```
[Out] 64*x**9/9 - 2*x**8 + x**7/7 + 64*x**6/3 + 112*x**5/5 - 4*x**4 + 64*x**3/3 +
 64*x**2 + 64*x
```

Giac [A] time = 1.10815, size = 59, normalized size = 1.09

$$\frac{64}{9}\,x^9 - 2\,x^8 + \frac{1}{7}\,x^7 + \frac{64}{3}\,x^6 + \frac{112}{5}\,x^5 - 4\,x^4 + \frac{64}{3}\,x^3 + 64\,x^2 + 64\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^4-x^3+8*x+8)^2,x, algorithm="giac")
```

```
[Out] 64/9*x^9 - 2*x^8 + 1/7*x^7 + 64/3*x^6 + 112/5*x^5 - 4*x^4 + 64/3*x^3 + 64*x
^2 + 64*x
```

3.48 $\int (8+8x-x^3+8x^4)\,dx$

Optimal. Leaf size=23

$$\frac{8x^5}{5}-\frac{x^4}{4}+4x^2+8x$$

```
[Out] 8*x + 4*x^2 - x^4/4 + (8*x^5)/5
```

Rubi [A] time = 0.0030522, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\frac{8x^5}{5}-\frac{x^4}{4}+4x^2+8x$$

Antiderivative was successfully verified.

```
[In] Int[8 + 8*x - x^3 + 8*x^4,x]
```

```
[Out] 8*x + 4*x^2 - x^4/4 + (8*x^5)/5
```

Rubi steps

$$\int (8+8x-x^3+8x^4)\,dx = 8x+4x^2-\frac{x^4}{4}+\frac{8x^5}{5}$$

Mathematica [A] time = 0.0000439, size = 23, normalized size = 1.

$$\frac{8x^5}{5}-\frac{x^4}{4}+4x^2+8x$$

Antiderivative was successfully verified.

```
[In] Integrate[8 + 8*x - x^3 + 8*x^4,x]
```

```
[Out] 8*x + 4*x^2 - x^4/4 + (8*x^5)/5
```

Maple [A] time = 0.001, size = 20, normalized size = 0.9

$$8\,x+4\,x^{2}-\frac{x^{4}}{4}+\frac{8\,x^{5}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(8*x^4-x^3+8*x+8,x)
```

```
[Out] 8*x+4*x^2-1/4*x^4+8/5*x^5
```

Maxima [A] time = 1.16148, size = 26, normalized size = 1.13

$$\frac{8}{5}\,x^{5}-\frac{1}{4}\,x^{4}+4\,x^{2}+8\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(8*x^4-x^3+8*x+8,x, algorithm="maxima")
```

```
[Out] 8/5*x^5 - 1/4*x^4 + 4*x^2 + 8*x
```

Fricas [A] time = 1.08171, size = 45, normalized size = 1.96

$$\frac{8}{5}x^{5}-\frac{1}{4}x^{4}+4x^{2}+8x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(8*x^4-x^3+8*x+8,x, algorithm="fricas")
```

```
[Out] 8/5*x^5 - 1/4*x^4 + 4*x^2 + 8*x
```

Sympy [A] time = 0.053158, size = 19, normalized size = 0.83

$$\frac{8x^{5}}{5}-\frac{x^{4}}{4}+4x^{2}+8x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(8*x**4-x**3+8*x+8,x)
```

```
[Out] 8*x**5/5 - x**4/4 + 4*x**2 + 8*x
```

Giac [A] time = 1.12515, size = 26, normalized size = 1.13

$$\frac{8}{5}x^5 - \frac{1}{4}x^4 + 4x^2 + 8x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(8*x^4-x^3+8*x+8,x, algorithm="giac")
```

```
[Out] 8/5*x^5 - 1/4*x^4 + 4*x^2 + 8*x
```

3.49 $\int \frac{1}{8+8x-x^3+8x^4}\,dx$

Optimal. Leaf size=268

$$-\frac{1}{24}\sqrt{\frac{67\sqrt{29}-109}{1218}}\log\left(\left(\frac{4}{x}+1\right)^2-\sqrt{6\left(1+\sqrt{29}\right)}\left(\frac{4}{x}+1\right)+3\sqrt{29}\right)+\frac{1}{24}\sqrt{\frac{67\sqrt{29}-109}{1218}}\log\left(\left(\frac{4}{x}+1\right)^2+\sqrt{6\left(1+\sqrt{2\right.}\right.}$$

```
[Out] -ArcTan[(3 - (1 + 4/x)^2)/(6*Sqrt[7])]/(12*Sqrt[7]) - (Sqrt[(109 + 67*Sqrt[
29])/1218]*ArcTan[(2 - Sqrt[6*(1 + Sqrt[29])] + 8/x)/Sqrt[6*(-1 + Sqrt[29])
]])/12 - (Sqrt[(109 + 67*Sqrt[29])/1218]*ArcTan[(2 + Sqrt[6*(1 + Sqrt[29])]
 + 8/x)/Sqrt[6*(-1 + Sqrt[29])]]])/12 - (Sqrt[(-109 + 67*Sqrt[29])/1218]*Log
[3*Sqrt[29] - Sqrt[6*(1 + Sqrt[29])]*(1 + 4/x) + (1 + 4/x)^2])/24 + (Sqrt[(
-109 + 67*Sqrt[29])/1218]*Log[3*Sqrt[29] + Sqrt[6*(1 + Sqrt[29])]*(1 + 4/x)
 + (1 + 4/x)^2])/24
```

Rubi [A] time = 0.39625, antiderivative size = 268, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {2069, 12, 1673, 1169, 634, 618, 204, 628, 1107}

$$-\frac{1}{24}\sqrt{\frac{67\sqrt{29}-109}{1218}}\log\left(\left(\frac{4}{x}+1\right)^2-\sqrt{6\left(1+\sqrt{29}\right)}\left(\frac{4}{x}+1\right)+3\sqrt{29}\right)+\frac{1}{24}\sqrt{\frac{67\sqrt{29}-109}{1218}}\log\left(\left(\frac{4}{x}+1\right)^2+\sqrt{6\left(1+\sqrt{2\right.}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(8 + 8*x - x^3 + 8*x^4)^(-1),x]
```

```
[Out] -ArcTan[(3 - (1 + 4/x)^2)/(6*Sqrt[7])]/(12*Sqrt[7]) - (Sqrt[(109 + 67*Sqrt[
29])/1218]*ArcTan[(2 - Sqrt[6*(1 + Sqrt[29])] + 8/x)/Sqrt[6*(-1 + Sqrt[29])
]])/12 - (Sqrt[(109 + 67*Sqrt[29])/1218]*ArcTan[(2 + Sqrt[6*(1 + Sqrt[29])]
 + 8/x)/Sqrt[6*(-1 + Sqrt[29])]]])/12 - (Sqrt[(-109 + 67*Sqrt[29])/1218]*Log
[3*Sqrt[29] - Sqrt[6*(1 + Sqrt[29])]*(1 + 4/x) + (1 + 4/x)^2])/24 + (Sqrt[(
-109 + 67*Sqrt[29])/1218]*Log[3*Sqrt[29] + Sqrt[6*(1 + Sqrt[29])]*(1 + 4/x)
 + (1 + 4/x)^2])/24
```

Rule 2069

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Dist[-16*
a^2, Subst[Int[(1*((a*(-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 256*a^3*e - 32*a^
2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4))/(b - 4*a*x)^4)^p)/(b - 4*a*x)^2, x],
x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a^
2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] &&  !IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\int \frac{1}{8+8x-x^3+8x^4}\,dx = -\left(1024\,\text{Subst}\left(\int \frac{(8-32x)^2}{8\left(1069056-393216x^2+1048576x^4\right)}\,dx, x, \frac{1}{4}+\frac{1}{x}\right)\right)$$

$$= -\left(128\,\text{Subst}\left(\int \frac{(8-32x)^2}{1069056-393216x^2+1048576x^4}\,dx, x, \frac{1}{4}+\frac{1}{x}\right)\right)$$

$$= -\left(128\,\text{Subst}\left(\int -\frac{512x}{1069056-393216x^2+1048576x^4}\,dx, x, \frac{1}{4}+\frac{1}{x}\right)\right) - 128\,\text{Subst}\left(\int \frac{}{1069\ldots}\right.$$

$$= 65536\,\text{Subst}\left(\int \frac{x}{1069056-393216x^2+1048576x^4}\,dx, x, \frac{1}{4}+\frac{1}{x}\right) - \frac{\text{Subst}\left(\int \frac{16\sqrt{6\left(1+\sqrt{29}\right)}-(\ldots}{\frac{3\sqrt{29}}{16}-\frac{1}{2}\sqrt{\frac{3}{2}\left(1\ldots\right.}}\right.}{768\sqrt{174\ldots}}$$

$$= 32768\,\text{Subst}\left(\int \frac{1}{1069056-393216x+1048576x^2}\,dx, x, \left(\frac{1}{4}+\frac{1}{x}\right)^2\right) - \frac{\left(87+\sqrt{29}\right)\text{Subst}\left(\int\ldots\right.}{\ldots}$$

$$= -\frac{1}{24}\sqrt{\frac{-109+67\sqrt{29}}{1218}}\log\left(3\sqrt{29}-\sqrt{6\left(1+\sqrt{29}\right)}\left(1+\frac{4}{x}\right)+\left(1+\frac{4}{x}\right)^2\right) + \frac{1}{24}\sqrt{\frac{-109+67\sqrt{2\ldots}}{1218}}$$

$$= -\frac{\tan^{-1}\left(\frac{3-\left(1+\frac{4}{x}\right)^2}{6\sqrt{7}}\right)}{12\sqrt{7}} - \frac{1}{12}\sqrt{\frac{109+67\sqrt{29}}{1218}}\tan^{-1}\left(\frac{2+\sqrt{6\left(1+\sqrt{29}\right)}+\frac{8}{x}}{\sqrt{6\left(-1+\sqrt{29}\right)}}\right) - \frac{1}{12}\sqrt{\frac{109+67\sqrt{2\ldots}}{1218}}$$

Mathematica [C] time = 0.0084298, size = 45, normalized size = 0.17

$$\text{RootSum}\left[8\#1^4-\#1^3+8\#1+8\&,\frac{\log(x-\#1)}{32\#1^3-3\#1^2+8}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(8 + 8*x - x^3 + 8*x^4)^(-1),x]
```

```
[Out] RootSum[8 + 8*#1 - #1^3 + 8*#1^4 & , Log[x - #1]/(8 - 3*#1^2 + 32*#1^3) & ]
```

Maple [C] time = 0.003, size = 41, normalized size = 0.2

$$\sum_{\_R=\text{RootOf}\left(8\_Z^4-\_Z^3+8\_Z+8\right)}\frac{\ln(x-\_R)}{32\_R^3-3\_R^2+8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(8*x^4-x^3+8*x+8),x)
```

```
[Out] sum(1/(32*_R^3-3*_R^2+8)*ln(x-_R),_R=RootOf(8*_Z^4-_Z^3+8*_Z+8))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{8\,x^4-x^3+8\,x+8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4-x^3+8*x+8),x, algorithm="maxima")
```

```
[Out] integrate(1/(8*x^4 - x^3 + 8*x + 8), x)
```

Fricas [C] time = 10.6498, size = 4797, normalized size = 17.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4-x^3+8*x+8),x, algorithm="fricas")
```

```
[Out] -1/168*(-I*sqrt(7) + 84*sqrt(65/43848*I*sqrt(7) - 109/87696))*log(287314195
392*(1/168*I*sqrt(7) - 1/2*sqrt(65/43848*I*sqrt(7) - 109/87696))^3 - 120389
06880*(1/168*I*sqrt(7) - 1/2*sqrt(65/43848*I*sqrt(7) - 109/87696))^2 + 1687
8104*x + 4897683*I*sqrt(7) - 411405372*sqrt(65/43848*I*sqrt(7) - 109/87696)
 + 6055613) - 1/168*(I*sqrt(7) + 84*sqrt(-65/43848*I*sqrt(7) - 109/87696))*
log(-35914274424*(1/168*I*sqrt(7) - 1/2*sqrt(65/43848*I*sqrt(7) - 109/87696
))^3 + 16443*(-1/168*I*sqrt(7) - 1/2*sqrt(-65/43848*I*sqrt(7) - 109/87696))
^2*(-13001*I*sqrt(7) + 1092084*sqrt(65/43848*I*sqrt(7) - 109/87696) - 91520
) + 609*(351027*(1/168*I*sqrt(7) - 1/2*sqrt(65/43848*I*sqrt(7) - 109/87696)
)^2 - 613)*(I*sqrt(7) + 84*sqrt(-65/43848*I*sqrt(7) - 109/87696)) + 2109763
*x - 1911147/8*I*sqrt(7) + 40134087/2*sqrt(65/43848*I*sqrt(7) - 109/87696)
- 1461344) + 1/1044*(sqrt(174)*sqrt(-4698*(1/168*I*sqrt(7) - 1/2*sqrt(65/43
848*I*sqrt(7) - 109/87696))^2 - 4698*(-1/168*I*sqrt(7) - 1/2*sqrt(-65/43848
*I*sqrt(7) - 109/87696))^2 - 87/784*(I*sqrt(7) + 84*sqrt(-65/43848*I*sqrt(7
) - 109/87696))*(-I*sqrt(7) + 84*sqrt(65/43848*I*sqrt(7) - 109/87696)) - 7)
 + 261*sqrt(65/43848*I*sqrt(7) - 109/87696) + 261*sqrt(-65/43848*I*sqrt(7)
- 109/87696))*log(-16443/2*(-1/168*I*sqrt(7) - 1/2*sqrt(-65/43848*I*sqrt(7)
 - 109/87696))^2*(-13001*I*sqrt(7) + 1092084*sqrt(65/43848*I*sqrt(7) - 109/
87696) - 91520) - 609/2*(351027*(1/168*I*sqrt(7) - 1/2*sqrt(65/43848*I*sqrt
(7) - 109/87696))^2 - 613)*(I*sqrt(7) + 84*sqrt(-65/43848*I*sqrt(7) - 109/8
7696)) + 752431680*(1/168*I*sqrt(7) - 1/2*sqrt(65/43848*I*sqrt(7) - 109/876
96))^2 + 1/32*(3*(13001*sqrt(174)*(-I*sqrt(7) + 84*sqrt(65/43848*I*sqrt(7)
- 109/87696)) - 91520*sqrt(174))*(I*sqrt(7) + 84*sqrt(-65/43848*I*sqrt(7) -
 109/87696)) - 274560*sqrt(174)*(-I*sqrt(7) + 84*sqrt(65/43848*I*sqrt(7) -
109/87696)) + 1922368*sqrt(174))*sqrt(-4698*(1/168*I*sqrt(7) - 1/2*sqrt(65/
43848*I*sqrt(7) - 109/87696))^2 - 4698*(-1/168*I*sqrt(7) - 1/2*sqrt(-65/438
48*I*sqrt(7) - 109/87696))^2 - 87/784*(I*sqrt(7) + 84*sqrt(-65/43848*I*sqrt
(7) - 109/87696))*(-I*sqrt(7) + 84*sqrt(65/43848*I*sqrt(7) - 109/87696)) -
7) + 2109763*x - 373317/2*I*sqrt(7) + 15679314*sqrt(65/43848*I*sqrt(7) - 10
9/87696) + 220336) - 1/1044*(sqrt(174)*sqrt(-4698*(1/168*I*sqrt(7) - 1/2*sq
rt(65/43848*I*sqrt(7) - 109/87696))^2 - 4698*(-1/168*I*sqrt(7) - 1/2*sqrt(-
65/43848*I*sqrt(7) - 109/87696))^2 - 87/784*(I*sqrt(7) + 84*sqrt(-65/43848*
I*sqrt(7) - 109/87696))*(-I*sqrt(7) + 84*sqrt(65/43848*I*sqrt(7) - 109/8769
6)) - 7) - 261*sqrt(65/43848*I*sqrt(7) - 109/87696) - 261*sqrt(-65/43848*I*
sqrt(7) - 109/87696))*log(-16443/2*(-1/168*I*sqrt(7) - 1/2*sqrt(-65/43848*I
*sqrt(7) - 109/87696))^2*(-13001*I*sqrt(7) + 1092084*sqrt(65/43848*I*sqrt(7
) - 109/87696) - 91520) - 609/2*(351027*(1/168*I*sqrt(7) - 1/2*sqrt(65/4384
8*I*sqrt(7) - 109/87696))^2 - 613)*(I*sqrt(7) + 84*sqrt(-65/43848*I*sqrt(7)
 - 109/87696)) + 752431680*(1/168*I*sqrt(7) - 1/2*sqrt(65/43848*I*sqrt(7) -
 109/87696))^2 - 1/32*(3*(13001*sqrt(174)*(-I*sqrt(7) + 84*sqrt(65/43848*I*
```

```
sqrt(7) - 109/87696)) - 91520*sqrt(174))*(I*sqrt(7) + 84*sqrt(-65/43848*I*s
qrt(7) - 109/87696)) - 274560*sqrt(174)*(-I*sqrt(7) + 84*sqrt(65/43848*I*sq
rt(7) - 109/87696)) + 1922368*sqrt(174))*sqrt(-4698*(1/168*I*sqrt(7) - 1/2*
sqrt(65/43848*I*sqrt(7) - 109/87696))^2 - 4698*(-1/168*I*sqrt(7) - 1/2*sqrt
(-65/43848*I*sqrt(7) - 109/87696))^2 - 87/784*(I*sqrt(7) + 84*sqrt(-65/4384
8*I*sqrt(7) - 109/87696))*(-I*sqrt(7) + 84*sqrt(65/43848*I*sqrt(7) - 109/87
696)) - 7) + 2109763*x - 373317/2*I*sqrt(7) + 15679314*sqrt(65/43848*I*sqrt
(7) - 109/87696) + 220336)
```

Sympy [A] time = 0.799543, size = 41, normalized size = 0.15

$$\mathrm{RootSum}\left(66298176t^4 + 74088t^2 + 4095t + 64, \left(t \mapsto t \log\left(\frac{35914274424t^3}{2109763} - \frac{1504863360t^2}{2109763} + \frac{102851343t}{2109763} + x + \frac{605}{168}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x**4-x**3+8*x+8),x)
```

```
[Out] RootSum(66298176*_t**4 + 74088*_t**2 + 4095*_t + 64, Lambda(_t, _t*log(3591
4274424*_t**3/2109763 - 1504863360*_t**2/2109763 + 102851343*_t/2109763 + x
 + 6055613/16878104)))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{8x^4 - x^3 + 8x + 8}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4-x^3+8*x+8),x, algorithm="giac")
```

```
[Out] integrate(1/(8*x^4 - x^3 + 8*x + 8), x)
```

3.50 $\int \frac{1}{\left(8+8x-x^3+8x^4\right)^2} \, dx$

Optimal. Leaf size=357

$$-\frac{29\left(\frac{4}{x}+1\right)^2+207}{336\left(\left(\frac{4}{x}+1\right)^4-6\left(\frac{4}{x}+1\right)^2+261\right)}+\frac{5\left(199\left(\frac{4}{x}+1\right)^2+5157\right)\left(\frac{4}{x}+1\right)}{87696\left(\left(\frac{4}{x}+1\right)^4-6\left(\frac{4}{x}+1\right)^2+261\right)}-\frac{\sqrt{\frac{45923327\sqrt{29}-180983329}{1218}}\log\left(\left(\frac{4}{x}+1\right)^2-\right.}{175392}$$

```
[Out] -(207 + 29*(1 + 4/x)^2)/(336*(261 - 6*(1 + 4/x)^2 + (1 + 4/x)^4)) + (5*(515
7 + 199*(1 + 4/x)^2)*(1 + 4/x))/(87696*(261 - 6*(1 + 4/x)^2 + (1 + 4/x)^4))
 - (17*ArcTan[(3 - (1 + 4/x)^2)/(6*Sqrt[7])])/(1008*Sqrt[7]) - (Sqrt[(18098
3329 + 45923327*Sqrt[29])/1218]*ArcTan[(2 - Sqrt[6*(1 + Sqrt[29])] + 8/x)/S
qrt[6*(-1 + Sqrt[29])]])/87696 - (Sqrt[(180983329 + 45923327*Sqrt[29])/1218
]*ArcTan[(2 + Sqrt[6*(1 + Sqrt[29])] + 8/x)/Sqrt[6*(-1 + Sqrt[29])]])/87696
 - (Sqrt[(-180983329 + 45923327*Sqrt[29])/1218]*Log[3*Sqrt[29] - Sqrt[6*(1
+ Sqrt[29])]*(1 + 4/x) + (1 + 4/x)^2])/175392 + (Sqrt[(-180983329 + 4592332
7*Sqrt[29])/1218]*Log[3*Sqrt[29] + Sqrt[6*(1 + Sqrt[29])]*(1 + 4/x) + (1 +
4/x)^2])/175392
```

Rubi [A] time = 0.396672, antiderivative size = 357, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 11, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.647, Rules used = {2069, 12, 1673, 1678, 1169, 634, 618, 204, 628, 1663, 1660}

$$-\frac{29\left(\frac{4}{x}+1\right)^2+207}{336\left(\left(\frac{4}{x}+1\right)^4-6\left(\frac{4}{x}+1\right)^2+261\right)}+\frac{5\left(199\left(\frac{4}{x}+1\right)^2+5157\right)\left(\frac{4}{x}+1\right)}{87696\left(\left(\frac{4}{x}+1\right)^4-6\left(\frac{4}{x}+1\right)^2+261\right)}-\frac{\sqrt{\frac{45923327\sqrt{29}-180983329}{1218}}\log\left(\left(\frac{4}{x}+1\right)^2-\right.}{175392}$$

Antiderivative was successfully verified.

```
[In] Int[(8 + 8*x - x^3 + 8*x^4)^(-2),x]
```

```
[Out] -(207 + 29*(1 + 4/x)^2)/(336*(261 - 6*(1 + 4/x)^2 + (1 + 4/x)^4)) + (5*(515
7 + 199*(1 + 4/x)^2)*(1 + 4/x))/(87696*(261 - 6*(1 + 4/x)^2 + (1 + 4/x)^4))
 - (17*ArcTan[(3 - (1 + 4/x)^2)/(6*Sqrt[7])])/(1008*Sqrt[7]) - (Sqrt[(18098
3329 + 45923327*Sqrt[29])/1218]*ArcTan[(2 - Sqrt[6*(1 + Sqrt[29])] + 8/x)/S
qrt[6*(-1 + Sqrt[29])]])/87696 - (Sqrt[(180983329 + 45923327*Sqrt[29])/1218
]*ArcTan[(2 + Sqrt[6*(1 + Sqrt[29])] + 8/x)/Sqrt[6*(-1 + Sqrt[29])]])/87696
```

```
 - (Sqrt[(-180983329 + 45923327*Sqrt[29])/1218]*Log[3*Sqrt[29] - Sqrt[6*(1
+ Sqrt[29])]*(1 + 4/x) + (1 + 4/x)^2])/175392 + (Sqrt[(-180983329 + 4592332
7*Sqrt[29])/1218]*Log[3*Sqrt[29] + Sqrt[6*(1 + Sqrt[29])]*(1 + 4/x) + (1 +
4/x)^2])/175392
```

Rule 2069

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Dist[-16*
a^2, Subst[Int[(1*((a*(-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 256*a^3*e - 32*a^
2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4))/(b - 4*a*x)^4)^p)/(b - 4*a*x)^2, x],
x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a^
2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] &&  !IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(8+8x-x^3+8x^4\right)^2}\,dx = -\left(1024\,\text{Subst}\left(\int \frac{(8-32x)^6}{64\left(1069056-393216x^2+1048576x^4\right)^2}\,dx,x,\frac{1}{4}+\frac{1}{x}\right)\right)$$

$$= -\left(16\,\text{Subst}\left(\int \frac{(8-32x)^6}{\left(1069056-393216x^2+1048576x^4\right)^2}\,dx,x,\frac{1}{4}+\frac{1}{x}\right)\right)$$

$$= -\left(16\,\text{Subst}\left(\int \frac{x\left(-6291456-335544320x^2-1610612736x^4\right)}{\left(1069056-393216x^2+1048576x^4\right)^2}\,dx,x,\frac{1}{4}+\frac{1}{x}\right)\right)-16\,\text{Subst}$$

$$= \frac{5\left(5157+199\left(1+\frac{4}{x}\right)^2\right)\left(1+\frac{4}{x}\right)}{87696\left(261-6\left(1+\frac{4}{x}\right)^2+\left(1+\frac{4}{x}\right)^4\right)} - \frac{\text{Subst}\left(\int \frac{278927740761415247462 4+775800880449947330150}{1069056-393216x^2+1048576x^4}\right.}{578536630256664576}$$

$$= -\frac{207+29\left(1+\frac{4}{x}\right)^2}{336\left(261-6\left(1+\frac{4}{x}\right)^2+\left(1+\frac{4}{x}\right)^4\right)} + \frac{5\left(5157+199\left(1+\frac{4}{x}\right)^2\right)\left(1+\frac{4}{x}\right)}{87696\left(261-6\left(1+\frac{4}{x}\right)^2+\left(1+\frac{4}{x}\right)^4\right)} - \frac{\text{Subst}\left(\int -\right.}{}$$

$$= -\frac{207+29\left(1+\frac{4}{x}\right)^2}{336\left(261-6\left(1+\frac{4}{x}\right)^2+\left(1+\frac{4}{x}\right)^4\right)} + \frac{5\left(5157+199\left(1+\frac{4}{x}\right)^2\right)\left(1+\frac{4}{x}\right)}{87696\left(261-6\left(1+\frac{4}{x}\right)^2+\left(1+\frac{4}{x}\right)^4\right)} + \frac{139264}{21}\,\text{Sub}$$

$$= -\frac{207+29\left(1+\frac{4}{x}\right)^2}{336\left(261-6\left(1+\frac{4}{x}\right)^2+\left(1+\frac{4}{x}\right)^4\right)} + \frac{5\left(5157+199\left(1+\frac{4}{x}\right)^2\right)\left(1+\frac{4}{x}\right)}{87696\left(261-6\left(1+\frac{4}{x}\right)^2+\left(1+\frac{4}{x}\right)^4\right)} - \frac{\sqrt{\frac{-180983329}{1}}}{}$$

$$= -\frac{207+29\left(1+\frac{4}{x}\right)^2}{336\left(261-6\left(1+\frac{4}{x}\right)^2+\left(1+\frac{4}{x}\right)^4\right)} + \frac{5\left(5157+199\left(1+\frac{4}{x}\right)^2\right)\left(1+\frac{4}{x}\right)}{87696\left(261-6\left(1+\frac{4}{x}\right)^2+\left(1+\frac{4}{x}\right)^4\right)} - \frac{17\tan^{-1}\left(\frac{3-}{}\right.}{1008\sqrt{}}$$

Mathematica [C] time = 0.0157122, size = 113, normalized size = 0.32

$$\frac{\text{RootSum}\left[8\#1^4-\#1^3+8\#1+8\&,\frac{392\#1^2\log(x-\#1)-1097\#1\log(x-\#1)+2243\log(x-\#1)}{32\#1^3-3\#1^2+8}\&\right]}{21924}+\frac{784x^3-1146x^2+1539x+544}{43848\left(8x^4-x^3+8x+8\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(8 + 8*x - x^3 + 8*x^4)^(-2),x]
```

```
[Out] (544 + 1539*x - 1146*x^2 + 784*x^3)/(43848*(8 + 8*x - x^3 + 8*x^4)) + RootS
um[8 + 8*#1 - #1^3 + 8*#1^4 & , (2243*Log[x - #1] - 1097*Log[x - #1]*#1 + 3
92*Log[x - #1]*#1^2)/(8 - 3*#1^2 + 32*#1^3) & ]/21924
```

Maple [C] time = 0.007, size = 83, normalized size = 0.2

$$\left(\frac{7x^3}{3132}-\frac{191x^2}{58464}+\frac{57x}{12992}+\frac{17}{10962}\right)\left(x^4-\frac{x^3}{8}+x+1\right)^{-1}+\frac{1}{21924}\sum_{\_R=\mathrm{RootOf}\left(8\_Z^4-\_Z^3+8\_Z+8\right)}\frac{\left(392\_R^2-1097\_R+224\right.}{32\_R^3-3\_R^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(8*x^4-x^3+8*x+8)^2,x)
```

```
[Out] (7/3132*x^3-191/58464*x^2+57/12992*x+17/10962)/(x^4-1/8*x^3+x+1)+1/21924*su
m((392*_R^2-1097*_R+2243)/(32*_R^3-3*_R^2+8)*ln(x-_R),_R=RootOf(8*_Z^4-_Z^3
+8*_Z+8))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{784x^3-1146x^2+1539x+544}{43848\left(8x^4-x^3+8x+8\right)}+\frac{1}{21924}\int\frac{392x^2-1097x+2243}{8x^4-x^3+8x+8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4-x^3+8*x+8)^2,x, algorithm="maxima")
```

```
[Out] 1/43848*(784*x^3 - 1146*x^2 + 1539*x + 544)/(8*x^4 - x^3 + 8*x + 8) + 1/219
24*integrate((392*x^2 - 1097*x + 2243)/(8*x^4 - x^3 + 8*x + 8), x)
```

Fricas [C] time = 12.6073, size = 8227, normalized size = 23.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4-x^3+8*x+8)^2,x, algorithm="fricas")
```

```
[Out] 1/213627456*(3819648*x^3 - 15138*(8*x^4 - x^3 + 8*x + 8)*(-17*I*sqrt(7) + 7
056*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/4683568345344))*log(621
785056787306565435997385932 8*(17/14112*I*sqrt(7) - 1/2*sqrt(4550065/3345405
96096*I*sqrt(7) - 180983329/4683568345344))^3 - 100287672431797174786327756
80*(17/14112*I*sqrt(7) - 1/2*sqrt(4550065/334540596096*I*sqrt(7) - 18098332
9/4683568345344))^2 + 674816656554692870314 16*x + 320944207138750561964778*
I*sqrt(7) - 133210725033589645013145504*sqrt(4550065/334540596096*I*sqrt(7)
 - 180983329/4683568345344) + 333979081113202533090737) - 15138*(8*x^4 - x^
3 + 8*x + 8)*(17*I*sqrt(7) + 7056*sqrt(-4550065/334540596096*I*sqrt(7) - 18
0983329/4683568345344))*log(-77723132098413320679499673241 6*(17/14112*I*sqr
t(7) - 1/2*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/4683568345344))^
3 + 878169064752*(-17/14112*I*sqrt(7) - 1/2*sqrt(-4550065/334540596096*I*sq
rt(7) - 180983329/4683568345344))^2*(-1066184864424603*I*sqrt(7) + 44252943
5492941104*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/4683568345344) -
 1427510892508480) + 7569*(7276511507810430573072*(17/14112*I*sqrt(7) - 1/2
*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/4683568345344))^2 - 233594
23554371543)*(17*I*sqrt(7) + 7056*sqrt(-4550065/334540596096*I*sqrt(7) - 18
0983329/4683568345344)) + 843520820693366087892 7*x - 1484491951413286827726
33/4*I*sqrt(7) + 154037870723119880241720 36*sqrt(4550065/334540596096*I*sqr
t(7) - 180983329/4683568345344) - 473936066066965950676 16) - 5583312*x^2 +
(56*sqrt(87)*(8*x^4 - x^3 + 8*x + 8)*sqrt(-125452723536*(17/14112*I*sqrt(7)
 - 1/2*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/4683568345344))^2 -
125452723536*(-17/14112*I*sqrt(7) - 1/2*sqrt(-4550065/334540596096*I*sqrt(7
) - 180983329/4683568345344))^2 - 658503/1568*(17*I*sqrt(7) + 7056*sqrt(-45
50065/334540596096*I*sqrt(7) - 180983329/4683568345344))*(-17*I*sqrt(7) + 7
056*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/4683568345344)) - 66301
91) + 7569*(8*x^4 - x^3 + 8*x + 8)*(17*I*sqrt(7) + 7056*sqrt(-4550065/33454
0596096*I*sqrt(7) - 180983329/4683568345344)) + 7569*(8*x^4 - x^3 + 8*x + 8
)*(-17*I*sqrt(7) + 7056*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/468
3568345344)))*log(-439084532376*(-17/14112*I*sqrt(7) - 1/2*sqrt(-4550065/33
4540596096*I*sqrt(7) - 180983329/4683568345344))^2*(-1066184864424603*I*sqr
t(7) + 442529435492941104*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/4
683568345344) - 1427510892508480) - 7569/2*(7276511507810430573072*(17/1411
2*I*sqrt(7) - 1/2*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/468356834
5344))^2 - 23359423554371543)*(17*I*sqrt(7) + 7056*sqrt(-4550065/3345405960
96*I*sqrt(7) - 180983329/4683568345344)) + 626797952698732342414548480*(17/
14112*I*sqrt(7) - 1/2*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/46835
68345344))^2 + 1/16*(261*(62716756730859*sqrt(87)*(-17*I*sqrt(7) + 7056*sqr
t(4550065/334540596096*I*sqrt(7) - 180983329/4683568345344)) - 142751089250
8480*sqrt(87))*(17*I*sqrt(7) + 7056*sqrt(-4550065/334540596096*I*sqrt(7) -
180983329/4683568345344)) - 372580342944713280*sqrt(87)*(-17*I*sqrt(7) + 70
56*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/4683568345344)) + 104650
```

```
21752358451264*sqrt(87))*sqrt(-125452723536*(17/14112*I*sqrt(7) - 1/2*sqrt(
4550065/334540596096*I*sqrt(7) - 180983329/4683568345344))^2 - 125452723536
*(-17/14112*I*sqrt(7) - 1/2*sqrt(-4550065/334540596096*I*sqrt(7) - 18098332
9/4683568345344))^2 - 658503/1568*(17*I*sqrt(7) + 7056*sqrt(-4550065/334540
596096*I*sqrt(7) - 180983329/4683568345344))*(-17*I*sqrt(7) + 7056*sqrt(455
0065/334540596096*I*sqrt(7) - 180983329/4683568345344)) - 6630191) + 843520
8206933660878927*x - 3005727107011649552439/2*I*sqrt(7) + 62377677844335880
1235576*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/4683568345344) + 22
959102208397854107O4) - (56*sqrt(87)*(8*x^4 - x^3 + 8*x + 8)*sqrt(-12545272
3536*(17/14112*I*sqrt(7) - 1/2*sqrt(4550065/334540596096*I*sqrt(7) - 180983
329/4683568345344))^2 - 125452723536*(-17/14112*I*sqrt(7) - 1/2*sqrt(-45500
65/334540596096*I*sqrt(7) - 180983329/4683568345344))^2 - 658503/1568*(17*I
*sqrt(7) + 7056*sqrt(-4550065/334540596096*I*sqrt(7) - 180983329/4683568345
344))*(-17*I*sqrt(7) + 7056*sqrt(4550065/334540596096*I*sqrt(7) - 180983329
/4683568345344)) - 6630191) - 7569*(8*x^4 - x^3 + 8*x + 8)*(17*I*sqrt(7) +
7056*sqrt(-4550065/334540596096*I*sqrt(7) - 180983329/4683568345344)) - 756
9*(8*x^4 - x^3 + 8*x + 8)*(-17*I*sqrt(7) + 7056*sqrt(4550065/334540596096*I
*sqrt(7) - 180983329/4683568345344)))*log(-439084532376*(-17/14112*I*sqrt(7
) - 1/2*sqrt(-4550065/334540596096*I*sqrt(7) - 180983329/4683568345344))^2*
(-1066184864424603*I*sqrt(7) + 4425294354929411O4*sqrt(4550065/334540596096
*I*sqrt(7) - 180983329/4683568345344) - 1427510892508480) - 7569/2*(7276511
507810430573072*(17/14112*I*sqrt(7) - 1/2*sqrt(4550065/334540596096*I*sqrt(
7) - 180983329/4683568345344))^2 - 23359423554371543)*(17*I*sqrt(7) + 7056*
sqrt(-4550065/334540596096*I*sqrt(7) - 180983329/4683568345344)) + 62679795
2698732342414548480*(17/14112*I*sqrt(7) - 1/2*sqrt(4550065/334540596096*I*s
qrt(7) - 180983329/4683568345344))^2 - 1/16*(261*(62716756730859*sqrt(87)*(
-17*I*sqrt(7) + 7056*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/468356
8345344)) - 1427510892508480*sqrt(87))*(17*I*sqrt(7) + 7056*sqrt(-4550065/3
34540596096*I*sqrt(7) - 180983329/4683568345344)) - 372580342944713280*sqrt
(87)*(-17*I*sqrt(7) + 7056*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/
4683568345344)) + 104650217523584512 64*sqrt(87))*sqrt(-125452723536*(17/141
12*I*sqrt(7) - 1/2*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/46835683
45344))^2 - 125452723536*(-17/14112*I*sqrt(7) - 1/2*sqrt(-4550065/334540596
096*I*sqrt(7) - 180983329/4683568345344))^2 - 658503/1568*(17*I*sqrt(7) + 7
056*sqrt(-4550065/334540596096*I*sqrt(7) - 180983329/4683568345344))*(-17*I
*sqrt(7) + 7056*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/46835683453
44)) - 6630191) + 8435208206933660878927*x - 3005727107011649552439/2*I*sqr
t(7) + 623776778443358801235576*sqrt(4550065/334540596096*I*sqrt(7) - 18098
3329/4683568345344) + 22959102208397854107O4) + 7498008*x + 2650368)/(8*x^4
 - x^3 + 8*x + 8)
```

Sympy [A] time = 0.916419, size = 71, normalized size = 0.2

$$\frac{784x^3 - 1146x^2 + 1539x + 544}{350784x^4 - 43848x^3 + 350784x + 350784} + \text{RootSum}\left(562133862743150960064t^4 + 2228162991905088t^2 + 64471372\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x**4-x**3+8*x+8)**2,x)
```

```
[Out] (784*x**3 - 1146*x**2 + 1539*x + 544)/(350784*x**4 - 43848*x**3 + 350784*x
+ 350784) + RootSum(562133862743150960064*_t**4 + 2228162991905088*_t**2 + 6
447137250645*_t + 4563337216, Lambda(_t, _t*log(777231320984133206794996732
416*_t**3/8435208206933660878927 - 1253595905397464684829096960*_t**2/84352
08206933660878927 + 900072466443173277115848*_t/227978600187396239971 + x +
 333979081113202533090737/67481665655469287031416)))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(8\,x^4 - x^3 + 8\,x + 8\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4-x^3+8*x+8)^2,x, algorithm="giac")
```

```
[Out] integrate((8*x^4 - x^3 + 8*x + 8)^(-2), x)
```

3.51 $\int \left(1+4x+4x^2+4x^4\right)^4 \, dx$

Optimal. Leaf size=97

$$\frac{256x^{17}}{17}+\frac{1024x^{15}}{15}+\frac{512x^{14}}{7}+\frac{1792x^{13}}{13}+256x^{12}+\frac{3328x^{11}}{11}+384x^{10}+\frac{4192x^9}{9}+448x^8+\frac{2752x^7}{7}+\frac{992x^6}{3}+\frac{1136x^5}{5}$$

```
[Out] x + 8*x^2 + (112*x^3)/3 + 112*x^4 + (1136*x^5)/5 + (992*x^6)/3 + (2752*x^7)
/7 + 448*x^8 + (4192*x^9)/9 + 384*x^10 + (3328*x^11)/11 + 256*x^12 + (1792*
x^13)/13 + (512*x^14)/7 + (1024*x^15)/15 + (256*x^17)/17
```

Rubi [A] time = 0.027978, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2061}

$$\frac{256x^{17}}{17}+\frac{1024x^{15}}{15}+\frac{512x^{14}}{7}+\frac{1792x^{13}}{13}+256x^{12}+\frac{3328x^{11}}{11}+384x^{10}+\frac{4192x^9}{9}+448x^8+\frac{2752x^7}{7}+\frac{992x^6}{3}+\frac{1136x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 4*x + 4*x^2 + 4*x^4)^4,x]
```

```
[Out] x + 8*x^2 + (112*x^3)/3 + 112*x^4 + (1136*x^5)/5 + (992*x^6)/3 + (2752*x^7)
/7 + 448*x^8 + (4192*x^9)/9 + 384*x^10 + (3328*x^11)/11 + 256*x^12 + (1792*
x^13)/13 + (512*x^14)/7 + (1024*x^15)/15 + (256*x^17)/17
```

Rule 2061

```
Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I
GtQ[p, 0]
```

Rubi steps

$$\int \left(1+4x+4x^2+4x^4\right)^4 \, dx = \int \left(1+16x+112x^2+448x^3+1136x^4+1984x^5+2752x^6+3584x^7+4192x^8+3840x^9$$

$$= x+8x^2+\frac{112x^3}{3}+112x^4+\frac{1136x^5}{5}+\frac{992x^6}{3}+\frac{2752x^7}{7}+448x^8+\frac{4192x^9}{9}+384x^{10}+$$

Mathematica [A] time = 0.0014482, size = 97, normalized size = 1.

$$\frac{256x^{17}}{17}+\frac{1024x^{15}}{15}+\frac{512x^{14}}{7}+\frac{1792x^{13}}{13}+256x^{12}+\frac{3328x^{11}}{11}+384x^{10}+\frac{4192x^9}{9}+448x^8+\frac{2752x^7}{7}+\frac{992x^6}{3}+\frac{1136x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 4*x + 4*x^2 + 4*x^4)^4,x]
```

```
[Out] x + 8*x^2 + (112*x^3)/3 + 112*x^4 + (1136*x^5)/5 + (992*x^6)/3 + (2752*x^7)
/7 + 448*x^8 + (4192*x^9)/9 + 384*x^10 + (3328*x^11)/11 + 256*x^12 + (1792*
x^13)/13 + (512*x^14)/7 + (1024*x^15)/15 + (256*x^17)/17
```

Maple [A] time = 0.001, size = 78, normalized size = 0.8

$$x+8\,x^{2}+\frac{112\,x^{3}}{3}+112\,x^{4}+\frac{1136\,x^{5}}{5}+\frac{992\,x^{6}}{3}+\frac{2752\,x^{7}}{7}+448\,x^{8}+\frac{4192\,x^{9}}{9}+384\,x^{10}+\frac{3328\,x^{11}}{11}+256\,x^{12}+\frac{1792\,x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^4+4*x^2+4*x+1)^4,x)
```

```
[Out] x+8*x^2+112/3*x^3+112*x^4+1136/5*x^5+992/3*x^6+2752/7*x^7+448*x^8+4192/9*x^
9+384*x^10+3328/11*x^11+256*x^12+1792/13*x^13+512/7*x^14+1024/15*x^15+256/1
7*x^17
```

Maxima [A] time = 1.15065, size = 104, normalized size = 1.07

$$\frac{256}{17}\,x^{17}+\frac{1024}{15}\,x^{15}+\frac{512}{7}\,x^{14}+\frac{1792}{13}\,x^{13}+256\,x^{12}+\frac{3328}{11}\,x^{11}+384\,x^{10}+\frac{4192}{9}\,x^{9}+448\,x^{8}+\frac{2752}{7}\,x^{7}+\frac{992}{3}\,x^{6}+\frac{113}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4+4*x^2+4*x+1)^4,x, algorithm="maxima")
```

```
[Out] 256/17*x^17 + 1024/15*x^15 + 512/7*x^14 + 1792/13*x^13 + 256*x^12 + 3328/11
*x^11 + 384*x^10 + 4192/9*x^9 + 448*x^8 + 2752/7*x^7 + 992/3*x^6 + 1136/5*x
^5 + 112*x^4 + 112/3*x^3 + 8*x^2 + x
```

Fricas [A] time = 1.11911, size = 254, normalized size = 2.62

$$\frac{256}{17}x^{17}+\frac{1024}{15}x^{15}+\frac{512}{7}x^{14}+\frac{1792}{13}x^{13}+256x^{12}+\frac{3328}{11}x^{11}+384x^{10}+\frac{4192}{9}x^{9}+448x^{8}+\frac{2752}{7}x^{7}+\frac{992}{3}x^{6}+\frac{1136}{5}x^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4+4*x^2+4*x+1)^4,x, algorithm="fricas")
```

```
[Out] 256/17*x^17 + 1024/15*x^15 + 512/7*x^14 + 1792/13*x^13 + 256*x^12 + 3328/11
*x^11 + 384*x^10 + 4192/9*x^9 + 448*x^8 + 2752/7*x^7 + 992/3*x^6 + 1136/5*x
^5 + 112*x^4 + 112/3*x^3 + 8*x^2 + x
```

Sympy [A] time = 0.067993, size = 94, normalized size = 0.97

$$\frac{256x^{17}}{17} + \frac{1024x^{15}}{15} + \frac{512x^{14}}{7} + \frac{1792x^{13}}{13} + 256x^{12} + \frac{3328x^{11}}{11} + 384x^{10} + \frac{4192x^{9}}{9} + 448x^{8} + \frac{2752x^{7}}{7} + \frac{992x^{6}}{3} + \frac{1136x^{5}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**4+4*x**2+4*x+1)**4,x)
```

```
[Out] 256*x**17/17 + 1024*x**15/15 + 512*x**14/7 + 1792*x**13/13 + 256*x**12 + 33
28*x**11/11 + 384*x**10 + 4192*x**9/9 + 448*x**8 + 2752*x**7/7 + 992*x**6/3
 + 1136*x**5/5 + 112*x**4 + 112*x**3/3 + 8*x**2 + x
```

Giac [A] time = 1.11347, size = 104, normalized size = 1.07

$$\frac{256}{17}\,x^{17} + \frac{1024}{15}\,x^{15} + \frac{512}{7}\,x^{14} + \frac{1792}{13}\,x^{13} + 256\,x^{12} + \frac{3328}{11}\,x^{11} + 384\,x^{10} + \frac{4192}{9}\,x^{9} + 448\,x^{8} + \frac{2752}{7}\,x^{7} + \frac{992}{3}\,x^{6} + \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4+4*x^2+4*x+1)^4,x, algorithm="giac")
```

```
[Out] 256/17*x^17 + 1024/15*x^15 + 512/7*x^14 + 1792/13*x^13 + 256*x^12 + 3328/11
*x^11 + 384*x^10 + 4192/9*x^9 + 448*x^8 + 2752/7*x^7 + 992/3*x^6 + 1136/5*x
^5 + 112*x^4 + 112/3*x^3 + 8*x^2 + x
```

3.52 $\int \left(1+4x+4x^2+4x^4\right)^3 \, dx$

Optimal. Leaf size=69

$$\frac{64x^{13}}{13}+\frac{192x^{11}}{11}+\frac{96x^{10}}{5}+\frac{80x^9}{3}+48x^8+\frac{352x^7}{7}+48x^6+\frac{252x^5}{5}+40x^4+20x^3+6x^2+x$$

```
[Out] x + 6*x^2 + 20*x^3 + 40*x^4 + (252*x^5)/5 + 48*x^6 + (352*x^7)/7 + 48*x^8 +
 (80*x^9)/3 + (96*x^10)/5 + (192*x^11)/11 + (64*x^13)/13
```

Rubi [A] time = 0.0200803, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2061}

$$\frac{64x^{13}}{13}+\frac{192x^{11}}{11}+\frac{96x^{10}}{5}+\frac{80x^9}{3}+48x^8+\frac{352x^7}{7}+48x^6+\frac{252x^5}{5}+40x^4+20x^3+6x^2+x$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 4*x + 4*x^2 + 4*x^4)^3,x]
```

```
[Out] x + 6*x^2 + 20*x^3 + 40*x^4 + (252*x^5)/5 + 48*x^6 + (352*x^7)/7 + 48*x^8 +
 (80*x^9)/3 + (96*x^10)/5 + (192*x^11)/11 + (64*x^13)/13
```

Rule 2061

```
Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I
GtQ[p, 0]
```

Rubi steps

$$\int \left(1+4x+4x^2+4x^4\right)^3 \, dx = \int \left(1+12x+60x^2+160x^3+252x^4+288x^5+352x^6+384x^7+240x^8+192x^9+192x^{10}\right.$$
$$= x+6x^2+20x^3+40x^4+\frac{252x^5}{5}+48x^6+\frac{352x^7}{7}+48x^8+\frac{80x^9}{3}+\frac{96x^{10}}{5}+\frac{192x^{11}}{11}+\frac{64x}{13}$$

Mathematica [A] time = 0.0009704, size = 69, normalized size = 1.

$$\frac{64x^{13}}{13}+\frac{192x^{11}}{11}+\frac{96x^{10}}{5}+\frac{80x^9}{3}+48x^8+\frac{352x^7}{7}+48x^6+\frac{252x^5}{5}+40x^4+20x^3+6x^2+x$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 4*x + 4*x^2 + 4*x^4)^3,x]
```

```
[Out] x + 6*x^2 + 20*x^3 + 40*x^4 + (252*x^5)/5 + 48*x^6 + (352*x^7)/7 + 48*x^8 +
 (80*x^9)/3 + (96*x^10)/5 + (192*x^11)/11 + (64*x^13)/13
```

Maple [A] time = 0., size = 58, normalized size = 0.8

$$x+6\,x^{2}+20\,x^{3}+40\,x^{4}+\frac{252\,x^{5}}{5}+48\,x^{6}+\frac{352\,x^{7}}{7}+48\,x^{8}+\frac{80\,x^{9}}{3}+\frac{96\,x^{10}}{5}+\frac{192\,x^{11}}{11}+\frac{64\,x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^4+4*x^2+4*x+1)^3,x)
```

```
[Out] x+6*x^2+20*x^3+40*x^4+252/5*x^5+48*x^6+352/7*x^7+48*x^8+80/3*x^9+96/5*x^10+
192/11*x^11+64/13*x^13
```

Maxima [A] time = 1.15995, size = 77, normalized size = 1.12

$$\frac{64}{13}\,x^{13}+\frac{192}{11}\,x^{11}+\frac{96}{5}\,x^{10}+\frac{80}{3}\,x^{9}+48\,x^{8}+\frac{352}{7}\,x^{7}+48\,x^{6}+\frac{252}{5}\,x^{5}+40\,x^{4}+20\,x^{3}+6\,x^{2}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4+4*x^2+4*x+1)^3,x, algorithm="maxima")
```

```
[Out] 64/13*x^13 + 192/11*x^11 + 96/5*x^10 + 80/3*x^9 + 48*x^8 + 352/7*x^7 + 48*x
^6 + 252/5*x^5 + 40*x^4 + 20*x^3 + 6*x^2 + x
```

Fricas [A] time = 1.12574, size = 163, normalized size = 2.36

$$\frac{64}{13}x^{13}+\frac{192}{11}x^{11}+\frac{96}{5}x^{10}+\frac{80}{3}x^{9}+48x^{8}+\frac{352}{7}x^{7}+48x^{6}+\frac{252}{5}x^{5}+40x^{4}+20x^{3}+6x^{2}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4+4*x^2+4*x+1)^3,x, algorithm="fricas")
```

```
[Out] 64/13*x^13 + 192/11*x^11 + 96/5*x^10 + 80/3*x^9 + 48*x^8 + 352/7*x^7 + 48*x
^6 + 252/5*x^5 + 40*x^4 + 20*x^3 + 6*x^2 + x
```

Sympy [A] time = 0.065145, size = 66, normalized size = 0.96

$$\frac{64x^{13}}{13} + \frac{192x^{11}}{11} + \frac{96x^{10}}{5} + \frac{80x^9}{3} + 48x^8 + \frac{352x^7}{7} + 48x^6 + \frac{252x^5}{5} + 40x^4 + 20x^3 + 6x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**4+4*x**2+4*x+1)**3,x)
```

```
[Out] 64*x**13/13 + 192*x**11/11 + 96*x**10/5 + 80*x**9/3 + 48*x**8 + 352*x**7/7
+ 48*x**6 + 252*x**5/5 + 40*x**4 + 20*x**3 + 6*x**2 + x
```

Giac [A] time = 1.12538, size = 77, normalized size = 1.12

$$\frac{64}{13}\,x^{13} + \frac{192}{11}\,x^{11} + \frac{96}{5}\,x^{10} + \frac{80}{3}\,x^{9} + 48\,x^{8} + \frac{352}{7}\,x^{7} + 48\,x^{6} + \frac{252}{5}\,x^{5} + 40\,x^{4} + 20\,x^{3} + 6\,x^{2} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4+4*x^2+4*x+1)^3,x, algorithm="giac")
```

```
[Out] 64/13*x^13 + 192/11*x^11 + 96/5*x^10 + 80/3*x^9 + 48*x^8 + 352/7*x^7 + 48*x
^6 + 252/5*x^5 + 40*x^4 + 20*x^3 + 6*x^2 + x
```

3.53 $\int \left(1+4x+4x^2+4x^4\right)^2 \, dx$

Optimal. Leaf size=45

$$\frac{16x^9}{9}+\frac{32x^7}{7}+\frac{16x^6}{3}+\frac{24x^5}{5}+8x^4+8x^3+4x^2+x$$

```
[Out] x + 4*x^2 + 8*x^3 + 8*x^4 + (24*x^5)/5 + (16*x^6)/3 + (32*x^7)/7 + (16*x^9)
/9
```

Rubi [A] time = 0.0148994, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2061}

$$\frac{16x^9}{9}+\frac{32x^7}{7}+\frac{16x^6}{3}+\frac{24x^5}{5}+8x^4+8x^3+4x^2+x$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 4*x + 4*x^2 + 4*x^4)^2,x]
```

```
[Out] x + 4*x^2 + 8*x^3 + 8*x^4 + (24*x^5)/5 + (16*x^6)/3 + (32*x^7)/7 + (16*x^9)
/9
```

Rule 2061

```
Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I
GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \left(1+4x+4x^2+4x^4\right)^2 \, dx &= \int \left(1+8x+24x^2+32x^3+24x^4+32x^5+32x^6+16x^8\right) dx\\ &= x+4x^2+8x^3+8x^4+\frac{24x^5}{5}+\frac{16x^6}{3}+\frac{32x^7}{7}+\frac{16x^9}{9}\end{aligned}$$

Mathematica [A] time = 0.001273, size = 45, normalized size = 1.

$$\frac{16x^9}{9}+\frac{32x^7}{7}+\frac{16x^6}{3}+\frac{24x^5}{5}+8x^4+8x^3+4x^2+x$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 4*x + 4*x^2 + 4*x^4)^2,x]
```

```
[Out] x + 4*x^2 + 8*x^3 + 8*x^4 + (24*x^5)/5 + (16*x^6)/3 + (32*x^7)/7 + (16*x^9)
/9
```

Maple [A] time = 0.002, size = 38, normalized size = 0.8

$$x+4x^2+8x^3+8x^4+\frac{24x^5}{5}+\frac{16x^6}{3}+\frac{32x^7}{7}+\frac{16x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^4+4*x^2+4*x+1)^2,x)
```

```
[Out] x+4*x^2+8*x^3+8*x^4+24/5*x^5+16/3*x^6+32/7*x^7+16/9*x^9
```

Maxima [A] time = 1.78079, size = 50, normalized size = 1.11

$$\frac{16}{9}x^9+\frac{32}{7}x^7+\frac{16}{3}x^6+\frac{24}{5}x^5+8x^4+8x^3+4x^2+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4+4*x^2+4*x+1)^2,x, algorithm="maxima")
```

```
[Out] 16/9*x^9 + 32/7*x^7 + 16/3*x^6 + 24/5*x^5 + 8*x^4 + 8*x^3 + 4*x^2 + x
```

Fricas [A] time = 1.0902, size = 96, normalized size = 2.13

$$\frac{16}{9}x^9+\frac{32}{7}x^7+\frac{16}{3}x^6+\frac{24}{5}x^5+8x^4+8x^3+4x^2+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4+4*x^2+4*x+1)^2,x, algorithm="fricas")
```

```
[Out] 16/9*x^9 + 32/7*x^7 + 16/3*x^6 + 24/5*x^5 + 8*x^4 + 8*x^3 + 4*x^2 + x
```

Sympy [A] time = 0.058814, size = 42, normalized size = 0.93

$$\frac{16x^9}{9}+\frac{32x^7}{7}+\frac{16x^6}{3}+\frac{24x^5}{5}+8x^4+8x^3+4x^2+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**4+4*x**2+4*x+1)**2,x)
```

```
[Out] 16*x**9/9 + 32*x**7/7 + 16*x**6/3 + 24*x**5/5 + 8*x**4 + 8*x**3 + 4*x**2 +
x
```

Giac [A] time = 1.12127, size = 50, normalized size = 1.11

$$\frac{16}{9}\,x^9+\frac{32}{7}\,x^7+\frac{16}{3}\,x^6+\frac{24}{5}\,x^5+8\,x^4+8\,x^3+4\,x^2+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4+4*x^2+4*x+1)^2,x, algorithm="giac")
```

```
[Out] 16/9*x^9 + 32/7*x^7 + 16/3*x^6 + 24/5*x^5 + 8*x^4 + 8*x^3 + 4*x^2 + x
```

3.54 $\int \left(1+4x+4x^2+4x^4\right)\,dx$

Optimal. Leaf size=21

$$\frac{4x^5}{5}+\frac{4x^3}{3}+2x^2+x$$

```
[Out] x + 2*x^2 + (4*x^3)/3 + (4*x^5)/5
```

Rubi [A] time = 0.0026362, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\frac{4x^5}{5}+\frac{4x^3}{3}+2x^2+x$$

Antiderivative was successfully verified.

```
[In] Int[1 + 4*x + 4*x^2 + 4*x^4,x]
```

```
[Out] x + 2*x^2 + (4*x^3)/3 + (4*x^5)/5
```

Rubi steps

$$\int \left(1+4x+4x^2+4x^4\right)\,dx=x+2x^2+\frac{4x^3}{3}+\frac{4x^5}{5}$$

Mathematica [A] time = 0.0000427, size = 21, normalized size = 1.

$$\frac{4x^5}{5}+\frac{4x^3}{3}+2x^2+x$$

Antiderivative was successfully verified.

```
[In] Integrate[1 + 4*x + 4*x^2 + 4*x^4,x]
```

```
[Out] x + 2*x^2 + (4*x^3)/3 + (4*x^5)/5
```

Maple [A] time = 0.001, size = 18, normalized size = 0.9

$$x + 2x^2 + \frac{4x^3}{3} + \frac{4x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(4*x^4+4*x^2+4*x+1,x)
```

```
[Out] x+2*x^2+4/3*x^3+4/5*x^5
```

Maxima [A] time = 1.09257, size = 23, normalized size = 1.1

$$\frac{4}{5}x^5 + \frac{4}{3}x^3 + 2x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*x^4+4*x^2+4*x+1,x, algorithm="maxima")
```

```
[Out] 4/5*x^5 + 4/3*x^3 + 2*x^2 + x
```

Fricas [A] time = 1.09068, size = 42, normalized size = 2.

$$\frac{4}{5}x^5 + \frac{4}{3}x^3 + 2x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*x^4+4*x^2+4*x+1,x, algorithm="fricas")
```

```
[Out] 4/5*x^5 + 4/3*x^3 + 2*x^2 + x
```

Sympy [A] time = 0.052716, size = 19, normalized size = 0.9

$$\frac{4x^5}{5} + \frac{4x^3}{3} + 2x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*x**4+4*x**2+4*x+1,x)
```

```
[Out] 4*x**5/5 + 4*x**3/3 + 2*x**2 + x
```

Giac [A] time = 1.14291, size = 23, normalized size = 1.1

$$\frac{4}{5}x^5 + \frac{4}{3}x^3 + 2x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*x^4+4*x^2+4*x+1,x, algorithm="giac")
```

```
[Out] 4/5*x^5 + 4/3*x^3 + 2*x^2 + x
```

3.55 $\int \frac{1}{1+4x+4x^2+4x^4}\,dx$

Optimal. Leaf size=234

$$-\frac{1}{4}\sqrt{\frac{1}{5}\left(\sqrt{5}-2\right)}\log\left(\left(\frac{1}{x}+1\right)^2-\sqrt{2\left(1+\sqrt{5}\right)}\left(\frac{1}{x}+1\right)+\sqrt{5}\right)+\frac{1}{4}\sqrt{\frac{1}{5}\left(\sqrt{5}-2\right)}\log\left(\left(\frac{1}{x}+1\right)^2+\sqrt{2\left(1+\sqrt{5}\right)}\left(\frac{1}{x}+1\right)\right.$$

```
[Out] ArcTan[(-1 + (1 + x^(-1))^2)/2]/2 - (Sqrt[(2 + Sqrt[5])/5]*ArcTan[(2 - Sqrt
[2*(1 + Sqrt[5])] + 2/x)/Sqrt[2*(-1 + Sqrt[5])]])/2 - (Sqrt[(2 + Sqrt[5])/5
]*ArcTan[(2 + Sqrt[2*(1 + Sqrt[5])] + 2/x)/Sqrt[2*(-1 + Sqrt[5])]])/2 - (Sq
rt[(-2 + Sqrt[5])/5]*Log[Sqrt[5] - Sqrt[2*(1 + Sqrt[5])]*(1 + x^(-1)) + (1
+ x^(-1))^2])/4 + (Sqrt[(-2 + Sqrt[5])/5]*Log[Sqrt[5] + Sqrt[2*(1 + Sqrt[5]
)]*(1 + x^(-1)) + (1 + x^(-1))^2])/4
```

Rubi [A] time = 0.315422, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {2069, 1673, 1169, 634, 618, 204, 628, 12, 1107}

$$-\frac{1}{4}\sqrt{\frac{1}{5}\left(\sqrt{5}-2\right)}\log\left(\left(\frac{1}{x}+1\right)^2-\sqrt{2\left(1+\sqrt{5}\right)}\left(\frac{1}{x}+1\right)+\sqrt{5}\right)+\frac{1}{4}\sqrt{\frac{1}{5}\left(\sqrt{5}-2\right)}\log\left(\left(\frac{1}{x}+1\right)^2+\sqrt{2\left(1+\sqrt{5}\right)}\left(\frac{1}{x}+1\right)\right.$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 4*x + 4*x^2 + 4*x^4)^(-1),x]
```

```
[Out] ArcTan[(-1 + (1 + x^(-1))^2)/2]/2 - (Sqrt[(2 + Sqrt[5])/5]*ArcTan[(2 - Sqrt
[2*(1 + Sqrt[5])] + 2/x)/Sqrt[2*(-1 + Sqrt[5])]])/2 - (Sqrt[(2 + Sqrt[5])/5
]*ArcTan[(2 + Sqrt[2*(1 + Sqrt[5])] + 2/x)/Sqrt[2*(-1 + Sqrt[5])]])/2 - (Sq
rt[(-2 + Sqrt[5])/5]*Log[Sqrt[5] - Sqrt[2*(1 + Sqrt[5])]*(1 + x^(-1)) + (1
+ x^(-1))^2])/4 + (Sqrt[(-2 + Sqrt[5])/5]*Log[Sqrt[5] + Sqrt[2*(1 + Sqrt[5]
)]*(1 + x^(-1)) + (1 + x^(-1))^2])/4
```

Rule 2069

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Dist[-16*
a^2, Subst[Int[(1*((a*(-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 256*a^3*e - 32*a^
2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4))/(b - 4*a*x)^4)^p)/(b - 4*a*x)^2, x],
x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a^
```

```
2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] &&  !IGtQ[p, 0]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\int \frac{1}{1+4x+4x^2+4x^4}\,dx = -\left(16\,\text{Subst}\left(\int \frac{(4-4x)^2}{1280-512x^2+256x^4}\,dx, x, 1+\frac{1}{x}\right)\right)$$

$$= -\left(16\,\text{Subst}\left(\int -\frac{32x}{1280-512x^2+256x^4}\,dx, x, 1+\frac{1}{x}\right)\right) - 16\,\text{Subst}\left(\int \frac{16+16x^2}{1280-512x^2+256\ldots}\right.$$

$$= 512\,\text{Subst}\left(\int \frac{x}{1280-512x^2+256x^4}\,dx, x, 1+\frac{1}{x}\right) - \frac{\text{Subst}\left(\int \frac{16\sqrt{2\left(1+\sqrt{5}\right)}-\left(16-16\sqrt{5}\right)x}{\sqrt{5}-\sqrt{2\left(1+\sqrt{5}\right)}x+x^2}\,dx, x, \ldots\right.}{32\sqrt{10\left(1+\sqrt{5}\right)}}$$

$$= 256\,\text{Subst}\left(\int \frac{1}{1280-512x+256x^2}\,dx, x, \left(1+\frac{1}{x}\right)^2\right) + \frac{\left(1-\sqrt{5}\right)\text{Subst}\left(\int \frac{-\sqrt{2\left(1+\sqrt{5}\right)}+2x}{\sqrt{5}-\sqrt{2\left(1+\sqrt{5}\right)}x+x^2}\ldots\right.}{4\sqrt{10\left(1+\sqrt{5}\right)}}$$

$$= -\frac{1}{4}\sqrt{-\frac{2}{5}+\frac{1}{\sqrt{5}}}\log\left(\sqrt{5}-\sqrt{2\left(1+\sqrt{5}\right)}\left(1+\frac{1}{x}\right)+\left(1+\frac{1}{x}\right)^2\right)+\frac{1}{4}\sqrt{-\frac{2}{5}+\frac{1}{\sqrt{5}}}\log\left(\sqrt{5}+\sqrt{2}\ldots\right.$$

$$= \frac{1}{2}\tan^{-1}\left(\frac{1}{2}\left(-1+\left(1+\frac{1}{x}\right)^2\right)\right) - \frac{\left(1+\sqrt{5}\right)^{3/2}\tan^{-1}\left(\frac{2-\sqrt{2\left(1+\sqrt{5}\right)}+\frac{2}{x}}{\sqrt{2\left(-1+\sqrt{5}\right)}}\right)}{4\sqrt{10}} - \frac{\left(1+\sqrt{5}\right)^{3/2}\tan^{-1}\left(\frac{2+\ldots}{\ldots}\right.}{4\sqrt{10}}$$

Mathematica [C] time = 0.0145809, size = 47, normalized size = 0.2

$$\frac{1}{4}\text{RootSum}\left[4\#1^4+4\#1^2+4\#1+1\&, \frac{\log(x-\#1)}{4\#1^3+2\#1+1}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 4*x + 4*x^2 + 4*x^4)^(-1),x]
```

```
[Out] RootSum[1 + 4*#1 + 4*#1^2 + 4*#1^4 & , Log[x - #1]/(1 + 2*#1 + 4*#1^3) & ]/
4
```

Maple [C] time = 0.004, size = 41, normalized size = 0.2

$$\frac{1}{4}\sum_{\_R=\mathrm{RootOf}\left(4\_Z^4+4\_Z^2+4\_Z+1\right)}\frac{\ln\left(x-\_R\right)}{4\_R^3+2\_R+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x^4+4*x^2+4*x+1),x)
```

```
[Out] 1/4*sum(1/(4*_R^3+2*_R+1)*ln(x-_R),_R=RootOf(4*_Z^4+4*_Z^2+4*_Z+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{4\,x^4+4\,x^2+4\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^4+4*x^2+4*x+1),x, algorithm="maxima")
```

```
[Out] integrate(1/(4*x^4 + 4*x^2 + 4*x + 1), x)
```

Fricas [C] time = 8.50128, size = 2303, normalized size = 9.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^4+4*x^2+4*x+1),x, algorithm="fricas")
```

```
[Out] -1/20*(sqrt(10)*sqrt(-15/8*(2*sqrt(1/10*I - 1/5) - I)^2 - 5/4*(2*sqrt(1/10*
I - 1/5) - I)*(2*sqrt(-1/10*I - 1/5) + I) - 15/8*(2*sqrt(-1/10*I - 1/5) + I
)^2 - 9) - 5*sqrt(1/10*I - 1/5) - 5*sqrt(-1/10*I - 1/5))*log(5/2*(2*sqrt(1/
10*I - 1/5) - I)^2*(12*sqrt(-1/10*I - 1/5) + 6*I - 1) + 15*(2*sqrt(1/10*I -
```

```
 1/5) - I)*(2*sqrt(-1/10*I - 1/5) + I)^2 - 5/2*(2*sqrt(-1/10*I - 1/5) + I)^
2 + ((6*sqrt(10)*(2*sqrt(-1/10*I - 1/5) + I) - sqrt(10))*(2*sqrt(1/10*I - 1
/5) - I) - sqrt(10)*(2*sqrt(-1/10*I - 1/5) + I))*sqrt(-15/8*(2*sqrt(1/10*I
- 1/5) - I)^2 - 5/4*(2*sqrt(1/10*I - 1/5) - I)*(2*sqrt(-1/10*I - 1/5) + I)
- 15/8*(2*sqrt(-1/10*I - 1/5) + I)^2 - 9) + 8*x + 3) + 1/20*(sqrt(10)*sqrt(
-15/8*(2*sqrt(1/10*I - 1/5) - I)^2 - 5/4*(2*sqrt(1/10*I - 1/5) - I)*(2*sqrt
(-1/10*I - 1/5) + I) - 15/8*(2*sqrt(-1/10*I - 1/5) + I)^2 - 9) + 5*sqrt(1/1
0*I - 1/5) + 5*sqrt(-1/10*I - 1/5))*log(5/2*(2*sqrt(1/10*I - 1/5) - I)^2*(1
2*sqrt(-1/10*I - 1/5) + 6*I - 1) + 15*(2*sqrt(1/10*I - 1/5) - I)*(2*sqrt(-1
/10*I - 1/5) + I)^2 - 5/2*(2*sqrt(-1/10*I - 1/5) + I)^2 - ((6*sqrt(10)*(2*s
qrt(-1/10*I - 1/5) + I) - sqrt(10))*(2*sqrt(1/10*I - 1/5) - I) - sqrt(10)*(
2*sqrt(-1/10*I - 1/5) + I))*sqrt(-15/8*(2*sqrt(1/10*I - 1/5) - I)^2 - 5/4*(
2*sqrt(1/10*I - 1/5) - I)*(2*sqrt(-1/10*I - 1/5) + I) - 15/8*(2*sqrt(-1/10*
I - 1/5) + I)^2 - 9) + 8*x + 3) - 1/4*(2*sqrt(1/10*I - 1/5) - I)*log(-5*(2*
sqrt(1/10*I - 1/5) - I)^2*(12*sqrt(-1/10*I - 1/5) + 6*I - 1) - 30*(2*sqrt(1
/10*I - 1/5) - I)*(2*sqrt(-1/10*I - 1/5) + I)^2 - 30*(2*sqrt(-1/10*I - 1/5)
 + I)^3 + 8*x - 216*sqrt(-1/10*I - 1/5) - 108*I + 21) - 1/4*(2*sqrt(-1/10*I
 - 1/5) + I)*log(30*(2*sqrt(-1/10*I - 1/5) + I)^3 + 5*(2*sqrt(-1/10*I - 1/5
) + I)^2 + 8*x + 216*sqrt(-1/10*I - 1/5) + 108*I - 27)
```

Sympy [A] time = 0.724326, size = 36, normalized size = 0.15

$$\operatorname{RootSum}\left(1280t^4 + 288t^2 + 32t + 1, \left(t \mapsto t\log\left(-240t^3 + 10t^2 - 54t + x - \frac{27}{8}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x**4+4*x**2+4*x+1),x)
```

```
[Out] RootSum(1280*_t**4 + 288*_t**2 + 32*_t + 1, Lambda(_t, _t*log(-240*_t**3 +
10*_t**2 - 54*_t + x - 27/8)))
```

Giac [A] time = 1.15943, size = 374, normalized size = 1.6

$$\frac{\left(\sqrt{\sqrt{5}-2}\left(\frac{i}{\sqrt{5}-2}+1\right)+2\,i+1\right)\log\left(2\,(7\,i+3)x+\sqrt{29\,\sqrt{5}+62}\left(\frac{19\,i}{29\,\sqrt{5}+62}+1\right)+3\,i-7\right)}{4\,(i-2)} - \frac{\left(\sqrt{\sqrt{5}-2}\left(\frac{i}{\sqrt{5}-2}+1\right)-2\,i-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^4+4*x^2+4*x+1),x, algorithm="giac")
```

```
[Out] 1/4*(sqrt(sqrt(5) - 2)*(i/(sqrt(5) - 2) + 1) + 2*i + 1)*log(2*(7*i + 3)*x +
 sqrt(29*sqrt(5) + 62)*(19*i/(29*sqrt(5) + 62) + 1) + 3*i - 7)/(i - 2) - 1/
4*(sqrt(sqrt(5) - 2)*(i/(sqrt(5) - 2) + 1) - 2*i - 1)*log(2*(7*i + 3)*x - s
qrt(29*sqrt(5) + 62)*(19*i/(29*sqrt(5) + 62) + 1) + 3*i - 7)/(i - 2) - 1/4*
(sqrt(sqrt(5) + 2)*(i/(sqrt(5) + 2) + 1) + i + 2)*log(2*(i + 5)*x + sqrt(13
*sqrt(5) - 22)*(19*i/(13*sqrt(5) - 22) + 1) - 5*i + 1)/(2*i - 1) + 1/4*(sqr
t(sqrt(5) + 2)*(i/(sqrt(5) + 2) + 1) - i - 2)*log(2*(i + 5)*x - sqrt(13*sqr
t(5) - 22)*(19*i/(13*sqrt(5) - 22) + 1) - 5*i + 1)/(2*i - 1)
```

3.56 $\int \frac{1}{\left(1+4x+4x^2+4x^4\right)^2}\,dx$

Optimal. Leaf size=317

$$-\frac{17-\left(\frac{1}{x}+1\right)^2}{2\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)}+\frac{\left(59-17\left(\frac{1}{x}+1\right)^2\right)\left(\frac{1}{x}+1\right)}{10\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)}+\frac{1}{40}\sqrt{\frac{1}{10}\left(2665\sqrt{5}-5959\right)}\log\left(\left(\frac{1}{x}+1\right)^2-\sqrt{2\left(1+\right.}\right.$$

```
[Out] -(17 - (1 + x^(-1))^2)/(2*(5 - 2*(1 + x^(-1))^2 + (1 + x^(-1))^4)) + ((59 -
 17*(1 + x^(-1))^2)*(1 + x^(-1)))/(10*(5 - 2*(1 + x^(-1))^2 + (1 + x^(-1))^
4)) + (7*ArcTan[(-1 + (1 + x^(-1))^2)/2])/4 - (Sqrt[(5959 + 2665*Sqrt[5])/1
0]*ArcTan[(2 - Sqrt[2*(1 + Sqrt[5])] + 2/x)/Sqrt[2*(-1 + Sqrt[5])]])/20 - (
Sqrt[(5959 + 2665*Sqrt[5])/10]*ArcTan[(2 + Sqrt[2*(1 + Sqrt[5])] + 2/x)/Sqr
t[2*(-1 + Sqrt[5])]])/20 + (Sqrt[(-5959 + 2665*Sqrt[5])/10]*Log[Sqrt[5] - S
qrt[2*(1 + Sqrt[5])]*(1 + x^(-1)) + (1 + x^(-1))^2])/40 - (Sqrt[(-5959 + 26
65*Sqrt[5])/10]*Log[Sqrt[5] + Sqrt[2*(1 + Sqrt[5])]*(1 + x^(-1)) + (1 + x^(
-1))^2])/40
```

Rubi [A] time = 0.334724, antiderivative size = 317, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 11, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.647, Rules used = {2069, 1673, 1678, 1169, 634, 618, 204, 628, 1663, 1660, 12}

$$-\frac{17-\left(\frac{1}{x}+1\right)^2}{2\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)}+\frac{\left(59-17\left(\frac{1}{x}+1\right)^2\right)\left(\frac{1}{x}+1\right)}{10\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)}+\frac{1}{40}\sqrt{\frac{1}{10}\left(2665\sqrt{5}-5959\right)}\log\left(\left(\frac{1}{x}+1\right)^2-\sqrt{2\left(1+\right.}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 4*x + 4*x^2 + 4*x^4)^(-2),x]
```

```
[Out] -(17 - (1 + x^(-1))^2)/(2*(5 - 2*(1 + x^(-1))^2 + (1 + x^(-1))^4)) + ((59 -
 17*(1 + x^(-1))^2)*(1 + x^(-1)))/(10*(5 - 2*(1 + x^(-1))^2 + (1 + x^(-1))^
4)) + (7*ArcTan[(-1 + (1 + x^(-1))^2)/2])/4 - (Sqrt[(5959 + 2665*Sqrt[5])/1
0]*ArcTan[(2 - Sqrt[2*(1 + Sqrt[5])] + 2/x)/Sqrt[2*(-1 + Sqrt[5])]])/20 - (
Sqrt[(5959 + 2665*Sqrt[5])/10]*ArcTan[(2 + Sqrt[2*(1 + Sqrt[5])] + 2/x)/Sqr
t[2*(-1 + Sqrt[5])]])/20 + (Sqrt[(-5959 + 2665*Sqrt[5])/10]*Log[Sqrt[5] - S
qrt[2*(1 + Sqrt[5])]*(1 + x^(-1)) + (1 + x^(-1))^2])/40 - (Sqrt[(-5959 + 26
65*Sqrt[5])/10]*Log[Sqrt[5] + Sqrt[2*(1 + Sqrt[5])]*(1 + x^(-1)) + (1 + x^(
```

```
-1))^2])/40
```

Rule 2069

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Dist[-16*
a^2, Subst[Int[(1*((a*(-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 256*a^3*e - 32*a^
2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4))/(b - 4*a*x)^4)^p)/(b - 4*a*x)^2, x],
x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a^
2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] &&  !IGtQ[p, 0]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rubi steps

$$\int \frac{1}{\left(1+4x+4x^2+4x^4\right)^2}\,dx = -\left(16\,\mathrm{Subst}\left(\int \frac{(4-4x)^6}{\left(1280-512x^2+256x^4\right)^2}\,dx, x, 1+\frac{1}{x}\right)\right)$$

$$= -\left(16\,\mathrm{Subst}\left(\int \frac{x\left(-24576-81920x^2-24576x^4\right)}{\left(1280-512x^2+256x^4\right)^2}\,dx, x, 1+\frac{1}{x}\right)\right) - 16\,\mathrm{Subst}\left(\int \frac{4096+6}{(12}\right.$$

$$= \frac{\left(59-17\left(1+\frac{1}{x}\right)^2\right)\left(1+\frac{1}{x}\right)}{10\left(5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4\right)} - \frac{\mathrm{Subst}\left(\int \frac{261993005056+115964116992x^2}{1280-512x^2+256x^4}\,dx, x, 1+\frac{1}{x}\right)}{167772160} - 8\,\mathrm{Sul}$$

$$= -\frac{17-\left(1+\frac{1}{x}\right)^2}{2\left(5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4\right)} + \frac{\left(59-17\left(1+\frac{1}{x}\right)^2\right)\left(1+\frac{1}{x}\right)}{10\left(5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4\right)} - \frac{\mathrm{Subst}\left(\int -\frac{11744051}{1280-512x+2}\right.}{1310}$$

$$= -\frac{17-\left(1+\frac{1}{x}\right)^2}{2\left(5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4\right)} + \frac{\left(59-17\left(1+\frac{1}{x}\right)^2\right)\left(1+\frac{1}{x}\right)}{10\left(5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4\right)} + 896\,\mathrm{Subst}\left(\int \frac{}{1280-}\right.$$

$$= -\frac{17-\left(1+\frac{1}{x}\right)^2}{2\left(5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4\right)} + \frac{\left(59-17\left(1+\frac{1}{x}\right)^2\right)\left(1+\frac{1}{x}\right)}{10\left(5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4\right)} + \frac{1}{40}\sqrt{-\frac{5959}{10}+\frac{533\sqrt{5}}{2}}$$

$$= -\frac{17-\left(1+\frac{1}{x}\right)^2}{2\left(5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4\right)} + \frac{\left(59-17\left(1+\frac{1}{x}\right)^2\right)\left(1+\frac{1}{x}\right)}{10\left(5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4\right)} + \frac{7}{4}\tan^{-1}\left(\frac{1}{2}\left(-1+\left(1+\right.\right.\right.$$

Mathematica [C] time = 0.0233888, size = 108, normalized size = 0.34

$$\frac{1}{40}\left(\mathrm{RootSum}\left[4\#1^4+4\#1^2+4\#1+1\&, \frac{18\#1^2\log(x-\#1)-16\#1\log(x-\#1)+27\log(x-\#1)}{4\#1^3+2\#1+1}\&\right] + \frac{72x^3-32x^2+84}{4x^4+4x^2+4x}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 4*x + 4*x^2 + 4*x^4)^(-2),x]
```

```
[Out] ((38 + 84*x - 32*x^2 + 72*x^3)/(1 + 4*x + 4*x^2 + 4*x^4) + RootSum[1 + 4*#1
 + 4*#1^2 + 4*#1^4 & , (27*Log[x - #1] - 16*Log[x - #1]*#1 + 18*Log[x - #1]
```

```
*#1^2)/(1 + 2*#1 + 4*#1^3) & ])/40
```

Maple [C] time = 0.008, size = 79, normalized size = 0.3

$$\left(\frac{9x^3}{20}-\frac{x^2}{5}+\frac{21x}{40}+\frac{19}{80}\right)\left(x^4+x^2+x+\frac{1}{4}\right)^{-1}+\frac{1}{40}\sum_{\_R=\mathrm{RootOf}\left(4\_Z^4+4\_Z^2+4\_Z+1\right)}\frac{\left(18\_R^2-16\_R+27\right)\ln\left(x-\_R\right)}{4\_R^3+2\_R+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x^4+4*x^2+4*x+1)^2,x)
```

```
[Out] (9/20*x^3-1/5*x^2+21/40*x+19/80)/(x^4+x^2+x+1/4)+1/40*sum((18*_R^2-16*_R+27
)/(4*_R^3+2*_R+1)*ln(x-_R),_R=RootOf(4*_Z^4+4*_Z^2+4*_Z+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{36x^3-16x^2+42x+19}{20\left(4x^4+4x^2+4x+1\right)}+\frac{1}{10}\int\frac{18x^2-16x+27}{4x^4+4x^2+4x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^4+4*x^2+4*x+1)^2,x, algorithm="maxima")
```

```
[Out] 1/20*(36*x^3 - 16*x^2 + 42*x + 19)/(4*x^4 + 4*x^2 + 4*x + 1) + 1/10*integra
te((18*x^2 - 16*x + 27)/(4*x^4 + 4*x^2 + 4*x + 1), x)
```

Fricas [C] time = 9.9968, size = 4027, normalized size = 12.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^4+4*x^2+4*x+1)^2,x, algorithm="fricas")
```

```
[Out] 1/400*(720*x^3 - 50*(4*x^4 + 4*x^2 + 4*x + 1)*(4*sqrt(19/1000*I - 5959/2000
) + 7*I)*log(33368250*(4*sqrt(19/1000*I - 5959/2000) + 7*I)^3 - 11755375/4*
```

```
(4*sqrt(19/1000*I - 5959/2000) + 7*I)^2 + 541735337*x + 25784243612*sqrt(19
/1000*I - 5959/2000) + 45122426321*I - 71080995) - 50*(4*x^4 + 4*x^2 + 4*x
+ 1)*(4*sqrt(-19/1000*I - 5959/2000) - 7*I)*log(-33368250*(4*sqrt(19/1000*I
 - 5959/2000) + 7*I)^3 - 125/4*(4271136*sqrt(19/1000*I - 5959/2000) + 74744
88*I + 94043)*(4*sqrt(-19/1000*I - 5959/2000) - 7*I)^2 - 25*(1334730*(4*sqr
t(19/1000*I - 5959/2000) + 7*I)^2 + 219601)*(4*sqrt(-19/1000*I - 5959/2000)
 - 7*I) + 541735337*x - 25806203712*sqrt(19/1000*I - 5959/2000) - 451608564
96*I - 355111539) - 320*x^2 - (4*sqrt(10)*(4*x^4 + 4*x^2 + 4*x + 1)*sqrt(-3
75/32*(4*sqrt(19/1000*I - 5959/2000) + 7*I)^2 - 125/16*(4*sqrt(19/1000*I -
5959/2000) + 7*I)*(4*sqrt(-19/1000*I - 5959/2000) - 7*I) - 375/32*(4*sqrt(-
19/1000*I - 5959/2000) - 7*I)^2 - 3021) - 25*(4*x^4 + 4*x^2 + 4*x + 1)*(4*s
qrt(19/1000*I - 5959/2000) + 7*I) - 25*(4*x^4 + 4*x^2 + 4*x + 1)*(4*sqrt(-1
9/1000*I - 5959/2000) - 7*I))*log(125/8*(4271136*sqrt(19/1000*I - 5959/2000
) + 7474488*I + 94043)*(4*sqrt(-19/1000*I - 5959/2000) - 7*I)^2 + 11755375/
8*(4*sqrt(19/1000*I - 5959/2000) + 7*I)^2 + 25/2*(1334730*(4*sqrt(19/1000*I
 - 5959/2000) + 7*I)^2 + 219601)*(4*sqrt(-19/1000*I - 5959/2000) - 7*I) + 1
/2*sqrt(-375/32*(4*sqrt(19/1000*I - 5959/2000) + 7*I)^2 - 125/16*(4*sqrt(19
/1000*I - 5959/2000) + 7*I)*(4*sqrt(-19/1000*I - 5959/2000) - 7*I) - 375/32
*(4*sqrt(-19/1000*I - 5959/2000) - 7*I)^2 - 3021)*(5*(1067784*sqrt(10)*(4*s
qrt(19/1000*I - 5959/2000) + 7*I) + 94043*sqrt(10))*(4*sqrt(-19/1000*I - 59
59/2000) - 7*I) + 470215*sqrt(10)*(4*sqrt(19/1000*I - 5959/2000) + 7*I) - 8
78404*sqrt(10)) + 541735337*x + 10980050*sqrt(19/1000*I - 5959/2000) + 3843
0175/2*I + 213096267) + (4*sqrt(10)*(4*x^4 + 4*x^2 + 4*x + 1)*sqrt(-375/32*
(4*sqrt(19/1000*I - 5959/2000) + 7*I)^2 - 125/16*(4*sqrt(19/1000*I - 5959/2
000) + 7*I)*(4*sqrt(-19/1000*I - 5959/2000) - 7*I) - 375/32*(4*sqrt(-19/100
0*I - 5959/2000) - 7*I)^2 - 3021) + 25*(4*x^4 + 4*x^2 + 4*x + 1)*(4*sqrt(19
/1000*I - 5959/2000) + 7*I) + 25*(4*x^4 + 4*x^2 + 4*x + 1)*(4*sqrt(-19/1000
*I - 5959/2000) - 7*I))*log(125/8*(4271136*sqrt(19/1000*I - 5959/2000) + 74
74488*I + 94043)*(4*sqrt(-19/1000*I - 5959/2000) - 7*I)^2 + 11755375/8*(4*s
qrt(19/1000*I - 5959/2000) + 7*I)^2 + 25/2*(1334730*(4*sqrt(19/1000*I - 595
9/2000) + 7*I)^2 + 219601)*(4*sqrt(-19/1000*I - 5959/2000) - 7*I) - 1/2*sqr
t(-375/32*(4*sqrt(19/1000*I - 5959/2000) + 7*I)^2 - 125/16*(4*sqrt(19/1000*
I - 5959/2000) + 7*I)*(4*sqrt(-19/1000*I - 5959/2000) - 7*I) - 375/32*(4*sq
rt(-19/1000*I - 5959/2000) - 7*I)^2 - 3021)*(5*(1067784*sqrt(10)*(4*sqrt(19
/1000*I - 5959/2000) + 7*I) + 94043*sqrt(10))*(4*sqrt(-19/1000*I - 5959/200
0) - 7*I) + 470215*sqrt(10)*(4*sqrt(19/1000*I - 5959/2000) + 7*I) - 878404*
sqrt(10)) + 541735337*x + 10980050*sqrt(19/1000*I - 5959/2000) + 38430175/2
*I + 213096267) + 840*x + 380)/(4*x^4 + 4*x^2 + 4*x + 1)
```

Sympy [A] time = 0.875286, size = 71, normalized size = 0.22

$$\frac{36x^3 - 16x^2 + 42x + 19}{80x^4 + 80x^2 + 80x + 20} + \operatorname{RootSum}\left(64000t^4 + 193344t^2 - 1064t + 29, \left(t \mapsto t \log\left(-\frac{17084544000t^3}{541735337} - \frac{188086000t^2}{541735337}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x**4+4*x**2+4*x+1)**2,x)
```

```
[Out] (36*x**3 - 16*x**2 + 42*x + 19)/(80*x**4 + 80*x**2 + 80*x + 20) + RootSum(6
4000*_t**4 + 193344*_t**2 - 1064*_t + 29, Lambda(_t, _t*log(-17084544000*_t
**3/541735337 - 188086000*_t**2/541735337 - 51568487224*_t/541735337 + x -
71080995/541735337)))
```

<u>Giac</u> [A] time = 1.2926, size = 463, normalized size = 1.46

$$-\frac{\left(\sqrt{2665\sqrt{5}+2062}\left(\frac{5591\,i}{2665\sqrt{5}+2062}+1\right)+63\,i+91\right)\log\left(11182\,(3765\,i+113)x+5591\sqrt{7093997\sqrt{5}+6230338}\left(\frac{14\cdots}{709399\cdots}\right.\right.}{8\,(13\,i-9)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^4+4*x^2+4*x+1)^2,x, algorithm="giac")
```

```
[Out] -1/8*(sqrt(2665*sqrt(5) + 2062)*(5591*i/(2665*sqrt(5) + 2062) + 1) + 63*i +
 91)*log(11182*(3765*i + 113)*x + 5591*sqrt(7093997*sqrt(5) + 6230338)*(145
87901*i/(7093997*sqrt(5) + 6230338) - 1) - 631783*i + 21050115)/(13*i - 9)
+ 1/8*(sqrt(2665*sqrt(5) + 2062)*(5591*i/(2665*sqrt(5) + 2062) + 1) - 63*i
- 91)*log(11182*(3765*i + 113)*x - 5591*sqrt(7093997*sqrt(5) + 6230338)*(14
587901*i/(7093997*sqrt(5) + 6230338) - 1) - 631783*i + 21050115)/(13*i - 9)
 - 1/8*(sqrt(2665*sqrt(5) - 2062)*(5591*i/(2665*sqrt(5) - 2062) + 1) - 91*i
 - 63)*log(11182*(125*i + 3769)*x + 5591*sqrt(7110493*sqrt(5) - 6152618)*(1
4660861*i/(7110493*sqrt(5) - 6152618) - 1) + 21072479*i - 698875)/(9*i - 13
) + 1/8*(sqrt(2665*sqrt(5) - 2062)*(5591*i/(2665*sqrt(5) - 2062) + 1) + 91*
i + 63)*log(11182*(125*i + 3769)*x - 5591*sqrt(7110493*sqrt(5) - 6152618)*(
14660861*i/(7110493*sqrt(5) - 6152618) - 1) + 21072479*i - 698875)/(9*i - 1
3) + 1/20*(36*x^3 - 16*x^2 + 42*x + 19)/(4*x^4 + 4*x^2 + 4*x + 1)
```

3.57 $\int \left(8+24x+8x^2-15x^3+8x^4\right)^4 \, dx$

Optimal. Leaf size=104

$$\frac{4096x^{17}}{17}-1920x^{16}+\frac{102784x^{15}}{15}-\frac{75504x^{14}}{7}-\frac{12095x^{13}}{13}+31128x^{12}-\frac{331040x^{11}}{11}-\frac{169584x^{10}}{5}+\frac{641152x^9}{9}+36384x^8$$

```
[Out] 4096*x + 24576*x^2 + (237568*x^3)/3 + 139776*x^4 + (538624*x^5)/5 - 30720*x
^6 - (566912*x^7)/7 + 36384*x^8 + (641152*x^9)/9 - (169584*x^10)/5 - (33104
0*x^11)/11 + 31128*x^12 - (12095*x^13)/13 - (75504*x^14)/7 + (102784*x^15)/
15 - 1920*x^16 + (4096*x^17)/17
```

Rubi [A] time = 0.0337367, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {2061}

$$\frac{4096x^{17}}{17}-1920x^{16}+\frac{102784x^{15}}{15}-\frac{75504x^{14}}{7}-\frac{12095x^{13}}{13}+31128x^{12}-\frac{331040x^{11}}{11}-\frac{169584x^{10}}{5}+\frac{641152x^9}{9}+36384x^8$$

Antiderivative was successfully verified.

```
[In] Int[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^4,x]
```

```
[Out] 4096*x + 24576*x^2 + (237568*x^3)/3 + 139776*x^4 + (538624*x^5)/5 - 30720*x
^6 - (566912*x^7)/7 + 36384*x^8 + (641152*x^9)/9 - (169584*x^10)/5 - (33104
0*x^11)/11 + 31128*x^12 - (12095*x^13)/13 - (75504*x^14)/7 + (102784*x^15)/
15 - 1920*x^16 + (4096*x^17)/17
```

Rule 2061

```
Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I
GtQ[p, 0]
```

Rubi steps

$$\int \left(8+24x+8x^2-15x^3+8x^4\right)^4 \, dx = \int \left(4096+49152x+237568x^2+559104x^3+538624x^4-184320x^5-566912x^6\right.$$

$$= 4096x+24576x^2+\frac{237568x^3}{3}+139776x^4+\frac{538624x^5}{5}-30720x^6-\frac{566912x^7}{7}$$

Mathematica [A] time = 0.0016699, size = 104, normalized size = 1.

$$\frac{4096x^{17}}{17}-1920x^{16}+\frac{102784x^{15}}{15}-\frac{75504x^{14}}{7}-\frac{12095x^{13}}{13}+31128x^{12}-\frac{331040x^{11}}{11}-\frac{169584x^{10}}{5}+\frac{641152x^9}{9}+36384$$

Antiderivative was successfully verified.

```
[In] Integrate[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^4,x]
```

```
[Out] 4096*x + 24576*x^2 + (237568*x^3)/3 + 139776*x^4 + (538624*x^5)/5 - 30720*x
^6 - (566912*x^7)/7 + 36384*x^8 + (641152*x^9)/9 - (169584*x^10)/5 - (33104
0*x^11)/11 + 31128*x^12 - (12095*x^13)/13 - (75504*x^14)/7 + (102784*x^15)/
15 - 1920*x^16 + (4096*x^17)/17
```

Maple [A] time = 0.003, size = 85, normalized size = 0.8

$$4096\,x+24576\,x^2+\frac{237568\,x^3}{3}+139776\,x^4+\frac{538624\,x^5}{5}-30720\,x^6-\frac{566912\,x^7}{7}+36384\,x^8+\frac{641152\,x^9}{9}-\frac{169584\,x}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*x^4-15*x^3+8*x^2+24*x+8)^4,x)
```

```
[Out] 4096*x+24576*x^2+237568/3*x^3+139776*x^4+538624/5*x^5-30720*x^6-566912/7*x^
7+36384*x^8+641152/9*x^9-169584/5*x^10-331040/11*x^11+31128*x^12-12095/13*x
^13-75504/7*x^14+102784/15*x^15-1920*x^16+4096/17*x^17
```

Maxima [A] time = 1.20434, size = 113, normalized size = 1.09

$$\frac{4096}{17}\,x^{17}-1920\,x^{16}+\frac{102784}{15}\,x^{15}-\frac{75504}{7}\,x^{14}-\frac{12095}{13}\,x^{13}+31128\,x^{12}-\frac{331040}{11}\,x^{11}-\frac{169584}{5}\,x^{10}+\frac{641152}{9}\,x^9+36$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^4-15*x^3+8*x^2+24*x+8)^4,x, algorithm="maxima")
```

```
[Out] 4096/17*x^17 - 1920*x^16 + 102784/15*x^15 - 75504/7*x^14 - 12095/13*x^13 +
31128*x^12 - 331040/11*x^11 - 169584/5*x^10 + 641152/9*x^9 + 36384*x^8 - 56
6912/7*x^7 - 30720*x^6 + 538624/5*x^5 + 139776*x^4 + 237568/3*x^3 + 24576*x
```

```
^2 + 4096*x
```

Fricas [A] time = 1.23255, size = 321, normalized size = 3.09

$$\frac{4096}{17}x^{17} - 1920x^{16} + \frac{102784}{15}x^{15} - \frac{75504}{7}x^{14} - \frac{12095}{13}x^{13} + 31128x^{12} - \frac{331040}{11}x^{11} - \frac{169584}{5}x^{10} + \frac{641152}{9}x^{9} + 36384x^{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^4-15*x^3+8*x^2+24*x+8)^4,x, algorithm="fricas")
```

```
[Out] 4096/17*x^17 - 1920*x^16 + 102784/15*x^15 - 75504/7*x^14 - 12095/13*x^13 +
31128*x^12 - 331040/11*x^11 - 169584/5*x^10 + 641152/9*x^9 + 36384*x^8 - 56
6912/7*x^7 - 30720*x^6 + 538624/5*x^5 + 139776*x^4 + 237568/3*x^3 + 24576*x
^2 + 4096*x
```

Sympy [A] time = 0.077191, size = 100, normalized size = 0.96

$$\frac{4096x^{17}}{17} - 1920x^{16} + \frac{102784x^{15}}{15} - \frac{75504x^{14}}{7} - \frac{12095x^{13}}{13} + 31128x^{12} - \frac{331040x^{11}}{11} - \frac{169584x^{10}}{5} + \frac{641152x^{9}}{9} + 36384x^{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x**4-15*x**3+8*x**2+24*x+8)**4,x)
```

```
[Out] 4096*x**17/17 - 1920*x**16 + 102784*x**15/15 - 75504*x**14/7 - 12095*x**13/
13 + 31128*x**12 - 331040*x**11/11 - 169584*x**10/5 + 641152*x**9/9 + 36384
*x**8 - 566912*x**7/7 - 30720*x**6 + 538624*x**5/5 + 139776*x**4 + 237568*x
**3/3 + 24576*x**2 + 4096*x
```

Giac [A] time = 1.1173, size = 113, normalized size = 1.09

$$\frac{4096}{17}x^{17} - 1920x^{16} + \frac{102784}{15}x^{15} - \frac{75504}{7}x^{14} - \frac{12095}{13}x^{13} + 31128x^{12} - \frac{331040}{11}x^{11} - \frac{169584}{5}x^{10} + \frac{641152}{9}x^{9} + 3638$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^4-15*x^3+8*x^2+24*x+8)^4,x, algorithm="giac")
```

```
[Out] 4096/17*x^17 - 1920*x^16 + 102784/15*x^15 - 75504/7*x^14 - 12095/13*x^13 +
31128*x^12 - 331040/11*x^11 - 169584/5*x^10 + 641152/9*x^9 + 36384*x^8 - 56
6912/7*x^7 - 30720*x^6 + 538624/5*x^5 + 139776*x^4 + 237568/3*x^3 + 24576*x
^2 + 4096*x
```

3.58 $\int \left(8+24x+8x^2-15x^3+8x^4\right)^3 \, dx$

Optimal. Leaf size=76

$$\frac{512x^{13}}{13}-240x^{12}+\frac{6936x^{11}}{11}-\frac{4527x^{10}}{10}-\frac{2936x^9}{3}+2097x^8+\frac{5528x^7}{7}-2976x^6-\frac{384x^5}{5}+5040x^4+5120x^3+2304x^2+$$

```
[Out] 512*x + 2304*x^2 + 5120*x^3 + 5040*x^4 - (384*x^5)/5 - 2976*x^6 + (5528*x^7
)/7 + 2097*x^8 - (2936*x^9)/3 - (4527*x^10)/10 + (6936*x^11)/11 - 240*x^12
+ (512*x^13)/13
```

Rubi [A] time = 0.023682, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {2061}

$$\frac{512x^{13}}{13}-240x^{12}+\frac{6936x^{11}}{11}-\frac{4527x^{10}}{10}-\frac{2936x^9}{3}+2097x^8+\frac{5528x^7}{7}-2976x^6-\frac{384x^5}{5}+5040x^4+5120x^3+2304x^2+$$

Antiderivative was successfully verified.

```
[In] Int[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^3,x]
```

```
[Out] 512*x + 2304*x^2 + 5120*x^3 + 5040*x^4 - (384*x^5)/5 - 2976*x^6 + (5528*x^7
)/7 + 2097*x^8 - (2936*x^9)/3 - (4527*x^10)/10 + (6936*x^11)/11 - 240*x^12
+ (512*x^13)/13
```

Rule 2061

```
Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I
GtQ[p, 0]
```

Rubi steps

$$\int \left(8+24x+8x^2-15x^3+8x^4\right)^3 \, dx = \int \left(512+4608x+15360x^2+20160x^3-384x^4-17856x^5+5528x^6+16776x^7-\right.$$

$$= 512x+2304x^2+5120x^3+5040x^4-\frac{384x^5}{5}-2976x^6+\frac{5528x^7}{7}+2097x^8-\frac{293}{3}$$

Mathematica [A] time = 0.0015606, size = 76, normalized size = 1.

$$\frac{512x^{13}}{13}-240x^{12}+\frac{6936x^{11}}{11}-\frac{4527x^{10}}{10}-\frac{2936x^9}{3}+2097x^8+\frac{5528x^7}{7}-2976x^6-\frac{384x^5}{5}+5040x^4+5120x^3+2304x^2+$$

Antiderivative was successfully verified.

```
[In] Integrate[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^3,x]
```

```
[Out] 512*x + 2304*x^2 + 5120*x^3 + 5040*x^4 - (384*x^5)/5 - 2976*x^6 + (5528*x^7
)/7 + 2097*x^8 - (2936*x^9)/3 - (4527*x^10)/10 + (6936*x^11)/11 - 240*x^12
+ (512*x^13)/13
```

Maple [A] time = 0., size = 65, normalized size = 0.9

$$512\,x+2304\,x^{2}+5120\,x^{3}+5040\,x^{4}-\frac{384\,x^{5}}{5}-2976\,x^{6}+\frac{5528\,x^{7}}{7}+2097\,x^{8}-\frac{2936\,x^{9}}{3}-\frac{4527\,x^{10}}{10}+\frac{6936\,x^{11}}{11}-240\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*x^4-15*x^3+8*x^2+24*x+8)^3,x)
```

```
[Out] 512*x+2304*x^2+5120*x^3+5040*x^4-384/5*x^5-2976*x^6+5528/7*x^7+2097*x^8-293
6/3*x^9-4527/10*x^10+6936/11*x^11-240*x^12+512/13*x^13
```

Maple [A] time = 1.1588, size = 86, normalized size = 1.13

$$\frac{512}{13}\,x^{13}-240\,x^{12}+\frac{6936}{11}\,x^{11}-\frac{4527}{10}\,x^{10}-\frac{2936}{3}\,x^{9}+2097\,x^{8}+\frac{5528}{7}\,x^{7}-2976\,x^{6}-\frac{384}{5}\,x^{5}+5040\,x^{4}+5120\,x^{3}+23$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^4-15*x^3+8*x^2+24*x+8)^3,x, algorithm="maxima")
```

```
[Out] 512/13*x^13 - 240*x^12 + 6936/11*x^11 - 4527/10*x^10 - 2936/3*x^9 + 2097*x^
8 + 5528/7*x^7 - 2976*x^6 - 384/5*x^5 + 5040*x^4 + 5120*x^3 + 2304*x^2 + 51
2*x
```

Fricas [A] time = 1.25592, size = 209, normalized size = 2.75

$$\frac{512}{13}x^{13}-240x^{12}+\frac{6936}{11}x^{11}-\frac{4527}{10}x^{10}-\frac{2936}{3}x^{9}+2097x^{8}+\frac{5528}{7}x^{7}-2976x^{6}-\frac{384}{5}x^{5}+5040x^{4}+5120x^{3}+2304x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^4-15*x^3+8*x^2+24*x+8)^3,x, algorithm="fricas")
```

```
[Out] 512/13*x^13 - 240*x^12 + 6936/11*x^11 - 4527/10*x^10 - 2936/3*x^9 + 2097*x^
8 + 5528/7*x^7 - 2976*x^6 - 384/5*x^5 + 5040*x^4 + 5120*x^3 + 2304*x^2 + 51
2*x
```

Sympy [A] time = 0.067505, size = 73, normalized size = 0.96

$$\frac{512x^{13}}{13} - 240x^{12} + \frac{6936x^{11}}{11} - \frac{4527x^{10}}{10} - \frac{2936x^{9}}{3} + 2097x^{8} + \frac{5528x^{7}}{7} - 2976x^{6} - \frac{384x^{5}}{5} + 5040x^{4} + 5120x^{3} + 2304x^{2} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x**4-15*x**3+8*x**2+24*x+8)**3,x)
```

```
[Out] 512*x**13/13 - 240*x**12 + 6936*x**11/11 - 4527*x**10/10 - 2936*x**9/3 + 20
97*x**8 + 5528*x**7/7 - 2976*x**6 - 384*x**5/5 + 5040*x**4 + 5120*x**3 + 23
04*x**2 + 512*x
```

Giac [A] time = 1.13712, size = 86, normalized size = 1.13

$$\frac{512}{13}\,x^{13} - 240\,x^{12} + \frac{6936}{11}\,x^{11} - \frac{4527}{10}\,x^{10} - \frac{2936}{3}\,x^{9} + 2097\,x^{8} + \frac{5528}{7}\,x^{7} - 2976\,x^{6} - \frac{384}{5}\,x^{5} + 5040\,x^{4} + 5120\,x^{3} + 230$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^4-15*x^3+8*x^2+24*x+8)^3,x, algorithm="giac")
```

```
[Out] 512/13*x^13 - 240*x^12 + 6936/11*x^11 - 4527/10*x^10 - 2936/3*x^9 + 2097*x^
8 + 5528/7*x^7 - 2976*x^6 - 384/5*x^5 + 5040*x^4 + 5120*x^3 + 2304*x^2 + 51
2*x
```

3.59 $\int \left(8+24x+8x^2-15x^3+8x^4\right)^2 \, dx$

Optimal. Leaf size=52

$$\frac{64x^9}{9}-30x^8+\frac{353x^7}{7}+24x^6-\frac{528x^5}{5}+36x^4+\frac{704x^3}{3}+192x^2+64x$$

```
[Out] 64*x + 192*x^2 + (704*x^3)/3 + 36*x^4 - (528*x^5)/5 + 24*x^6 + (353*x^7)/7
- 30*x^8 + (64*x^9)/9
```

Rubi [A] time = 0.0160692, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {2061}

$$\frac{64x^9}{9}-30x^8+\frac{353x^7}{7}+24x^6-\frac{528x^5}{5}+36x^4+\frac{704x^3}{3}+192x^2+64x$$

Antiderivative was successfully verified.

```
[In] Int[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^2,x]
```

```
[Out] 64*x + 192*x^2 + (704*x^3)/3 + 36*x^4 - (528*x^5)/5 + 24*x^6 + (353*x^7)/7
- 30*x^8 + (64*x^9)/9
```

Rule 2061

```
Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I
GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \left(8+24x+8x^2-15x^3+8x^4\right)^2 \, dx &= \int \left(64+384x+704x^2+144x^3-528x^4+144x^5+353x^6-240x^7+64x^8\right) dx\\ &= 64x+192x^2+\frac{704x^3}{3}+36x^4-\frac{528x^5}{5}+24x^6+\frac{353x^7}{7}-30x^8+\frac{64x^9}{9}\end{aligned}$$

Mathematica [A] time = 0.0014015, size = 52, normalized size = 1.

$$\frac{64x^9}{9}-30x^8+\frac{353x^7}{7}+24x^6-\frac{528x^5}{5}+36x^4+\frac{704x^3}{3}+192x^2+64x$$

Antiderivative was successfully verified.

```
[In] Integrate[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^2,x]
```

```
[Out] 64*x + 192*x^2 + (704*x^3)/3 + 36*x^4 - (528*x^5)/5 + 24*x^6 + (353*x^7)/7
- 30*x^8 + (64*x^9)/9
```

Maple [A] time = 0., size = 45, normalized size = 0.9

$$64\,x+192\,x^{2}+\frac{704\,x^{3}}{3}+36\,x^{4}-\frac{528\,x^{5}}{5}+24\,x^{6}+\frac{353\,x^{7}}{7}-30\,x^{8}+\frac{64\,x^{9}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*x^4-15*x^3+8*x^2+24*x+8)^2,x)
```

```
[Out] 64*x+192*x^2+704/3*x^3+36*x^4-528/5*x^5+24*x^6+353/7*x^7-30*x^8+64/9*x^9
```

Maxima [A] time = 1.33984, size = 59, normalized size = 1.13

$$\frac{64}{9}\,x^{9}-30\,x^{8}+\frac{353}{7}\,x^{7}+24\,x^{6}-\frac{528}{5}\,x^{5}+36\,x^{4}+\frac{704}{3}\,x^{3}+192\,x^{2}+64\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^4-15*x^3+8*x^2+24*x+8)^2,x, algorithm="maxima")
```

```
[Out] 64/9*x^9 - 30*x^8 + 353/7*x^7 + 24*x^6 - 528/5*x^5 + 36*x^4 + 704/3*x^3 + 1
92*x^2 + 64*x
```

Fricas [A] time = 1.33198, size = 122, normalized size = 2.35

$$\frac{64}{9}x^{9}-30x^{8}+\frac{353}{7}x^{7}+24x^{6}-\frac{528}{5}x^{5}+36x^{4}+\frac{704}{3}x^{3}+192x^{2}+64x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^4-15*x^3+8*x^2+24*x+8)^2,x, algorithm="fricas")
```

```
[Out] 64/9*x^9 - 30*x^8 + 353/7*x^7 + 24*x^6 - 528/5*x^5 + 36*x^4 + 704/3*x^3 + 1
92*x^2 + 64*x
```

Sympy [A] time = 0.06325, size = 49, normalized size = 0.94

$$\frac{64x^9}{9} - 30x^8 + \frac{353x^7}{7} + 24x^6 - \frac{528x^5}{5} + 36x^4 + \frac{704x^3}{3} + 192x^2 + 64x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x**4-15*x**3+8*x**2+24*x+8)**2,x)
```

```
[Out] 64*x**9/9 - 30*x**8 + 353*x**7/7 + 24*x**6 - 528*x**5/5 + 36*x**4 + 704*x**
3/3 + 192*x**2 + 64*x
```

Giac [A] time = 1.11621, size = 59, normalized size = 1.13

$$\frac{64}{9}\,x^9 - 30\,x^8 + \frac{353}{7}\,x^7 + 24\,x^6 - \frac{528}{5}\,x^5 + 36\,x^4 + \frac{704}{3}\,x^3 + 192\,x^2 + 64\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^4-15*x^3+8*x^2+24*x+8)^2,x, algorithm="giac")
```

```
[Out] 64/9*x^9 - 30*x^8 + 353/7*x^7 + 24*x^6 - 528/5*x^5 + 36*x^4 + 704/3*x^3 + 1
92*x^2 + 64*x
```

3.60 $\int \left(8+24x+8x^2-15x^3+8x^4\right) dx$

Optimal. Leaf size=30

$$\frac{8x^5}{5}-\frac{15x^4}{4}+\frac{8x^3}{3}+12x^2+8x$$

```
[Out] 8*x + 12*x^2 + (8*x^3)/3 - (15*x^4)/4 + (8*x^5)/5
```

Rubi [A] time = 0.0038845, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\frac{8x^5}{5}-\frac{15x^4}{4}+\frac{8x^3}{3}+12x^2+8x$$

Antiderivative was successfully verified.

```
[In] Int[8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4,x]
```

```
[Out] 8*x + 12*x^2 + (8*x^3)/3 - (15*x^4)/4 + (8*x^5)/5
```

Rubi steps

$$\int \left(8+24x+8x^2-15x^3+8x^4\right) dx = 8x+12x^2+\frac{8x^3}{3}-\frac{15x^4}{4}+\frac{8x^5}{5}$$

Mathematica [A] time = 0.0000704, size = 30, normalized size = 1.

$$\frac{8x^5}{5}-\frac{15x^4}{4}+\frac{8x^3}{3}+12x^2+8x$$

Antiderivative was successfully verified.

```
[In] Integrate[8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4,x]
```

```
[Out] 8*x + 12*x^2 + (8*x^3)/3 - (15*x^4)/4 + (8*x^5)/5
```

Maple [A] time = 0.002, size = 25, normalized size = 0.8

$$8x+12x^2+\frac{8x^3}{3}-\frac{15x^4}{4}+\frac{8x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(8*x^4-15*x^3+8*x^2+24*x+8,x)
```

```
[Out] 8*x+12*x^2+8/3*x^3-15/4*x^4+8/5*x^5
```

Maxima [A] time = 1.19578, size = 32, normalized size = 1.07

$$\frac{8}{5}x^5-\frac{15}{4}x^4+\frac{8}{3}x^3+12x^2+8x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(8*x^4-15*x^3+8*x^2+24*x+8,x, algorithm="maxima")
```

```
[Out] 8/5*x^5 - 15/4*x^4 + 8/3*x^3 + 12*x^2 + 8*x
```

Fricas [A] time = 1.2414, size = 61, normalized size = 2.03

$$\frac{8}{5}x^5-\frac{15}{4}x^4+\frac{8}{3}x^3+12x^2+8x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(8*x^4-15*x^3+8*x^2+24*x+8,x, algorithm="fricas")
```

```
[Out] 8/5*x^5 - 15/4*x^4 + 8/3*x^3 + 12*x^2 + 8*x
```

Sympy [A] time = 0.052436, size = 27, normalized size = 0.9

$$\frac{8x^5}{5}-\frac{15x^4}{4}+\frac{8x^3}{3}+12x^2+8x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(8*x**4-15*x**3+8*x**2+24*x+8,x)
```

```
[Out] 8*x**5/5 - 15*x**4/4 + 8*x**3/3 + 12*x**2 + 8*x
```

Giac [A] time = 1.12765, size = 32, normalized size = 1.07

$$\frac{8}{5}x^5 - \frac{15}{4}x^4 + \frac{8}{3}x^3 + 12x^2 + 8x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(8*x^4-15*x^3+8*x^2+24*x+8,x, algorithm="giac")
```

```
[Out] 8/5*x^5 - 15/4*x^4 + 8/3*x^3 + 12*x^2 + 8*x
```

3.61 $\int \frac{1}{8+24x+8x^2-15x^3+8x^4}\,dx$

Optimal. Leaf size=263

$$\frac{1}{4}\sqrt{\frac{3}{13}}\tan^{-1}\left(\frac{-5x^2+12x+8}{\sqrt{39}x^2}\right)-\frac{1}{8}\sqrt{\frac{235\sqrt{517}-5167}{40326}}\log\left(\left(\frac{4}{x}+3\right)^2-\sqrt{2\left(19+\sqrt{517}\right)}\left(\frac{4}{x}+3\right)+\sqrt{517}\right)+\frac{1}{8}\sqrt{\frac{235\ldots}{}}$$

```
[Out] -(Sqrt[(5167 + 235*Sqrt[517])/40326]*ArcTan[(6 - Sqrt[2*(19 + Sqrt[517])] +
 8/x)/Sqrt[2*(-19 + Sqrt[517])]])/4 - (Sqrt[(5167 + 235*Sqrt[517])/40326]*A
rcTan[(6 + Sqrt[2*(19 + Sqrt[517])] + 8/x)/Sqrt[2*(-19 + Sqrt[517])]])/4 +
(Sqrt[3/13]*ArcTan[(8 + 12*x - 5*x^2)/(Sqrt[39]*x^2)])/4 - (Sqrt[(-5167 + 2
35*Sqrt[517])/40326]*Log[Sqrt[517] - Sqrt[2*(19 + Sqrt[517])]*(3 + 4/x) + (
3 + 4/x)^2])/8 + (Sqrt[(-5167 + 235*Sqrt[517])/40326]*Log[Sqrt[517] + Sqrt[
2*(19 + Sqrt[517])]*(3 + 4/x) + (3 + 4/x)^2])/8
```

Rubi [A] time = 0.492824, antiderivative size = 263, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {2069, 12, 1673, 1169, 634, 618, 204, 628, 1107}

$$\frac{1}{4}\sqrt{\frac{3}{13}}\tan^{-1}\left(\frac{-5x^2+12x+8}{\sqrt{39}x^2}\right)-\frac{1}{8}\sqrt{\frac{235\sqrt{517}-5167}{40326}}\log\left(\left(\frac{4}{x}+3\right)^2-\sqrt{2\left(19+\sqrt{517}\right)}\left(\frac{4}{x}+3\right)+\sqrt{517}\right)+\frac{1}{8}\sqrt{\frac{235\ldots}{}}$$

Antiderivative was successfully verified.

```
[In] Int[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^(-1),x]
```


```
[Out] -(Sqrt[(5167 + 235*Sqrt[517])/40326]*ArcTan[(6 - Sqrt[2*(19 + Sqrt[517])] +
 8/x)/Sqrt[2*(-19 + Sqrt[517])]])/4 - (Sqrt[(5167 + 235*Sqrt[517])/40326]*A
rcTan[(6 + Sqrt[2*(19 + Sqrt[517])] + 8/x)/Sqrt[2*(-19 + Sqrt[517])]])/4 +
(Sqrt[3/13]*ArcTan[(8 + 12*x - 5*x^2)/(Sqrt[39]*x^2)])/4 - (Sqrt[(-5167 + 2
35*Sqrt[517])/40326]*Log[Sqrt[517] - Sqrt[2*(19 + Sqrt[517])]*(3 + 4/x) + (
3 + 4/x)^2])/8 + (Sqrt[(-5167 + 235*Sqrt[517])/40326]*Log[Sqrt[517] + Sqrt[
2*(19 + Sqrt[517])]*(3 + 4/x) + (3 + 4/x)^2])/8
```


Rule 2069

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Dist[-16*
a^2, Subst[Int[(1*((a*(-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 256*a^3*e - 32*a^
```

```
2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4))/(b - 4*a*x)^4)^p)/(b - 4*a*x)^2, x],
x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a^
2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] &&  !IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\int \frac{1}{8+24x+8x^2-15x^3+8x^4}\,dx = -\left(1024\,\mathrm{Subst}\left(\int \frac{(24-32x)^2}{8\left(2117632-2490368x^2+1048576x^4\right)}\,dx, x, \frac{3}{4}+\frac{1}{x}\right)\right)$$

$$= -\left(128\,\mathrm{Subst}\left(\int \frac{(24-32x)^2}{2117632-2490368x^2+1048576x^4}\,dx, x, \frac{3}{4}+\frac{1}{x}\right)\right)$$

$$= -\left(128\,\mathrm{Subst}\left(\int -\frac{1536x}{2117632-2490368x^2+1048576x^4}\,dx, x, \frac{3}{4}+\frac{1}{x}\right)\right) - 128\,\mathrm{Subs}$$

$$= 196608\,\mathrm{Subst}\left(\int \frac{x}{2117632-2490368x^2+1048576x^4}\,dx, x, \frac{3}{4}+\frac{1}{x}\right) - \frac{\mathrm{Subst}\left(\int \frac{1}{-}\right.}{}$$

$$= 98304\,\mathrm{Subst}\left(\int \frac{1}{2117632-2490368x+1048576x^2}\,dx, x, \left(\frac{3}{4}+\frac{1}{x}\right)^2\right) - \frac{\left(517+9\sqrt{\right.}}{}$$

$$= -\frac{1}{8}\sqrt{\frac{-5167+235\sqrt{517}}{40326}}\log\left(\sqrt{517}-\sqrt{2\left(19+\sqrt{517}\right)}\left(3+\frac{4}{x}\right)+\left(3+\frac{4}{x}\right)^2\right)+\frac{1}{8}\sqrt{}$$

$$= -\frac{1}{4}\sqrt{\frac{3}{13}}\tan^{-1}\left(\frac{19-\left(3+\frac{4}{x}\right)^2}{2\sqrt{39}}\right) - \frac{1}{4}\sqrt{\frac{5167+235\sqrt{517}}{40326}}\tan^{-1}\left(\frac{6+\sqrt{2\left(19+\sqrt{51}\right.}}{\sqrt{2\left(-19+\sqrt{51}\right.}}\right.$$

Mathematica [C] time = 0.0096563, size = 55, normalized size = 0.21

$$\mathrm{RootSum}\left[8\#1^4-15\#1^3+8\#1^2+24\#1+8\&, \frac{\log(x-\#1)}{32\#1^3-45\#1^2+16\#1+24}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^(-1),x]
```

```
[Out] RootSum[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , Log[x - #1]/(24 + 16*#1 -
 45*#1^2 + 32*#1^3) & ]
```

Maple [C] time = 0.005, size = 49, normalized size = 0.2

$$\sum_{\_R=\mathrm{RootOf}\left(8\_Z^4-15\_Z^3+8\_Z^2+24\_Z+8\right)} \frac{\ln(x-\_R)}{32\_R^3-45\_R^2+16\_R+24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(8*x^4-15*x^3+8*x^2+24*x+8),x)
```

```
[Out] sum(1/(32*_R^3-45*_R^2+16*_R+24)*ln(x-_R),_R=RootOf(8*_Z^4-15*_Z^3+8*_Z^2+2
4*_Z+8))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{8x^4-15x^3+8x^2+24x+8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8),x, algorithm="maxima")
```

```
[Out] integrate(1/(8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 0.851635, size = 41, normalized size = 0.16

$$\operatorname{RootSum}\left(50326848t^4 + 765960t^2 + 12753t + 64, \left(t \mapsto t\log\left(\frac{100785893208t^3}{4758335} - \frac{1430512512t^2}{4758335} + \frac{72982352521t}{223641745} + x\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x**4-15*x**3+8*x**2+24*x+8),x)
```

```
[Out] RootSum(50326848*_t**4 + 765960*_t**2 + 12753*_t + 64, Lambda(_t, _t*log(10
0785893208*_t**3/4758335 - 1430512512*_t**2/4758335 + 72982352521*_t/223641
745 + x + 2270349121/1789133960)))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{8\,x^4 - 15\,x^3 + 8\,x^2 + 24\,x + 8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8),x, algorithm="giac")
```

```
[Out] integrate(1/(8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8), x)
```

3.62 $\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^2}\,dx$

Optimal. Leaf size=366

$$\frac{73}{208}\sqrt{\frac{3}{13}}\tan^{-1}\left(\frac{-5x^2+12x+8}{\sqrt{39}x^2}\right)-\frac{3\left(3359-107\left(\frac{4}{x}+3\right)^2\right)}{208\left(\left(\frac{4}{x}+3\right)^4-38\left(\frac{4}{x}+3\right)^2+517\right)}+\frac{\left(3327931-129631\left(\frac{4}{x}+3\right)^2\right)\left(\frac{4}{x}+3\right)}{322608\left(\left(\frac{4}{x}+3\right)^4-38\left(\frac{4}{x}+3\right)^2+517\right)}-\frac{\sqrt{}}{}$$

```
[Out] (-3*(3359 - 107*(3 + 4/x)^2))/(208*(517 - 38*(3 + 4/x)^2 + (3 + 4/x)^4)) +
((3327931 - 129631*(3 + 4/x)^2)*(3 + 4/x))/(322608*(517 - 38*(3 + 4/x)^2 +
(3 + 4/x)^4)) - (Sqrt[(19 + Sqrt[517])/40326]*(1678181 + 74897*Sqrt[517])*A
rcTan[(6 - Sqrt[2*(19 + Sqrt[517])] + 8/x)/Sqrt[2*(-19 + Sqrt[517])]])/6452
16 - (Sqrt[(19 + Sqrt[517])/40326]*(1678181 + 74897*Sqrt[517])*ArcTan[(6 +
Sqrt[2*(19 + Sqrt[517])] + 8/x)/Sqrt[2*(-19 + Sqrt[517])]])/645216 + (73*Sq
rt[3/13]*ArcTan[(8 + 12*x - 5*x^2)/(Sqrt[39]*x^2)])/208 - (Sqrt[(-596441146
71451 + 2623170438295*Sqrt[517])/40326]*Log[Sqrt[517] - Sqrt[2*(19 + Sqrt[5
17])]*(3 + 4/x) + (3 + 4/x)^2])/645216 + (Sqrt[(-59644114671451 + 262317043
8295*Sqrt[517])/40326]*Log[Sqrt[517] + Sqrt[2*(19 + Sqrt[517])]*(3 + 4/x) +
 (3 + 4/x)^2])/645216
```

Rubi [A] time = 0.507629, antiderivative size = 366, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2069, 12, 1673, 1678, 1169, 634, 618, 204, 628, 1663, 1660}

$$\frac{73}{208}\sqrt{\frac{3}{13}}\tan^{-1}\left(\frac{-5x^2+12x+8}{\sqrt{39}x^2}\right)-\frac{3\left(3359-107\left(\frac{4}{x}+3\right)^2\right)}{208\left(\left(\frac{4}{x}+3\right)^4-38\left(\frac{4}{x}+3\right)^2+517\right)}+\frac{\left(3327931-129631\left(\frac{4}{x}+3\right)^2\right)\left(\frac{4}{x}+3\right)}{322608\left(\left(\frac{4}{x}+3\right)^4-38\left(\frac{4}{x}+3\right)^2+517\right)}-\frac{\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^(-2),x]
```


```
[Out] (-3*(3359 - 107*(3 + 4/x)^2))/(208*(517 - 38*(3 + 4/x)^2 + (3 + 4/x)^4)) +
((3327931 - 129631*(3 + 4/x)^2)*(3 + 4/x))/(322608*(517 - 38*(3 + 4/x)^2 +
(3 + 4/x)^4)) - (Sqrt[(19 + Sqrt[517])/40326]*(1678181 + 74897*Sqrt[517])*A
rcTan[(6 - Sqrt[2*(19 + Sqrt[517])] + 8/x)/Sqrt[2*(-19 + Sqrt[517])]])/6452
16 - (Sqrt[(19 + Sqrt[517])/40326]*(1678181 + 74897*Sqrt[517])*ArcTan[(6 +
```

```
Sqrt[2*(19 + Sqrt[517])] + 8/x)/Sqrt[2*(-19 + Sqrt[517])]])/645216 + (73*Sq
rt[3/13]*ArcTan[(8 + 12*x - 5*x^2)/(Sqrt[39]*x^2)])/208 - (Sqrt[(-596441146
71451 + 2623170438295*Sqrt[517])/40326]*Log[Sqrt[517] - Sqrt[2*(19 + Sqrt[5
17])]*(3 + 4/x) + (3 + 4/x)^2])/645216 + (Sqrt[(-59644114671451 + 262317043
8295*Sqrt[517])/40326]*Log[Sqrt[517] + Sqrt[2*(19 + Sqrt[517])]*(3 + 4/x) +
 (3 + 4/x)^2])/645216
```

Rule 2069

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Dist[-16*
a^2, Subst[Int[(1*((a*(-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 256*a^3*e - 32*a^
2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4))/(b - 4*a*x)^4)^p)/(b - 4*a*x)^2, x],
x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a^
2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] &&  !IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
```

[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
```

```
- 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(8+24x+8x^2-15x^3+8x^4\right)^2}\,dx = -\left(1024\,\text{Subst}\left(\int \frac{(24-32x)^6}{64\left(2117632-2490368x^2+1048576x^4\right)^2}\,dx, x, \frac{3}{4}+\frac{1}{x}\right)\right)$$

$$= -\left(16\,\text{Subst}\left(\int \frac{(24-32x)^6}{\left(2117632-2490368x^2+1048576x^4\right)^2}\,dx, x, \frac{3}{4}+\frac{1}{x}\right)\right)$$

$$= -\left(16\,\text{Subst}\left(\int \frac{x\left(-1528823808-9059696640x^2-4831838208x^4\right)}{\left(2117632-2490368x^2+1048576x^4\right)^2}\,dx, x, \frac{3}{4}+\right.\right.$$

$$= \frac{\left(3327931-129631\left(3+\frac{4}{x}\right)^2\right)\left(3+\frac{4}{x}\right)}{322608\left(517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4\right)} - \frac{\text{Subst}\left(\int \frac{1209256852201639415644416+8635}{2117632-2490368x^2}\right.}{70942249442}$$

$$= -\frac{3\left(3359-107\left(3+\frac{4}{x}\right)^2\right)}{208\left(517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4\right)} + \frac{\left(3327931-129631\left(3+\frac{4}{x}\right)^2\right)\left(3+\frac{4}{x}\right)}{322608\left(517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4\right)}$$

$$= -\frac{3\left(3359-107\left(3+\frac{4}{x}\right)^2\right)}{208\left(517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4\right)} + \frac{\left(3327931-129631\left(3+\frac{4}{x}\right)^2\right)\left(3+\frac{4}{x}\right)}{322608\left(517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4\right)}$$

$$= -\frac{3\left(3359-107\left(3+\frac{4}{x}\right)^2\right)}{208\left(517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4\right)} + \frac{\left(3327931-129631\left(3+\frac{4}{x}\right)^2\right)\left(3+\frac{4}{x}\right)}{322608\left(517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4\right)}$$

$$= -\frac{3\left(3359-107\left(3+\frac{4}{x}\right)^2\right)}{208\left(517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4\right)} + \frac{\left(3327931-129631\left(3+\frac{4}{x}\right)^2\right)\left(3+\frac{4}{x}\right)}{322608\left(517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4\right)}$$

Mathematica [C] time = 0.0185367, size = 128, normalized size = 0.35

$$\frac{\text{RootSum}\left[8\#1^4-15\#1^3+8\#1^2+24\#1+8\&, \frac{19640\#1^2\log(x-\#1)-57489\#1\log(x-\#1)+74897\log(x-\#1)}{32\#1^3-45\#1^2+16\#1+24}\&\right]}{80652} + \frac{39280x^3-94314x^2}{161304\left(8x^4-15x^3\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^(-2),x]
```

```
[Out] (72888 + 89033*x - 94314*x^2 + 39280*x^3)/(161304*(8 + 24*x + 8*x^2 - 15*x^
3 + 8*x^4)) + RootSum[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , (74897*Log[
x - #1] - 57489*Log[x - #1]*#1 + 19640*Log[x - #1]*#1^2)/(24 + 16*#1 - 45*#
1^2 + 32*#1^3) & ]/80652
```

Maple [C] time = 0.007, size = 96, normalized size = 0.3

$$\left(\frac{2455x^3}{80652}-\frac{1429x^2}{19552}+\frac{89033x}{1290432}+\frac{3037}{53768}\right)\left(x^4-\frac{15x^3}{8}+x^2+3x+1\right)^{-1}+\frac{1}{80652}\sum_{\_R=\mathrm{RootOf}\left(8\_Z^4-15\_Z^3+8\_Z^2+24\_Z+8\right)}\frac{(19$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(8*x^4-15*x^3+8*x^2+24*x+8)^2,x)
```

```
[Out] (2455/80652*x^3-1429/19552*x^2+89033/1290432*x+3037/53768)/(x^4-15/8*x^3+x^
2+3*x+1)+1/80652*sum((19640*_R^2-57489*_R+74897)/(32*_R^3-45*_R^2+16*_R+24)
*ln(x-_R),_R=RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{39280x^3-94314x^2+89033x+72888}{161304\left(8x^4-15x^3+8x^2+24x+8\right)}+\frac{1}{80652}\int\frac{19640x^2-57489x+74897}{8x^4-15x^3+8x^2+24x+8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8)^2,x, algorithm="maxima")
```

```
[Out] 1/161304*(39280*x^3 - 94314*x^2 + 89033*x + 72888)/(8*x^4 - 15*x^3 + 8*x^2
+ 24*x + 8) + 1/80652*integrate((19640*x^2 - 57489*x + 74897)/(8*x^4 - 15*x
^3 + 8*x^2 + 24*x + 8), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 0.956513, size = 76, normalized size = 0.21

$$\frac{39280x^3 - 94314x^2 + 89033x + 72888}{1290432x^4 - 2419560x^3 + 1290432x^2 + 3871296x + 1290432} + \mathrm{RootSum}\Big(19916784274892443361 28t^4 + 566107340$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x**4-15*x**3+8*x**2+24*x+8)**2,x)
```

```
[Out] (39280*x**3 - 94314*x**2 + 89033*x + 72888)/(1290432*x**4 - 2419560*x**3 +
1290432*x**2 + 3871296*x + 1290432) + RootSum(1991678427489244336128*_t**4
+ 56610734087162189376*_t**2 + 20948104645409331*_t + 1938464112640, Lambda
(_t, _t*log(-705077742393966388453254545830232274432*_t**3/5031017713433135
9960511301071755 + 126981475823989945260152267904580608*_t**2/5031017713433
1359960511301071755 - 20040865325746858989799932658629535256*_t/50310177134
331359960511301071755 + x - 18148095975820500157416495488749859/24148885024
4790527810454245144424)))
```

Giac [F] time = 0., size = 0, normalized size = 0.

undef

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8)^2,x, algorithm="giac")
```

```
[Out] undef
```

3.63 $\int \left(a^5+5 a^4 b x+10 a^3 b^2 x^2+10 a^2 b^3 x^3+5 a b^4 x^4+b^5 x^5\right)^3 \, dx$

Optimal. Leaf size=14

$$\frac{(a+bx)^{16}}{16b}$$

```
[Out] (a + b*x)^16/(16*b)
```

Rubi [A] time = 0.0169574, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.039, Rules used = {2059, 32}

$$\frac{(a+bx)^{16}}{16b}$$

Antiderivative was successfully verified.

```
[In] Int[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4 + b^5*
x^5)^3,x]
```

```
[Out] (a + b*x)^16/(16*b)
```

Rule 2059

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[u^p, x] /;  !SumQ[Non
freeFactors[u, x]]] /; PolyQ[P, x] && IntegerQ[p]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \left(a^5+5 a^4 b x+10 a^3 b^2 x^2+10 a^2 b^3 x^3+5 a b^4 x^4+b^5 x^5\right)^3 \, dx &= \int (a+bx)^{15} \, dx \\ &= \frac{(a+bx)^{16}}{16b} \end{aligned}$$

Mathematica [A] time = 0.0013318, size = 14, normalized size = 1.

$$\frac{(a+bx)^{16}}{16b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4
+ b^5*x^5)^3,x]
```

```
[Out] (a + b*x)^16/(16*b)
```

Maple [B] time = 0.001, size = 164, normalized size = 11.7

$$\frac{b^{15}x^{16}}{16}+ab^{14}x^{15}+\frac{15\,a^2b^{13}x^{14}}{2}+35\,a^3b^{12}x^{13}+\frac{455\,a^4b^{11}x^{12}}{4}+273\,a^5b^{10}x^{11}+\frac{1001\,a^6b^9x^{10}}{2}+715\,a^7b^8x^9+\frac{6435\,a^8b^7}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5)^3,x)
```

```
[Out] 1/16*b^15*x^16+a*b^14*x^15+15/2*a^2*b^13*x^14+35*a^3*b^12*x^13+455/4*a^4*b^
11*x^12+273*a^5*b^10*x^11+1001/2*a^6*b^9*x^10+715*a^7*b^8*x^9+6435/8*a^8*b^
7*x^8+715*a^9*b^6*x^7+1001/2*a^10*b^5*x^6+273*a^11*b^4*x^5+455/4*a^12*b^3*x
^4+35*a^13*b^2*x^3+15/2*a^14*b*x^2+a^15*x
```

Maxima [B] time = 1.19535, size = 799, normalized size = 57.07

$$\frac{1}{16}\,b^{15}x^{16}+ab^{14}x^{15}+\frac{75}{14}\,a^2b^{13}x^{14}+\frac{125}{13}\,a^3b^{12}x^{13}+100\,a^6b^9x^{10}+\frac{1000}{7}\,a^9b^6x^7+\frac{125}{4}\,a^{12}b^3x^4+a^{15}x+\frac{1}{2}\left(b^5x^6+6\,ab^4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5)
^3,x, algorithm="maxima")
```

```
[Out] 1/16*b^15*x^16 + a*b^14*x^15 + 75/14*a^2*b^13*x^14 + 125/13*a^3*b^12*x^13 +
 100*a^6*b^9*x^10 + 1000/7*a^9*b^6*x^7 + 125/4*a^12*b^3*x^4 + a^15*x + 1/2*
(b^5*x^6 + 6*a*b^4*x^5 + 15*a^2*b^3*x^4 + 20*a^3*b^2*x^3 + 15*a^4*b*x^2)*a^
10 + 25/56*(21*b^5*x^8 + 120*a*b^4*x^7 + 280*a^2*b^3*x^6 + 336*a^3*b^2*x^5)
```

```
*a^8*b^2 + 5/3*(18*b^5*x^10 + 100*a*b^4*x^9 + 225*a^2*b^3*x^8)*a^6*b^4 + 25
/11*(11*b^5*x^12 + 60*a*b^4*x^11)*a^4*b^6 + 1/462*(126*b^10*x^11 + 1386*a*b
^9*x^10 + 3850*a^2*b^8*x^9 + 19800*a^4*b^6*x^7 + 27720*a^6*b^4*x^5 + 11550*
a^8*b^2*x^3 + 330*(6*b^5*x^7 + 35*a*b^4*x^6 + 84*a^2*b^3*x^5 + 105*a^3*b^2*
x^4)*a^4*b + 165*(21*b^5*x^8 + 120*a*b^4*x^7 + 280*a^2*b^3*x^6)*a^3*b^2 + 3
85*(8*b^5*x^9 + 45*a*b^4*x^8)*a^2*b^3)*a^5 + 5/308*(77*b^10*x^12 + 840*a*b^
9*x^11 + 4158*a^2*b^8*x^10 + 12320*a^3*b^7*x^9 + 23100*a^4*b^6*x^8 + 26400*
a^5*b^5*x^7 + 15400*a^6*b^4*x^6)*a^4*b + 5/429*(198*b^10*x^13 + 2145*a*b^9*
x^12 + 10530*a^2*b^8*x^11 + 25740*a^3*b^7*x^10 + 28600*a^4*b^6*x^9)*a^3*b^2
 + 5/182*(78*b^10*x^14 + 840*a*b^9*x^13 + 2275*a^2*b^8*x^12)*a^2*b^3
```

Fricas [B] time = 1.59064, size = 402, normalized size = 28.71

$$\frac{1}{16}x^{16}b^{15} + x^{15}b^{14}a + \frac{15}{2}x^{14}b^{13}a^2 + 35x^{13}b^{12}a^3 + \frac{455}{4}x^{12}b^{11}a^4 + 273x^{11}b^{10}a^5 + \frac{1001}{2}x^{10}b^9a^6 + 715x^9b^8a^7 + \frac{6435}{8}x^8b^7a^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5)
^3,x, algorithm="fricas")
```

```
[Out] 1/16*x^16*b^15 + x^15*b^14*a + 15/2*x^14*b^13*a^2 + 35*x^13*b^12*a^3 + 455/
4*x^12*b^11*a^4 + 273*x^11*b^10*a^5 + 1001/2*x^10*b^9*a^6 + 715*x^9*b^8*a^7
 + 6435/8*x^8*b^7*a^8 + 715*x^7*b^6*a^9 + 1001/2*x^6*b^5*a^10 + 273*x^5*b^4
*a^11 + 455/4*x^4*b^3*a^12 + 35*x^3*b^2*a^13 + 15/2*x^2*b*a^14 + x*a^15
```

Sympy [B] time = 0.105733, size = 185, normalized size = 13.21

$$a^{15}x + \frac{15a^{14}bx^2}{2} + 35a^{13}b^2x^3 + \frac{455a^{12}b^3x^4}{4} + 273a^{11}b^4x^5 + \frac{1001a^{10}b^5x^6}{2} + 715a^9b^6x^7 + \frac{6435a^8b^7x^8}{8} + 715a^7b^8x^9 + \frac{100}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**5*x**5+5*a*b**4*x**4+10*a**2*b**3*x**3+10*a**3*b**2*x**2+5*a*
*4*b*x+a**5)**3,x)
```

```
[Out] a**15*x + 15*a**14*b*x**2/2 + 35*a**13*b**2*x**3 + 455*a**12*b**3*x**4/4 +
273*a**11*b**4*x**5 + 1001*a**10*b**5*x**6/2 + 715*a**9*b**6*x**7 + 6435*a*
*8*b**7*x**8/8 + 715*a**7*b**8*x**9 + 1001*a**6*b**9*x**10/2 + 273*a**5*b**
10*x**11 + 455*a**4*b**11*x**12/4 + 35*a**3*b**12*x**13 + 15*a**2*b**13*x**
```

```
14/2 + a*b**14*x**15 + b**15*x**16/16
```

Giac [B] time = 1.14015, size = 220, normalized size = 15.71

$$\frac{1}{16}\,b^{15}x^{16} + ab^{14}x^{15} + \frac{15}{2}\,a^{2}b^{13}x^{14} + 35\,a^{3}b^{12}x^{13} + \frac{455}{4}\,a^{4}b^{11}x^{12} + 273\,a^{5}b^{10}x^{11} + \frac{1001}{2}\,a^{6}b^{9}x^{10} + 715\,a^{7}b^{8}x^{9} + \frac{6435}{8}\,a^{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5)
^3,x, algorithm="giac")
```

```
[Out] 1/16*b^15*x^16 + a*b^14*x^15 + 15/2*a^2*b^13*x^14 + 35*a^3*b^12*x^13 + 455/
4*a^4*b^11*x^12 + 273*a^5*b^10*x^11 + 1001/2*a^6*b^9*x^10 + 715*a^7*b^8*x^9
 + 6435/8*a^8*b^7*x^8 + 715*a^9*b^6*x^7 + 1001/2*a^10*b^5*x^6 + 273*a^11*b^
4*x^5 + 455/4*a^12*b^3*x^4 + 35*a^13*b^2*x^3 + 15/2*a^14*b*x^2 + a^15*x
```

3.64 $\int \left(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5\right)^2 \, dx$

Optimal. Leaf size=14

$$\frac{(a+bx)^{11}}{11b}$$

```
[Out] (a + b*x)^11/(11*b)
```

Rubi [A] time = 0.017942, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.039, Rules used = {2059, 32}

$$\frac{(a+bx)^{11}}{11b}$$

Antiderivative was successfully verified.

```
[In] Int[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4 + b^5*
x^5)^2,x]
```

```
[Out] (a + b*x)^11/(11*b)
```

Rule 2059

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[u^p, x] /;  !SumQ[Non
freeFactors[u, x]]] /; PolyQ[P, x] && IntegerQ[p]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \left(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5\right)^2 \, dx &= \int (a+bx)^{10} \, dx\\ &= \frac{(a+bx)^{11}}{11b}\end{aligned}$$

Mathematica [A] time = 0.0014514, size = 14, normalized size = 1.

$$\frac{(a+bx)^{11}}{11b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4
+ b^5*x^5)^2,x]
```

```
[Out] (a + b*x)^11/(11*b)
```

Maple [B] time = 0.001, size = 109, normalized size = 7.8

$$\frac{b^{10}x^{11}}{11}+ab^9x^{10}+5a^2b^8x^9+15a^3b^7x^8+30a^4b^6x^7+42a^5b^5x^6+42a^6b^4x^5+30a^7b^3x^4+15a^8b^2x^3+5a^9bx^2+a^{10}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5)^2,x)
```

```
[Out] 1/11*b^10*x^11+a*b^9*x^10+5*a^2*b^8*x^9+15*a^3*b^7*x^8+30*a^4*b^6*x^7+42*a^
5*b^5*x^6+42*a^6*b^4*x^5+30*a^7*b^3*x^4+15*a^8*b^2*x^3+5*a^9*b*x^2+a^10*x
```

Maxima [B] time = 1.15947, size = 308, normalized size = 22.

$$\frac{1}{11}b^{10}x^{11}+ab^9x^{10}+\frac{25}{9}a^2b^8x^9+\frac{100}{7}a^4b^6x^7+20a^6b^4x^5+\frac{25}{3}a^8b^2x^3+a^{10}x+\frac{1}{3}\left(b^5x^6+6ab^4x^5+15a^2b^3x^4+20a^3b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5)
^2,x, algorithm="maxima")
```

```
[Out] 1/11*b^10*x^11 + a*b^9*x^10 + 25/9*a^2*b^8*x^9 + 100/7*a^4*b^6*x^7 + 20*a^6
*b^4*x^5 + 25/3*a^8*b^2*x^3 + a^10*x + 1/3*(b^5*x^6 + 6*a*b^4*x^5 + 15*a^2*
b^3*x^4 + 20*a^3*b^2*x^3 + 15*a^4*b*x^2)*a^5 + 5/21*(6*b^5*x^7 + 35*a*b^4*x
^6 + 84*a^2*b^3*x^5 + 105*a^3*b^2*x^4)*a^4*b + 5/42*(21*b^5*x^8 + 120*a*b^4
*x^7 + 280*a^2*b^3*x^6)*a^3*b^2 + 5/18*(8*b^5*x^9 + 45*a*b^4*x^8)*a^2*b^3
```

Fricas [B] time = 1.53508, size = 230, normalized size = 16.43

$$\frac{1}{11}x^{11}b^{10} + x^{10}b^9a + 5x^9b^8a^2 + 15x^8b^7a^3 + 30x^7b^6a^4 + 42x^6b^5a^5 + 42x^5b^4a^6 + 30x^4b^3a^7 + 15x^3b^2a^8 + 5x^2ba^9 + xa^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5)
^2,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*b^10 + x^10*b^9*a + 5*x^9*b^8*a^2 + 15*x^8*b^7*a^3 + 30*x^7*b^6*a
^4 + 42*x^6*b^5*a^5 + 42*x^5*b^4*a^6 + 30*x^4*b^3*a^7 + 15*x^3*b^2*a^8 + 5*
x^2*b*a^9 + x*a^10
```

Sympy [B] time = 0.086512, size = 114, normalized size = 8.14

$$a^{10}x + 5a^9bx^2 + 15a^8b^2x^3 + 30a^7b^3x^4 + 42a^6b^4x^5 + 42a^5b^5x^6 + 30a^4b^6x^7 + 15a^3b^7x^8 + 5a^2b^8x^9 + ab^9x^{10} + \frac{b^{10}x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**5*x**5+5*a*b**4*x**4+10*a**2*b**3*x**3+10*a**3*b**2*x**2+5*a*
*4*b*x+a**5)**2,x)
```

```
[Out] a**10*x + 5*a**9*b*x**2 + 15*a**8*b**2*x**3 + 30*a**7*b**3*x**4 + 42*a**6*b
**4*x**5 + 42*a**5*b**5*x**6 + 30*a**4*b**6*x**7 + 15*a**3*b**7*x**8 + 5*a*
*2*b**8*x**9 + a*b**9*x**10 + b**10*x**11/11
```

Giac [B] time = 1.10229, size = 146, normalized size = 10.43

$$\frac{1}{11}b^{10}x^{11} + ab^9x^{10} + 5a^2b^8x^9 + 15a^3b^7x^8 + 30a^4b^6x^7 + 42a^5b^5x^6 + 42a^6b^4x^5 + 30a^7b^3x^4 + 15a^8b^2x^3 + 5a^9bx^2 + a^{10}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5)
^2,x, algorithm="giac")
```

```
[Out] 1/11*b^10*x^11 + a*b^9*x^10 + 5*a^2*b^8*x^9 + 15*a^3*b^7*x^8 + 30*a^4*b^6*x
^7 + 42*a^5*b^5*x^6 + 42*a^6*b^4*x^5 + 30*a^7*b^3*x^4 + 15*a^8*b^2*x^3 + 5*
a^9*b*x^2 + a^10*x
```

3.65 $\int \left(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5\right)\,dx$

Optimal. Leaf size=14

$$\frac{(a+bx)^6}{6b}$$

```
[Out] (a + b*x)^6/(6*b)
```

Rubi [B] time = 0.014024, antiderivative size = 61, normalized size of antiderivative = 4.36, number of steps used = 1, number of rules used = 0, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\frac{5}{2}a^2b^3x^4+\frac{10}{3}a^3b^2x^3+\frac{5}{2}a^4bx^2+a^5x+ab^4x^5+\frac{b^5x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4 + b^5*x
^5,x]
```

```
[Out] a^5*x + (5*a^4*b*x^2)/2 + (10*a^3*b^2*x^3)/3 + (5*a^2*b^3*x^4)/2 + a*b^4*x^
5 + (b^5*x^6)/6
```

Rubi steps

$$\int \left(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5\right)\,dx=a^5x+\frac{5}{2}a^4bx^2+\frac{10}{3}a^3b^2x^3+\frac{5}{2}a^2b^3x^4+ab^4x^5+\frac{b^5x^6}{6}$$

Mathematica [B] time = 0.0000577, size = 61, normalized size = 4.36

$$\frac{5}{2}a^2b^3x^4+\frac{10}{3}a^3b^2x^3+\frac{5}{2}a^4bx^2+a^5x+ab^4x^5+\frac{b^5x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4 +
 b^5*x^5,x]
```

```
[Out] a^5*x + (5*a^4*b*x^2)/2 + (10*a^3*b^2*x^3)/3 + (5*a^2*b^3*x^4)/2 + a*b^4*x^
5 + (b^5*x^6)/6
```

Maple [B] time = 0.001, size = 54, normalized size = 3.9

$$a^5x + \frac{5\,a^4bx^2}{2} + \frac{10\,a^3b^2x^3}{3} + \frac{5\,a^2b^3x^4}{2} + ab^4x^5 + \frac{b^5x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5,x)
```

```
[Out] a^5*x+5/2*a^4*b*x^2+10/3*a^3*b^2*x^3+5/2*a^2*b^3*x^4+a*b^4*x^5+1/6*b^5*x^6
```

Maxima [B] time = 1.17789, size = 72, normalized size = 5.14

$$\frac{1}{6}\,b^5x^6 + ab^4x^5 + \frac{5}{2}\,a^2b^3x^4 + \frac{10}{3}\,a^3b^2x^3 + \frac{5}{2}\,a^4bx^2 + a^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5,x
, algorithm="maxima")
```

```
[Out] 1/6*b^5*x^6 + a*b^4*x^5 + 5/2*a^2*b^3*x^4 + 10/3*a^3*b^2*x^3 + 5/2*a^4*b*x^
2 + a^5*x
```

Fricas [B] time = 1.48361, size = 116, normalized size = 8.29

$$\frac{1}{6}x^6b^5 + x^5b^4a + \frac{5}{2}x^4b^3a^2 + \frac{10}{3}x^3b^2a^3 + \frac{5}{2}x^2ba^4 + xa^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5,x
, algorithm="fricas")
```

```
[Out] 1/6*x^6*b^5 + x^5*b^4*a + 5/2*x^4*b^3*a^2 + 10/3*x^3*b^2*a^3 + 5/2*x^2*b*a^
4 + x*a^5
```

Sympy [B] time = 0.070587, size = 60, normalized size = 4.29

$$a^5x + \frac{5a^4bx^2}{2} + \frac{10a^3b^2x^3}{3} + \frac{5a^2b^3x^4}{2} + ab^4x^5 + \frac{b^5x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b**5*x**5+5*a*b**4*x**4+10*a**2*b**3*x**3+10*a**3*b**2*x**2+5*a**
4*b*x+a**5,x)
```

```
[Out] a**5*x + 5*a**4*b*x**2/2 + 10*a**3*b**2*x**3/3 + 5*a**2*b**3*x**4/2 + a*b**
4*x**5 + b**5*x**6/6
```

Giac [B] time = 1.1179, size = 72, normalized size = 5.14

$$\frac{1}{6}\,b^5x^6 + ab^4x^5 + \frac{5}{2}\,a^2b^3x^4 + \frac{10}{3}\,a^3b^2x^3 + \frac{5}{2}\,a^4bx^2 + a^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5,x
, algorithm="giac")
```

```
[Out] 1/6*b^5*x^6 + a*b^4*x^5 + 5/2*a^2*b^3*x^4 + 10/3*a^3*b^2*x^3 + 5/2*a^4*b*x^
2 + a^5*x
```

3.66 $\int \frac{1}{a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5}\,dx$

Optimal. Leaf size=14

$$-\frac{1}{4b(a+bx)^4}$$

```
[Out] -1/(4*b*(a + b*x)^4)
```

Rubi [A] time = 0.0176062, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.039, Rules used = {2058, 32}

$$-\frac{1}{4b(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4 + b^5*
x^5)^(-1),x]
```

```
[Out] -1/(4*b*(a + b*x)^4)
```

Rule 2058

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5}\,dx &= \int \frac{1}{(a+bx)^5}\,dx\\ &= -\frac{1}{4b(a+bx)^4}\end{aligned}$$

Mathematica [A] time = 0.0037967, size = 14, normalized size = 1.

$$-\frac{1}{4b(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4
+ b^5*x^5)^(-1),x]
```

```
[Out] -1/(4*b*(a + b*x)^4)
```

Maple [A] time = 0.004, size = 13, normalized size = 0.9

$$-\frac{1}{4\,b\,(bx+a)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5),x)
```

```
[Out] -1/4/b/(b*x+a)^4
```

Maxima [B] time = 1.20622, size = 62, normalized size = 4.43

$$-\frac{1}{4\left(b^5x^4+4\,ab^4x^3+6\,a^2b^3x^2+4\,a^3b^2x+a^4b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^
5),x, algorithm="maxima")
```

```
[Out] -1/4/(b^5*x^4 + 4*a*b^4*x^3 + 6*a^2*b^3*x^2 + 4*a^3*b^2*x + a^4*b)
```

Fricas [B] time = 1.69806, size = 92, normalized size = 6.57

$$-\frac{1}{4\left(b^5x^4+4\,ab^4x^3+6\,a^2b^3x^2+4\,a^3b^2x+a^4b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^
5),x, algorithm="fricas")
```

```
[Out] -1/4/(b^5*x^4 + 4*a*b^4*x^3 + 6*a^2*b^3*x^2 + 4*a^3*b^2*x + a^4*b)
```

Sympy [B] time = 0.443923, size = 49, normalized size = 3.5

$$-\frac{1}{4a^4b + 16a^3b^2x + 24a^2b^3x^2 + 16ab^4x^3 + 4b^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**5*x**5+5*a*b**4*x**4+10*a**2*b**3*x**3+10*a**3*b**2*x**2+5*
a**4*b*x+a**5),x)
```

```
[Out] -1/(4*a**4*b + 16*a**3*b**2*x + 24*a**2*b**3*x**2 + 16*a*b**4*x**3 + 4*b**5
*x**4)
```

Giac [A] time = 1.09632, size = 16, normalized size = 1.14

$$-\frac{1}{4\,(bx+a)^4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^
5),x, algorithm="giac")
```

```
[Out] -1/4/((b*x + a)^4*b)
```

3.67 $\int \frac{1}{(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5)^2}\,dx$

Optimal. Leaf size=14

$$-\frac{1}{9b(a+bx)^9}$$

```
[Out] -1/(9*b*(a + b*x)^9)
```

Rubi [A] time = 0.0167872, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.039, Rules used = {2058, 32}

$$-\frac{1}{9b(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4 + b^5*
x^5)^(-2),x]
```

```
[Out] -1/(9*b*(a + b*x)^9)
```

Rule 2058

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5\right)^2}\,dx &= \int \frac{1}{(a+bx)^{10}}\,dx\\ &= -\frac{1}{9b(a+bx)^9}\end{aligned}$$

Mathematica [A] time = 0.0035355, size = 14, normalized size = 1.

$$-\frac{1}{9b(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4
+ b^5*x^5)^(-2),x]
```

```
[Out] -1/(9*b*(a + b*x)^9)
```

Maple [A] time = 0.003, size = 13, normalized size = 0.9

$$-\frac{1}{9\,b\,(bx+a)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5)^2,x
)
```

```
[Out] -1/9/b/(b*x+a)^9
```

Maxima [B] time = 1.26305, size = 136, normalized size = 9.71

$$-\frac{1}{9\left(b^{10}x^9+9\,ab^9x^8+36\,a^2b^8x^7+84\,a^3b^7x^6+126\,a^4b^6x^5+126\,a^5b^5x^4+84\,a^6b^4x^3+36\,a^7b^3x^2+9\,a^8b^2x+a^9b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^
5)^2,x, algorithm="maxima")
```

```
[Out] -1/9/(b^10*x^9 + 9*a*b^9*x^8 + 36*a^2*b^8*x^7 + 84*a^3*b^7*x^6 + 126*a^4*b^
6*x^5 + 126*a^5*b^5*x^4 + 84*a^6*b^4*x^3 + 36*a^7*b^3*x^2 + 9*a^8*b^2*x + a
^9*b)
```

Fricas [B] time = 1.62422, size = 212, normalized size = 15.14

$$-\frac{1}{9\left(b^{10}x^9+9\,ab^9x^8+36\,a^2b^8x^7+84\,a^3b^7x^6+126\,a^4b^6x^5+126\,a^5b^5x^4+84\,a^6b^4x^3+36\,a^7b^3x^2+9\,a^8b^2x+a^9b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^
5)^2,x, algorithm="fricas")
```

```
[Out] -1/9/(b^10*x^9 + 9*a*b^9*x^8 + 36*a^2*b^8*x^7 + 84*a^3*b^7*x^6 + 126*a^4*b^
6*x^5 + 126*a^5*b^5*x^4 + 84*a^6*b^4*x^3 + 36*a^7*b^3*x^2 + 9*a^8*b^2*x + a
^9*b)
```

Sympy [B] time = 0.840877, size = 109, normalized size = 7.79

$$-\frac{1}{9a^9b+81a^8b^2x+324a^7b^3x^2+756a^6b^4x^3+1134a^5b^5x^4+1134a^4b^6x^5+756a^3b^7x^6+324a^2b^8x^7+81ab^9x^8+9b^{10}x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**5*x**5+5*a*b**4*x**4+10*a**2*b**3*x**3+10*a**3*b**2*x**2+5*
a**4*b*x+a**5)**2,x)
```

```
[Out] -1/(9*a**9*b + 81*a**8*b**2*x + 324*a**7*b**3*x**2 + 756*a**6*b**4*x**3 + 1
134*a**5*b**5*x**4 + 1134*a**4*b**6*x**5 + 756*a**3*b**7*x**6 + 324*a**2*b*
*8*x**7 + 81*a*b**9*x**8 + 9*b**10*x**9)
```

Giac [A] time = 1.13431, size = 16, normalized size = 1.14

$$-\frac{1}{9\,(bx+a)^9b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^
5)^2,x, algorithm="giac")
```

```
[Out] -1/9/((b*x + a)^9*b)
```

3.68 $\int \frac{1}{\left(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5\right)^3}\,dx$

Optimal. Leaf size=14

$$-\frac{1}{14b(a+bx)^{14}}$$

```
[Out] -1/(14*b*(a + b*x)^14)
```

Rubi [A] time = 0.0181846, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.039, Rules used = {2058, 32}

$$-\frac{1}{14b(a+bx)^{14}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4 + b^5*
x^5)^(-3),x]
```

```
[Out] -1/(14*b*(a + b*x)^14)
```

Rule 2058

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5\right)^3}\,dx &= \int \frac{1}{(a+bx)^{15}}\,dx\\ &= -\frac{1}{14b(a+bx)^{14}}\end{aligned}$$

Mathematica [A] time = 0.0039681, size = 14, normalized size = 1.

$$-\frac{1}{14b(a+bx)^{14}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4
+ b^5*x^5)^(-3),x]
```

```
[Out] -1/(14*b*(a + b*x)^14)
```

Maple [A] time = 0.003, size = 13, normalized size = 0.9

$$-\frac{1}{14\,b\,(bx+a)^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5)^3,x
)
```

```
[Out] -1/14/b/(b*x+a)^14
```

Maxima [B] time = 1.66979, size = 211, normalized size = 15.07

$$-\frac{1}{14\left(b^{15}x^{14}+14\,ab^{14}x^{13}+91\,a^{2}b^{13}x^{12}+364\,a^{3}b^{12}x^{11}+1001\,a^{4}b^{11}x^{10}+2002\,a^{5}b^{10}x^{9}+3003\,a^{6}b^{9}x^{8}+3432\,a^{7}b^{8}x^{7}+30\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^
5)^3,x, algorithm="maxima")
```

```
[Out] -1/14/(b^15*x^14 + 14*a*b^14*x^13 + 91*a^2*b^13*x^12 + 364*a^3*b^12*x^11 +
1001*a^4*b^11*x^10 + 2002*a^5*b^10*x^9 + 3003*a^6*b^9*x^8 + 3432*a^7*b^8*x^
7 + 3003*a^8*b^7*x^6 + 2002*a^9*b^6*x^5 + 1001*a^10*b^5*x^4 + 364*a^11*b^4*
x^3 + 91*a^12*b^3*x^2 + 14*a^13*b^2*x + a^14*b)
```

Fricas [B] time = 1.76439, size = 370, normalized size = 26.43

$$-\frac{1}{14\left(b^{15}x^{14}+14\,ab^{14}x^{13}+91\,a^{2}b^{13}x^{12}+364\,a^{3}b^{12}x^{11}+1001\,a^{4}b^{11}x^{10}+2002\,a^{5}b^{10}x^{9}+3003\,a^{6}b^{9}x^{8}+3432\,a^{7}b^{8}x^{7}+3\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^
5)^3,x, algorithm="fricas")
```

```
[Out] -1/14/(b^15*x^14 + 14*a*b^14*x^13 + 91*a^2*b^13*x^12 + 364*a^3*b^12*x^11 +
1001*a^4*b^11*x^10 + 2002*a^5*b^10*x^9 + 3003*a^6*b^9*x^8 + 3432*a^7*b^8*x^
7 + 3003*a^8*b^7*x^6 + 2002*a^9*b^6*x^5 + 1001*a^10*b^5*x^4 + 364*a^11*b^4*
x^3 + 91*a^12*b^3*x^2 + 14*a^13*b^2*x + a^14*b)
```

Sympy [B] time = 1.55131, size = 168, normalized size = 12.

$$-\frac{1}{14a^{14}b+196a^{13}b^{2}x+1274a^{12}b^{3}x^{2}+5096a^{11}b^{4}x^{3}+14014a^{10}b^{5}x^{4}+28028a^{9}b^{6}x^{5}+42042a^{8}b^{7}x^{6}+48048a^{7}b^{8}x^{7}+4\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**5*x**5+5*a*b**4*x**4+10*a**2*b**3*x**3+10*a**3*b**2*x**2+5*
a**4*b*x+a**5)**3,x)
```

```
[Out] -1/(14*a**14*b + 196*a**13*b**2*x + 1274*a**12*b**3*x**2 + 5096*a**11*b**4*
x**3 + 14014*a**10*b**5*x**4 + 28028*a**9*b**6*x**5 + 42042*a**8*b**7*x**6
+ 48048*a**7*b**8*x**7 + 42042*a**6*b**9*x**8 + 28028*a**5*b**10*x**9 + 140
14*a**4*b**11*x**10 + 5096*a**3*b**12*x**11 + 1274*a**2*b**13*x**12 + 196*a
*b**14*x**13 + 14*b**15*x**14)
```

Giac [A] time = 1.14627, size = 16, normalized size = 1.14

$$-\frac{1}{14\,(bx+a)^{14}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^
5)^3,x, algorithm="giac")
```

```
[Out] -1/14/((b*x + a)^14*b)
```

3.69 $\int \frac{1}{1+x^2+x^3+x^5}\,dx$

Optimal. Leaf size=38

$$\frac{1}{4}\log\left(x^2+1\right)-\frac{1}{3}\log\left(x^2-x+1\right)+\frac{1}{6}\log(x+1)+\frac{1}{2}\tan^{-1}(x)$$

```
[Out] ArcTan[x]/2 + Log[1 + x]/6 + Log[1 + x^2]/4 - Log[1 - x + x^2]/3
```

Rubi [A] time = 0.0249394, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2058, 635, 203, 260, 628}

$$\frac{1}{4}\log\left(x^2+1\right)-\frac{1}{3}\log\left(x^2-x+1\right)+\frac{1}{6}\log(x+1)+\frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2 + x^3 + x^5)^(-1),x]
```

```
[Out] ArcTan[x]/2 + Log[1 + x]/6 + Log[1 + x^2]/4 - Log[1 - x + x^2]/3
```

Rule 2058

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{1+x^2+x^3+x^5}\,dx &= \int \left(\frac{1}{6(1+x)}+\frac{1+x}{2\left(1+x^2\right)}+\frac{1-2x}{3\left(1-x+x^2\right)}\right)dx\\
&= \frac{1}{6}\log(1+x)+\frac{1}{3}\int \frac{1-2x}{1-x+x^2}\,dx+\frac{1}{2}\int \frac{1+x}{1+x^2}\,dx\\
&= \frac{1}{6}\log(1+x)-\frac{1}{3}\log\left(1-x+x^2\right)+\frac{1}{2}\int \frac{1}{1+x^2}\,dx+\frac{1}{2}\int \frac{x}{1+x^2}\,dx\\
&= \frac{1}{2}\tan^{-1}(x)+\frac{1}{6}\log(1+x)+\frac{1}{4}\log\left(1+x^2\right)-\frac{1}{3}\log\left(1-x+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.0070307, size = 38, normalized size = 1.

$$\frac{1}{4}\log\left(x^2+1\right)-\frac{1}{3}\log\left(x^2-x+1\right)+\frac{1}{6}\log(x+1)+\frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2 + x^3 + x^5)^(-1),x]
```

```
[Out] ArcTan[x]/2 + Log[1 + x]/6 + Log[1 + x^2]/4 - Log[1 - x + x^2]/3
```

Maple [A] time = 0.007, size = 31, normalized size = 0.8

$$\frac{\arctan(x)}{2}+\frac{\ln(1+x)}{6}+\frac{\ln\left(x^2+1\right)}{4}-\frac{\ln\left(x^2-x+1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5+x^3+x^2+1),x)
```

```
[Out] 1/2*arctan(x)+1/6*ln(1+x)+1/4*ln(x^2+1)-1/3*ln(x^2-x+1)
```

Maxima [A] time = 2.70635, size = 41, normalized size = 1.08

$$\frac{1}{2}\arctan(x) - \frac{1}{3}\log\left(x^2 - x + 1\right) + \frac{1}{4}\log\left(x^2 + 1\right) + \frac{1}{6}\log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^5+x^3+x^2+1),x, algorithm="maxima")
```

```
[Out] 1/2*arctan(x) - 1/3*log(x^2 - x + 1) + 1/4*log(x^2 + 1) + 1/6*log(x + 1)
```

Fricas [A] time = 1.79973, size = 100, normalized size = 2.63

$$\frac{1}{2}\arctan(x) - \frac{1}{3}\log\left(x^2 - x + 1\right) + \frac{1}{4}\log\left(x^2 + 1\right) + \frac{1}{6}\log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^5+x^3+x^2+1),x, algorithm="fricas")
```

```
[Out] 1/2*arctan(x) - 1/3*log(x^2 - x + 1) + 1/4*log(x^2 + 1) + 1/6*log(x + 1)
```

Sympy [A] time = 0.141915, size = 29, normalized size = 0.76

$$\frac{\log(x + 1)}{6} + \frac{\log\left(x^2 + 1\right)}{4} - \frac{\log\left(x^2 - x + 1\right)}{3} + \frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**5+x**3+x**2+1),x)
```

```
[Out] log(x + 1)/6 + log(x**2 + 1)/4 - log(x**2 - x + 1)/3 + atan(x)/2
```

Giac [A] time = 1.14541, size = 42, normalized size = 1.11

$$\frac{1}{2}\arctan\left(x\right)-\frac{1}{3}\log\left(x^{2}-x+1\right)+\frac{1}{4}\log\left(x^{2}+1\right)+\frac{1}{6}\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^5+x^3+x^2+1),x, algorithm="giac")
```

```
[Out] 1/2*arctan(x) - 1/3*log(x^2 - x + 1) + 1/4*log(x^2 + 1) + 1/6*log(abs(x + 1
))
```

3.70 $\int \left(3-19x^2+32x^4-16x^6\right)^4 \, dx$

Optimal. Leaf size=84

$$\frac{65536x^{25}}{25}-\frac{524288x^{23}}{23}+\frac{1884160x^{21}}{21}-\frac{4014080x^{19}}{19}+\frac{5633536x^{17}}{17}-\frac{1094656x^{15}}{3}+\frac{3764416x^{13}}{13}-\frac{1841600x^{11}}{11}+\frac{63\ldots}{}$$

```
[Out] 81*x - 684*x^3 + 4590*x^5 - (149700*x^7)/7 + (634321*x^9)/9 - (1841600*x^11
)/11 + (3764416*x^13)/13 - (1094656*x^15)/3 + (5633536*x^17)/17 - (4014080*
x^19)/19 + (1884160*x^21)/21 - (524288*x^23)/23 + (65536*x^25)/25
```

Rubi [A] time = 0.0855259, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2059, 517, 521}

$$\frac{65536x^{25}}{25}-\frac{524288x^{23}}{23}+\frac{1884160x^{21}}{21}-\frac{4014080x^{19}}{19}+\frac{5633536x^{17}}{17}-\frac{1094656x^{15}}{3}+\frac{3764416x^{13}}{13}-\frac{1841600x^{11}}{11}+\frac{63\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 19*x^2 + 32*x^4 - 16*x^6)^4,x]
```


```
[Out] 81*x - 684*x^3 + 4590*x^5 - (149700*x^7)/7 + (634321*x^9)/9 - (1841600*x^11
)/11 + (3764416*x^13)/13 - (1094656*x^15)/3 + (5633536*x^17)/17 - (4014080*
x^19)/19 + (1884160*x^21)/21 - (524288*x^23)/23 + (65536*x^25)/25
```


Rule 2059

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[u^p, x] /;  !SumQ[Non
freeFactors[u, x]]] /; PolyQ[P, x] && IntegerQ[p]
```

Rule 517

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :> Int[u*(a1*a2 + b1*b2
*x^n)^p*(c + d*x^n)^q, x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && E
qQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && Gt
Q[a2, 0]))
```

Rule 521

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c +
d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[p
, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rubi steps

$$\begin{aligned}\int \left(3-19x^2+32x^4-16x^6\right)^4 dx &= \int (-1+x)^4(1+x)^4(-1+2x)^4(1+2x)^4\left(-3+4x^2\right)^4 dx\\ &= \int (-1+2x)^4(1+2x)^4\left(-1+x^2\right)^4\left(-3+4x^2\right)^4 dx\\ &= \int \left(-1+x^2\right)^4\left(-3+4x^2\right)^4\left(-1+4x^2\right)^4 dx\\ &= \int \left(81-2052x^2+22950x^4-149700x^6+634321x^8-1841600x^{10}+3764416x^{12}-54\right.\\ &= 81x-684x^3+4590x^5-\frac{149700x^7}{7}+\frac{634321x^9}{9}-\frac{1841600x^{11}}{11}+\frac{3764416x^{13}}{13}-\frac{1094}{}\end{aligned}$$

Mathematica [A] time = 0.001617, size = 84, normalized size = 1.

$$\frac{65536x^{25}}{25}-\frac{524288x^{23}}{23}+\frac{1884160x^{21}}{21}-\frac{4014080x^{19}}{19}+\frac{5633536x^{17}}{17}-\frac{1094656x^{15}}{3}+\frac{3764416x^{13}}{13}-\frac{1841600x^{11}}{11}+\frac{634\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 19*x^2 + 32*x^4 - 16*x^6)^4,x]
```

```
[Out] 81*x - 684*x^3 + 4590*x^5 - (149700*x^7)/7 + (634321*x^9)/9 - (1841600*x^11
)/11 + (3764416*x^13)/13 - (1094656*x^15)/3 + (5633536*x^17)/17 - (4014080*
x^19)/19 + (1884160*x^21)/21 - (524288*x^23)/23 + (65536*x^25)/25
```

Maple [A] time = 0.002, size = 65, normalized size = 0.8

$$81\,x-684\,x^3+4590\,x^5-\frac{149700\,x^7}{7}+\frac{634321\,x^9}{9}-\frac{1841600\,x^{11}}{11}+\frac{3764416\,x^{13}}{13}-\frac{1094656\,x^{15}}{3}+\frac{5633536\,x^{17}}{17}-\frac{40140\cdots}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-16*x^6+32*x^4-19*x^2+3)^4,x)
```

```
[Out] 81*x-684*x^3+4590*x^5-149700/7*x^7+634321/9*x^9-1841600/11*x^11+3764416/13*
x^13-1094656/3*x^15+5633536/17*x^17-4014080/19*x^19+1884160/21*x^21-524288/
23*x^23+65536/25*x^25
```

Maxima [A] time = 1.75358, size = 86, normalized size = 1.02

$$\frac{65536}{25}\,x^{25}-\frac{524288}{23}\,x^{23}+\frac{1884160}{21}\,x^{21}-\frac{4014080}{19}\,x^{19}+\frac{5633536}{17}\,x^{17}-\frac{1094656}{3}\,x^{15}+\frac{3764416}{13}\,x^{13}-\frac{1841600}{11}\,x^{11}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-16*x^6+32*x^4-19*x^2+3)^4,x, algorithm="maxima")
```

```
[Out] 65536/25*x^25 - 524288/23*x^23 + 1884160/21*x^21 - 4014080/19*x^19 + 563353
6/17*x^17 - 1094656/3*x^15 + 3764416/13*x^13 - 1841600/11*x^11 + 634321/9*x
^9 - 149700/7*x^7 + 4590*x^5 - 684*x^3 + 81*x
```

Fricas [A] time = 1.54675, size = 266, normalized size = 3.17

$$\frac{65536}{25}x^{25}-\frac{524288}{23}x^{23}+\frac{1884160}{21}x^{21}-\frac{4014080}{19}x^{19}+\frac{5633536}{17}x^{17}-\frac{1094656}{3}x^{15}+\frac{3764416}{13}x^{13}-\frac{1841600}{11}x^{11}+\frac{63}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-16*x^6+32*x^4-19*x^2+3)^4,x, algorithm="fricas")
```

```
[Out] 65536/25*x^25 - 524288/23*x^23 + 1884160/21*x^21 - 4014080/19*x^19 + 563353
6/17*x^17 - 1094656/3*x^15 + 3764416/13*x^13 - 1841600/11*x^11 + 634321/9*x
^9 - 149700/7*x^7 + 4590*x^5 - 684*x^3 + 81*x
```

Sympy [A] time = 0.0718, size = 80, normalized size = 0.95

$$\frac{65536x^{25}}{25}-\frac{524288x^{23}}{23}+\frac{1884160x^{21}}{21}-\frac{4014080x^{19}}{19}+\frac{5633536x^{17}}{17}-\frac{1094656x^{15}}{3}+\frac{3764416x^{13}}{13}-\frac{1841600x^{11}}{11}+\frac{63}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-16*x**6+32*x**4-19*x**2+3)**4,x)
```

```
[Out] 65536*x**25/25 - 524288*x**23/23 + 1884160*x**21/21 - 4014080*x**19/19 + 56
33536*x**17/17 - 1094656*x**15/3 + 3764416*x**13/13 - 1841600*x**11/11 + 63
4321*x**9/9 - 149700*x**7/7 + 4590*x**5 - 684*x**3 + 81*x
```

Giac [A] time = 1.1395, size = 86, normalized size = 1.02

$$\frac{65536}{25}x^{25} - \frac{524288}{23}x^{23} + \frac{1884160}{21}x^{21} - \frac{4014080}{19}x^{19} + \frac{5633536}{17}x^{17} - \frac{1094656}{3}x^{15} + \frac{3764416}{13}x^{13} - \frac{1841600}{11}x^{11} + \frac{6}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-16*x^6+32*x^4-19*x^2+3)^4,x, algorithm="giac")
```

```
[Out] 65536/25*x^25 - 524288/23*x^23 + 1884160/21*x^21 - 4014080/19*x^19 + 563353
6/17*x^17 - 1094656/3*x^15 + 3764416/13*x^13 - 1841600/11*x^11 + 634321/9*x
^9 - 149700/7*x^7 + 4590*x^5 - 684*x^3 + 81*x
```

3.71 $\int \left(3-19x^2+32x^4-16x^6\right)^3 \, dx$

Optimal. Leaf size=63

$$-\frac{4096x^{19}}{19}+\frac{24576x^{17}}{17}-\frac{21248x^{15}}{5}+\frac{93440x^{13}}{13}-\frac{84912x^{11}}{11}+\frac{16448x^9}{3}-2605x^7+\frac{4113x^5}{5}-171x^3+27x$$

```
[Out] 27*x - 171*x^3 + (4113*x^5)/5 - 2605*x^7 + (16448*x^9)/3 - (84912*x^11)/11
+ (93440*x^13)/13 - (21248*x^15)/5 + (24576*x^17)/17 - (4096*x^19)/19
```

Rubi [A] time = 0.0757429, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2059, 517, 521}

$$-\frac{4096x^{19}}{19}+\frac{24576x^{17}}{17}-\frac{21248x^{15}}{5}+\frac{93440x^{13}}{13}-\frac{84912x^{11}}{11}+\frac{16448x^9}{3}-2605x^7+\frac{4113x^5}{5}-171x^3+27x$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 19*x^2 + 32*x^4 - 16*x^6)^3,x]
```

```
[Out] 27*x - 171*x^3 + (4113*x^5)/5 - 2605*x^7 + (16448*x^9)/3 - (84912*x^11)/11
+ (93440*x^13)/13 - (21248*x^15)/5 + (24576*x^17)/17 - (4096*x^19)/19
```

Rule 2059

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[u^p, x] /;  !SumQ[Non
freeFactors[u, x]]] /; PolyQ[P, x] && IntegerQ[p]
```

Rule 517

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :> Int[u*(a1*a2 + b1*b2
*x^n)^p*(c + d*x^n)^q, x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && E
qQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && Gt
Q[a2, 0]))
```

Rule 521

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c +
d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[p
```

```
, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rubi steps

$$\begin{aligned}\int \left(3-19x^2+32x^4-16x^6\right)^3\,dx &= -\int (-1+x)^3(1+x)^3(-1+2x)^3(1+2x)^3\left(-3+4x^2\right)^3\,dx\\ &= -\int (-1+2x)^3(1+2x)^3\left(-1+x^2\right)^3\left(-3+4x^2\right)^3\,dx\\ &= -\int \left(-1+x^2\right)^3\left(-3+4x^2\right)^3\left(-1+4x^2\right)^3\,dx\\ &= -\int \left(-27+513x^2-4113x^4+18235x^6-49344x^8+84912x^{10}-93440x^{12}+63744x^1\right.\\ &= 27x-171x^3+\frac{4113x^5}{5}-2605x^7+\frac{16448x^9}{3}-\frac{84912x^{11}}{11}+\frac{93440x^{13}}{13}-\frac{21248x^{15}}{5}+\end{aligned}$$

Mathematica [A] time = 0.0020171, size = 63, normalized size = 1.

$$-\frac{4096x^{19}}{19}+\frac{24576x^{17}}{17}-\frac{21248x^{15}}{5}+\frac{93440x^{13}}{13}-\frac{84912x^{11}}{11}+\frac{16448x^9}{3}-2605x^7+\frac{4113x^5}{5}-171x^3+27x$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 19*x^2 + 32*x^4 - 16*x^6)^3,x]

[Out] 27*x - 171*x^3 + (4113*x^5)/5 - 2605*x^7 + (16448*x^9)/3 - (84912*x^11)/11
+ (93440*x^13)/13 - (21248*x^15)/5 + (24576*x^17)/17 - (4096*x^19)/19
```

Maple [A] time = 0.001, size = 50, normalized size = 0.8

$$27\,x-171\,x^3+\frac{4113\,x^5}{5}-2605\,x^7+\frac{16448\,x^9}{3}-\frac{84912\,x^{11}}{11}+\frac{93440\,x^{13}}{13}-\frac{21248\,x^{15}}{5}+\frac{24576\,x^{17}}{17}-\frac{4096\,x^{19}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-16*x^6+32*x^4-19*x^2+3)^3,x)

[Out] 27*x-171*x^3+4113/5*x^5-2605*x^7+16448/3*x^9-84912/11*x^11+93440/13*x^13-21
248/5*x^15+24576/17*x^17-4096/19*x^19
```

Maxima [A] time = 1.20483, size = 66, normalized size = 1.05

$$-\frac{4096}{19}\,x^{19}+\frac{24576}{17}\,x^{17}-\frac{21248}{5}\,x^{15}+\frac{93440}{13}\,x^{13}-\frac{84912}{11}\,x^{11}+\frac{16448}{3}\,x^{9}-2605\,x^{7}+\frac{4113}{5}\,x^{5}-171\,x^{3}+27\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-16*x^6+32*x^4-19*x^2+3)^3,x, algorithm="maxima")
```

```
[Out] -4096/19*x^19 + 24576/17*x^17 - 21248/5*x^15 + 93440/13*x^13 - 84912/11*x^1
1 + 16448/3*x^9 - 2605*x^7 + 4113/5*x^5 - 171*x^3 + 27*x
```

Fricas [A] time = 1.5149, size = 180, normalized size = 2.86

$$-\frac{4096}{19}x^{19}+\frac{24576}{17}x^{17}-\frac{21248}{5}x^{15}+\frac{93440}{13}x^{13}-\frac{84912}{11}x^{11}+\frac{16448}{3}x^{9}-2605x^{7}+\frac{4113}{5}x^{5}-171x^{3}+27x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-16*x^6+32*x^4-19*x^2+3)^3,x, algorithm="fricas")
```

```
[Out] -4096/19*x^19 + 24576/17*x^17 - 21248/5*x^15 + 93440/13*x^13 - 84912/11*x^1
1 + 16448/3*x^9 - 2605*x^7 + 4113/5*x^5 - 171*x^3 + 27*x
```

Sympy [A] time = 0.068592, size = 60, normalized size = 0.95

$$-\frac{4096x^{19}}{19}+\frac{24576x^{17}}{17}-\frac{21248x^{15}}{5}+\frac{93440x^{13}}{13}-\frac{84912x^{11}}{11}+\frac{16448x^{9}}{3}-2605x^{7}+\frac{4113x^{5}}{5}-171x^{3}+27x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-16*x**6+32*x**4-19*x**2+3)**3,x)
```

```
[Out] -4096*x**19/19 + 24576*x**17/17 - 21248*x**15/5 + 93440*x**13/13 - 84912*x*
*11/11 + 16448*x**9/3 - 2605*x**7 + 4113*x**5/5 - 171*x**3 + 27*x
```

Giac [A] time = 1.11731, size = 66, normalized size = 1.05

$$-\frac{4096}{19}\,x^{19}+\frac{24576}{17}\,x^{17}-\frac{21248}{5}\,x^{15}+\frac{93440}{13}\,x^{13}-\frac{84912}{11}\,x^{11}+\frac{16448}{3}\,x^{9}-2605\,x^{7}+\frac{4113}{5}\,x^{5}-171\,x^{3}+27\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-16*x^6+32*x^4-19*x^2+3)^3,x, algorithm="giac")
```

```
[Out] -4096/19*x^19 + 24576/17*x^17 - 21248/5*x^15 + 93440/13*x^13 - 84912/11*x^1
1 + 16448/3*x^9 - 2605*x^7 + 4113/5*x^5 - 171*x^3 + 27*x
```

3.72 $\int \left(3-19x^2+32x^4-16x^6\right)^2\,dx$

Optimal. Leaf size=44

$$\frac{256x^{13}}{13}-\frac{1024x^{11}}{11}+\frac{544x^9}{3}-\frac{1312x^7}{7}+\frac{553x^5}{5}-38x^3+9x$$

```
[Out] 9*x - 38*x^3 + (553*x^5)/5 - (1312*x^7)/7 + (544*x^9)/3 - (1024*x^11)/11 +
(256*x^13)/13
```

Rubi [A] time = 0.0693192, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2059, 517, 521}

$$\frac{256x^{13}}{13}-\frac{1024x^{11}}{11}+\frac{544x^9}{3}-\frac{1312x^7}{7}+\frac{553x^5}{5}-38x^3+9x$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 19*x^2 + 32*x^4 - 16*x^6)^2,x]
```

```
[Out] 9*x - 38*x^3 + (553*x^5)/5 - (1312*x^7)/7 + (544*x^9)/3 - (1024*x^11)/11 +
(256*x^13)/13
```

Rule 2059

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[u^p, x] /;  !SumQ[Non
freeFactors[u, x]]] /; PolyQ[P, x] && IntegerQ[p]
```

Rule 517

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :> Int[u*(a1*a2 + b1*b2
*x^n)^p*(c + d*x^n)^q, x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && E
qQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && Gt
Q[a2, 0]))
```

Rule 521

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c +
d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[p
```

```
, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rubi steps

$$
\begin{aligned}
\int \left(3-19x^2+32x^4-16x^6\right)^2\,dx &= \int (-1+x)^2(1+x)^2(-1+2x)^2(1+2x)^2\left(-3+4x^2\right)^2\,dx\\
&= \int (-1+2x)^2(1+2x)^2\left(-1+x^2\right)^2\left(-3+4x^2\right)^2\,dx\\
&= \int \left(-1+x^2\right)^2\left(-3+4x^2\right)^2\left(-1+4x^2\right)^2\,dx\\
&= \int \left(9-114x^2+553x^4-1312x^6+1632x^8-1024x^{10}+256x^{12}\right)dx\\
&= 9x-38x^3+\frac{553x^5}{5}-\frac{1312x^7}{7}+\frac{544x^9}{3}-\frac{1024x^{11}}{11}+\frac{256x^{13}}{13}
\end{aligned}
$$

Mathematica [A] time = 0.0007037, size = 44, normalized size = 1.

$$\frac{256x^{13}}{13}-\frac{1024x^{11}}{11}+\frac{544x^9}{3}-\frac{1312x^7}{7}+\frac{553x^5}{5}-38x^3+9x$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 19*x^2 + 32*x^4 - 16*x^6)^2,x]
```

```
[Out] 9*x - 38*x^3 + (553*x^5)/5 - (1312*x^7)/7 + (544*x^9)/3 - (1024*x^11)/11 +
(256*x^13)/13
```

Maple [A] time = 0., size = 35, normalized size = 0.8

$$9\,x-38\,x^3+\frac{553\,x^5}{5}-\frac{1312\,x^7}{7}+\frac{544\,x^9}{3}-\frac{1024\,x^{11}}{11}+\frac{256\,x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-16*x^6+32*x^4-19*x^2+3)^2,x)
```

```
[Out] 9*x-38*x^3+553/5*x^5-1312/7*x^7+544/3*x^9-1024/11*x^11+256/13*x^13
```

Maxima [A] time = 1.14573, size = 46, normalized size = 1.05

$$\frac{256}{13}\,x^{13} - \frac{1024}{11}\,x^{11} + \frac{544}{3}\,x^{9} - \frac{1312}{7}\,x^{7} + \frac{553}{5}\,x^{5} - 38\,x^{3} + 9\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-16*x^6+32*x^4-19*x^2+3)^2,x, algorithm="maxima")
```

```
[Out] 256/13*x^13 - 1024/11*x^11 + 544/3*x^9 - 1312/7*x^7 + 553/5*x^5 - 38*x^3 +
9*x
```

Fricas [A] time = 1.5264, size = 108, normalized size = 2.45

$$\frac{256}{13}x^{13} - \frac{1024}{11}x^{11} + \frac{544}{3}x^{9} - \frac{1312}{7}x^{7} + \frac{553}{5}x^{5} - 38x^{3} + 9x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-16*x^6+32*x^4-19*x^2+3)^2,x, algorithm="fricas")
```

```
[Out] 256/13*x^13 - 1024/11*x^11 + 544/3*x^9 - 1312/7*x^7 + 553/5*x^5 - 38*x^3 +
9*x
```

Sympy [A] time = 0.061542, size = 41, normalized size = 0.93

$$\frac{256x^{13}}{13} - \frac{1024x^{11}}{11} + \frac{544x^{9}}{3} - \frac{1312x^{7}}{7} + \frac{553x^{5}}{5} - 38x^{3} + 9x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-16*x**6+32*x**4-19*x**2+3)**2,x)
```

```
[Out] 256*x**13/13 - 1024*x**11/11 + 544*x**9/3 - 1312*x**7/7 + 553*x**5/5 - 38*x
**3 + 9*x
```

Giac [A] time = 1.12267, size = 46, normalized size = 1.05

$$\frac{256}{13}\,x^{13} - \frac{1024}{11}\,x^{11} + \frac{544}{3}\,x^{9} - \frac{1312}{7}\,x^{7} + \frac{553}{5}\,x^{5} - 38\,x^{3} + 9\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-16*x^6+32*x^4-19*x^2+3)^2,x, algorithm="giac")
```

```
[Out] 256/13*x^13 - 1024/11*x^11 + 544/3*x^9 - 1312/7*x^7 + 553/5*x^5 - 38*x^3 +
9*x
```

3.73 $\int \left(3-19x^2+32x^4-16x^6\right)\,dx$

Optimal. Leaf size=25

$$-\frac{16x^7}{7}+\frac{32x^5}{5}-\frac{19x^3}{3}+3x$$

```
[Out] 3*x - (19*x^3)/3 + (32*x^5)/5 - (16*x^7)/7
```

Rubi [A] time = 0.0031791, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$-\frac{16x^7}{7}+\frac{32x^5}{5}-\frac{19x^3}{3}+3x$$

Antiderivative was successfully verified.

```
[In] Int[3 - 19*x^2 + 32*x^4 - 16*x^6,x]

[Out] 3*x - (19*x^3)/3 + (32*x^5)/5 - (16*x^7)/7
```

Rubi steps

$$\int \left(3-19x^2+32x^4-16x^6\right)\,dx=3x-\frac{19x^3}{3}+\frac{32x^5}{5}-\frac{16x^7}{7}$$

Mathematica [A] time = 0.0000443, size = 25, normalized size = 1.

$$-\frac{16x^7}{7}+\frac{32x^5}{5}-\frac{19x^3}{3}+3x$$

Antiderivative was successfully verified.

```
[In] Integrate[3 - 19*x^2 + 32*x^4 - 16*x^6,x]

[Out] 3*x - (19*x^3)/3 + (32*x^5)/5 - (16*x^7)/7
```

Maple [A] time = 0., size = 20, normalized size = 0.8

$$3\,x-\frac{19\,x^{3}}{3}+\frac{32\,x^{5}}{5}-\frac{16\,x^{7}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-16*x^6+32*x^4-19*x^2+3,x)
```

```
[Out] 3*x-19/3*x^3+32/5*x^5-16/7*x^7
```

Maxima [A] time = 1.17189, size = 26, normalized size = 1.04

$$-\frac{16}{7}\,x^{7}+\frac{32}{5}\,x^{5}-\frac{19}{3}\,x^{3}+3\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-16*x^6+32*x^4-19*x^2+3,x, algorithm="maxima")
```

```
[Out] -16/7*x^7 + 32/5*x^5 - 19/3*x^3 + 3*x
```

Fricas [A] time = 1.43674, size = 53, normalized size = 2.12

$$-\frac{16}{7}x^{7}+\frac{32}{5}x^{5}-\frac{19}{3}x^{3}+3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-16*x^6+32*x^4-19*x^2+3,x, algorithm="fricas")
```

```
[Out] -16/7*x^7 + 32/5*x^5 - 19/3*x^3 + 3*x
```

Sympy [A] time = 0.05443, size = 22, normalized size = 0.88

$$-\frac{16x^{7}}{7}+\frac{32x^{5}}{5}-\frac{19x^{3}}{3}+3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-16*x**6+32*x**4-19*x**2+3,x)
```

```
[Out] -16*x**7/7 + 32*x**5/5 - 19*x**3/3 + 3*x
```

Giac [A] time = 1.12437, size = 26, normalized size = 1.04

$$-\frac{16}{7}x^7 + \frac{32}{5}x^5 - \frac{19}{3}x^3 + 3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-16*x^6+32*x^4-19*x^2+3,x, algorithm="giac")
```

```
[Out] -16/7*x^7 + 32/5*x^5 - 19/3*x^3 + 3*x
```

3.74 $\int \frac{1}{3-19x^2+32x^4-16x^6}\,dx$

Optimal. Leaf size=31

$$\frac{1}{3}\tanh^{-1}(x)+\frac{1}{3}\tanh^{-1}(2x)-\frac{\tanh^{-1}\left(\frac{2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] ArcTanh[x]/3 + ArcTanh[2*x]/3 - ArcTanh[(2*x)/Sqrt[3]]/Sqrt[3]
```

Rubi [A] time = 0.0228096, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2057, 207}

$$\frac{1}{3}\tanh^{-1}(x)+\frac{1}{3}\tanh^{-1}(2x)-\frac{\tanh^{-1}\left(\frac{2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 19*x^2 + 32*x^4 - 16*x^6)^(-1),x]
```

```
[Out] ArcTanh[x]/3 + ArcTanh[2*x]/3 - ArcTanh[(2*x)/Sqrt[3]]/Sqrt[3]
```

Rule 2057

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P /. x -> Sqrt[x]]}, Int[Expan
dIntegrand[(u /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; Po
lyQ[P, x^2] && ILtQ[p, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{3-19x^2+32x^4-16x^6}\,dx = \int \left(-\frac{1}{3\left(-1+x^2\right)} + \frac{2}{-3+4x^2} - \frac{2}{3\left(-1+4x^2\right)}\right) dx$$
$$= -\left(\frac{1}{3}\int \frac{1}{-1+x^2}\,dx\right) - \frac{2}{3}\int \frac{1}{-1+4x^2}\,dx + 2\int \frac{1}{-3+4x^2}\,dx$$
$$= \frac{1}{3}\tanh^{-1}(x) + \frac{1}{3}\tanh^{-1}(2x) - \frac{\tanh^{-1}\left(\frac{2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Mathematica [A] time = 0.0134657, size = 62, normalized size = 2.

$$\frac{1}{6}\left(-\log\left(2x^2-3x+1\right) + \log\left(2x^2+3x+1\right) + \sqrt{3}\log\left(\sqrt{3}-2x\right) - \sqrt{3}\log\left(2x+\sqrt{3}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 19*x^2 + 32*x^4 - 16*x^6)^(-1),x]
```

```
[Out] (Sqrt[3]*Log[Sqrt[3] - 2*x] - Sqrt[3]*Log[Sqrt[3] + 2*x] - Log[1 - 3*x + 2*
x^2] + Log[1 + 3*x + 2*x^2])/6
```

Maple [A] time = 0.01, size = 42, normalized size = 1.4

$$-\frac{\ln(x-1)}{6} - \frac{\sqrt{3}}{3}\mathrm{Artanh}\left(\frac{2x\sqrt{3}}{3}\right) - \frac{\ln(2x-1)}{6} + \frac{\ln(1+2x)}{6} + \frac{\ln(1+x)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-16*x^6+32*x^4-19*x^2+3),x)
```

```
[Out] -1/6*ln(x-1)-1/3*arctanh(2/3*x*3^(1/2))*3^(1/2)-1/6*ln(2*x-1)+1/6*ln(1+2*x)
+1/6*ln(1+x)
```

Maxima [B] time = 1.73327, size = 73, normalized size = 2.35

$$\frac{1}{6}\sqrt{3}\log\left(\frac{2x-\sqrt{3}}{2x+\sqrt{3}}\right) + \frac{1}{6}\log(2x+1) - \frac{1}{6}\log(2x-1) + \frac{1}{6}\log(x+1) - \frac{1}{6}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-16*x^6+32*x^4-19*x^2+3),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*log((2*x - sqrt(3))/(2*x + sqrt(3))) + 1/6*log(2*x + 1) - 1/6*l
og(2*x - 1) + 1/6*log(x + 1) - 1/6*log(x - 1)
```

Fricas [B] time = 1.63208, size = 149, normalized size = 4.81

$$\frac{1}{6}\sqrt{3}\log\left(\frac{4x^2-4\sqrt{3}x+3}{4x^2-3}\right)+\frac{1}{6}\log\left(2x^2+3x+1\right)-\frac{1}{6}\log\left(2x^2-3x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-16*x^6+32*x^4-19*x^2+3),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*log((4*x^2 - 4*sqrt(3)*x + 3)/(4*x^2 - 3)) + 1/6*log(2*x^2 + 3*
x + 1) - 1/6*log(2*x^2 - 3*x + 1)
```

Sympy [B] time = 0.139756, size = 63, normalized size = 2.03

$$\frac{\sqrt{3}\log\left(x-\frac{\sqrt{3}}{2}\right)}{6}-\frac{\sqrt{3}\log\left(x+\frac{\sqrt{3}}{2}\right)}{6}-\frac{\log\left(x^2-\frac{3x}{2}+\frac{1}{2}\right)}{6}+\frac{\log\left(x^2+\frac{3x}{2}+\frac{1}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-16*x**6+32*x**4-19*x**2+3),x)
```

```
[Out] sqrt(3)*log(x - sqrt(3)/2)/6 - sqrt(3)*log(x + sqrt(3)/2)/6 - log(x**2 - 3*
x/2 + 1/2)/6 + log(x**2 + 3*x/2 + 1/2)/6
```

Giac [B] time = 1.12844, size = 84, normalized size = 2.71

$$\frac{1}{6}\sqrt{3}\log\left(\frac{\left|8x-4\sqrt{3}\right|}{\left|8x+4\sqrt{3}\right|}\right)+\frac{1}{6}\log\left(|2x+1|\right)-\frac{1}{6}\log\left(|2x-1|\right)+\frac{1}{6}\log\left(|x+1|\right)-\frac{1}{6}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-16*x^6+32*x^4-19*x^2+3),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*log(abs(8*x - 4*sqrt(3))/abs(8*x + 4*sqrt(3))) + 1/6*log(abs(2*
x + 1)) - 1/6*log(abs(2*x - 1)) + 1/6*log(abs(x + 1)) - 1/6*log(abs(x - 1))
```

3.75 $\int \frac{1}{(3-19x^2+32x^4-16x^6)^2}\,dx$

Optimal. Leaf size=89

$$\frac{2x}{3(3-4x^2)}+\frac{1}{18(1-2x)}+\frac{1}{36(1-x)}-\frac{1}{36(x+1)}-\frac{1}{18(2x+1)}+\frac{67}{54}\tanh^{-1}(x)-\frac{7}{27}\tanh^{-1}(2x)-\frac{5\tanh^{-1}\left(\frac{2x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] 1/(18*(1 - 2*x)) + 1/(36*(1 - x)) - 1/(36*(1 + x)) - 1/(18*(1 + 2*x)) + (2*
x)/(3*(3 - 4*x^2)) + (67*ArcTanh[x])/54 - (7*ArcTanh[2*x])/27 - (5*ArcTanh[
(2*x)/Sqrt[3]])/(3*Sqrt[3])
```

Rubi [A] time = 0.0614828, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2057, 207, 199}

$$\frac{2x}{3(3-4x^2)}+\frac{1}{18(1-2x)}+\frac{1}{36(1-x)}-\frac{1}{36(x+1)}-\frac{1}{18(2x+1)}+\frac{67}{54}\tanh^{-1}(x)-\frac{7}{27}\tanh^{-1}(2x)-\frac{5\tanh^{-1}\left(\frac{2x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 19*x^2 + 32*x^4 - 16*x^6)^(-2),x]
```

```
[Out] 1/(18*(1 - 2*x)) + 1/(36*(1 - x)) - 1/(36*(1 + x)) - 1/(18*(1 + 2*x)) + (2*
x)/(3*(3 - 4*x^2)) + (67*ArcTanh[x])/54 - (7*ArcTanh[2*x])/27 - (5*ArcTanh[
(2*x)/Sqrt[3]])/(3*Sqrt[3])
```

Rule 2057

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P /. x -> Sqrt[x]]}, Int[Expan
dIntegrand[(u /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; Po
lyQ[P, x^2] && ILtQ[p, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rubi steps

$$\int \frac{1}{\left(3-19x^2+32x^4-16x^6\right)^2}\,dx = \int \left(\frac{1}{36(-1+x)^2}+\frac{1}{36(1+x)^2}+\frac{1}{9(-1+2x)^2}+\frac{1}{9(1+2x)^2}-\frac{67}{54\left(-1+x^2\right)}+\frac{4}{\left(-3+\right.}\right.$$

$$=\frac{1}{18(1-2x)}+\frac{1}{36(1-x)}-\frac{1}{36(1+x)}-\frac{1}{18(1+2x)}+\frac{14}{27}\int \frac{1}{-1+4x^2}\,dx-\frac{67}{54}\int \frac{1}{-1}$$

$$=\frac{1}{18(1-2x)}+\frac{1}{36(1-x)}-\frac{1}{36(1+x)}-\frac{1}{18(1+2x)}+\frac{2x}{3\left(3-4x^2\right)}+\frac{67}{54}\tanh^{-1}(x)-$$

$$=\frac{1}{18(1-2x)}+\frac{1}{36(1-x)}-\frac{1}{36(1+x)}-\frac{1}{18(1+2x)}+\frac{2x}{3\left(3-4x^2\right)}+\frac{67}{54}\tanh^{-1}(x)-$$

Mathematica [A] time = 0.0510452, size = 103, normalized size = 1.16

$$\frac{1}{108}\left(-\frac{6x\left(80x^4-104x^2+27\right)}{16x^6-32x^4+19x^2-3}+14\log(1-2x)+30\sqrt{3}\log\left(\sqrt{3}-2x\right)-67\log(1-x)+67\log(x+1)-14\log(2x+1)\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 19*x^2 + 32*x^4 - 16*x^6)^(-2),x]
```

```
[Out] ((-6*x*(27 - 104*x^2 + 80*x^4))/(-3 + 19*x^2 - 32*x^4 + 16*x^6) + 14*Log[1
- 2*x] + 30*Sqrt[3]*Log[Sqrt[3] - 2*x] - 67*Log[1 - x] + 67*Log[1 + x] - 14
*Log[1 + 2*x] - 30*Sqrt[3]*Log[Sqrt[3] + 2*x])/108
```

Maple [A] time = 0.02, size = 84, normalized size = 0.9

$$-\frac{1}{36\,x-36}-\frac{67\,\ln(x-1)}{108}-\frac{x}{6}\left(x^2-\frac{3}{4}\right)^{-1}-\frac{5\,\sqrt{3}}{9}\mathrm{Artanh}\left(\frac{2\,x\sqrt{3}}{3}\right)-\frac{1}{36\,x-18}+\frac{7\,\ln(2\,x-1)}{54}-\frac{1}{18+36\,x}-\frac{7\,\ln(1}{54}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-16*x^6+32*x^4-19*x^2+3)^2,x)
```

```
[Out] -1/36/(x-1)-67/108*ln(x-1)-1/6*x/(x^2-3/4)-5/9*arctanh(2/3*x*3^(1/2))*3^(1/
2)-1/18/(2*x-1)+7/54*ln(2*x-1)-1/18/(1+2*x)-7/54*ln(1+2*x)-1/36/(1+x)+67/10
8*ln(1+x)
```

Maxima [A] time = 1.74873, size = 120, normalized size = 1.35

$$\frac{5}{18}\sqrt{3}\log\left(\frac{2x-\sqrt{3}}{2x+\sqrt{3}}\right)-\frac{80x^5-104x^3+27x}{18\left(16x^6-32x^4+19x^2-3\right)}-\frac{7}{54}\log\left(2x+1\right)+\frac{7}{54}\log\left(2x-1\right)+\frac{67}{108}\log\left(x+1\right)-\frac{67}{108}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-16*x^6+32*x^4-19*x^2+3)^2,x, algorithm="maxima")
```

```
[Out] 5/18*sqrt(3)*log((2*x - sqrt(3))/(2*x + sqrt(3))) - 1/18*(80*x^5 - 104*x^3
+ 27*x)/(16*x^6 - 32*x^4 + 19*x^2 - 3) - 7/54*log(2*x + 1) + 7/54*log(2*x -
 1) + 67/108*log(x + 1) - 67/108*log(x - 1)
```

Fricas [B] time = 1.7795, size = 467, normalized size = 5.25

$$-\frac{480x^5-624x^3-30\sqrt{3}\left(16x^6-32x^4+19x^2-3\right)\log\left(\frac{4x^2-4\sqrt{3}x+3}{4x^2-3}\right)+14\left(16x^6-32x^4+19x^2-3\right)\log\left(2x+1\right)-14\,(}{108\,(16x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-16*x^6+32*x^4-19*x^2+3)^2,x, algorithm="fricas")
```

```
[Out] -1/108*(480*x^5 - 624*x^3 - 30*sqrt(3)*(16*x^6 - 32*x^4 + 19*x^2 - 3)*log((
4*x^2 - 4*sqrt(3)*x + 3)/(4*x^2 - 3)) + 14*(16*x^6 - 32*x^4 + 19*x^2 - 3)*l
og(2*x + 1) - 14*(16*x^6 - 32*x^4 + 19*x^2 - 3)*log(2*x - 1) - 67*(16*x^6 -
 32*x^4 + 19*x^2 - 3)*log(x + 1) + 67*(16*x^6 - 32*x^4 + 19*x^2 - 3)*log(x
- 1) + 162*x)/(16*x^6 - 32*x^4 + 19*x^2 - 3)
```

Sympy [A] time = 1.10252, size = 104, normalized size = 1.17

$$-\frac{80x^5-104x^3+27x}{288x^6-576x^4+342x^2-54}-\frac{67\log(x-1)}{108}+\frac{7\log\left(x-\frac{1}{2}\right)}{54}-\frac{7\log\left(x+\frac{1}{2}\right)}{54}+\frac{67\log(x+1)}{108}+\frac{5\sqrt{3}\log\left(x-\frac{\sqrt{3}}{2}\right)}{18}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-16*x**6+32*x**4-19*x**2+3)**2,x)
```

```
[Out] -(80*x**5 - 104*x**3 + 27*x)/(288*x**6 - 576*x**4 + 342*x**2 - 54) - 67*log
(x - 1)/108 + 7*log(x - 1/2)/54 - 7*log(x + 1/2)/54 + 67*log(x + 1)/108 + 5
*sqrt(3)*log(x - sqrt(3)/2)/18 - 5*sqrt(3)*log(x + sqrt(3)/2)/18
```

Giac [A] time = 1.1205, size = 131, normalized size = 1.47

$$\frac{5}{18}\sqrt{3}\log\left(\frac{\left|8\,x-4\,\sqrt{3}\right|}{\left|8\,x+4\,\sqrt{3}\right|}\right)-\frac{80\,x^5-104\,x^3+27\,x}{18\left(16\,x^6-32\,x^4+19\,x^2-3\right)}-\frac{7}{54}\,\log\left(\left|2\,x+1\right|\right)+\frac{7}{54}\,\log\left(\left|2\,x-1\right|\right)+\frac{67}{108}\,\log\left(\left|x+1\right|\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-16*x^6+32*x^4-19*x^2+3)^2,x, algorithm="giac")
```

```
[Out] 5/18*sqrt(3)*log(abs(8*x - 4*sqrt(3))/abs(8*x + 4*sqrt(3))) - 1/18*(80*x^5
- 104*x^3 + 27*x)/(16*x^6 - 32*x^4 + 19*x^2 - 3) - 7/54*log(abs(2*x + 1)) +
 7/54*log(abs(2*x - 1)) + 67/108*log(abs(x + 1)) - 67/108*log(abs(x - 1))
```

3.76 $\int \frac{1}{(3-19x^2+32x^4-16x^6)^3} \, dx$

Optimal. Leaf size=161

$$\frac{5x}{3(3-4x^2)} - \frac{2x}{3(3-4x^2)^2} - \frac{7}{108(1-2x)} + \frac{67}{432(1-x)} - \frac{67}{432(x+1)} + \frac{7}{108(2x+1)} + \frac{1}{108(1-2x)^2} + \frac{1}{432(1-x)^2} - \frac{}{432}$$

```
[Out] 1/(108*(1 - 2*x)^2) - 7/(108*(1 - 2*x)) + 1/(432*(1 - x)^2) + 67/(432*(1 -
x)) - 1/(432*(1 + x)^2) - 67/(432*(1 + x)) - 1/(108*(1 + 2*x)^2) + 7/(108*(
1 + 2*x)) - (2*x)/(3*(3 - 4*x^2)^2) + (5*x)/(3*(3 - 4*x^2)) + (3913*ArcTanh
[x])/648 + (67*ArcTanh[2*x])/162 + (5*ArcTanh[(2*x)/Sqrt[3]])/(6*Sqrt[3]) -
 4*Sqrt[3]*ArcTanh[(2*x)/Sqrt[3]]
```

Rubi [A] time = 0.119377, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2057, 207, 199}

$$\frac{5x}{3(3-4x^2)} - \frac{2x}{3(3-4x^2)^2} - \frac{7}{108(1-2x)} + \frac{67}{432(1-x)} - \frac{67}{432(x+1)} + \frac{7}{108(2x+1)} + \frac{1}{108(1-2x)^2} + \frac{1}{432(1-x)^2} - \frac{}{432}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 19*x^2 + 32*x^4 - 16*x^6)^(-3),x]
```

```
[Out] 1/(108*(1 - 2*x)^2) - 7/(108*(1 - 2*x)) + 1/(432*(1 - x)^2) + 67/(432*(1 -
x)) - 1/(432*(1 + x)^2) - 67/(432*(1 + x)) - 1/(108*(1 + 2*x)^2) + 7/(108*(
1 + 2*x)) - (2*x)/(3*(3 - 4*x^2)^2) + (5*x)/(3*(3 - 4*x^2)) + (3913*ArcTanh
[x])/648 + (67*ArcTanh[2*x])/162 + (5*ArcTanh[(2*x)/Sqrt[3]])/(6*Sqrt[3]) -
 4*Sqrt[3]*ArcTanh[(2*x)/Sqrt[3]]
```

Rule 2057

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P /. x -> Sqrt[x]]}, Int[Expan
dIntegrand[(u /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; Po
lyQ[P, x^2] && ILtQ[p, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rubi steps

$$\int \frac{1}{\left(3-19x^2+32x^4-16x^6\right)^3}\,dx = \int \left(-\frac{1}{216(-1+x)^3}+\frac{67}{432(-1+x)^2}+\frac{1}{216(1+x)^3}+\frac{67}{432(1+x)^2}-\frac{1}{27(-1+2x)^3}-\right.$$

$$=\frac{1}{108(1-2x)^2}-\frac{7}{108(1-2x)}+\frac{1}{432(1-x)^2}+\frac{67}{432(1-x)}-\frac{1}{432(1+x)^2}-\frac{67}{432(1+x}$$

$$=\frac{1}{108(1-2x)^2}-\frac{7}{108(1-2x)}+\frac{1}{432(1-x)^2}+\frac{67}{432(1-x)}-\frac{1}{432(1+x)^2}-\frac{67}{432(1+x}$$

$$=\frac{1}{108(1-2x)^2}-\frac{7}{108(1-2x)}+\frac{1}{432(1-x)^2}+\frac{67}{432(1-x)}-\frac{1}{432(1+x)^2}-\frac{67}{432(1+x}$$

$$=\frac{1}{108(1-2x)^2}-\frac{7}{108(1-2x)}+\frac{1}{432(1-x)^2}+\frac{67}{432(1-x)}-\frac{1}{432(1+x)^2}-\frac{67}{432(1+x}$$

Mathematica [A] time = 0.084386, size = 137, normalized size = 0.85

$$\frac{\frac{36x\left(80x^4-104x^2+27\right)}{\left(-16x^6+32x^4-19x^2+3\right)^2}-\frac{6x\left(2288x^4-2384x^2+345\right)}{16x^6-32x^4+19x^2-3}-268\log(1-2x)+2412\sqrt{3}\log\left(\sqrt{3}-2x\right)-3913\log(1-x)+3913\log(x+1}{1296}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 19*x^2 + 32*x^4 - 16*x^6)^(-3),x]
```

```
[Out] ((36*x*(27 - 104*x^2 + 80*x^4))/(3 - 19*x^2 + 32*x^4 - 16*x^6)^2 - (6*x*(34
5 - 2384*x^2 + 2288*x^4))/(-3 + 19*x^2 - 32*x^4 + 16*x^6) - 268*Log[1 - 2*x
```

```
] + 2412*Sqrt[3]*Log[Sqrt[3] - 2*x] - 3913*Log[1 - x] + 3913*Log[1 + x] + 2
68*Log[1 + 2*x] - 2412*Sqrt[3]*Log[Sqrt[3] + 2*x])/1296
```

Maple [A] time = 0.021, size = 126, normalized size = 0.8

$$\frac{1}{432 (x-1)^2}-\frac{67}{432 x-432}-\frac{3913 \ln (x-1)}{1296}+64 \frac{1}{\left(4 x^2-3\right)^2}\left(-\frac{5 x^3}{48}+\frac{13 x}{192}\right)-\frac{67 \sqrt{3}}{18}\mathrm{Artanh}\left(\frac{2 x\sqrt{3}}{3}\right)+\frac{1}{108 (2 x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-16*x^6+32*x^4-19*x^2+3)^3,x)
```

```
[Out] 1/432/(x-1)^2-67/432/(x-1)-3913/1296*ln(x-1)+64*(-5/48*x^3+13/192*x)/(4*x^2
-3)^2-67/18*arctanh(2/3*x*3^(1/2))*3^(1/2)+1/108/(2*x-1)^2+7/108/(2*x-1)-67
/324*ln(2*x-1)-1/108/(1+2*x)^2+7/108/(1+2*x)+67/324*ln(1+2*x)-1/432/(1+x)^2
-67/432/(1+x)+3913/1296*ln(1+x)
```

Maxima [A] time = 1.85468, size = 161, normalized size = 1.

$$\frac{67}{36}\sqrt{3}\log\left(\frac{2 x-\sqrt{3}}{2 x+\sqrt{3}}\right)-\frac{36608 x^{11}-111360 x^9+125280 x^7-63680 x^5+14331 x^3-1197 x}{216\left(256 x^{12}-1024 x^{10}+1632 x^8-1312 x^6+553 x^4-114 x^2+9\right)}+\frac{67}{324}\log(2 x+1)-\frac{67}{324}\mathrm{l}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-16*x^6+32*x^4-19*x^2+3)^3,x, algorithm="maxima")
```

```
[Out] 67/36*sqrt(3)*log((2*x - sqrt(3))/(2*x + sqrt(3))) - 1/216*(36608*x^11 - 11
1360*x^9 + 125280*x^7 - 63680*x^5 + 14331*x^3 - 1197*x)/(256*x^12 - 1024*x^
10 + 1632*x^8 - 1312*x^6 + 553*x^4 - 114*x^2 + 9) + 67/324*log(2*x + 1) - 6
7/324*log(2*x - 1) + 3913/1296*log(x + 1) - 3913/1296*log(x - 1)
```

Fricas [B] time = 1.75742, size = 849, normalized size = 5.27

$$-\frac{219648 x^{11}-668160 x^9+751680 x^7-382080 x^5+85986 x^3-2412 \sqrt{3}\left(256 x^{12}-1024 x^{10}+1632 x^8-1312 x^6+553 x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-16*x^6+32*x^4-19*x^2+3)^3,x, algorithm="fricas")
```

```
[Out] -1/1296*(219648*x^11 - 668160*x^9 + 751680*x^7 - 382080*x^5 + 85986*x^3 - 2
412*sqrt(3)*(256*x^12 - 1024*x^10 + 1632*x^8 - 1312*x^6 + 553*x^4 - 114*x^2
 + 9)*log((4*x^2 - 4*sqrt(3)*x + 3)/(4*x^2 - 3)) - 268*(256*x^12 - 1024*x^1
0 + 1632*x^8 - 1312*x^6 + 553*x^4 - 114*x^2 + 9)*log(2*x + 1) + 268*(256*x^
12 - 1024*x^10 + 1632*x^8 - 1312*x^6 + 553*x^4 - 114*x^2 + 9)*log(2*x - 1)
- 3913*(256*x^12 - 1024*x^10 + 1632*x^8 - 1312*x^6 + 553*x^4 - 114*x^2 + 9)
*log(x + 1) + 3913*(256*x^12 - 1024*x^10 + 1632*x^8 - 1312*x^6 + 553*x^4 -
114*x^2 + 9)*log(x - 1) - 7182*x)/(256*x^12 - 1024*x^10 + 1632*x^8 - 1312*x
^6 + 553*x^4 - 114*x^2 + 9)
```

Sympy [A] time = 1.2412, size = 134, normalized size = 0.83

$$-\frac{36608x^{11}-111360x^{9}+125280x^{7}-63680x^{5}+14331x^{3}-1197x}{55296x^{12}-221184x^{10}+352512x^{8}-283392x^{6}+119448x^{4}-24624x^{2}+1944}-\frac{3913\log(x-1)}{1296}-\frac{67\log\left(x-\frac{1}{2}\right)}{324}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-16*x**6+32*x**4-19*x**2+3)**3,x)
```

```
[Out] -(36608*x**11 - 111360*x**9 + 125280*x**7 - 63680*x**5 + 14331*x**3 - 1197*
x)/(55296*x**12 - 221184*x**10 + 352512*x**8 - 283392*x**6 + 119448*x**4 -
24624*x**2 + 1944) - 3913*log(x - 1)/1296 - 67*log(x - 1/2)/324 + 67*log(x
+ 1/2)/324 + 3913*log(x + 1)/1296 + 67*sqrt(3)*log(x - sqrt(3)/2)/36 - 67*s
qrt(3)*log(x + sqrt(3)/2)/36
```

Giac [A] time = 1.15641, size = 151, normalized size = 0.94

$$\frac{67}{36}\sqrt{3}\log\left(\frac{\left|8x-4\sqrt{3}\right|}{\left|8x+4\sqrt{3}\right|}\right)-\frac{36608x^{11}-111360x^{9}+125280x^{7}-63680x^{5}+14331x^{3}-1197x}{216\left(16x^{6}-32x^{4}+19x^{2}-3\right)^{2}}+\frac{67}{324}\log\left(\left|2x+1\right|\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-16*x^6+32*x^4-19*x^2+3)^3,x, algorithm="giac")
```

```
[Out] 67/36*sqrt(3)*log(abs(8*x - 4*sqrt(3))/abs(8*x + 4*sqrt(3))) - 1/216*(36608
*x^11 - 111360*x^9 + 125280*x^7 - 63680*x^5 + 14331*x^3 - 1197*x)/(16*x^6 -
 32*x^4 + 19*x^2 - 3)^2 + 67/324*log(abs(2*x + 1)) - 67/324*log(abs(2*x - 1
)) + 3913/1296*log(abs(x + 1)) - 3913/1296*log(abs(x - 1))
```

3.77 $\int \frac{1}{\left(-1+7x^2-7x^4+x^6\right)^2} dx$

Optimal. Leaf size=91

$$\frac{x}{32\left(1-x^2\right)}+\frac{\left(99-17x^2\right)x}{128\left(x^4-6x^2+1\right)}+\frac{5}{32}\tanh^{-1}(x)+\frac{1}{512}\left(3\sqrt{2}-4\right)\tanh^{-1}\left(\left(\sqrt{2}-1\right)x\right)+\frac{1}{512}\left(4+3\sqrt{2}\right)\tanh^{-1}\left(\left(1+\right.\right.$$

```
[Out] x/(32*(1 - x^2)) + (x*(99 - 17*x^2))/(128*(1 - 6*x^2 + x^4)) + (5*ArcTanh[x
])/32 + ((-4 + 3*Sqrt[2])*ArcTanh[(-1 + Sqrt[2])*x])/512 + ((4 + 3*Sqrt[2])
*ArcTanh[(1 + Sqrt[2])*x])/512
```

Rubi [B] time = 0.130008, antiderivative size = 205, normalized size of antiderivative = 2.25, number of steps used = 15, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {2057, 207, 638, 618, 206, 632, 31}

$$-\frac{41-17x}{256\left(-x^2+2x+1\right)}+\frac{17x+41}{256\left(-x^2-2x+1\right)}+\frac{1}{64(1-x)}-\frac{1}{64(x+1)}+\frac{1}{512}\left(2-7\sqrt{2}\right)\log\left(-x-\sqrt{2}+1\right)+\frac{1}{512}\left(2+7\right.$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 7*x^2 - 7*x^4 + x^6)^(-2),x]
```

```
[Out] 1/(64*(1 - x)) - 1/(64*(1 + x)) + (41 + 17*x)/(256*(1 - 2*x - x^2)) - (41 -
 17*x)/(256*(1 + 2*x - x^2)) - (17*ArcTanh[(1 - x)/Sqrt[2]])/(256*Sqrt[2])
+ (5*ArcTanh[x])/32 + (17*ArcTanh[(1 + x)/Sqrt[2]])/(256*Sqrt[2]) + ((2 - 7
*Sqrt[2])*Log[1 - Sqrt[2] - x])/512 + ((2 + 7*Sqrt[2])*Log[1 + Sqrt[2] - x]
)/512 - ((2 - 7*Sqrt[2])*Log[1 - Sqrt[2] + x])/512 - ((2 + 7*Sqrt[2])*Log[1
 + Sqrt[2] + x])/512
```

Rule 2057

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P /. x -> Sqrt[x]]}, Int[Expan
dIntegrand[(u /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; Po
lyQ[P, x^2] && ILtQ[p, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{\left(-1+7x^2-7x^4+x^6\right)^2}\,dx = \int \left(\frac{1}{64(-1+x)^2}+\frac{1}{64(1+x)^2}-\frac{5}{32\left(-1+x^2\right)}+\frac{29-12x}{64\left(-1-2x+x^2\right)^2}+\frac{6+x}{128\left(-1-2x+x\right.}\right.$$

$$= \frac{1}{64(1-x)}-\frac{1}{64(1+x)}+\frac{1}{128}\int \frac{6+x}{-1-2x+x^2}\,dx+\frac{1}{128}\int \frac{6-x}{-1+2x+x^2}\,dx+\frac{1}{64}\int$$

$$= \frac{1}{64(1-x)}-\frac{1}{64(1+x)}+\frac{41+17x}{256\left(1-2x-x^2\right)}-\frac{41-17x}{256\left(1+2x-x^2\right)}+\frac{5}{32}\tanh^{-1}(x)-\frac{17}{256}$$

$$= \frac{1}{64(1-x)}-\frac{1}{64(1+x)}+\frac{41+17x}{256\left(1-2x-x^2\right)}-\frac{41-17x}{256\left(1+2x-x^2\right)}+\frac{5}{32}\tanh^{-1}(x)+\frac{1}{512}$$

$$= \frac{1}{64(1-x)}-\frac{1}{64(1+x)}+\frac{41+17x}{256\left(1-2x-x^2\right)}-\frac{41-17x}{256\left(1+2x-x^2\right)}-\frac{17\tanh^{-1}\left(\frac{1-x}{\sqrt{2}}\right)}{256\sqrt{2}}+\frac{5}{3}$$

Mathematica [A] time = 0.0883109, size = 132, normalized size = 1.45

$$\frac{-\frac{8x\left(21x^4-140x^2+103\right)}{x^6-7x^4+7x^2-1}-80\log(1-x)-\left(4+3\sqrt{2}\right)\log\left(-x+\sqrt{2}-1\right)+\left(4-3\sqrt{2}\right)\log\left(-x+\sqrt{2}+1\right)+80\log(x+1)+\left(4\right.}{1024}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + 7*x^2 - 7*x^4 + x^6)^(-2),x]
```

```
[Out] ((-8*x*(103 - 140*x^2 + 21*x^4))/(-1 + 7*x^2 - 7*x^4 + x^6) - 80*Log[1 - x]
 - (4 + 3*Sqrt[2])*Log[-1 + Sqrt[2] - x] + (4 - 3*Sqrt[2])*Log[1 + Sqrt[2]
- x] + 80*Log[1 + x] + (4 + 3*Sqrt[2])*Log[-1 + Sqrt[2] + x] + (-4 + 3*Sqrt
[2])*Log[1 + Sqrt[2] + x])/1024
```

Maple [A] time = 0.019, size = 116, normalized size = 1.3

$$\frac{1}{128\,x^2-256\,x-128}\left(-\frac{17\,x}{2}+\frac{41}{2}\right)+\frac{\ln\left(x^2-2\,x-1\right)}{256}+\frac{3\,\sqrt{2}}{512}\mathrm{Artanh}\left(\frac{(2\,x-2)\,\sqrt{2}}{4}\right)-\frac{1}{64\,x-64}-\frac{5\,\ln(x-1)}{64}-\frac{}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6-7*x^4+7*x^2-1)^2,x)
```

```
[Out] 1/128*(-17/2*x+41/2)/(x^2-2*x-1)+1/256*ln(x^2-2*x-1)+3/512*2^(1/2)*arctanh(
1/4*(2*x-2)*2^(1/2))-1/64/(x-1)-5/64*ln(x-1)-1/128*(17/2*x+41/2)/(x^2+2*x-1
)-1/256*ln(x^2+2*x-1)+3/512*2^(1/2)*arctanh(1/4*(2+2*x)*2^(1/2))-1/64/(1+x)
+5/64*ln(1+x)
```

Maxima [A] time = 1.95656, size = 154, normalized size = 1.69

$$-\frac{3}{1024}\sqrt{2}\log\left(\frac{x-\sqrt{2}+1}{x+\sqrt{2}+1}\right)-\frac{3}{1024}\sqrt{2}\log\left(\frac{x-\sqrt{2}-1}{x+\sqrt{2}-1}\right)-\frac{21\,x^{5}-140\,x^{3}+103\,x}{128\left(x^{6}-7\,x^{4}+7\,x^{2}-1\right)}-\frac{1}{256}\log\left(x^{2}+2\,x-1\right)+\frac{1}{256}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6-7*x^4+7*x^2-1)^2,x, algorithm="maxima")
```

```
[Out] -3/1024*sqrt(2)*log((x - sqrt(2) + 1)/(x + sqrt(2) + 1)) - 3/1024*sqrt(2)*l
og((x - sqrt(2) - 1)/(x + sqrt(2) - 1)) - 1/128*(21*x^5 - 140*x^3 + 103*x)/
(x^6 - 7*x^4 + 7*x^2 - 1) - 1/256*log(x^2 + 2*x - 1) + 1/256*log(x^2 - 2*x
- 1) + 5/64*log(x + 1) - 5/64*log(x - 1)
```

Fricas [B] time = 1.78809, size = 589, normalized size = 6.47

$$-\frac{168\,x^{5}-1120\,x^{3}-3\,\sqrt{2}\left(x^{6}-7\,x^{4}+7\,x^{2}-1\right)\log\left(\frac{x^{2}+2\,\sqrt{2}(x+1)+2\,x+3}{x^{2}+2\,x-1}\right)-3\,\sqrt{2}\left(x^{6}-7\,x^{4}+7\,x^{2}-1\right)\log\left(\frac{x^{2}+2\,\sqrt{2}(x-1)-2\,x+3}{x^{2}-2\,x-1}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6-7*x^4+7*x^2-1)^2,x, algorithm="fricas")
```

```
[Out] -1/1024*(168*x^5 - 1120*x^3 - 3*sqrt(2)*(x^6 - 7*x^4 + 7*x^2 - 1)*log((x^2
+ 2*sqrt(2)*(x + 1) + 2*x + 3)/(x^2 + 2*x - 1)) - 3*sqrt(2)*(x^6 - 7*x^4 +
7*x^2 - 1)*log((x^2 + 2*sqrt(2)*(x - 1) - 2*x + 3)/(x^2 - 2*x - 1)) + 4*(x^
6 - 7*x^4 + 7*x^2 - 1)*log(x^2 + 2*x - 1) - 4*(x^6 - 7*x^4 + 7*x^2 - 1)*log
(x^2 - 2*x - 1) - 80*(x^6 - 7*x^4 + 7*x^2 - 1)*log(x + 1) + 80*(x^6 - 7*x^4
 + 7*x^2 - 1)*log(x - 1) + 824*x)/(x^6 - 7*x^4 + 7*x^2 - 1)
```

Sympy [B] time = 1.11242, size = 296, normalized size = 3.25

$$-\frac{21x^5-140x^3+103x}{128x^6-896x^4+896x^2-128}-\frac{5\log(x-1)}{64}+\frac{5\log(x+1)}{64}+\left(-\frac{1}{256}+\frac{3\sqrt{2}}{1024}\right)\log\left(x-\frac{8071264001}{202624020}-\frac{471550901878}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**6-7*x**4+7*x**2-1)**2,x)
```

```
[Out] -(21*x**5 - 140*x**3 + 103*x)/(128*x**6 - 896*x**4 + 896*x**2 - 128) - 5*lo
g(x - 1)/64 + 5*log(x + 1)/64 + (-1/256 + 3*sqrt(2)/1024)*log(x - 807126400
1/202624020 - 471550901878784*(-1/256 + 3*sqrt(2)/1024)**3/2979765 + 129955
2375287054336*(-1/256 + 3*sqrt(2)/1024)**5/50656005 + 8071264001*sqrt(2)/27
0165360) + (-3*sqrt(2)/1024 - 1/256)*log(x - 8071264001*sqrt(2)/270165360 -
 8071264001/202624020 + 1299552375287054336*(-3*sqrt(2)/1024 - 1/256)**5/50
656005 - 471550901878784*(-3*sqrt(2)/1024 - 1/256)**3/2979765) + (1/256 - 3
*sqrt(2)/1024)*log(x - 8071264001*sqrt(2)/270165360 + 1299552375287054336*(
1/256 - 3*sqrt(2)/1024)**5/50656005 - 471550901878784*(1/256 - 3*sqrt(2)/10
24)**3/2979765 + 8071264001/202624020) + (1/256 + 3*sqrt(2)/1024)*log(x - 4
71550901878784*(1/256 + 3*sqrt(2)/1024)**3/2979765 + 1299552375287054336*(1
/256 + 3*sqrt(2)/1024)**5/50656005 + 8071264001/202624020 + 8071264001*sqrt
(2)/270165360)
```

Giac [A] time = 1.11241, size = 181, normalized size = 1.99

$$-\frac{3}{1024}\sqrt{2}\log\left(\frac{\left|2x-2\sqrt{2}+2\right|}{\left|2x+2\sqrt{2}+2\right|}\right)-\frac{3}{1024}\sqrt{2}\log\left(\frac{\left|2x-2\sqrt{2}-2\right|}{\left|2x+2\sqrt{2}-2\right|}\right)-\frac{21x^5-140x^3+103x}{128\left(x^6-7x^4+7x^2-1\right)}-\frac{1}{256}\log\left(\left|x^2+2x-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6-7*x^4+7*x^2-1)^2,x, algorithm="giac")
```

```
[Out] -3/1024*sqrt(2)*log(abs(2*x - 2*sqrt(2) + 2)/abs(2*x + 2*sqrt(2) + 2)) - 3/
1024*sqrt(2)*log(abs(2*x - 2*sqrt(2) - 2)/abs(2*x + 2*sqrt(2) - 2)) - 1/128
*(21*x^5 - 140*x^3 + 103*x)/(x^6 - 7*x^4 + 7*x^2 - 1) - 1/256*log(abs(x^2 +
 2*x - 1)) + 1/256*log(abs(x^2 - 2*x - 1)) + 5/64*log(abs(x + 1)) - 5/64*lo
g(abs(x - 1))
```

3.78 $\int \frac{x^3}{c+(a+bx)^2}\,dx$

Optimal. Leaf size=78

$$\frac{(3a^2-c)\log\left((a+bx)^2+c\right)}{2b^4}-\frac{a\left(a^2-3c\right)\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{b^4\sqrt{c}}+\frac{(a+bx)^2}{2b^4}-\frac{3ax}{b^3}$$

```
[Out] (-3*a*x)/b^3 + (a + b*x)^2/(2*b^4) - (a*(a^2 - 3*c)*ArcTan[(a + b*x)/Sqrt[c
]])/(b^4*Sqrt[c]) + ((3*a^2 - c)*Log[c + (a + b*x)^2])/(2*b^4)
```

Rubi [A] time = 0.0630097, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {371, 702, 635, 203, 260}

$$\frac{(3a^2-c)\log\left((a+bx)^2+c\right)}{2b^4}-\frac{a\left(a^2-3c\right)\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{b^4\sqrt{c}}+\frac{(a+bx)^2}{2b^4}-\frac{3ax}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(c + (a + b*x)^2),x]
```


```
[Out] (-3*a*x)/b^3 + (a + b*x)^2/(2*b^4) - (a*(a^2 - 3*c)*ArcTan[(a + b*x)/Sqrt[c
]])/(b^4*Sqrt[c]) + ((3*a^2 - c)*Log[c + (a + b*x)^2])/(2*b^4)
```


Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 702

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{x^3}{c+(a+bx)^2}\,dx = \frac{\text{Subst}\left(\int \frac{(-a+x)^3}{c+x^2}\,dx, x, a+bx\right)}{b^4}$$

$$= \frac{\text{Subst}\left(\int \left(-3a+x-\frac{a^3-3ac-\left(3a^2-c\right)x}{c+x^2}\right)dx, x, a+bx\right)}{b^4}$$

$$= -\frac{3ax}{b^3}+\frac{(a+bx)^2}{2b^4}-\frac{\text{Subst}\left(\int \frac{a^3-3ac-\left(3a^2-c\right)x}{c+x^2}\,dx, x, a+bx\right)}{b^4}$$

$$= -\frac{3ax}{b^3}+\frac{(a+bx)^2}{2b^4}-\frac{\left(a\left(a^2-3c\right)\right)\text{Subst}\left(\int \frac{1}{c+x^2}\,dx, x, a+bx\right)}{b^4}+\frac{\left(3a^2-c\right)\text{Subst}\left(\int \frac{x}{c+x^2}\,dx, x, a\right.}{b^4}$$

$$= -\frac{3ax}{b^3}+\frac{(a+bx)^2}{2b^4}-\frac{a\left(a^2-3c\right)\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{b^4\sqrt{c}}+\frac{\left(3a^2-c\right)\log\left(c+(a+bx)^2\right)}{2b^4}$$

Mathematica [A] time = 0.0496587, size = 73, normalized size = 0.94

$$\frac{\left(3a^2-c\right)\log\left(a^2+2abx+b^2x^2+c\right)-\frac{2\left(a^3-3ac\right)\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}}+bx(bx-4a)}{2b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(c + (a + b*x)^2),x]
```

```
[Out] (b*x*(-4*a + b*x) - (2*(a^3 - 3*a*c)*ArcTan[(a + b*x)/Sqrt[c]])/Sqrt[c] + (
3*a^2 - c)*Log[a^2 + c + 2*a*b*x + b^2*x^2])/(2*b^4)
```

Maple [A] time = 0.005, size = 127, normalized size = 1.6

$$\frac{x^2}{2\,b^2}-2\,\frac{ax}{b^3}+\frac{3\,\ln\left(b^2x^2+2\,abx+a^2+c\right)a^2}{2\,b^4}-\frac{\ln\left(b^2x^2+2\,abx+a^2+c\right)c}{2\,b^4}-\frac{a^3}{b^4}\arctan\left(\frac{2\,b^2x+2\,ab}{2\,b}\frac{1}{\sqrt{c}}\right)\frac{1}{\sqrt{c}}+3\,\frac{\sqrt{c}a}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c+(b*x+a)^2),x)
```

```
[Out] 1/2/b^2*x^2-2*a*x/b^3+3/2/b^4*ln(b^2*x^2+2*a*b*x+a^2+c)*a^2-1/2/b^4*ln(b^2*
x^2+2*a*b*x+a^2+c)*c-1/b^4/c^(1/2)*arctan(1/2*(2*b^2*x+2*a*b)/b/c^(1/2))*a^
3+3/b^4*c^(1/2)*arctan(1/2*(2*b^2*x+2*a*b)/b/c^(1/2))*a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c+(b*x+a)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.8337, size = 466, normalized size = 5.97

$$\left[\frac{b^2cx^2-4\,abcx+\left(a^3-3\,ac\right)\sqrt{-c}\log\left(\frac{b^2x^2+2\,abx+a^2-2\,(bx+a)\sqrt{-c}-c}{b^2x^2+2\,abx+a^2+c}\right)+\left(3\,a^2c-c^2\right)\log\left(b^2x^2+2\,abx+a^2+c\right)}{2\,b^4c},\frac{b^2cx^2-4\,abc}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c+(b*x+a)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(b^2*c*x^2 - 4*a*b*c*x + (a^3 - 3*a*c)*sqrt(-c)*log((b^2*x^2 + 2*a*b*x
 + a^2 - 2*(b*x + a)*sqrt(-c) - c)/(b^2*x^2 + 2*a*b*x + a^2 + c)) + (3*a^2*
c - c^2)*log(b^2*x^2 + 2*a*b*x + a^2 + c))/(b^4*c), 1/2*(b^2*c*x^2 - 4*a*b*
c*x - 2*(a^3 - 3*a*c)*sqrt(c)*arctan((b*x + a)/sqrt(c)) + (3*a^2*c - c^2)*l
og(b^2*x^2 + 2*a*b*x + a^2 + c))/(b^4*c)]
```

Sympy [B] time = 0.760163, size = 209, normalized size = 2.68

$$-\frac{2ax}{b^3} + \left(-\frac{a\sqrt{-c}\left(a^2-3c\right)}{2b^4c} + \frac{3a^2-c}{2b^4}\right)\log\left(x + \frac{a^4 - 2b^4c\left(-\frac{a\sqrt{-c}\left(a^2-3c\right)}{2b^4c} + \frac{3a^2-c}{2b^4}\right) - c^2}{a^3b - 3abc}\right) + \left(\frac{a\sqrt{-c}\left(a^2-3c\right)}{2b^4c} + \frac{3a^2-c}{2b^4}\right)\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c+(b*x+a)**2),x)
```

```
[Out] -2*a*x/b**3 + (-a*sqrt(-c)*(a**2 - 3*c)/(2*b**4*c) + (3*a**2 - c)/(2*b**4))
*log(x + (a**4 - 2*b**4*c*(-a*sqrt(-c)*(a**2 - 3*c)/(2*b**4*c) + (3*a**2 -
c)/(2*b**4)) - c**2)/(a**3*b - 3*a*b*c)) + (a*sqrt(-c)*(a**2 - 3*c)/(2*b**4
*c) + (3*a**2 - c)/(2*b**4))*log(x + (a**4 - 2*b**4*c*(a*sqrt(-c)*(a**2 - 3
*c)/(2*b**4*c) + (3*a**2 - c)/(2*b**4)) - c**2)/(a**3*b - 3*a*b*c)) + x**2/
(2*b**2)
```

Giac [A] time = 1.11854, size = 104, normalized size = 1.33

$$\frac{\left(3\,a^2-c\right)\log\left(b^2x^2+2\,abx+a^2+c\right)}{2\,b^4} - \frac{\left(a^3-3\,ac\right)\arctan\left(\frac{bx+a}{\sqrt{c}}\right)}{b^4\sqrt{c}} + \frac{b^2x^2-4\,abx}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c+(b*x+a)^2),x, algorithm="giac")
```

```
[Out] 1/2*(3*a^2 - c)*log(b^2*x^2 + 2*a*b*x + a^2 + c)/b^4 - (a^3 - 3*a*c)*arctan
((b*x + a)/sqrt(c))/(b^4*sqrt(c)) + 1/2*(b^2*x^2 - 4*a*b*x)/b^4
```

3.79 $\int \frac{x^2}{c+(a+bx)^2} dx$

Optimal. Leaf size=50

$$\frac{(a^2-c)\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{b^3\sqrt{c}} - \frac{a\log\left((a+bx)^2+c\right)}{b^3} + \frac{x}{b^2}$$

```
[Out] x/b^2 + ((a^2 - c)*ArcTan[(a + b*x)/Sqrt[c]])/(b^3*Sqrt[c]) - (a*Log[c + (a
 + b*x)^2])/b^3
```

Rubi [A] time = 0.0387091, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {371, 702, 635, 203, 260}

$$\frac{(a^2-c)\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{b^3\sqrt{c}} - \frac{a\log\left((a+bx)^2+c\right)}{b^3} + \frac{x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(c + (a + b*x)^2),x]
```

```
[Out] x/b^2 + ((a^2 - c)*ArcTan[(a + b*x)/Sqrt[c]])/(b^3*Sqrt[c]) - (a*Log[c + (a
 + b*x)^2])/b^3
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 702

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{c+(a+bx)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{(-a+x)^2}{c+x^2}\,dx, x, a+bx\right)}{b^3}\\
&= \frac{\text{Subst}\left(\int \left(1+\frac{a^2-c-2ax}{c+x^2}\right)dx, x, a+bx\right)}{b^3}\\
&= \frac{x}{b^2}+\frac{\text{Subst}\left(\int \frac{a^2-c-2ax}{c+x^2}\,dx, x, a+bx\right)}{b^3}\\
&= \frac{x}{b^2}-\frac{(2a)\,\text{Subst}\left(\int \frac{x}{c+x^2}\,dx, x, a+bx\right)}{b^3}+\frac{\left(a^2-c\right)\text{Subst}\left(\int \frac{1}{c+x^2}\,dx, x, a+bx\right)}{b^3}\\
&= \frac{x}{b^2}+\frac{\left(a^2-c\right)\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{b^3\sqrt{c}}-\frac{a\log\left(c+(a+bx)^2\right)}{b^3}\end{aligned}$$

Mathematica [A] time = 0.0276808, size = 54, normalized size = 1.08

$$\frac{-a\log\left(a^2+2abx+b^2x^2+c\right)+\frac{\left(a^2-c\right)\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}}+bx}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(c + (a + b*x)^2),x]
```

```
[Out] (b*x + ((a^2 - c)*ArcTan[(a + b*x)/Sqrt[c]])/Sqrt[c] - a*Log[a^2 + c + 2*a*
b*x + b^2*x^2])/b^3
```

Maple [A] time = 0.002, size = 89, normalized size = 1.8

$$\frac{x}{b^2}-\frac{a\ln\left(b^2x^2+2\,abx+a^2+c\right)}{b^3}+\frac{a^2}{b^3}\arctan\left(\frac{2\,b^2x+2\,ab}{2\,b}\frac{1}{\sqrt{c}}\right)\frac{1}{\sqrt{c}}-\frac{1}{b^3}\sqrt{c}\arctan\left(\frac{2\,b^2x+2\,ab}{2\,b}\frac{1}{\sqrt{c}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c+(b*x+a)^2),x)
```

```
[Out] x/b^2-1/b^3*a*ln(b^2*x^2+2*a*b*x+a^2+c)+1/b^3/c^(1/2)*arctan(1/2*(2*b^2*x+2
*a*b)/b/c^(1/2))*a^2-1/b^3*c^(1/2)*arctan(1/2*(2*b^2*x+2*a*b)/b/c^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c+(b*x+a)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.77995, size = 377, normalized size = 7.54

$$\left[\frac{2\,bcx-2\,ac\log\left(b^2x^2+2\,abx+a^2+c\right)+\left(a^2-c\right)\sqrt{-c}\log\left(\frac{b^2x^2+2\,abx+a^2+2\,(bx+a)\sqrt{-c}-c}{b^2x^2+2\,abx+a^2+c}\right)}{2\,b^3c},\frac{bcx-ac\log\left(b^2x^2+2\,abx+a^2+\right.}{b^3c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c+(b*x+a)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*b*c*x - 2*a*c*log(b^2*x^2 + 2*a*b*x + a^2 + c) + (a^2 - c)*sqrt(-c)
*log((b^2*x^2 + 2*a*b*x + a^2 + 2*(b*x + a)*sqrt(-c) - c)/(b^2*x^2 + 2*a*b*
```

```
x + a^2 + c)))/(b^3*c), (b*c*x - a*c*log(b^2*x^2 + 2*a*b*x + a^2 + c) + (a^
2 - c)*sqrt(c)*arctan((b*x + a)/sqrt(c)))/(b^3*c)]
```

Sympy [B] time = 0.577375, size = 153, normalized size = 3.06

$$\left(-\frac{a}{b^3}-\frac{\sqrt{-c}\left(a^2-c\right)}{2b^3c}\right)\log\left(x+\frac{a^3+ac+2b^3c\left(-\frac{a}{b^3}-\frac{\sqrt{-c}\left(a^2-c\right)}{2b^3c}\right)}{a^2b-bc}\right)+\left(-\frac{a}{b^3}+\frac{\sqrt{-c}\left(a^2-c\right)}{2b^3c}\right)\log\left(x+\frac{a^3+ac+2b^3c\left(-\frac{a}{b^3}\right.}{a^2b-bc}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c+(b*x+a)**2),x)
```

```
[Out] (-a/b**3 - sqrt(-c)*(a**2 - c)/(2*b**3*c))*log(x + (a**3 + a*c + 2*b**3*c*(
-a/b**3 - sqrt(-c)*(a**2 - c)/(2*b**3*c)))/(a**2*b - b*c)) + (-a/b**3 + sqr
t(-c)*(a**2 - c)/(2*b**3*c))*log(x + (a**3 + a*c + 2*b**3*c*(-a/b**3 + sqrt
(-c)*(a**2 - c)/(2*b**3*c)))/(a**2*b - b*c)) + x/b**2
```

Giac [A] time = 1.17294, size = 73, normalized size = 1.46

$$\frac{x}{b^2}-\frac{a\log\left(b^2x^2+2\,abx+a^2+c\right)}{b^3}+\frac{\left(a^2-c\right)\arctan\left(\frac{bx+a}{\sqrt{c}}\right)}{b^3\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c+(b*x+a)^2),x, algorithm="giac")
```

```
[Out] x/b^2 - a*log(b^2*x^2 + 2*a*b*x + a^2 + c)/b^3 + (a^2 - c)*arctan((b*x + a)
/sqrt(c))/(b^3*sqrt(c))
```

3.80 $\int \frac{x}{c+(a+bx)^2} \, dx$

Optimal. Leaf size=41

$$\frac{\log \left((a+bx)^2+c\right)}{2b^2}-\frac{a \tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{b^2\sqrt{c}}$$

```
[Out] -((a*ArcTan[(a + b*x)/Sqrt[c]])/(b^2*Sqrt[c])) + Log[c + (a + b*x)^2]/(2*b^
2)
```

Rubi [A] time = 0.0212057, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {371, 635, 203, 260}

$$\frac{\log \left((a+bx)^2+c\right)}{2b^2}-\frac{a \tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{b^2\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(c + (a + b*x)^2),x]
```

```
[Out] -((a*ArcTan[(a + b*x)/Sqrt[c]])/(b^2*Sqrt[c])) + Log[c + (a + b*x)^2]/(2*b^
2)
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{c+(a+bx)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{-a+x}{c+x^2}\,dx,x,a+bx\right)}{b^2}\\ &= \frac{\text{Subst}\left(\int \frac{x}{c+x^2}\,dx,x,a+bx\right)}{b^2}-\frac{a\,\text{Subst}\left(\int \frac{1}{c+x^2}\,dx,x,a+bx\right)}{b^2}\\ &= -\frac{a\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{b^2\sqrt{c}}+\frac{\log\left(c+(a+bx)^2\right)}{2b^2}\end{aligned}$$

Mathematica [A] time = 0.0138319, size = 38, normalized size = 0.93

$$\frac{\log\left((a+bx)^2+c\right)-\frac{2a\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}}}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(c + (a + b*x)^2),x]
```

```
[Out] ((-2*a*ArcTan[(a + b*x)/Sqrt[c]])/Sqrt[c] + Log[c + (a + b*x)^2])/(2*b^2)
```

Maple [A] time = 0.003, size = 54, normalized size = 1.3

$$\frac{\ln\left(b^2x^2+2\,abx+a^2+c\right)}{2\,b^2}-\frac{a}{b^2}\arctan\left(\frac{2\,b^2x+2\,ab}{2\,b}\frac{1}{\sqrt{c}}\right)\frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c+(b*x+a)^2),x)
```

```
[Out] 1/2/b^2*ln(b^2*x^2+2*a*b*x+a^2+c)-a/b^2/c^(1/2)*arctan(1/2*(2*b^2*x+2*a*b)/
b/c^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c+(b*x+a)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.83059, size = 333, normalized size = 8.12

$$\left[-\frac{a\sqrt{-c}\log\left(\frac{b^2x^2+2\,abx+a^2+2\,(bx+a)\sqrt{-c}-c}{b^2x^2+2\,abx+a^2+c}\right)-c\log\left(b^2x^2+2\,abx+a^2+c\right)}{2\,b^2c}, -\frac{2\,a\sqrt{c}\arctan\left(\frac{bx+a}{\sqrt{c}}\right)-c\log\left(b^2x^2+2\,abx+a^2+c\right)}{2\,b^2c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c+(b*x+a)^2),x, algorithm="fricas")
```

```
[Out] [-1/2*(a*sqrt(-c)*log((b^2*x^2 + 2*a*b*x + a^2 + 2*(b*x + a)*sqrt(-c) - c)/
(b^2*x^2 + 2*a*b*x + a^2 + c)) - c*log(b^2*x^2 + 2*a*b*x + a^2 + c))/(b^2*c
), -1/2*(2*a*sqrt(c)*arctan((b*x + a)/sqrt(c)) - c*log(b^2*x^2 + 2*a*b*x +
a^2 + c))/(b^2*c)]
```

Sympy [B] time = 0.229962, size = 124, normalized size = 3.02

$$\left(-\frac{a\sqrt{-c}}{2b^2c}+\frac{1}{2b^2}\right)\log\left(x+\frac{a^2-2b^2c\left(-\frac{a\sqrt{-c}}{2b^2c}+\frac{1}{2b^2}\right)+c}{ab}\right)+\left(\frac{a\sqrt{-c}}{2b^2c}+\frac{1}{2b^2}\right)\log\left(x+\frac{a^2-2b^2c\left(\frac{a\sqrt{-c}}{2b^2c}+\frac{1}{2b^2}\right)+c}{ab}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c+(b*x+a)**2),x)
```

```
[Out] (-a*sqrt(-c)/(2*b**2*c) + 1/(2*b**2))*log(x + (a**2 - 2*b**2*c*(-a*sqrt(-c)
/(2*b**2*c) + 1/(2*b**2)) + c)/(a*b)) + (a*sqrt(-c)/(2*b**2*c) + 1/(2*b**2)
)*log(x + (a**2 - 2*b**2*c*(a*sqrt(-c)/(2*b**2*c) + 1/(2*b**2)) + c)/(a*b))
```

Giac [A] time = 1.14193, size = 58, normalized size = 1.41

$$-\frac{a \arctan\left(\frac{bx+a}{\sqrt{c}}\right)}{b^2\sqrt{c}} + \frac{\log\left(b^2x^2 + 2\,abx + a^2 + c\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c+(b*x+a)^2),x, algorithm="giac")
```

```
[Out] -a*arctan((b*x + a)/sqrt(c))/(b^2*sqrt(c)) + 1/2*log(b^2*x^2 + 2*a*b*x + a^
2 + c)/b^2
```

3.81 $\int \frac{1}{c+(a+bx)^2} dx$

Optimal. Leaf size=21

$$\frac{\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{b\sqrt{c}}$$

```
[Out] ArcTan[(a + b*x)/Sqrt[c]]/(b*Sqrt[c])
```

Rubi [A] time = 0.0081073, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {247, 203}

$$\frac{\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{b\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + (a + b*x)^2)^(-1),x]
```

```
[Out] ArcTan[(a + b*x)/Sqrt[c]]/(b*Sqrt[c])
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{c+(a+bx)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{c+x^2}\,dx, x, a+bx\right)}{b}$$
$$= \frac{\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{b\sqrt{c}}$$

Mathematica [A] time = 0.0033573, size = 21, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{b\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + (a + b*x)^2)^(-1),x]
```

```
[Out] ArcTan[(a + b*x)/Sqrt[c]]/(b*Sqrt[c])
```

Maple [A] time = 0.003, size = 28, normalized size = 1.3

$$\frac{1}{b}\arctan\left(\frac{2\,b^2x+2\,ab}{2\,b}\frac{1}{\sqrt{c}}\right)\frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+(b*x+a)^2),x)
```

```
[Out] 1/b/c^(1/2)*arctan(1/2*(2*b^2*x+2*a*b)/b/c^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+(b*x+a)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.74942, size = 205, normalized size = 9.76

$$\left[-\frac{\sqrt{-c}\log\left(\frac{b^2x^2+2\,abx+a^2-2\,(bx+a)\sqrt{-c}-c}{b^2x^2+2\,abx+a^2+c}\right)}{2\,bc}, \frac{\arctan\left(\frac{bx+a}{\sqrt{c}}\right)}{b\sqrt{c}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+(b*x+a)^2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-c)*log((b^2*x^2 + 2*a*b*x + a^2 - 2*(b*x + a)*sqrt(-c) - c)/(b^
2*x^2 + 2*a*b*x + a^2 + c))/(b*c), arctan((b*x + a)/sqrt(c))/(b*sqrt(c))]
```

Sympy [B] time = 0.175442, size = 54, normalized size = 2.57

$$\frac{-\frac{\sqrt{-\frac{1}{c}}\log\left(x+\frac{a-c\sqrt{-\frac{1}{c}}}{b}\right)}{2}+\frac{\sqrt{-\frac{1}{c}}\log\left(x+\frac{a+c\sqrt{-\frac{1}{c}}}{b}\right)}{2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+(b*x+a)**2),x)
```

```
[Out] (-sqrt(-1/c)*log(x + (a - c*sqrt(-1/c))/b)/2 + sqrt(-1/c)*log(x + (a + c*sq
rt(-1/c))/b)/2)/b
```

Giac [A] time = 1.12321, size = 23, normalized size = 1.1

$$\frac{\arctan\left(\frac{bx+a}{\sqrt{c}}\right)}{b\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+(b*x+a)^2),x, algorithm="giac")
```

```
[Out] arctan((b*x + a)/sqrt(c))/(b*sqrt(c))
```

3.82 $\int \frac{1}{x\left(c+(a+bx)^2\right)}\,dx$

Optimal. Leaf size=59

$$-\frac{\log\left((a+bx)^2+c\right)}{2\left(a^2+c\right)}-\frac{a\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}\left(a^2+c\right)}+\frac{\log(x)}{a^2+c}$$

```
[Out] -((a*ArcTan[(a + b*x)/Sqrt[c]])/(Sqrt[c]*(a^2 + c))) + Log[x]/(a^2 + c) - L
og[c + (a + b*x)^2]/(2*(a^2 + c))
```

Rubi [A] time = 0.0351043, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {371, 706, 31, 635, 203, 260}

$$-\frac{\log\left((a+bx)^2+c\right)}{2\left(a^2+c\right)}-\frac{a\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}\left(a^2+c\right)}+\frac{\log(x)}{a^2+c}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(c + (a + b*x)^2)),x]
```

```
[Out] -((a*ArcTan[(a + b*x)/Sqrt[c]])/(Sqrt[c]*(a^2 + c))) + Log[x]/(a^2 + c) - L
og[c + (a + b*x)^2]/(2*(a^2 + c))
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 706

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x\left(c+(a+bx)^2\right)}\,dx &= \text{Subst}\left(\int \frac{1}{(-a+x)\left(c+x^2\right)}\,dx,x,a+bx\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{-a+x}\,dx,x,a+bx\right)}{a^2+c}+\frac{\text{Subst}\left(\int \frac{-a-x}{c+x^2}\,dx,x,a+bx\right)}{a^2+c}\\
&= \frac{\log(x)}{a^2+c}-\frac{\text{Subst}\left(\int \frac{x}{c+x^2}\,dx,x,a+bx\right)}{a^2+c}-\frac{a\,\text{Subst}\left(\int \frac{1}{c+x^2}\,dx,x,a+bx\right)}{a^2+c}\\
&= -\frac{a\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}\left(a^2+c\right)}+\frac{\log(x)}{a^2+c}-\frac{\log\left(c+(a+bx)^2\right)}{2\left(a^2+c\right)}
\end{aligned}
$$

Mathematica [A] time = 0.0328941, size = 48, normalized size = 0.81

$$
-\frac{\log\left((a+bx)^2+c\right)+\frac{2a\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}}-2\log(bx)}{2\left(a^2+c\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(c + (a + b*x)^2)),x]
```

```
[Out] -((2*a*ArcTan[(a + b*x)/Sqrt[c]])/Sqrt[c] - 2*Log[b*x] + Log[c + (a + b*x)^
2])/(2*(a^2 + c))
```

Maple [A] time = 0.006, size = 72, normalized size = 1.2

$$\frac{\ln(x)}{a^2+c} - \frac{\ln\left(b^2x^2+2\,abx+a^2+c\right)}{2\,a^2+2\,c} - \frac{a}{a^2+c}\arctan\left(\frac{2\,b^2x+2\,ab}{2\,b}\frac{1}{\sqrt{c}}\right)\frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c+(b*x+a)^2),x)
```

```
[Out] ln(x)/(a^2+c)-1/2/(a^2+c)*ln(b^2*x^2+2*a*b*x+a^2+c)-1/(a^2+c)*a/c^(1/2)*arc
tan(1/2*(2*b^2*x+2*a*b)/b/c^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c+(b*x+a)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.86333, size = 385, normalized size = 6.53

$$\left[-\frac{a\sqrt{-c}\log\left(\frac{b^2x^2+2\,abx+a^2+2\,(bx+a)\sqrt{-c}-c}{b^2x^2+2\,abx+a^2+c}\right)+c\log\left(b^2x^2+2\,abx+a^2+c\right)-2\,c\log(x)}{2\left(a^2c+c^2\right)}, -\frac{2\,a\sqrt{c}\arctan\left(\frac{bx+a}{\sqrt{c}}\right)+c\log\left(b^2x^2+\right.}{2\left(a^2c+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c+(b*x+a)^2),x, algorithm="fricas")
```

```
[Out] [-1/2*(a*sqrt(-c)*log((b^2*x^2 + 2*a*b*x + a^2 + 2*(b*x + a)*sqrt(-c) - c)/
(b^2*x^2 + 2*a*b*x + a^2 + c)) + c*log(b^2*x^2 + 2*a*b*x + a^2 + c) - 2*c*l
og(x))/(a^2*c + c^2), -1/2*(2*a*sqrt(c)*arctan((b*x + a)/sqrt(c)) + c*log(b
^2*x^2 + 2*a*b*x + a^2 + c) - 2*c*log(x))/(a^2*c + c^2)]
```

Sympy [B] time = 1.70819, size = 738, normalized size = 12.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c+(b*x+a)**2),x)
```

```
[Out] (-a*sqrt(-c)/(2*c*(a**2 + c)) - 1/(2*(a**2 + c)))*log(x + (-4*a**6*c*(-a*sq
rt(-c)/(2*c*(a**2 + c)) - 1/(2*(a**2 + c)))**2 + 4*a**4*c**2*(-a*sqrt(-c)/(
2*c*(a**2 + c)) - 1/(2*(a**2 + c)))**2 - 6*a**4*c*(-a*sqrt(-c)/(2*c*(a**2 +
 c)) - 1/(2*(a**2 + c))) + 20*a**2*c**3*(-a*sqrt(-c)/(2*c*(a**2 + c)) - 1/(
2*(a**2 + c)))**2 - 12*a**2*c**2*(-a*sqrt(-c)/(2*c*(a**2 + c)) - 1/(2*(a**2
 + c))) + 10*a**2*c + 12*c**4*(-a*sqrt(-c)/(2*c*(a**2 + c)) - 1/(2*(a**2 +
c)))**2 - 6*c**3*(-a*sqrt(-c)/(2*c*(a**2 + c)) - 1/(2*(a**2 + c))) - 6*c**2
)/(a**3*b + 9*a*b*c)) + (a*sqrt(-c)/(2*c*(a**2 + c)) - 1/(2*(a**2 + c)))*lo
g(x + (-4*a**6*c*(a*sqrt(-c)/(2*c*(a**2 + c)) - 1/(2*(a**2 + c)))**2 + 4*a*
*4*c**2*(a*sqrt(-c)/(2*c*(a**2 + c)) - 1/(2*(a**2 + c)))**2 - 6*a**4*c*(a*s
qrt(-c)/(2*c*(a**2 + c)) - 1/(2*(a**2 + c))) + 20*a**2*c**3*(a*sqrt(-c)/(2*
c*(a**2 + c)) - 1/(2*(a**2 + c)))**2 - 12*a**2*c**2*(a*sqrt(-c)/(2*c*(a**2
+ c)) - 1/(2*(a**2 + c))) + 10*a**2*c + 12*c**4*(a*sqrt(-c)/(2*c*(a**2 + c)
) - 1/(2*(a**2 + c)))**2 - 6*c**3*(a*sqrt(-c)/(2*c*(a**2 + c)) - 1/(2*(a**2
 + c))) - 6*c**2)/(a**3*b + 9*a*b*c)) + log(x + (-4*a**6*c/(a**2 + c)**2 +
4*a**4*c**2/(a**2 + c)**2 - 6*a**4*c/(a**2 + c) + 20*a**2*c**3/(a**2 + c)**
2 - 12*a**2*c**2/(a**2 + c) + 10*a**2*c + 12*c**4/(a**2 + c)**2 - 6*c**3/(a
**2 + c) - 6*c**2)/(a**3*b + 9*a*b*c))/(a**2 + c)
```

Giac [A] time = 1.12384, size = 84, normalized size = 1.42

$$-\frac{a \arctan\left(\frac{bx+a}{\sqrt{c}}\right)}{\left(a^2+c\right)\sqrt{c}} - \frac{\log\left(b^2x^2+2\,abx+a^2+c\right)}{2\left(a^2+c\right)} + \frac{\log\left(|x|\right)}{a^2+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c+(b*x+a)^2),x, algorithm="giac")
```

```
[Out] -a*arctan((b*x + a)/sqrt(c))/((a^2 + c)*sqrt(c)) - 1/2*log(b^2*x^2 + 2*a*b*
x + a^2 + c)/(a^2 + c) + log(abs(x))/(a^2 + c)
```

3.83 $\int \frac{1}{x^2\left(c+(a+bx)^2\right)}\,dx$

Optimal. Leaf size=79

$$-\frac{2ab\log(x)}{\left(a^2+c\right)^2}+\frac{ab\log\left((a+bx)^2+c\right)}{\left(a^2+c\right)^2}+\frac{b\left(a^2-c\right)\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}\left(a^2+c\right)^2}-\frac{1}{x\left(a^2+c\right)}$$

```
[Out] -(1/((a^2 + c)*x)) + (b*(a^2 - c)*ArcTan[(a + b*x)/Sqrt[c]])/(Sqrt[c]*(a^2
+ c)^2) - (2*a*b*Log[x])/(a^2 + c)^2 + (a*b*Log[c + (a + b*x)^2])/(a^2 + c)
^2
```

Rubi [A] time = 0.0861113, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {371, 710, 801, 635, 203, 260}

$$-\frac{2ab\log(x)}{\left(a^2+c\right)^2}+\frac{ab\log\left((a+bx)^2+c\right)}{\left(a^2+c\right)^2}+\frac{b\left(a^2-c\right)\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}\left(a^2+c\right)^2}-\frac{1}{x\left(a^2+c\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(c + (a + b*x)^2)),x]
```

```
[Out] -(1/((a^2 + c)*x)) + (b*(a^2 - c)*ArcTan[(a + b*x)/Sqrt[c]])/(Sqrt[c]*(a^2
+ c)^2) - (2*a*b*Log[x])/(a^2 + c)^2 + (a*b*Log[c + (a + b*x)^2])/(a^2 + c)
^2
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 710

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(e*(d
 + e*x)^(m + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + Dist[c/(c*d^2 + a*e^2), In
t[((d + e*x)^(m + 1)*(d - e*x))/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m
```

```
}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{1}{x^2\left(c+(a+bx)^2\right)}\,dx = b\,\text{Subst}\left(\int \frac{1}{(-a+x)^2\left(c+x^2\right)}\,dx, x, a+bx\right)$$

$$= -\frac{1}{\left(a^2+c\right)x} + \frac{b\,\text{Subst}\left(\int \frac{-a-x}{(-a+x)\left(c+x^2\right)}\,dx, x, a+bx\right)}{a^2+c}$$

$$= -\frac{1}{\left(a^2+c\right)x} + \frac{b\,\text{Subst}\left(\int \left(\frac{2a}{\left(a^2+c\right)(a-x)} + \frac{a^2-c+2ax}{\left(a^2+c\right)\left(c+x^2\right)}\right)dx, x, a+bx\right)}{a^2+c}$$

$$= -\frac{1}{\left(a^2+c\right)x} - \frac{2ab\log(x)}{\left(a^2+c\right)^2} + \frac{b\,\text{Subst}\left(\int \frac{a^2-c+2ax}{c+x^2}\,dx, x, a+bx\right)}{\left(a^2+c\right)^2}$$

$$= -\frac{1}{\left(a^2+c\right)x} - \frac{2ab\log(x)}{\left(a^2+c\right)^2} + \frac{(2ab)\,\text{Subst}\left(\int \frac{x}{c+x^2}\,dx, x, a+bx\right)}{\left(a^2+c\right)^2} + \frac{\left(b\left(a^2-c\right)\right)\text{Subst}\left(\int \frac{1}{c+x^2}\,d\right.}{\left(a^2+c\right)^2}$$

$$= -\frac{1}{\left(a^2+c\right)x} + \frac{b\left(a^2-c\right)\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}\left(a^2+c\right)^2} - \frac{2ab\log(x)}{\left(a^2+c\right)^2} + \frac{ab\log\left(c+(a+bx)^2\right)}{\left(a^2+c\right)^2}$$

Mathematica [A] time = 0.0437477, size = 81, normalized size = 1.03

$$\frac{bx\left(a^2-c\right)\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right) - \sqrt{c}\left(-abx\log\left(a^2+2abx+b^2x^2+c\right)+a^2+2abx\log(x)+c\right)}{\sqrt{c}x\left(a^2+c\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(c + (a + b*x)^2)),x]
```

```
[Out] (b*(a^2 - c)*x*ArcTan[(a + b*x)/Sqrt[c]] - Sqrt[c]*(a^2 + c + 2*a*b*x*Log[x
] - a*b*x*Log[a^2 + c + 2*a*b*x + b^2*x^2]))/(Sqrt[c]*(a^2 + c)^2*x)
```

Maple [A] time = 0.007, size = 123, normalized size = 1.6

$$-\frac{1}{\left(a^2+c\right)x} - 2\frac{ab\ln(x)}{\left(a^2+c\right)^2} + \frac{ab\ln\left(b^2x^2+2\,abx+a^2+c\right)}{\left(a^2+c\right)^2} + \frac{ba^2}{\left(a^2+c\right)^2}\arctan\left(\frac{2\,b^2x+2\,ab}{2\,b}\frac{1}{\sqrt{c}}\right)\frac{1}{\sqrt{c}} - \frac{b}{\left(a^2+c\right)^2}\sqrt{c}\,\text{arct}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c+(b*x+a)^2),x)
```

```
[Out] -1/(a^2+c)/x-2*a*b*ln(x)/(a^2+c)^2+b/(a^2+c)^2*a*ln(b^2*x^2+2*a*b*x+a^2+c)+
b/(a^2+c)^2/c^(1/2)*arctan(1/2*(2*b^2*x+2*a*b)/b/c^(1/2))*a^2-b/(a^2+c)^2*c
^(1/2)*arctan(1/2*(2*b^2*x+2*a*b)/b/c^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c+(b*x+a)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.92129, size = 533, normalized size = 6.75

$$\left[\frac{2\,abcx\log\left(b^2x^2+2\,abx+a^2+c\right)-4\,abcx\log\left(x\right)+\left(a^2b-bc\right)\sqrt{-c}x\log\left(\frac{b^2x^2+2\,abx+a^2+2\,(bx+a)\sqrt{-c}-c}{b^2x^2+2\,abx+a^2+c}\right)-2\,a^2c-2\,c^2}{2\left(a^4c+2\,a^2c^2+c^3\right)x}, \frac{abcx\ \cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c+(b*x+a)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*b*c*x*log(b^2*x^2 + 2*a*b*x + a^2 + c) - 4*a*b*c*x*log(x) + (a^2*
b - b*c)*sqrt(-c)*x*log((b^2*x^2 + 2*a*b*x + a^2 + 2*(b*x + a)*sqrt(-c) - c
)/(b^2*x^2 + 2*a*b*x + a^2 + c)) - 2*a^2*c - 2*c^2)/((a^4*c + 2*a^2*c^2 + c
^3)*x), (a*b*c*x*log(b^2*x^2 + 2*a*b*x + a^2 + c) - 2*a*b*c*x*log(x) + (a^2
*b - b*c)*sqrt(c)*x*arctan((b*x + a)/sqrt(c)) - a^2*c - c^2)/((a^4*c + 2*a^
2*c^2 + c^3)*x)]
```

Sympy [B] time = 4.43378, size = 1620, normalized size = 20.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c+(b*x+a)**2),x)
```

```
[Out] -2*a*b*log(x + (-16*a**13*b**2*c/(a**2 + c)**4 + 48*a**11*b**2*c**2/(a**2 +
 c)**4 + 352*a**9*b**2*c**3/(a**2 + c)**4 - 20*a**9*b**2*c/(a**2 + c)**2 +
608*a**7*b**2*c**4/(a**2 + c)**4 - 64*a**7*b**2*c**2/(a**2 + c)**2 + 432*a*
*5*b**2*c**5/(a**2 + c)**4 - 72*a**5*b**2*c**3/(a**2 + c)**2 + 36*a**5*b**2
*c + 112*a**3*b**2*c**6/(a**2 + c)**4 - 32*a**3*b**2*c**4/(a**2 + c)**2 - 8
8*a**3*b**2*c**2 - 4*a*b**2*c**5/(a**2 + c)**2 + 4*a*b**2*c**3)/(a**6*b**3
+ 33*a**4*b**3*c - 33*a**2*b**3*c**2 - b**3*c**3))/(a**2 + c)**2 + (a*b/(a*
*2 + c)**2 - b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c + c**2)))*log(x +
(-4*a**11*c*(a*b/(a**2 + c)**2 - b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*
c + c**2)))**2 + 12*a**9*c**2*(a*b/(a**2 + c)**2 - b*sqrt(-c)*(a**2 - c)/(2
*c*(a**4 + 2*a**2*c + c**2)))**2 + 10*a**8*b*c*(a*b/(a**2 + c)**2 - b*sqrt(
-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c + c**2))) + 88*a**7*c**3*(a*b/(a**2 +
c)**2 - b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c + c**2)))**2 + 32*a**6*
b*c**2*(a*b/(a**2 + c)**2 - b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c + c
**2))) + 36*a**5*b**2*c + 152*a**5*c**4*(a*b/(a**2 + c)**2 - b*sqrt(-c)*(a*
*2 - c)/(2*c*(a**4 + 2*a**2*c + c**2)))**2 + 36*a**4*b*c**3*(a*b/(a**2 + c)
**2 - b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c + c**2))) - 88*a**3*b**2*
c**2 + 108*a**3*c**5*(a*b/(a**2 + c)**2 - b*sqrt(-c)*(a**2 - c)/(2*c*(a**4
+ 2*a**2*c + c**2)))**2 + 16*a**2*b*c**4*(a*b/(a**2 + c)**2 - b*sqrt(-c)*(a
**2 - c)/(2*c*(a**4 + 2*a**2*c + c**2))) + 4*a*b**2*c**3 + 28*a*c**6*(a*b/(
a**2 + c)**2 - b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c + c**2)))**2 + 2
*b*c**5*(a*b/(a**2 + c)**2 - b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c +
c**2))))/(a**6*b**3 + 33*a**4*b**3*c - 33*a**2*b**3*c**2 - b**3*c**3)) + (a
*b/(a**2 + c)**2 + b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c + c**2)))*lo
g(x + (-4*a**11*c*(a*b/(a**2 + c)**2 + b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2
*a**2*c + c**2)))**2 + 12*a**9*c**2*(a*b/(a**2 + c)**2 + b*sqrt(-c)*(a**2 -
 c)/(2*c*(a**4 + 2*a**2*c + c**2)))**2 + 10*a**8*b*c*(a*b/(a**2 + c)**2 + b
*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c + c**2))) + 88*a**7*c**3*(a*b/(a
**2 + c)**2 + b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c + c**2)))**2 + 32
*a**6*b*c**2*(a*b/(a**2 + c)**2 + b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2
*c + c**2))) + 36*a**5*b**2*c + 152*a**5*c**4*(a*b/(a**2 + c)**2 + b*sqrt(-
c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c + c**2)))**2 + 36*a**4*b*c**3*(a*b/(a**
2 + c)**2 + b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c + c**2))) - 88*a**3
*b**2*c**2 + 108*a**3*c**5*(a*b/(a**2 + c)**2 + b*sqrt(-c)*(a**2 - c)/(2*c*
(a**4 + 2*a**2*c + c**2)))**2 + 16*a**2*b*c**4*(a*b/(a**2 + c)**2 + b*sqrt(
-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c + c**2))) + 4*a*b**2*c**3 + 28*a*c**6*
(a*b/(a**2 + c)**2 + b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c + c**2)))*
*2 + 2*b*c**5*(a*b/(a**2 + c)**2 + b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**
2*c + c**2))))/(a**6*b**3 + 33*a**4*b**3*c - 33*a**2*b**3*c**2 - b**3*c**3)
) - 1/(x*(a**2 + c))
```

Giac [A] time = 1.11608, size = 158, normalized size = 2.

$$\frac{ab\log\left(b^2x^2+2\,abx+a^2+c\right)}{a^4+2\,a^2c+c^2}-\frac{2\,ab\log\left(|x|\right)}{a^4+2\,a^2c+c^2}+\frac{\left(a^2b^2-b^2c\right)\arctan\left(\frac{bx+a}{\sqrt{c}}\right)}{\left(a^4+2\,a^2c+c^2\right)b\sqrt{c}}-\frac{1}{\left(a^2+c\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c+(b*x+a)^2),x, algorithm="giac")
```

```
[Out] a*b*log(b^2*x^2 + 2*a*b*x + a^2 + c)/(a^4 + 2*a^2*c + c^2) - 2*a*b*log(abs(
x))/(a^4 + 2*a^2*c + c^2) + (a^2*b^2 - b^2*c)*arctan((b*x + a)/sqrt(c))/((a
^4 + 2*a^2*c + c^2)*b*sqrt(c)) - 1/((a^2 + c)*x)
```

3.84 $\int \frac{1}{x^3\left(c+(a+bx)^2\right)}\,dx$

Optimal. Leaf size=121

$$\frac{b^2\left(3a^2-c\right)\log(x)}{\left(a^2+c\right)^3}-\frac{b^2\left(3a^2-c\right)\log\left((a+bx)^2+c\right)}{2\left(a^2+c\right)^3}-\frac{ab^2\left(a^2-3c\right)\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}\left(a^2+c\right)^3}+\frac{2ab}{x\left(a^2+c\right)^2}-\frac{1}{2x^2\left(a^2+c\right)}$$

```
[Out] -1/(2*(a^2 + c)*x^2) + (2*a*b)/((a^2 + c)^2*x) - (a*b^2*(a^2 - 3*c)*ArcTan[
(a + b*x)/Sqrt[c]])/(Sqrt[c]*(a^2 + c)^3) + (b^2*(3*a^2 - c)*Log[x])/(a^2 +
 c)^3 - (b^2*(3*a^2 - c)*Log[c + (a + b*x)^2])/(2*(a^2 + c)^3)
```

Rubi [A] time = 0.125346, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {371, 710, 801, 635, 203, 260}

$$\frac{b^2\left(3a^2-c\right)\log(x)}{\left(a^2+c\right)^3}-\frac{b^2\left(3a^2-c\right)\log\left((a+bx)^2+c\right)}{2\left(a^2+c\right)^3}-\frac{ab^2\left(a^2-3c\right)\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}\left(a^2+c\right)^3}+\frac{2ab}{x\left(a^2+c\right)^2}-\frac{1}{2x^2\left(a^2+c\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(c + (a + b*x)^2)),x]
```

```
[Out] -1/(2*(a^2 + c)*x^2) + (2*a*b)/((a^2 + c)^2*x) - (a*b^2*(a^2 - 3*c)*ArcTan[
(a + b*x)/Sqrt[c]])/(Sqrt[c]*(a^2 + c)^3) + (b^2*(3*a^2 - c)*Log[x])/(a^2 +
 c)^3 - (b^2*(3*a^2 - c)*Log[c + (a + b*x)^2])/(2*(a^2 + c)^3)
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 710

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(e*(d
 + e*x)^(m + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + Dist[c/(c*d^2 + a*e^2), In
t[((d + e*x)^(m + 1)*(d - e*x))/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m
```

```
}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{1}{x^3\left(c+(a+bx)^2\right)}\,dx = b^2\,\mathrm{Subst}\left(\int \frac{1}{(-a+x)^3\left(c+x^2\right)}\,dx, x, a+bx\right)$$

$$= -\frac{1}{2\left(a^2+c\right)x^2} + \frac{b^2\,\mathrm{Subst}\left(\int \frac{-a-x}{(-a+x)^2\left(c+x^2\right)}\,dx, x, a+bx\right)}{a^2+c}$$

$$= -\frac{1}{2\left(a^2+c\right)x^2} + \frac{b^2\,\mathrm{Subst}\left(\int \left(-\frac{2a}{\left(a^2+c\right)(a-x)^2} + \frac{-3a^2+c}{\left(a^2+c\right)^2(a-x)} + \frac{-a\left(a^2-3c\right)-\left(3a^2-c\right)x}{\left(a^2+c\right)^2\left(c+x^2\right)}\right)dx, x, a+bx\right)}{a^2+c}$$

$$= -\frac{1}{2\left(a^2+c\right)x^2} + \frac{2ab}{\left(a^2+c\right)^2 x} + \frac{b^2\left(3a^2-c\right)\log(x)}{\left(a^2+c\right)^3} + \frac{b^2\,\mathrm{Subst}\left(\int \frac{-a\left(a^2-3c\right)-\left(3a^2-c\right)x}{c+x^2}\,dx, x, a+\right.}{\left(a^2+c\right)^3}$$

$$= -\frac{1}{2\left(a^2+c\right)x^2} + \frac{2ab}{\left(a^2+c\right)^2 x} + \frac{b^2\left(3a^2-c\right)\log(x)}{\left(a^2+c\right)^3} - \frac{\left(ab^2\left(a^2-3c\right)\right)\mathrm{Subst}\left(\int \frac{1}{c+x^2}\,dx, x, a+\right.}{\left(a^2+c\right)^3}$$

$$= -\frac{1}{2\left(a^2+c\right)x^2} + \frac{2ab}{\left(a^2+c\right)^2 x} - \frac{ab^2\left(a^2-3c\right)\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}\left(a^2+c\right)^3} + \frac{b^2\left(3a^2-c\right)\log(x)}{\left(a^2+c\right)^3} - \frac{b^2\left(3a^2-c\right.}{2}$$

Mathematica [A] time = 0.128787, size = 106, normalized size = 0.88

$$-\frac{b^2\left(3a^2-c\right)\log\left(a^2+2abx+b^2x^2+c\right)+2b^2\left(c-3a^2\right)\log(x)+\frac{2ab^2\left(a^2-3c\right)\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}}+\frac{\left(a^2+c\right)\left(a^2-4abx+c\right)}{x^2}}{2\left(a^2+c\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(c + (a + b*x)^2)),x]
```

```
[Out] -(((a^2 + c)*(a^2 + c - 4*a*b*x))/x^2 + (2*a*b^2*(a^2 - 3*c)*ArcTan[(a + b*
x)/Sqrt[c]])/Sqrt[c] + 2*b^2*(-3*a^2 + c)*Log[x] + b^2*(3*a^2 - c)*Log[a^2
+ c + 2*a*b*x + b^2*x^2])/(2*(a^2 + c)^3)
```

Maple [A] time = 0.01, size = 198, normalized size = 1.6

$$-\frac{1}{\left(2a^2+2c\right)x^2} + 3\frac{b^2\ln(x)a^2}{\left(a^2+c\right)^3} - \frac{b^2\ln(x)c}{\left(a^2+c\right)^3} + 2\frac{ab}{\left(a^2+c\right)^2 x} - \frac{3b^2\ln\left(b^2x^2+2abx+a^2+c\right)a^2}{2\left(a^2+c\right)^3} + \frac{b^2\ln\left(b^2x^2+2abx+\right.}{2\left(a^2+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(c+(b*x+a)^2),x)
```

```
[Out] -1/2/(a^2+c)/x^2+3*b^2/(a^2+c)^3*ln(x)*a^2-b^2/(a^2+c)^3*ln(x)*c+2*a*b/(a^2
+c)^2/x-3/2*b^2/(a^2+c)^3*ln(b^2*x^2+2*a*b*x+a^2+c)*a^2+1/2*b^2/(a^2+c)^3*l
n(b^2*x^2+2*a*b*x+a^2+c)*c-b^2/(a^2+c)^3/c^(1/2)*arctan(1/2*(2*b^2*x+2*a*b)
/b/c^(1/2))*a^3+3*b^2/(a^2+c)^3*c^(1/2)*arctan(1/2*(2*b^2*x+2*a*b)/b/c^(1/2
))*a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c+(b*x+a)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.92599, size = 811, normalized size = 6.7

$$\left[-\frac{a^4c - \left(a^3b^2 - 3\,ab^2c\right)\sqrt{-c}x^2\log\left(\frac{b^2x^2+2\,abx+a^2-2\,(bx+a)\sqrt{-c}-c}{b^2x^2+2\,abx+a^2+c}\right) + 2\,a^2c^2 + \left(3\,a^2b^2c - b^2c^2\right)x^2\log\left(b^2x^2 + 2\,abx + a^2 + c\right) - 2}{2\left(a^6c + 3\,a^4c^2 + 3\,a^2c^3 + c^4\right)x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c+(b*x+a)^2),x, algorithm="fricas")
```

```
[Out] [-1/2*(a^4*c - (a^3*b^2 - 3*a*b^2*c)*sqrt(-c)*x^2*log((b^2*x^2 + 2*a*b*x +
a^2 - 2*(b*x + a)*sqrt(-c) - c)/(b^2*x^2 + 2*a*b*x + a^2 + c)) + 2*a^2*c^2
+ (3*a^2*b^2*c - b^2*c^2)*x^2*log(b^2*x^2 + 2*a*b*x + a^2 + c) - 2*(3*a^2*b
^2*c - b^2*c^2)*x^2*log(x) + c^3 - 4*(a^3*b*c + a*b*c^2)*x)/((a^6*c + 3*a^4
*c^2 + 3*a^2*c^3 + c^4)*x^2), -1/2*(a^4*c + 2*(a^3*b^2 - 3*a*b^2*c)*sqrt(c)
*x^2*arctan((b*x + a)/sqrt(c)) + 2*a^2*c^2 + (3*a^2*b^2*c - b^2*c^2)*x^2*lo
g(b^2*x^2 + 2*a*b*x + a^2 + c) - 2*(3*a^2*b^2*c - b^2*c^2)*x^2*log(x) + c^3
 - 4*(a^3*b*c + a*b*c^2)*x)/((a^6*c + 3*a^4*c^2 + 3*a^2*c^3 + c^4)*x^2)]
```

Sympy [B] time = 7.24595, size = 3284, normalized size = 27.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(c+(b*x+a)**2),x)
```

```
[Out] b**2*(3*a**2 - c)*log(x + (-4*a**16*b**4*c*(3*a**2 - c)**2/(a**2 + c)**6 +
24*a**14*b**4*c**2*(3*a**2 - c)**2/(a**2 + c)**6 + 216*a**12*b**4*c**3*(3*a
**2 - c)**2/(a**2 + c)**6 - 14*a**12*b**4*c*(3*a**2 - c)/(a**2 + c)**3 + 56
8*a**10*b**4*c**4*(3*a**2 - c)**2/(a**2 + c)**6 - 44*a**10*b**4*c**2*(3*a**
2 - c)/(a**2 + c)**3 + 720*a**8*b**4*c**5*(3*a**2 - c)**2/(a**2 + c)**6 - 4
2*a**8*b**4*c**3*(3*a**2 - c)/(a**2 + c)**3 + 78*a**8*b**4*c + 456*a**6*b**
4*c**6*(3*a**2 - c)**2/(a**2 + c)**6 - 8*a**6*b**4*c**4*(3*a**2 - c)/(a**2
+ c)**3 - 464*a**6*b**4*c**2 + 104*a**4*b**4*c**7*(3*a**2 - c)**2/(a**2 + c
)**6 - 2*a**4*b**4*c**5*(3*a**2 - c)/(a**2 + c)**3 + 380*a**4*b**4*c**3 - 2
4*a**2*b**4*c**8*(3*a**2 - c)**2/(a**2 + c)**6 - 12*a**2*b**4*c**6*(3*a**2
- c)/(a**2 + c)**3 - 96*a**2*b**4*c**4 - 12*b**4*c**9*(3*a**2 - c)**2/(a**2
 + c)**6 - 6*b**4*c**7*(3*a**2 - c)/(a**2 + c)**3 + 6*b**4*c**5)/(a**9*b**5
 + 72*a**7*b**5*c - 270*a**5*b**5*c**2 + 144*a**3*b**5*c**3 - 27*a*b**5*c**
4))/(a**2 + c)**3 + (-a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c +
3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3))*log(x + (-4*a**
16*c*(-a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c
**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3))**2 + 24*a**14*c**2*(-a*b**2*sq
rt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*
a**2 - c)/(2*(a**2 + c)**3))**2 - 14*a**12*b**2*c*(-a*b**2*sqrt(-c)*(a**2 -
 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(
a**2 + c)**3)) + 216*a**12*c**3*(-a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 +
 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3))**2
- 44*a**10*b**2*c**2*(-a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c +
 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3)) + 568*a**10*c*
*4*(-a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**
3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3))**2 + 78*a**8*b**4*c - 42*a**8*b*
*2*c**3*(-a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2
+ c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3)) + 720*a**8*c**5*(-a*b**2*sq
rt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*
a**2 - c)/(2*(a**2 + c)**3))**2 - 464*a**6*b**4*c**2 - 8*a**6*b**2*c**4*(-a
*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) -
b**2*(3*a**2 - c)/(2*(a**2 + c)**3)) + 456*a**6*c**6*(-a*b**2*sqrt(-c)*(a**
2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(
2*(a**2 + c)**3))**2 + 380*a**4*b**4*c**3 - 2*a**4*b**2*c**5*(-a*b**2*sqrt(
-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**
```

```
2 - c)/(2*(a**2 + c)**3)) + 104*a**4*c**7*(-a*b**2*sqrt(-c)*(a**2 - 3*c)/(2
*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c
)**3))**2 - 96*a**2*b**4*c**4 - 12*a**2*b**2*c**6*(-a*b**2*sqrt(-c)*(a**2 -
 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(
a**2 + c)**3)) - 24*a**2*c**8*(-a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3
*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3))**2 +
6*b**4*c**5 - 6*b**2*c**7*(-a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**
4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3)) - 12*c**9
*(-a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)
) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3))**2)/(a**9*b**5 + 72*a**7*b**5*c -
270*a**5*b**5*c**2 + 144*a**3*b**5*c**3 - 27*a*b**5*c**4)) + (a*b**2*sqrt(-
c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2
 - c)/(2*(a**2 + c)**3))*log(x + (-4*a**16*c*(a*b**2*sqrt(-c)*(a**2 - 3*c)/
(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2 +
 c)**3))**2 + 24*a**14*c**2*(a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a*
*4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3))**2 - 14*
a**12*b**2*c*(a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c
**2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3)) + 216*a**12*c**3*(a*b**
2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2
*(3*a**2 - c)/(2*(a**2 + c)**3))**2 - 44*a**10*b**2*c**2*(a*b**2*sqrt(-c)*(
a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c
)/(2*(a**2 + c)**3)) + 568*a**10*c**4*(a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a
**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3)
)**2 + 78*a**8*b**4*c - 42*a**8*b**2*c**3*(a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*
c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)
**3)) + 720*a**8*c**5*(a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c +
 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3))**2 - 464*a**6*
b**4*c**2 - 8*a**6*b**2*c**4*(a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a
**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3)) + 456*a
**6*c**6*(a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2
+ c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3))**2 + 380*a**4*b**4*c**3 - 2
*a**4*b**2*c**5*(a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**
2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3)) + 104*a**4*c**7*(a*b
**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b*
*2*(3*a**2 - c)/(2*(a**2 + c)**3))**2 - 96*a**2*b**4*c**4 - 12*a**2*b**2*c*
*6*(a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3
)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3)) - 24*a**2*c**8*(a*b**2*sqrt(-c)*(
a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c
)/(2*(a**2 + c)**3))**2 + 6*b**4*c**5 - 6*b**2*c**7*(a*b**2*sqrt(-c)*(a**2
- 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*
(a**2 + c)**3)) - 12*c**9*(a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4
*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3))**2)/(a**9*
b**5 + 72*a**7*b**5*c - 270*a**5*b**5*c**2 + 144*a**3*b**5*c**3 - 27*a*b**5
*c**4)) + (-a**2 + 4*a*b*x - c)/(x**2*(2*a**4 + 4*a**2*c + 2*c**2))
```

Giac [A] time = 1.16012, size = 263, normalized size = 2.17

$$-\frac{\left(3\,a^{2}b^{2}-b^{2}c\right)\log\left(b^{2}x^{2}+2\,abx+a^{2}+c\right)}{2\left(a^{6}+3\,a^{4}c+3\,a^{2}c^{2}+c^{3}\right)}+\frac{\left(3\,a^{2}b^{2}-b^{2}c\right)\log\left(|x|\right)}{a^{6}+3\,a^{4}c+3\,a^{2}c^{2}+c^{3}}-\frac{\left(a^{3}b^{3}-3\,ab^{3}c\right)\arctan\left(\frac{bx+a}{\sqrt{c}}\right)}{\left(a^{6}+3\,a^{4}c+3\,a^{2}c^{2}+c^{3}\right)b\sqrt{c}}-\frac{a^{4}+2\,a^{2}c+c}{2\left(a^{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c+(b*x+a)^2),x, algorithm="giac")
```

```
[Out] -1/2*(3*a^2*b^2 - b^2*c)*log(b^2*x^2 + 2*a*b*x + a^2 + c)/(a^6 + 3*a^4*c +
3*a^2*c^2 + c^3) + (3*a^2*b^2 - b^2*c)*log(abs(x))/(a^6 + 3*a^4*c + 3*a^2*c
^2 + c^3) - (a^3*b^3 - 3*a*b^3*c)*arctan((b*x + a)/sqrt(c))/((a^6 + 3*a^4*c
 + 3*a^2*c^2 + c^3)*b*sqrt(c)) - 1/2*(a^4 + 2*a^2*c + c^2 - 4*(a^3*b + a*b*
c)*x)/((a^2 + c)^3*x^2)
```

3.85 $\int \frac{1}{a+b(c+dx)^2} \, dx$

Optimal. Leaf size=31

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}d}$$

```
[Out] ArcTan[(Sqrt[b]*(c + d*x))/Sqrt[a]]/(Sqrt[a]*Sqrt[b]*d)
```

Rubi [A] time = 0.0235124, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {247, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*(c + d*x)^2)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[b]*(c + d*x))/Sqrt[a]]/(Sqrt[a]*Sqrt[b]*d)
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{a+b(c+dx)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,c+dx\right)}{d}$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{b}(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}d}$$

Mathematica [A] time = 0.0090773, size = 31, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*(c + d*x)^2)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[b]*(c + d*x))/Sqrt[a]]/(Sqrt[a]*Sqrt[b]*d)
```

Maple [A] time = 0.006, size = 34, normalized size = 1.1

$$\frac{1}{d}\arctan\left(\frac{2\,bd^2x+2\,bcd}{2\,d}\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*(d*x+c)^2),x)
```

```
[Out] 1/d/(a*b)^(1/2)*arctan(1/2*(2*b*d^2*x+2*b*c*d)/d/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.67146, size = 243, normalized size = 7.84

$$\left[-\frac{\sqrt{-ab}\log\left(\frac{bd^2x^2+2\,bcdx+bc^2-2\,\sqrt{-ab}(dx+c)-a}{bd^2x^2+2\,bcdx+bc^2+a}\right)}{2\,abd}, \frac{\sqrt{ab}\arctan\left(\frac{\sqrt{ab}(dx+c)}{a}\right)}{abd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a*b)*log((b*d^2*x^2 + 2*b*c*d*x + b*c^2 - 2*sqrt(-a*b)*(d*x + c
) - a)/(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a))/(a*b*d), sqrt(a*b)*arctan(sqrt(
a*b)*(d*x + c)/a)/(a*b*d)]
```

Sympy [B] time = 0.201686, size = 61, normalized size = 1.97

$$\frac{-\frac{\sqrt{-\frac{1}{ab}}\log\left(x+\frac{-a\sqrt{-\frac{1}{ab}}+c}{d}\right)}{2}+\frac{\sqrt{-\frac{1}{ab}}\log\left(x+\frac{a\sqrt{-\frac{1}{ab}}+c}{d}\right)}{2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)**2),x)
```

```
[Out] (-sqrt(-1/(a*b))*log(x + (-a*sqrt(-1/(a*b)) + c)/d)/2 + sqrt(-1/(a*b))*log(
x + (a*sqrt(-1/(a*b)) + c)/d)/2)/d
```

Giac [A] time = 1.12971, size = 32, normalized size = 1.03

$$\frac{\arctan\left(\frac{bdx+bc}{\sqrt{ab}}\right)}{\sqrt{ab}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^2),x, algorithm="giac")
```

```
[Out] arctan((b*d*x + b*c)/sqrt(a*b))/(sqrt(a*b)*d)
```

3.86 $\int \frac{1}{\left(a+b(c+dx)^2\right)^2}\,dx$

Optimal. Leaf size=63

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}d}+\frac{c+dx}{2ad\left(a+b(c+dx)^2\right)}$$

```
[Out] (c + d*x)/(2*a*d*(a + b*(c + d*x)^2)) + ArcTan[(Sqrt[b]*(c + d*x))/Sqrt[a]]
/(2*a^(3/2)*Sqrt[b]*d)
```

Rubi [A] time = 0.0329842, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {247, 199, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}d}+\frac{c+dx}{2ad\left(a+b(c+dx)^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*(c + d*x)^2)^(-2),x]
```

```
[Out] (c + d*x)/(2*a*d*(a + b*(c + d*x)^2)) + ArcTan[(Sqrt[b]*(c + d*x))/Sqrt[a]]
/(2*a^(3/2)*Sqrt[b]*d)
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(a+b(c+dx)^2\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)^2}\,dx, x, c+dx\right)}{d}$$

$$= \frac{c+dx}{2ad\left(a+b(c+dx)^2\right)} + \frac{\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, c+dx\right)}{2ad}$$

$$= \frac{c+dx}{2ad\left(a+b(c+dx)^2\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{b}(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}d}$$

Mathematica [A] time = 0.0247364, size = 60, normalized size = 0.95

$$\frac{\frac{\sqrt{a}(c+dx)}{a+b(c+dx)^2} + \frac{\tan^{-1}\left(\frac{\sqrt{b}(c+dx)}{\sqrt{a}}\right)}{\sqrt{b}}}{2a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*(c + d*x)^2)^(-2),x]
```

```
[Out] ((Sqrt[a]*(c + d*x))/(a + b*(c + d*x)^2) + ArcTan[(Sqrt[b]*(c + d*x))/Sqrt[
a]]/Sqrt[b])/(2*a^(3/2)*d)
```

Maple [A] time = 0.004, size = 86, normalized size = 1.4

$$\frac{2\,bd^2x+2\,bcd}{4\,abd^2\left(bd^2x^2+2\,bcdx+c^2b+a\right)} + \frac{1}{2\,ad}\arctan\left(\frac{2\,bd^2x+2\,bcd}{2\,d}\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*(d*x+c)^2)^2,x)
```

```
[Out] 1/4*(2*b*d^2*x+2*b*c*d)/a/b/d^2/(b*d^2*x^2+2*b*c*d*x+b*c^2+a)+1/2/d/a/(a*b)
^(1/2)*arctan(1/2*(2*b*d^2*x+2*b*c*d)/d/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.8116, size = 555, normalized size = 8.81

$$\left[\frac{2\,abdx + 2\,abc - \left(bd^2x^2 + 2\,bcdx + bc^2 + a\right)\sqrt{-ab}\log\left(\frac{bd^2x^2+2\,bcdx+bc^2-2\,\sqrt{-ab}(dx+c)-a}{bd^2x^2+2\,bcdx+bc^2+a}\right)}{4\left(a^2b^2d^3x^2 + 2\,a^2b^2cd^2x + \left(a^2b^2c^2 + a^3b\right)d\right)}, \frac{abdx + abc + \left(bd^2x^2 + 2\,bcdx + bc\right.}{2\left(a^2b^2d^3x^2 + 2\,a^2b^2cd^2x\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*a*b*d*x + 2*a*b*c - (b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)*sqrt(-a*b)*
log((b*d^2*x^2 + 2*b*c*d*x + b*c^2 - 2*sqrt(-a*b)*(d*x + c) - a)/(b*d^2*x^2
 + 2*b*c*d*x + b*c^2 + a)))/(a^2*b^2*d^3*x^2 + 2*a^2*b^2*c*d^2*x + (a^2*b^2
*c^2 + a^3*b)*d), 1/2*(a*b*d*x + a*b*c + (b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a
)*sqrt(a*b)*arctan(sqrt(a*b)*(d*x + c)/a))/(a^2*b^2*d^3*x^2 + 2*a^2*b^2*c*d
^2*x + (a^2*b^2*c^2 + a^3*b)*d)]
```

Sympy [B] time = 0.727365, size = 117, normalized size = 1.86

$$\frac{c + dx}{2a^2d + 2abc^2d + 4abcd^2x + 2abd^3x^2} + \frac{-\frac{\sqrt{-\frac{1}{a^3b}}\log\left(x+\frac{-a^2\sqrt{-\frac{1}{a^3b}}+c}{d}\right)}{4} + \frac{\sqrt{-\frac{1}{a^3b}}\log\left(x+\frac{a^2\sqrt{-\frac{1}{a^3b}}+c}{d}\right)}{4}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)**2)**2,x)
```

```
[Out] (c + d*x)/(2*a**2*d + 2*a*b*c**2*d + 4*a*b*c*d**2*x + 2*a*b*d**3*x**2) + (-
sqrt(-1/(a**3*b))*log(x + (-a**2*sqrt(-1/(a**3*b)) + c)/d)/4 + sqrt(-1/(a**
3*b))*log(x + (a**2*sqrt(-1/(a**3*b)) + c)/d)/4)/d
```

Giac [A] time = 1.11284, size = 88, normalized size = 1.4

$$\frac{\arctan\left(\frac{bdx+bc}{\sqrt{ab}}\right)}{2\sqrt{ab}ad}+\frac{dx+c}{2\left(bd^2x^2+2\,bcdx+bc^2+a\right)ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*arctan((b*d*x + b*c)/sqrt(a*b))/(sqrt(a*b)*a*d) + 1/2*(d*x + c)/((b*d^2
*x^2 + 2*b*c*d*x + b*c^2 + a)*a*d)
```

3.87 $\int \frac{1}{\left(a+b(c+dx)^2\right)^3} dx$

Optimal. Leaf size=91

$$\frac{3(c+dx)}{8a^2d\left(a+b(c+dx)^2\right)}+\frac{3\tan^{-1}\left(\frac{\sqrt{b}(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}d}+\frac{c+dx}{4ad\left(a+b(c+dx)^2\right)^2}$$

```
[Out] (c + d*x)/(4*a*d*(a + b*(c + d*x)^2)^2) + (3*(c + d*x))/(8*a^2*d*(a + b*(c
+ d*x)^2)) + (3*ArcTan[(Sqrt[b]*(c + d*x))/Sqrt[a]])/(8*a^(5/2)*Sqrt[b]*d)
```

Rubi [A] time = 0.0482007, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {247, 199, 205}

$$\frac{3(c+dx)}{8a^2d\left(a+b(c+dx)^2\right)}+\frac{3\tan^{-1}\left(\frac{\sqrt{b}(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}d}+\frac{c+dx}{4ad\left(a+b(c+dx)^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*(c + d*x)^2)^(-3),x]
```



```
[Out] (c + d*x)/(4*a*d*(a + b*(c + d*x)^2)^2) + (3*(c + d*x))/(8*a^2*d*(a + b*(c
+ d*x)^2)) + (3*ArcTan[(Sqrt[b]*(c + d*x))/Sqrt[a]])/(8*a^(5/2)*Sqrt[b]*d)
```


Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(a+b(c+dx)^2\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)^3}\,dx, x, c+dx\right)}{d}$$

$$= \frac{c+dx}{4ad\left(a+b(c+dx)^2\right)^2} + \frac{3\,\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)^2}\,dx, x, c+dx\right)}{4ad}$$

$$= \frac{c+dx}{4ad\left(a+b(c+dx)^2\right)^2} + \frac{3(c+dx)}{8a^2d\left(a+b(c+dx)^2\right)} + \frac{3\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, c+dx\right)}{8a^2d}$$

$$= \frac{c+dx}{4ad\left(a+b(c+dx)^2\right)^2} + \frac{3(c+dx)}{8a^2d\left(a+b(c+dx)^2\right)} + \frac{3\tan^{-1}\left(\frac{\sqrt{b}(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}d}$$

Mathematica [A] time = 0.0601837, size = 75, normalized size = 0.82

$$\frac{\frac{\sqrt{a}(c+dx)\left(5a+3b(c+dx)^2\right)}{\left(a+b(c+dx)^2\right)^2} + \frac{3\tan^{-1}\left(\frac{\sqrt{b}(c+dx)}{\sqrt{a}}\right)}{\sqrt{b}}}{8a^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*(c + d*x)^2)^(-3),x]
```

```
[Out] ((Sqrt[a]*(c + d*x)*(5*a + 3*b*(c + d*x)^2))/(a + b*(c + d*x)^2)^2 + (3*Arc
Tan[(Sqrt[b]*(c + d*x))/Sqrt[a]])/Sqrt[b])/(8*a^(5/2)*d)
```

Maple [A] time = 0.005, size = 147, normalized size = 1.6

$$\frac{2\,bd^2x+2\,bcd}{8\,abd^2\left(bd^2x^2+2\,bcdx+c^2b+a\right)^2} + \frac{3\,x}{8\,a^2\left(bd^2x^2+2\,bcdx+c^2b+a\right)} + \frac{3\,c}{8\,a^2d\left(bd^2x^2+2\,bcdx+c^2b+a\right)} + \frac{3}{8\,a^2d}\,\text{arcta}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*(d*x+c)^2)^3,x)
```

```
[Out] 1/8*(2*b*d^2*x+2*b*c*d)/a/b/d^2/(b*d^2*x^2+2*b*c*d*x+b*c^2+a)^2+3/8/a^2/(b*
d^2*x^2+2*b*c*d*x+b*c^2+a)*x+3/8/a^2/d/(b*d^2*x^2+2*b*c*d*x+b*c^2+a)*c+3/8/
a^2/d/(a*b)^(1/2)*arctan(1/2*(2*b*d^2*x+2*b*c*d)/d/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.94832, size = 1231, normalized size = 13.53

$$\left[\frac{6\,ab^2d^3x^3 + 18\,ab^2cd^2x^2 + 6\,ab^2c^3 + 10\,a^2bc + 2\left(9\,ab^2c^2 + 5\,a^2b\right)dx - 3\left(b^2d^4x^4 + 4\,b^2cd^3x^3 + b^2c^4 + 2\left(3\,b^2c^2 + ab\right)d^2x^2\right.}{16\left(a^3b^3d^5x^4 + 4\,a^3b^3cd^4x^3 + 2\left(3\,a^3b^3c^2 + a^4b^2\right)d^3x^2 + 4\left(a^3b^3c^3 + a^4b^2c\right)d^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(6*a*b^2*d^3*x^3 + 18*a*b^2*c*d^2*x^2 + 6*a*b^2*c^3 + 10*a^2*b*c + 2*
(9*a*b^2*c^2 + 5*a^2*b)*d*x - 3*(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + b^2*c^4 +
2*(3*b^2*c^2 + a*b)*d^2*x^2 + 2*a*b*c^2 + 4*(b^2*c^3 + a*b*c)*d*x + a^2)*sq
rt(-a*b)*log((b*d^2*x^2 + 2*b*c*d*x + b*c^2 - 2*sqrt(-a*b)*(d*x + c) - a)/(
b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)))/(a^3*b^3*d^5*x^4 + 4*a^3*b^3*c*d^4*x^3
 + 2*(3*a^3*b^3*c^2 + a^4*b^2)*d^3*x^2 + 4*(a^3*b^3*c^3 + a^4*b^2*c)*d^2*x
+ (a^3*b^3*c^4 + 2*a^4*b^2*c^2 + a^5*b)*d), 1/8*(3*a*b^2*d^3*x^3 + 9*a*b^2*
c*d^2*x^2 + 3*a*b^2*c^3 + 5*a^2*b*c + (9*a*b^2*c^2 + 5*a^2*b)*d*x + 3*(b^2*
d^4*x^4 + 4*b^2*c*d^3*x^3 + b^2*c^4 + 2*(3*b^2*c^2 + a*b)*d^2*x^2 + 2*a*b*c
^2 + 4*(b^2*c^3 + a*b*c)*d*x + a^2)*sqrt(a*b)*arctan(sqrt(a*b)*(d*x + c)/a)
)/(a^3*b^3*d^5*x^4 + 4*a^3*b^3*c*d^4*x^3 + 2*(3*a^3*b^3*c^2 + a^4*b^2)*d^3*
```

```
x^2 + 4*(a^3*b^3*c^3 + a^4*b^2*c)*d^2*x + (a^3*b^3*c^4 + 2*a^4*b^2*c^2 + a^
5*b)*d)]
```

Sympy [B] time = 1.65176, size = 257, normalized size = 2.82

$$\frac{5ac+3bc^3+9bcd^2x^2+3bd^3x^3+x\left(5ad+9bc^2d\right)}{8a^4d+16a^3bc^2d+8a^2b^2c^4d+32a^2b^2cd^4x^3+8a^2b^2d^5x^4+x^2\left(16a^3bd^3+48a^2b^2c^2d^3\right)+x\left(32a^3bcd^2+32a^2b^2c^3d^2\right)}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)**2)**3,x)
```

```
[Out] (5*a*c + 3*b*c**3 + 9*b*c*d**2*x**2 + 3*b*d**3*x**3 + x*(5*a*d + 9*b*c**2*d
))/(8*a**4*d + 16*a**3*b*c**2*d + 8*a**2*b**2*c**4*d + 32*a**2*b**2*c*d**4*
x**3 + 8*a**2*b**2*d**5*x**4 + x**2*(16*a**3*b*d**3 + 48*a**2*b**2*c**2*d**
3) + x*(32*a**3*b*c*d**2 + 32*a**2*b**2*c**3*d**2)) + (-3*sqrt(-1/(a**5*b))
*log(x + (-3*a**3*sqrt(-1/(a**5*b)) + 3*c)/(3*d))/16 + 3*sqrt(-1/(a**5*b))*
log(x + (3*a**3*sqrt(-1/(a**5*b)) + 3*c)/(3*d))/16)/d
```

Giac [A] time = 1.09979, size = 139, normalized size = 1.53

$$\frac{3\arctan\left(\frac{bdx+bc}{\sqrt{ab}}\right)}{8\sqrt{ab}a^2d}+\frac{3\,bd^3x^3+9\,bcd^2x^2+9\,bc^2dx+3\,bc^3+5\,adx+5\,ac}{8\left(bd^2x^2+2\,bcdx+bc^2+a\right)^2a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 3/8*arctan((b*d*x + b*c)/sqrt(a*b))/(sqrt(a*b)*a^2*d) + 1/8*(3*b*d^3*x^3 +
9*b*c*d^2*x^2 + 9*b*c^2*d*x + 3*b*c^3 + 5*a*d*x + 5*a*c)/((b*d^2*x^2 + 2*b*
c*d*x + b*c^2 + a)^2*a^2*d)
```

3.88 $\int \frac{1}{\sqrt{-a}+b(c+dx)^2}\,dx$

Optimal. Leaf size=35

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}(c+dx)}{\sqrt[4]{-a}}\right)}{\sqrt[4]{-a}\sqrt{b}d}$$

```
[Out] ArcTan[(Sqrt[b]*(c + d*x))/(-a)^(1/4)]/((-a)^(1/4)*Sqrt[b]*d)
```

Rubi [A] time = 0.0353452, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {247, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}(c+dx)}{\sqrt[4]{-a}}\right)}{\sqrt[4]{-a}\sqrt{b}d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-a] + b*(c + d*x)^2)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[b]*(c + d*x))/(-a)^(1/4)]/((-a)^(1/4)*Sqrt[b]*d)
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\sqrt{-a}+b(c+dx)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\sqrt{-a}+bx^2}\,dx, x, c+dx\right)}{d}$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{b}(c+dx)}{\sqrt[4]{-a}}\right)}{\sqrt[4]{-a}\sqrt{b}d}$$

Mathematica [A] time = 0.0136602, size = 35, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}(c+dx)}{\sqrt[4]{-a}}\right)}{\sqrt[4]{-a}\sqrt{b}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-a] + b*(c + d*x)^2)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[b]*(c + d*x))/(-a)^(1/4)]/((-a)^(1/4)*Sqrt[b]*d)
```

Maple [A] time = 0.006, size = 42, normalized size = 1.2

$$\frac{1}{d}\arctan\left(\frac{2\,bd^2x+2\,bcd}{2\,d}\frac{1}{\sqrt{\sqrt{-a}b}}\right)\frac{1}{\sqrt{\sqrt{-a}b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*(d*x+c)^2+(-a)^(1/2)),x)
```

```
[Out] 1/d/((-a)^(1/2)*b)^(1/2)*arctan(1/2*(2*b*d^2*x+2*b*c*d)/d/((-a)^(1/2)*b)^(1
/2))
```

Maxima [B] time = 1.65837, size = 89, normalized size = 2.54

$$\frac{\log\left(\frac{bd^2x+bcd-\sqrt{-\sqrt{-a}b}d}{bd^2x+bcd+\sqrt{-\sqrt{-a}b}d}\right)}{2\sqrt{-\sqrt{-a}b}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*(d*x+c)^2+(-a)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/2*log((b*d^2*x + b*c*d - sqrt(-sqrt(-a)*b)*d)/(b*d^2*x + b*c*d + sqrt(-sq
rt(-a)*b)*d))/(sqrt(-sqrt(-a)*b)*d)
```

Fricas [A] time = 2.11544, size = 576, normalized size = 16.46

$$\left[\frac{\sqrt{\frac{\sqrt{-a}}{ab}}\log\left(\frac{b^2d^4x^4+4\,b^2cd^3x^3+6\,b^2c^2d^2x^2+4\,b^2c^3dx+b^2c^4-2\left(bd^2x^2+2\,bcdx+bc^2\right)\sqrt{-a}+2\left(abdx+abc+\left(b^2d^3x^3+3\,b^2cd^2x^2+3\,b^2c^2dx+b^2c^3\right)\sqrt{-a}\right)\sqrt{\frac{\sqrt{-a}}{ab}}-a}{b^2d^4x^4+4\,b^2cd^3x^3+6\,b^2c^2d^2x^2+4\,b^2c^3dx+b^2c^4+a}\right)}{2\,d},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*(d*x+c)^2+(-a)^(1/2)),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(sqrt(-a)/(a*b))*log((b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^
2*x^2 + 4*b^2*c^3*d*x + b^2*c^4 - 2*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sqrt(-a
) + 2*(a*b*d*x + a*b*c + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b
^2*c^3)*sqrt(-a))*sqrt(sqrt(-a)/(a*b)) - a)/(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3
+ 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4 + a))/d, sqrt(-sqrt(-a)/(a*b)
)*arctan((b*d*x + b*c)*sqrt(-sqrt(-a)/(a*b)))/d]
```

Sympy [B] time = 0.198407, size = 92, normalized size = 2.63

$$\frac{-\frac{\sqrt{-\frac{1}{b\sqrt{-a}}}\log\left(x+\frac{c-\sqrt{-a}\sqrt{-\frac{1}{b\sqrt{-a}}}}{d}\right)}{2}+\frac{\sqrt{-\frac{1}{b\sqrt{-a}}}\log\left(x+\frac{c+\sqrt{-a}\sqrt{-\frac{1}{b\sqrt{-a}}}}{d}\right)}{2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*(d*x+c)**2+(-a)**(1/2)),x)
```

```
[Out] (-sqrt(-1/(b*sqrt(-a)))*log(x + (c - sqrt(-a)*sqrt(-1/(b*sqrt(-a))))/d)/2 +
 sqrt(-1/(b*sqrt(-a)))*log(x + (c + sqrt(-a)*sqrt(-1/(b*sqrt(-a))))/d)/2)/d
```

Giac [A] time = 1.14624, size = 41, normalized size = 1.17

$$\frac{\arctan\left(\frac{bdx+bc}{(-a)^{\frac{1}{4}}\sqrt{b}}\right)}{(-a)^{\frac{1}{4}}\sqrt{b}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*(d*x+c)^2+(-a)^(1/2)),x, algorithm="giac")
```

```
[Out] arctan((b*d*x + b*c)/((-a)^(1/4)*sqrt(b)))/((-a)^(1/4)*sqrt(b)*d)
```

3.89 $\int \frac{1}{1+(c+dx)^2}\,dx$

Optimal. Leaf size=10

$$\frac{\tan^{-1}(c+dx)}{d}$$

```
[Out] ArcTan[c + d*x]/d
```

Rubi [A] time = 0.002813, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {247, 203}

$$\frac{\tan^{-1}(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + (c + d*x)^2)^(-1),x]
```

```
[Out] ArcTan[c + d*x]/d
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1+(c+dx)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, c+dx\right)}{d}\\ &= \frac{\tan^{-1}(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.0044184, size = 10, normalized size = 1.

$$\frac{\tan^{-1}(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + (c + d*x)^2)^(-1),x]
```

```
[Out] ArcTan[c + d*x]/d
```

Maple [A] time = 0.003, size = 11, normalized size = 1.1

$$\frac{\arctan(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+(d*x+c)^2),x)
```

```
[Out] arctan(d*x+c)/d
```

Maxima [A] time = 1.5606, size = 24, normalized size = 2.4

$$\frac{\arctan\left(\frac{d^2x+cd}{d}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] arctan((d^2*x + c*d)/d)/d
```

Fricas [A] time = 1.68576, size = 26, normalized size = 2.6

$$\frac{\arctan(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] arctan(d*x + c)/d
```

Sympy [C] time = 0.158548, size = 24, normalized size = 2.4

$$\frac{-\frac{i\log\left(x+\frac{c-i}{d}\right)}{2}+\frac{i\log\left(x+\frac{c+i}{d}\right)}{2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(d*x+c)**2),x)
```

```
[Out] (-I*log(x + (c - I)/d)/2 + I*log(x + (c + I)/d)/2)/d
```

Giac [A] time = 1.12743, size = 14, normalized size = 1.4

$$\frac{\arctan\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(d*x+c)^2),x, algorithm="giac")
```

```
[Out] arctan(d*x + c)/d
```

3.90 $\int \frac{1}{\left(1+(c+dx)^2\right)^2}\,dx$

Optimal. Leaf size=37

$$\frac{c+dx}{2d\left((c+dx)^2+1\right)}+\frac{\tan^{-1}(c+dx)}{2d}$$

```
[Out] (c + d*x)/(2*d*(1 + (c + d*x)^2)) + ArcTan[c + d*x]/(2*d)
```

Rubi [A] time = 0.0095384, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {247, 199, 203}

$$\frac{c+dx}{2d\left((c+dx)^2+1\right)}+\frac{\tan^{-1}(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + (c + d*x)^2)^(-2),x]
```

```
[Out] (c + d*x)/(2*d*(1 + (c + d*x)^2)) + ArcTan[c + d*x]/(2*d)
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(1+(c+dx)^2\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)^2}\,dx, x, c+dx\right)}{d}$$

$$= \frac{c+dx}{2d\left(1+(c+dx)^2\right)} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, c+dx\right)}{2d}$$

$$= \frac{c+dx}{2d\left(1+(c+dx)^2\right)} + \frac{\tan^{-1}(c+dx)}{2d}$$

Mathematica [A] time = 0.011791, size = 31, normalized size = 0.84

$$\frac{\frac{c+dx}{(c+dx)^2+1} + \tan^{-1}(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + (c + d*x)^2)^(-2),x]
```

```
[Out] ((c + d*x)/(1 + (c + d*x)^2) + ArcTan[c + d*x])/(2*d)
```

Maple [A] time = 0.004, size = 59, normalized size = 1.6

$$\frac{2\,d^2x+2\,cd}{4\,d^2\left(d^2x^2+2\,cdx+c^2+1\right)} + \frac{1}{2\,d}\arctan\left(\frac{2\,d^2x+2\,cd}{2\,d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+(d*x+c)^2)^2,x)
```

```
[Out] 1/4*(2*d^2*x+2*c*d)/d^2/(d^2*x^2+2*c*d*x+c^2+1)+1/2/d*arctan(1/2*(2*d^2*x+2
*c*d)/d)
```

Maxima [A] time = 1.45754, size = 69, normalized size = 1.86

$$\frac{dx+c}{2\left(d^3x^2+2\,cd^2x+\left(c^2+1\right)d\right)}+\frac{\arctan\left(\frac{d^2x+cd}{d}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(d*x + c)/(d^3*x^2 + 2*c*d^2*x + (c^2 + 1)*d) + 1/2*arctan((d^2*x + c*d
)/d)/d
```

Fricas [A] time = 1.80018, size = 134, normalized size = 3.62

$$\frac{dx+\left(d^2x^2+2\,cdx+c^2+1\right)\arctan\left(dx+c\right)+c}{2\left(d^3x^2+2\,cd^2x+\left(c^2+1\right)d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(d*x + (d^2*x^2 + 2*c*d*x + c^2 + 1)*arctan(d*x + c) + c)/(d^3*x^2 + 2*
c*d^2*x + (c^2 + 1)*d)
```

Sympy [C] time = 0.58651, size = 56, normalized size = 1.51

$$\frac{c+dx}{2c^2d+4cd^2x+2d^3x^2+2d}+\frac{-\frac{i\log\left(x+\frac{c-i}{d}\right)}{4}+\frac{i\log\left(x+\frac{c+i}{d}\right)}{4}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(d*x+c)**2)**2,x)
```

```
[Out] (c + d*x)/(2*c**2*d + 4*c*d**2*x + 2*d**3*x**2 + 2*d) + (-I*log(x + (c - I)
/d)/4 + I*log(x + (c + I)/d)/4)/d
```

<u>Giac</u> [A] time = 1.1549, size = 55, normalized size = 1.49

$$\frac{\arctan(dx+c)}{2d}+\frac{dx+c}{2\left(d^2x^2+2\,cdx+c^2+1\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*arctan(d*x + c)/d + 1/2*(d*x + c)/((d^2*x^2 + 2*c*d*x + c^2 + 1)*d)
```

3.91 $\int \frac{1}{\left(1+(c+dx)^2\right)^3} dx$

Optimal. Leaf size=60

$$\frac{3(c+dx)}{8d\left((c+dx)^2+1\right)}+\frac{c+dx}{4d\left((c+dx)^2+1\right)^2}+\frac{3\tan^{-1}(c+dx)}{8d}$$

```
[Out] (c + d*x)/(4*d*(1 + (c + d*x)^2)^2) + (3*(c + d*x))/(8*d*(1 + (c + d*x)^2))
 + (3*ArcTan[c + d*x])/(8*d)
```

Rubi [A] time = 0.0163135, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {247, 199, 203}

$$\frac{3(c+dx)}{8d\left((c+dx)^2+1\right)}+\frac{c+dx}{4d\left((c+dx)^2+1\right)^2}+\frac{3\tan^{-1}(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + (c + d*x)^2)^(-3),x]
```

```
[Out] (c + d*x)/(4*d*(1 + (c + d*x)^2)^2) + (3*(c + d*x))/(8*d*(1 + (c + d*x)^2))
 + (3*ArcTan[c + d*x])/(8*d)
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(1+(c+dx)^2\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)^3}\,dx, x, c+dx\right)}{d}$$

$$= \frac{c+dx}{4d\left(1+(c+dx)^2\right)^2} + \frac{3\,\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)^2}\,dx, x, c+dx\right)}{4d}$$

$$= \frac{c+dx}{4d\left(1+(c+dx)^2\right)^2} + \frac{3(c+dx)}{8d\left(1+(c+dx)^2\right)} + \frac{3\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, c+dx\right)}{8d}$$

$$= \frac{c+dx}{4d\left(1+(c+dx)^2\right)^2} + \frac{3(c+dx)}{8d\left(1+(c+dx)^2\right)} + \frac{3\tan^{-1}(c+dx)}{8d}$$

Mathematica [A] time = 0.0152632, size = 52, normalized size = 0.87

$$\frac{\frac{3(c+dx)}{(c+dx)^2+1} + \frac{2(c+dx)}{\left((c+dx)^2+1\right)^2} + 3\tan^{-1}(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + (c + d*x)^2)^(-3),x]
```

```
[Out] ((2*(c + d*x))/(1 + (c + d*x)^2)^2 + (3*(c + d*x))/(1 + (c + d*x)^2) + 3*Ar
cTan[c + d*x])/(8*d)
```

Maple [A] time = 0.003, size = 94, normalized size = 1.6

$$\frac{2\,d^2x+2\,cd}{8\,d^2\left(d^2x^2+2\,cdx+c^2+1\right)^2} + \frac{6\,d^2x+6\,cd}{16\,d^2\left(d^2x^2+2\,cdx+c^2+1\right)} + \frac{3}{8\,d}\arctan\left(\frac{2\,d^2x+2\,cd}{2\,d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+(d*x+c)^2)^3,x)
```

```
[Out] 1/8*(2*d^2*x+2*c*d)/d^2/(d^2*x^2+2*c*d*x+c^2+1)^2+3/16*(2*d^2*x+2*c*d)/d^2/
(d^2*x^2+2*c*d*x+c^2+1)+3/8/d*arctan(1/2*(2*d^2*x+2*c*d)/d)
```

Maxima [B] time = 1.48005, size = 155, normalized size = 2.58

$$\frac{3\,d^3x^3+9\,cd^2x^2+3\,c^3+\left(9\,c^2+5\right)dx+5\,c}{8\left(d^5x^4+4\,cd^4x^3+2\left(3\,c^2+1\right)d^3x^2+4\left(c^3+c\right)d^2x+\left(c^4+2\,c^2+1\right)d\right)}+\frac{3\,\arctan\left(\frac{d^2x+cd}{d}\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(3*d^3*x^3 + 9*c*d^2*x^2 + 3*c^3 + (9*c^2 + 5)*d*x + 5*c)/(d^5*x^4 + 4*
c*d^4*x^3 + 2*(3*c^2 + 1)*d^3*x^2 + 4*(c^3 + c)*d^2*x + (c^4 + 2*c^2 + 1)*d
) + 3/8*arctan((d^2*x + c*d)/d)/d
```

Fricas [B] time = 1.74752, size = 347, normalized size = 5.78

$$\frac{3\,d^3x^3+9\,cd^2x^2+3\,c^3+\left(9\,c^2+5\right)dx+3\left(d^4x^4+4\,cd^3x^3+2\left(3\,c^2+1\right)d^2x^2+c^4+4\left(c^3+c\right)dx+2\,c^2+1\right)\arctan\left(dx\right.}{8\left(d^5x^4+4\,cd^4x^3+2\left(3\,c^2+1\right)d^3x^2+4\left(c^3+c\right)d^2x+\left(c^4+2\,c^2+1\right)d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/8*(3*d^3*x^3 + 9*c*d^2*x^2 + 3*c^3 + (9*c^2 + 5)*d*x + 3*(d^4*x^4 + 4*c*d
^3*x^3 + 2*(3*c^2 + 1)*d^2*x^2 + c^4 + 4*(c^3 + c)*d*x + 2*c^2 + 1)*arctan(
d*x + c) + 5*c)/(d^5*x^4 + 4*c*d^4*x^3 + 2*(3*c^2 + 1)*d^3*x^2 + 4*(c^3 + c
)*d^2*x + (c^4 + 2*c^2 + 1)*d)
```

Sympy [C] time = 1.2406, size = 146, normalized size = 2.43

$$\frac{3c^3+9cd^2x^2+5c+3d^3x^3+x\left(9c^2d+5d\right)}{8c^4d+16c^2d+32cd^4x^3+8d^5x^4+8d+x^2\left(48c^2d^3+16d^3\right)+x\left(32c^3d^2+32cd^2\right)}+\frac{-\frac{3i\log\left(x+\frac{3c-3i}{3d}\right)}{16}+\frac{3i\log\left(x+\frac{3c+3i}{3d}\right)}{16}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(d*x+c)**2)**3,x)
```

```
[Out] (3*c**3 + 9*c*d**2*x**2 + 5*c + 3*d**3*x**3 + x*(9*c**2*d + 5*d))/(8*c**4*d
 + 16*c**2*d + 32*c*d**4*x**3 + 8*d**5*x**4 + 8*d + x**2*(48*c**2*d**3 + 16
*d**3) + x*(32*c**3*d**2 + 32*c*d**2)) + (-3*I*log(x + (3*c - 3*I)/(3*d))/1
6 + 3*I*log(x + (3*c + 3*I)/(3*d))/16)/d
```

Giac [A] time = 1.11363, size = 99, normalized size = 1.65

$$\frac{3\arctan\left(dx+c\right)}{8d}+\frac{3d^3x^3+9cd^2x^2+9c^2dx+3c^3+5dx+5c}{8\left(d^2x^2+2cdx+c^2+1\right)^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 3/8*arctan(d*x + c)/d + 1/8*(3*d^3*x^3 + 9*c*d^2*x^2 + 9*c^2*d*x + 3*c^3 +
5*d*x + 5*c)/((d^2*x^2 + 2*c*d*x + c^2 + 1)^2*d)
```

3.92 $\int \frac{1}{1-(c+dx)^2}\, dx$

Optimal. Leaf size=10

$$\frac{\tanh^{-1}(c+dx)}{d}$$

```
[Out] ArcTanh[c + d*x]/d
```

Rubi [A] time = 0.0029759, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {247, 206}

$$\frac{\tanh^{-1}(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - (c + d*x)^2)^(-1),x]
```

```
[Out] ArcTanh[c + d*x]/d
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{1-(c+dx)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, c+dx\right)}{d}$$
$$= \frac{\tanh^{-1}(c+dx)}{d}$$

Mathematica [B] time = 0.0052695, size = 32, normalized size = 3.2

$$\frac{\log(c+dx+1)}{2d} - \frac{\log(-c-dx+1)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - (c + d*x)^2)^(-1),x]
```

```
[Out] -Log[1 - c - d*x]/(2*d) + Log[1 + c + d*x]/(2*d)
```

Maple [B] time = 0.007, size = 26, normalized size = 2.6

$$\frac{\ln(dx+c+1)}{2d} - \frac{\ln(dx+c-1)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-(d*x+c)^2),x)
```

```
[Out] 1/2/d*ln(d*x+c+1)-1/2/d*ln(d*x+c-1)
```

Maxima [B] time = 0.958947, size = 34, normalized size = 3.4

$$\frac{\log(dx+c+1)}{2d} - \frac{\log(dx+c-1)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/2*log(d*x + c + 1)/d - 1/2*log(d*x + c - 1)/d
```

Fricas [B] time = 1.742, size = 61, normalized size = 6.1

$$\frac{\log(dx+c+1)-\log(dx+c-1)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/2*(log(d*x + c + 1) - log(d*x + c - 1))/d
```

Sympy [B] time = 0.159961, size = 22, normalized size = 2.2

$$-\frac{\frac{\log\left(x+\frac{c-1}{d}\right)}{2}-\frac{\log\left(x+\frac{c+1}{d}\right)}{2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(d*x+c)**2),x)
```

```
[Out] -(log(x + (c - 1)/d)/2 - log(x + (c + 1)/d)/2)/d
```

Giac [B] time = 1.11608, size = 36, normalized size = 3.6

$$\frac{\log(|dx+c+1|)}{2d}-\frac{\log(|dx+c-1|)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(d*x + c + 1))/d - 1/2*log(abs(d*x + c - 1))/d
```

3.93 $\int \frac{1}{\left(1-(c+dx)^2\right)^2} dx$

Optimal. Leaf size=39

$$\frac{c+dx}{2d\left(1-(c+dx)^2\right)}+\frac{\tanh^{-1}(c+dx)}{2d}$$

```
[Out] (c + d*x)/(2*d*(1 - (c + d*x)^2)) + ArcTanh[c + d*x]/(2*d)
```

Rubi [A] time = 0.0126135, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {247, 199, 206}

$$\frac{c+dx}{2d\left(1-(c+dx)^2\right)}+\frac{\tanh^{-1}(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - (c + d*x)^2)^(-2),x]
```

```
[Out] (c + d*x)/(2*d*(1 - (c + d*x)^2)) + ArcTanh[c + d*x]/(2*d)
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(1-(c+dx)^2\right)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^2}\,dx, x, c+dx\right)}{d}\\ &= \frac{c+dx}{2d\left(1-(c+dx)^2\right)}+\frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, c+dx\right)}{2d}\\ &= \frac{c+dx}{2d\left(1-(c+dx)^2\right)}+\frac{\tanh^{-1}(c+dx)}{2d}\end{aligned}$$

Mathematica [A] time = 0.0197483, size = 45, normalized size = 1.15

$$\frac{-\frac{2(c+dx)}{(c+dx)^2-1}-\log(-c-dx+1)+\log(c+dx+1)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - (c + d*x)^2)^(-2),x]
```

```
[Out] ((-2*(c + d*x))/(-1 + (c + d*x)^2) - Log[1 - c - d*x] + Log[1 + c + d*x])/(
4*d)
```

Maple [A] time = 0.008, size = 52, normalized size = 1.3

$$-\frac{1}{4\,d\,(dx+c+1)}+\frac{\ln(dx+c+1)}{4\,d}-\frac{1}{4\,d\,(dx+c-1)}-\frac{\ln(dx+c-1)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-(d*x+c)^2)^2,x)
```

```
[Out] -1/4/d/(d*x+c+1)+1/4/d*ln(d*x+c+1)-1/4/d/(d*x+c-1)-1/4/d*ln(d*x+c-1)
```

Maxima [A] time = 1.05549, size = 76, normalized size = 1.95

$$-\frac{dx+c}{2\left(d^3x^2+2\,cd^2x+\left(c^2-1\right)d\right)}+\frac{\log\left(dx+c+1\right)}{4\,d}-\frac{\log\left(dx+c-1\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(d*x + c)/(d^3*x^2 + 2*c*d^2*x + (c^2 - 1)*d) + 1/4*log(d*x + c + 1)/d
 - 1/4*log(d*x + c - 1)/d
```

Fricas [B] time = 1.71631, size = 208, normalized size = 5.33

$$-\frac{2\,dx-\left(d^2x^2+2\,cdx+c^2-1\right)\log\left(dx+c+1\right)+\left(d^2x^2+2\,cdx+c^2-1\right)\log\left(dx+c-1\right)+2\,c}{4\left(d^3x^2+2\,cd^2x+\left(c^2-1\right)d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*d*x - (d^2*x^2 + 2*c*d*x + c^2 - 1)*log(d*x + c + 1) + (d^2*x^2 + 2
*c*d*x + c^2 - 1)*log(d*x + c - 1) + 2*c)/(d^3*x^2 + 2*c*d^2*x + (c^2 - 1)*
d)
```

Sympy [A] time = 0.595988, size = 53, normalized size = 1.36

$$-\frac{c+dx}{2c^2d+4cd^2x+2d^3x^2-2d}+\frac{-\frac{\log\left(x+\frac{c-1}{d}\right)}{4}+\frac{\log\left(x+\frac{c+1}{d}\right)}{4}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(d*x+c)**2)**2,x)
```

```
[Out] -(c + d*x)/(2*c**2*d + 4*c*d**2*x + 2*d**3*x**2 - 2*d) + (-log(x + (c - 1)/
d)/4 + log(x + (c + 1)/d)/4)/d
```

Giac [A] time = 1.15003, size = 76, normalized size = 1.95

$$\frac{\log\left(|dx+c+1|\right)}{4\,d}-\frac{\log\left(|dx+c-1|\right)}{4\,d}-\frac{dx+c}{2\left(d^2x^2+2\,cdx+c^2-1\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/4*log(abs(d*x + c + 1))/d - 1/4*log(abs(d*x + c - 1))/d - 1/2*(d*x + c)/(
(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)
```

3.94 $\int \frac{1}{\left(1-(c+dx)^2\right)^3}\,dx$

Optimal. Leaf size=64

$$\frac{3(c+dx)}{8d\left(1-(c+dx)^2\right)}+\frac{c+dx}{4d\left(1-(c+dx)^2\right)^2}+\frac{3\tanh^{-1}(c+dx)}{8d}$$

```
[Out] (c + d*x)/(4*d*(1 - (c + d*x)^2)^2) + (3*(c + d*x))/(8*d*(1 - (c + d*x)^2))
 + (3*ArcTanh[c + d*x])/(8*d)
```

Rubi [A] time = 0.0223646, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {247, 199, 206}

$$\frac{3(c+dx)}{8d\left(1-(c+dx)^2\right)}+\frac{c+dx}{4d\left(1-(c+dx)^2\right)^2}+\frac{3\tanh^{-1}(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - (c + d*x)^2)^(-3),x]
```

```
[Out] (c + d*x)/(4*d*(1 - (c + d*x)^2)^2) + (3*(c + d*x))/(8*d*(1 - (c + d*x)^2))
 + (3*ArcTanh[c + d*x])/(8*d)
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(1-(c+dx)^2\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^3}\,dx,x,c+dx\right)}{d}$$

$$= \frac{c+dx}{4d\left(1-(c+dx)^2\right)^2} + \frac{3\,\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^2}\,dx,x,c+dx\right)}{4d}$$

$$= \frac{c+dx}{4d\left(1-(c+dx)^2\right)^2} + \frac{3(c+dx)}{8d\left(1-(c+dx)^2\right)} + \frac{3\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,c+dx\right)}{8d}$$

$$= \frac{c+dx}{4d\left(1-(c+dx)^2\right)^2} + \frac{3(c+dx)}{8d\left(1-(c+dx)^2\right)} + \frac{3\tanh^{-1}(c+dx)}{8d}$$

Mathematica [A] time = 0.0266841, size = 65, normalized size = 1.02

$$\frac{-\frac{6(c+dx)}{(c+dx)^2-1}+\frac{4(c+dx)}{\left((c+dx)^2-1\right)^2}-3\log(-c-dx+1)+3\log(c+dx+1)}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - (c + d*x)^2)^(-3),x]
```

```
[Out] ((4*(c + d*x))/(-1 + (c + d*x)^2)^2 - (6*(c + d*x))/(-1 + (c + d*x)^2) - 3*
Log[1 - c - d*x] + 3*Log[1 + c + d*x])/(16*d)
```

Maple [A] time = 0.008, size = 78, normalized size = 1.2

$$-\frac{1}{16\,d\,(dx+c+1)^2}-\frac{3}{16\,d\,(dx+c+1)}+\frac{3\,\ln(dx+c+1)}{16\,d}+\frac{1}{16\,d\,(dx+c-1)^2}-\frac{3}{16\,d\,(dx+c-1)}-\frac{3\,\ln(dx+c-1)}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-(d*x+c)^2)^3,x)
```

```
[Out] -1/16/d/(d*x+c+1)^2-3/16/d/(d*x+c+1)+3/16/d*ln(d*x+c+1)+1/16/d/(d*x+c-1)^2-
3/16/d/(d*x+c-1)-3/16/d*ln(d*x+c-1)
```

Maxima [B] time = 1.01374, size = 165, normalized size = 2.58

$$-\frac{3\,d^3x^3+9\,cd^2x^2+3\,c^3+\left(9\,c^2-5\right)dx-5\,c}{8\left(d^5x^4+4\,cd^4x^3+2\left(3\,c^2-1\right)d^3x^2+4\left(c^3-c\right)d^2x+\left(c^4-2\,c^2+1\right)d\right)}+\frac{3\,\log\left(dx+c+1\right)}{16\,d}-\frac{3\,\log\left(dx+c-1\right)}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(3*d^3*x^3 + 9*c*d^2*x^2 + 3*c^3 + (9*c^2 - 5)*d*x - 5*c)/(d^5*x^4 + 4
*c*d^4*x^3 + 2*(3*c^2 - 1)*d^3*x^2 + 4*(c^3 - c)*d^2*x + (c^4 - 2*c^2 + 1)*
d) + 3/16*log(d*x + c + 1)/d - 3/16*log(d*x + c - 1)/d
```

Fricas [B] time = 1.69552, size = 498, normalized size = 7.78

$$-\frac{6\,d^3x^3+18\,cd^2x^2+6\,c^3+2\left(9\,c^2-5\right)dx-3\left(d^4x^4+4\,cd^3x^3+2\left(3\,c^2-1\right)d^2x^2+c^4+4\left(c^3-c\right)dx-2\,c^2+1\right)\log\left(dx+\right.}{16\left(d^5x^4+4\,cd^4x^3+2\left(3\,c^2-1\right)d^3x^2+4\left(c^3-\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(6*d^3*x^3 + 18*c*d^2*x^2 + 6*c^3 + 2*(9*c^2 - 5)*d*x - 3*(d^4*x^4 +
4*c*d^3*x^3 + 2*(3*c^2 - 1)*d^2*x^2 + c^4 + 4*(c^3 - c)*d*x - 2*c^2 + 1)*lo
g(d*x + c + 1) + 3*(d^4*x^4 + 4*c*d^3*x^3 + 2*(3*c^2 - 1)*d^2*x^2 + c^4 + 4
*(c^3 - c)*d*x - 2*c^2 + 1)*log(d*x + c - 1) - 10*c)/(d^5*x^4 + 4*c*d^4*x^3
 + 2*(3*c^2 - 1)*d^3*x^2 + 4*(c^3 - c)*d^2*x + (c^4 - 2*c^2 + 1)*d)
```

Sympy [B] time = 1.30668, size = 141, normalized size = 2.2

$$-\frac{3c^3+9cd^2x^2-5c+3d^3x^3+x\left(9c^2d-5d\right)}{8c^4d-16c^2d+32cd^4x^3+8d^5x^4+8d+x^2\left(48c^2d^3-16d^3\right)+x\left(32c^3d^2-32cd^2\right)}-\frac{\frac{3\log\left(x+\frac{3c-3}{3d}\right)}{16}-\frac{3\log\left(x+\frac{3c+3}{3d}\right)}{16}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(d*x+c)**2)**3,x)
```

```
[Out] -(3*c**3 + 9*c*d**2*x**2 - 5*c + 3*d**3*x**3 + x*(9*c**2*d - 5*d))/(8*c**4*
d - 16*c**2*d + 32*c*d**4*x**3 + 8*d**5*x**4 + 8*d + x**2*(48*c**2*d**3 - 1
6*d**3) + x*(32*c**3*d**2 - 32*c*d**2)) - (3*log(x + (3*c - 3)/(3*d))/16 -
3*log(x + (3*c + 3)/(3*d))/16)/d
```

Giac [A] time = 1.13959, size = 119, normalized size = 1.86

$$\frac{3\,\log\left(|dx+c+1|\right)}{16\,d}-\frac{3\,\log\left(|dx+c-1|\right)}{16\,d}-\frac{3\,d^3x^3+9\,cd^2x^2+9\,c^2dx+3\,c^3-5\,dx-5\,c}{8\left(d^2x^2+2\,cdx+c^2-1\right)^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 3/16*log(abs(d*x + c + 1))/d - 3/16*log(abs(d*x + c - 1))/d - 1/8*(3*d^3*x^
3 + 9*c*d^2*x^2 + 9*c^2*d*x + 3*c^3 - 5*d*x - 5*c)/((d^2*x^2 + 2*c*d*x + c^
2 - 1)^2*d)
```

3.95 $\int \frac{1}{1-(1+x)^2} \, dx$

Optimal. Leaf size=4

$$\tanh^{-1}(x+1)$$

```
[Out] ArcTanh[1 + x]
```

Rubi [A] time = 0.0018798, antiderivative size = 4, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {247, 206}

$$\tanh^{-1}(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - (1 + x)^2)^(-1),x]
```

```
[Out] ArcTanh[1 + x]
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{1}{1-(1+x)^2} \, dx &= \text{Subst}\left(\int \frac{1}{1-x^2} \, dx, x, 1+x\right) \\ &= \tanh^{-1}(1+x) \end{aligned}$$

Mathematica [B] time = 0.0021521, size = 15, normalized size = 3.75

$$\frac{1}{2}\log(x+2)-\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - (1 + x)^2)^(-1),x]
```

```
[Out] -Log[x]/2 + Log[2 + x]/2
```

Maple [B] time = 0.005, size = 12, normalized size = 3.

$$-\frac{\ln(x)}{2}+\frac{\ln(2+x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-(1+x)^2),x)
```

```
[Out] -1/2*ln(x)+1/2*ln(2+x)
```

Maxima [B] time = 1.00659, size = 15, normalized size = 3.75

$$\frac{1}{2}\log(x+2)-\frac{1}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(1+x)^2),x, algorithm="maxima")
```

```
[Out] 1/2*log(x + 2) - 1/2*log(x)
```

Fricas [B] time = 1.66796, size = 39, normalized size = 9.75

$$\frac{1}{2}\log(x+2)-\frac{1}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(1+x)^2),x, algorithm="fricas")
```

```
[Out] 1/2*log(x + 2) - 1/2*log(x)
```

Sympy [B] time = 0.093003, size = 10, normalized size = 2.5

$$-\frac{\log(x)}{2}+\frac{\log(x+2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(1+x)**2),x)
```

```
[Out] -log(x)/2 + log(x + 2)/2
```

Giac [B] time = 1.11716, size = 18, normalized size = 4.5

$$\frac{1}{2}\log(|x+2|)-\frac{1}{2}\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(1+x)^2),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(x + 2)) - 1/2*log(abs(x))
```

3.96 $\int \frac{1}{\left(1-(1+x)^2\right)^2} dx$

Optimal. Leaf size=27

$$\frac{x+1}{2\left(1-(x+1)^2\right)}+\frac{1}{2}\tanh^{-1}(x+1)$$

```
[Out] (1 + x)/(2*(1 - (1 + x)^2)) + ArcTanh[1 + x]/2
```

Rubi [A] time = 0.0072031, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {247, 199, 206}

$$\frac{x+1}{2\left(1-(x+1)^2\right)}+\frac{1}{2}\tanh^{-1}(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - (1 + x)^2)^(-2),x]
```

```
[Out] (1 + x)/(2*(1 - (1 + x)^2)) + ArcTanh[1 + x]/2
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(1-(1+x)^2\right)^2}\,dx &= \text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^2}\,dx,x,1+x\right)\\ &= \frac{1+x}{2\left(1-(1+x)^2\right)}+\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,1+x\right)\\ &= \frac{1+x}{2\left(1-(1+x)^2\right)}+\frac{1}{2}\tanh^{-1}(1+x)\end{aligned}$$

Mathematica [A] time = 0.0162995, size = 26, normalized size = 0.96

$$\frac{1}{4}\left(-\frac{2(x+1)}{x(x+2)}-\log(x)+\log(x+2)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - (1 + x)^2)^(-2),x]
```

```
[Out] ((-2*(1 + x))/(x*(2 + x)) - Log[x] + Log[2 + x])/4
```

Maple [A] time = 0.007, size = 24, normalized size = 0.9

$$-\frac{1}{4\,x}-\frac{\ln(x)}{4}-\frac{1}{8+4\,x}+\frac{\ln(2+x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-(1+x)^2)^2,x)
```

```
[Out] -1/4/x-1/4*ln(x)-1/4/(2+x)+1/4*ln(2+x)
```

Maxima [A] time = 1.16811, size = 34, normalized size = 1.26

$$-\frac{x+1}{2\left(x^2+2\,x\right)}+\frac{1}{4}\,\log(x+2)-\frac{1}{4}\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(1+x)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(x + 1)/(x^2 + 2*x) + 1/4*log(x + 2) - 1/4*log(x)
```

Fricas [A] time = 1.67117, size = 99, normalized size = 3.67

$$\frac{\left(x^2+2\,x\right)\log\left(x+2\right)-\left(x^2+2\,x\right)\log\left(x\right)-2\,x-2}{4\left(x^2+2\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(1+x)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/4*((x^2 + 2*x)*log(x + 2) - (x^2 + 2*x)*log(x) - 2*x - 2)/(x^2 + 2*x)
```

Sympy [A] time = 0.10968, size = 22, normalized size = 0.81

$$-\frac{x+1}{2x^2+4x}-\frac{\log\left(x\right)}{4}+\frac{\log\left(x+2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(1+x)**2)**2,x)
```

```
[Out] -(x + 1)/(2*x**2 + 4*x) - log(x)/4 + log(x + 2)/4
```

Giac [A] time = 1.11523, size = 36, normalized size = 1.33

$$-\frac{x+1}{2\left(x^2+2\,x\right)}+\frac{1}{4}\,\log\left(|x+2|\right)-\frac{1}{4}\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(1+x)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(x + 1)/(x^2 + 2*x) + 1/4*log(abs(x + 2)) - 1/4*log(abs(x))
```

3.97 $\int \frac{1}{\left(1-(1+x)^2\right)^3} dx$

Optimal. Leaf size=45

$$\frac{3(x+1)}{8\left(1-(x+1)^2\right)}+\frac{x+1}{4\left(1-(x+1)^2\right)^2}+\frac{3}{8}\tanh^{-1}(x+1)$$

```
[Out] (1 + x)/(4*(1 - (1 + x)^2)^2) + (3*(1 + x))/(8*(1 - (1 + x)^2)) + (3*ArcTan
h[1 + x])/8
```

Rubi [A] time = 0.0122762, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {247, 199, 206}

$$\frac{3(x+1)}{8\left(1-(x+1)^2\right)}+\frac{x+1}{4\left(1-(x+1)^2\right)^2}+\frac{3}{8}\tanh^{-1}(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - (1 + x)^2)^(-3),x]
```

```
[Out] (1 + x)/(4*(1 - (1 + x)^2)^2) + (3*(1 + x))/(8*(1 - (1 + x)^2)) + (3*ArcTan
h[1 + x])/8
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(1-(1+x)^2\right)^3}\,dx &= \text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^3}\,dx, x, 1+x\right)\\
&= \frac{1+x}{4\left(1-(1+x)^2\right)^2} + \frac{3}{4}\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^2}\,dx, x, 1+x\right)\\
&= \frac{1+x}{4\left(1-(1+x)^2\right)^2} + \frac{3(1+x)}{8\left(1-(1+x)^2\right)} + \frac{3}{8}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, 1+x\right)\\
&= \frac{1+x}{4\left(1-(1+x)^2\right)^2} + \frac{3(1+x)}{8\left(1-(1+x)^2\right)} + \frac{3}{8}\tanh^{-1}(1+x)
\end{aligned}$$

Mathematica [A] time = 0.0174647, size = 37, normalized size = 0.82

$$\frac{1}{16}\left(\frac{1}{x^2} - \frac{3}{x} - \frac{3}{x+2} - \frac{1}{(x+2)^2} - 3\log(x) + 3\log(x+2)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - (1 + x)^2)^(-3),x]
```

```
[Out] (x^(-2) - 3/x - (2 + x)^(-2) - 3/(2 + x) - 3*Log[x] + 3*Log[2 + x])/16
```

Maple [A] time = 0.009, size = 36, normalized size = 0.8

$$\frac{1}{16\,x^2} - \frac{3}{16\,x} - \frac{3\,\ln(x)}{16} - \frac{1}{16\,(2+x)^2} - \frac{3}{32+16\,x} + \frac{3\,\ln(2+x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-(1+x)^2)^3,x)
```

```
[Out] 1/16/x^2-3/16/x-3/16*ln(x)-1/16/(2+x)^2-3/16/(2+x)+3/16*ln(2+x)
```

Maxima [A] time = 1.18182, size = 59, normalized size = 1.31

$$-\frac{3\,x^3+9\,x^2+4\,x-2}{8\left(x^4+4\,x^3+4\,x^2\right)}+\frac{3}{16}\,\log\left(x+2\right)-\frac{3}{16}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(1+x)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(3*x^3 + 9*x^2 + 4*x - 2)/(x^4 + 4*x^3 + 4*x^2) + 3/16*log(x + 2) - 3/
16*log(x)
```

Fricas [B] time = 1.7459, size = 170, normalized size = 3.78

$$-\frac{6\,x^3+18\,x^2-3\left(x^4+4\,x^3+4\,x^2\right)\log\left(x+2\right)+3\left(x^4+4\,x^3+4\,x^2\right)\log\left(x\right)+8\,x-4}{16\left(x^4+4\,x^3+4\,x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(1+x)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(6*x^3 + 18*x^2 - 3*(x^4 + 4*x^3 + 4*x^2)*log(x + 2) + 3*(x^4 + 4*x^3
 + 4*x^2)*log(x) + 8*x - 4)/(x^4 + 4*x^3 + 4*x^2)
```

Sympy [A] time = 0.138179, size = 44, normalized size = 0.98

$$-\frac{3\log\left(x\right)}{16}+\frac{3\log\left(x+2\right)}{16}-\frac{3x^3+9x^2+4x-2}{8x^4+32x^3+32x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(1+x)**2)**3,x)
```

```
[Out] -3*log(x)/16 + 3*log(x + 2)/16 - (3*x**3 + 9*x**2 + 4*x - 2)/(8*x**4 + 32*x
**3 + 32*x**2)
```

Giac [A] time = 1.14151, size = 53, normalized size = 1.18

$$-\frac{3\,x^{3}+9\,x^{2}+4\,x-2}{8\,\left(x^{2}+2\,x\right)^{2}}+\frac{3}{16}\,\log\left(|x+2|\right)-\frac{3}{16}\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(1+x)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*(3*x^3 + 9*x^2 + 4*x - 2)/(x^2 + 2*x)^2 + 3/16*log(abs(x + 2)) - 3/16*
log(abs(x))
```

3.98 $\int \frac{\left(1+(a+bx)^2\right)^2}{x}\,dx$

Optimal. Leaf size=59

$$\frac{1}{2}\left(a^2+2\right)(a+bx)^2+a\left(a^2+2\right)bx+\left(a^2+1\right)^2\log(x)+\frac{1}{4}(a+bx)^4+\frac{1}{3}a(a+bx)^3$$

```
[Out] a*(2 + a^2)*b*x + ((2 + a^2)*(a + b*x)^2)/2 + (a*(a + b*x)^3)/3 + (a + b*x)
^4/4 + (1 + a^2)^2*Log[x]
```

Rubi [A] time = 0.0553684, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {371, 697}

$$\frac{1}{2}\left(a^2+2\right)(a+bx)^2+a\left(a^2+2\right)bx+\left(a^2+1\right)^2\log(x)+\frac{1}{4}(a+bx)^4+\frac{1}{3}a(a+bx)^3$$

Antiderivative was successfully verified.

```
[In] Int[(1 + (a + b*x)^2)^2/x,x]
```

```
[Out] a*(2 + a^2)*b*x + ((2 + a^2)*(a + b*x)^2)/2 + (a*(a + b*x)^3)/3 + (a + b*x)
^4/4 + (1 + a^2)^2*Log[x]
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(1+(a+bx)^2\right)^2}{x}\,dx = \text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{-a+x}\,dx, x, a+bx\right)$$
$$= \text{Subst}\left(\int \left(a\left(2+a^2\right) - \frac{\left(1+a^2\right)^2}{a-x} + \left(2+a^2\right)x + ax^2 + x^3\right)dx, x, a+bx\right)$$
$$= a\left(2+a^2\right)bx + \frac{1}{2}\left(2+a^2\right)(a+bx)^2 + \frac{1}{3}a(a+bx)^3 + \frac{1}{4}(a+bx)^4 + \left(1+a^2\right)^2\log(x)$$

Mathematica [A] time = 0.0214168, size = 64, normalized size = 1.08

$$\frac{1}{2}\left(a^2+2\right)(a+bx)^2 + a\left(a^2+2\right)(a+bx) + \left(a^2+1\right)^2\log(bx) + \frac{1}{4}(a+bx)^4 + \frac{1}{3}a(a+bx)^3$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + (a + b*x)^2)^2/x,x]
```

```
[Out] a*(2 + a^2)*(a + b*x) + ((2 + a^2)*(a + b*x)^2)/2 + (a*(a + b*x)^3)/3 + (a
+ b*x)^4/4 + (1 + a^2)^2*Log[b*x]
```

Maple [A] time = 0.003, size = 64, normalized size = 1.1

$$\frac{b^4x^4}{4} + \frac{4\,ab^3x^3}{3} + 3\,x^2a^2b^2 + 4\,a^3bx + b^2x^2 + 4\,abx + \ln(x)\,a^4 + 2\,\ln(x)\,a^2 + \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+(b*x+a)^2)^2/x,x)
```

```
[Out] 1/4*b^4*x^4+4/3*a*b^3*x^3+3*x^2*a^2*b^2+4*a^3*b*x+b^2*x^2+4*a*b*x+ln(x)*a^4
+2*ln(x)*a^2+ln(x)
```

Maxima [A] time = 1.12995, size = 73, normalized size = 1.24

$$\frac{1}{4}\,b^4x^4 + \frac{4}{3}\,ab^3x^3 + \left(3\,a^2+1\right)b^2x^2 + 4\left(a^3+a\right)bx + \left(a^4+2\,a^2+1\right)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b*x+a)^2)^2/x,x, algorithm="maxima")
```

```
[Out] 1/4*b^4*x^4 + 4/3*a*b^3*x^3 + (3*a^2 + 1)*b^2*x^2 + 4*(a^3 + a)*b*x + (a^4
+ 2*a^2 + 1)*log(x)
```

Fricas [A] time = 1.77001, size = 130, normalized size = 2.2

$$\frac{1}{4}\,b^4x^4 + \frac{4}{3}\,ab^3x^3 + \left(3\,a^2 + 1\right)b^2x^2 + 4\left(a^3 + a\right)bx + \left(a^4 + 2\,a^2 + 1\right)\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b*x+a)^2)^2/x,x, algorithm="fricas")
```

```
[Out] 1/4*b^4*x^4 + 4/3*a*b^3*x^3 + (3*a^2 + 1)*b^2*x^2 + 4*(a^3 + a)*b*x + (a^4
+ 2*a^2 + 1)*log(x)
```

Sympy [A] time = 0.312202, size = 58, normalized size = 0.98

$$\frac{4ab^3x^3}{3} + \frac{b^4x^4}{4} + x^2\left(3a^2b^2 + b^2\right) + x\left(4a^3b + 4ab\right) + \left(a^2 + 1\right)^2\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b*x+a)**2)**2/x,x)
```

```
[Out] 4*a*b**3*x**3/3 + b**4*x**4/4 + x**2*(3*a**2*b**2 + b**2) + x*(4*a**3*b + 4
*a*b) + (a**2 + 1)**2*log(x)
```

Giac [A] time = 1.12301, size = 84, normalized size = 1.42

$$\frac{1}{4}\,b^4x^4 + \frac{4}{3}\,ab^3x^3 + 3\,a^2b^2x^2 + 4\,a^3bx + b^2x^2 + 4\,abx + \left(a^4 + 2\,a^2 + 1\right)\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b*x+a)^2)^2/x,x, algorithm="giac")
```

```
[Out] 1/4*b^4*x^4 + 4/3*a*b^3*x^3 + 3*a^2*b^2*x^2 + 4*a^3*b*x + b^2*x^2 + 4*a*b*x
 + (a^4 + 2*a^2 + 1)*log(abs(x))
```

3.99 $\int \frac{x^2}{1+(-1+x)^2} \, dx$

Optimal. Leaf size=10

$$x + \log\left((x-1)^2 + 1\right)$$

```
[Out] x + Log[1 + (-1 + x)^2]
```

Rubi [A] time = 0.0112678, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {371, 702, 260}

$$x + \log\left((x-1)^2 + 1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(1 + (-1 + x)^2),x]
```

```
[Out] x + Log[1 + (-1 + x)^2]
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 702

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2}{1+(-1+x)^2}\,dx &= \text{Subst}\left(\int \frac{(1+x)^2}{1+x^2}\,dx, x, -1+x\right)\\
&= \text{Subst}\left(\int \left(1+\frac{2x}{1+x^2}\right)dx, x, -1+x\right)\\
&= x + 2\,\text{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, -1+x\right)\\
&= x + \log\left(1+(-1+x)^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.0062418, size = 11, normalized size = 1.1

$$\log\left(x^2-2x+2\right)+x$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(1 + (-1 + x)^2),x]
```

```
[Out] x + Log[2 - 2*x + x^2]
```

Maple [A] time = 0.002, size = 12, normalized size = 1.2

$$x+\ln\left(x^2-2\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(1+(x-1)^2),x)
```

```
[Out] x+ln(x^2-2*x+2)
```

Maxima [A] time = 1.1136, size = 15, normalized size = 1.5

$$x+\log\left(x^2-2\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+(-1+x)^2),x, algorithm="maxima")
```

```
[Out] x + log(x^2 - 2*x + 2)
```

Fricas [A] time = 1.64335, size = 32, normalized size = 3.2

$$x + \log\left(x^2 - 2\,x + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+(-1+x)^2),x, algorithm="fricas")
```

```
[Out] x + log(x^2 - 2*x + 2)
```

Sympy [A] time = 0.081725, size = 10, normalized size = 1.

$$x + \log\left(x^2 - 2x + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(1+(-1+x)**2),x)
```

```
[Out] x + log(x**2 - 2*x + 2)
```

Giac [A] time = 1.13048, size = 15, normalized size = 1.5

$$x + \log\left(x^2 - 2\,x + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+(-1+x)^2),x, algorithm="giac")
```

```
[Out] x + log(x^2 - 2*x + 2)
```

3.100 $\int \frac{x^2}{\sqrt{1-(1+x)^2}} \, dx$

Optimal. Leaf size=44

$$-\frac{1}{2}\sqrt{1-(x+1)^2}x+\frac{3}{2}\sqrt{1-(x+1)^2}+\frac{3}{2}\sin^{-1}(x+1)$$

```
[Out] (3*Sqrt[1 - (1 + x)^2])/2 - (x*Sqrt[1 - (1 + x)^2])/2 + (3*ArcSin[1 + x])/2
```

Rubi [A] time = 0.0258208, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {371, 671, 641, 216}

$$-\frac{1}{2}\sqrt{1-(x+1)^2}x+\frac{3}{2}\sqrt{1-(x+1)^2}+\frac{3}{2}\sin^{-1}(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[1 - (1 + x)^2],x]
```

```
[Out] (3*Sqrt[1 - (1 + x)^2])/2 - (x*Sqrt[1 - (1 + x)^2])/2 + (3*ArcSin[1 + x])/2
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
```

```
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt{1-(1+x)^2}}\,dx &= \text{Subst}\left(\int \frac{(-1+x)^2}{\sqrt{1-x^2}}\,dx, x, 1+x\right)\\ &= -\frac{1}{2}x\sqrt{1-(1+x)^2}-\frac{3}{2}\,\text{Subst}\left(\int \frac{-1+x}{\sqrt{1-x^2}}\,dx, x, 1+x\right)\\ &= \frac{3}{2}\sqrt{1-(1+x)^2}-\frac{1}{2}x\sqrt{1-(1+x)^2}+\frac{3}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, 1+x\right)\\ &= \frac{3}{2}\sqrt{1-(1+x)^2}-\frac{1}{2}x\sqrt{1-(1+x)^2}+\frac{3}{2}\sin^{-1}(1+x)\end{aligned}$$

Mathematica [A] time = 0.0214099, size = 51, normalized size = 1.16

$$\frac{x\left(x^2-x-6\right)+6\sqrt{x}\sqrt{x+2}\sinh^{-1}\left(\frac{\sqrt{x}}{\sqrt{2}}\right)}{2\sqrt{-x(x+2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[1 - (1 + x)^2],x]
```

```
[Out] (x*(-6 - x + x^2) + 6*Sqrt[x]*Sqrt[2 + x]*ArcSinh[Sqrt[x]/Sqrt[2]])/(2*Sqrt
[-(x*(2 + x))])
```

Maple [A] time = 0.002, size = 35, normalized size = 0.8

$$-\frac{x}{2}\sqrt{-x^2-2\,x}+\frac{3}{2}\sqrt{-x^2-2\,x}+\frac{3\ \arcsin\left(1+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(1-(1+x)^2)^(1/2),x)
```

```
[Out] -1/2*x*(-x^2-2*x)^(1/2)+3/2*(-x^2-2*x)^(1/2)+3/2*arcsin(1+x)
```

Maxima [A] time = 1.65103, size = 49, normalized size = 1.11

$$-\frac{1}{2}\sqrt{-x^2-2\,x}x+\frac{3}{2}\sqrt{-x^2-2\,x}-\frac{3}{2}\arcsin\left(-x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1-(1+x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(-x^2 - 2*x)*x + 3/2*sqrt(-x^2 - 2*x) - 3/2*arcsin(-x - 1)
```

Fricas [A] time = 1.76868, size = 84, normalized size = 1.91

$$-\frac{1}{2}\sqrt{-x^2-2\,x}(x-3)-3\arctan\left(\frac{\sqrt{-x^2-2\,x}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1-(1+x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(-x^2 - 2*x)*(x - 3) - 3*arctan(sqrt(-x^2 - 2*x)/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^2}{\sqrt{-x\,(x+2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(1-(1+x)**2)**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(-x*(x + 2)), x)
```

<u>Giac</u> [A] time = 1.11091, size = 31, normalized size = 0.7

$$-\frac{1}{2}\sqrt{-(x+1)^2+1}(x-3)+\frac{3}{2}\arcsin(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1-(1+x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(-(x + 1)^2 + 1)*(x - 3) + 3/2*arcsin(x + 1)
```

3.101 $\int \frac{x^2}{\sqrt{1-(a+bx)^2}} dx$

Optimal. Leaf size=67

$$\frac{(2a^2+1)\sin^{-1}(a+bx)}{2b^3}+\frac{3a\sqrt{1-(a+bx)^2}}{2b^3}-\frac{x\sqrt{1-(a+bx)^2}}{2b^2}$$

```
[Out] (3*a*Sqrt[1 - (a + b*x)^2])/(2*b^3) - (x*Sqrt[1 - (a + b*x)^2])/(2*b^2) + (
(1 + 2*a^2)*ArcSin[a + b*x])/(2*b^3)
```

Rubi [A] time = 0.0485348, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {371, 743, 641, 216}

$$\frac{(2a^2+1)\sin^{-1}(a+bx)}{2b^3}+\frac{3a\sqrt{1-(a+bx)^2}}{2b^3}-\frac{x\sqrt{1-(a+bx)^2}}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[1 - (a + b*x)^2],x]
```

```
[Out] (3*a*Sqrt[1 - (a + b*x)^2])/(2*b^3) - (x*Sqrt[1 - (a + b*x)^2])/(2*b^2) + (
(1 + 2*a^2)*ArcSin[a + b*x])/(2*b^3)
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 743

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt{1-(a+bx)^2}}\,dx &= \frac{\text{Subst}\left(\int \frac{(-a+x)^2}{\sqrt{1-x^2}}\,dx, x, a+bx\right)}{b^3}\\ &= -\frac{x\sqrt{1-(a+bx)^2}}{2b^2} - \frac{\text{Subst}\left(\int \frac{-1-2a^2+3ax}{\sqrt{1-x^2}}\,dx, x, a+bx\right)}{2b^3}\\ &= \frac{3a\sqrt{1-(a+bx)^2}}{2b^3} - \frac{x\sqrt{1-(a+bx)^2}}{2b^2} + \frac{\left(1+2a^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, a+bx\right)}{2b^3}\\ &= \frac{3a\sqrt{1-(a+bx)^2}}{2b^3} - \frac{x\sqrt{1-(a+bx)^2}}{2b^2} + \frac{\left(1+2a^2\right)\sin^{-1}(a+bx)}{2b^3}\end{aligned}$$

Mathematica [A] time = 0.069198, size = 55, normalized size = 0.82

$$\frac{\sqrt{-a^2-2abx-b^2x^2+1}(3a-bx)+\left(2a^2+1\right)\sin^{-1}(a+bx)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[1 - (a + b*x)^2],x]
```

```
[Out] ((3*a - b*x)*Sqrt[1 - a^2 - 2*a*b*x - b^2*x^2] + (1 + 2*a^2)*ArcSin[a + b*x
])/(2*b^3)
```

Maple [B] time = 0.016, size = 152, normalized size = 2.3

$$-\frac{x}{2\,b^2}\sqrt{-b^2x^2-2\,abx-a^2+1}+\frac{3\,a}{2\,b^3}\sqrt{-b^2x^2-2\,abx-a^2+1}+\frac{a^2}{b^2}\arctan\left(\sqrt{b^2}\left(\frac{a}{b}+x\right)\frac{1}{\sqrt{-b^2x^2-2\,abx-a^2+1}}\right)\frac{1}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(1-(b*x+a)^2)^(1/2),x)
```

```
[Out] -1/2*x/b^2*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+3/2*a/b^3*(-b^2*x^2-2*a*b*x-a^2+1
)^(1/2)+a^2/b^2/(b^2)^(1/2)*arctan((b^2)^(1/2)*(a/b+x)/(-b^2*x^2-2*a*b*x-a^
2+1)^(1/2))+1/2/b^2/(b^2)^(1/2)*arctan((b^2)^(1/2)*(a/b+x)/(-b^2*x^2-2*a*b*
x-a^2+1)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1-(b*x+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.87414, size = 211, normalized size = 3.15

$$-\frac{\left(2\,a^2+1\right)\arctan\left(\frac{\sqrt{-b^2x^2-2\,abx-a^2+1}(bx+a)}{b^2x^2+2\,abx+a^2-1}\right)+\sqrt{-b^2x^2-2\,abx-a^2+1}(bx-3\,a)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1-(b*x+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*((2*a^2 + 1)*arctan(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(b*x + a)/(b^2*
x^2 + 2*a*b*x + a^2 - 1)) + sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(b*x - 3*a))
/b^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{-(a+bx-1)(a+bx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(1-(b*x+a)**2)**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(-(a + b*x - 1)*(a + b*x + 1)), x)
```

Giac [A] time = 1.16016, size = 74, normalized size = 1.1

$$-\frac{1}{2}\sqrt{-(bx+a)^2+1}\left(\frac{x}{b^2}-\frac{3\,a}{b^3}\right)-\frac{\left(2\,a^2+1\right)\arcsin\left(-bx-a\right)\operatorname{sgn}\left(b\right)}{2\,b^2|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1-(b*x+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(-(b*x + a)^2 + 1)*(x/b^2 - 3*a/b^3) - 1/2*(2*a^2 + 1)*arcsin(-b*x
 - a)*sgn(b)/(b^2*abs(b))
```

3.102 $\int \frac{x^2}{\sqrt{1+(a+bx)^2}}\,dx$

Optimal. Leaf size=63

$$-\frac{(1-2a^2)\sinh^{-1}(a+bx)}{2b^3}-\frac{3a\sqrt{(a+bx)^2+1}}{2b^3}+\frac{x\sqrt{(a+bx)^2+1}}{2b^2}$$

```
[Out] (-3*a*Sqrt[1 + (a + b*x)^2])/(2*b^3) + (x*Sqrt[1 + (a + b*x)^2])/(2*b^2) -
((1 - 2*a^2)*ArcSinh[a + b*x])/(2*b^3)
```

Rubi [A] time = 0.0404596, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {371, 743, 641, 215}

$$-\frac{(1-2a^2)\sinh^{-1}(a+bx)}{2b^3}-\frac{3a\sqrt{(a+bx)^2+1}}{2b^3}+\frac{x\sqrt{(a+bx)^2+1}}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[1 + (a + b*x)^2],x]
```

```
[Out] (-3*a*Sqrt[1 + (a + b*x)^2])/(2*b^3) + (x*Sqrt[1 + (a + b*x)^2])/(2*b^2) -
((1 - 2*a^2)*ArcSinh[a + b*x])/(2*b^3)
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 743

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt{1+(a+bx)^2}}\,dx &= \frac{\text{Subst}\left(\int \frac{(-a+x)^2}{\sqrt{1+x^2}}\,dx,x,a+bx\right)}{b^3}\\ &= \frac{x\sqrt{1+(a+bx)^2}}{2b^2}+\frac{\text{Subst}\left(\int \frac{-1+2a^2-3ax}{\sqrt{1+x^2}}\,dx,x,a+bx\right)}{2b^3}\\ &= -\frac{3a\sqrt{1+(a+bx)^2}}{2b^3}+\frac{x\sqrt{1+(a+bx)^2}}{2b^2}-\frac{\left(1-2a^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx,x,a+bx\right)}{2b^3}\\ &= -\frac{3a\sqrt{1+(a+bx)^2}}{2b^3}+\frac{x\sqrt{1+(a+bx)^2}}{2b^2}-\frac{\left(1-2a^2\right)\sinh^{-1}(a+bx)}{2b^3}\end{aligned}$$

Mathematica [A] time = 0.0565015, size = 51, normalized size = 0.81

$$\frac{\sqrt{a^2+2abx+b^2x^2+1}(bx-3a)+\left(2a^2-1\right)\sinh^{-1}(a+bx)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[1 + (a + b*x)^2],x]
```

```
[Out] ((-3*a + b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2] + (-1 + 2*a^2)*ArcSinh[a +
b*x])/(2*b^3)
```

Maple [B] time = 0.01, size = 146, normalized size = 2.3

$$\frac{x}{2\,b^2}\sqrt{b^2x^2+2\,abx+a^2+1}-\frac{3\,a}{2\,b^3}\sqrt{b^2x^2+2\,abx+a^2+1}+\frac{a^2}{b^2}\ln\left((b^2x+ab)\frac{1}{\sqrt{b^2}}+\sqrt{b^2x^2+2\,abx+a^2+1}\right)\frac{1}{\sqrt{b^2}}-\frac{1}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(1+(b*x+a)^2)^(1/2),x)
```

```
[Out] 1/2*x/b^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-3/2*a/b^3*(b^2*x^2+2*a*b*x+a^2+1)^(
1/2)+a^2/b^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2
)^(1/2)-1/2/b^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(
b^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+(b*x+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.74386, size = 165, normalized size = 2.62

$$-\frac{\left(2\,a^2-1\right)\log\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)-\sqrt{b^2x^2+2\,abx+a^2+1}(bx-3\,a)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+(b*x+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*((2*a^2 - 1)*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - sqrt(
b^2*x^2 + 2*a*b*x + a^2 + 1)*(b*x - 3*a))/b^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{a^2+2abx+b^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(1+(b*x+a)**2)**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1), x)
```

Giac [A] time = 1.1905, size = 95, normalized size = 1.51

$$\frac{1}{2}\sqrt{(bx+a)^2+1}\left(\frac{x}{b^2}-\frac{3a}{b^3}\right)-\frac{\left(2a^2-1\right)\log\left(-ab-\left(x|b|-\sqrt{(bx+a)^2+1}\right)|b|\right)}{2b^2|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+(b*x+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt((b*x + a)^2 + 1)*(x/b^2 - 3*a/b^3) - 1/2*(2*a^2 - 1)*log(-a*b - (x
*abs(b) - sqrt((b*x + a)^2 + 1))*abs(b))/(b^2*abs(b))
```

3.103 $\int \frac{x^3}{a+b(c+dx)^3}\,dx$

Optimal. Leaf size=234

$$-\frac{\left(3\sqrt[3]{a}b^{2/3}c^2+a+bc^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}b^{4/3}d^4}+\frac{\left(3\sqrt[3]{a}b^{2/3}c^2+a+bc^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}b^{4/3}d^4}+\frac{(-}{}$$

```
[Out] x/(b*d^3) + ((a - 3*a^(1/3)*b^(2/3)*c^2 + b*c^3)*ArcTan[(a^(1/3) - 2*b^(1/3
)*(c + d*x))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*b^(4/3)*d^4) - ((a + 3*a^
(1/3)*b^(2/3)*c^2 + b*c^3)*Log[a^(1/3) + b^(1/3)*(c + d*x)])/(3*a^(2/3)*b^(
4/3)*d^4) + ((a + 3*a^(1/3)*b^(2/3)*c^2 + b*c^3)*Log[a^(2/3) - a^(1/3)*b^(1
/3)*(c + d*x) + b^(2/3)*(c + d*x)^2])/(6*a^(2/3)*b^(4/3)*d^4) - (c*Log[a +
b*(c + d*x)^3])/(b*d^4)
```

Rubi [A] time = 0.371634, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.588, Rules used = {371, 1887, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{\left(3\sqrt[3]{a}b^{2/3}c^2+a+bc^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}b^{4/3}d^4}+\frac{\left(3\sqrt[3]{a}b^{2/3}c^2+a+bc^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}b^{4/3}d^4}+\frac{(-}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*(c + d*x)^3),x]
```

```
[Out] x/(b*d^3) + ((a - 3*a^(1/3)*b^(2/3)*c^2 + b*c^3)*ArcTan[(a^(1/3) - 2*b^(1/3
)*(c + d*x))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*b^(4/3)*d^4) - ((a + 3*a^
(1/3)*b^(2/3)*c^2 + b*c^3)*Log[a^(1/3) + b^(1/3)*(c + d*x)])/(3*a^(2/3)*b^(
4/3)*d^4) + ((a + 3*a^(1/3)*b^(2/3)*c^2 + b*c^3)*Log[a^(2/3) - a^(1/3)*b^(1
/3)*(c + d*x) + b^(2/3)*(c + d*x)^2])/(6*a^(2/3)*b^(4/3)*d^4) - (c*Log[a +
b*(c + d*x)^3])/(b*d^4)
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{x^3}{a+b(c+dx)^3}\,dx = \frac{\text{Subst}\left(\int \frac{(-c+x)^3}{a+bx^3}\,dx, x, c+dx\right)}{d^4}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{b} - \frac{a+bc^3-3bc^2x+3bcx^2}{b\left(a+bx^3\right)}\right)dx, x, c+dx\right)}{d^4}$$

$$= \frac{x}{bd^3} - \frac{\text{Subst}\left(\int \frac{a+bc^3-3bc^2x+3bcx^2}{a+bx^3}\,dx, x, c+dx\right)}{bd^4}$$

$$= \frac{x}{bd^3} - \frac{\text{Subst}\left(\int \frac{a+bc^3-3bc^2x}{a+bx^3}\,dx, x, c+dx\right)}{bd^4} - \frac{(3c)\,\text{Subst}\left(\int \frac{x^2}{a+bx^3}\,dx, x, c+dx\right)}{d^4}$$

$$= \frac{x}{bd^3} - \frac{c\log\left(a+b(c+dx)^3\right)}{bd^4} - \frac{\text{Subst}\left(\int \frac{\sqrt[3]{a}\left(-3\sqrt[3]{a}bc^2+2\sqrt[3]{b}\left(a+bc^3\right)\right)+\sqrt[3]{b}\left(-3\sqrt[3]{a}bc^2-\sqrt[3]{b}\left(a+bc^3\right)\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx, x, c+\right.}{3a^{2/3}b^{4/3}d^4}$$

$$= \frac{x}{bd^3} - \frac{\left(a+3\sqrt[3]{a}b^{2/3}c^2+bc^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}b^{4/3}d^4} - \frac{c\log\left(a+b(c+dx)^3\right)}{bd^4} - \frac{\left(a-3\sqrt[3]{a}b^{2/3}c^2+\right.}{}$$

$$= \frac{x}{bd^3} - \frac{\left(a+3\sqrt[3]{a}b^{2/3}c^2+bc^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}b^{4/3}d^4} + \frac{\left(a+3\sqrt[3]{a}b^{2/3}c^2+bc^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(\right.}{6a^{2/3}b^{4/3}d^4}$$

$$= \frac{x}{bd^3} + \frac{\left(a-3\sqrt[3]{a}b^{2/3}c^2+bc^3\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}a^{2/3}b^{4/3}d^4} - \frac{\left(a+3\sqrt[3]{a}b^{2/3}c^2+bc^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}b^{4/3}d^4}$$

Mathematica [C] time = 0.0474439, size = 132, normalized size = 0.56

$$-\frac{\text{RootSum}\left[3\#1^2bcd^2+\#1^3bd^3+3\#1bc^2d+a+bc^3\&,\frac{3\#1^2bcd^2\log(x-\#1)+a\log(x-\#1)+3\#1bc^2d\log(x-\#1)+bc^3\log(x-\#1)}{\#1^2d^2+2\#1cd+c^2}\&\right]-3bdx}{3b^2d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*(c + d*x)^3),x]
```

```
[Out] -(-3*b*d*x + RootSum[a + b*c^3 + 3*b*c^2*d*#1 + 3*b*c*d^2*#1^2 + b*d^3*#1^3
 & , (a*Log[x - #1] + b*c^3*Log[x - #1] + 3*b*c^2*d*Log[x - #1]*#1 + 3*b*c*
d^2*Log[x - #1]*#1^2)/(c^2 + 2*c*d*#1 + d^2*#1^2) & ])/(3*b^2*d^4)
```

Maple [C] time = 0.006, size = 108, normalized size = 0.5

$$\frac{x}{bd^3}+\frac{1}{3\,b^2d^4}\sum_{\_R=\text{RootOf}\left(bd^3\_Z^3+3\,bcd^2\_Z^2+3\,bc^2d\_Z+bc^3+a\right)}\frac{\left(-3\_R^2bcd^2-3\_R\,bc^2d-bc^3-a\right)\ln(x-\_R)}{d^2\_R^2+2\,cd\_R+c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+b*(d*x+c)^3),x)
```

```
[Out] x/b/d^3+1/3/b^2/d^4*sum((-3*_R^2*b*c*d^2-3*_R*b*c^2*d-b*c^3-a)/(_R^2*d^2+2*
_R*c*d+c^2)*ln(x-_R),_R=RootOf(_Z^3*b*d^3+3*_Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c^3
+a))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] Exception raised: AttributeError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 2.52364, size = 238, normalized size = 1.02

$$\operatorname{RootSum}\left(27t^3a^2b^4d^{12} + 81t^2a^2b^3cd^8 + t\left(54a^2b^2c^2d^4 - 27ab^3c^5d^4\right) + a^3 + 3a^2bc^3 + 3ab^2c^6 + b^3c^9, \left(t \mapsto t\log\left(x + \frac{-2\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b*(d*x+c)**3),x)
```

```
[Out] RootSum(27*_t**3*a**2*b**4*d**12 + 81*_t**2*a**2*b**3*c*d**8 + _t*(54*a**2*
b**2*c**2*d**4 - 27*a*b**3*c**5*d**4) + a**3 + 3*a**2*b*c**3 + 3*a*b**2*c**
6 + b**3*c**9, Lambda(_t, _t*log(x + (-27*_t**2*a**2*b**3*c**2*d**8 - 3*_t*
a**3*b*d**4 - 60*_t*a**2*b**2*c**3*d**4 - 3*_t*a*b**3*c**6*d**4 - 2*a**3*c
- 12*a**2*b*c**4 - 9*a*b**2*c**7 + b**3*c**10)/(a**3*d + 3*a**2*b*c**3*d -
24*a*b**2*c**6*d + b**3*c**9*d)))) + x/(b*d**3)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{(dx+c)^3b+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(x^3/((d*x + c)^3*b + a), x)
```

3.104 $\int \frac{x^2}{a+b(c+dx)^3}\,dx$

Optimal. Leaf size=210

$$\frac{c\left(2\sqrt[3]{a}+\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}b^{2/3}d^3}-\frac{c\left(2\sqrt[3]{a}+\sqrt[3]{b}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}b^{2/3}d^3}+\frac{c\left(2\sqrt[3]{a}-\sqrt[3]{b}c\right)\tan^{-}}{\sqrt{3}a^{2/3}b}$$

```
[Out] (c*(2*a^(1/3) - b^(1/3)*c)*ArcTan[(a^(1/3) - 2*b^(1/3)*(c + d*x))/(Sqrt[3]*
a^(1/3))])/(Sqrt[3]*a^(2/3)*b^(2/3)*d^3) + (c*(2*a^(1/3) + b^(1/3)*c)*Log[a
^(1/3) + b^(1/3)*(c + d*x)])/(3*a^(2/3)*b^(2/3)*d^3) - (c*(2*a^(1/3) + b^(1
/3)*c)*Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2])/(6*a
^(2/3)*b^(2/3)*d^3) + Log[a + b*(c + d*x)^3]/(3*b*d^3)
```

Rubi [A] time = 0.228165, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {371, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{c\left(2\sqrt[3]{a}+\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}b^{2/3}d^3}-\frac{c\left(2\sqrt[3]{a}+\sqrt[3]{b}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}b^{2/3}d^3}+\frac{c\left(2\sqrt[3]{a}-\sqrt[3]{b}c\right)\tan^{-}}{\sqrt{3}a^{2/3}b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*(c + d*x)^3),x]
```

```
[Out] (c*(2*a^(1/3) - b^(1/3)*c)*ArcTan[(a^(1/3) - 2*b^(1/3)*(c + d*x))/(Sqrt[3]*
a^(1/3))])/(Sqrt[3]*a^(2/3)*b^(2/3)*d^3) + (c*(2*a^(1/3) + b^(1/3)*c)*Log[a
^(1/3) + b^(1/3)*(c + d*x)])/(3*a^(2/3)*b^(2/3)*d^3) - (c*(2*a^(1/3) + b^(1
/3)*c)*Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2])/(6*a
^(2/3)*b^(2/3)*d^3) + Log[a + b*(c + d*x)^3]/(3*b*d^3)
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{x^2}{a+b(c+dx)^3}\,dx = \frac{\text{Subst}\left(\int \frac{(-c+x)^2}{a+bx^3}\,dx, x, c+dx\right)}{d^3}$$

$$= \frac{\text{Subst}\left(\int \frac{x^2}{a+bx^3}\,dx, x, c+dx\right)}{d^3} + \frac{\text{Subst}\left(\int \frac{c^2-2cx}{a+bx^3}\,dx, x, c+dx\right)}{d^3}$$

$$= \frac{\log\left(a+b(c+dx)^3\right)}{3bd^3} + \frac{\text{Subst}\left(\int \frac{\sqrt[3]{a}\left(-2\sqrt[3]{a}c+2\sqrt[3]{b}c^2\right)+\sqrt[3]{b}\left(-2\sqrt[3]{a}c-\sqrt[3]{b}c^2\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx, x, c+dx\right)}{3a^{2/3}\sqrt[3]{b}d^3} + -\frac{\left(-2\sqrt[3]{a}c-\sqrt[3]{b}\right.}{}$$

$$= \frac{c\left(2\sqrt[3]{a}+\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}b^{2/3}d^3} + \frac{\log\left(a+b(c+dx)^3\right)}{3bd^3} - \frac{\left(c\left(\frac{2}{\sqrt[3]{b}}-\frac{c}{\sqrt[3]{a}}\right)\right)\text{Subst}\left(\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{}}\right.}{2d^3}$$

$$= \frac{c\left(2\sqrt[3]{a}+\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}b^{2/3}d^3} - \frac{c\left(2\sqrt[3]{a}+\sqrt[3]{b}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}b^{2/3}d^3}$$

$$= \frac{c\left(2\sqrt[3]{a}-\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}a^{2/3}b^{2/3}d^3} + \frac{c\left(2\sqrt[3]{a}+\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}b^{2/3}d^3} - \frac{c\left(2\sqrt[3]{a}+\sqrt[3]{b}c\right)\log\left(\right.}{}$$

Mathematica [C] time = 0.0293863, size = 81, normalized size = 0.39

$$\frac{\text{RootSum}\left[3\#1^2bcd^2+\#1^3bd^3+3\#1bc^2d+a+bc^3\&, \frac{\#1^2\log(x-\#1)}{\#1^2d^2+2\#1cd+c^2}\&\right]}{3bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*(c + d*x)^3),x]
```

```
[Out] RootSum[a + b*c^3 + 3*b*c^2*d*#1 + 3*b*c*d^2*#1^2 + b*d^3*#1^3 & , (Log[x -
 #1]*#1^2)/(c^2 + 2*c*d*#1 + d^2*#1^2) & ]/(3*b*d)
```

Maple [C] time = 0.002, size = 74, normalized size = 0.4

$$\frac{1}{3\,bd}\sum_{\_R=\mathrm{RootOf}\left(\_Z^3bd^3+3\_Z^2bcd^2+3\_Z\,bc^2d+bc^3+a\right)}\frac{\_R^2\ln\left(x-\_R\right)}{d^2\_R^2+2\,cd\_R+c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*(d*x+c)^3),x)
```

```
[Out] 1/3/b/d*sum(_R^2/(_R^2*d^2+2*_R*c*d+c^2)*ln(x-_R),_R=RootOf(_Z^3*b*d^3+3*_Z
^2*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^2}{(dx+c)^3b+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((d*x + c)^3*b + a), x)
```

Fricas [C] time = 8.12535, size = 10654, normalized size = 50.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] -1/12*(2*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((2*b*c^3 - a)/(a*b^2*d^6) + 1/(b^
2*d^6))/((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/
(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) + (1/2)^(1/3)*
(I*sqrt(3) + 1)*((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d
^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) - 2/(b
*d^3))*b*d^3*log(-1/2*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((2*b*c^3 - a)/(a*b^2
*d^6) + 1/(b^2*d^6))/((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*
```

```
b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) +
 (1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3
- a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))
^(1/3) - 2/(b*d^3))^2*a^2*b^2*d^6 + b^2*c^6 - a*b*c^3 - 1/2*(a*b^2*c^3 + 4*
a^2*b)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((2*b*c^3 - a)/(a*b^2*d^6) + 1/(b^2*
d^6))/((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b
^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) + (1/2)^(1/3)*(I
*sqrt(3) + 1)*((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9
) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) - 2/(b*d
^3))*d^3 + (b^2*c^5 - 8*a*b*c^2)*d*x - 2*a^2) - ((2*(1/2)^(2/3)*(-I*sqrt(3)
 + 1)*((2*b*c^3 - a)/(a*b^2*d^6) + 1/(b^2*d^6))/((b*c^3 - 8*a)*c^3/(a^2*b^2
*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 +
a^2)/(a^2*b^3*d^9))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 - 8*a)*c^3/
(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*
b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) - 2/(b*d^3))*b*d^3 - 3*sqrt(1/3)*b*d^3*sq
rt(-((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((2*b*c^3 - a)/(a*b^2*d^6) + 1/(b^2*d^
6))/((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3
*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) + (1/2)^(1/3)*(I*s
qrt(3) + 1)*((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9)
+ 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) - 2/(b*d^3
))^2*a*b^2*d^6 + 4*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((2*b*c^3 - a)/(a*b^2*d^
6) + 1/(b^2*d^6))/((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3
*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) + (1
/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a
)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1
/3) - 2/(b*d^3))*a*b*d^3 - 32*b*c^3 + 4*a)/(a*b^2*d^6)) + 6)*log(1/2*(2*(1/
2)^(2/3)*(-I*sqrt(3) + 1)*((2*b*c^3 - a)/(a*b^2*d^6) + 1/(b^2*d^6))/((b*c^3
 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^
2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)
*((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^
9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) - 2/(b*d^3))^2*a^2*b^
2*d^6 + 2*b^2*c^6 - 23*a*b*c^3 + 1/2*(a*b^2*c^3 + 4*a^2*b)*(2*(1/2)^(2/3)*(
-I*sqrt(3) + 1)*((2*b*c^3 - a)/(a*b^2*d^6) + 1/(b^2*d^6))/((b*c^3 - 8*a)*c^
3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*
a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 -
 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*
c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) - 2/(b*d^3))*d^3 + 2*(b^2*c^5 -
 8*a*b*c^2)*d*x + 2*a^2 + 3/2*sqrt(1/3)*((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((
2*b*c^3 - a)/(a*b^2*d^6) + 1/(b^2*d^6))/((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) +
3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^
2*b^3*d^9))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 - 8*a)*c^3/(a^2*b^2
*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 +
a^2)/(a^2*b^3*d^9))^(1/3) - 2/(b*d^3))*a^2*b^2*d^6 - (a*b^2*c^3 - 2*a^2*b)*
d^3)*sqrt(-((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((2*b*c^3 - a)/(a*b^2*d^6) + 1/
(b^2*d^6))/((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) +
```

```
2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) + (1/2)^(1/
3)*(I*sqrt(3) + 1)*((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^
3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) - 2
/(b*d^3))^2*a*b^2*d^6 + 4*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((2*b*c^3 - a)/(a
*b^2*d^6) + 1/(b^2*d^6))/((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)
/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/
3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*
c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d
^9))^(1/3) - 2/(b*d^3))*a*b*d^3 - 32*b*c^3 + 4*a)/(a*b^2*d^6))) - ((2*(1/2)
^(2/3)*(-I*sqrt(3) + 1)*((2*b*c^3 - a)/(a*b^2*d^6) + 1/(b^2*d^6))/((b*c^3 -
 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*
c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(
(b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9)
 + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) - 2/(b*d^3))*b*d^3 + 3*
sqrt(1/3)*b*d^3*sqrt(-((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((2*b*c^3 - a)/(a*b^
2*d^6) + 1/(b^2*d^6))/((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a
*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3)
+ (1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3
 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9)
)^(1/3) - 2/(b*d^3))^2*a*b^2*d^6 + 4*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((2*b*
c^3 - a)/(a*b^2*d^6) + 1/(b^2*d^6))/((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2
*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^
3*d^9))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9
) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)
/(a^2*b^3*d^9))^(1/3) - 2/(b*d^3))*a*b*d^3 - 32*b*c^3 + 4*a)/(a*b^2*d^6)) +
 6)*log(1/2*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((2*b*c^3 - a)/(a*b^2*d^6) + 1/
(b^2*d^6))/((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) +
 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) + (1/2)^(1/
3)*(I*sqrt(3) + 1)*((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^
3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) - 2
/(b*d^3))^2*a^2*b^2*d^6 + 2*b^2*c^6 - 23*a*b*c^3 + 1/2*(a*b^2*c^3 + 4*a^2*b
)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((2*b*c^3 - a)/(a*b^2*d^6) + 1/(b^2*d^6))
/((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^
9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) + (1/2)^(1/3)*(I*sqrt
(3) + 1)*((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2
/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) - 2/(b*d^3))*
d^3 + 2*(b^2*c^5 - 8*a*b*c^2)*d*x + 2*a^2 - 3/2*sqrt(1/3)*((2*(1/2)^(2/3)*(
-I*sqrt(3) + 1)*((2*b*c^3 - a)/(a*b^2*d^6) + 1/(b^2*d^6))/((b*c^3 - 8*a)*c^
3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*
a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 -
 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*
c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) - 2/(b*d^3))*a^2*b^2*d^6 - (a*b
^2*c^3 - 2*a^2*b)*d^3)*sqrt(-((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((2*b*c^3 - a
)/(a*b^2*d^6) + 1/(b^2*d^6))/((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3
- a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))
```

```
^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(
2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b
^3*d^9))^(1/3) - 2/(b*d^3))^2*a*b^2*d^6 + 4*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)
*((2*b*c^3 - a)/(a*b^2*d^6) + 1/(b^2*d^6))/((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9)
 + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/
(a^2*b^3*d^9))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 - 8*a)*c^3/(a^2*
b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3
 + a^2)/(a^2*b^3*d^9))^(1/3) - 2/(b*d^3))*a*b*d^3 - 32*b*c^3 + 4*a)/(a*b^2*
d^6))))/(b*d^3)
```

Sympy [A] time = 0.952662, size = 158, normalized size = 0.75

$$\operatorname{RootSum}\left(27t^3a^2b^3d^9 - 27t^2a^2b^2d^6 + t\left(9a^2bd^3 - 18ab^2c^3d^3\right) - a^2 - 2abc^3 - b^2c^6, \left(t \mapsto t\log\left(x + \frac{18t^2a^2b^2d^6 - 12ta^2bd^3}{8a}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*(d*x+c)**3),x)
```

```
[Out] RootSum(27*_t**3*a**2*b**3*d**9 - 27*_t**2*a**2*b**2*d**6 + _t*(9*a**2*b*d*
*3 - 18*a*b**2*c**3*d**3) - a**2 - 2*a*b*c**3 - b**2*c**6, Lambda(_t, _t*lo
g(x + (18*_t**2*a**2*b**2*d**6 - 12*_t*a**2*b*d**3 - 3*_t*a*b**2*c**3*d**3
+ 2*a**2 + a*b*c**3 - b**2*c**6)/(8*a*b*c**2*d - b**2*c**5*d))))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(dx+c)^3b+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(x^2/((d*x + c)^3*b + a), x)
```

3.105 $\int \frac{x}{a+b(c+dx)^3}\,dx$

Optimal. Leaf size=180

$$-\frac{\left(\sqrt[3]{a}+\sqrt[3]{bc}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}b^{2/3}d^2}+\frac{\left(\sqrt[3]{a}+\sqrt[3]{bc}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}b^{2/3}d^2}-\frac{\left(\sqrt[3]{a}-\sqrt[3]{bc}\right)\tan^{-1}\left(\sqrt[3]{\cdot}\right.}{\sqrt{3}a^{2/3}b^{2/3}d}$$

```
[Out] -(((a^(1/3) - b^(1/3)*c)*ArcTan[(a^(1/3) - 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^
(1/3))])/(Sqrt[3]*a^(2/3)*b^(2/3)*d^2)) - ((a^(1/3) + b^(1/3)*c)*Log[a^(1/3
) + b^(1/3)*(c + d*x)])/(3*a^(2/3)*b^(2/3)*d^2) + ((a^(1/3) + b^(1/3)*c)*Lo
g[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2])/(6*a^(2/3)*b^
(2/3)*d^2)
```

Rubi [A] time = 0.156257, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {371, 1860, 31, 634, 617, 204, 628}

$$-\frac{\left(\sqrt[3]{a}+\sqrt[3]{bc}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}b^{2/3}d^2}+\frac{\left(\sqrt[3]{a}+\sqrt[3]{bc}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}b^{2/3}d^2}-\frac{\left(\sqrt[3]{a}-\sqrt[3]{bc}\right)\tan^{-1}\left(\sqrt[3]{\cdot}\right.}{\sqrt{3}a^{2/3}b^{2/3}d}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*(c + d*x)^3),x]
```

```
[Out] -(((a^(1/3) - b^(1/3)*c)*ArcTan[(a^(1/3) - 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^
(1/3))])/(Sqrt[3]*a^(2/3)*b^(2/3)*d^2)) - ((a^(1/3) + b^(1/3)*c)*Log[a^(1/3
) + b^(1/3)*(c + d*x)])/(3*a^(2/3)*b^(2/3)*d^2) + ((a^(1/3) + b^(1/3)*c)*Lo
g[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2])/(6*a^(2/3)*b^
(2/3)*d^2)
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x}{a+b(c+dx)^3}\,dx = \frac{\text{Subst}\left(\int \frac{-c+x}{a+bx^3}\,dx, x, c+dx\right)}{d^2}$$

$$= \frac{\text{Subst}\left(\int \frac{\sqrt[3]{a}\left(\sqrt[3]{a}-2\sqrt[3]{b}c\right)+\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}c\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx, x, c+dx\right)}{3a^{2/3}\sqrt[3]{b}d^2} - \frac{\left(\frac{\sqrt[3]{a}}{\sqrt[3]{b}}+c\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx, x, c+dx\right)}{3a^{2/3}d^2}$$

$$= -\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}b^{2/3}d^2} + \frac{\left(\frac{1}{\sqrt[3]{b}}-\frac{c}{\sqrt[3]{a}}\right)\text{Subst}\left(\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx, x, c+dx\right)}{2d^2} + \ldots$$

$$= -\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}b^{2/3}d^2} + \frac{\left(\sqrt[3]{a}+\sqrt[3]{b}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}b^{2/3}d^2} - \ldots$$

$$= -\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}a^{2/3}b^{2/3}d^2} - \frac{\left(\sqrt[3]{a}+\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}b^{2/3}d^2} + \frac{\left(\sqrt[3]{a}+\sqrt[3]{b}c\right)\log\left(a^{2/3}\ldots\right.}{}$$

Mathematica [C] time = 0.0215535, size = 79, normalized size = 0.44

$$\frac{\text{RootSum}\left[3\#1^2bcd^2+\#1^3bd^3+3\#1bc^2d+a+bc^3\&, \frac{\#1\log(x-\#1)}{\#1^2d^2+2\#1cd+c^2}\&\right]}{3bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*(c + d*x)^3),x]
```

```
[Out] RootSum[a + b*c^3 + 3*b*c^2*d*#1 + 3*b*c*d^2*#1^2 + b*d^3*#1^3 & , (Log[x -
 #1]*#1)/(c^2 + 2*c*d*#1 + d^2*#1^2) & ]/(3*b*d)
```

Maple [C] time = 0.001, size = 72, normalized size = 0.4

$$\frac{1}{3\,bd}\sum_{\_R=\text{RootOf}\left(\_Z^3bd^3+3\_Z^2bcd^2+3\_Z\,bc^2d+bc^3+a\right)}\frac{\_R\ln(x-\_R)}{d^2\_R^2+2\,cd\_R+c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*(d*x+c)^3),x)
```

```
[Out] 1/3/b/d*sum(_R/(_R^2*d^2+2*_R*c*d+c^2)*ln(x-_R),_R=RootOf(_Z^3*b*d^3+3*_Z^2
*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(dx+c)^3 b + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] integrate(x/((d*x + c)^3*b + a), x)
```

Fricas [C] time = 7.86129, size = 4685, normalized size = 26.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] 1/36*(9*(I*sqrt(3) + 1)*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)
/(a^2*b^2*d^6))^(1/3) - 3*sqrt(1/3)*sqrt(-((9*(I*sqrt(3) + 1)*(-1/54*(b*c^3
 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3) + c*(-I*sqrt(3)
 + 1)/(a*b*d^4*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2
*d^6))^(1/3)))^2*a*b*d^4 - 144*c)/(a*b*d^4)) + c*(-I*sqrt(3) + 1)/(a*b*d^4*
(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3)))*
log(1/36*(9*(I*sqrt(3) + 1)*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3
- a)/(a^2*b^2*d^6))^(1/3) + c*(-I*sqrt(3) + 1)/(a*b*d^4*(-1/54*(b*c^3 + a)/
(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3)))^2*a^2*b*d^4 - 1/6*(
9*(I*sqrt(3) + 1)*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*
b^2*d^6))^(1/3) + c*(-I*sqrt(3) + 1)/(a*b*d^4*(-1/54*(b*c^3 + a)/(a^2*b^2*d
^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3)))*a*b*c^2*d^2 + 2*b*c^4 + 2*(b*
c^3 - a)*d*x - 4*a*c + 1/12*sqrt(1/3)*((9*(I*sqrt(3) + 1)*(-1/54*(b*c^3 + a
)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3) + c*(-I*sqrt(3) + 1
)/(a*b*d^4*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6
))^(1/3)))*a^2*b*d^4 + 6*a*b*c^2*d^2)*sqrt(-((9*(I*sqrt(3) + 1)*(-1/54*(b*c
^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3) + c*(-I*sqrt(
3) + 1)/(a*b*d^4*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b
```

```
^2*d^6))^(1/3)))^2*a*b*d^4 - 144*c)/(a*b*d^4))) + 1/36*(9*(I*sqrt(3) + 1)*(
-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3) + 3
*sqrt(1/3)*sqrt(-((9*(I*sqrt(3) + 1)*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/5
4*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3) + c*(-I*sqrt(3) + 1)/(a*b*d^4*(-1/54*(b*
c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3)))^2*a*b*d^4
- 144*c)/(a*b*d^4)) + c*(-I*sqrt(3) + 1)/(a*b*d^4*(-1/54*(b*c^3 + a)/(a^2*b
^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3)))*log(1/36*(9*(I*sqrt(3) +
1)*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3)
 + c*(-I*sqrt(3) + 1)/(a*b*d^4*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c
^3 - a)/(a^2*b^2*d^6))^(1/3)))^2*a^2*b*d^4 - 1/6*(9*(I*sqrt(3) + 1)*(-1/54*
(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3) + c*(-I*s
qrt(3) + 1)/(a*b*d^4*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a
^2*b^2*d^6))^(1/3)))*a*b*c^2*d^2 + 2*b*c^4 + 2*(b*c^3 - a)*d*x - 4*a*c - 1/
12*sqrt(1/3)*((9*(I*sqrt(3) + 1)*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b
*c^3 - a)/(a^2*b^2*d^6))^(1/3) + c*(-I*sqrt(3) + 1)/(a*b*d^4*(-1/54*(b*c^3
+ a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3)))*a^2*b*d^4 + 6*
a*b*c^2*d^2)*sqrt(-((9*(I*sqrt(3) + 1)*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1
/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3) + c*(-I*sqrt(3) + 1)/(a*b*d^4*(-1/54*(
b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3)))^2*a*b*d^
4 - 144*c)/(a*b*d^4))) - 1/18*(9*(I*sqrt(3) + 1)*(-1/54*(b*c^3 + a)/(a^2*b^
2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3) + c*(-I*sqrt(3) + 1)/(a*b*d^
4*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3))
)*log(-1/36*(9*(I*sqrt(3) + 1)*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c
^3 - a)/(a^2*b^2*d^6))^(1/3) + c*(-I*sqrt(3) + 1)/(a*b*d^4*(-1/54*(b*c^3 +
a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3)))^2*a^2*b*d^4 + 1/
6*(9*(I*sqrt(3) + 1)*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a
^2*b^2*d^6))^(1/3) + c*(-I*sqrt(3) + 1)/(a*b*d^4*(-1/54*(b*c^3 + a)/(a^2*b^
2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3)))*a*b*c^2*d^2 + b*c^4 + (b*c
^3 - a)*d*x + a*c)
```

Sympy [A] time = 0.659871, size = 83, normalized size = 0.46

$$\mathrm{RootSum}\left(27t^3a^2b^2d^6 - 9tabcd^2 + a + bc^3, \left(t \mapsto t\log\left(x + \frac{9t^2a^2bd^4 + 3tabc^2d^2 - ac - bc^4}{ad - bc^3d}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(d*x+c)**3),x)
```

```
[Out] RootSum(27*_t**3*a**2*b**2*d**6 - 9*_t*a*b*c*d**2 + a + b*c**3, Lambda(_t,
_t*log(x + (9*_t**2*a**2*b*d**4 + 3*_t*a*b*c**2*d**2 - a*c - b*c**4)/(a*d -
 b*c**3*d))))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(dx+c)^3 b + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(x/((d*x + c)^3*b + a), x)
```

3.106 $\int \frac{1}{a+b(c+dx)^3}\,dx$

Optimal. Leaf size=140

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}\sqrt[3]{b}d}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}d}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\sqrt[3]{b}d}$$

```
[Out] -(ArcTan[(a^(1/3) - 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(2/3
)*b^(1/3)*d)) + Log[a^(1/3) + b^(1/3)*(c + d*x)]/(3*a^(2/3)*b^(1/3)*d) - Lo
g[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2]/(6*a^(2/3)*b^(
1/3)*d)
```

Rubi [A] time = 0.107231, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {247, 200, 31, 634, 617, 204, 628}

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}\sqrt[3]{b}d}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}d}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\sqrt[3]{b}d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*(c + d*x)^3)^(-1),x]
```

```
[Out] -(ArcTan[(a^(1/3) - 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(2/3
)*b^(1/3)*d)) + Log[a^(1/3) + b^(1/3)*(c + d*x)]/(3*a^(2/3)*b^(1/3)*d) - Lo
g[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2]/(6*a^(2/3)*b^(
1/3)*d)
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
```

```
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{a+b(c+dx)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{a+bx^3}\,dx, x, c+dx\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx, x, c+dx\right)}{3a^{2/3}d} + \frac{\text{Subst}\left(\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx, x, c+dx\right)}{3a^{2/3}d}$$
$$= \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}d} + \frac{\text{Subst}\left(\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx, x, c+dx\right)}{2\sqrt[3]{a}d} - \frac{\text{Subst}\left(\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\right.}{6a^{2/3}\sqrt[3]{b}d}$$
$$= \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}d} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}\sqrt[3]{b}d} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\right.}{a^{2/3}\sqrt[3]{b}d}$$
$$= -\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}a^{2/3}\sqrt[3]{b}d} + \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}d} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}\sqrt[3]{b}d}$$

Mathematica [A] time = 0.0301893, size = 116, normalized size = 0.83

$$\frac{-\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)+2\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)+2\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{b}(c+dx)-\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{6a^{2/3}\sqrt[3]{b}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*(c + d*x)^3)^(-1),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(-a^(1/3) + 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))] + 2*L
og[a^(1/3) + b^(1/3)*(c + d*x)] - Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) +
 b^(2/3)*(c + d*x)^2])/(6*a^(2/3)*b^(1/3)*d)
```

Maple [C] time = 0.003, size = 71, normalized size = 0.5

$$\frac{1}{3\,bd}\sum_{\_R=\text{RootOf}\left(\_Z^3bd^3+3\_Z^2bcd^2+3\_Z\,bc^2d+bc^3+a\right)}\frac{\ln(x-\_R)}{d^2\_R^2+2\,cd\_R+c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*(d*x+c)^3),x)
```

```
[Out] 1/3/b/d*sum(1/(_R^2*d^2+2*_R*c*d+c^2)*ln(x-_R),_R=RootOf(_Z^3*b*d^3+3*_Z^2*
b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx+c)^3 b + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x + c)^3*b + a), x)
```

Fricas [A] time = 1.37389, size = 1079, normalized size = 7.71

$$\left[\frac{3\sqrt{\frac{1}{3}}ab\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}\log\left(\frac{2abd^3x^3+6abcd^2x^2+6abc^2dx+2abc^3-a^2+3\sqrt{\frac{1}{3}}\left(2abd^2x^2+4abcdx+2abc^2+\left(a^2b\right)^{\frac{2}{3}}(dx+c)-\left(a^2b\right)^{\frac{1}{3}}a\right)\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}-3\left(a^2b\right)^{\frac{1}{3}}(adx+ac}{bd^3x^3+3bcd^2x^2+3bc^2dx+bc^3+a}\right.}{6a^2bd}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*a*b*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*d^3*x^3 + 6*a*b*c*d
^2*x^2 + 6*a*b*c^2*d*x + 2*a*b*c^3 - a^2 + 3*sqrt(1/3)*(2*a*b*d^2*x^2 + 4*a
*b*c*d*x + 2*a*b*c^2 + (a^2*b)^(2/3)*(d*x + c) - (a^2*b)^(1/3)*a)*sqrt(-(a^
2*b)^(1/3)/b) - 3*(a^2*b)^(1/3)*(a*d*x + a*c))/(b*d^3*x^3 + 3*b*c*d^2*x^2 +
 3*b*c^2*d*x + b*c^3 + a)) - (a^2*b)^(2/3)*log(a*b*d^2*x^2 + 2*a*b*c*d*x +
a*b*c^2 - (a^2*b)^(2/3)*(d*x + c) + (a^2*b)^(1/3)*a) + 2*(a^2*b)^(2/3)*log(
a*b*d*x + a*b*c + (a^2*b)^(2/3)))/(a^2*b*d), 1/6*(6*sqrt(1/3)*a*b*sqrt((a^2
```

```
*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*(d*x + c) - (a^2*b)^(1/3)*a)
*sqrt((a^2*b)^(1/3)/b)/a^2) - (a^2*b)^(2/3)*log(a*b*d^2*x^2 + 2*a*b*c*d*x +
 a*b*c^2 - (a^2*b)^(2/3)*(d*x + c) + (a^2*b)^(1/3)*a) + 2*(a^2*b)^(2/3)*log
(a*b*d*x + a*b*c + (a^2*b)^(2/3)))/(a^2*b*d)]
```

Sympy [A] time = 0.249505, size = 26, normalized size = 0.19

$$\frac{\text{RootSum}\left(27t^3a^2b-1,\left(t\mapsto t\log\left(x+\frac{3ta+c}{d}\right)\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)**3),x)
```

```
[Out] RootSum(27*_t**3*a**2*b - 1, Lambda(_t, _t*log(x + (3*_t*a + c)/d)))/d
```

Giac [A] time = 1.13296, size = 211, normalized size = 1.51

$$\frac{1}{3}\sqrt{3}\left(\frac{1}{a^2bd^3}\right)^{\frac{1}{3}}\arctan\left(-\frac{bdx+bc+\left(ab^2\right)^{\frac{1}{3}}}{\sqrt{3}bdx+\sqrt{3}bc-\sqrt{3}\left(ab^2\right)^{\frac{1}{3}}}\right)-\frac{1}{6}\left(\frac{1}{a^2bd^3}\right)^{\frac{1}{3}}\log\left(\left(\sqrt{3}bdx+\sqrt{3}bc-\sqrt{3}\left(ab^2\right)^{\frac{1}{3}}\right)^2+\left(bdx+bc\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^3),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*(1/(a^2*b*d^3))^(1/3)*arctan(-(b*d*x + b*c + (a*b^2)^(1/3))/(sq
rt(3)*b*d*x + sqrt(3)*b*c - sqrt(3)*(a*b^2)^(1/3))) - 1/6*(1/(a^2*b*d^3))^(
1/3)*log((sqrt(3)*b*d*x + sqrt(3)*b*c - sqrt(3)*(a*b^2)^(1/3))^2 + (b*d*x +
 b*c + (a*b^2)^(1/3))^2) + 1/3*(1/(a^2*b*d^3))^(1/3)*log(abs(b*d*x + b*c +
(a*b^2)^(1/3)))
```

3.107 $\int \frac{1}{x\left(a+b(c+dx)^3\right)}\,dx$

Optimal. Leaf size=224

$$-\frac{\left(2\sqrt[3]{a}-\sqrt[3]{b}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}c+b^{2/3}c^2\right)}+\frac{\sqrt[3]{b}c\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}c+b^{2/3}c^2\right)}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}c\right)}+\cdots$$

```
[Out] (b^(1/3)*c*ArcTan[(a^(1/3) - 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))])/(Sqrt
[3]*a^(2/3)*(a^(2/3) - a^(1/3)*b^(1/3)*c + b^(2/3)*c^2)) + Log[x]/(a + b*c^
3) - Log[a^(1/3) + b^(1/3)*(c + d*x)]/(3*a^(2/3)*(a^(1/3) + b^(1/3)*c)) - (
(2*a^(1/3) - b^(1/3)*c)*Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(
c + d*x)^2])/(6*a^(2/3)*(a^(2/3) - a^(1/3)*b^(1/3)*c + b^(2/3)*c^2))
```

Rubi [A] time = 0.481797, antiderivative size = 238, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 10, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.588, Rules used = {371, 6725, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{\sqrt[3]{b}c\left(\sqrt[3]{a}-\sqrt[3]{b}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}\left(a+bc^3\right)}+\frac{\sqrt[3]{b}c\left(\sqrt[3]{a}-\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\left(a+bc^3\right)}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*(c + d*x)^3)),x]
```

```
[Out] (b^(1/3)*c*(a^(1/3) + b^(1/3)*c)*ArcTan[(a^(1/3) - 2*b^(1/3)*(c + d*x))/(Sq
rt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*(a + b*c^3)) + Log[x]/(a + b*c^3) + (b^(1
/3)*c*(a^(1/3) - b^(1/3)*c)*Log[a^(1/3) + b^(1/3)*(c + d*x)])/(3*a^(2/3)*(a
 + b*c^3)) - (b^(1/3)*c*(a^(1/3) - b^(1/3)*c)*Log[a^(2/3) - a^(1/3)*b^(1/3)
*(c + d*x) + b^(2/3)*(c + d*x)^2])/(6*a^(2/3)*(a + b*c^3)) - Log[a + b*(c +
 d*x)^3]/(3*(a + b*c^3))
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{1}{x\left(a+b(c+dx)^3\right)}\,dx = \text{Subst}\left(\int \frac{1}{(-c+x)\left(a+bx^3\right)}\,dx, x, c+dx\right)$$

$$= \text{Subst}\left(\int\left(-\frac{1}{\left(a+bc^3\right)(c-x)} - \frac{b\left(c^2+cx+x^2\right)}{\left(a+bc^3\right)\left(a+bx^3\right)}\right)dx, x, c+dx\right)$$

$$= \frac{\log(x)}{a+bc^3} - \frac{b\,\text{Subst}\left(\int \frac{c^2+cx+x^2}{a+bx^3}\,dx, x, c+dx\right)}{a+bc^3}$$

$$= \frac{\log(x)}{a+bc^3} - \frac{b\,\text{Subst}\left(\int \frac{x^2}{a+bx^3}\,dx, x, c+dx\right)}{a+bc^3} - \frac{b\,\text{Subst}\left(\int \frac{c^2+cx}{a+bx^3}\,dx, x, c+dx\right)}{a+bc^3}$$

$$= \frac{\log(x)}{a+bc^3} - \frac{\log\left(a+b(c+dx)^3\right)}{3\left(a+bc^3\right)} - \frac{b^{2/3}\,\text{Subst}\left(\int \frac{\sqrt[3]{a}\left(\sqrt[3]{a}c+2\sqrt[3]{b}c^2\right)+\sqrt[3]{b}\left(\sqrt[3]{a}c-\sqrt[3]{b}c^2\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx, x, c+dx\right)}{3a^{2/3}\left(a+bc^3\right)} + \cdots$$

$$= \frac{\log(x)}{a+bc^3} + \frac{\sqrt[3]{b}c\left(\sqrt[3]{a}-\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\left(a+bc^3\right)} - \frac{\log\left(a+b(c+dx)^3\right)}{3\left(a+bc^3\right)} - \frac{\left(\sqrt[3]{b}c\left(\sqrt[3]{a}-\sqrt[3]{b}c\right)\cdots\right.}{\cdots}$$

$$= \frac{\log(x)}{a+bc^3} + \frac{\sqrt[3]{b}c\left(\sqrt[3]{a}-\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\left(a+bc^3\right)} - \frac{\sqrt[3]{b}c\left(\sqrt[3]{a}-\sqrt[3]{b}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+d\cdots\right.}{6a^{2/3}\left(a+bc^3\right)}$$

$$= \frac{\sqrt[3]{b}c\left(\sqrt[3]{a}+\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}a^{2/3}\left(a+bc^3\right)} + \frac{\log(x)}{a+bc^3} + \frac{\sqrt[3]{b}c\left(\sqrt[3]{a}-\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\left(a+bc^3\right)} - \cdots$$

Mathematica [C] time = 0.0484136, size = 119, normalized size = 0.53

$$-\frac{\text{RootSum}\left[3\#1^2bcd^2+\#1^3bd^3+3\#1bc^2d+a+bc^3\&,\frac{\#1^2d^2\log(x-\#1)+3c^2\log(x-\#1)+3\#1cd\log(x-\#1)}{\#1^2d^2+2\#1cd+c^2}\&\right]-3\log(x)}{3\left(a+bc^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*(c + d*x)^3)),x]
```

```
[Out] -(-3*Log[x] + RootSum[a + b*c^3 + 3*b*c^2*d*#1 + 3*b*c*d^2*#1^2 + b*d^3*#1^
3 & , (3*c^2*Log[x - #1] + 3*c*d*Log[x - #1]*#1 + d^2*Log[x - #1]*#1^2)/(c^
2 + 2*c*d*#1 + d^2*#1^2) & ])/(3*(a + b*c^3))
```

Maple [C] time = 0.007, size = 105, normalized size = 0.5

$$-\frac{1}{3\,bc^3+3\,a}\sum_{\_R=\text{RootOf}\left(\_Z^3bd^3+3\_Z^2bcd^2+3\_Z\,bc^2d+bc^3+a\right)}\frac{\left(d^2\_R^2+3\,cd\_R+3\,c^2\right)\ln\left(x-\_R\right)}{d^2\_R^2+2\,cd\_R+c^2}+\frac{\ln\left(x\right)}{bc^3+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b*(d*x+c)^3),x)
```

```
[Out] -1/3*sum((_R^2*d^2+3*_R*c*d+3*c^2)/(_R^2*d^2+2*_R*c*d+c^2)*ln(x-_R),_R=Root
Of(_Z^3*b*d^3+3*_Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))/(b*c^3+a)+ln(x)/(b*c^3+
a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] Exception raised: AttributeError
```

Fricas [C] time = 8.20721, size = 9806, normalized size = 43.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] 1/12*(2*(b*c^3 + a)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(1/(a*b*c^3 + a^2) - 1/
(b*c^3 + a)^2)/(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c
^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - (1/2)^(1/3)*(I*sqrt(3) +
1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b
*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - 2/(b*c^3 + a))*log(b*c^2*d*x + b*c^3
+ 1/4*(a^2*b*c^3 + a^3)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(1/(a*b*c^3 + a^2)
- 1/(b*c^3 + a)^2)/(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a
*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - (1/2)^(1/3)*(I*sqrt(3
) + 1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2
)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - 2/(b*c^3 + a))^2 - 1/2*(a*b*c^3 -
 2*a^2)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(1/(a*b*c^3 + a^2) - 1/(b*c^3 + a)^
2)/(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b
*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - (1/2)^(1/3)*(I*sqrt(3) + 1)*(b*c^3/((
b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) -
 2/(b*c^3 + a)^3)^(1/3) - 2/(b*c^3 + a)) + a) - ((b*c^3 + a)*(2*(1/2)^(2/3)
*(-I*sqrt(3) + 1)*(1/(a*b*c^3 + a^2) - 1/(b*c^3 + a)^2)/(b*c^3/((b*c^3 + a)
^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3
+ a)^3)^(1/3) - (1/2)^(1/3)*(I*sqrt(3) + 1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/
(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3
) - 2/(b*c^3 + a)) + 3*sqrt(1/3)*(b*c^3 + a)*sqrt(-(16*b*c^3 + (a*b^2*c^6 +
 2*a^2*b*c^3 + a^3)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(1/(a*b*c^3 + a^2) - 1/
(b*c^3 + a)^2)/(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c
^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - (1/2)^(1/3)*(I*sqrt(3) +
1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b
*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - 2/(b*c^3 + a))^2 + 4*(a*b*c^3 + a^2)*
(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(1/(a*b*c^3 + a^2) - 1/(b*c^3 + a)^2)/(b*c^
3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a
)) - 2/(b*c^3 + a)^3)^(1/3) - (1/2)^(1/3)*(I*sqrt(3) + 1)*(b*c^3/((b*c^3 +
a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^
3 + a)^3)^(1/3) - 2/(b*c^3 + a)) + 4*a)/(a*b^2*c^6 + 2*a^2*b*c^3 + a^3)) +
6)*log(2*b*c^2*d*x + 2*b*c^3 - 1/4*(a^2*b*c^3 + a^3)*(2*(1/2)^(2/3)*(-I*sqr
t(3) + 1)*(1/(a*b*c^3 + a^2) - 1/(b*c^3 + a)^2)/(b*c^3/((b*c^3 + a)^2*a^2)
- 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^
(1/3) - (1/2)^(1/3)*(I*sqrt(3) + 1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c
^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - 2/(b
*c^3 + a))^2 + 1/2*(a*b*c^3 - 2*a^2)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(1/(a*
```

```
b*c^3 + a^2) - 1/(b*c^3 + a)^2)/(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 +
 a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - (1/2)^(1
/3)*(I*sqrt(3) + 1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((
a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - 2/(b*c^3 + a)) + 3/4
*sqrt(1/3)*(2*a*b*c^3 + (a^2*b*c^3 + a^3)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(
1/(a*b*c^3 + a^2) - 1/(b*c^3 + a)^2)/(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*
c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - (1/
2)^(1/3)*(I*sqrt(3) + 1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) +
 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - 2/(b*c^3 + a))
+ 2*a^2)*sqrt(-(16*b*c^3 + (a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(2*(1/2)^(2/3)*(
-I*sqrt(3) + 1)*(1/(a*b*c^3 + a^2) - 1/(b*c^3 + a)^2)/(b*c^3/((b*c^3 + a)^2
*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 +
a)^3)^(1/3) - (1/2)^(1/3)*(I*sqrt(3) + 1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a
^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3)
- 2/(b*c^3 + a))^2 + 4*(a*b*c^3 + a^2)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(1/(
a*b*c^3 + a^2) - 1/(b*c^3 + a)^2)/(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3
 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - (1/2)^
(1/3)*(I*sqrt(3) + 1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/
((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - 2/(b*c^3 + a)) + 4
*a)/(a*b^2*c^6 + 2*a^2*b*c^3 + a^3)) - a) - ((b*c^3 + a)*(2*(1/2)^(2/3)*(-I
*sqrt(3) + 1)*(1/(a*b*c^3 + a^2) - 1/(b*c^3 + a)^2)/(b*c^3/((b*c^3 + a)^2*a
^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)
^3)^(1/3) - (1/2)^(1/3)*(I*sqrt(3) + 1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2
*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) -
2/(b*c^3 + a)) - 3*sqrt(1/3)*(b*c^3 + a)*sqrt(-(16*b*c^3 + (a*b^2*c^6 + 2*a
^2*b*c^3 + a^3)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(1/(a*b*c^3 + a^2) - 1/(b*c
^3 + a)^2)/(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 +
 a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - (1/2)^(1/3)*(I*sqrt(3) + 1)*(
b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3
 + a)) - 2/(b*c^3 + a)^3)^(1/3) - 2/(b*c^3 + a))^2 + 4*(a*b*c^3 + a^2)*(2*(
1/2)^(2/3)*(-I*sqrt(3) + 1)*(1/(a*b*c^3 + a^2) - 1/(b*c^3 + a)^2)/(b*c^3/((
b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) -
 2/(b*c^3 + a)^3)^(1/3) - (1/2)^(1/3)*(I*sqrt(3) + 1)*(b*c^3/((b*c^3 + a)^2
*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 +
a)^3)^(1/3) - 2/(b*c^3 + a)) + 4*a)/(a*b^2*c^6 + 2*a^2*b*c^3 + a^3)) + 6)*l
og(2*b*c^2*d*x + 2*b*c^3 - 1/4*(a^2*b*c^3 + a^3)*(2*(1/2)^(2/3)*(-I*sqrt(3)
 + 1)*(1/(a*b*c^3 + a^2) - 1/(b*c^3 + a)^2)/(b*c^3/((b*c^3 + a)^2*a^2) - 1/
(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3
) - (1/2)^(1/3)*(I*sqrt(3) + 1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 +
 a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - 2/(b*c^3
 + a))^2 + 1/2*(a*b*c^3 - 2*a^2)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(1/(a*b*c^
3 + a^2) - 1/(b*c^3 + a)^2)/(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3
) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - (1/2)^(1/3)*
(I*sqrt(3) + 1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*
c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - 2/(b*c^3 + a)) - 3/4*sqr
```

```
t(1/3)*(2*a*b*c^3 + (a^2*b*c^3 + a^3)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(1/(a
*b*c^3 + a^2) - 1/(b*c^3 + a)^2)/(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3
+ a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - (1/2)^(
1/3)*(I*sqrt(3) + 1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/(
(a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - 2/(b*c^3 + a)) + 2*
a^2)*sqrt(-(16*b*c^3 + (a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(2*(1/2)^(2/3)*(-I*s
qrt(3) + 1)*(1/(a*b*c^3 + a^2) - 1/(b*c^3 + a)^2)/(b*c^3/((b*c^3 + a)^2*a^2
) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3
)^(1/3) - (1/2)^(1/3)*(I*sqrt(3) + 1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b
*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - 2/
(b*c^3 + a))^2 + 4*(a*b*c^3 + a^2)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(1/(a*b*
c^3 + a^2) - 1/(b*c^3 + a)^2)/(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a
^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - (1/2)^(1/3
)*(I*sqrt(3) + 1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*
b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - 2/(b*c^3 + a)) + 4*a)/
(a*b^2*c^6 + 2*a^2*b*c^3 + a^3)) - a) + 12*log(x))/(b*c^3 + a)
```

Sympy [B] time = 17.9407, size = 559, normalized size = 2.5

$$\text{RootSum}\left(t^3\left(27a^3+27a^2bc^3\right)+27t^2a^2+9ta+1,\left(t\mapsto t\log\left(x+\frac{-432t^3a^6-837t^3a^5bc^3-405t^3a^4b^2c^6-27t^3a^3b^3c^9-}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(d*x+c)**3),x)
```

```
[Out] RootSum(_t**3*(27*a**3 + 27*a**2*b*c**3) + 27*_t**2*a**2 + 9*_t*a + 1, Lamb
da(_t, _t*log(x + (-432*_t**3*a**6 - 837*_t**3*a**5*b*c**3 - 405*_t**3*a**4
*b**2*c**6 - 27*_t**3*a**3*b**3*c**9 - 27*_t**3*a**2*b**4*c**12 + 144*_t**2
*a**5 + 270*_t**2*a**4*b*c**3 + 108*_t**2*a**3*b**2*c**6 - 18*_t**2*a**2*b*
*3*c**9 + 240*_t*a**4 - 261*_t*a**3*b*c**3 - 27*_t*a**2*b**2*c**6 - 12*_t*a
*b**3*c**9 + 48*a**3 + 60*a**2*b*c**3 + 12*a*b**2*c**6)/(64*a**2*b*c**2*d +
 11*a*b**2*c**5*d + b**3*c**8*d)))) + log(x + (-432*a**6/(a + b*c**3)**3 -
837*a**5*b*c**3/(a + b*c**3)**3 + 144*a**5/(a + b*c**3)**2 - 405*a**4*b**2*
c**6/(a + b*c**3)**3 + 270*a**4*b*c**3/(a + b*c**3)**2 + 240*a**4/(a + b*c*
*3) - 27*a**3*b**3*c**9/(a + b*c**3)**3 + 108*a**3*b**2*c**6/(a + b*c**3)**
2 - 261*a**3*b*c**3/(a + b*c**3) + 48*a**3 - 27*a**2*b**4*c**12/(a + b*c**3
)**3 - 18*a**2*b**3*c**9/(a + b*c**3)**2 - 27*a**2*b**2*c**6/(a + b*c**3) +
 60*a**2*b*c**3 - 12*a*b**3*c**9/(a + b*c**3) + 12*a*b**2*c**6)/(64*a**2*b*
```

```
c**2*d + 11*a*b**2*c**5*d + b**3*c**8*d))/(a + b*c**3)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left((dx+c)^3b+a\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(1/(((d*x + c)^3*b + a)*x), x)
```

3.108 $\int \frac{1}{x^2\left(a+b(c+dx)^3\right)}\,dx$

Optimal. Leaf size=314

$$-\frac{\sqrt[3]{b}d\left(\sqrt[3]{a}\left(a-2bc^3\right)-\sqrt[3]{b}c\left(2a-bc^3\right)\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}\left(a+bc^3\right)^2}+\frac{\sqrt[3]{b}d\left(\sqrt[3]{a}\left(a-2bc^3\right)-\sqrt[3]{b}c\left(2a-bc^3\right.\right.}{3a^{2/3}\left(a+bc^3\right.}$$

```
[Out] -(1/((a + b*c^3)*x)) + (b^(1/3)*(a^(1/3) - b^(1/3)*c)*(a^(1/3) + b^(1/3)*c)
^3*d*ArcTan[(a^(1/3) - 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^
(2/3)*(a + b*c^3)^2) - (3*b*c^2*d*Log[x])/(a + b*c^3)^2 + (b^(1/3)*(a^(1/3)
*(a - 2*b*c^3) - b^(1/3)*c*(2*a - b*c^3))*d*Log[a^(1/3) + b^(1/3)*(c + d*x)
])/(3*a^(2/3)*(a + b*c^3)^2) - (b^(1/3)*(a^(1/3)*(a - 2*b*c^3) - b^(1/3)*c*
(2*a - b*c^3))*d*Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x
)^2])/(6*a^(2/3)*(a + b*c^3)^2) + (b*c^2*d*Log[a + b*(c + d*x)^3])/(a + b*c
^3)^2
```

Rubi [A] time = 0.544747, antiderivative size = 312, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 10, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.588, Rules used = {371, 6725, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{b^{2/3}d\left(-\frac{\sqrt[3]{a}\left(a-2bc^3\right)}{\sqrt[3]{b}}+2ac-bc^4\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}\left(a+bc^3\right)^2}+\frac{\sqrt[3]{b}d\left(\sqrt[3]{a}\left(a-2bc^3\right)-\sqrt[3]{b}c\left(2a-bc^3\right)\right)\log\left(\sqrt[3]{\right.}{3a^{2/3}\left(a+bc^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*(c + d*x)^3)),x]
```

```
[Out] -(1/((a + b*c^3)*x)) + (b^(1/3)*(a^(1/3) - b^(1/3)*c)*(a^(1/3) + b^(1/3)*c)
^3*d*ArcTan[(a^(1/3) - 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^
(2/3)*(a + b*c^3)^2) - (3*b*c^2*d*Log[x])/(a + b*c^3)^2 + (b^(1/3)*(a^(1/3)
*(a - 2*b*c^3) - b^(1/3)*c*(2*a - b*c^3))*d*Log[a^(1/3) + b^(1/3)*(c + d*x)
])/(3*a^(2/3)*(a + b*c^3)^2) + (b^(2/3)*(2*a*c - b*c^4 - (a^(1/3)*(a - 2*b*
c^3))/b^(1/3))*d*Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x
)^2])/(6*a^(2/3)*(a + b*c^3)^2) + (b*c^2*d*Log[a + b*(c + d*x)^3])/(a + b*c
^3)^2
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+b(c+dx)^3\right)}\,dx = d\,\text{Subst}\left(\int \frac{1}{(-c+x)^2\left(a+bx^3\right)}\,dx, x, c+dx\right)$$

$$= d\,\text{Subst}\left(\int\left(\frac{1}{\left(a+bc^3\right)(c-x)^2}+\frac{3bc^2}{\left(a+bc^3\right)^2(c-x)}+\frac{b\left(-c\left(2a-bc^3\right)-\left(a-2bc^3\right)x+3bc\right.}{\left(a+bc^3\right)^2\left(a+bx^3\right)}\right.\right.$$

$$= -\frac{1}{\left(a+bc^3\right)x}-\frac{3bc^2d\log(x)}{\left(a+bc^3\right)^2}+\frac{(bd)\,\text{Subst}\left(\int \frac{-c\left(2a-bc^3\right)-\left(a-2bc^3\right)x+3bc^2x^2}{a+bx^3}\,dx, x, c+dx\right)}{\left(a+bc^3\right)^2}$$

$$= -\frac{1}{\left(a+bc^3\right)x}-\frac{3bc^2d\log(x)}{\left(a+bc^3\right)^2}+\frac{(bd)\,\text{Subst}\left(\int \frac{-c\left(2a-bc^3\right)+\left(-a+2bc^3\right)x}{a+bx^3}\,dx, x, c+dx\right)}{\left(a+bc^3\right)^2}+\frac{\left(3b^2c^2d\right)}{}$$

$$= -\frac{1}{\left(a+bc^3\right)x}-\frac{3bc^2d\log(x)}{\left(a+bc^3\right)^2}+\frac{bc^2d\log\left(a+b(c+dx)^3\right)}{\left(a+bc^3\right)^2}+\frac{\left(b^{2/3}d\right)\text{Subst}\left(\int \frac{\sqrt[3]{a}\left(-2\sqrt[3]{b}c\left(2a-bc^3\right.\right.}{}\right.}{}$$

$$= -\frac{1}{\left(a+bc^3\right)x}-\frac{3bc^2d\log(x)}{\left(a+bc^3\right)^2}-\frac{b^{2/3}\left(2ac-bc^4-\frac{\sqrt[3]{a}\left(a-2bc^3\right)}{\sqrt[3]{b}}\right)d\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\left(a+bc^3\right)^2}+\frac{bc^2d}{}$$

$$= -\frac{1}{\left(a+bc^3\right)x}-\frac{3bc^2d\log(x)}{\left(a+bc^3\right)^2}-\frac{b^{2/3}\left(2ac-bc^4-\frac{\sqrt[3]{a}\left(a-2bc^3\right)}{\sqrt[3]{b}}\right)d\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\left(a+bc^3\right)^2}+\frac{b^{2/3}}{}$$

$$= -\frac{1}{\left(a+bc^3\right)x}+\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-\sqrt[3]{b}c\right)\left(\sqrt[3]{a}+\sqrt[3]{b}c\right)^3 d\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}a^{2/3}\left(a+bc^3\right)^2}-\frac{3bc^2d\log(x)}{\left(a+bc^3\right)^2}-\frac{b^{2/3}\left(2a\right.}{}$$

Mathematica [C] time = 0.0896584, size = 173, normalized size = 0.55

$$\frac{dx\text{RootSum}\left[3\#1^2bcd^2+\#1^3bd^3+3\#1bc^2d+a+bc^3\&, \frac{3\#1^2bc^2d^2\log(x-\#1)-3ac\log(x-\#1)-\#1ad\log(x-\#1)+8\#1bc^3d\log(x-\#1)+6bc^4\log}{\#1^2d^2+2\#1cd+c^2}\right.}{3x\left(a+bc^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*(c + d*x)^3)),x]
```

```
[Out] (-3*(a + b*c^3 + 3*b*c^2*d*x*Log[x]) + d*x*RootSum[a + b*c^3 + 3*b*c^2*d*#1
 + 3*b*c*d^2*#1^2 + b*d^3*#1^3 & , (-3*a*c*Log[x - #1] + 6*b*c^4*Log[x - #1
] - a*d*Log[x - #1]*#1 + 8*b*c^3*d*Log[x - #1]*#1 + 3*b*c^2*d^2*Log[x - #1]
*#1^2)/(c^2 + 2*c*d*#1 + d^2*#1^2) & ])/(3*(a + b*c^3)^2*x)
```

Maple [C] time = 0.007, size = 144, normalized size = 0.5

$$\frac{d}{3\left(bc^3+a\right)^2}\sum_{\_R=\mathrm{RootOf}\left(\_Z^3bd^3+3\_Z^2bcd^2+3\_Z\,bc^2d+bc^3+a\right)}\frac{\left(3\_R^2bc^2d^2+8\_R\,bc^3d+6\,bc^4-\_R\,ad-3\,ac\right)\ln\left(x-\_R\right)}{d^2\_R^2+2\,cd\_R+c^2}-\frac{}{\left(bc^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+b*(d*x+c)^3),x)
```

```
[Out] 1/3*d*sum((3*_R^2*b*c^2*d^2+8*_R*b*c^3*d+6*b*c^4-_R*a*d-3*a*c)/(_R^2*d^2+2*
_R*c*d+c^2)*ln(x-_R),_R=RootOf(_Z^3*b*d^3+3*_Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c^3
+a))/(b*c^3+a)^2-1/(b*c^3+a)/x-3*b*c^2*d*ln(x)/(b*c^3+a)^2
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [C] time = 11.2615, size = 18572, normalized size = 59.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] -1/12*(36*b*c^2*d*x*log(x) + 12*b*c^3 - 2*(b^2*c^6 + 2*a*b*c^3 + a^2)*(6*b*
c^2*d/(b^2*c^6 + 2*a*b*c^3 + a^2) - 2*(1/2)^(2/3)*(9*b^2*c^4*d^2/(b^2*c^6 +
 2*a*b*c^3 + a^2)^2 - 2*b*c*d^2/(a*b^2*c^6 + 2*a^2*b*c^3 + a^3))*(-I*sqrt(3
) + 1)/(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((a*b
^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^2*c
^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^(1/3) - (1
/2)^(1/3)*(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((
a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^
2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^(1/3)*(
I*sqrt(3) + 1))*x*log((b^3*c^6 - a^2*b)*d^3*x + 1/4*(2*a^2*b^3*c^9 + 3*a^3*
b^2*c^6 - a^5)*(6*b*c^2*d/(b^2*c^6 + 2*a*b*c^3 + a^2) - 2*(1/2)^(2/3)*(9*b^
2*c^4*d^2/(b^2*c^6 + 2*a*b*c^3 + a^2)^2 - 2*b*c*d^2/(a*b^2*c^6 + 2*a^2*b*c^
3 + a^3))*(-I*sqrt(3) + 1)/(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 -
18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)
) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a
)^3*a^2))^(1/3) - (1/2)^(1/3)*(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3
 - 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a
^2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3
+ a)^3*a^2))^(1/3)*(I*sqrt(3) + 1))^2 + 1/2*(a*b^3*c^8 - 16*a^2*b^2*c^5 + 1
0*a^3*b*c^2)*(6*b*c^2*d/(b^2*c^6 + 2*a*b*c^3 + a^2) - 2*(1/2)^(2/3)*(9*b^2*
c^4*d^2/(b^2*c^6 + 2*a*b*c^3 + a^2)^2 - 2*b*c*d^2/(a*b^2*c^6 + 2*a^2*b*c^3
+ a^3))*(-I*sqrt(3) + 1)/(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18
*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2))
+ b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^
3*a^2))^(1/3) - (1/2)^(1/3)*(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 -
 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2
)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 +
a)^3*a^2))^(1/3)*(I*sqrt(3) + 1))*d + (b^3*c^7 + 5*a*b^2*c^4 - 5*a^2*b*c)*d
^2) - (18*b*c^2*d*x - (b^2*c^6 + 2*a*b*c^3 + a^2)*(6*b*c^2*d/(b^2*c^6 + 2*a
*b*c^3 + a^2) - 2*(1/2)^(2/3)*(9*b^2*c^4*d^2/(b^2*c^6 + 2*a*b*c^3 + a^2)^2
- 2*b*c*d^2/(a*b^2*c^6 + 2*a^2*b*c^3 + a^3))*(-I*sqrt(3) + 1)/(54*b^3*c^6*d
^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3
 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a
^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^(1/3) - (1/2)^(1/3)*(54*b^3*c^
6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*
c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3
+ a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^(1/3)*(I*sqrt(3) + 1))*x -
3*sqrt(1/3)*(b^2*c^6 + 2*a*b*c^3 + a^2)*x*sqrt(-((a*b^4*c^12 + 4*a^2*b^3*c^
9 + 6*a^3*b^2*c^6 + 4*a^4*b*c^3 + a^5)*(6*b*c^2*d/(b^2*c^6 + 2*a*b*c^3 + a^
2) - 2*(1/2)^(2/3)*(9*b^2*c^4*d^2/(b^2*c^6 + 2*a*b*c^3 + a^2)^2 - 2*b*c*d^2
/(a*b^2*c^6 + 2*a^2*b*c^3 + a^3))*(-I*sqrt(3) + 1)/(54*b^3*c^6*d^3/(b^2*c^6
 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^
2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^
3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^(1/3) - (1/2)^(1/3)*(54*b^3*c^6*d^3/(b^2*
c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*
```

(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b
*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^(1/3)*(I*sqrt(3) + 1))^2 - 12*(a*b^3*c
^8 + 2*a^2*b^2*c^5 + a^3*b*c^2)*(6*b*c^2*d/(b^2*c^6 + 2*a*b*c^3 + a^2) - 2*
(1/2)^(2/3)*(9*b^2*c^4*d^2/(b^2*c^6 + 2*a*b*c^3 + a^2)^2 - 2*b*c*d^2/(a*b^2
*c^6 + 2*a^2*b*c^3 + a^3))*(-I*sqrt(3) + 1)/(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*
b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 +
 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*
b*d^3/((b*c^3 + a)^3*a^2))^(1/3) - (1/2)^(1/3)*(54*b^3*c^6*d^3/(b^2*c^6 + 2
*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^
6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 -
a)*b*d^3/((b*c^3 + a)^3*a^2))^(1/3)*(I*sqrt(3) + 1))*d + 4*(8*b^3*c^7 - 11*
a*b^2*c^4 + 8*a^2*b*c)*d^2)/(a*b^4*c^12 + 4*a^2*b^3*c^9 + 6*a^3*b^2*c^6 + 4
*a^4*b*c^3 + a^5)))*log(2*(b^3*c^6 - a^2*b)*d^3*x - 1/4*(2*a^2*b^3*c^9 + 3*
a^3*b^2*c^6 - a^5)*(6*b*c^2*d/(b^2*c^6 + 2*a*b*c^3 + a^2) - 2*(1/2)^(2/3)*(
9*b^2*c^4*d^2/(b^2*c^6 + 2*a*b*c^3 + a^2)^2 - 2*b*c*d^2/(a*b^2*c^6 + 2*a^2*
b*c^3 + a^3))*(-I*sqrt(3) + 1)/(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^
3 - 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 +
a^2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3
 + a)^3*a^2))^(1/3) - (1/2)^(1/3)*(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^
2)^3 - 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3
 + a^2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*
c^3 + a)^3*a^2))^(1/3)*(I*sqrt(3) + 1))^2 - 1/2*(a*b^3*c^8 - 16*a^2*b^2*c^5
 + 10*a^3*b*c^2)*(6*b*c^2*d/(b^2*c^6 + 2*a*b*c^3 + a^2) - 2*(1/2)^(2/3)*(9*
b^2*c^4*d^2/(b^2*c^6 + 2*a*b*c^3 + a^2)^2 - 2*b*c*d^2/(a*b^2*c^6 + 2*a^2*b*
c^3 + a^3))*(-I*sqrt(3) + 1)/(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3
- 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^
2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 +
 a)^3*a^2))^(1/3) - (1/2)^(1/3)*(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)
^3 - 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 +
 a^2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^
3 + a)^3*a^2))^(1/3)*(I*sqrt(3) + 1))*d + (2*b^3*c^7 - 5*a*b^2*c^4 + 2*a^2*
b*c)*d^2 + 3/4*sqrt(1/3)*((2*a^2*b^3*c^9 + 3*a^3*b^2*c^6 - a^5)*(6*b*c^2*d/
(b^2*c^6 + 2*a*b*c^3 + a^2) - 2*(1/2)^(2/3)*(9*b^2*c^4*d^2/(b^2*c^6 + 2*a*b
*c^3 + a^2)^2 - 2*b*c*d^2/(a*b^2*c^6 + 2*a^2*b*c^3 + a^3))*(-I*sqrt(3) + 1)
/(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((a*b^2*c^6
 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^2*c^6 + 2
*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^(1/3) - (1/2)^(1
/3)*(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((a*b^2*
c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^2*c^6
+ 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^(1/3)*(I*sqrt
(3) + 1)) - 2*(a*b^3*c^8 + 2*a^2*b^2*c^5 + a^3*b*c^2)*d)*sqrt(-((a*b^4*c^12
 + 4*a^2*b^3*c^9 + 6*a^3*b^2*c^6 + 4*a^4*b*c^3 + a^5)*(6*b*c^2*d/(b^2*c^6 +
 2*a*b*c^3 + a^2) - 2*(1/2)^(2/3)*(9*b^2*c^4*d^2/(b^2*c^6 + 2*a*b*c^3 + a^2
)^2 - 2*b*c*d^2/(a*b^2*c^6 + 2*a^2*b*c^3 + a^3))*(-I*sqrt(3) + 1)/(54*b^3*c
^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b

```
*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3
 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^(1/3) - (1/2)^(1/3)*(54*b^
3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^
2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*
c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^(1/3)*(I*sqrt(3) + 1))^
2 - 12*(a*b^3*c^8 + 2*a^2*b^2*c^5 + a^3*b*c^2)*(6*b*c^2*d/(b^2*c^6 + 2*a*b*
c^3 + a^2) - 2*(1/2)^(2/3)*(9*b^2*c^4*d^2/(b^2*c^6 + 2*a*b*c^3 + a^2)^2 - 2
*b*c*d^2/(a*b^2*c^6 + 2*a^2*b*c^3 + a^3))*(-I*sqrt(3) + 1)/(54*b^3*c^6*d^3/
(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 +
a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4)
 + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^(1/3) - (1/2)^(1/3)*(54*b^3*c^6*d
^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3
 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a
^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^(1/3)*(I*sqrt(3) + 1))*d + 4*(
8*b^3*c^7 - 11*a*b^2*c^4 + 8*a^2*b*c)*d^2)/(a*b^4*c^12 + 4*a^2*b^3*c^9 + 6*
a^3*b^2*c^6 + 4*a^4*b*c^3 + a^5))) - (18*b*c^2*d*x - (b^2*c^6 + 2*a*b*c^3 +
 a^2)*(6*b*c^2*d/(b^2*c^6 + 2*a*b*c^3 + a^2) - 2*(1/2)^(2/3)*(9*b^2*c^4*d^2
/(b^2*c^6 + 2*a*b*c^3 + a^2)^2 - 2*b*c*d^2/(a*b^2*c^6 + 2*a^2*b*c^3 + a^3))
*(-I*sqrt(3) + 1)/(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^
3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3
/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))
^(1/3) - (1/2)^(1/3)*(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2
*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*
d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^
2))^(1/3)*(I*sqrt(3) + 1))*x + 3*sqrt(1/3)*(b^2*c^6 + 2*a*b*c^3 + a^2)*x*sq
rt(-((a*b^4*c^12 + 4*a^2*b^3*c^9 + 6*a^3*b^2*c^6 + 4*a^4*b*c^3 + a^5)*(6*b*
c^2*d/(b^2*c^6 + 2*a*b*c^3 + a^2) - 2*(1/2)^(2/3)*(9*b^2*c^4*d^2/(b^2*c^6 +
 2*a*b*c^3 + a^2)^2 - 2*b*c*d^2/(a*b^2*c^6 + 2*a^2*b*c^3 + a^3))*(-I*sqrt(3
) + 1)/(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((a*b
^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^2*c
^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^(1/3) - (1
/2)^(1/3)*(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((
a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^
2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^(1/3)*(
I*sqrt(3) + 1))^2 - 12*(a*b^3*c^8 + 2*a^2*b^2*c^5 + a^3*b*c^2)*(6*b*c^2*d/(
b^2*c^6 + 2*a*b*c^3 + a^2) - 2*(1/2)^(2/3)*(9*b^2*c^4*d^2/(b^2*c^6 + 2*a*b*
c^3 + a^2)^2 - 2*b*c*d^2/(a*b^2*c^6 + 2*a^2*b*c^3 + a^3))*(-I*sqrt(3) + 1)/
(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((a*b^2*c^6
+ 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^2*c^6 + 2*
a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^(1/3) - (1/2)^(1/
3)*(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((a*b^2*c
^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^2*c^6 +
 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^(1/3)*(I*sqrt(
3) + 1))*d + 4*(8*b^3*c^7 - 11*a*b^2*c^4 + 8*a^2*b*c)*d^2)/(a*b^4*c^12 + 4*
a^2*b^3*c^9 + 6*a^3*b^2*c^6 + 4*a^4*b*c^3 + a^5)))*log(2*(b^3*c^6 - a^2*b)*
```

d^3*x - 1/4*(2*a^2*b^3*c^9 + 3*a^3*b^2*c^6 - a^5)*(6*b*c^2*d/(b^2*c^6 + 2*a*b*c^3 + a^2) - 2*(1/2)^(2/3)*(9*b^2*c^4*d^2/(b^2*c^6 + 2*a*b*c^3 + a^2)^2 - 2*b*c*d^2/(a*b^2*c^6 + 2*a^2*b*c^3 + a^3))*(-I*sqrt(3) + 1)/(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^(1/3) - (1/2)^(1/3)*(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^(1/3)*(I*sqrt(3) + 1))^2 - 1/2*(a*b^3*c^8 - 16*a^2*b^2*c^5 + 10*a^3*b*c^2)*(6*b*c^2*d/(b^2*c^6 + 2*a*b*c^3 + a^2) - 2*(1/2)^(2/3)*(9*b^2*c^4*d^2/(b^2*c^6 + 2*a*b*c^3 + a^2)^2 - 2*b*c*d^2/(a*b^2*c^6 + 2*a^2*b*c^3 + a^3))*(-I*sqrt(3) + 1)/(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^(1/3) - (1/2)^(1/3)*(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^(1/3)*(I*sqrt(3) + 1))*d + (2*b^3*c^7 - 5*a*b^2*c^4 + 2*a^2*b*c)*d^2 - 3/4*sqrt(1/3)*((2*a^2*b^3*c^9 + 3*a^3*b^2*c^6 - a^5)*(6*b*c^2*d/(b^2*c^6 + 2*a*b*c^3 + a^2) - 2*(1/2)^(2/3)*(9*b^2*c^4*d^2/(b^2*c^6 + 2*a*b*c^3 + a^2)^2 - 2*b*c*d^2/(a*b^2*c^6 + 2*a^2*b*c^3 + a^3))*(-I*sqrt(3) + 1)/(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^(1/3) - (1/2)^(1/3)*(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^(1/3)*(I*sqrt(3) + 1)) - 2*(a*b^3*c^8 + 2*a^2*b^2*c^5 + a^3*b*c^2)*d)*sqrt(-((a*b^4*c^12 + 4*a^2*b^3*c^9 + 6*a^3*b^2*c^6 + 4*a^4*b*c^3 + a^5)*(6*b*c^2*d/(b^2*c^6 + 2*a*b*c^3 + a^2) - 2*(1/2)^(2/3)*(9*b^2*c^4*d^2/(b^2*c^6 + 2*a*b*c^3 + a^2)^2 - 2*b*c*d^2/(a*b^2*c^6 + 2*a^2*b*c^3 + a^3))*(-I*sqrt(3) + 1)/(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^(1/3) - (1/2)^(1/3)*(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^(1/3)*(I*sqrt(3) + 1))^2 - 12*(a*b^3*c^8 + 2*a^2*b^2*c^5 + a^3*b*c^2)*(6*b*c^2*d/(b^2*c^6 + 2*a*b*c^3 + a^2) - 2*(1/2)^(2/3)*(9*b^2*c^4*d^2/(b^2*c^6 + 2*a*b*c^3 + a^2)^2 - 2*b*c*d^2/(a*b^2*c^6 + 2*a^2*b*c^3 + a^3))*(-I*sqrt(3) + 1)/(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^(1/3) - (1/2)^(1/3)*(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3

```
/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))
^(1/3)*(I*sqrt(3) + 1))*d + 4*(8*b^3*c^7 - 11*a*b^2*c^4 + 8*a^2*b*c)*d^2)/(
a*b^4*c^12 + 4*a^2*b^3*c^9 + 6*a^3*b^2*c^6 + 4*a^4*b*c^3 + a^5))) + 12*a)/(
(b^2*c^6 + 2*a*b*c^3 + a^2)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*(d*x+c)**3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left((dx+c)^3b+a\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(1/(((d*x + c)^3*b + a)*x^2), x)
```

3.109 $\int \frac{1}{x^3\left(a+b(c+dx)^3\right)}\,dx$

Optimal. Leaf size=393

$$-\frac{b^{2/3}d^2\left(6a^{4/3}b^{2/3}c^2+a^2-3\sqrt[3]{a}b^{5/3}c^5-7abc^3+b^2c^6\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\left(a+bc^3\right)^3}+\frac{b^{2/3}d^2\left(6a^{4/3}b^{2/3}c^2+a^2-3\sqrt[3]{a}b^{5/3}c^5-7ab\right.}{6a}$$

```
[Out] -1/(2*(a + b*c^3)*x^2) + (3*b*c^2*d)/((a + b*c^3)^2*x) + (b^(2/3)*(a^(1/3)
+ b^(1/3)*c)^3*(a - 3*a^(2/3)*b^(1/3)*c + b*c^3)*d^2*ArcTan[(a^(1/3) - 2*b^
(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*(a + b*c^3)^3) - (3*b
*c*(a - 2*b*c^3)*d^2*Log[x])/(a + b*c^3)^3 - (b^(2/3)*(a^2 + 6*a^(4/3)*b^(2
/3)*c^2 - 7*a*b*c^3 - 3*a^(1/3)*b^(5/3)*c^5 + b^2*c^6)*d^2*Log[a^(1/3) + b^
(1/3)*(c + d*x)])/(3*a^(2/3)*(a + b*c^3)^3) + (b^(2/3)*(a^2 + 6*a^(4/3)*b^(
2/3)*c^2 - 7*a*b*c^3 - 3*a^(1/3)*b^(5/3)*c^5 + b^2*c^6)*d^2*Log[a^(2/3) - a
^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2])/(6*a^(2/3)*(a + b*c^3)^3)
+ (b*c*(a - 2*b*c^3)*d^2*Log[a + b*(c + d*x)^3])/(a + b*c^3)^3
```

Rubi [A] time = 0.603958, antiderivative size = 393, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.588, Rules used = {371, 6725, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{b^{2/3}d^2\left(6a^{4/3}b^{2/3}c^2+a^2-3\sqrt[3]{a}b^{5/3}c^5-7abc^3+b^2c^6\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\left(a+bc^3\right)^3}+\frac{b^{2/3}d^2\left(6a^{4/3}b^{2/3}c^2+a^2-3\sqrt[3]{a}b^{5/3}c^5-7ab\right.}{6a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*(c + d*x)^3)),x]
```

```
[Out] -1/(2*(a + b*c^3)*x^2) + (3*b*c^2*d)/((a + b*c^3)^2*x) + (b^(2/3)*(a^(1/3)
+ b^(1/3)*c)^3*(a - 3*a^(2/3)*b^(1/3)*c + b*c^3)*d^2*ArcTan[(a^(1/3) - 2*b^
(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*(a + b*c^3)^3) - (3*b
*c*(a - 2*b*c^3)*d^2*Log[x])/(a + b*c^3)^3 - (b^(2/3)*(a^2 + 6*a^(4/3)*b^(2
/3)*c^2 - 7*a*b*c^3 - 3*a^(1/3)*b^(5/3)*c^5 + b^2*c^6)*d^2*Log[a^(1/3) + b^
(1/3)*(c + d*x)])/(3*a^(2/3)*(a + b*c^3)^3) + (b^(2/3)*(a^2 + 6*a^(4/3)*b^(
2/3)*c^2 - 7*a*b*c^3 - 3*a^(1/3)*b^(5/3)*c^5 + b^2*c^6)*d^2*Log[a^(2/3) - a
^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2])/(6*a^(2/3)*(a + b*c^3)^3)
+ (b*c*(a - 2*b*c^3)*d^2*Log[a + b*(c + d*x)^3])/(a + b*c^3)^3
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{1}{x^3\left(a+b(c+dx)^3\right)}\,dx = d^2\,\mathrm{Subst}\left(\int \frac{1}{(-c+x)^3\left(a+bx^3\right)}\,dx, x, c+dx\right)$$

$$= d^2\,\mathrm{Subst}\left(\int\left(-\frac{1}{\left(a+bc^3\right)(c-x)^3} - \frac{3bc^2}{\left(a+bc^3\right)^2(c-x)^2} - \frac{3bc\left(-a+2bc^3\right)}{\left(a+bc^3\right)^3(c-x)} + \frac{b\left(-a^2+7ab\right.}{}\right.\right.$$

$$= -\frac{1}{2\left(a+bc^3\right)x^2} + \frac{3bc^2d}{\left(a+bc^3\right)^2x} - \frac{3bc\left(a-2bc^3\right)d^2\log(x)}{\left(a+bc^3\right)^3} + \frac{\left(bd^2\right)\mathrm{Subst}\left(\int \frac{-a^2+7abc^3-b^2c^6+3}{}\right.}{(a}$$

$$= -\frac{1}{2\left(a+bc^3\right)x^2} + \frac{3bc^2d}{\left(a+bc^3\right)^2x} - \frac{3bc\left(a-2bc^3\right)d^2\log(x)}{\left(a+bc^3\right)^3} + \frac{\left(bd^2\right)\mathrm{Subst}\left(\int \frac{-a^2+7abc^3-b^2c^6+3}{a+bx^3}\right.}{\left(a+bc^3\right)}$$

$$= -\frac{1}{2\left(a+bc^3\right)x^2} + \frac{3bc^2d}{\left(a+bc^3\right)^2x} - \frac{3bc\left(a-2bc^3\right)d^2\log(x)}{\left(a+bc^3\right)^3} + \frac{bc\left(a-2bc^3\right)d^2\log\left(a+b(c+\right.}{\left(a+bc^3\right)^3}$$

$$= -\frac{1}{2\left(a+bc^3\right)x^2} + \frac{3bc^2d}{\left(a+bc^3\right)^2x} - \frac{3bc\left(a-2bc^3\right)d^2\log(x)}{\left(a+bc^3\right)^3} - \frac{b^{2/3}\left(a^2+6a^{4/3}b^{2/3}c^2-7abc^3-\right.}{3}$$

$$= -\frac{1}{2\left(a+bc^3\right)x^2} + \frac{3bc^2d}{\left(a+bc^3\right)^2x} - \frac{3bc\left(a-2bc^3\right)d^2\log(x)}{\left(a+bc^3\right)^3} - \frac{b^{2/3}\left(a^2+6a^{4/3}b^{2/3}c^2-7abc^3-\right.}{3}$$

$$= -\frac{1}{2\left(a+bc^3\right)x^2} + \frac{3bc^2d}{\left(a+bc^3\right)^2x} + \frac{b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}c\right)^3\left(a-3a^{2/3}\sqrt[3]{b}c+bc^3\right)d^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}(c+dx}{\sqrt[3]{a}}}{\sqrt{3}}\right.}{\sqrt{3}a^{2/3}\left(a+bc^3\right)^3}$$

<u>Mathematica</u> [C] time = 0.143322, size = 244, normalized size = 0.62

$$-\frac{2d^2x^2\mathrm{RootSum}\left[3\#1^2bcd^2+\#1^3bd^3+3\#1bc^2d+a+bc^3\&, \frac{-3\#1^2abcd^2\log(x-\#1)+6\#1^2b^2c^4d^2\log(x-\#1)+a^2\log(x-\#1)-12\#1abc^2d\log}{\#1^2d^2+2\#1c}\right.}{6x^2\left(a+\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*(c + d*x)^3)),x]
```

```
[Out] -(3*(a + b*c^3)*(a + b*c^2*(c - 6*d*x)) + 18*b*c*(a - 2*b*c^3)*d^2*x^2*Log[
x] + 2*d^2*x^2*RootSum[a + b*c^3 + 3*b*c^2*d*#1 + 3*b*c*d^2*#1^2 + b*d^3*#1
^3 & , (a^2*Log[x - #1] - 16*a*b*c^3*Log[x - #1] + 10*b^2*c^6*Log[x - #1] -
 12*a*b*c^2*d*Log[x - #1]*#1 + 15*b^2*c^5*d*Log[x - #1]*#1 - 3*a*b*c*d^2*Lo
g[x - #1]*#1^2 + 6*b^2*c^4*d^2*Log[x - #1]*#1^2)/(c^2 + 2*c*d*#1 + d^2*#1^2
) & ])/(6*(a + b*c^3)^3*x^2)
```

Maple [C] time = 0.01, size = 217, normalized size = 0.6

$$\frac{d^2}{3\left(bc^3+a\right)^3}\sum_{\_R=\mathrm{RootOf}\left(\_Z^3bd^3+3\_Z^2bcd^2+3\_Z\,bc^2d+bc^3+a\right)}\frac{\left(-6\_R^2b^2c^4d^2-15\_R\,b^2c^5d-10\,b^2c^6+3\_R^2abcd^2+12\_R\,abc^2 d}{d^2\_R^2+2\,cd\_R+c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a+b*(d*x+c)^3),x)
```

```
[Out] 1/3*d^2*sum((-6*_R^2*b^2*c^4*d^2-15*_R*b^2*c^5*d-10*b^2*c^6+3*_R^2*a*b*c*d^
2+12*_R*a*b*c^2*d+16*a*b*c^3-a^2)/(_R^2*d^2+2*_R*c*d+c^2)*ln(x-_R),_R=RootO
f(_Z^3*b*d^3+3*_Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))/(b*c^3+a)^3-1/2/(b*c^3+a
)/x^2+3*b*c^2*d/(b*c^3+a)^2/x+6*b^2*c^4*d^2/(b*c^3+a)^3*ln(x)-3*b*c*d^2/(b*
c^3+a)^3*ln(x)*a
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [C] time = 53.0366, size = 29965, normalized size = 76.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] -1/12*(6*b^2*c^6 - 36*(2*b^2*c^4 - a*b*c)*d^2*x^2*log(x) + 12*a*b*c^3 - 2*(
b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)*(6*(1/2)^(2/3)*(b^2*c^2*d^4/(a*b
^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4) - 3*(2*b^2*c^4*d^2 - a*b*c*d^2)
^2/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^2)*(-I*sqrt(3) + 1)/(27*(2*b
^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c
^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*c
^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b
*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/
(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)^(1/3) - (1/2)^(1/3)*(27*(2*b
^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c
^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*c
^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b
*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/
(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)^(1/3)*(I*sqrt(3) + 1) - 6*(2
*b^2*c^4*d^2 - a*b*c*d^2)/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3))*x^2*
log((b^4*c^9 + 3*a*b^3*c^6 - 24*a^2*b^2*c^3 + a^3*b)*d^5*x + (b^4*c^10 + 15
*a*b^3*c^7 - 63*a^2*b^2*c^4 + 4*a^3*b*c)*d^4 - 1/2*(a*b^4*c^12 - 50*a^2*b^3
*c^9 + 141*a^3*b^2*c^6 - 50*a^4*b*c^3 + a^5)*(6*(1/2)^(2/3)*(b^2*c^2*d^4/(a
*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4) - 3*(2*b^2*c^4*d^2 - a*b*c*d^
2)^2/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^2)*(-I*sqrt(3) + 1)/(27*(2
*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b
*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3
*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2
*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^
3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)^(1/3) - (1/2)^(1/3)*(27*(2
*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b
*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3
*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2
*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^
3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)^(1/3)*(I*sqrt(3) + 1) - 6*
(2*b^2*c^4*d^2 - a*b*c*d^2)/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3))*d^
2 + 3/4*(a^2*b^4*c^14 + a^3*b^3*c^11 - 3*a^4*b^2*c^8 - 5*a^5*b*c^5 - 2*a^6*
c^2)*(6*(1/2)^(2/3)*(b^2*c^2*d^4/(a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 +
 a^4) - 3*(2*b^2*c^4*d^2 - a*b*c*d^2)^2/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^
3 + a^3)^2)*(-I*sqrt(3) + 1)/(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((
a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a
^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5
) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2
) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 +
 a^3)^3)^(1/3) - (1/2)^(1/3)*(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((
a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a
^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5
```

```
) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2
) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 +
 a^3)^3)^(1/3)*(I*sqrt(3) + 1) - 6*(2*b^2*c^4*d^2 - a*b*c*d^2)/(b^3*c^9 + 3
*a*b^2*c^6 + 3*a^2*b*c^3 + a^3))^2) - 36*(b^2*c^5 + a*b*c^2)*d*x + 6*a^2 +
(18*(2*b^2*c^4 - a*b*c)*d^2*x^2 + (b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^
3)*(6*(1/2)^(2/3)*(b^2*c^2*d^4/(a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a
^4) - 3*(2*b^2*c^4*d^2 - a*b*c*d^2)^2/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3
+ a^3)^2)*(-I*sqrt(3) + 1)/(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*
b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2
*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5)
+ (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2)
- 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a
^3)^3)^(1/3) - (1/2)^(1/3)*(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*
b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2
*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5)
+ (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2)
- 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a
^3)^3)^(1/3)*(I*sqrt(3) + 1) - 6*(2*b^2*c^4*d^2 - a*b*c*d^2)/(b^3*c^9 + 3*a
*b^2*c^6 + 3*a^2*b*c^3 + a^3))*x^2 + 3*sqrt(1/3)*(b^3*c^9 + 3*a*b^2*c^6 + 3
*a^2*b*c^3 + a^3)*x^2*sqrt(-(12*(4*b^5*c^11 - 24*a*b^4*c^8 + 48*a^2*b^3*c^5
 - 5*a^3*b^2*c^2)*d^4 + 12*(2*a*b^5*c^13 + 5*a^2*b^4*c^10 + 3*a^3*b^3*c^7 -
 a^4*b^2*c^4 - a^5*b*c)*(6*(1/2)^(2/3)*(b^2*c^2*d^4/(a*b^3*c^9 + 3*a^2*b^2*
c^6 + 3*a^3*b*c^3 + a^4) - 3*(2*b^2*c^4*d^2 - a*b*c*d^2)^2/(b^3*c^9 + 3*a*b
^2*c^6 + 3*a^2*b*c^3 + a^3)^2)*(-I*sqrt(3) + 1)/(27*(2*b^2*c^4*d^2 - a*b*c*
d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9
+ 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6
+ 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6
/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*
c^6 + 3*a^2*b*c^3 + a^3)^3)^(1/3) - (1/2)^(1/3)*(27*(2*b^2*c^4*d^2 - a*b*c*
d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9
+ 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6
+ 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6
/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*
c^6 + 3*a^2*b*c^3 + a^3)^3)^(1/3)*(I*sqrt(3) + 1) - 6*(2*b^2*c^4*d^2 - a*b*
c*d^2)/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3))*d^2 + (a*b^6*c^18 + 6*a
^2*b^5*c^15 + 15*a^3*b^4*c^12 + 20*a^4*b^3*c^9 + 15*a^5*b^2*c^6 + 6*a^6*b*c
^3 + a^7)*(6*(1/2)^(2/3)*(b^2*c^2*d^4/(a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*
c^3 + a^4) - 3*(2*b^2*c^4*d^2 - a*b*c*d^2)^2/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2
*b*c^3 + a^3)^2)*(-I*sqrt(3) + 1)/(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d
^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6
+ 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3
+ a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^
6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*
c^3 + a^3)^3)^(1/3) - (1/2)^(1/3)*(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d
^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6
```

+ 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3
+ a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^
6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*
c^3 + a^3)^3)^(1/3)*(I*sqrt(3) + 1) - 6*(2*b^2*c^4*d^2 - a*b*c*d^2)/(b^3*c^
9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3))^2)/(a*b^6*c^18 + 6*a^2*b^5*c^15 + 15*
a^3*b^4*c^12 + 20*a^4*b^3*c^9 + 15*a^5*b^2*c^6 + 6*a^6*b*c^3 + a^7)))*log(2
*(b^4*c^9 + 3*a*b^3*c^6 - 24*a^2*b^2*c^3 + a^3*b)*d^5*x + (2*b^4*c^10 - 6*a
*b^3*c^7 - 9*a^2*b^2*c^4 - a^3*b*c)*d^4 + 1/2*(a*b^4*c^12 - 50*a^2*b^3*c^9
+ 141*a^3*b^2*c^6 - 50*a^4*b*c^3 + a^5)*(6*(1/2)^(2/3)*(b^2*c^2*d^4/(a*b^3*
c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4) - 3*(2*b^2*c^4*d^2 - a*b*c*d^2)^2/
(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^2)*(-I*sqrt(3) + 1)/(27*(2*b^2*
c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3
+ a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9
+ 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^
3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^
3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)^(1/3) - (1/2)^(1/3)*(27*(2*b^2*
c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3
+ a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9
+ 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^
3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^
3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)^(1/3)*(I*sqrt(3) + 1) - 6*(2*b^
2*c^4*d^2 - a*b*c*d^2)/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3))*d^2 - 3
/4*(a^2*b^4*c^14 + a^3*b^3*c^11 - 3*a^4*b^2*c^8 - 5*a^5*b*c^5 - 2*a^6*c^2)*
(6*(1/2)^(2/3)*(b^2*c^2*d^4/(a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)
 - 3*(2*b^2*c^4*d^2 - a*b*c*d^2)^2/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a
^3)^2)*(-I*sqrt(3) + 1)/(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3
*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*
c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (
b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 5
4*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)
^3)^(1/3) - (1/2)^(1/3)*(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3
*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*
c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (
b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 5
4*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)
^3)^(1/3)*(I*sqrt(3) + 1) - 6*(2*b^2*c^4*d^2 - a*b*c*d^2)/(b^3*c^9 + 3*a*b^
2*c^6 + 3*a^2*b*c^3 + a^3))^2 + 3/4*sqrt(1/3)*(2*(a*b^4*c^12 + 4*a^2*b^3*c^
9 + 6*a^3*b^2*c^6 + 4*a^4*b*c^3 + a^5)*d^2 + 3*(a^2*b^4*c^14 + a^3*b^3*c^11
 - 3*a^4*b^2*c^8 - 5*a^5*b*c^5 - 2*a^6*c^2)*(6*(1/2)^(2/3)*(b^2*c^2*d^4/(a*
b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4) - 3*(2*b^2*c^4*d^2 - a*b*c*d^2
)^2/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^2)*(-I*sqrt(3) + 1)/(27*(2*
b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*
c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*
c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*
b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3

/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)^(1/3) - (1/2)^(1/3)*(27*(2*
b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*
c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*
c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*
b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3
/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)^(1/3)*(I*sqrt(3) + 1) - 6*(
2*b^2*c^4*d^2 - a*b*c*d^2)/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)))*sq
rt(-(12*(4*b^5*c^11 - 24*a*b^4*c^8 + 48*a^2*b^3*c^5 - 5*a^3*b^2*c^2)*d^4 +
12*(2*a*b^5*c^13 + 5*a^2*b^4*c^10 + 3*a^3*b^3*c^7 - a^4*b^2*c^4 - a^5*b*c)*
(6*(1/2)^(2/3)*(b^2*c^2*d^4/(a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)
 - 3*(2*b^2*c^4*d^2 - a*b*c*d^2)^2/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a
^3)^2)*(-I*sqrt(3) + 1)/(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3
*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*
c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (
b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 5
4*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)
^3)^(1/3) - (1/2)^(1/3)*(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3
*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*
c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (
b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 5
4*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)
^3)^(1/3)*(I*sqrt(3) + 1) - 6*(2*b^2*c^4*d^2 - a*b*c*d^2)/(b^3*c^9 + 3*a*b^
2*c^6 + 3*a^2*b*c^3 + a^3))*d^2 + (a*b^6*c^18 + 6*a^2*b^5*c^15 + 15*a^3*b^4
*c^12 + 20*a^4*b^3*c^9 + 15*a^5*b^2*c^6 + 6*a^6*b*c^3 + a^7)*(6*(1/2)^(2/3)
*(b^2*c^2*d^4/(a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4) - 3*(2*b^2*c^
4*d^2 - a*b*c*d^2)^2/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^2)*(-I*sqr
t(3) + 1)/(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b
^2*c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) -
b^2*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*
b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d
^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)^(1/3) - (1
/2)^(1/3)*(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b
^2*c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) -
b^2*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*
b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d
^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)^(1/3)*(I*s
qrt(3) + 1) - 6*(2*b^2*c^4*d^2 - a*b*c*d^2)/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*
b*c^3 + a^3))^2)/(a*b^6*c^18 + 6*a^2*b^5*c^15 + 15*a^3*b^4*c^12 + 20*a^4*b^
3*c^9 + 15*a^5*b^2*c^6 + 6*a^6*b*c^3 + a^7))) + (18*(2*b^2*c^4 - a*b*c)*d^2
*x^2 + (b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)*(6*(1/2)^(2/3)*(b^2*c^2*
d^4/(a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4) - 3*(2*b^2*c^4*d^2 - a*
b*c*d^2)^2/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^2)*(-I*sqrt(3) + 1)/
(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3
*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a
^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 -

24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c
*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)^(1/3) - (1/2)^(1/3)*
(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3
*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a
^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 -
24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c
*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)^(1/3)*(I*sqrt(3) + 1
) - 6*(2*b^2*c^4*d^2 - a*b*c*d^2)/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^
3))*x^2 - 3*sqrt(1/3)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)*x^2*sqrt(
-(12*(4*b^5*c^11 - 24*a*b^4*c^8 + 48*a^2*b^3*c^5 - 5*a^3*b^2*c^2)*d^4 + 12*
(2*a*b^5*c^13 + 5*a^2*b^4*c^10 + 3*a^3*b^3*c^7 - a^4*b^2*c^4 - a^5*b*c)*(6*
(1/2)^(2/3)*(b^2*c^2*d^4/(a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4) -
3*(2*b^2*c^4*d^2 - a*b*c*d^2)^2/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)
^2)*(-I*sqrt(3) + 1)/(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^
9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3
 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3
*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(
2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)
^(1/3) - (1/2)^(1/3)*(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^
9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3
 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3
*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(
2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)
^(1/3)*(I*sqrt(3) + 1) - 6*(2*b^2*c^4*d^2 - a*b*c*d^2)/(b^3*c^9 + 3*a*b^2*c
^6 + 3*a^2*b*c^3 + a^3))*d^2 + (a*b^6*c^18 + 6*a^2*b^5*c^15 + 15*a^3*b^4*c^
12 + 20*a^4*b^3*c^9 + 15*a^5*b^2*c^6 + 6*a^6*b*c^3 + a^7)*(6*(1/2)^(2/3)*(b
^2*c^2*d^4/(a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4) - 3*(2*b^2*c^4*d
^2 - a*b*c*d^2)^2/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^2)*(-I*sqrt(3
) + 1)/(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*
c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2
*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2
*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2
- a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)^(1/3) - (1/2)
^(1/3)*(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*
c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2
*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2
*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2
- a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)^(1/3)*(I*sqrt
(3) + 1) - 6*(2*b^2*c^4*d^2 - a*b*c*d^2)/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c
^3 + a^3))^2)/(a*b^6*c^18 + 6*a^2*b^5*c^15 + 15*a^3*b^4*c^12 + 20*a^4*b^3*c
^9 + 15*a^5*b^2*c^6 + 6*a^6*b*c^3 + a^7)))*log(2*(b^4*c^9 + 3*a*b^3*c^6 - 2
4*a^2*b^2*c^3 + a^3*b)*d^5*x + (2*b^4*c^10 - 6*a*b^3*c^7 - 9*a^2*b^2*c^4 -
a^3*b*c)*d^4 + 1/2*(a*b^4*c^12 - 50*a^2*b^3*c^9 + 141*a^3*b^2*c^6 - 50*a^4*
b*c^3 + a^5)*(6*(1/2)^(2/3)*(b^2*c^2*d^4/(a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3
*b*c^3 + a^4) - 3*(2*b^2*c^4*d^2 - a*b*c*d^2)^2/(b^3*c^9 + 3*a*b^2*c^6 + 3*

a^2*b*c^3 + a^3)^2)*(-I*sqrt(3) + 1)/(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^
2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c
^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c
^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 +
a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2
*b*c^3 + a^3)^3)^(1/3) - (1/2)^(1/3)*(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^
2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c
^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c
^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 +
a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2
*b*c^3 + a^3)^3)^(1/3)*(I*sqrt(3) + 1) - 6*(2*b^2*c^4*d^2 - a*b*c*d^2)/(b^3
*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3))*d^2 - 3/4*(a^2*b^4*c^14 + a^3*b^3*
c^11 - 3*a^4*b^2*c^8 - 5*a^5*b*c^5 - 2*a^6*c^2)*(6*(1/2)^(2/3)*(b^2*c^2*d^4
/(a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4) - 3*(2*b^2*c^4*d^2 - a*b*c
*d^2)^2/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^2)*(-I*sqrt(3) + 1)/(27
*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^
3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*
b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*
a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^
2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)^(1/3) - (1/2)^(1/3)*(27
*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^
3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*
b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*
a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^
2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)^(1/3)*(I*sqrt(3) + 1) -
 6*(2*b^2*c^4*d^2 - a*b*c*d^2)/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3))
^2 - 3/4*sqrt(1/3)*(2*(a*b^4*c^12 + 4*a^2*b^3*c^9 + 6*a^3*b^2*c^6 + 4*a^4*b
*c^3 + a^5)*d^2 + 3*(a^2*b^4*c^14 + a^3*b^3*c^11 - 3*a^4*b^2*c^8 - 5*a^5*b*
c^5 - 2*a^6*c^2)*(6*(1/2)^(2/3)*(b^2*c^2*d^4/(a*b^3*c^9 + 3*a^2*b^2*c^6 + 3
*a^3*b*c^3 + a^4) - 3*(2*b^2*c^4*d^2 - a*b*c*d^2)^2/(b^3*c^9 + 3*a*b^2*c^6
+ 3*a^2*b*c^3 + a^3)^2)*(-I*sqrt(3) + 1)/(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^
2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b
^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4
*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^
3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3
*a^2*b*c^3 + a^3)^3)^(1/3) - (1/2)^(1/3)*(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^
2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b
^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4
*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^
3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3
*a^2*b*c^3 + a^3)^3)^(1/3)*(I*sqrt(3) + 1) - 6*(2*b^2*c^4*d^2 - a*b*c*d^2)/
(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)))*sqrt(-(12*(4*b^5*c^11 - 24*a*
b^4*c^8 + 48*a^2*b^3*c^5 - 5*a^3*b^2*c^2)*d^4 + 12*(2*a*b^5*c^13 + 5*a^2*b^
4*c^10 + 3*a^3*b^3*c^7 - a^4*b^2*c^4 - a^5*b*c)*(6*(1/2)^(2/3)*(b^2*c^2*d^4
/(a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4) - 3*(2*b^2*c^4*d^2 - a*b*c

```
*d^2)^2/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^2)*(-I*sqrt(3) + 1)/(27
*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^
3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*
b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*
a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^
2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)^(1/3) - (1/2)^(1/3)*(27
*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^
3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*
b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*
a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^
2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)^(1/3)*(I*sqrt(3) + 1) -
 6*(2*b^2*c^4*d^2 - a*b*c*d^2)/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3))
*d^2 + (a*b^6*c^18 + 6*a^2*b^5*c^15 + 15*a^3*b^4*c^12 + 20*a^4*b^3*c^9 + 15
*a^5*b^2*c^6 + 6*a^6*b*c^3 + a^7)*(6*(1/2)^(2/3)*(b^2*c^2*d^4/(a*b^3*c^9 +
3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4) - 3*(2*b^2*c^4*d^2 - a*b*c*d^2)^2/(b^3*c
^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^2)*(-I*sqrt(3) + 1)/(27*(2*b^2*c^4*d^
2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)
*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3*a^
3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^3 + a^
3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9
+ 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)^(1/3) - (1/2)^(1/3)*(27*(2*b^2*c^4*d^
2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)
*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3*a^
3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^3 + a^
3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9
+ 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)^(1/3)*(I*sqrt(3) + 1) - 6*(2*b^2*c^4*
d^2 - a*b*c*d^2)/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3))^2)/(a*b^6*c^1
8 + 6*a^2*b^5*c^15 + 15*a^3*b^4*c^12 + 20*a^4*b^3*c^9 + 15*a^5*b^2*c^6 + 6*
a^6*b*c^3 + a^7))))/((b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a+b*(d*x+c)**3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left((dx+c)^3b+a\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(1/(((d*x + c)^3*b + a)*x^3), x)
```

3.110 $\int \frac{x^3}{a+b(c+dx)^4}\,dx$

Optimal. Leaf size=356

$$-\frac{c\left(3\sqrt{a}-\sqrt{b}c^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}d^4}+\frac{c\left(3\sqrt{a}-\sqrt{b}c^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+\right.}{4\sqrt{2}a^{3/4}b^{3/4}d^4}$$

```
[Out] (3*c^2*ArcTan[(Sqrt[b]*(c + d*x)^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[b]*d^4) + (c*
(3*Sqrt[a] + Sqrt[b]*c^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)])/
(2*Sqrt[2]*a^(3/4)*b^(3/4)*d^4) - (c*(3*Sqrt[a] + Sqrt[b]*c^2)*ArcTan[1 + (
Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)*d^4) - (c*(
3*Sqrt[a] - Sqrt[b]*c^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) +
Sqrt[b]*(c + d*x)^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)*d^4) + (c*(3*Sqrt[a] - Sqr
t[b]*c^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*
x)^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)*d^4) + Log[a + b*(c + d*x)^4]/(4*b*d^4)
```

Rubi [A] time = 0.425632, antiderivative size = 356, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 12, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.706, Rules used = {371, 1876, 1248, 635, 205, 260, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{c\left(3\sqrt{a}-\sqrt{b}c^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}d^4}+\frac{c\left(3\sqrt{a}-\sqrt{b}c^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+\right.}{4\sqrt{2}a^{3/4}b^{3/4}d^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*(c + d*x)^4),x]
```

```
[Out] (3*c^2*ArcTan[(Sqrt[b]*(c + d*x)^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[b]*d^4) + (c*
(3*Sqrt[a] + Sqrt[b]*c^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)])/
(2*Sqrt[2]*a^(3/4)*b^(3/4)*d^4) - (c*(3*Sqrt[a] + Sqrt[b]*c^2)*ArcTan[1 + (
Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)*d^4) - (c*(
3*Sqrt[a] - Sqrt[b]*c^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) +
Sqrt[b]*(c + d*x)^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)*d^4) + (c*(3*Sqrt[a] - Sqr
t[b]*c^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*
x)^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)*d^4) + Log[a + b*(c + d*x)^4]/(4*b*d^4)
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^3}{a+b(c+dx)^4}\,dx = \frac{\text{Subst}\left(\int \frac{(-c+x)^3}{a+bx^4}\,dx, x, c+dx\right)}{d^4}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{x(3c^2+x^2)}{a+bx^4} + \frac{-c^3-3cx^2}{a+bx^4}\right) dx, x, c+dx\right)}{d^4}$$

$$= \frac{\text{Subst}\left(\int \frac{x(3c^2+x^2)}{a+bx^4}\,dx, x, c+dx\right)}{d^4} + \frac{\text{Subst}\left(\int \frac{-c^3-3cx^2}{a+bx^4}\,dx, x, c+dx\right)}{d^4}$$

$$= \frac{\text{Subst}\left(\int \frac{3c^2+x}{a+bx^2}\,dx, x, (c+dx)^2\right)}{2d^4} + \frac{\left(c\left(3-\frac{\sqrt{b}c^2}{\sqrt{a}}\right)\right)\text{Subst}\left(\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}\,dx, x, c+dx\right)}{2bd^4} - \frac{\left(c\left(3+\frac{\sqrt{b}c^2}{\sqrt{a}}\right.\right.}{}$$

$$= \frac{\text{Subst}\left(\int \frac{x}{a+bx^2}\,dx, x, (c+dx)^2\right)}{2d^4} + \frac{(3c^2)\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, (c+dx)^2\right)}{2d^4} - \frac{\left(c\left(3\sqrt{a}-\sqrt{b}c^2\right)\right)\text{Sub}}{4}$$

$$= \frac{3c^2\tan^{-1}\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}d^4} - \frac{c\left(3\sqrt{a}-\sqrt{b}c^2\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}d^4} + \frac{c\left(3\sqrt{a}-\right.}{}$$

$$= \frac{3c^2\tan^{-1}\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}d^4} + \frac{c\left(3\sqrt{a}+\sqrt{b}c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}d^4} - \frac{c\left(3\sqrt{a}+\sqrt{b}c^2\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{}}{}\right.}{2\sqrt{2}a^{3/4}b^{3/4}d^4}$$

Mathematica [C] time = 0.0420945, size = 106, normalized size = 0.3

$$\frac{\text{RootSum}\left[6\#1^2bc^2d^2+4\#1^3bcd^3+\#1^4bd^4+4\#1bc^3d+a+bc^4\&, \frac{\#1^3\log(x-\#1)}{3\#1^2cd^2+\#1^3d^3+3\#1c^2d+c^3}\&\right]}{4bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*(c + d*x)^4),x]
```

```
[Out] RootSum[a + b*c^4 + 4*b*c^3*d*#1 + 6*b*c^2*d^2*#1^2 + 4*b*c*d^3*#1^3 + b*d^
4*#1^4 & , (Log[x - #1]*#1^3)/(c^3 + 3*c^2*d*#1 + 3*c*d^2*#1^2 + d^3*#1^3)
& ]/(4*b*d)
```

Maple [C] time = 0.012, size = 97, normalized size = 0.3

$$\frac{1}{4\,bd}\sum_{\_R=\mathrm{RootOf}\left(bd^4\_Z^4+4\,bcd^3\_Z^3+6\,bc^2d^2\_Z^2+4\,bc^3d\_Z+bc^4+a\right)}\frac{\_R^3\ln\left(x-\_R\right)}{d^3\_R^3+3\,cd^2\_R^2+3\,\_R\,c^2d+c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+b*(d*x+c)^4),x)
```

```
[Out] 1/4/b/d*sum(_R^3/(_R^3*d^3+3*_R^2*c*d^2+3*_R*c^2*d+c^3)*ln(x-_R),_R=RootOf(
_Z^4*b*d^4+4*_Z^3*b*c*d^3+6*_Z^2*b*c^2*d^2+4*_Z*b*c^3*d+b*c^4+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{(dx+c)^4b+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] integrate(x^3/((d*x + c)^4*b + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 3.39874, size = 374, normalized size = 1.05

$$\mathrm{RootSum}\Big(256t^4a^3b^4d^{16}-256t^3a^3b^3d^{12}+t^2\left(96a^3b^2d^8+480a^2b^3c^4d^8\right)+t\left(-16a^3bd^4+192a^2b^2c^4d^4-48ab^3c^8d^4\right)+a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b*(d*x+c)**4),x)
```

```
[Out] RootSum(256*_t**4*a**3*b**4*d**16 - 256*_t**3*a**3*b**3*d**12 + _t**2*(96*a
**3*b**2*d**8 + 480*a**2*b**3*c**4*d**8) + _t*(-16*a**3*b*d**4 + 192*a**2*b
**2*c**4*d**4 - 48*a*b**3*c**8*d**4) + a**3 + 3*a**2*b*c**4 + 3*a*b**2*c**8
 + b**3*c**12, Lambda(_t, _t*log(x + (-1728*_t**3*a**4*b**3*d**12 - 960*_t*
*3*a**3*b**4*c**4*d**12 + 1296*_t**2*a**4*b**2*d**8 + 2016*_t**2*a**3*b**3*
c**4*d**8 - 48*_t**2*a**2*b**4*c**8*d**8 - 324*_t*a**4*b*d**4 - 4716*_t*a**
3*b**2*c**4*d**4 - 1452*_t*a**2*b**3*c**8*d**4 - 4*_t*a*b**4*c**12*d**4 + 2
7*a**4 - 390*a**3*b*c**4 - 444*a**2*b**2*c**8 - 26*a*b**3*c**12 + b**4*c**1
6)/(729*a**3*b*c**3*d - 1053*a**2*b**2*c**7*d - 117*a*b**3*c**11*d + b**4*c
**15*d))))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{(dx+c)^4 b + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*(d*x+c)^4),x, algorithm="giac")
```

```
[Out] integrate(x^3/((d*x + c)^4*b + a), x)
```

3.111 $\int \frac{x^2}{a+b(c+dx)^4} dx$

Optimal. Leaf size=318

$$\frac{\left(\sqrt{a}-\sqrt{b}c^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}d^3}-\frac{\left(\sqrt{a}-\sqrt{b}c^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}d^3}$$

```
[Out] -((c*ArcTan[(Sqrt[b]*(c + d*x)^2)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]*d^3)) - ((Sqrt
[a] + Sqrt[b]*c^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)])/(2*Sqrt
[2]*a^(3/4)*b^(3/4)*d^3) + ((Sqrt[a] + Sqrt[b]*c^2)*ArcTan[1 + (Sqrt[2]*b^(
1/4)*(c + d*x))/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)*d^3) + ((Sqrt[a] - Sqr
t[b]*c^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*
x)^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)*d^3) - ((Sqrt[a] - Sqrt[b]*c^2)*Log[Sqrt[
a] + Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*x)^2])/(4*Sqrt[2]*a
^(3/4)*b^(3/4)*d^3)
```

Rubi [A] time = 0.312481, antiderivative size = 318, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.588, Rules used = {371, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$\frac{\left(\sqrt{a}-\sqrt{b}c^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}d^3}-\frac{\left(\sqrt{a}-\sqrt{b}c^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}d^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*(c + d*x)^4),x]
```


```
[Out] -((c*ArcTan[(Sqrt[b]*(c + d*x)^2)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]*d^3)) - ((Sqrt
[a] + Sqrt[b]*c^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)])/(2*Sqrt
[2]*a^(3/4)*b^(3/4)*d^3) + ((Sqrt[a] + Sqrt[b]*c^2)*ArcTan[1 + (Sqrt[2]*b^(
1/4)*(c + d*x))/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)*d^3) + ((Sqrt[a] - Sqr
t[b]*c^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*
x)^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)*d^3) - ((Sqrt[a] - Sqrt[b]*c^2)*Log[Sqrt[
a] + Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*x)^2])/(4*Sqrt[2]*a
^(3/4)*b^(3/4)*d^3)
```


Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^2}{a+b(c+dx)^4}\,dx = \frac{\text{Subst}\left(\int \frac{(-c+x)^2}{a+bx^4}\,dx, x, c+dx\right)}{d^3}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{2cx}{a+bx^4}+\frac{c^2+x^2}{a+bx^4}\right)dx, x, c+dx\right)}{d^3}$$
$$= \frac{\text{Subst}\left(\int \frac{c^2+x^2}{a+bx^4}\,dx, x, c+dx\right)}{d^3} - \frac{(2c)\,\text{Subst}\left(\int \frac{x}{a+bx^4}\,dx, x, c+dx\right)}{d^3}$$
$$= -\frac{c\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, (c+dx)^2\right)}{d^3} - \frac{\left(1-\frac{\sqrt{b}c^2}{\sqrt{a}}\right)\text{Subst}\left(\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}\,dx, x, c+dx\right)}{2bd^3} + \frac{\left(1+\frac{\sqrt{b}c^2}{\sqrt{a}}\right)\text{Su}}{}$$
$$= -\frac{c\tan^{-1}\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}d^3} + \frac{\left(\sqrt{a}-\sqrt{b}c^2\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx, x, c+dx\right)}{4\sqrt{2}a^{3/4}b^{3/4}d^3} + \frac{\left(\sqrt{a}-\sqrt{b}c^2\right)\text{Subst}}{4\sqrt{}}$$
$$= -\frac{c\tan^{-1}\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}d^3} + \frac{\left(\sqrt{a}-\sqrt{b}c^2\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}d^3} - \frac{\left(\sqrt{a}-\sqrt{b}c^2\right)\text{l}}{}$$
$$= -\frac{c\tan^{-1}\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}d^3} - \frac{\left(\sqrt{a}+\sqrt{b}c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}d^3} + \frac{\left(\sqrt{a}+\sqrt{b}c^2\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}d^3}$$

Mathematica [C] time = 0.0325013, size = 106, normalized size = 0.33

$$\frac{\text{RootSum}\left[6\#1^2bc^2d^2+4\#1^3bcd^3+\#1^4bd^4+4\#1bc^3d+a+bc^4\&, \frac{\#1^2\log(x-\#1)}{3\#1^2cd^2+\#1^3d^3+3\#1c^2d+c^3}\&\right]}{4bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*(c + d*x)^4),x]
```

```
[Out] RootSum[a + b*c^4 + 4*b*c^3*d*#1 + 6*b*c^2*d^2*#1^2 + 4*b*c*d^3*#1^3 + b*d^
4*#1^4 & , (Log[x - #1]*#1^2)/(c^3 + 3*c^2*d*#1 + 3*c*d^2*#1^2 + d^3*#1^3)
& ]/(4*b*d)
```

Maple [C] time = 0.003, size = 97, normalized size = 0.3

$$\frac{1}{4\,bd}\sum_{\_R=\mathrm{RootOf}\left(\_Z^4bd^4+4\_Z^3bcd^3+6\_Z^2bc^2d^2+4\_Z\,bc^3d+bc^4+a\right)}\frac{\_R^2\ln\left(x-\_R\right)}{d^3\_R^3+3\,cd^2\_R^2+3\_R\,c^2d+c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*(d*x+c)^4),x)
```

```
[Out] 1/4/b/d*sum(_R^2/(_R^3*d^3+3*_R^2*c*d^2+3*_R*c^2*d+c^3)*ln(x-_R),_R=RootOf(
_Z^4*b*d^4+4*_Z^3*b*c*d^3+6*_Z^2*b*c^2*d^2+4*_Z*b*c^3*d+b*c^4+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^2}{(dx+c)^4b+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((d*x + c)^4*b + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 2.54595, size = 274, normalized size = 0.86

$$\mathrm{RootSum}\left(256t^4a^3b^3d^{12}+192t^2a^2b^2c^2d^6+t\left(-32a^2bcd^3+32ab^2c^5d^3\right)+a^2+2abc^4+b^2c^8,\left(t\mapsto t\log\left(x+\frac{64t^3a^4b^2d}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*(d*x+c)**4),x)
```

```
[Out] RootSum(256*_t**4*a**3*b**3*d**12 + 192*_t**2*a**2*b**2*c**2*d**6 + _t*(-32
*a**2*b*c*d**3 + 32*a*b**2*c**5*d**3) + a**2 + 2*a*b*c**4 + b**2*c**8, Lamb
da(_t, _t*log(x + (64*_t**3*a**4*b**2*d**9 + 448*_t**3*a**3*b**3*c**4*d**9
+ 160*_t**2*a**3*b**2*c**3*d**6 - 32*_t**2*a**2*b**3*c**7*d**6 + 60*_t*a**3
*b*c**2*d**3 + 256*_t*a**2*b**2*c**6*d**3 + 4*_t*a*b**3*c**10*d**3 - 5*a**3
*c - 9*a**2*b*c**5 - 3*a*b**2*c**9 + b**3*c**13)/(a**3*d - 33*a**2*b*c**4*d
 - 33*a*b**2*c**8*d + b**3*c**12*d))))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(dx+c)^4 b + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*(d*x+c)^4),x, algorithm="giac")
```

```
[Out] integrate(x^2/((d*x + c)^4*b + a), x)
```

3.112 $\int \frac{x}{a+b(c+dx)^4}\,dx$

Optimal. Leaf size=261

$$\frac{c\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}d^2}-\frac{c\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}d^2}+\frac{c\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+d}{\sqrt[4]{a}}\right.}{2\sqrt{2}a^{3/4}\sqrt[4]{b}d^2}$$

```
[Out] ArcTan[(Sqrt[b]*(c + d*x)^2)/Sqrt[a]]/(2*Sqrt[a]*Sqrt[b]*d^2) + (c*ArcTan[1
 - (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(1/4)*d^2) -
(c*ArcTan[1 + (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(1
/4)*d^2) + (c*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c
+ d*x)^2])/(4*Sqrt[2]*a^(3/4)*b^(1/4)*d^2) - (c*Log[Sqrt[a] + Sqrt[2]*a^(1/
4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*x)^2])/(4*Sqrt[2]*a^(3/4)*b^(1/4)*d^2
)
```

Rubi [A] time = 0.26372, antiderivative size = 261, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {371, 1876, 211, 1165, 628, 1162, 617, 204, 275, 205}

$$\frac{c\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}d^2}-\frac{c\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}d^2}+\frac{c\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+d}{\sqrt[4]{a}}\right.}{2\sqrt{2}a^{3/4}\sqrt[4]{b}d^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*(c + d*x)^4),x]
```

```
[Out] ArcTan[(Sqrt[b]*(c + d*x)^2)/Sqrt[a]]/(2*Sqrt[a]*Sqrt[b]*d^2) + (c*ArcTan[1
 - (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(1/4)*d^2) -
(c*ArcTan[1 + (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(1
/4)*d^2) + (c*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c
+ d*x)^2])/(4*Sqrt[2]*a^(3/4)*b^(1/4)*d^2) - (c*Log[Sqrt[a] + Sqrt[2]*a^(1/
4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*x)^2])/(4*Sqrt[2]*a^(3/4)*b^(1/4)*d^2
)
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
```

```
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x}{a+b(c+dx)^4}\,dx = \frac{\text{Subst}\left(\int \frac{-c+x}{a+bx^4}\,dx, x, c+dx\right)}{d^2}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{c}{a+bx^4}+\frac{x}{a+bx^4}\right)dx, x, c+dx\right)}{d^2}$$

$$= \frac{\text{Subst}\left(\int \frac{x}{a+bx^4}\,dx, x, c+dx\right)}{d^2} - \frac{c\,\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, c+dx\right)}{d^2}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, (c+dx)^2\right)}{2d^2} - \frac{c\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx, x, c+dx\right)}{2\sqrt{a}d^2} - \frac{c\,\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx, x\right.}{2\sqrt{a}d^2}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}d^2} - \frac{c\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, c+dx\right)}{4\sqrt{a}\sqrt{b}d^2} - \frac{c\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, c+\right.}{4\sqrt{a}\sqrt{b}d^2}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}d^2} + \frac{c\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}d^2} - \frac{c\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+\right.}{4\sqrt{2}a^{3/4}\sqrt[4]{b}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}d^2} + \frac{c\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}d^2} - \frac{c\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}d^2} + \frac{c\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\right.}{4\sqrt{2}a}$$

Mathematica [C] time = 0.0272071, size = 104, normalized size = 0.4

$$\frac{\text{RootSum}\left[6\#1^2bc^2d^2+4\#1^3bcd^3+\#1^4bd^4+4\#1bc^3d+a+bc^4\&,\frac{\#1\log(x-\#1)}{3\#1^2cd^2+\#1^3d^3+3\#1c^2d+c^3}\&\right]}{4bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*(c + d*x)^4),x]
```

```
[Out] RootSum[a + b*c^4 + 4*b*c^3*d*#1 + 6*b*c^2*d^2*#1^2 + 4*b*c*d^3*#1^3 + b*d^
4*#1^4 & , (Log[x - #1]*#1)/(c^3 + 3*c^2*d*#1 + 3*c*d^2*#1^2 + d^3*#1^3) &
]/(4*b*d)
```

Maple [C] time = 0.003, size = 95, normalized size = 0.4

$$\frac{1}{4\,bd}\sum_{\_R=\text{RootOf}\left(\_Z^4bd^4+4\_Z^3bcd^3+6\_Z^2bc^2d^2+4\_Z\,bc^3d+bc^4+a\right)}\frac{\_R\ln(x-\_R)}{d^3\_R^3+3\,cd^2\_R^2+3\_R\,c^2d+c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*(d*x+c)^4),x)
```

```
[Out] 1/4/b/d*sum(_R/(_R^3*d^3+3*_R^2*c*d^2+3*_R*c^2*d+c^3)*ln(x-_R),_R=RootOf(_Z
^4*b*d^4+4*_Z^3*b*c*d^3+6*_Z^2*b*c^2*d^2+4*_Z*b*c^3*d+b*c^4+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x}{(dx+c)^4b+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] integrate(x/((d*x + c)^4*b + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 0.901808, size = 131, normalized size = 0.5

$$\mathrm{RootSum}\left(256t^4a^3b^2d^8 + 32t^2a^2bd^4 - 16tabc^2d^2 + a + bc^4, \left(t \mapsto t\log\left(x + \frac{128t^3a^3bd^6 + 16t^2a^2bc^2d^4 + 8ta^2d^2 + 4tabc}{4acd - bc^5d}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(d*x+c)**4),x)
```

```
[Out] RootSum(256*_t**4*a**3*b**2*d**8 + 32*_t**2*a**2*b*d**4 - 16*_t*a*b*c**2*d*
*2 + a + b*c**4, Lambda(_t, _t*log(x + (128*_t**3*a**3*b*d**6 + 16*_t**2*a*
*2*b*c**2*d**4 + 8*_t*a**2*d**2 + 4*_t*a*b*c**4*d**2 - a*c**2 - b*c**6)/(4*
a*c*d - b*c**5*d))))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(dx+c)^4b+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(d*x+c)^4),x, algorithm="giac")
```

```
[Out] integrate(x/((d*x + c)^4*b + a), x)
```

3.113 $\int \frac{1}{a+b(c+dx)^4}\,dx$

Optimal. Leaf size=221

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}d}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}d}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}d}$$

```
[Out] -ArcTan[1 - (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)]/(2*Sqrt[2]*a^(3/4)*b^(1/4)
*d) + ArcTan[1 + (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)]/(2*Sqrt[2]*a^(3/4)*b^
(1/4)*d) - Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d
*x)^2]/(4*Sqrt[2]*a^(3/4)*b^(1/4)*d) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4
)*(c + d*x) + Sqrt[b]*(c + d*x)^2]/(4*Sqrt[2]*a^(3/4)*b^(1/4)*d)
```

Rubi [A] time = 0.185432, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {247, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}d}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}d}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*(c + d*x)^4)^(-1),x]
```

```
[Out] -ArcTan[1 - (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)]/(2*Sqrt[2]*a^(3/4)*b^(1/4)
*d) + ArcTan[1 + (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)]/(2*Sqrt[2]*a^(3/4)*b^
(1/4)*d) - Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d
*x)^2]/(4*Sqrt[2]*a^(3/4)*b^(1/4)*d) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4
)*(c + d*x) + Sqrt[b]*(c + d*x)^2]/(4*Sqrt[2]*a^(3/4)*b^(1/4)*d)
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{a+b(c+dx)^4}\,dx = \frac{\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx, x, c+dx\right)}{2\sqrt{a}d} + \frac{\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx, x, c+dx\right)}{2\sqrt{a}d}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, c+dx\right)}{4\sqrt{a}\sqrt{b}d} + \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, c+dx\right)}{4\sqrt{a}\sqrt{b}d} - \frac{\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{}}{\sqrt[4]{b}}}\right.}{4\sqrt{2}a^3}$$

$$= -\frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}d} + \frac{\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}d} + \frac{\text{S}}{}$$

$$= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}d} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}d} - \frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}d}$$

Mathematica [A] time = 0.0709959, size = 161, normalized size = 0.73

$$\frac{-\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)+\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)-2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*(c + d*x)^4)^(-1),x]
```

```
[Out] (-2*ArcTan[1 - (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)] + 2*ArcTan[1 + (Sqrt[2]
*b^(1/4)*(c + d*x))/a^(1/4)] - Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*(c + d
*x) + Sqrt[b]*(c + d*x)^2] + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x
) + Sqrt[b]*(c + d*x)^2])/(4*Sqrt[2]*a^(3/4)*b^(1/4)*d)
```

Maple [C] time = 0.003, size = 94, normalized size = 0.4

$$\frac{1}{4\,bd}\sum_{\_R=\text{RootOf}\left(\_Z^4bd^4+4\_Z^3bcd^3+6\_Z^2bc^2d^2+4\_Z\,bc^3d+bc^4+a\right)}\frac{\ln(x-\_R)}{d^3\_R^3+3\,cd^2\_R^2+3\_R\,c^2d+c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*(d*x+c)^4),x)
```

```
[Out] 1/4/b/d*sum(1/(_R^3*d^3+3*_R^2*c*d^2+3*_R*c^2*d+c^3)*ln(x-_R),_R=RootOf(_Z^
4*b*d^4+4*_Z^3*b*c*d^3+6*_Z^2*b*c^2*d^2+4*_Z*b*c^3*d+b*c^4+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx+c)^4 b + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x + c)^4*b + a), x)
```

Fricas [A] time = 1.55748, size = 443, normalized size = 2.

$$\left(-\frac{1}{a^3bd^4}\right)^{\frac{1}{4}} \arctan\left(a^2bd^4\sqrt{\frac{a^2d^2\sqrt{-\frac{1}{a^3bd^4}} + d^2x^2 + 2\,cdx + c^2}{d^2}}\left(-\frac{1}{a^3bd^4}\right)^{\frac{3}{4}} - \left(a^2bd^4x + a^2bcd^3\right)\left(-\frac{1}{a^3bd^4}\right)^{\frac{3}{4}}\right) + \frac{1}{4}\left(-\frac{1}{a^3bd^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] (-1/(a^3*b*d^4))^(1/4)*arctan(a^2*b*d^4*sqrt((a^2*d^2*sqrt(-1/(a^3*b*d^4))
+ d^2*x^2 + 2*c*d*x + c^2)/d^2)*(-1/(a^3*b*d^4))^(3/4) - (a^2*b*d^4*x + a^2
*b*c*d^3)*(-1/(a^3*b*d^4))^(3/4)) + 1/4*(-1/(a^3*b*d^4))^(1/4)*log(a*d*(-1/
(a^3*b*d^4))^(1/4) + d*x + c) - 1/4*(-1/(a^3*b*d^4))^(1/4)*log(-a*d*(-1/(a^
3*b*d^4))^(1/4) + d*x + c)
```

Sympy [A] time = 0.284529, size = 26, normalized size = 0.12

$$\frac{\operatorname{RootSum}\left(256t^4a^3b + 1, \left(t \mapsto t\log\left(x + \frac{4ta+c}{d}\right)\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)**4),x)
```

```
[Out] RootSum(256*_t**4*a**3*b + 1, Lambda(_t, _t*log(x + (4*_t*a + c)/d)))/d
```

Giac [A] time = 1.13681, size = 196, normalized size = 0.89

$$\frac{1}{4}\,i\left(-\frac{1}{a^3bd^4}\right)^{\frac{1}{4}}\log\left(bdix+bci-\left(-ab^3\right)^{\frac{1}{4}}\right)-\frac{1}{4}\,i\left(-\frac{1}{a^3bd^4}\right)^{\frac{1}{4}}\log\left(-bdix-bci-\left(-ab^3\right)^{\frac{1}{4}}\right)+\frac{1}{4}\left(-\frac{1}{a^3bd^4}\right)^{\frac{1}{4}}\log\left(\left|bdx+bc+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^4),x, algorithm="giac")
```

```
[Out] 1/4*i*(-1/(a^3*b*d^4))^(1/4)*log(b*d*i*x + b*c*i - (-a*b^3)^(1/4)) - 1/4*i*
(-1/(a^3*b*d^4))^(1/4)*log(-b*d*i*x - b*c*i - (-a*b^3)^(1/4)) + 1/4*(-1/(a^
3*b*d^4))^(1/4)*log(abs(b*d*x + b*c + (-a*b^3)^(1/4))) - 1/4*(-1/(a^3*b*d^4
))^(1/4)*log(abs(-b*d*x - b*c + (-a*b^3)^(1/4)))
```

3.114 $\int \frac{1}{x\left(a+b(c+dx)^4\right)}\,dx$

Optimal. Leaf size=393

$$-\frac{\sqrt[4]{b}c\left(\sqrt{a}-\sqrt{b}c^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\left(a+bc^4\right)}+\frac{\sqrt[4]{b}c\left(\sqrt{a}-\sqrt{b}c^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}\right.}{4\sqrt{2}a^{3/4}\left(a+bc^4\right)}$$

```
[Out] -(Sqrt[b]*c^2*ArcTan[(Sqrt[b]*(c + d*x)^2)/Sqrt[a]])/(2*Sqrt[a]*(a + b*c^4)
) + (b^(1/4)*c*(Sqrt[a] + Sqrt[b]*c^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*(c + d*x
))/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(a + b*c^4)) - (b^(1/4)*c*(Sqrt[a] + Sqrt[b
]*c^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*
(a + b*c^4)) + Log[x]/(a + b*c^4) - (b^(1/4)*c*(Sqrt[a] - Sqrt[b]*c^2)*Log[
Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*x)^2])/(4*Sqrt
[2]*a^(3/4)*(a + b*c^4)) + (b^(1/4)*c*(Sqrt[a] - Sqrt[b]*c^2)*Log[Sqrt[a] +
 Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*x)^2])/(4*Sqrt[2]*a^(3/
4)*(a + b*c^4)) - Log[a + b*(c + d*x)^4]/(4*(a + b*c^4))
```

Rubi [A] time = 0.466281, antiderivative size = 393, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 13, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.765, Rules used = {371, 6725, 1876, 1248, 635, 205, 260, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{\sqrt[4]{b}c\left(\sqrt{a}-\sqrt{b}c^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\left(a+bc^4\right)}+\frac{\sqrt[4]{b}c\left(\sqrt{a}-\sqrt{b}c^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}\right.}{4\sqrt{2}a^{3/4}\left(a+bc^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*(c + d*x)^4)),x]
```

```
[Out] -(Sqrt[b]*c^2*ArcTan[(Sqrt[b]*(c + d*x)^2)/Sqrt[a]])/(2*Sqrt[a]*(a + b*c^4)
) + (b^(1/4)*c*(Sqrt[a] + Sqrt[b]*c^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*(c + d*x
))/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(a + b*c^4)) - (b^(1/4)*c*(Sqrt[a] + Sqrt[b
]*c^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*
(a + b*c^4)) + Log[x]/(a + b*c^4) - (b^(1/4)*c*(Sqrt[a] - Sqrt[b]*c^2)*Log[
Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*x)^2])/(4*Sqrt
[2]*a^(3/4)*(a + b*c^4)) + (b^(1/4)*c*(Sqrt[a] - Sqrt[b]*c^2)*Log[Sqrt[a] +
 Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*x)^2])/(4*Sqrt[2]*a^(3/
4)*(a + b*c^4)) - Log[a + b*(c + d*x)^4]/(4*(a + b*c^4))
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{x\left(a+b(c+dx)^4\right)}\,dx = \text{Subst}\left(\int \frac{1}{(-c+x)\left(a+bx^4\right)}\,dx, x, c+dx\right)$$

$$= \text{Subst}\left(\int \left(-\frac{1}{\left(a+bc^4\right)(c-x)} - \frac{b\left(c^3+c^2x+cx^2+x^3\right)}{\left(a+bc^4\right)\left(a+bx^4\right)}\right)dx, x, c+dx\right)$$

$$= \frac{\log(x)}{a+bc^4} - \frac{b\,\text{Subst}\left(\int \frac{c^3+c^2x+cx^2+x^3}{a+bx^4}\,dx, x, c+dx\right)}{a+bc^4}$$

$$= \frac{\log(x)}{a+bc^4} - \frac{b\,\text{Subst}\left(\int \left(\frac{x\left(c^2+x^2\right)}{a+bx^4} + \frac{c^3+cx^2}{a+bx^4}\right)dx, x, c+dx\right)}{a+bc^4}$$

$$= \frac{\log(x)}{a+bc^4} - \frac{b\,\text{Subst}\left(\int \frac{x\left(c^2+x^2\right)}{a+bx^4}\,dx, x, c+dx\right)}{a+bc^4} - \frac{b\,\text{Subst}\left(\int \frac{c^3+cx^2}{a+bx^4}\,dx, x, c+dx\right)}{a+bc^4}$$

$$= \frac{\log(x)}{a+bc^4} - \frac{b\,\text{Subst}\left(\int \frac{c^2+x}{a+bx^2}\,dx, x, (c+dx)^2\right)}{2\left(a+bc^4\right)} + \frac{\left(c\left(1-\frac{\sqrt{b}c^2}{\sqrt{a}}\right)\right)\text{Subst}\left(\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}\,dx, x, c+dx\right)}{2\left(a+bc^4\right)}$$

$$= \frac{\log(x)}{a+bc^4} - \frac{b\,\text{Subst}\left(\int \frac{x}{a+bx^2}\,dx, x, (c+dx)^2\right)}{2\left(a+bc^4\right)} - \frac{\left(bc^2\right)\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, (c+dx)^2\right)}{2\left(a+bc^4\right)} - \frac{\left(\sqrt[4]{b}c\left(\cdots\right.\right.}{\cdots}$$

$$= -\frac{\sqrt{b}c^2\tan^{-1}\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{2\sqrt{a}\left(a+bc^4\right)} + \frac{\log(x)}{a+bc^4} - \frac{\sqrt[4]{b}c\left(\sqrt{a}-\sqrt{b}c^2\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{b}(c+\cdots\right.}{4\sqrt{2}a^{3/4}\left(a+bc^4\right)}$$

$$= -\frac{\sqrt{b}c^2\tan^{-1}\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{2\sqrt{a}\left(a+bc^4\right)} + \frac{\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{b}c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(a+bc^4\right)} - \frac{\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{b}c^2\right)\tan^{-}\cdots}{2\sqrt{2}a^{3/4}\left(a\cdots\right.}$$

Mathematica [C] time = 0.0632193, size = 163, normalized size = 0.41

$$-\frac{\text{RootSum}\left[6\#1^2bc^2d^2+4\#1^3bcd^3+\#1^4bd^4+4\#1bc^3d+a+bc^4\&, \frac{4\#1^2cd^2\log(x-\#1)+\#1^3d^3\log(x-\#1)+6\#1c^2d\log(x-\#1)+4c^3\log(x-\cdots}{3\#1^2cd^2+\#1^3d^3+3\#1c^2d+c^3}\right.}{4\left(a+bc^4\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*(c + d*x)^4)),x]
```

```
[Out] -(-4*Log[x] + RootSum[a + b*c^4 + 4*b*c^3*d*#1 + 6*b*c^2*d^2*#1^2 + 4*b*c*d
^3*#1^3 + b*d^4*#1^4 & , (4*c^3*Log[x - #1] + 6*c^2*d*Log[x - #1]*#1 + 4*c*
```

```
d^2*Log[x - #1]*#1^2 + d^3*Log[x - #1]*#1^3)/(c^3 + 3*c^2*d*#1 + 3*c*d^2*#1
^2 + d^3*#1^3) & ])/(4*(a + b*c^4))
```

Maple [C] time = 0.008, size = 139, normalized size = 0.4

$$\frac{\ln(x)}{bc^4+a}-\frac{1}{4\,bc^4+4\,a}\sum_{\_R=\mathrm{RootOf}\left(\_Z^4bd^4+4\_Z^3bcd^3+6\_Z^2bc^2d^2+4\_Z\,bc^3d+bc^4+a\right)}\frac{\left(\_R^3d^3+4\_R^2cd^2+6\_R\,c^2d+4\,c^3\right)\ln(x-\_R)}{\_R^3d^3+3\_R^2cd^2+3\_R\,c^2d+c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b*(d*x+c)^4),x)
```

```
[Out] ln(x)/(b*c^4+a)-1/4/(b*c^4+a)*sum((_R^3*d^3+4*_R^2*c*d^2+6*_R*c^2*d+4*c^3)/
(_R^3*d^3+3*_R^2*c*d^2+3*_R*c^2*d+c^3)*ln(x-_R),_R=RootOf(_Z^4*b*d^4+4*_Z^3
*b*c*d^3+6*_Z^2*b*c^2*d^2+4*_Z*b*c^3*d+b*c^4+a))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] Exception raised: AttributeError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left((dx+c)^4b+a\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(d*x+c)^4),x, algorithm="giac")
```

```
[Out] integrate(1/(((d*x + c)^4*b + a)*x), x)
```

3.115 $\int \frac{1}{x^2\left(a+b(c+dx)^4\right)}\,dx$

Optimal. Leaf size=496

$$-\frac{\sqrt[4]{b}d\left(\sqrt{a}\left(a-3bc^4\right)-\sqrt{b}c^2\left(3a-bc^4\right)\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\left(a+bc^4\right)^2}+\frac{\sqrt[4]{b}d\left(\sqrt{a}\left(a-3bc^4\right)-\sqrt{b}c^2\left(3\right.\right.}{}$$

```
[Out] -(1/((a + b*c^4)*x)) - (Sqrt[b]*c*(a - b*c^4)*d*ArcTan[(Sqrt[b]*(c + d*x)^2
)/Sqrt[a]])/(Sqrt[a]*(a + b*c^4)^2) + (b^(1/4)*(Sqrt[a]*(a - 3*b*c^4) + Sqr
t[b]*c^2*(3*a - b*c^4))*d*ArcTan[1 - (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)])/
(2*Sqrt[2]*a^(3/4)*(a + b*c^4)^2) - (b^(1/4)*(Sqrt[a]*(a - 3*b*c^4) + Sqrt[
b]*c^2*(3*a - b*c^4))*d*ArcTan[1 + (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)])/(2
*Sqrt[2]*a^(3/4)*(a + b*c^4)^2) - (4*b*c^3*d*Log[x])/(a + b*c^4)^2 - (b^(1/
4)*(Sqrt[a]*(a - 3*b*c^4) - Sqrt[b]*c^2*(3*a - b*c^4))*d*Log[Sqrt[a] - Sqrt
[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*x)^2])/(4*Sqrt[2]*a^(3/4)*(a
 + b*c^4)^2) + (b^(1/4)*(Sqrt[a]*(a - 3*b*c^4) - Sqrt[b]*c^2*(3*a - b*c^4))
*d*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*x)^2])/
(4*Sqrt[2]*a^(3/4)*(a + b*c^4)^2) + (b*c^3*d*Log[a + b*(c + d*x)^4])/(a + b
*c^4)^2
```

Rubi [A] time = 0.894138, antiderivative size = 496, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 13, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.765, Rules used = {371, 6725, 1876, 1248, 635, 205, 260, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{\sqrt[4]{b}d\left(\sqrt{a}\left(a-3bc^4\right)-\sqrt{b}c^2\left(3a-bc^4\right)\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\left(a+bc^4\right)^2}+\frac{\sqrt[4]{b}d\left(\sqrt{a}\left(a-3bc^4\right)-\sqrt{b}c^2\left(3\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*(c + d*x)^4)),x]
```


```
[Out] -(1/((a + b*c^4)*x)) - (Sqrt[b]*c*(a - b*c^4)*d*ArcTan[(Sqrt[b]*(c + d*x)^2
)/Sqrt[a]])/(Sqrt[a]*(a + b*c^4)^2) + (b^(1/4)*(Sqrt[a]*(a - 3*b*c^4) + Sqr
t[b]*c^2*(3*a - b*c^4))*d*ArcTan[1 - (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)])/
(2*Sqrt[2]*a^(3/4)*(a + b*c^4)^2) - (b^(1/4)*(Sqrt[a]*(a - 3*b*c^4) + Sqrt[
b]*c^2*(3*a - b*c^4))*d*ArcTan[1 + (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)])/(2
*Sqrt[2]*a^(3/4)*(a + b*c^4)^2) - (4*b*c^3*d*Log[x])/(a + b*c^4)^2 - (b^(1/
4)*(Sqrt[a]*(a - 3*b*c^4) - Sqrt[b]*c^2*(3*a - b*c^4))*d*Log[Sqrt[a] - Sqrt
```

```
[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*x)^2])/(4*Sqrt[2]*a^(3/4)*(a
 + b*c^4)^2) + (b^(1/4)*(Sqrt[a]*(a - 3*b*c^4) - Sqrt[b]*c^2*(3*a - b*c^4))
*d*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*x)^2])/
(4*Sqrt[2]*a^(3/4)*(a + b*c^4)^2) + (b*c^3*d*Log[a + b*(c + d*x)^4])/(a + b
*c^4)^2
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+b(c+dx)^4\right)}\,dx = d\,\mathrm{Subst}\left(\int \frac{1}{(-c+x)^2\left(a+bx^4\right)}\,dx, x, c+dx\right)$$

$$= d\,\mathrm{Subst}\left(\int\left(\frac{1}{\left(a+bc^4\right)(c-x)^2}+\frac{4bc^3}{\left(a+bc^4\right)^2(c-x)}+\frac{b\left(-c^2\left(3a-bc^4\right)-2c\left(a-bc^4\right)x-\left(a\right.\right.}{\left(a+bc^4\right)^2\left(a+bx^4\right.}\right.\right.$$

$$= -\frac{1}{\left(a+bc^4\right)x}-\frac{4bc^3d\log(x)}{\left(a+bc^4\right)^2}+\frac{(bd)\,\mathrm{Subst}\left(\int\frac{-c^2\left(3a-bc^4\right)-2c\left(a-bc^4\right)x-\left(a-3bc^4\right)x^2+4bc^3x^3}{a+bx^4}\,dx, x, c+d\right.}{\left(a+bc^4\right)^2}$$

$$= -\frac{1}{\left(a+bc^4\right)x}-\frac{4bc^3d\log(x)}{\left(a+bc^4\right)^2}+\frac{(bd)\,\mathrm{Subst}\left(\int\left(\frac{x\left(-2c\left(a-bc^4\right)+4bc^3x^2\right)}{a+bx^4}+\frac{-c^2\left(3a-bc^4\right)+\left(-a+3bc^4\right)x^2}{a+bx^4}\right)dx\right.}{\left(a+bc^4\right)^2}$$

$$= -\frac{1}{\left(a+bc^4\right)x}-\frac{4bc^3d\log(x)}{\left(a+bc^4\right)^2}+\frac{(bd)\,\mathrm{Subst}\left(\int\frac{x\left(-2c\left(a-bc^4\right)+4bc^3x^2\right)}{a+bx^4}\,dx, x, c+dx\right)}{\left(a+bc^4\right)^2}+\frac{(bd)\,\mathrm{Subst}\left(\right.}{}$$

$$= -\frac{1}{\left(a+bc^4\right)x}-\frac{4bc^3d\log(x)}{\left(a+bc^4\right)^2}+\frac{(bd)\,\mathrm{Subst}\left(\int\frac{-2c\left(a-bc^4\right)+4bc^3x}{a+bx^2}\,dx, x, (c+dx)^2\right)}{2\left(a+bc^4\right)^2}+\frac{\left(\left(a-3bc^4-\right.\right.}{}$$

$$= -\frac{1}{\left(a+bc^4\right)x}-\frac{4bc^3d\log(x)}{\left(a+bc^4\right)^2}+\frac{\left(2b^2c^3d\right)\mathrm{Subst}\left(\int\frac{x}{a+bx^2}\,dx, x, (c+dx)^2\right)}{\left(a+bc^4\right)^2}-\frac{\left(bc\left(a-bc^4\right)d\right)\mathrm{Su}}{}$$

$$= -\frac{1}{\left(a+bc^4\right)x}-\frac{\sqrt{b}c\left(a-bc^4\right)d\tan^{-1}\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{\sqrt{a}\left(a+bc^4\right)^2}-\frac{4bc^3d\log(x)}{\left(a+bc^4\right)^2}-\frac{\sqrt[4]{b}\left(a-3bc^4-\frac{\sqrt{b}c^2\left(3a-bc^4\right)}{\sqrt{a}}\right.}{}$$

$$= -\frac{1}{\left(a+bc^4\right)x}-\frac{\sqrt{b}c\left(a-bc^4\right)d\tan^{-1}\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{\sqrt{a}\left(a+bc^4\right)^2}+\frac{\sqrt[4]{b}\left(a-3bc^4+\frac{\sqrt{b}c^2\left(3a-bc^4\right)}{\sqrt{a}}\right)d\tan^{-1}\left(1-\right.}{2\sqrt{2}\sqrt[4]{a}\left(a+bc^4\right)^2}$$

Mathematica [C] time = 0.124412, size = 238, normalized size = 0.48

$$\frac{dx\mathrm{RootSum}\left[6\#1^2bc^2d^2+4\#1^3bcd^3+\#1^4bd^4+4\#1bc^3d+a+bc^4\&,\frac{-\#1^2ad^2\log(x-\#1)+15\#1^2bc^4d^2\log(x-\#1)+4\#1^3bc^3d^3\log(x-\#1)-}{3\#1^2cd^2+}\right.}{4x\left(a+bc^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*(c + d*x)^4)),x]
```

```
[Out] (-4*(a + b*c^4 + 4*b*c^3*d*x*Log[x]) + d*x*RootSum[a + b*c^4 + 4*b*c^3*d*#1
 + 6*b*c^2*d^2*#1^2 + 4*b*c*d^3*#1^3 + b*d^4*#1^4 & , (-6*a*c^2*Log[x - #1]
 + 10*b*c^6*Log[x - #1] - 4*a*c*d*Log[x - #1]*#1 + 20*b*c^5*d*Log[x - #1]*#
1 - a*d^2*Log[x - #1]*#1^2 + 15*b*c^4*d^2*Log[x - #1]*#1^2 + 4*b*c^3*d^3*Lo
g[x - #1]*#1^3)/(c^3 + 3*c^2*d*#1 + 3*c*d^2*#1^2 + d^3*#1^3) & ])/(4*(a + b
*c^4)^2*x)
```

Maple [C] time = 0.01, size = 188, normalized size = 0.4

$$-\frac{1}{\left(bc^4+a\right)x}-4\,\frac{bc^3d\ln\left(x\right)}{\left(bc^4+a\right)^2}+\frac{d}{4\,\left(bc^4+a\right)^2}\sum_{\_R=\mathrm{RootOf}\left(\_Z^4bd^4+4\_Z^3bcd^3+6\_Z^2bc^2d^2+4\_Z\,bc^3d+bc^4+a\right)}\frac{\left(4\,bd^3c^3\_R^3+d^2\left(15\,bc^4\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+b*(d*x+c)^4),x)
```

```
[Out] -1/(b*c^4+a)/x-4*b*c^3*d*ln(x)/(b*c^4+a)^2+1/4*d/(b*c^4+a)^2*sum((4*b*d^3*c
^3*_R^3+d^2*(15*b*c^4-a)*_R^2+4*c*d*(5*b*c^4-a)*_R+10*b*c^6-6*c^2*a)/(_R^3*
d^3+3*_R^2*c*d^2+3*_R*c^2*d+c^3)*ln(x-_R),_R=RootOf(_Z^4*b*d^4+4*_Z^3*b*c*d
^3+6*_Z^2*b*c^2*d^2+4*_Z*b*c^3*d+b*c^4+a))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] Exception raised: AttributeError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left((dx+c)^4 b+a\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*(d*x+c)^4),x, algorithm="giac")
```

```
[Out] integrate(1/(((d*x + c)^4*b + a)*x^2), x)
```

3.116 $\int \left(a+8x-8x^2+4x^3-x^4\right)^4 \, dx$

Optimal. Leaf size=123

$$-\frac{2}{9}\left(-3a^2+6a+37\right)(x-1)^9+\frac{4}{13}(3-a)(x-1)^{13}-\frac{8}{11}(3a+5)(x-1)^{11}+\frac{8}{7}(a+3)(3a+5)(x-1)^7+\frac{4}{5}(3-a)(a+3)^2($$

```
[Out] (-8*(3 + a)^3*(-1 + x)^3)/3 + (4*(3 - a)*(3 + a)^2*(-1 + x)^5)/5 + (8*(3 +
a)*(5 + 3*a)*(-1 + x)^7)/7 - (2*(37 + 6*a - 3*a^2)*(-1 + x)^9)/9 - (8*(5 +
3*a)*(-1 + x)^11)/11 + (4*(3 - a)*(-1 + x)^13)/13 + (8*(-1 + x)^15)/15 + (-
1 + x)^17/17 + (3 + a)^4*x
```

Rubi [A] time = 0.239745, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1106, 1090}

$$-\frac{2}{9}\left(-3a^2+6a+37\right)(x-1)^9+\frac{4}{13}(3-a)(x-1)^{13}-\frac{8}{11}(3a+5)(x-1)^{11}+\frac{8}{7}(a+3)(3a+5)(x-1)^7+\frac{4}{5}(3-a)(a+3)^2($$

Antiderivative was successfully verified.

```
[In] Int[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^4,x]
```


```
[Out] (-8*(3 + a)^3*(-1 + x)^3)/3 + (4*(3 - a)*(3 + a)^2*(-1 + x)^5)/5 + (8*(3 +
a)*(5 + 3*a)*(-1 + x)^7)/7 - (2*(37 + 6*a - 3*a^2)*(-1 + x)^9)/9 - (8*(5 +
3*a)*(-1 + x)^11)/11 + (4*(3 - a)*(-1 + x)^13)/13 + (8*(-1 + x)^15)/15 + (-
1 + x)^17/17 + (3 + a)^4*x
```


Rule 1106

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1090

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandInteg
rand[(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int\left(a+8x-8x^2+4x^3-x^4\right)^4dx=\text{Subst}\left(\int\left(3+a-2x^2-x^4\right)^4dx,x,-1+x\right)$$
$$=\text{Subst}\left(\int\left(81\left(1+\frac{1}{81}a\left(108+54a+12a^2+a^3\right)\right)-216\left(1+a\left(1+\frac{1}{27}a(9+a)\right)\right)x^2\right.\right.$$
$$=-\frac{8}{3}(3+a)^3(-1+x)^3+\frac{4}{5}(3-a)(3+a)^2(-1+x)^5+\frac{8}{7}(3+a)(5+3a)(-1+x)^7-\frac{2}{9}\left(3\right.$$

Mathematica [A] time = 0.0280276, size = 195, normalized size = 1.59

$$\frac{2}{9}\left(3a^2-1536a+20480\right)x^9-6\left(a^2-128a+896\right)x^8+\frac{64}{7}\left(3a^2-140a+512\right)x^7-\frac{16}{3}\left(15a^2-288a+512\right)x^6-\frac{4}{5}\left(a^3-\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^4,x]
```

```
[Out] a^4*x + 16*a^3*x^2 - (32*(-12 + a)*a^2*x^3)/3 + 4*a*(128 - 48*a + a^2)*x^4
- (4*(-1024 + 1536*a - 192*a^2 + a^3)*x^5)/5 - (16*(512 - 288*a + 15*a^2)*x
^6)/3 + (64*(512 - 140*a + 3*a^2)*x^7)/7 - 6*(896 - 128*a + a^2)*x^8 + (2*(
20480 - 1536*a + 3*a^2)*x^9)/9 + (16*(-928 + 35*a)*x^10)/5 - (32*(-524 + 9*
a)*x^11)/11 + (4*(-464 + 3*a)*x^12)/3 - (4*(-640 + a)*x^13)/13 - 48*x^14 +
(128*x^15)/15 - x^16 + x^17/17
```

Maple [B] time = 0.002, size = 264, normalized size = 2.2

$$\frac{x^{17}}{17}-x^{16}+\frac{128\,x^{15}}{15}-48\,x^{14}+\frac{(-4\,a+2560)\,x^{13}}{13}+\frac{(48\,a-7424)\,x^{12}}{12}+\frac{(-288\,a+16768)\,x^{11}}{11}+\frac{(1120\,a-29696)\,x^{10}}{10}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+4*x^3-8*x^2+a+8*x)^4,x)
```

```
[Out] 1/17*x^17-x^16+128/15*x^15-48*x^14+1/13*(-4*a+2560)*x^13+1/12*(48*a-7424)*x
^12+1/11*(-288*a+16768)*x^11+1/10*(1120*a-29696)*x^10+1/9*(2*a^2-2560*a+245
76+(-2*a+128)^2)*x^9+1/8*(-16*a^2+3584*a-10240+2*(8*a-128)*(-2*a+128))*x^8+
1/7*(64*a^2-2560*a+2*(-16*a+64)*(-2*a+128)+(8*a-128)^2)*x^7+1/6*(-160*a^2+3
2*a*(-2*a+128)+2*(-16*a+64)*(8*a-128))*x^6+1/5*(2*a^2*(-2*a+128)+32*a*(8*a-
```

```
128)+(-16*a+64)^2)*x^5+1/4*(2*a^2*(8*a-128)+32*a*(-16*a+64))*x^4+1/3*(2*a^2
*(-16*a+64)+256*a^2)*x^3+16*a^3*x^2+a^4*x
```

Maxima [A] time = 1.1306, size = 259, normalized size = 2.11

$$\frac{1}{17}x^{17} - x^{16} + \frac{128}{15}x^{15} - 48x^{14} + \frac{2560}{13}x^{13} - \frac{1856}{3}x^{12} + \frac{16768}{11}x^{11} - \frac{14848}{5}x^{10} + \frac{40960}{9}x^{9} - 5376x^{8} + \frac{32768}{7}x^{7} - $$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+4*x^3-8*x^2+a+8*x)^4,x, algorithm="maxima")
```

```
[Out] 1/17*x^17 - x^16 + 128/15*x^15 - 48*x^14 + 2560/13*x^13 - 1856/3*x^12 + 167
68/11*x^11 - 14848/5*x^10 + 40960/9*x^9 - 5376*x^8 + 32768/7*x^7 - 8192/3*x
^6 + a^4*x + 4096/5*x^5 - 4/15*(3*x^5 - 15*x^4 + 40*x^3 - 60*x^2)*a^3 + 2/1
05*(35*x^9 - 315*x^8 + 1440*x^7 - 4200*x^6 + 8064*x^5 - 10080*x^4 + 6720*x^
3)*a^2 - 4/2145*(165*x^13 - 2145*x^12 + 14040*x^11 - 60060*x^10 + 183040*x^
9 - 411840*x^8 + 686400*x^7 - 823680*x^6 + 658944*x^5 - 274560*x^4)*a
```

Fricas [B] time = 1.26237, size = 624, normalized size = 5.07

$$\frac{1}{17}x^{17} - x^{16} + \frac{128}{15}x^{15} - 48x^{14} - \frac{4}{13}x^{13}a + \frac{2560}{13}x^{13} + 4x^{12}a - \frac{1856}{3}x^{12} - \frac{288}{11}x^{11}a + \frac{16768}{11}x^{11} + 112x^{10}a + \frac{2}{3}x^{9}a^{2} - $$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+4*x^3-8*x^2+a+8*x)^4,x, algorithm="fricas")
```

```
[Out] 1/17*x^17 - x^16 + 128/15*x^15 - 48*x^14 - 4/13*x^13*a + 2560/13*x^13 + 4*x
^12*a - 1856/3*x^12 - 288/11*x^11*a + 16768/11*x^11 + 112*x^10*a + 2/3*x^9*
a^2 - 14848/5*x^10 - 1024/3*x^9*a - 6*x^8*a^2 + 40960/9*x^9 + 768*x^8*a + 1
92/7*x^7*a^2 - 5376*x^8 - 1280*x^7*a - 80*x^6*a^2 - 4/5*x^5*a^3 + 32768/7*x
^7 + 1536*x^6*a + 768/5*x^5*a^2 + 4*x^4*a^3 - 8192/3*x^6 - 6144/5*x^5*a - 1
92*x^4*a^2 - 32/3*x^3*a^3 + 4096/5*x^5 + 512*x^4*a + 128*x^3*a^2 + 16*x^2*a
^3 + x*a^4
```

Sympy [A] time = 0.112054, size = 199, normalized size = 1.62

$$a^{4}x + 16a^{3}x^{2} + \frac{x^{17}}{17} - x^{16} + \frac{128x^{15}}{15} - 48x^{14} + x^{13}\left(\frac{2560}{13} - \frac{4a}{13}\right) + x^{12}\left(4a - \frac{1856}{3}\right) + x^{11}\left(\frac{16768}{11} - \frac{288a}{11}\right) + x^{10}\left(112a - \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+4*x**3-8*x**2+a+8*x)**4,x)
```

```
[Out] a**4*x + 16*a**3*x**2 + x**17/17 - x**16 + 128*x**15/15 - 48*x**14 + x**13*
(2560/13 - 4*a/13) + x**12*(4*a - 1856/3) + x**11*(16768/11 - 288*a/11) + x
**10*(112*a - 14848/5) + x**9*(2*a**2/3 - 1024*a/3 + 40960/9) + x**8*(-6*a*
*2 + 768*a - 5376) + x**7*(192*a**2/7 - 1280*a + 32768/7) + x**6*(-80*a**2
+ 1536*a - 8192/3) + x**5*(-4*a**3/5 + 768*a**2/5 - 6144*a/5 + 4096/5) + x*
*4*(4*a**3 - 192*a**2 + 512*a) + x**3*(-32*a**3/3 + 128*a**2)
```

Giac [B] time = 1.25165, size = 296, normalized size = 2.41

$$\frac{1}{17}x^{17} - x^{16} + \frac{128}{15}x^{15} - \frac{4}{13}ax^{13} - 48x^{14} + 4ax^{12} + \frac{2560}{13}x^{13} - \frac{288}{11}ax^{11} - \frac{1856}{3}x^{12} + \frac{2}{3}a^2x^9 + 112ax^{10} + \frac{16768}{11}x^{11} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+4*x^3-8*x^2+a+8*x)^4,x, algorithm="giac")
```

```
[Out] 1/17*x^17 - x^16 + 128/15*x^15 - 4/13*a*x^13 - 48*x^14 + 4*a*x^12 + 2560/13
*x^13 - 288/11*a*x^11 - 1856/3*x^12 + 2/3*a^2*x^9 + 112*a*x^10 + 16768/11*x
^11 - 6*a^2*x^8 - 1024/3*a*x^9 - 14848/5*x^10 + 192/7*a^2*x^7 + 768*a*x^8 +
 40960/9*x^9 - 4/5*a^3*x^5 - 80*a^2*x^6 - 1280*a*x^7 - 5376*x^8 + 4*a^3*x^4
 + 768/5*a^2*x^5 + 1536*a*x^6 + 32768/7*x^7 - 32/3*a^3*x^3 - 192*a^2*x^4 -
6144/5*a*x^5 - 8192/3*x^6 + a^4*x + 16*a^3*x^2 + 128*a^2*x^3 + 512*a*x^4 +
4096/5*x^5
```

3.117 $\int \left(a+8x-8x^2+4x^3-x^4\right)^3 \, dx$

Optimal. Leaf size=120

$$-\frac{3}{5}\left(a^2-128a+512\right)x^5+\left(3a^2-96a+128\right)x^4+12a^2x^2+a^3x-\frac{1}{3}(256-a)x^9+3(64-a)x^8-\frac{32}{7}(70-3a)x^7+8(48$$

```
[Out] a^3*x + 12*a^2*x^2 + 8*(8 - a)*a*x^3 + (128 - 96*a + 3*a^2)*x^4 - (3*(512 -
 128*a + a^2)*x^5)/5 + 8*(48 - 5*a)*x^6 - (32*(70 - 3*a)*x^7)/7 + 3*(64 - a
)*x^8 - ((256 - a)*x^9)/3 + 28*x^10 - (72*x^11)/11 + x^12 - x^13/13
```

Rubi [A] time = 0.0631218, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {2061}

$$-\frac{3}{5}\left(a^2-128a+512\right)x^5+\left(3a^2-96a+128\right)x^4+12a^2x^2+a^3x-\frac{1}{3}(256-a)x^9+3(64-a)x^8-\frac{32}{7}(70-3a)x^7+8(48$$

Antiderivative was successfully verified.

```
[In] Int[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x]
```

```
[Out] a^3*x + 12*a^2*x^2 + 8*(8 - a)*a*x^3 + (128 - 96*a + 3*a^2)*x^4 - (3*(512 -
 128*a + a^2)*x^5)/5 + 8*(48 - 5*a)*x^6 - (32*(70 - 3*a)*x^7)/7 + 3*(64 - a
)*x^8 - ((256 - a)*x^9)/3 + 28*x^10 - (72*x^11)/11 + x^12 - x^13/13
```

Rule 2061

```
Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I
GtQ[p, 0]
```

Rubi steps

$$\int \left(a+8x-8x^2+4x^3-x^4\right)^3 \, dx=\int \left(a^3+24a^2x+24(8-a)ax^2+4\left(128-96a+3a^2\right)x^3-3\left(512-128a+a^2\right)x^4+$$

$$=a^3x+12a^2x^2+8(8-a)ax^3+\left(128-96a+3a^2\right)x^4-\frac{3}{5}\left(512-128a+a^2\right)x^5+8(48$$

Mathematica [A] time = 0.0127656, size = 114, normalized size = 0.95

$$-\frac{3}{5}\left(a^2-128a+512\right)x^5+\left(3a^2-96a+128\right)x^4+12a^2x^2+a^3x+\frac{1}{3}(a-256)x^9-3(a-64)x^8+\frac{32}{7}(3a-70)x^7-8(5a$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x]
```

```
[Out] a^3*x + 12*a^2*x^2 - 8*(-8 + a)*a*x^3 + (128 - 96*a + 3*a^2)*x^4 - (3*(512
- 128*a + a^2)*x^5)/5 - 8*(-48 + 5*a)*x^6 + (32*(-70 + 3*a)*x^7)/7 - 3*(-64
 + a)*x^8 + ((-256 + a)*x^9)/3 + 28*x^10 - (72*x^11)/11 + x^12 - x^13/13
```

Maple [A] time = 0.002, size = 138, normalized size = 1.2

$$-\frac{x^{13}}{13}+x^{12}-\frac{72\,x^{11}}{11}+28\,x^{10}+\frac{(3\,a-768)\,x^{9}}{9}+\frac{(-24\,a+1536)\,x^{8}}{8}+\frac{(96\,a-2240)\,x^{7}}{7}+\frac{(-240\,a+2304)\,x^{6}}{6}+\frac{\left(a\,(-2\,a+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+4*x^3-8*x^2+a+8*x)^3,x)
```

```
[Out] -1/13*x^13+x^12-72/11*x^11+28*x^10+1/9*(3*a-768)*x^9+1/8*(-24*a+1536)*x^8+1
/7*(96*a-2240)*x^7+1/6*(-240*a+2304)*x^6+1/5*(a*(-2*a+128)+256*a-1536-a^2)*
x^5+1/4*(a*(8*a-128)-256*a+512+4*a^2)*x^4+1/3*(a*(-16*a+64)+128*a-8*a^2)*x^
3+12*a^2*x^2+a^3*x
```

Maxima [A] time = 1.19175, size = 161, normalized size = 1.34

$$-\frac{1}{13}\,x^{13}+x^{12}-\frac{72}{11}\,x^{11}+28\,x^{10}-\frac{256}{3}\,x^{9}+192\,x^{8}-320\,x^{7}+384\,x^{6}-\frac{1536}{5}\,x^{5}+a^{3}x+128\,x^{4}-\frac{1}{5}\left(3\,x^{5}-15\,x^{4}+40\,x^{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="maxima")
```

```
[Out] -1/13*x^13 + x^12 - 72/11*x^11 + 28*x^10 - 256/3*x^9 + 192*x^8 - 320*x^7 +
384*x^6 - 1536/5*x^5 + a^3*x + 128*x^4 - 1/5*(3*x^5 - 15*x^4 + 40*x^3 - 60*
x^2)*a^2 + 1/105*(35*x^9 - 315*x^8 + 1440*x^7 - 4200*x^6 + 8064*x^5 - 10080
*x^4 + 6720*x^3)*a
```

Fricas [A] time = 1.28333, size = 335, normalized size = 2.79

$$-\frac{1}{13}x^{13} + x^{12} - \frac{72}{11}x^{11} + 28x^{10} + \frac{1}{3}x^9a - \frac{256}{3}x^9 - 3x^8a + 192x^8 + \frac{96}{7}x^7a - 320x^7 - 40x^6a - \frac{3}{5}x^5a^2 + 384x^6 + \frac{384}{5}x^5a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="fricas")
```

```
[Out] -1/13*x^13 + x^12 - 72/11*x^11 + 28*x^10 + 1/3*x^9*a - 256/3*x^9 - 3*x^8*a
+ 192*x^8 + 96/7*x^7*a - 320*x^7 - 40*x^6*a - 3/5*x^5*a^2 + 384*x^6 + 384/5
*x^5*a + 3*x^4*a^2 - 1536/5*x^5 - 96*x^4*a - 8*x^3*a^2 + 128*x^4 + 64*x^3*a
 + 12*x^2*a^2 + x*a^3
```

Sympy [A] time = 0.086457, size = 114, normalized size = 0.95

$$a^3x + 12a^2x^2 - \frac{x^{13}}{13} + x^{12} - \frac{72x^{11}}{11} + 28x^{10} + x^9\left(\frac{a}{3} - \frac{256}{3}\right) + x^8(192 - 3a) + x^7\left(\frac{96a}{7} - 320\right) + x^6(384 - 40a) + x^5\left(-\frac{3}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+4*x**3-8*x**2+a+8*x)**3,x)
```

```
[Out] a**3*x + 12*a**2*x**2 - x**13/13 + x**12 - 72*x**11/11 + 28*x**10 + x**9*(a
/3 - 256/3) + x**8*(192 - 3*a) + x**7*(96*a/7 - 320) + x**6*(384 - 40*a) +
x**5*(-3*a**2/5 + 384*a/5 - 1536/5) + x**4*(3*a**2 - 96*a + 128) + x**3*(-8
*a**2 + 64*a)
```

Giac [A] time = 1.11707, size = 173, normalized size = 1.44

$$-\frac{1}{13}x^{13} + x^{12} - \frac{72}{11}x^{11} + \frac{1}{3}ax^9 + 28x^{10} - 3ax^8 - \frac{256}{3}x^9 + \frac{96}{7}ax^7 + 192x^8 - \frac{3}{5}a^2x^5 - 40ax^6 - 320x^7 + 3a^2x^4 + \frac{384}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="giac")
```

```
[Out] -1/13*x^13 + x^12 - 72/11*x^11 + 1/3*a*x^9 + 28*x^10 - 3*a*x^8 - 256/3*x^9
+ 96/7*a*x^7 + 192*x^8 - 3/5*a^2*x^5 - 40*a*x^6 - 320*x^7 + 3*a^2*x^4 + 384
/5*a*x^5 + 384*x^6 - 8*a^2*x^3 - 96*a*x^4 - 1536/5*x^5 + a^3*x + 12*a^2*x^2
 + 64*a*x^3 + 128*x^4
```

3.118 $\int \left(a+8x-8x^2+4x^3-x^4\right)^2\,dx$

Optimal. Leaf size=72

$$a^2x+\frac{2}{5}(64-a)x^5-2(16-a)x^4+\frac{16}{3}(4-a)x^3+8ax^2+\frac{x^9}{9}-x^8+\frac{32x^7}{7}-\frac{40x^6}{3}$$

```
[Out] a^2*x + 8*a*x^2 + (16*(4 - a)*x^3)/3 - 2*(16 - a)*x^4 + (2*(64 - a)*x^5)/5
- (40*x^6)/3 + (32*x^7)/7 - x^8 + x^9/9
```

Rubi [A] time = 0.0297305, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {2061}

$$a^2x+\frac{2}{5}(64-a)x^5-2(16-a)x^4+\frac{16}{3}(4-a)x^3+8ax^2+\frac{x^9}{9}-x^8+\frac{32x^7}{7}-\frac{40x^6}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]
```

```
[Out] a^2*x + 8*a*x^2 + (16*(4 - a)*x^3)/3 - 2*(16 - a)*x^4 + (2*(64 - a)*x^5)/5
- (40*x^6)/3 + (32*x^7)/7 - x^8 + x^9/9
```

Rule 2061

```
Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I
GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+8x-8x^2+4x^3-x^4\right)^2\,dx &= \int \left(a^2+16ax+16(4-a)x^2-8(16-a)x^3+2(64-a)x^4-80x^5+32x^6-8x^7+x^8\right) d\\ &= a^2x+8ax^2+\frac{16}{3}(4-a)x^3-2(16-a)x^4+\frac{2}{5}(64-a)x^5-\frac{40x^6}{3}+\frac{32x^7}{7}-x^8+\frac{x^9}{9}\end{aligned}$$

Mathematica [A] time = 0.0070781, size = 66, normalized size = 0.92

$$a^2x-\frac{2}{5}(a-64)x^5+2(a-16)x^4-\frac{16}{3}(a-4)x^3+8ax^2+\frac{x^9}{9}-x^8+\frac{32x^7}{7}-\frac{40x^6}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]
```

```
[Out] a^2*x + 8*a*x^2 - (16*(-4 + a)*x^3)/3 + 2*(-16 + a)*x^4 - (2*(-64 + a)*x^5)
/5 - (40*x^6)/3 + (32*x^7)/7 - x^8 + x^9/9
```

Maple [A] time = 0.001, size = 63, normalized size = 0.9

$$\frac{x^9}{9} - x^8 + \frac{32\,x^7}{7} - \frac{40\,x^6}{3} + \frac{(-2\,a + 128)\,x^5}{5} + \frac{(8\,a - 128)\,x^4}{4} + \frac{(-16\,a + 64)\,x^3}{3} + 8\,ax^2 + xa^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+4*x^3-8*x^2+a+8*x)^2,x)
```

```
[Out] 1/9*x^9-x^8+32/7*x^7-40/3*x^6+1/5*(-2*a+128)*x^5+1/4*(8*a-128)*x^4+1/3*(-16
*a+64)*x^3+8*a*x^2+x*a^2
```

Maxima [A] time = 1.08178, size = 88, normalized size = 1.22

$$\frac{1}{9}\,x^9 - x^8 + \frac{32}{7}\,x^7 - \frac{40}{3}\,x^6 + \frac{128}{5}\,x^5 - 32\,x^4 + a^2x + \frac{64}{3}\,x^3 - \frac{2}{15}\left(3\,x^5 - 15\,x^4 + 40\,x^3 - 60\,x^2\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="maxima")
```

```
[Out] 1/9*x^9 - x^8 + 32/7*x^7 - 40/3*x^6 + 128/5*x^5 - 32*x^4 + a^2*x + 64/3*x^3
 - 2/15*(3*x^5 - 15*x^4 + 40*x^3 - 60*x^2)*a
```

Fricas [A] time = 1.33987, size = 165, normalized size = 2.29

$$\frac{1}{9}x^9 - x^8 + \frac{32}{7}x^7 - \frac{40}{3}x^6 - \frac{2}{5}x^5a + \frac{128}{5}x^5 + 2x^4a - 32x^4 - \frac{16}{3}x^3a + \frac{64}{3}x^3 + 8x^2a + xa^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="fricas")
```

```
[Out] 1/9*x^9 - x^8 + 32/7*x^7 - 40/3*x^6 - 2/5*x^5*a + 128/5*x^5 + 2*x^4*a - 32*
x^4 - 16/3*x^3*a + 64/3*x^3 + 8*x^2*a + x*a^2
```

Sympy [A] time = 0.068419, size = 65, normalized size = 0.9

$$a^2x + 8ax^2 + \frac{x^9}{9} - x^8 + \frac{32x^7}{7} - \frac{40x^6}{3} + x^5\left(\frac{128}{5} - \frac{2a}{5}\right) + x^4(2a - 32) + x^3\left(\frac{64}{3} - \frac{16a}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+4*x**3-8*x**2+a+8*x)**2,x)
```

```
[Out] a**2*x + 8*a*x**2 + x**9/9 - x**8 + 32*x**7/7 - 40*x**6/3 + x**5*(128/5 - 2
*a/5) + x**4*(2*a - 32) + x**3*(64/3 - 16*a/3)
```

Giac [A] time = 1.11347, size = 88, normalized size = 1.22

$$\frac{1}{9}x^9 - x^8 + \frac{32}{7}x^7 - \frac{2}{5}ax^5 - \frac{40}{3}x^6 + 2ax^4 + \frac{128}{5}x^5 - \frac{16}{3}ax^3 - 32x^4 + a^2x + 8ax^2 + \frac{64}{3}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="giac")
```

```
[Out] 1/9*x^9 - x^8 + 32/7*x^7 - 2/5*a*x^5 - 40/3*x^6 + 2*a*x^4 + 128/5*x^5 - 16/
3*a*x^3 - 32*x^4 + a^2*x + 8*a*x^2 + 64/3*x^3
```

3.119 $\int \left(a+8x-8x^2+4x^3-x^4\right) dx$

Optimal. Leaf size=26

$$ax-\frac{x^5}{5}+x^4-\frac{8x^3}{3}+4x^2$$

```
[Out] a*x + 4*x^2 - (8*x^3)/3 + x^4 - x^5/5
```

Rubi [A] time = 0.00396, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$ax-\frac{x^5}{5}+x^4-\frac{8x^3}{3}+4x^2$$

Antiderivative was successfully verified.

```
[In] Int[a + 8*x - 8*x^2 + 4*x^3 - x^4,x]
```

```
[Out] a*x + 4*x^2 - (8*x^3)/3 + x^4 - x^5/5
```

Rubi steps

$$\int \left(a+8x-8x^2+4x^3-x^4\right) dx = ax+4x^2-\frac{8x^3}{3}+x^4-\frac{x^5}{5}$$

Mathematica [A] time = 0.0000663, size = 26, normalized size = 1.

$$ax-\frac{x^5}{5}+x^4-\frac{8x^3}{3}+4x^2$$

Antiderivative was successfully verified.

```
[In] Integrate[a + 8*x - 8*x^2 + 4*x^3 - x^4,x]
```

```
[Out] a*x + 4*x^2 - (8*x^3)/3 + x^4 - x^5/5
```

Maple [A] time = 0., size = 23, normalized size = 0.9

$$ax+4\,x^{2}-\frac{8\,x^{3}}{3}+x^{4}-\frac{x^{5}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^4+4*x^3-8*x^2+a+8*x,x)
```

```
[Out] a*x+4*x^2-8/3*x^3+x^4-1/5*x^5
```

Maxima [A] time = 1.06695, size = 30, normalized size = 1.15

$$-\frac{1}{5}\,x^{5}+x^{4}-\frac{8}{3}\,x^{3}+ax+4\,x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^4+4*x^3-8*x^2+a+8*x,x, algorithm="maxima")
```

```
[Out] -1/5*x^5 + x^4 - 8/3*x^3 + a*x + 4*x^2
```

Fricas [A] time = 1.27597, size = 54, normalized size = 2.08

$$-\frac{1}{5}x^{5}+x^{4}-\frac{8}{3}x^{3}+4x^{2}+xa$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^4+4*x^3-8*x^2+a+8*x,x, algorithm="fricas")
```

```
[Out] -1/5*x^5 + x^4 - 8/3*x^3 + 4*x^2 + x*a
```

Sympy [A] time = 0.056983, size = 22, normalized size = 0.85

$$ax-\frac{x^{5}}{5}+x^{4}-\frac{8x^{3}}{3}+4x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x**4+4*x**3-8*x**2+a+8*x,x)
```

```
[Out] a*x - x**5/5 + x**4 - 8*x**3/3 + 4*x**2
```

Giac [A] time = 1.10444, size = 30, normalized size = 1.15

$$-\frac{1}{5}x^5 + x^4 - \frac{8}{3}x^3 + ax + 4x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^4+4*x^3-8*x^2+a+8*x,x, algorithm="giac")
```

```
[Out] -1/5*x^5 + x^4 - 8/3*x^3 + a*x + 4*x^2
```

3.120 $\int \frac{1}{a+8x-8x^2+4x^3-x^4} dx$

Optimal. Leaf size=89

$$\frac{\tan^{-1}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{2\sqrt{a+4}\sqrt{\sqrt{a+4}+1}}-\frac{\tan^{-1}\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{2\sqrt{a+4}\sqrt{1-\sqrt{a+4}}}$$

```
[Out] -ArcTan[(-1 + x)/Sqrt[1 - Sqrt[4 + a]]]/(2*Sqrt[4 + a]*Sqrt[1 - Sqrt[4 + a]
]) + ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]]/(2*Sqrt[4 + a]*Sqrt[1 + Sqrt[4
+ a]])
```

Rubi [A] time = 0.086782, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1106, 1093, 204}

$$\frac{\tan^{-1}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{2\sqrt{a+4}\sqrt{\sqrt{a+4}+1}}-\frac{\tan^{-1}\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{2\sqrt{a+4}\sqrt{1-\sqrt{a+4}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-1),x]
```

```
[Out] -ArcTan[(-1 + x)/Sqrt[1 - Sqrt[4 + a]]]/(2*Sqrt[4 + a]*Sqrt[1 - Sqrt[4 + a]
]) + ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]]/(2*Sqrt[4 + a]*Sqrt[1 + Sqrt[4
+ a]])
```

Rule 1106

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
```

```
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{a+8x-8x^2+4x^3-x^4}\,dx &= \text{Subst}\left(\int \frac{1}{3+a-2x^2-x^4}\,dx, x, -1+x\right)\\
&= -\frac{\text{Subst}\left(\int \frac{1}{-1-\sqrt{4+a}-x^2}\,dx, x, -1+x\right)}{2\sqrt{4+a}} + \frac{\text{Subst}\left(\int \frac{1}{-1+\sqrt{4+a}-x^2}\,dx, x, -1+x\right)}{2\sqrt{4+a}}\\
&= \frac{\tan^{-1}\left(\frac{1-x}{\sqrt{1-\sqrt{4+a}}}\right)}{2\sqrt{4+a}\sqrt{1-\sqrt{4+a}}} - \frac{\tan^{-1}\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)}{2\sqrt{4+a}\sqrt{1+\sqrt{4+a}}}
\end{aligned}$$

Mathematica [C] time = 0.0142267, size = 57, normalized size = 0.64

$$-\frac{1}{4}\text{RootSum}\left[-\#1^4+4\#1^3-8\#1^2+8\#1+a\&, \frac{\log(x-\#1)}{\#1^3-3\#1^2+4\#1-2}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-1),x]
```

```
[Out] -RootSum[a + 8*#1 - 8*#1^2 + 4*#1^3 - #1^4 & , Log[x - #1]/(-2 + 4*#1 - 3*#
1^2 + #1^3) & ]/4
```

Maple [C] time = 0.013, size = 49, normalized size = 0.6

$$-\frac{1}{4}\sum_{\_R=\text{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)}\frac{\ln(x-\_R)}{\_R^3-3\_R^2+4\_R-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^4+4*x^3-8*x^2+a+8*x),x)
```

```
[Out] -1/4*sum(1/(_R^3-3*_R^2+4*_R-2)*ln(x-_R),_R=RootOf(_Z^4-4*_Z^3+8*_Z^2-8*_Z-
a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{x^4 - 4\,x^3 + 8\,x^2 - a - 8\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="maxima")
```

```
[Out] -integrate(1/(x^4 - 4*x^3 + 8*x^2 - a - 8*x), x)
```

Fricas [B] time = 1.54701, size = 1254, normalized size = 14.09

$$\frac{1}{4}\sqrt{\frac{\frac{a^2+7\,a+12}{\sqrt{a^3+10\,a^2+33\,a+36}}+1}{a^2+7\,a+12}}\log\left(\left(a-\frac{a^2+7\,a+12}{\sqrt{a^3+10\,a^2+33\,a+36}}+4\right)\sqrt{\frac{\frac{a^2+7\,a+12}{\sqrt{a^3+10\,a^2+33\,a+36}}+1}{a^2+7\,a+12}}+x-1\right)-\frac{1}{4}\sqrt{\frac{\frac{a^2+7\,a+12}{\sqrt{a^3+10\,a^2+33\,a+36}}+}{a^2+7\,a+12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(((a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36) + 1)/(a^2 + 7*a +
 12))*log((a - (a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36) + 4)*sqrt(((
a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36) + 1)/(a^2 + 7*a + 12)) + x -
 1) - 1/4*sqrt(((a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36) + 1)/(a^2 +
 7*a + 12))*log(-(a - (a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36) + 4)*
sqrt(((a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36) + 1)/(a^2 + 7*a + 12)
) + x - 1) + 1/4*sqrt(-((a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36) - 1
)/(a^2 + 7*a + 12))*log((a + (a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36
) + 4)*sqrt(-((a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36) - 1)/(a^2 + 7
*a + 12)) + x - 1) - 1/4*sqrt(-((a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a +
 36) - 1)/(a^2 + 7*a + 12))*log(-(a + (a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 +
33*a + 36) + 4)*sqrt(-((a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36) - 1)
```

```
/(a^2 + 7*a + 12)) + x - 1)
```

Sympy [A] time = 0.705436, size = 66, normalized size = 0.74

$$-\operatorname{RootSum}\left(t^4\left(256a^3+2816a^2+10240a+12288\right)+t^2\left(-32a-128\right)-1,\left(t\mapsto t\log\left(64t^3a^2+448t^3a+768t^3-4ta-\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**4+4*x**3-8*x**2+a+8*x),x)
```

```
[Out] -RootSum(_t**4*(256*a**3 + 2816*a**2 + 10240*a + 12288) + _t**2*(-32*a - 12
8) - 1, Lambda(_t, _t*log(64*_t**3*a**2 + 448*_t**3*a + 768*_t**3 - 4*_t*a
- 20*_t + x - 1)))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.121 $\int \frac{1}{\left(a+8x-8x^2+4x^3-x^4\right)^2}\,dx$

Optimal. Leaf size=169

$$\frac{(x-1)\left(a+(x-1)^2+5\right)}{4\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)}-\frac{\left(3a+\sqrt{a+4}+10\right)\tan^{-1}\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{8(a+3)(a+4)^{3/2}\sqrt{1-\sqrt{a+4}}}+\frac{\left(3a-\sqrt{a+4}+10\right)\tan^{-1}\left(\frac{x-}{\sqrt{\sqrt{a+}}}\right.}{8(a+3)(a+4)^{3/2}\sqrt{\sqrt{a+4}+}}$$

```
[Out] ((5 + a + (-1 + x)^2)*(-1 + x))/(4*(12 + 7*a + a^2)*(3 + a - 2*(-1 + x)^2 -
 (-1 + x)^4)) - ((10 + 3*a + Sqrt[4 + a])*ArcTan[(-1 + x)/Sqrt[1 - Sqrt[4 +
 a]]])/(8*(3 + a)*(4 + a)^(3/2)*Sqrt[1 - Sqrt[4 + a]]) + ((10 + 3*a - Sqrt[
4 + a])*ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]])/(8*(3 + a)*(4 + a)^(3/2)*Sq
rt[1 + Sqrt[4 + a]])
```

Rubi [A] time = 0.291298, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1106, 1092, 1166, 204}

$$\frac{(x-1)\left(a+(x-1)^2+5\right)}{4\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)}-\frac{\left(3a+\sqrt{a+4}+10\right)\tan^{-1}\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{8(a+3)(a+4)^{3/2}\sqrt{1-\sqrt{a+4}}}+\frac{\left(3a-\sqrt{a+4}+10\right)\tan^{-1}\left(\frac{x-}{\sqrt{\sqrt{a+}}}\right.}{8(a+3)(a+4)^{3/2}\sqrt{\sqrt{a+4}+}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-2),x]
```

```
[Out] ((5 + a + (-1 + x)^2)*(-1 + x))/(4*(12 + 7*a + a^2)*(3 + a - 2*(-1 + x)^2 -
 (-1 + x)^4)) - ((10 + 3*a + Sqrt[4 + a])*ArcTan[(-1 + x)/Sqrt[1 - Sqrt[4 +
 a]]])/(8*(3 + a)*(4 + a)^(3/2)*Sqrt[1 - Sqrt[4 + a]]) + ((10 + 3*a - Sqrt[
4 + a])*ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]])/(8*(3 + a)*(4 + a)^(3/2)*Sq
rt[1 + Sqrt[4 + a]])
```

Rule 1106

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
```

```
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1092

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x*(b^2 -
 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2
 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(a+8x-8x^2+4x^3-x^4\right)^2}\,dx = \text{Subst}\left(\int \frac{1}{\left(3+a-2x^2-x^4\right)^2}\,dx, x, -1+x\right)$$

$$= \frac{\left(5+a+(-1+x)^2\right)(-1+x)}{4\left(12+7a+a^2\right)\left(3+a-2(-1+x)^2-(-1+x)^4\right)} - \frac{\text{Subst}\left(\int \frac{4+2(3+a)-2(4+4(3+a))-2x\cdots}{3+a-2x^2-x^4}\cdots\right.}{8\left(12+7a+a^2\right)\cdots}$$

$$= \frac{\left(5+a+(-1+x)^2\right)(-1+x)}{4\left(12+7a+a^2\right)\left(3+a-2(-1+x)^2-(-1+x)^4\right)} - \frac{\left(10+3a-\sqrt{4+a}\right)\text{Subst}\left(\int \cdots\right.}{8(3+a)(4+\cdots}$$

$$= \frac{\left(5+a+(-1+x)^2\right)(-1+x)}{4\left(12+7a+a^2\right)\left(3+a-2(-1+x)^2-(-1+x)^4\right)} + \frac{\left(10+3a+\sqrt{4+a}\right)\tan^{-1}\left(\frac{1\cdots}{\sqrt{1-\cdots}}\right.}{8(3+a)(4+a)^{3/2}\sqrt{1-\sqrt{4+\cdots}}}$$

Mathematica [C] time = 0.0535722, size = 150, normalized size = 0.89

$$\frac{(x-1)\left(a+x^2-2x+6\right)}{4(a+3)(a+4)\left(a-x\left(x^3-4x^2+8x-8\right)\right)}-\frac{\text{RootSum}\left[-\#1^4+4\#1^3-8\#1^2+8\#1+a\&,\frac{\#1^2\log(x-\#1)+3a\log(x-\#1)-2\#1\log(}{\#1^3-3\#1^2+4\#1-2}\right.}{16\left(a^2+7a+12\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-2),x]
```

```
[Out] ((-1 + x)*(6 + a - 2*x + x^2))/(4*(3 + a)*(4 + a)*(a - x*(-8 + 8*x - 4*x^2
+ x^3))) - RootSum[a + 8*#1 - 8*#1^2 + 4*#1^3 - #1^4 & , (12*Log[x - #1] +
3*a*Log[x - #1] - 2*Log[x - #1]*#1 + Log[x - #1]*#1^2)/(-2 + 4*#1 - 3*#1^2
+ #1^3) & ]/(16*(12 + 7*a + a^2))
```

Maple [C] time = 0.01, size = 158, normalized size = 0.9

$$\frac{1}{x^4-4\,x^3+8\,x^2-a-8\,x}\left(-\frac{x^3}{4\,a^2+28\,a+48}+\frac{3\,x^2}{4\,a^2+28\,a+48}-\frac{(8+a)\,x}{4\,a^2+28\,a+48}+\frac{6+a}{4\,a^2+28\,a+48}\right)+\frac{1}{(48+16\,a)\,(4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^4+4*x^3-8*x^2+a+8*x)^2,x)
```

```
[Out] (-1/4/(a^2+7*a+12)*x^3+3/4/(a^2+7*a+12)*x^2-1/4*(8+a)/(a^2+7*a+12)*x+1/4*(6
+a)/(a^2+7*a+12))/(x^4-4*x^3+8*x^2-a-8*x)+1/16/(3+a)/(4+a)*sum((-_R^2+2*_R-
3*a-12)/(_R^3-3*_R^2+4*_R-2)*ln(x-_R),_R=RootOf(_Z^4-4*_Z^3+8*_Z^2-8*_Z-a))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: AttributeError
```

Fricas [B] time = 1.7985, size = 5716, normalized size = 33.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(4*x^3 - ((a^2 + 7*a + 12)*x^4 - 4*(a^2 + 7*a + 12)*x^3 - a^3 + 8*(a^
2 + 7*a + 12)*x^2 - 7*a^2 - 8*(a^2 + 7*a + 12)*x - 12*a)*sqrt((15*a^2 + (a^
6 + 21*a^5 + 183*a^4 + 847*a^3 + 2196*a^2 + 3024*a + 1728)*sqrt((81*a^2 + 5
58*a + 961)/(a^9 + 30*a^8 + 399*a^7 + 3088*a^6 + 15327*a^5 + 50598*a^4 + 11
1105*a^3 + 156492*a^2 + 128304*a + 46656)) + 105*a + 184)/(a^6 + 21*a^5 + 1
83*a^4 + 847*a^3 + 2196*a^2 + 3024*a + 1728))*log(-81*a^2 + (81*a^2 + 567*a
 + 992)*x + (27*a^4 + 408*a^3 + 2309*a^2 - 2*(2*a^7 + 49*a^6 + 513*a^5 + 29
75*a^4 + 10321*a^3 + 21420*a^2 + 24624*a + 12096)*sqrt((81*a^2 + 558*a + 96
1)/(a^9 + 30*a^8 + 399*a^7 + 3088*a^6 + 15327*a^5 + 50598*a^4 + 111105*a^3
+ 156492*a^2 + 128304*a + 46656)) + 5800*a + 5456)*sqrt((15*a^2 + (a^6 + 21
*a^5 + 183*a^4 + 847*a^3 + 2196*a^2 + 3024*a + 1728)*sqrt((81*a^2 + 558*a +
 961)/(a^9 + 30*a^8 + 399*a^7 + 3088*a^6 + 15327*a^5 + 50598*a^4 + 111105*a
^3 + 156492*a^2 + 128304*a + 46656)) + 105*a + 184)/(a^6 + 21*a^5 + 183*a^4
 + 847*a^3 + 2196*a^2 + 3024*a + 1728)) - 567*a - 992) + ((a^2 + 7*a + 12)*
x^4 - 4*(a^2 + 7*a + 12)*x^3 - a^3 + 8*(a^2 + 7*a + 12)*x^2 - 7*a^2 - 8*(a^
2 + 7*a + 12)*x - 12*a)*sqrt((15*a^2 + (a^6 + 21*a^5 + 183*a^4 + 847*a^3 +
2196*a^2 + 3024*a + 1728)*sqrt((81*a^2 + 558*a + 961)/(a^9 + 30*a^8 + 399*a
^7 + 3088*a^6 + 15327*a^5 + 50598*a^4 + 111105*a^3 + 156492*a^2 + 128304*a
+ 46656)) + 105*a + 184)/(a^6 + 21*a^5 + 183*a^4 + 847*a^3 + 2196*a^2 + 302
4*a + 1728))*log(-81*a^2 + (81*a^2 + 567*a + 992)*x - (27*a^4 + 408*a^3 + 2
309*a^2 - 2*(2*a^7 + 49*a^6 + 513*a^5 + 2975*a^4 + 10321*a^3 + 21420*a^2 +
24624*a + 12096)*sqrt((81*a^2 + 558*a + 961)/(a^9 + 30*a^8 + 399*a^7 + 3088
*a^6 + 15327*a^5 + 50598*a^4 + 111105*a^3 + 156492*a^2 + 128304*a + 46656))
 + 5800*a + 5456)*sqrt((15*a^2 + (a^6 + 21*a^5 + 183*a^4 + 847*a^3 + 2196*a
^2 + 3024*a + 1728)*sqrt((81*a^2 + 558*a + 961)/(a^9 + 30*a^8 + 399*a^7 + 3
088*a^6 + 15327*a^5 + 50598*a^4 + 111105*a^3 + 156492*a^2 + 128304*a + 4665
6)) + 105*a + 184)/(a^6 + 21*a^5 + 183*a^4 + 847*a^3 + 2196*a^2 + 3024*a +
1728)) - 567*a - 992) - ((a^2 + 7*a + 12)*x^4 - 4*(a^2 + 7*a + 12)*x^3 - a^
3 + 8*(a^2 + 7*a + 12)*x^2 - 7*a^2 - 8*(a^2 + 7*a + 12)*x - 12*a)*sqrt((15*
a^2 - (a^6 + 21*a^5 + 183*a^4 + 847*a^3 + 2196*a^2 + 3024*a + 1728)*sqrt((8
1*a^2 + 558*a + 961)/(a^9 + 30*a^8 + 399*a^7 + 3088*a^6 + 15327*a^5 + 50598
*a^4 + 111105*a^3 + 156492*a^2 + 128304*a + 46656)) + 105*a + 184)/(a^6 + 2
1*a^5 + 183*a^4 + 847*a^3 + 2196*a^2 + 3024*a + 1728))*log(-81*a^2 + (81*a^
2 + 567*a + 992)*x + (27*a^4 + 408*a^3 + 2309*a^2 + 2*(2*a^7 + 49*a^6 + 513
*a^5 + 2975*a^4 + 10321*a^3 + 21420*a^2 + 24624*a + 12096)*sqrt((81*a^2 + 5
```

```
58*a + 961)/(a^9 + 30*a^8 + 399*a^7 + 3088*a^6 + 15327*a^5 + 50598*a^4 + 11
1105*a^3 + 156492*a^2 + 128304*a + 46656)) + 5800*a + 5456)*sqrt((15*a^2 -
(a^6 + 21*a^5 + 183*a^4 + 847*a^3 + 2196*a^2 + 3024*a + 1728)*sqrt((81*a^2
+ 558*a + 961)/(a^9 + 30*a^8 + 399*a^7 + 3088*a^6 + 15327*a^5 + 50598*a^4 +
 111105*a^3 + 156492*a^2 + 128304*a + 46656)) + 105*a + 184)/(a^6 + 21*a^5
+ 183*a^4 + 847*a^3 + 2196*a^2 + 3024*a + 1728)) - 567*a - 992) + ((a^2 + 7
*a + 12)*x^4 - 4*(a^2 + 7*a + 12)*x^3 - a^3 + 8*(a^2 + 7*a + 12)*x^2 - 7*a^
2 - 8*(a^2 + 7*a + 12)*x - 12*a)*sqrt((15*a^2 - (a^6 + 21*a^5 + 183*a^4 + 8
47*a^3 + 2196*a^2 + 3024*a + 1728)*sqrt((81*a^2 + 558*a + 961)/(a^9 + 30*a^
8 + 399*a^7 + 3088*a^6 + 15327*a^5 + 50598*a^4 + 111105*a^3 + 156492*a^2 +
128304*a + 46656)) + 105*a + 184)/(a^6 + 21*a^5 + 183*a^4 + 847*a^3 + 2196*
a^2 + 3024*a + 1728))*log(-81*a^2 + (81*a^2 + 567*a + 992)*x - (27*a^4 + 40
8*a^3 + 2309*a^2 + 2*(2*a^7 + 49*a^6 + 513*a^5 + 2975*a^4 + 10321*a^3 + 214
20*a^2 + 24624*a + 12096)*sqrt((81*a^2 + 558*a + 961)/(a^9 + 30*a^8 + 399*a
^7 + 3088*a^6 + 15327*a^5 + 50598*a^4 + 111105*a^3 + 156492*a^2 + 128304*a
+ 46656)) + 5800*a + 5456)*sqrt((15*a^2 - (a^6 + 21*a^5 + 183*a^4 + 847*a^3
 + 2196*a^2 + 3024*a + 1728)*sqrt((81*a^2 + 558*a + 961)/(a^9 + 30*a^8 + 39
9*a^7 + 3088*a^6 + 15327*a^5 + 50598*a^4 + 111105*a^3 + 156492*a^2 + 128304
*a + 46656)) + 105*a + 184)/(a^6 + 21*a^5 + 183*a^4 + 847*a^3 + 2196*a^2 +
3024*a + 1728)) - 567*a - 992) + 4*(a + 8)*x - 12*x^2 - 4*a - 24)/((a^2 + 7
*a + 12)*x^4 - 4*(a^2 + 7*a + 12)*x^3 - a^3 + 8*(a^2 + 7*a + 12)*x^2 - 7*a^
2 - 8*(a^2 + 7*a + 12)*x - 12*a)
```

Sympy [B] time = 4.60528, size = 292, normalized size = 1.73

$$-\frac{-a + x^3 - 3x^2 + x\,(a+8) - 6}{-4a^3 - 28a^2 - 48a + x^4\left(4a^2 + 28a + 48\right) + x^3\left(-16a^2 - 112a - 192\right) + x^2\left(32a^2 + 224a + 384\right) + x\left(-32a^2 - 224a - 384\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**4+4*x**3-8*x**2+a+8*x)**2,x)
```

```
[Out] -(-a + x**3 - 3*x**2 + x*(a + 8) - 6)/(-4*a**3 - 28*a**2 - 48*a + x**4*(4*a
**2 + 28*a + 48) + x**3*(-16*a**2 - 112*a - 192) + x**2*(32*a**2 + 224*a +
384) + x*(-32*a**2 - 224*a - 384)) + RootSum(_t**4*(65536*a**9 + 2162688*a*
*8 + 31653888*a**7 + 269680640*a**6 + 1473773568*a**5 + 5357174784*a**4 + 1
2952010752*a**3 + 20082327552*a**2 + 18119393280*a + 7247757312) + _t**2*(-
7680*a**5 - 145920*a**4 - 1107968*a**3 - 4202496*a**2 - 7962624*a - 6029312
) - 81*a**2 - 576*a - 1024, Lambda(_t, _t*log(x + (-16384*_t**3*a**7 - 4014
08*_t**3*a**6 - 4202496*_t**3*a**5 - 24371200*_t**3*a**4 - 84549632*_t**3*a
**3 - 175472640*_t**3*a**2 - 201719808*_t**3*a - 99090432*_t**3 + 432*_t*a*
*4 + 7488*_t*a**3 + 47024*_t*a**2 + 128096*_t*a + 128512*_t - 81*a**2 - 567
```

```
*a - 992)/(81*a**2 + 567*a + 992))))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.122 $\int \frac{1}{\left(a+8x-8x^2+4x^3-x^4\right)^3}\,dx$

Optimal. Leaf size=252

$$\frac{(x-1)\left(a+(x-1)^2+5\right)}{8\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^2}-\frac{3\left(7a^2+\left(4\sqrt{a+4}+47\right)a+14\sqrt{a+4}+80\right)\tan^{-1}\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{64(a+3)^2(a+4)^{5/2}\sqrt{1-\sqrt{a+4}}}-\frac{3\left(-\frac{7a}{}\right.}{}$$

```
[Out] ((5 + a + (-1 + x)^2)*(-1 + x))/(8*(12 + 7*a + a^2)*(3 + a - 2*(-1 + x)^2 -
 (-1 + x)^4)^2) + (((6 + a)*(25 + 7*a) + 6*(7 + 2*a)*(-1 + x)^2)*(-1 + x))/
(32*(3 + a)^2*(4 + a)^2*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)) - (3*(80 + 7*a
^2 + 14*Sqrt[4 + a] + a*(47 + 4*Sqrt[4 + a]))*ArcTan[(-1 + x)/Sqrt[1 - Sqrt
[4 + a]]])/(64*(3 + a)^2*(4 + a)^(5/2)*Sqrt[1 - Sqrt[4 + a]]) - (3*(14 + 4*
a - (80 + 47*a + 7*a^2)/Sqrt[4 + a])*ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]]
)/(64*(3 + a)^2*(4 + a)^2*Sqrt[1 + Sqrt[4 + a]])
```

Rubi [A] time = 0.531924, antiderivative size = 252, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1106, 1092, 1178, 1166, 204}

$$\frac{(x-1)\left(a+(x-1)^2+5\right)}{8\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^2}-\frac{3\left(7a^2+\left(4\sqrt{a+4}+47\right)a+14\sqrt{a+4}+80\right)\tan^{-1}\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{64(a+3)^2(a+4)^{5/2}\sqrt{1-\sqrt{a+4}}}-\frac{3\left(-\frac{7a}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-3),x]
```


```
[Out] ((5 + a + (-1 + x)^2)*(-1 + x))/(8*(12 + 7*a + a^2)*(3 + a - 2*(-1 + x)^2 -
 (-1 + x)^4)^2) + (((6 + a)*(25 + 7*a) + 6*(7 + 2*a)*(-1 + x)^2)*(-1 + x))/
(32*(3 + a)^2*(4 + a)^2*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)) - (3*(80 + 7*a
^2 + 14*Sqrt[4 + a] + a*(47 + 4*Sqrt[4 + a]))*ArcTan[(-1 + x)/Sqrt[1 - Sqrt
[4 + a]]])/(64*(3 + a)^2*(4 + a)^(5/2)*Sqrt[1 - Sqrt[4 + a]]) - (3*(14 + 4*
a - (80 + 47*a + 7*a^2)/Sqrt[4 + a])*ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]]
)/(64*(3 + a)^2*(4 + a)^2*Sqrt[1 + Sqrt[4 + a]])
```


Rule 1106

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1092

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x*(b^2 -
 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2
 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(a+8x-8x^2+4x^3-x^4\right)^3}\,dx = \text{Subst}\left(\int \frac{1}{\left(3+a-2x^2-x^4\right)^3}\,dx, x, -1+x\right)$$

$$= \frac{\left(5+a+(-1+x)^2\right)(-1+x)}{8\left(12+7a+a^2\right)\left(3+a-2(-1+x)^2-(-1+x)^4\right)^2} - \frac{\text{Subst}\left(\int \frac{4+2(3+a)-4(4+4(3+a))-10x}{\left(3+a-2x^2-x^4\right)^2}\right.}{16\left(12+7a+a^2\right.}$$

$$= -\frac{\left((6+a)(25+7a)+6(7+2a)(1-x)^2\right)(1-x)}{32\left(12+7a+a^2\right)^2\left(3+a-2(1-x)^2-(1-x)^4\right)} + \frac{\left(5+a+(-1+x)^2\right)(\cdot}{8\left(12+7a+a^2\right)\left(3+a-2(-1+\right.}$$

$$= -\frac{\left((6+a)(25+7a)+6(7+2a)(1-x)^2\right)(1-x)}{32\left(12+7a+a^2\right)^2\left(3+a-2(1-x)^2-(1-x)^4\right)} + \frac{\left(5+a+(-1+x)^2\right)(\cdot}{8\left(12+7a+a^2\right)\left(3+a-2(-1+\right.}$$

$$= -\frac{\left((6+a)(25+7a)+6(7+2a)(1-x)^2\right)(1-x)}{32\left(12+7a+a^2\right)^2\left(3+a-2(1-x)^2-(1-x)^4\right)} + \frac{\left(5+a+(-1+x)^2\right)(\cdot}{8\left(12+7a+a^2\right)\left(3+a-2(-1+\right.}$$

Mathematica [C] time = 0.118802, size = 254, normalized size = 1.01

$$\frac{1}{128}\left(-\frac{3\text{RootSum}\left[-\#1^4+4\#1^3-8\#1^2+8\#1+a\&, \frac{4\#1^2 a\log(x-\#1)+14\#1^2\log(x-\#1)+7a^2\log(x-\#1)+55a\log(x-\#1)-8\#1 a\log(x-\#1)+108}{\#1^3-3\#1^2+4\#1-2}\right.}{\left(a^2+7a+12\right)^2}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-3),x]
```

```
[Out] ((16*(-1 + x)*(6 + a - 2*x + x^2))/((3 + a)*(4 + a)*(a - x*(-8 + 8*x - 4*x^
2 + x^3))^2) + (4*(-1 + x)*(7*a^2 + 6*(32 - 14*x + 7*x^2) + a*(79 - 24*x +
12*x^2)))/((3 + a)^2*(4 + a)^2*(a - x*(-8 + 8*x - 4*x^2 + x^3))) - (3*RootS
um[a + 8*#1 - 8*#1^2 + 4*#1^3 - #1^4 & , (108*Log[x - #1] + 55*a*Log[x - #1
] + 7*a^2*Log[x - #1] - 28*Log[x - #1]*#1 - 8*a*Log[x - #1]*#1 + 14*Log[x -
 #1]*#1^2 + 4*a*Log[x - #1]*#1^2)/(-2 + 4*#1 - 3*#1^2 + #1^3) & ])/(12 + 7*
a + a^2)^2)/128
```

Maple [C] time = 0.016, size = 398, normalized size = 1.6

$$-\frac{1}{\left(x^4-4x^3+8x^2-a-8x\right)^2}\left(\frac{(6a+21)x^7}{16a^4+224a^3+1168a^2+2688a+2304}-\frac{(147+42a)x^6}{\left(16a^2+128a+256\right)\left(a^2+6a+9\right)}+\frac{}{32a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^4+4*x^3-8*x^2+a+8*x)^3,x)
```

```
[Out] -(3/16*(7+2*a)/(a^4+14*a^3+73*a^2+168*a+144)*x^7-21/16*(7+2*a)/(a^2+8*a+16)
/(a^2+6*a+9)*x^6+1/32*(7*a^2+343*a+1116)/(a^4+14*a^3+73*a^2+168*a+144)*x^5-
5/32*(7*a^2+175*a+528)/(a^4+14*a^3+73*a^2+168*a+144)*x^4+1/16*(34*a^2+679*a
+1968)/(a^4+14*a^3+73*a^2+168*a+144)*x^3-1/16*(32*a^2+623*a+1800)/(a^4+14*a
^3+73*a^2+168*a+144)*x^2-1/32*(11*a^3+107*a^2-84*a-1152)/(a^4+14*a^3+73*a^2
+168*a+144)*x+1/32*(11*a^3+131*a^2+408*a+288)/(a^4+14*a^3+73*a^2+168*a+144)
)/(x^4-4*x^3+8*x^2-a-8*x)^2-3/128/(a^3+10*a^2+33*a+36)/(4+a)*sum((108+2*(7+
2*a)*_R^2+4*(-2*a-7)*_R+7*a^2+55*a)/(_R^3-3*_R^2+4*_R-2)*ln(x-_R),_R=RootOf
(_Z^4-4*_Z^3+8*_Z^2-8*_Z-a))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: AttributeError
```

Fricas [B] time = 1.90238, size = 13437, normalized size = 53.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="fricas")
```

```
[Out] -1/128*(24*(2*a + 7)*x^7 - 168*(2*a + 7)*x^6 + 4*(7*a^2 + 343*a + 1116)*x^5
 - 20*(7*a^2 + 175*a + 528)*x^4 + 8*(34*a^2 + 679*a + 1968)*x^3 + 44*a^3 -
```

8*(32*a^2 + 623*a + 1800)*x^2 - 3*((a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^
8 - 8*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^7 + 32*(a^4 + 14*a^3 + 73*a^2
 + 168*a + 144)*x^6 + a^6 - 80*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^5 +
14*a^5 - 2*(a^5 - 50*a^4 - 823*a^3 - 4504*a^2 - 10608*a - 9216)*x^4 + 73*a^
4 + 8*(a^5 - 2*a^4 - 151*a^3 - 1000*a^2 - 2544*a - 2304)*x^3 + 168*a^3 - 16
*(a^5 + 10*a^4 + 17*a^3 - 124*a^2 - 528*a - 576)*x^2 + 144*a^2 + 16*(a^5 +
14*a^4 + 73*a^3 + 168*a^2 + 144*a)*x)*sqrt((105*a^4 + 1470*a^3 + 7749*a^2 +
 (a^10 + 35*a^9 + 550*a^8 + 5110*a^7 + 31085*a^6 + 129367*a^5 + 373020*a^4
+ 735840*a^3 + 950400*a^2 + 725760*a + 248832)*sqrt((2401*a^4 + 33124*a^3 +
 171966*a^2 + 398164*a + 346921)/(a^15 + 50*a^14 + 1165*a^13 + 16780*a^12 +
 167090*a^11 + 1218460*a^10 + 6722130*a^9 + 28570320*a^8 + 94320045*a^7 + 2
41870050*a^6 + 477857313*a^5 + 714317940*a^4 + 782071200*a^3 + 592064640*a^
2 + 277136640*a + 60466176)) + 18228*a + 16144)/(a^10 + 35*a^9 + 550*a^8 +
5110*a^7 + 31085*a^6 + 129367*a^5 + 373020*a^4 + 735840*a^3 + 950400*a^2 +
725760*a + 248832))*log(-64827*a^4 - 907578*a^3 - 4780647*a^2 + 27*(2401*a^
4 + 33614*a^3 + 177061*a^2 + 415884*a + 367536)*x + 27*(343*a^7 + 8981*a^6
+ 100811*a^5 + 628887*a^4 + 2354874*a^3 + 5293208*a^2 - (11*a^12 + 462*a^11
 + 8881*a^10 + 103320*a^9 + 810205*a^8 + 4511542*a^7 + 18292039*a^6 + 54410
692*a^5 + 117844800*a^4 + 181238400*a^3 + 187875072*a^2 + 117863424*a + 338
41152)*sqrt((2401*a^4 + 33124*a^3 + 171966*a^2 + 398164*a + 346921)/(a^15 +
 50*a^14 + 1165*a^13 + 16780*a^12 + 167090*a^11 + 1218460*a^10 + 6722130*a^
9 + 28570320*a^8 + 94320045*a^7 + 241870050*a^6 + 477857313*a^5 + 714317940
*a^4 + 782071200*a^3 + 592064640*a^2 + 277136640*a + 60466176)) + 6613472*a
 + 3543424)*sqrt((105*a^4 + 1470*a^3 + 7749*a^2 + (a^10 + 35*a^9 + 550*a^8
+ 5110*a^7 + 31085*a^6 + 129367*a^5 + 373020*a^4 + 735840*a^3 + 950400*a^2
+ 725760*a + 248832)*sqrt((2401*a^4 + 33124*a^3 + 171966*a^2 + 398164*a + 3
46921)/(a^15 + 50*a^14 + 1165*a^13 + 16780*a^12 + 167090*a^11 + 1218460*a^1
0 + 6722130*a^9 + 28570320*a^8 + 94320045*a^7 + 241870050*a^6 + 477857313*a
^5 + 714317940*a^4 + 782071200*a^3 + 592064640*a^2 + 277136640*a + 60466176
)) + 18228*a + 16144)/(a^10 + 35*a^9 + 550*a^8 + 5110*a^7 + 31085*a^6 + 129
367*a^5 + 373020*a^4 + 735840*a^3 + 950400*a^2 + 725760*a + 248832)) - 1122
8868*a - 9923472) + 3*((a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^8 - 8*(a^4 +
 14*a^3 + 73*a^2 + 168*a + 144)*x^7 + 32*(a^4 + 14*a^3 + 73*a^2 + 168*a + 1
44)*x^6 + a^6 - 80*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^5 + 14*a^5 - 2*(
a^5 - 50*a^4 - 823*a^3 - 4504*a^2 - 10608*a - 9216)*x^4 + 73*a^4 + 8*(a^5 -
 2*a^4 - 151*a^3 - 1000*a^2 - 2544*a - 2304)*x^3 + 168*a^3 - 16*(a^5 + 10*a
^4 + 17*a^3 - 124*a^2 - 528*a - 576)*x^2 + 144*a^2 + 16*(a^5 + 14*a^4 + 73*
a^3 + 168*a^2 + 144*a)*x)*sqrt((105*a^4 + 1470*a^3 + 7749*a^2 + (a^10 + 35*
a^9 + 550*a^8 + 5110*a^7 + 31085*a^6 + 129367*a^5 + 373020*a^4 + 735840*a^3
 + 950400*a^2 + 725760*a + 248832)*sqrt((2401*a^4 + 33124*a^3 + 171966*a^2
+ 398164*a + 346921)/(a^15 + 50*a^14 + 1165*a^13 + 16780*a^12 + 167090*a^11
 + 1218460*a^10 + 6722130*a^9 + 28570320*a^8 + 94320045*a^7 + 241870050*a^6
 + 477857313*a^5 + 714317940*a^4 + 782071200*a^3 + 592064640*a^2 + 27713664
0*a + 60466176)) + 18228*a + 16144)/(a^10 + 35*a^9 + 550*a^8 + 5110*a^7 + 3
1085*a^6 + 129367*a^5 + 373020*a^4 + 735840*a^3 + 950400*a^2 + 725760*a + 2

48832))*log(-64827*a^4 - 907578*a^3 - 4780647*a^2 + 27*(2401*a^4 + 33614*a^
3 + 177061*a^2 + 415884*a + 367536)*x - 27*(343*a^7 + 8981*a^6 + 100811*a^5
 + 628887*a^4 + 2354874*a^3 + 5293208*a^2 - (11*a^12 + 462*a^11 + 8881*a^10
 + 103320*a^9 + 810205*a^8 + 4511542*a^7 + 18292039*a^6 + 54410692*a^5 + 11
7844800*a^4 + 181238400*a^3 + 187875072*a^2 + 117863424*a + 33841152)*sqrt(
(2401*a^4 + 33124*a^3 + 171966*a^2 + 398164*a + 346921)/(a^15 + 50*a^14 + 1
165*a^13 + 16780*a^12 + 167090*a^11 + 1218460*a^10 + 6722130*a^9 + 28570320
*a^8 + 94320045*a^7 + 241870050*a^6 + 477857313*a^5 + 714317940*a^4 + 78207
1200*a^3 + 592064640*a^2 + 277136640*a + 60466176)) + 6613472*a + 3543424)*
sqrt((105*a^4 + 1470*a^3 + 7749*a^2 + (a^10 + 35*a^9 + 550*a^8 + 5110*a^7 +
 31085*a^6 + 129367*a^5 + 373020*a^4 + 735840*a^3 + 950400*a^2 + 725760*a +
 248832)*sqrt((2401*a^4 + 33124*a^3 + 171966*a^2 + 398164*a + 346921)/(a^15
 + 50*a^14 + 1165*a^13 + 16780*a^12 + 167090*a^11 + 1218460*a^10 + 6722130*
a^9 + 28570320*a^8 + 94320045*a^7 + 241870050*a^6 + 477857313*a^5 + 7143179
40*a^4 + 782071200*a^3 + 592064640*a^2 + 277136640*a + 60466176)) + 18228*a
 + 16144)/(a^10 + 35*a^9 + 550*a^8 + 5110*a^7 + 31085*a^6 + 129367*a^5 + 37
3020*a^4 + 735840*a^3 + 950400*a^2 + 725760*a + 248832)) - 11228868*a - 992
3472) - 3*((a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^8 - 8*(a^4 + 14*a^3 + 73
*a^2 + 168*a + 144)*x^7 + 32*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^6 + a^
6 - 80*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^5 + 14*a^5 - 2*(a^5 - 50*a^4
 - 823*a^3 - 4504*a^2 - 10608*a - 9216)*x^4 + 73*a^4 + 8*(a^5 - 2*a^4 - 151
*a^3 - 1000*a^2 - 2544*a - 2304)*x^3 + 168*a^3 - 16*(a^5 + 10*a^4 + 17*a^3
 - 124*a^2 - 528*a - 576)*x^2 + 144*a^2 + 16*(a^5 + 14*a^4 + 73*a^3 + 168*a^
2 + 144*a)*x)*sqrt((105*a^4 + 1470*a^3 + 7749*a^2 - (a^10 + 35*a^9 + 550*a^
8 + 5110*a^7 + 31085*a^6 + 129367*a^5 + 373020*a^4 + 735840*a^3 + 950400*a^
2 + 725760*a + 248832)*sqrt((2401*a^4 + 33124*a^3 + 171966*a^2 + 398164*a +
 346921)/(a^15 + 50*a^14 + 1165*a^13 + 16780*a^12 + 167090*a^11 + 1218460*a
^10 + 6722130*a^9 + 28570320*a^8 + 94320045*a^7 + 241870050*a^6 + 477857313
*a^5 + 714317940*a^4 + 782071200*a^3 + 592064640*a^2 + 277136640*a + 604661
76)) + 18228*a + 16144)/(a^10 + 35*a^9 + 550*a^8 + 5110*a^7 + 31085*a^6 + 1
29367*a^5 + 373020*a^4 + 735840*a^3 + 950400*a^2 + 725760*a + 248832))*log(
-64827*a^4 - 907578*a^3 - 4780647*a^2 + 27*(2401*a^4 + 33614*a^3 + 177061*a
^2 + 415884*a + 367536)*x + 27*(343*a^7 + 8981*a^6 + 100811*a^5 + 628887*a^
4 + 2354874*a^3 + 5293208*a^2 + (11*a^12 + 462*a^11 + 8881*a^10 + 103320*a^
9 + 810205*a^8 + 4511542*a^7 + 18292039*a^6 + 54410692*a^5 + 117844800*a^4
+ 181238400*a^3 + 187875072*a^2 + 117863424*a + 33841152)*sqrt((2401*a^4 +
33124*a^3 + 171966*a^2 + 398164*a + 346921)/(a^15 + 50*a^14 + 1165*a^13 + 1
6780*a^12 + 167090*a^11 + 1218460*a^10 + 6722130*a^9 + 28570320*a^8 + 94320
045*a^7 + 241870050*a^6 + 477857313*a^5 + 714317940*a^4 + 782071200*a^3 + 5
92064640*a^2 + 277136640*a + 60466176)) + 6613472*a + 3543424)*sqrt((105*a^
4 + 1470*a^3 + 7749*a^2 - (a^10 + 35*a^9 + 550*a^8 + 5110*a^7 + 31085*a^6 +
 129367*a^5 + 373020*a^4 + 735840*a^3 + 950400*a^2 + 725760*a + 248832)*sqr
t((2401*a^4 + 33124*a^3 + 171966*a^2 + 398164*a + 346921)/(a^15 + 50*a^14 +
 1165*a^13 + 16780*a^12 + 167090*a^11 + 1218460*a^10 + 6722130*a^9 + 285703
20*a^8 + 94320045*a^7 + 241870050*a^6 + 477857313*a^5 + 714317940*a^4 + 782

071200*a^3 + 592064640*a^2 + 277136640*a + 60466176)) + 18228*a + 16144)/(a
^10 + 35*a^9 + 550*a^8 + 5110*a^7 + 31085*a^6 + 129367*a^5 + 373020*a^4 + 7
35840*a^3 + 950400*a^2 + 725760*a + 248832)) - 11228868*a - 9923472) + 3*((
a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^8 - 8*(a^4 + 14*a^3 + 73*a^2 + 168*a
 + 144)*x^7 + 32*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^6 + a^6 - 80*(a^4
+ 14*a^3 + 73*a^2 + 168*a + 144)*x^5 + 14*a^5 - 2*(a^5 - 50*a^4 - 823*a^3 -
 4504*a^2 - 10608*a - 9216)*x^4 + 73*a^4 + 8*(a^5 - 2*a^4 - 151*a^3 - 1000*
a^2 - 2544*a - 2304)*x^3 + 168*a^3 - 16*(a^5 + 10*a^4 + 17*a^3 - 124*a^2 -
528*a - 576)*x^2 + 144*a^2 + 16*(a^5 + 14*a^4 + 73*a^3 + 168*a^2 + 144*a)*x
)*sqrt((105*a^4 + 1470*a^3 + 7749*a^2 - (a^10 + 35*a^9 + 550*a^8 + 5110*a^7
 + 31085*a^6 + 129367*a^5 + 373020*a^4 + 735840*a^3 + 950400*a^2 + 725760*a
 + 248832)*sqrt((2401*a^4 + 33124*a^3 + 171966*a^2 + 398164*a + 346921)/(a^
15 + 50*a^14 + 1165*a^13 + 16780*a^12 + 167090*a^11 + 1218460*a^10 + 672213
0*a^9 + 28570320*a^8 + 94320045*a^7 + 241870050*a^6 + 477857313*a^5 + 71431
7940*a^4 + 782071200*a^3 + 592064640*a^2 + 277136640*a + 60466176)) + 18228
*a + 16144)/(a^10 + 35*a^9 + 550*a^8 + 5110*a^7 + 31085*a^6 + 129367*a^5 +
373020*a^4 + 735840*a^3 + 950400*a^2 + 725760*a + 248832))*log(-64827*a^4 -
 907578*a^3 - 4780647*a^2 + 27*(2401*a^4 + 33614*a^3 + 177061*a^2 + 415884*
a + 367536)*x - 27*(343*a^7 + 8981*a^6 + 100811*a^5 + 628887*a^4 + 2354874*
a^3 + 5293208*a^2 + (11*a^12 + 462*a^11 + 8881*a^10 + 103320*a^9 + 810205*a
^8 + 4511542*a^7 + 18292039*a^6 + 54410692*a^5 + 117844800*a^4 + 181238400*
a^3 + 187875072*a^2 + 117863424*a + 33841152)*sqrt((2401*a^4 + 33124*a^3 +
171966*a^2 + 398164*a + 346921)/(a^15 + 50*a^14 + 1165*a^13 + 16780*a^12 +
167090*a^11 + 1218460*a^10 + 6722130*a^9 + 28570320*a^8 + 94320045*a^7 + 24
1870050*a^6 + 477857313*a^5 + 714317940*a^4 + 782071200*a^3 + 592064640*a^2
 + 277136640*a + 60466176)) + 6613472*a + 3543424)*sqrt((105*a^4 + 1470*a^3
 + 7749*a^2 - (a^10 + 35*a^9 + 550*a^8 + 5110*a^7 + 31085*a^6 + 129367*a^5
+ 373020*a^4 + 735840*a^3 + 950400*a^2 + 725760*a + 248832)*sqrt((2401*a^4
+ 33124*a^3 + 171966*a^2 + 398164*a + 346921)/(a^15 + 50*a^14 + 1165*a^13 +
 16780*a^12 + 167090*a^11 + 1218460*a^10 + 6722130*a^9 + 28570320*a^8 + 943
20045*a^7 + 241870050*a^6 + 477857313*a^5 + 714317940*a^4 + 782071200*a^3 +
 592064640*a^2 + 277136640*a + 60466176)) + 18228*a + 16144)/(a^10 + 35*a^9
 + 550*a^8 + 5110*a^7 + 31085*a^6 + 129367*a^5 + 373020*a^4 + 735840*a^3 +
950400*a^2 + 725760*a + 248832)) - 11228868*a - 9923472) + 524*a^2 - 4*(11*
a^3 + 107*a^2 - 84*a - 1152)*x + 1632*a + 1152)/((a^4 + 14*a^3 + 73*a^2 + 1
68*a + 144)*x^8 - 8*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^7 + 32*(a^4 + 1
4*a^3 + 73*a^2 + 168*a + 144)*x^6 + a^6 - 80*(a^4 + 14*a^3 + 73*a^2 + 168*a
 + 144)*x^5 + 14*a^5 - 2*(a^5 - 50*a^4 - 823*a^3 - 4504*a^2 - 10608*a - 921
6)*x^4 + 73*a^4 + 8*(a^5 - 2*a^4 - 151*a^3 - 1000*a^2 - 2544*a - 2304)*x^3
+ 168*a^3 - 16*(a^5 + 10*a^4 + 17*a^3 - 124*a^2 - 528*a - 576)*x^2 + 144*a^
2 + 16*(a^5 + 14*a^4 + 73*a^3 + 168*a^2 + 144*a)*x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**4+4*x**3-8*x**2+a+8*x)**3,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.123 $\int x\left(a+8x-8x^2+4x^3-x^4\right)^4\,dx$

Optimal. Leaf size=210

$$\frac{1}{5}\left(3a^2-1536a+20480\right)x^{10}-\frac{16}{3}\left(a^2-128a+896\right)x^9-\frac{32}{7}\left(15a^2-288a+512\right)x^7+\frac{2}{3}\left(-a^3+192a^2-1536a+1024\right)$$

```
[Out] (a^4*x^2)/2 + (32*a^3*x^3)/3 + 8*(12 - a)*a^2*x^4 + (16*a*(128 - 48*a + a^2
)*x^5)/5 + (2*(1024 - 1536*a + 192*a^2 - a^3)*x^6)/3 - (32*(512 - 288*a + 1
5*a^2)*x^7)/7 + 8*(128 - 3*a)*(4 - a)*x^8 - (16*(896 - 128*a + a^2)*x^9)/3
+ ((20480 - 1536*a + 3*a^2)*x^10)/5 - (32*(928 - 35*a)*x^11)/11 + (8*(524 -
 9*a)*x^12)/3 - (16*(464 - 3*a)*x^13)/13 + (2*(640 - a)*x^14)/7 - (224*x^15
)/5 + 8*x^16 - (16*x^17)/17 + x^18/18
```

Rubi [A] time = 0.225637, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {6742}

$$\frac{1}{5}\left(3a^2-1536a+20480\right)x^{10}-\frac{16}{3}\left(a^2-128a+896\right)x^9-\frac{32}{7}\left(15a^2-288a+512\right)x^7+\frac{2}{3}\left(-a^3+192a^2-1536a+1024\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^4,x]
```

```
[Out] (a^4*x^2)/2 + (32*a^3*x^3)/3 + 8*(12 - a)*a^2*x^4 + (16*a*(128 - 48*a + a^2
)*x^5)/5 + (2*(1024 - 1536*a + 192*a^2 - a^3)*x^6)/3 - (32*(512 - 288*a + 1
5*a^2)*x^7)/7 + 8*(128 - 3*a)*(4 - a)*x^8 - (16*(896 - 128*a + a^2)*x^9)/3
+ ((20480 - 1536*a + 3*a^2)*x^10)/5 - (32*(928 - 35*a)*x^11)/11 + (8*(524 -
 9*a)*x^12)/3 - (16*(464 - 3*a)*x^13)/13 + (2*(640 - a)*x^14)/7 - (224*x^15
)/5 + 8*x^16 - (16*x^17)/17 + x^18/18
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x\left(a+8x-8x^2+4x^3-x^4\right)^4\,dx = \int \left(a^4x+32a^3x^2-32(-12+a)a^2x^3+16a\left(128-48a+a^2\right)x^4-4\left(-1024+1536\right.\right.$$
$$= \frac{a^4x^2}{2}+\frac{32a^3x^3}{3}+8(12-a)a^2x^4+\frac{16}{5}a\left(128-48a+a^2\right)x^5+\frac{2}{3}\left(1024-1536a+1\right.$$

Mathematica [A] time = 0.0258348, size = 204, normalized size = 0.97

$$\frac{1}{5}\left(3a^2-1536a+20480\right)x^{10}-\frac{16}{3}\left(a^2-128a+896\right)x^9+8\left(3a^2-140a+512\right)x^8-\frac{32}{7}\left(15a^2-288a+512\right)x^7-\frac{2}{3}\left(a^3\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^4,x]
```

```
[Out] (a^4*x^2)/2 + (32*a^3*x^3)/3 - 8*(-12 + a)*a^2*x^4 + (16*a*(128 - 48*a + a^
2)*x^5)/5 - (2*(-1024 + 1536*a - 192*a^2 + a^3)*x^6)/3 - (32*(512 - 288*a +
 15*a^2)*x^7)/7 + 8*(512 - 140*a + 3*a^2)*x^8 - (16*(896 - 128*a + a^2)*x^9
)/3 + ((20480 - 1536*a + 3*a^2)*x^10)/5 + (32*(-928 + 35*a)*x^11)/11 - (8*(
-524 + 9*a)*x^12)/3 + (16*(-464 + 3*a)*x^13)/13 - (2*(-640 + a)*x^14)/7 - (
224*x^15)/5 + 8*x^16 - (16*x^17)/17 + x^18/18
```

Maple [A] time = 0.001, size = 267, normalized size = 1.3

$$\frac{x^{18}}{18}-\frac{16\,x^{17}}{17}+8\,x^{16}-\frac{224\,x^{15}}{5}+\frac{(-4\,a+2560)\,x^{14}}{14}+\frac{(48\,a-7424)\,x^{13}}{13}+\frac{(-288\,a+16768)\,x^{12}}{12}+\frac{(1120\,a-29696)\,x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-x^4+4*x^3-8*x^2+a+8*x)^4,x)
```

```
[Out] 1/18*x^18-16/17*x^17+8*x^16-224/5*x^15+1/14*(-4*a+2560)*x^14+1/13*(48*a-742
4)*x^13+1/12*(-288*a+16768)*x^12+1/11*(1120*a-29696)*x^11+1/10*(2*a^2-2560*
a+24576+(-2*a+128)^2)*x^10+1/9*(-16*a^2+3584*a-10240+2*(8*a-128)*(-2*a+128)
)*x^9+1/8*(64*a^2-2560*a+2*(-16*a+64)*(-2*a+128)+(8*a-128)^2)*x^8+1/7*(-160
*a^2+32*a*(-2*a+128)+2*(-16*a+64)*(8*a-128))*x^7+1/6*(2*a^2*(-2*a+128)+32*a
*(8*a-128)+(-16*a+64)^2)*x^6+1/5*(2*a^2*(8*a-128)+32*a*(-16*a+64))*x^5+1/4*
(2*a^2*(-16*a+64)+256*a^2)*x^4+32/3*a^3*x^3+1/2*a^4*x^2
```

Maxima [A] time = 1.10301, size = 246, normalized size = 1.17

$$\frac{1}{18}\,x^{18}-\frac{16}{17}\,x^{17}+8\,x^{16}-\frac{2}{7}\,(a-640)x^{14}-\frac{224}{5}\,x^{15}+\frac{16}{13}\,(3\,a-464)x^{13}-\frac{8}{3}\,(9\,a-524)x^{12}+\frac{32}{11}\,(35\,a-928)x^{11}+\frac{1}{5}\,\big(3\,a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^4,x, algorithm="maxima")
```

```
[Out] 1/18*x^18 - 16/17*x^17 + 8*x^16 - 2/7*(a - 640)*x^14 - 224/5*x^15 + 16/13*(
3*a - 464)*x^13 - 8/3*(9*a - 524)*x^12 + 32/11*(35*a - 928)*x^11 + 1/5*(3*a
^2 - 1536*a + 20480)*x^10 - 16/3*(a^2 - 128*a + 896)*x^9 + 8*(3*a^2 - 140*a
 + 512)*x^8 - 32/7*(15*a^2 - 288*a + 512)*x^7 - 2/3*(a^3 - 192*a^2 + 1536*a
 - 1024)*x^6 + 1/2*a^4*x^2 + 32/3*a^3*x^3 + 16/5*(a^3 - 48*a^2 + 128*a)*x^5
 - 8*(a^3 - 12*a^2)*x^4
```

Fricas [A] time = 1.31534, size = 655, normalized size = 3.12

$$\frac{1}{18}x^{18}-\frac{16}{17}x^{17}+8x^{16}-\frac{224}{5}x^{15}-\frac{2}{7}x^{14}a+\frac{1280}{7}x^{14}+\frac{48}{13}x^{13}a-\frac{7424}{13}x^{13}-24x^{12}a+\frac{4192}{3}x^{12}+\frac{1120}{11}x^{11}a+\frac{3}{5}x^{10}a^2-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^4,x, algorithm="fricas")
```

```
[Out] 1/18*x^18 - 16/17*x^17 + 8*x^16 - 224/5*x^15 - 2/7*x^14*a + 1280/7*x^14 + 4
8/13*x^13*a - 7424/13*x^13 - 24*x^12*a + 4192/3*x^12 + 1120/11*x^11*a + 3/5
*x^10*a^2 - 29696/11*x^11 - 1536/5*x^10*a - 16/3*x^9*a^2 + 4096*x^10 + 2048
/3*x^9*a + 24*x^8*a^2 - 14336/3*x^9 - 1120*x^8*a - 480/7*x^7*a^2 - 2/3*x^6*
a^3 + 4096*x^8 + 9216/7*x^7*a + 128*x^6*a^2 + 16/5*x^5*a^3 - 16384/7*x^7 -
1024*x^6*a - 768/5*x^5*a^2 - 8*x^4*a^3 + 2048/3*x^6 + 2048/5*x^5*a + 96*x^4
*a^2 + 32/3*x^3*a^3 + 1/2*x^2*a^4
```

Sympy [A] time = 0.14828, size = 212, normalized size = 1.01

$$\frac{a^4x^2}{2}+\frac{32a^3x^3}{3}+\frac{x^{18}}{18}-\frac{16x^{17}}{17}+8x^{16}-\frac{224x^{15}}{5}+x^{14}\left(\frac{1280}{7}-\frac{2a}{7}\right)+x^{13}\left(\frac{48a}{13}-\frac{7424}{13}\right)+x^{12}\left(\frac{4192}{3}-24a\right)+x^{11}\left(\frac{1120}{11}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x**4+4*x**3-8*x**2+a+8*x)**4,x)
```

```
[Out] a**4*x**2/2 + 32*a**3*x**3/3 + x**18/18 - 16*x**17/17 + 8*x**16 - 224*x**15
/5 + x**14*(1280/7 - 2*a/7) + x**13*(48*a/13 - 7424/13) + x**12*(4192/3 - 2
4*a) + x**11*(1120*a/11 - 29696/11) + x**10*(3*a**2/5 - 1536*a/5 + 4096) +
x**9*(-16*a**2/3 + 2048*a/3 - 14336/3) + x**8*(24*a**2 - 1120*a + 4096) + x
**7*(-480*a**2/7 + 9216*a/7 - 16384/7) + x**6*(-2*a**3/3 + 128*a**2 - 1024*
a + 2048/3) + x**5*(16*a**3/5 - 768*a**2/5 + 2048*a/5) + x**4*(-8*a**3 + 96
*a**2)
```

Giac [A] time = 1.14848, size = 300, normalized size = 1.43

$$\frac{1}{18}\,x^{18}-\frac{16}{17}\,x^{17}+8\,x^{16}-\frac{2}{7}\,ax^{14}-\frac{224}{5}\,x^{15}+\frac{48}{13}\,ax^{13}+\frac{1280}{7}\,x^{14}-24\,ax^{12}-\frac{7424}{13}\,x^{13}+\frac{3}{5}\,a^{2}x^{10}+\frac{1120}{11}\,ax^{11}+\frac{4192}{3}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^4,x, algorithm="giac")
```

```
[Out] 1/18*x^18 - 16/17*x^17 + 8*x^16 - 2/7*a*x^14 - 224/5*x^15 + 48/13*a*x^13 +
1280/7*x^14 - 24*a*x^12 - 7424/13*x^13 + 3/5*a^2*x^10 + 1120/11*a*x^11 + 41
92/3*x^12 - 16/3*a^2*x^9 - 1536/5*a*x^10 - 29696/11*x^11 + 24*a^2*x^8 + 204
8/3*a*x^9 + 4096*x^10 - 2/3*a^3*x^6 - 480/7*a^2*x^7 - 1120*a*x^8 - 14336/3*
x^9 + 16/5*a^3*x^5 + 128*a^2*x^6 + 9216/7*a*x^7 + 4096*x^8 - 8*a^3*x^4 - 76
8/5*a^2*x^5 - 1024*a*x^6 - 16384/7*x^7 + 1/2*a^4*x^2 + 32/3*a^3*x^3 + 96*a^
2*x^4 + 2048/5*a*x^5 + 2048/3*x^6
```

3.124 $\int x\left(a+8x-8x^2+4x^3-x^4\right)^3\,dx$

Optimal. Leaf size=134

$$-\frac{1}{2}\left(a^2-128a+512\right)x^6+\frac{4}{5}\left(3a^2-96a+128\right)x^5+8a^2x^3+\frac{a^3x^2}{2}-\frac{3}{10}(256-a)x^{10}+\frac{8}{3}(64-a)x^9-4(70-3a)x^8+\frac{48}{7}($$

```
[Out] (a^3*x^2)/2 + 8*a^2*x^3 + 6*(8 - a)*a*x^4 + (4*(128 - 96*a + 3*a^2)*x^5)/5
- ((512 - 128*a + a^2)*x^6)/2 + (48*(48 - 5*a)*x^7)/7 - 4*(70 - 3*a)*x^8 +
(8*(64 - a)*x^9)/3 - (3*(256 - a)*x^10)/10 + (280*x^11)/11 - 6*x^12 + (12*x
^13)/13 - x^14/14
```

Rubi [A] time = 0.140292, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {6742}

$$-\frac{1}{2}\left(a^2-128a+512\right)x^6+\frac{4}{5}\left(3a^2-96a+128\right)x^5+8a^2x^3+\frac{a^3x^2}{2}-\frac{3}{10}(256-a)x^{10}+\frac{8}{3}(64-a)x^9-4(70-3a)x^8+\frac{48}{7}($$

Antiderivative was successfully verified.

```
[In] Int[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x]
```


```
[Out] (a^3*x^2)/2 + 8*a^2*x^3 + 6*(8 - a)*a*x^4 + (4*(128 - 96*a + 3*a^2)*x^5)/5
- ((512 - 128*a + a^2)*x^6)/2 + (48*(48 - 5*a)*x^7)/7 - 4*(70 - 3*a)*x^8 +
(8*(64 - a)*x^9)/3 - (3*(256 - a)*x^10)/10 + (280*x^11)/11 - 6*x^12 + (12*x
^13)/13 - x^14/14
```


Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x\left(a+8x-8x^2+4x^3-x^4\right)^3\,dx=\int\left(a^3x+24a^2x^2-24(-8+a)ax^3+4\left(128-96a+3a^2\right)x^4-3\left(512-128a+a^2\right)x\right.$$
$$=\frac{a^3x^2}{2}+8a^2x^3+6(8-a)ax^4+\frac{4}{5}\left(128-96a+3a^2\right)x^5-\frac{1}{2}\left(512-128a+a^2\right)x^6+\frac{4}{7}$$

Mathematica [A] time = 0.0152381, size = 130, normalized size = 0.97

$$\frac{1}{2}\left(-a^2+128a-512\right)x^6+\frac{4}{5}\left(3a^2-96a+128\right)x^5+8a^2x^3+\frac{a^3x^2}{2}+\frac{3}{10}(a-256)x^{10}-\frac{8}{3}(a-64)x^9+4(3a-70)x^8-\frac{48}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x]
```

```
[Out] (a^3*x^2)/2 + 8*a^2*x^3 - 6*(-8 + a)*a*x^4 + (4*(128 - 96*a + 3*a^2)*x^5)/5
 + ((-512 + 128*a - a^2)*x^6)/2 - (48*(-48 + 5*a)*x^7)/7 + 4*(-70 + 3*a)*x^
8 - (8*(-64 + a)*x^9)/3 + (3*(-256 + a)*x^10)/10 + (280*x^11)/11 - 6*x^12 +
 (12*x^13)/13 - x^14/14
```

Maple [A] time = 0.001, size = 143, normalized size = 1.1

$$-\frac{x^{14}}{14}+\frac{12\,x^{13}}{13}-6\,x^{12}+\frac{280\,x^{11}}{11}+\frac{(3\,a-768)\,x^{10}}{10}+\frac{(-24\,a+1536)\,x^9}{9}+\frac{(96\,a-2240)\,x^8}{8}+\frac{(-240\,a+2304)\,x^7}{7}+\frac{(a(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-x^4+4*x^3-8*x^2+a+8*x)^3,x)
```

```
[Out] -1/14*x^14+12/13*x^13-6*x^12+280/11*x^11+1/10*(3*a-768)*x^10+1/9*(-24*a+153
6)*x^9+1/8*(96*a-2240)*x^8+1/7*(-240*a+2304)*x^7+1/6*(a*(-2*a+128)+256*a-15
36-a^2)*x^6+1/5*(a*(8*a-128)-256*a+512+4*a^2)*x^5+1/4*(a*(-16*a+64)+128*a-8
*a^2)*x^4+8*a^2*x^3+1/2*a^3*x^2
```

Maxima [A] time = 1.11751, size = 153, normalized size = 1.14

$$-\frac{1}{14}\,x^{14}+\frac{12}{13}\,x^{13}-6\,x^{12}+\frac{3}{10}\,(a-256)x^{10}+\frac{280}{11}\,x^{11}-\frac{8}{3}\,(a-64)x^9+4\,(3\,a-70)x^8-\frac{48}{7}\,(5\,a-48)x^7-\frac{1}{2}\left(a^2-128\,a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="maxima")
```

```
[Out] -1/14*x^14 + 12/13*x^13 - 6*x^12 + 3/10*(a - 256)*x^10 + 280/11*x^11 - 8/3*
(a - 64)*x^9 + 4*(3*a - 70)*x^8 - 48/7*(5*a - 48)*x^7 - 1/2*(a^2 - 128*a +
```

```
512)*x^6 + 4/5*(3*a^2 - 96*a + 128)*x^5 + 1/2*a^3*x^2 + 8*a^2*x^3 - 6*(a^2
- 8*a)*x^4
```

Fricas [A] time = 1.30467, size = 367, normalized size = 2.74

$$-\frac{1}{14}x^{14} + \frac{12}{13}x^{13} - 6x^{12} + \frac{280}{11}x^{11} + \frac{3}{10}x^{10}a - \frac{384}{5}x^{10} - \frac{8}{3}x^{9}a + \frac{512}{3}x^{9} + 12x^{8}a - 280x^{8} - \frac{240}{7}x^{7}a - \frac{1}{2}x^{6}a^{2} + \frac{2304}{7}x^{7} + 6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="fricas")
```

```
[Out] -1/14*x^14 + 12/13*x^13 - 6*x^12 + 280/11*x^11 + 3/10*x^10*a - 384/5*x^10 -
 8/3*x^9*a + 512/3*x^9 + 12*x^8*a - 280*x^8 - 240/7*x^7*a - 1/2*x^6*a^2 + 2
304/7*x^7 + 64*x^6*a + 12/5*x^5*a^2 - 256*x^6 - 384/5*x^5*a - 6*x^4*a^2 + 5
12/5*x^5 + 48*x^4*a + 8*x^3*a^2 + 1/2*x^2*a^3
```

Sympy [A] time = 0.124691, size = 128, normalized size = 0.96

$$\frac{a^{3}x^{2}}{2} + 8a^{2}x^{3} - \frac{x^{14}}{14} + \frac{12x^{13}}{13} - 6x^{12} + \frac{280x^{11}}{11} + x^{10}\left(\frac{3a}{10} - \frac{384}{5}\right) + x^{9}\left(\frac{512}{3} - \frac{8a}{3}\right) + x^{8}\left(12a - 280\right) + x^{7}\left(\frac{2304}{7} - \frac{240a}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x**4+4*x**3-8*x**2+a+8*x)**3,x)
```

```
[Out] a**3*x**2/2 + 8*a**2*x**3 - x**14/14 + 12*x**13/13 - 6*x**12 + 280*x**11/11
 + x**10*(3*a/10 - 384/5) + x**9*(512/3 - 8*a/3) + x**8*(12*a - 280) + x**7
*(2304/7 - 240*a/7) + x**6*(-a**2/2 + 64*a - 256) + x**5*(12*a**2/5 - 384*a
/5 + 512/5) + x**4*(-6*a**2 + 48*a)
```

Giac [A] time = 1.13084, size = 180, normalized size = 1.34

$$-\frac{1}{14}x^{14} + \frac{12}{13}x^{13} - 6x^{12} + \frac{3}{10}ax^{10} + \frac{280}{11}x^{11} - \frac{8}{3}ax^{9} - \frac{384}{5}x^{10} + 12ax^{8} + \frac{512}{3}x^{9} - \frac{1}{2}a^{2}x^{6} - \frac{240}{7}ax^{7} - 280x^{8} + \frac{12}{5}a^{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="giac")
```

```
[Out] -1/14*x^14 + 12/13*x^13 - 6*x^12 + 3/10*a*x^10 + 280/11*x^11 - 8/3*a*x^9 -
384/5*x^10 + 12*a*x^8 + 512/3*x^9 - 1/2*a^2*x^6 - 240/7*a*x^7 - 280*x^8 + 1
2/5*a^2*x^5 + 64*a*x^6 + 2304/7*x^7 - 6*a^2*x^4 - 384/5*a*x^5 - 256*x^6 + 1
/2*a^3*x^2 + 8*a^2*x^3 + 48*a*x^4 + 512/5*x^5
```

3.125 $\int x\left(a+8x-8x^2+4x^3-x^4\right)^2\,dx$

Optimal. Leaf size=79

$$\frac{a^2x^2}{2}+\frac{1}{3}(64-a)x^6-\frac{8}{5}(16-a)x^5+4(4-a)x^4+\frac{16ax^3}{3}+\frac{x^{10}}{10}-\frac{8x^9}{9}+4x^8-\frac{80x^7}{7}$$

```
[Out] (a^2*x^2)/2 + (16*a*x^3)/3 + 4*(4 - a)*x^4 - (8*(16 - a)*x^5)/5 + ((64 - a)
*x^6)/3 - (80*x^7)/7 + 4*x^8 - (8*x^9)/9 + x^10/10
```

Rubi [A] time = 0.0759102, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {6742}

$$\frac{a^2x^2}{2}+\frac{1}{3}(64-a)x^6-\frac{8}{5}(16-a)x^5+4(4-a)x^4+\frac{16ax^3}{3}+\frac{x^{10}}{10}-\frac{8x^9}{9}+4x^8-\frac{80x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]
```

```
[Out] (a^2*x^2)/2 + (16*a*x^3)/3 + 4*(4 - a)*x^4 - (8*(16 - a)*x^5)/5 + ((64 - a)
*x^6)/3 - (80*x^7)/7 + 4*x^8 - (8*x^9)/9 + x^10/10
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int x\left(a+8x-8x^2+4x^3-x^4\right)^2\,dx &= \int \left(a^2x+16ax^2-16(-4+a)x^3+8(-16+a)x^4-2(-64+a)x^5-80x^6+32x^7-8x\right.\\ &= \frac{a^2x^2}{2}+\frac{16ax^3}{3}+4(4-a)x^4-\frac{8}{5}(16-a)x^5+\frac{1}{3}(64-a)x^6-\frac{80x^7}{7}+4x^8-\frac{8x^9}{9}+\frac{x^{10}}{10}\end{aligned}$$

Mathematica [A] time = 0.0078411, size = 75, normalized size = 0.95

$$\frac{a^2x^2}{2}+\frac{1}{3}(64-a)x^6+\frac{8}{5}(a-16)x^5-4(a-4)x^4+\frac{16ax^3}{3}+\frac{x^{10}}{10}-\frac{8x^9}{9}+4x^8-\frac{80x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]
```

```
[Out] (a^2*x^2)/2 + (16*a*x^3)/3 - 4*(-4 + a)*x^4 + (8*(-16 + a)*x^5)/5 + ((64 -
a)*x^6)/3 - (80*x^7)/7 + 4*x^8 - (8*x^9)/9 + x^10/10
```

Maple [A] time = 0.001, size = 66, normalized size = 0.8

$$\frac{x^{10}}{10}-\frac{8\,x^{9}}{9}+4\,x^{8}-\frac{80\,x^{7}}{7}+\frac{(-2\,a+128)\,x^{6}}{6}+\frac{(8\,a-128)\,x^{5}}{5}+\frac{(-16\,a+64)\,x^{4}}{4}+\frac{16\,ax^{3}}{3}+\frac{a^{2}x^{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-x^4+4*x^3-8*x^2+a+8*x)^2,x)
```

```
[Out] 1/10*x^10-8/9*x^9+4*x^8-80/7*x^7+1/6*(-2*a+128)*x^6+1/5*(8*a-128)*x^5+1/4*(
-16*a+64)*x^4+16/3*a*x^3+1/2*a^2*x^2
```

Maxima [A] time = 1.07956, size = 80, normalized size = 1.01

$$\frac{1}{10}\,x^{10}-\frac{8}{9}\,x^{9}+4\,x^{8}-\frac{1}{3}\,(a-64)x^{6}-\frac{80}{7}\,x^{7}+\frac{8}{5}\,(a-16)x^{5}-4\,(a-4)x^{4}+\frac{1}{2}\,a^{2}x^{2}+\frac{16}{3}\,ax^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="maxima")
```

```
[Out] 1/10*x^10 - 8/9*x^9 + 4*x^8 - 1/3*(a - 64)*x^6 - 80/7*x^7 + 8/5*(a - 16)*x^
5 - 4*(a - 4)*x^4 + 1/2*a^2*x^2 + 16/3*a*x^3
```

Fricas [A] time = 1.34024, size = 180, normalized size = 2.28

$$\frac{1}{10}x^{10}-\frac{8}{9}x^{9}+4x^{8}-\frac{80}{7}x^{7}-\frac{1}{3}x^{6}a+\frac{64}{3}x^{6}+\frac{8}{5}x^{5}a-\frac{128}{5}x^{5}-4x^{4}a+16x^{4}+\frac{16}{3}x^{3}a+\frac{1}{2}x^{2}a^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="fricas")
```

```
[Out] 1/10*x^10 - 8/9*x^9 + 4*x^8 - 80/7*x^7 - 1/3*x^6*a + 64/3*x^6 + 8/5*x^5*a -
 128/5*x^5 - 4*x^4*a + 16*x^4 + 16/3*x^3*a + 1/2*x^2*a^2
```

Sympy [A] time = 0.072107, size = 70, normalized size = 0.89

$$\frac{a^2x^2}{2} + \frac{16ax^3}{3} + \frac{x^{10}}{10} - \frac{8x^9}{9} + 4x^8 - \frac{80x^7}{7} + x^6\left(\frac{64}{3} - \frac{a}{3}\right) + x^5\left(\frac{8a}{5} - \frac{128}{5}\right) + x^4(16 - 4a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x**4+4*x**3-8*x**2+a+8*x)**2,x)
```

```
[Out] a**2*x**2/2 + 16*a*x**3/3 + x**10/10 - 8*x**9/9 + 4*x**8 - 80*x**7/7 + x**6
*(64/3 - a/3) + x**5*(8*a/5 - 128/5) + x**4*(16 - 4*a)
```

Giac [A] time = 1.12922, size = 92, normalized size = 1.16

$$\frac{1}{10}x^{10} - \frac{8}{9}x^9 + 4x^8 - \frac{1}{3}ax^6 - \frac{80}{7}x^7 + \frac{8}{5}ax^5 + \frac{64}{3}x^6 - 4ax^4 - \frac{128}{5}x^5 + \frac{1}{2}a^2x^2 + \frac{16}{3}ax^3 + 16x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="giac")
```

```
[Out] 1/10*x^10 - 8/9*x^9 + 4*x^8 - 1/3*a*x^6 - 80/7*x^7 + 8/5*a*x^5 + 64/3*x^6 -
 4*a*x^4 - 128/5*x^5 + 1/2*a^2*x^2 + 16/3*a*x^3 + 16*x^4
```

3.126 $\int x\left(a+8x-8x^2+4x^3-x^4\right)\,dx$

Optimal. Leaf size=35

$$\frac{ax^2}{2}-\frac{x^6}{6}+\frac{4x^5}{5}-2x^4+\frac{8x^3}{3}$$

```
[Out] (a*x^2)/2 + (8*x^3)/3 - 2*x^4 + (4*x^5)/5 - x^6/6
```

Rubi [A] time = 0.0099643, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {14}

$$\frac{ax^2}{2}-\frac{x^6}{6}+\frac{4x^5}{5}-2x^4+\frac{8x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4),x]
```

```
[Out] (a*x^2)/2 + (8*x^3)/3 - 2*x^4 + (4*x^5)/5 - x^6/6
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x\left(a+8x-8x^2+4x^3-x^4\right)\,dx&=\int\left(ax+8x^2-8x^3+4x^4-x^5\right)\,dx\\&=\frac{ax^2}{2}+\frac{8x^3}{3}-2x^4+\frac{4x^5}{5}-\frac{x^6}{6}\end{aligned}$$

Mathematica [A] time = 0.0013309, size = 35, normalized size = 1.

$$\frac{ax^2}{2}-\frac{x^6}{6}+\frac{4x^5}{5}-2x^4+\frac{8x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4),x]
```

```
[Out] (a*x^2)/2 + (8*x^3)/3 - 2*x^4 + (4*x^5)/5 - x^6/6
```

Maple [A] time = 0.001, size = 28, normalized size = 0.8

$$\frac{ax^2}{2}+\frac{8\,x^3}{3}-2\,x^4+\frac{4\,x^5}{5}-\frac{x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-x^4+4*x^3-8*x^2+a+8*x),x)
```

```
[Out] 1/2*a*x^2+8/3*x^3-2*x^4+4/5*x^5-1/6*x^6
```

Maxima [A] time = 1.08354, size = 36, normalized size = 1.03

$$-\frac{1}{6}\,x^6+\frac{4}{5}\,x^5-2\,x^4+\frac{1}{2}\,ax^2+\frac{8}{3}\,x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="maxima")
```

```
[Out] -1/6*x^6 + 4/5*x^5 - 2*x^4 + 1/2*a*x^2 + 8/3*x^3
```

Fricas [A] time = 1.2997, size = 68, normalized size = 1.94

$$-\frac{1}{6}x^6+\frac{4}{5}x^5-2x^4+\frac{8}{3}x^3+\frac{1}{2}x^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="fricas")
```

```
[Out] -1/6*x^6 + 4/5*x^5 - 2*x^4 + 8/3*x^3 + 1/2*x^2*a
```

Sympy [A] time = 0.057822, size = 29, normalized size = 0.83

$$\frac{ax^2}{2} - \frac{x^6}{6} + \frac{4x^5}{5} - 2x^4 + \frac{8x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x**4+4*x**3-8*x**2+a+8*x),x)
```

```
[Out] a*x**2/2 - x**6/6 + 4*x**5/5 - 2*x**4 + 8*x**3/3
```

Giac [A] time = 1.13136, size = 36, normalized size = 1.03

$$-\frac{1}{6}\,x^6 + \frac{4}{5}\,x^5 - 2\,x^4 + \frac{1}{2}\,ax^2 + \frac{8}{3}\,x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="giac")
```

```
[Out] -1/6*x^6 + 4/5*x^5 - 2*x^4 + 1/2*a*x^2 + 8/3*x^3
```

3.127 $\int \frac{x}{a+8x-8x^2+4x^3-x^4} \, dx$

Optimal. Leaf size=116

$$-\frac{\tan^{-1}\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{2\sqrt{a+4}\sqrt{1-\sqrt{a+4}}}+\frac{\tan^{-1}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{2\sqrt{a+4}\sqrt{\sqrt{a+4}+1}}+\frac{\tanh^{-1}\left(\frac{(x-1)^2+1}{\sqrt{a+4}}\right)}{2\sqrt{a+4}}$$

```
[Out] -ArcTan[(-1 + x)/Sqrt[1 - Sqrt[4 + a]]]/(2*Sqrt[4 + a]*Sqrt[1 - Sqrt[4 + a]
]) + ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]]/(2*Sqrt[4 + a]*Sqrt[1 + Sqrt[4
+ a]]) + ArcTanh[(1 + (-1 + x)^2)/Sqrt[4 + a]]/(2*Sqrt[4 + a])
```

Rubi [A] time = 0.0830145, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1680, 1673, 1093, 204, 1107, 618, 206}

$$-\frac{\tan^{-1}\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{2\sqrt{a+4}\sqrt{1-\sqrt{a+4}}}+\frac{\tan^{-1}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{2\sqrt{a+4}\sqrt{\sqrt{a+4}+1}}+\frac{\tanh^{-1}\left(\frac{(x-1)^2+1}{\sqrt{a+4}}\right)}{2\sqrt{a+4}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4),x]
```


```
[Out] -ArcTan[(-1 + x)/Sqrt[1 - Sqrt[4 + a]]]/(2*Sqrt[4 + a]*Sqrt[1 - Sqrt[4 + a]
]) + ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]]/(2*Sqrt[4 + a]*Sqrt[1 + Sqrt[4
+ a]]) + ArcTanh[(1 + (-1 + x)^2)/Sqrt[4 + a]]/(2*Sqrt[4 + a])
```


Rule 1680

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -(d/(4*e)) + x)*(a + d^4/(256*e^3) - (
b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /;
 EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq,
x] && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x}{a+8x-8x^2+4x^3-x^4}\,dx = \text{Subst}\left(\int \frac{1+x}{3+a-2x^2-x^4}\,dx, x, -1+x\right)$$
$$= \text{Subst}\left(\int \frac{1}{3+a-2x^2-x^4}\,dx, x, -1+x\right) + \text{Subst}\left(\int \frac{x}{3+a-2x^2-x^4}\,dx, x, -1+x\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{3+a-2x-x^2}\,dx, x, (-1+x)^2\right) - \frac{\text{Subst}\left(\int \frac{1}{-1-\sqrt{4+a}-x^2}\,dx, x, -1+x\right)}{2\sqrt{4+a}} + -$$
$$= \frac{\tan^{-1}\left(\frac{1-x}{\sqrt{1-\sqrt{4+a}}}\right)}{2\sqrt{4+a}\sqrt{1-\sqrt{4+a}}} - \frac{\tan^{-1}\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)}{2\sqrt{4+a}\sqrt{1+\sqrt{4+a}}} - \text{Subst}\left(\int \frac{1}{4(4+a)-x^2}\,dx, x, -2(1+\right.$$
$$= \frac{\tan^{-1}\left(\frac{1-x}{\sqrt{1-\sqrt{4+a}}}\right)}{2\sqrt{4+a}\sqrt{1-\sqrt{4+a}}} - \frac{\tan^{-1}\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)}{2\sqrt{4+a}\sqrt{1+\sqrt{4+a}}} + \frac{\tanh^{-1}\left(\frac{1+(-1+x)^2}{\sqrt{4+a}}\right)}{2\sqrt{4+a}}$$

Mathematica [C] time = 0.0156602, size = 59, normalized size = 0.51

$$-\frac{1}{4}\text{RootSum}\left[-\#1^4+4\#1^3-8\#1^2+8\#1+a\&, \frac{\#1\log(x-\#1)}{\#1^3-3\#1^2+4\#1-2}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4),x]
```

```
[Out] -RootSum[a + 8*#1 - 8*#1^2 + 4*#1^3 - #1^4 & , (Log[x - #1]*#1)/(-2 + 4*#1
- 3*#1^2 + #1^3) & ]/4
```

Maple [C] time = 0.003, size = 50, normalized size = 0.4

$$-\frac{1}{4}\sum_{\_R=\text{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)} \frac{\_R\ln(x-\_R)}{\_R^3-3\_R^2+4\_R-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^4+4*x^3-8*x^2+a+8*x),x)
```

```
[Out] -1/4*sum(_R/(_R^3-3*_R^2+4*_R-2)*ln(x-_R),_R=RootOf(_Z^4-4*_Z^3+8*_Z^2-8*_Z
-a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{x^4 - 4x^3 + 8x^2 - a - 8x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="maxima")
```

```
[Out] -integrate(x/(x^4 - 4*x^3 + 8*x^2 - a - 8*x), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 2.57714, size = 155, normalized size = 1.34

$$-\operatorname{RootSum}\left(t^4\left(256a^3+2816a^2+10240a+12288\right)+t^2\left(-32a^2-256a-512\right)+t\left(-16a-64\right)+a,\left(t\mapsto t\log\left(x+\frac{-1\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**4+4*x**3-8*x**2+a+8*x),x)
```

```
[Out] -RootSum(_t**4*(256*a**3 + 2816*a**2 + 10240*a + 12288) + _t**2*(-32*a**2 -
 256*a - 512) + _t*(-16*a - 64) + a, Lambda(_t, _t*log(x + (-128*_t**3*a**4
 - 1728*_t**3*a**3 - 8640*_t**3*a**2 - 18944*_t**3*a - 15360*_t**3 + 48*_t*
*2*a**3 + 464*_t**2*a**2 + 1472*_t**2*a + 1536*_t**2 + 8*_t*a**3 + 88*_t*a*
```

```
*2 + 312*_t*a + 352*_t - a**2 - 2*a)/(4*a**2 + 21*a + 28))))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.128 $\int \frac{x}{\left(a+8x-8x^2+4x^3-x^4\right)^2}\,dx$

Optimal. Leaf size=231

$$\frac{(x-1)\left(a+(x-1)^2+5\right)}{4\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)}+\frac{(x-1)^2+1}{4(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)}-\frac{\left(3a+\sqrt{a+4}+10\right)\tan^{-1}\left(\frac{\cdots}{\sqrt{\cdots}}\right.}{8(a+3)(a+4)^{3/2}\sqrt{1-\sqrt{a\cdots}}}$$

```
[Out] (1 + (-1 + x)^2)/(4*(4 + a)*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)) + ((5 + a
+ (-1 + x)^2)*(-1 + x))/(4*(12 + 7*a + a^2)*(3 + a - 2*(-1 + x)^2 - (-1 + x
)^4)) - ((10 + 3*a + Sqrt[4 + a])*ArcTan[(-1 + x)/Sqrt[1 - Sqrt[4 + a]]])/(
8*(3 + a)*(4 + a)^(3/2)*Sqrt[1 - Sqrt[4 + a]]) + ((10 + 3*a - Sqrt[4 + a])*
ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]])/(8*(3 + a)*(4 + a)^(3/2)*Sqrt[1 + S
qrt[4 + a]]) + ArcTanh[(1 + (-1 + x)^2)/Sqrt[4 + a]]/(4*(4 + a)^(3/2))
```

Rubi [A] time = 0.2405, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1680, 1673, 1092, 1166, 204, 1107, 614, 618, 206}

$$\frac{(x-1)\left(a+(x-1)^2+5\right)}{4\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)}+\frac{(x-1)^2+1}{4(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)}-\frac{\left(3a+\sqrt{a+4}+10\right)\tan^{-1}\left(\frac{\cdots}{\sqrt{\cdots}}\right.}{8(a+3)(a+4)^{3/2}\sqrt{1-\sqrt{a\cdots}}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]
```


```
[Out] (1 + (-1 + x)^2)/(4*(4 + a)*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)) + ((5 + a
+ (-1 + x)^2)*(-1 + x))/(4*(12 + 7*a + a^2)*(3 + a - 2*(-1 + x)^2 - (-1 + x
)^4)) - ((10 + 3*a + Sqrt[4 + a])*ArcTan[(-1 + x)/Sqrt[1 - Sqrt[4 + a]]])/(
8*(3 + a)*(4 + a)^(3/2)*Sqrt[1 - Sqrt[4 + a]]) + ((10 + 3*a - Sqrt[4 + a])*
ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]])/(8*(3 + a)*(4 + a)^(3/2)*Sqrt[1 + S
qrt[4 + a]]) + ArcTanh[(1 + (-1 + x)^2)/Sqrt[4 + a]]/(4*(4 + a)^(3/2))
```


Rule 1680

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -(d/(4*e)) + x)*(a + d^4/(256*e^3) - (
```

```
b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /;
 EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq,
x] && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1092

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x*(b^2 -
 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2
 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
```

```
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x}{\left(a+8x-8x^2+4x^3-x^4\right)^2}\,dx = \text{Subst}\left(\int \frac{1+x}{\left(3+a-2x^2-x^4\right)^2}\,dx, x, -1+x\right)$$

$$= \text{Subst}\left(\int \frac{1}{\left(3+a-2x^2-x^4\right)^2}\,dx, x, -1+x\right) + \text{Subst}\left(\int \frac{x}{\left(3+a-2x^2-x^4\right)^2}\,dx, x$$

$$= \frac{\left(5+a+(-1+x)^2\right)(-1+x)}{4\left(12+7a+a^2\right)\left(3+a-2(-1+x)^2-(-1+x)^4\right)} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{\left(3+a-2x-x^2\right)^2}$$

$$= \frac{1+(-1+x)^2}{4(4+a)\left(3+a-2(1-x)^2-(1-x)^4\right)} + \frac{\left(5+a+(-1+x)^2\right)(-1+x)}{4\left(12+7a+a^2\right)\left(3+a-2(-1+x)^2-(-1\right.}$$

$$= \frac{1+(-1+x)^2}{4(4+a)\left(3+a-2(1-x)^2-(1-x)^4\right)} + \frac{\left(5+a+(-1+x)^2\right)(-1+x)}{4\left(12+7a+a^2\right)\left(3+a-2(-1+x)^2-(-1\right.}$$

$$= \frac{1+(-1+x)^2}{4(4+a)\left(3+a-2(1-x)^2-(1-x)^4\right)} + \frac{\left(5+a+(-1+x)^2\right)(-1+x)}{4\left(12+7a+a^2\right)\left(3+a-2(-1+x)^2-(-1\right.}$$

Mathematica [C] time = 0.0595318, size = 166, normalized size = 0.72

$$\frac{ax^2-ax+a+x^3+2x}{4(a+3)(a+4)\left(a-x\left(x^3-4x^2+8x-8\right)\right)}-\frac{\text{RootSum}\left[-\#1^4+4\#1^3-8\#1^2+8\#1+a\&,\frac{\#1^2\log(x-\#1)+2\#1a\log(x-\#1)+a\log(}{\#1^3-3\#1^2-}\right.}{16\left(a^2+7a+12\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]
```

```
[Out] (a + 2*x - a*x + a*x^2 + x^3)/(4*(3 + a)*(4 + a)*(a - x*(-8 + 8*x - 4*x^2 +
 x^3))) - RootSum[a + 8*#1 - 8*#1^2 + 4*#1^3 - #1^4 & , (6*Log[x - #1] + a*
Log[x - #1] + 4*Log[x - #1]*#1 + 2*a*Log[x - #1]*#1 + Log[x - #1]*#1^2)/(-2
 + 4*#1 - 3*#1^2 + #1^3) & ]/(16*(12 + 7*a + a^2))
```

Maple [C] time = 0.01, size = 162, normalized size = 0.7

$$\frac{1}{x^4-4x^3+8x^2-a-8x}\left(-\frac{x^3}{4a^2+28a+48}-\frac{ax^2}{4a^2+28a+48}+\frac{(a-2)x}{4a^2+28a+48}-\frac{a}{4a^2+28a+48}\right)+\frac{1}{16a^2+112a+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^4+4*x^3-8*x^2+a+8*x)^2,x)
```

```
[Out] (-1/4/(a^2+7*a+12)*x^3-1/4*a/(a^2+7*a+12)*x^2+1/4*(a-2)/(a^2+7*a+12)*x-1/4*
a/(a^2+7*a+12))/(x^4-4*x^3+8*x^2-a-8*x)+1/16/(a^2+7*a+12)*sum((-6-_R^2+2*(-
a-2)*_R-a)/(_R^3-3*_R^2+4*_R-2)*ln(x-_R),_R=RootOf(_Z^4-4*_Z^3+8*_Z^2-8*_Z-
a))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: AttributeError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [B] time = 16.4623, size = 539, normalized size = 2.33

$$-\frac{ax^2 + a + x^3 + x(2-a)}{-4a^3 - 28a^2 - 48a + x^4\left(4a^2 + 28a + 48\right) + x^3\left(-16a^2 - 112a - 192\right) + x^2\left(32a^2 + 224a + 384\right) + x\left(-32a^2 - 224a - 3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**4+4*x**3-8*x**2+a+8*x)**2,x)
```

```
[Out] -(a*x**2 + a + x**3 + x*(2 - a))/(-4*a**3 - 28*a**2 - 48*a + x**4*(4*a**2 +
 28*a + 48) + x**3*(-16*a**2 - 112*a - 192) + x**2*(32*a**2 + 224*a + 384)
+ x*(-32*a**2 - 224*a - 384)) + RootSum(_t**4*(65536*a**9 + 2162688*a**8 +
31653888*a**7 + 269680640*a**6 + 1473773568*a**5 + 5357174784*a**4 + 129520
10752*a**3 + 20082327552*a**2 + 18119393280*a + 7247757312) + _t**2*(-2048*
a**6 - 50688*a**5 - 520704*a**4 - 2842624*a**3 - 8699904*a**2 - 14155776*a
- 9568256) + _t*(1152*a**4 + 17792*a**3 + 102912*a**2 + 264192*a + 253952)
+ 16*a**3 - 57*a**2 - 984*a - 2064, Lambda(_t, _t*log(x + (98304*_t**3*a**1
2 + 3948544*_t**3*a**11 + 72196096*_t**3*a**10 + 793837568*_t**3*a**9 + 583
9372288*_t**3*a**8 + 30226464768*_t**3*a**7 + 112668450816*_t**3*a**6 + 303
864643584*_t**3*a**5 + 586157391872*_t**3*a**4 + 784017129472*_t**3*a**3 +
683648483328*_t**3*a**2 + 343136010240*_t**3*a + 72477573120*_t**3 + 30208*
_t**2*a**10 + 986624*_t**2*a**9 + 14420992*_t**2*a**8 + 124156928*_t**2*a**
7 + 696815104*_t**2*a**6 + 2661758464*_t**2*a**5 + 7001485312*_t**2*a**4 +
12506562560*_t**2*a**3 + 14494924800*_t**2*a**2 + 9820569600*_t**2*a + 2944
401408*_t**2 - 1536*_t*a**9 - 52048*_t*a**8 - 757040*_t*a**7 - 6200656*_t*a
**6 - 31380496*_t*a**5 - 100736416*_t*a**4 - 200813696*_t*a**3 - 228144640*
_t*a**2 - 114632704*_t*a - 2490368*_t + 248*a**7 + 6797*a**6 + 71132*a**5 +
 369745*a**4 + 987758*a**3 + 1128896*a**2 - 129568*a - 956416)/(576*a**7 +
10985*a**6 + 88746*a**5 + 396609*a**4 + 1076268*a**3 + 1826304*a**2 + 18677
```

```
76*a + 917504))))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.129 $\int \frac{x}{\left(a+8x-8x^2+4x^3-x^4\right)^3}\,dx$

Optimal. Leaf size=349

$$\frac{(x-1)\left(a+(x-1)^2+5\right)}{8\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^2}-\frac{3\left(7a^2+\left(4\sqrt{a+4}+47\right)a+14\sqrt{a+4}+80\right)\tan^{-1}\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{64(a+3)^2(a+4)^{5/2}\sqrt{1-\sqrt{a+4}}}-\frac{3\left(-\cdots\right.}{\cdots}$$

```
[Out] (1 + (-1 + x)^2)/(8*(4 + a)*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)^2) + (3*(1
+ (-1 + x)^2))/(16*(4 + a)^2*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)) + ((5 + a
 + (-1 + x)^2)*(-1 + x))/(8*(12 + 7*a + a^2)*(3 + a - 2*(-1 + x)^2 - (-1 +
x)^4)^2) + (((6 + a)*(25 + 7*a) + 6*(7 + 2*a)*(-1 + x)^2)*(-1 + x))/(32*(3
+ a)^2*(4 + a)^2*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)) - (3*(80 + 7*a^2 + 14
*Sqrt[4 + a] + a*(47 + 4*Sqrt[4 + a]))*ArcTan[(-1 + x)/Sqrt[1 - Sqrt[4 + a]
]])/(64*(3 + a)^2*(4 + a)^(5/2)*Sqrt[1 - Sqrt[4 + a]]) - (3*(14 + 4*a - (80
 + 47*a + 7*a^2)/Sqrt[4 + a])*ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]])/(64*(
3 + a)^2*(4 + a)^2*Sqrt[1 + Sqrt[4 + a]]) + (3*ArcTanh[(1 + (-1 + x)^2)/Sqr
t[4 + a]])/(16*(4 + a)^(5/2))
```

Rubi [A] time = 0.368768, antiderivative size = 349, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {1680, 1673, 1092, 1178, 1166, 204, 1107, 614, 618, 206}

$$\frac{(x-1)\left(a+(x-1)^2+5\right)}{8\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^2}-\frac{3\left(7a^2+\left(4\sqrt{a+4}+47\right)a+14\sqrt{a+4}+80\right)\tan^{-1}\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{64(a+3)^2(a+4)^{5/2}\sqrt{1-\sqrt{a+4}}}-\frac{3\left(-\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x]
```

```
[Out] (1 + (-1 + x)^2)/(8*(4 + a)*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)^2) + (3*(1
+ (-1 + x)^2))/(16*(4 + a)^2*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)) + ((5 + a
 + (-1 + x)^2)*(-1 + x))/(8*(12 + 7*a + a^2)*(3 + a - 2*(-1 + x)^2 - (-1 +
x)^4)^2) + (((6 + a)*(25 + 7*a) + 6*(7 + 2*a)*(-1 + x)^2)*(-1 + x))/(32*(3
+ a)^2*(4 + a)^2*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)) - (3*(80 + 7*a^2 + 14
*Sqrt[4 + a] + a*(47 + 4*Sqrt[4 + a]))*ArcTan[(-1 + x)/Sqrt[1 - Sqrt[4 + a]
]])/(64*(3 + a)^2*(4 + a)^(5/2)*Sqrt[1 - Sqrt[4 + a]]) - (3*(14 + 4*a - (80
 + 47*a + 7*a^2)/Sqrt[4 + a])*ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]])/(64*(
```

```
3 + a)^2*(4 + a)^2*Sqrt[1 + Sqrt[4 + a]]) + (3*ArcTanh[(1 + (-1 + x)^2)/Sqr
t[4 + a]])/(16*(4 + a)^(5/2))
```

Rule 1680

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -(d/(4*e)) + x)*(a + d^4/(256*e^3) - (
b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /;
 EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq,
x] && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1092

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x*(b^2 -
 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2
 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
```

```
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^3}\,dx = \text{Subst}\left(\int \frac{1+x}{(3+a-2x^2-x^4)^3}\,dx, x, -1+x\right)$$

$$= \text{Subst}\left(\int \frac{1}{(3+a-2x^2-x^4)^3}\,dx, x, -1+x\right) + \text{Subst}\left(\int \frac{x}{(3+a-2x^2-x^4)^3}\,dx, x, -\right.$$

$$= \frac{(5+a+(-1+x)^2)(-1+x)}{8(12+7a+a^2)(3+a-2(-1+x)^2-(-1+x)^4)^2} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{(3+a-2x-x^2)^3}\right.$$

$$= \frac{1+(-1+x)^2}{8(4+a)(3+a-2(1-x)^2-(1-x)^4)^2} - \frac{((6+a)(25+7a)+6(7+2a)(1-x)^2)(1-}{32(12+7a+a^2)^2(3+a-2(1-x)^2-(1-}$$

$$= \frac{1+(-1+x)^2}{8(4+a)(3+a-2(1-x)^2-(1-x)^4)^2} + \frac{3(1+(-1+x)^2)}{16(4+a)^2(3+a-2(1-x)^2-(1-x)^4)} -$$

$$= \frac{1+(-1+x)^2}{8(4+a)(3+a-2(1-x)^2-(1-x)^4)^2} + \frac{3(1+(-1+x)^2)}{16(4+a)^2(3+a-2(1-x)^2-(1-x)^4)} -$$

$$= \frac{1+(-1+x)^2}{8(4+a)(3+a-2(1-x)^2-(1-x)^4)^2} + \frac{3(1+(-1+x)^2)}{16(4+a)^2(3+a-2(1-x)^2-(1-x)^4)} -$$

Mathematica [C] time = 0.119132, size = 284, normalized size = 0.81

$$\frac{1}{128}\left(-\frac{3\text{RootSum}\left[-\#1^4+4\#1^3-8\#1^2+8\#1+a\&, \frac{4\#1^2 a\log(x-\#1)+14\#1^2\log(x-\#1)+3a^2\log(x-\#1)+4\#1a^2\log(x-\#1)+31a\log(x-\#1)+16}{\#1^3-3\#1^2+4\#1-2}\right.}{(a^2+7a+12)^2}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x]
```

```
[Out] ((16*(a + 2*x - a*x + a*x^2 + x^3))/((3 + a)*(4 + a)*(a - x*(-8 + 8*x - 4*x
^2 + x^3))^2) + (4*(a^2*(5 - 5*x + 6*x^2) + 6*(-14 + 28*x - 12*x^2 + 7*x^3)
 + a*(-7 + 31*x + 12*x^3)))/((3 + a)^2*(4 + a)^2*(a - x*(-8 + 8*x - 4*x^2 +
 x^3))) - (3*RootSum[a + 8*#1 - 8*#1^2 + 4*#1^3 - #1^4 & , (72*Log[x - #1]
```

```
+ 31*a*Log[x - #1] + 3*a^2*Log[x - #1] + 8*Log[x - #1]*#1 + 16*a*Log[x - #1
]*#1 + 4*a^2*Log[x - #1]*#1 + 14*Log[x - #1]*#1^2 + 4*a*Log[x - #1]*#1^2)/(
-2 + 4*#1 - 3*#1^2 + #1^3) & ])/(12 + 7*a + a^2)^2)/128
```

Maple [C] time = 0.016, size = 405, normalized size = 1.2

$$-\frac{1}{\left(x^4-4x^3+8x^2-a-8x\right)^2}\left(\frac{(6a+21)x^7}{16a^4+224a^3+1168a^2+2688a+2304}+\frac{\left(3a^2-24a-120\right)x^6}{16a^4+224a^3+1168a^2+2688a+2304}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^4+4*x^3-8*x^2+a+8*x)^3,x)
```

```
[Out] -(3/16*(7+2*a)/(a^4+14*a^3+73*a^2+168*a+144)*x^7+3/16*(a^2-8*a-40)/(a^4+14*
a^3+73*a^2+168*a+144)*x^6-1/32*(29*a^2-127*a-792)/(a^4+14*a^3+73*a^2+168*a+
144)*x^5+1/32*(73*a^2-227*a-1668)/(a^4+14*a^3+73*a^2+168*a+144)*x^4-1/16*(6
2*a^2-103*a-1104)/(a^4+14*a^3+73*a^2+168*a+144)*x^3-1/16*(5*a^3-26*a^2+140*
a+1008)/(a^4+14*a^3+73*a^2+168*a+144)*x^2+3/32*(3*a^3-17*a^2-40*a+192)/(a^4
+14*a^3+73*a^2+168*a+144)*x-3/32*a*(3*a^2+7*a-12)/(a^4+14*a^3+73*a^2+168*a+
144))/(x^4-4*x^3+8*x^2-a-8*x)^2-3/128/(a^4+14*a^3+73*a^2+168*a+144)*sum((72
+2*(7+2*a)*_R^2+4*(a^2+4*a+2)*_R+3*a^2+31*a)/(_R^3-3*_R^2+4*_R-2)*ln(x-_R),
_R=RootOf(_Z^4-4*_Z^3+8*_Z^2-8*_Z-a))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: AttributeError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**4+4*x**3-8*x**2+a+8*x)**3,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.130 $\int x^2 \left(a+8x-8x^2+4x^3-x^4\right)^4 \, dx$

Optimal. Leaf size=210

$$\frac{2}{11}\left(3a^2-1536a+20480\right)x^{11}-\frac{24}{5}\left(a^2-128a+896\right)x^{10}-4\left(15a^2-288a+512\right)x^8+\frac{4}{7}\left(-a^3+192a^2-1536a+1024\right.$$

```
[Out] (a^4*x^3)/3 + 8*a^3*x^4 + (32*(12 - a)*a^2*x^5)/5 + (8*a*(128 - 48*a + a^2)
*x^6)/3 + (4*(1024 - 1536*a + 192*a^2 - a^3)*x^7)/7 - 4*(512 - 288*a + 15*a
^2)*x^8 + (64*(128 - 3*a)*(4 - a)*x^9)/9 - (24*(896 - 128*a + a^2)*x^10)/5
+ (2*(20480 - 1536*a + 3*a^2)*x^11)/11 - (8*(928 - 35*a)*x^12)/3 + (32*(524
 - 9*a)*x^13)/13 - (8*(464 - 3*a)*x^14)/7 + (4*(640 - a)*x^15)/15 - 42*x^16
 + (128*x^17)/17 - (8*x^18)/9 + x^19/19
```

Rubi [A] time = 0.163692, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {6742}

$$\frac{2}{11}\left(3a^2-1536a+20480\right)x^{11}-\frac{24}{5}\left(a^2-128a+896\right)x^{10}-4\left(15a^2-288a+512\right)x^8+\frac{4}{7}\left(-a^3+192a^2-1536a+1024\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^4,x]
```


```
[Out] (a^4*x^3)/3 + 8*a^3*x^4 + (32*(12 - a)*a^2*x^5)/5 + (8*a*(128 - 48*a + a^2)
*x^6)/3 + (4*(1024 - 1536*a + 192*a^2 - a^3)*x^7)/7 - 4*(512 - 288*a + 15*a
^2)*x^8 + (64*(128 - 3*a)*(4 - a)*x^9)/9 - (24*(896 - 128*a + a^2)*x^10)/5
+ (2*(20480 - 1536*a + 3*a^2)*x^11)/11 - (8*(928 - 35*a)*x^12)/3 + (32*(524
 - 9*a)*x^13)/13 - (8*(464 - 3*a)*x^14)/7 + (4*(640 - a)*x^15)/15 - 42*x^16
 + (128*x^17)/17 - (8*x^18)/9 + x^19/19
```


Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x^2\left(a+8x-8x^2+4x^3-x^4\right)^4\,dx=\int\left(a^4x^2+32a^3x^3-32(-12+a)a^2x^4+16a\left(128-48a+a^2\right)x^5-4\left(-1024+1536\right.\right.$$
$$=\frac{a^4x^3}{3}+8a^3x^4+\frac{32}{5}(12-a)a^2x^5+\frac{8}{3}a\left(128-48a+a^2\right)x^6+\frac{4}{7}\left(1024-1536a+192\right.$$

Mathematica [A] time = 0.0269374, size = 204, normalized size = 0.97

$$\frac{2}{11}\left(3a^2-1536a+20480\right)x^{11}-\frac{24}{5}\left(a^2-128a+896\right)x^{10}+\frac{64}{9}\left(3a^2-140a+512\right)x^9-4\left(15a^2-288a+512\right)x^8-\frac{4}{7}\left(a^3\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^4,x]
```

```
[Out] (a^4*x^3)/3 + 8*a^3*x^4 - (32*(-12 + a)*a^2*x^5)/5 + (8*a*(128 - 48*a + a^2
)*x^6)/3 - (4*(-1024 + 1536*a - 192*a^2 + a^3)*x^7)/7 - 4*(512 - 288*a + 15
*a^2)*x^8 + (64*(512 - 140*a + 3*a^2)*x^9)/9 - (24*(896 - 128*a + a^2)*x^10
)/5 + (2*(20480 - 1536*a + 3*a^2)*x^11)/11 + (8*(-928 + 35*a)*x^12)/3 - (32
*(-524 + 9*a)*x^13)/13 + (8*(-464 + 3*a)*x^14)/7 - (4*(-640 + a)*x^15)/15 -
 42*x^16 + (128*x^17)/17 - (8*x^18)/9 + x^19/19
```

Maple [A] time = 0.002, size = 267, normalized size = 1.3

$$\frac{x^{19}}{19}-\frac{8\,x^{18}}{9}+\frac{128\,x^{17}}{17}-42\,x^{16}+\frac{(-4\,a+2560)\,x^{15}}{15}+\frac{(48\,a-7424)\,x^{14}}{14}+\frac{(-288\,a+16768)\,x^{13}}{13}+\frac{(1120\,a-29696)\,x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^4,x)
```

```
[Out] 1/19*x^19-8/9*x^18+128/17*x^17-42*x^16+1/15*(-4*a+2560)*x^15+1/14*(48*a-742
4)*x^14+1/13*(-288*a+16768)*x^13+1/12*(1120*a-29696)*x^12+1/11*(2*a^2-2560*
a+24576+(-2*a+128)^2)*x^11+1/10*(-16*a^2+3584*a-10240+2*(8*a-128)*(-2*a+128
))*x^10+1/9*(64*a^2-2560*a+2*(-16*a+64)*(-2*a+128)+(8*a-128)^2)*x^9+1/8*(-1
60*a^2+32*a*(-2*a+128)+2*(-16*a+64)*(8*a-128))*x^8+1/7*(2*a^2*(-2*a+128)+32
*a*(8*a-128)+(-16*a+64)^2)*x^7+1/6*(2*a^2*(8*a-128)+32*a*(-16*a+64))*x^6+1/
5*(2*a^2*(-16*a+64)+256*a^2)*x^5+8*a^3*x^4+1/3*a^4*x^3
```

Maxima [A] time = 1.03212, size = 246, normalized size = 1.17

$$\frac{1}{19}\,x^{19} - \frac{8}{9}\,x^{18} + \frac{128}{17}\,x^{17} - \frac{4}{15}\,(a-640)x^{15} - 42\,x^{16} + \frac{8}{7}\,(3\,a-464)x^{14} - \frac{32}{13}\,(9\,a-524)x^{13} + \frac{8}{3}\,(35\,a-928)x^{12} + \frac{2}{11}\,\big(3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^4,x, algorithm="maxima")
```

```
[Out] 1/19*x^19 - 8/9*x^18 + 128/17*x^17 - 4/15*(a - 640)*x^15 - 42*x^16 + 8/7*(3
*a - 464)*x^14 - 32/13*(9*a - 524)*x^13 + 8/3*(35*a - 928)*x^12 + 2/11*(3*a
^2 - 1536*a + 20480)*x^11 - 24/5*(a^2 - 128*a + 896)*x^10 + 64/9*(3*a^2 - 1
40*a + 512)*x^9 - 4*(15*a^2 - 288*a + 512)*x^8 - 4/7*(a^3 - 192*a^2 + 1536*
a - 1024)*x^7 + 1/3*a^4*x^3 + 8*a^3*x^4 + 8/3*(a^3 - 48*a^2 + 128*a)*x^6 -
32/5*(a^3 - 12*a^2)*x^5
```

Fricas [A] time = 1.30445, size = 671, normalized size = 3.2

$$\frac{1}{19}x^{19} - \frac{8}{9}x^{18} + \frac{128}{17}x^{17} - 42x^{16} - \frac{4}{15}x^{15}a + \frac{512}{3}x^{15} + \frac{24}{7}x^{14}a - \frac{3712}{7}x^{14} - \frac{288}{13}x^{13}a + \frac{16768}{13}x^{13} + \frac{280}{3}x^{12}a + \frac{6}{11}x^{11}a^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^4,x, algorithm="fricas")
```

```
[Out] 1/19*x^19 - 8/9*x^18 + 128/17*x^17 - 42*x^16 - 4/15*x^15*a + 512/3*x^15 + 2
4/7*x^14*a - 3712/7*x^14 - 288/13*x^13*a + 16768/13*x^13 + 280/3*x^12*a + 6
/11*x^11*a^2 - 7424/3*x^12 - 3072/11*x^11*a - 24/5*x^10*a^2 + 40960/11*x^11
 + 3072/5*x^10*a + 64/3*x^9*a^2 - 21504/5*x^10 - 8960/9*x^9*a - 60*x^8*a^2
- 4/7*x^7*a^3 + 32768/9*x^9 + 1152*x^8*a + 768/7*x^7*a^2 + 8/3*x^6*a^3 - 20
48*x^8 - 6144/7*x^7*a - 128*x^6*a^2 - 32/5*x^5*a^3 + 4096/7*x^7 + 1024/3*x^
6*a + 384/5*x^5*a^2 + 8*x^4*a^3 + 1/3*x^3*a^4
```

Sympy [A] time = 0.117434, size = 219, normalized size = 1.04

$$\frac{a^4x^3}{3} + 8a^3x^4 + \frac{x^{19}}{19} - \frac{8x^{18}}{9} + \frac{128x^{17}}{17} - 42x^{16} + x^{15}\left(\frac{512}{3} - \frac{4a}{15}\right) + x^{14}\left(\frac{24a}{7} - \frac{3712}{7}\right) + x^{13}\left(\frac{16768}{13} - \frac{288a}{13}\right) + x^{12}\left(\frac{280}{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-x**4+4*x**3-8*x**2+a+8*x)**4,x)
```

```
[Out] a**4*x**3/3 + 8*a**3*x**4 + x**19/19 - 8*x**18/9 + 128*x**17/17 - 42*x**16
+ x**15*(512/3 - 4*a/15) + x**14*(24*a/7 - 3712/7) + x**13*(16768/13 - 288*
a/13) + x**12*(280*a/3 - 7424/3) + x**11*(6*a**2/11 - 3072*a/11 + 40960/11)
 + x**10*(-24*a**2/5 + 3072*a/5 - 21504/5) + x**9*(64*a**2/3 - 8960*a/9 + 3
2768/9) + x**8*(-60*a**2 + 1152*a - 2048) + x**7*(-4*a**3/7 + 768*a**2/7 -
6144*a/7 + 4096/7) + x**6*(8*a**3/3 - 128*a**2 + 1024*a/3) + x**5*(-32*a**3
/5 + 384*a**2/5)
```

Giac [A] time = 1.10342, size = 300, normalized size = 1.43

$$\frac{1}{19}\,x^{19} - \frac{8}{9}\,x^{18} + \frac{128}{17}\,x^{17} - \frac{4}{15}\,ax^{15} - 42\,x^{16} + \frac{24}{7}\,ax^{14} + \frac{512}{3}\,x^{15} - \frac{288}{13}\,ax^{13} - \frac{3712}{7}\,x^{14} + \frac{6}{11}\,a^2x^{11} + \frac{280}{3}\,ax^{12} + \frac{16768}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^4,x, algorithm="giac")
```

```
[Out] 1/19*x^19 - 8/9*x^18 + 128/17*x^17 - 4/15*a*x^15 - 42*x^16 + 24/7*a*x^14 +
512/3*x^15 - 288/13*a*x^13 - 3712/7*x^14 + 6/11*a^2*x^11 + 280/3*a*x^12 + 1
6768/13*x^13 - 24/5*a^2*x^10 - 3072/11*a*x^11 - 7424/3*x^12 + 64/3*a^2*x^9
+ 3072/5*a*x^10 + 40960/11*x^11 - 4/7*a^3*x^7 - 60*a^2*x^8 - 8960/9*a*x^9 -
 21504/5*x^10 + 8/3*a^3*x^6 + 768/7*a^2*x^7 + 1152*a*x^8 + 32768/9*x^9 - 32
/5*a^3*x^5 - 128*a^2*x^6 - 6144/7*a*x^7 - 2048*x^8 + 1/3*a^4*x^3 + 8*a^3*x^
4 + 384/5*a^2*x^5 + 1024/3*a*x^6 + 4096/7*x^7
```

3.131 $\int x^2 \left(a+8x-8x^2+4x^3-x^4\right)^3\,dx$

Optimal. Leaf size=138

$$-\frac{3}{7}\left(a^2-128a+512\right)x^7+\frac{2}{3}\left(3a^2-96a+128\right)x^6+6a^2x^4+\frac{a^3x^3}{3}-\frac{3}{11}(256-a)x^{11}+\frac{12}{5}(64-a)x^{10}-\frac{32}{9}(70-3a)x^9$$

```
[Out] (a^3*x^3)/3 + 6*a^2*x^4 + (24*(8 - a)*a*x^5)/5 + (2*(128 - 96*a + 3*a^2)*x^
6)/3 - (3*(512 - 128*a + a^2)*x^7)/7 + 6*(48 - 5*a)*x^8 - (32*(70 - 3*a)*x^
9)/9 + (12*(64 - a)*x^10)/5 - (3*(256 - a)*x^11)/11 + (70*x^12)/3 - (72*x^1
3)/13 + (6*x^14)/7 - x^15/15
```

Rubi [A] time = 0.120811, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {6742}

$$-\frac{3}{7}\left(a^2-128a+512\right)x^7+\frac{2}{3}\left(3a^2-96a+128\right)x^6+6a^2x^4+\frac{a^3x^3}{3}-\frac{3}{11}(256-a)x^{11}+\frac{12}{5}(64-a)x^{10}-\frac{32}{9}(70-3a)x^9$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x]
```


```
[Out] (a^3*x^3)/3 + 6*a^2*x^4 + (24*(8 - a)*a*x^5)/5 + (2*(128 - 96*a + 3*a^2)*x^
6)/3 - (3*(512 - 128*a + a^2)*x^7)/7 + 6*(48 - 5*a)*x^8 - (32*(70 - 3*a)*x^
9)/9 + (12*(64 - a)*x^10)/5 - (3*(256 - a)*x^11)/11 + (70*x^12)/3 - (72*x^1
3)/13 + (6*x^14)/7 - x^15/15
```


Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x^2\left(a+8x-8x^2+4x^3-x^4\right)^3\,dx=\int\left(a^3x^2+24a^2x^3-24(-8+a)ax^4+4\left(128-96a+3a^2\right)x^5-3\left(512-128a+a\right.\right.$$

$$=\frac{a^3x^3}{3}+6a^2x^4+\frac{24}{5}(8-a)ax^5+\frac{2}{3}\left(128-96a+3a^2\right)x^6-\frac{3}{7}\left(512-128a+a^2\right)x^7$$

Mathematica [A] time = 0.014738, size = 132, normalized size = 0.96

$$-\frac{3}{7}\left(a^2-128a+512\right)x^7+\frac{2}{3}\left(3a^2-96a+128\right)x^6+6a^2x^4+\frac{a^3x^3}{3}+\frac{3}{11}(a-256)x^{11}-\frac{12}{5}(a-64)x^{10}+\frac{32}{9}(3a-70)x^9-$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x]
```

```
[Out] (a^3*x^3)/3 + 6*a^2*x^4 - (24*(-8 + a)*a*x^5)/5 + (2*(128 - 96*a + 3*a^2)*x
^6)/3 - (3*(512 - 128*a + a^2)*x^7)/7 - 6*(-48 + 5*a)*x^8 + (32*(-70 + 3*a)
*x^9)/9 - (12*(-64 + a)*x^10)/5 + (3*(-256 + a)*x^11)/11 + (70*x^12)/3 - (7
2*x^13)/13 + (6*x^14)/7 - x^15/15
```

Maple [A] time = 0.001, size = 143, normalized size = 1.

$$-\frac{x^{15}}{15}+\frac{6\,x^{14}}{7}-\frac{72\,x^{13}}{13}+\frac{70\,x^{12}}{3}+\frac{(3\,a-768)\,x^{11}}{11}+\frac{(-24\,a+1536)\,x^{10}}{10}+\frac{(96\,a-2240)\,x^9}{9}+\frac{(-240\,a+2304)\,x^8}{8}+\frac{(a\,(-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^3,x)
```

```
[Out] -1/15*x^15+6/7*x^14-72/13*x^13+70/3*x^12+1/11*(3*a-768)*x^11+1/10*(-24*a+15
36)*x^10+1/9*(96*a-2240)*x^9+1/8*(-240*a+2304)*x^8+1/7*(a*(-2*a+128)+256*a-
1536-a^2)*x^7+1/6*(a*(8*a-128)-256*a+512+4*a^2)*x^6+1/5*(a*(-16*a+64)+128*a
-8*a^2)*x^5+6*a^2*x^4+1/3*a^3*x^3
```

Maxima [A] time = 1.00186, size = 153, normalized size = 1.11

$$-\frac{1}{15}\,x^{15}+\frac{6}{7}\,x^{14}-\frac{72}{13}\,x^{13}+\frac{3}{11}\,(a-256)x^{11}+\frac{70}{3}\,x^{12}-\frac{12}{5}\,(a-64)x^{10}+\frac{32}{9}\,(3\,a-70)x^9-6\,(5\,a-48)x^8-\frac{3}{7}\left(a^2-128\,a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="maxima")
```

```
[Out] -1/15*x^15 + 6/7*x^14 - 72/13*x^13 + 3/11*(a - 256)*x^11 + 70/3*x^12 - 12/5
*(a - 64)*x^10 + 32/9*(3*a - 70)*x^9 - 6*(5*a - 48)*x^8 - 3/7*(a^2 - 128*a
```

```
+ 512)*x^7 + 2/3*(3*a^2 - 96*a + 128)*x^6 + 1/3*a^3*x^3 + 6*a^2*x^4 - 24/5*
(a^2 - 8*a)*x^5
```

Fricas [A] time = 1.25447, size = 379, normalized size = 2.75

$$-\frac{1}{15}x^{15} + \frac{6}{7}x^{14} - \frac{72}{13}x^{13} + \frac{70}{3}x^{12} + \frac{3}{11}x^{11}a - \frac{768}{11}x^{11} - \frac{12}{5}x^{10}a + \frac{768}{5}x^{10} + \frac{32}{3}x^{9}a - \frac{2240}{9}x^{9} - 30x^{8}a - \frac{3}{7}x^{7}a^{2} + 288x^{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="fricas")
```

```
[Out] -1/15*x^15 + 6/7*x^14 - 72/13*x^13 + 70/3*x^12 + 3/11*x^11*a - 768/11*x^11
- 12/5*x^10*a + 768/5*x^10 + 32/3*x^9*a - 2240/9*x^9 - 30*x^8*a - 3/7*x^7*a
^2 + 288*x^8 + 384/7*x^7*a + 2*x^6*a^2 - 1536/7*x^7 - 64*x^6*a - 24/5*x^5*a
^2 + 256/3*x^6 + 192/5*x^5*a + 6*x^4*a^2 + 1/3*x^3*a^3
```

Sympy [A] time = 0.091309, size = 134, normalized size = 0.97

$$\frac{a^{3}x^{3}}{3} + 6a^{2}x^{4} - \frac{x^{15}}{15} + \frac{6x^{14}}{7} - \frac{72x^{13}}{13} + \frac{70x^{12}}{3} + x^{11}\left(\frac{3a}{11} - \frac{768}{11}\right) + x^{10}\left(\frac{768}{5} - \frac{12a}{5}\right) + x^{9}\left(\frac{32a}{3} - \frac{2240}{9}\right) + x^{8}(288 - 30a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-x**4+4*x**3-8*x**2+a+8*x)**3,x)
```

```
[Out] a**3*x**3/3 + 6*a**2*x**4 - x**15/15 + 6*x**14/7 - 72*x**13/13 + 70*x**12/3
 + x**11*(3*a/11 - 768/11) + x**10*(768/5 - 12*a/5) + x**9*(32*a/3 - 2240/9
) + x**8*(288 - 30*a) + x**7*(-3*a**2/7 + 384*a/7 - 1536/7) + x**6*(2*a**2
- 64*a + 256/3) + x**5*(-24*a**2/5 + 192*a/5)
```

Giac [A] time = 1.13438, size = 180, normalized size = 1.3

$$-\frac{1}{15}x^{15} + \frac{6}{7}x^{14} - \frac{72}{13}x^{13} + \frac{3}{11}ax^{11} + \frac{70}{3}x^{12} - \frac{12}{5}ax^{10} - \frac{768}{11}x^{11} + \frac{32}{3}ax^{9} + \frac{768}{5}x^{10} - \frac{3}{7}a^{2}x^{7} - 30\,ax^{8} - \frac{2240}{9}x^{9} + 2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="giac")
```

```
[Out] -1/15*x^15 + 6/7*x^14 - 72/13*x^13 + 3/11*a*x^11 + 70/3*x^12 - 12/5*a*x^10
- 768/11*x^11 + 32/3*a*x^9 + 768/5*x^10 - 3/7*a^2*x^7 - 30*a*x^8 - 2240/9*x
^9 + 2*a^2*x^6 + 384/7*a*x^7 + 288*x^8 - 24/5*a^2*x^5 - 64*a*x^6 - 1536/7*x
^7 + 1/3*a^3*x^3 + 6*a^2*x^4 + 192/5*a*x^5 + 256/3*x^6
```

3.132 $\int x^2 \left(a+8x-8x^2+4x^3-x^4\right)^2 \, dx$

Optimal. Leaf size=79

$$\frac{a^2x^3}{3}+\frac{2}{7}(64-a)x^7-\frac{4}{3}(16-a)x^6+\frac{16}{5}(4-a)x^5+4ax^4+\frac{x^{11}}{11}-\frac{4x^{10}}{5}+\frac{32x^9}{9}-10x^8$$

```
[Out] (a^2*x^3)/3 + 4*a*x^4 + (16*(4 - a)*x^5)/5 - (4*(16 - a)*x^6)/3 + (2*(64 -
a)*x^7)/7 - 10*x^8 + (32*x^9)/9 - (4*x^10)/5 + x^11/11
```

Rubi [A] time = 0.0767837, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {6742}

$$\frac{a^2x^3}{3}+\frac{2}{7}(64-a)x^7-\frac{4}{3}(16-a)x^6+\frac{16}{5}(4-a)x^5+4ax^4+\frac{x^{11}}{11}-\frac{4x^{10}}{5}+\frac{32x^9}{9}-10x^8$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]
```

```
[Out] (a^2*x^3)/3 + 4*a*x^4 + (16*(4 - a)*x^5)/5 - (4*(16 - a)*x^6)/3 + (2*(64 -
a)*x^7)/7 - 10*x^8 + (32*x^9)/9 - (4*x^10)/5 + x^11/11
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x^2\left(a+8x-8x^2+4x^3-x^4\right)^2\,dx=\int\left(a^2x^2+16ax^3-16(-4+a)x^4+8(-16+a)x^5-2(-64+a)x^6-80x^7+32x^8-\right.$$

$$=\frac{a^2x^3}{3}+4ax^4+\frac{16}{5}(4-a)x^5-\frac{4}{3}(16-a)x^6+\frac{2}{7}(64-a)x^7-10x^8+\frac{32x^9}{9}-\frac{4x^{10}}{5}+$$

Mathematica [A] time = 0.0083944, size = 73, normalized size = 0.92

$$\frac{a^2x^3}{3}-\frac{2}{7}(a-64)x^7+\frac{4}{3}(a-16)x^6-\frac{16}{5}(a-4)x^5+4ax^4+\frac{x^{11}}{11}-\frac{4x^{10}}{5}+\frac{32x^9}{9}-10x^8$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]
```

```
[Out] (a^2*x^3)/3 + 4*a*x^4 - (16*(-4 + a)*x^5)/5 + (4*(-16 + a)*x^6)/3 - (2*(-64
 + a)*x^7)/7 - 10*x^8 + (32*x^9)/9 - (4*x^10)/5 + x^11/11
```

Maple [A] time = 0.001, size = 66, normalized size = 0.8

$$\frac{x^{11}}{11}-\frac{4\,x^{10}}{5}+\frac{32\,x^{9}}{9}-10\,x^{8}+\frac{(-2\,a+128)\,x^{7}}{7}+\frac{(8\,a-128)\,x^{6}}{6}+\frac{(-16\,a+64)\,x^{5}}{5}+4\,ax^{4}+\frac{a^{2}x^{3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^2,x)
```

```
[Out] 1/11*x^11-4/5*x^10+32/9*x^9-10*x^8+1/7*(-2*a+128)*x^7+1/6*(8*a-128)*x^6+1/5
*(-16*a+64)*x^5+4*a*x^4+1/3*a^2*x^3
```

Maxima [A] time = 1.08814, size = 80, normalized size = 1.01

$$\frac{1}{11}\,x^{11}-\frac{4}{5}\,x^{10}+\frac{32}{9}\,x^{9}-\frac{2}{7}\,(a-64)x^{7}-10\,x^{8}+\frac{4}{3}\,(a-16)x^{6}-\frac{16}{5}\,(a-4)x^{5}+\frac{1}{3}\,a^{2}x^{3}+4\,ax^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="maxima")
```

```
[Out] 1/11*x^11 - 4/5*x^10 + 32/9*x^9 - 2/7*(a - 64)*x^7 - 10*x^8 + 4/3*(a - 16)*
x^6 - 16/5*(a - 4)*x^5 + 1/3*a^2*x^3 + 4*a*x^4
```

Fricas [A] time = 1.27528, size = 185, normalized size = 2.34

$$\frac{1}{11}x^{11}-\frac{4}{5}x^{10}+\frac{32}{9}x^{9}-10x^{8}-\frac{2}{7}x^{7}a+\frac{128}{7}x^{7}+\frac{4}{3}x^{6}a-\frac{64}{3}x^{6}-\frac{16}{5}x^{5}a+\frac{64}{5}x^{5}+4x^{4}a+\frac{1}{3}x^{3}a^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="fricas")
```

```
[Out] 1/11*x^11 - 4/5*x^10 + 32/9*x^9 - 10*x^8 - 2/7*x^7*a + 128/7*x^7 + 4/3*x^6*
a - 64/3*x^6 - 16/5*x^5*a + 64/5*x^5 + 4*x^4*a + 1/3*x^3*a^2
```

Sympy [A] time = 0.07261, size = 73, normalized size = 0.92

$$\frac{a^2x^3}{3} + 4ax^4 + \frac{x^{11}}{11} - \frac{4x^{10}}{5} + \frac{32x^9}{9} - 10x^8 + x^7\left(\frac{128}{7} - \frac{2a}{7}\right) + x^6\left(\frac{4a}{3} - \frac{64}{3}\right) + x^5\left(\frac{64}{5} - \frac{16a}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-x**4+4*x**3-8*x**2+a+8*x)**2,x)
```

```
[Out] a**2*x**3/3 + 4*a*x**4 + x**11/11 - 4*x**10/5 + 32*x**9/9 - 10*x**8 + x**7*
(128/7 - 2*a/7) + x**6*(4*a/3 - 64/3) + x**5*(64/5 - 16*a/5)
```

Giac [A] time = 1.14009, size = 92, normalized size = 1.16

$$\frac{1}{11}\,x^{11} - \frac{4}{5}\,x^{10} + \frac{32}{9}\,x^9 - \frac{2}{7}\,ax^7 - 10\,x^8 + \frac{4}{3}\,ax^6 + \frac{128}{7}\,x^7 - \frac{16}{5}\,ax^5 - \frac{64}{3}\,x^6 + \frac{1}{3}\,a^2x^3 + 4\,ax^4 + \frac{64}{5}\,x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="giac")
```

```
[Out] 1/11*x^11 - 4/5*x^10 + 32/9*x^9 - 2/7*a*x^7 - 10*x^8 + 4/3*a*x^6 + 128/7*x^
7 - 16/5*a*x^5 - 64/3*x^6 + 1/3*a^2*x^3 + 4*a*x^4 + 64/5*x^5
```

3.133 $\int x^2 \left(a+8x-8x^2+4x^3-x^4\right) dx$

Optimal. Leaf size=35

$$\frac{ax^3}{3}-\frac{x^7}{7}+\frac{2x^6}{3}-\frac{8x^5}{5}+2x^4$$

```
[Out] (a*x^3)/3 + 2*x^4 - (8*x^5)/5 + (2*x^6)/3 - x^7/7
```

Rubi [A] time = 0.0090193, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {14}

$$\frac{ax^3}{3}-\frac{x^7}{7}+\frac{2x^6}{3}-\frac{8x^5}{5}+2x^4$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4),x]
```

```
[Out] (a*x^3)/3 + 2*x^4 - (8*x^5)/5 + (2*x^6)/3 - x^7/7
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^2 \left(a+8x-8x^2+4x^3-x^4\right) dx &= \int \left(ax^2+8x^3-8x^4+4x^5-x^6\right) dx\\ &= \frac{ax^3}{3}+2x^4-\frac{8x^5}{5}+\frac{2x^6}{3}-\frac{x^7}{7}\end{aligned}$$

Mathematica [A] time = 0.0015729, size = 35, normalized size = 1.

$$\frac{ax^3}{3}-\frac{x^7}{7}+\frac{2x^6}{3}-\frac{8x^5}{5}+2x^4$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4),x]
```

```
[Out] (a*x^3)/3 + 2*x^4 - (8*x^5)/5 + (2*x^6)/3 - x^7/7
```

Maple [A] time = 0., size = 28, normalized size = 0.8

$$\frac{a x^{3}}{3}+2 x^{4}-\frac{8 x^{5}}{5}+\frac{2 x^{6}}{3}-\frac{x^{7}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-x^4+4*x^3-8*x^2+a+8*x),x)
```

```
[Out] 1/3*a*x^3+2*x^4-8/5*x^5+2/3*x^6-1/7*x^7
```

Maxima [A] time = 1.13278, size = 36, normalized size = 1.03

$$-\frac{1}{7} x^{7}+\frac{2}{3} x^{6}-\frac{8}{5} x^{5}+\frac{1}{3} a x^{3}+2 x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="maxima")
```

```
[Out] -1/7*x^7 + 2/3*x^6 - 8/5*x^5 + 1/3*a*x^3 + 2*x^4
```

Fricas [A] time = 1.29859, size = 68, normalized size = 1.94

$$-\frac{1}{7}x^{7}+\frac{2}{3}x^{6}-\frac{8}{5}x^{5}+2x^{4}+\frac{1}{3}x^{3}a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="fricas")
```

```
[Out] -1/7*x^7 + 2/3*x^6 - 8/5*x^5 + 2*x^4 + 1/3*x^3*a
```

Sympy [A] time = 0.057391, size = 29, normalized size = 0.83

$$\frac{ax^3}{3} - \frac{x^7}{7} + \frac{2x^6}{3} - \frac{8x^5}{5} + 2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-x**4+4*x**3-8*x**2+a+8*x),x)
```

```
[Out] a*x**3/3 - x**7/7 + 2*x**6/3 - 8*x**5/5 + 2*x**4
```

Giac [A] time = 1.14173, size = 36, normalized size = 1.03

$$-\frac{1}{7}\,x^7 + \frac{2}{3}\,x^6 - \frac{8}{5}\,x^5 + \frac{1}{3}\,ax^3 + 2\,x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="giac")
```

```
[Out] -1/7*x^7 + 2/3*x^6 - 8/5*x^5 + 1/3*a*x^3 + 2*x^4
```

3.134 $\int \frac{x^2}{a+8x-8x^2+4x^3-x^4} \, dx$

Optimal. Leaf size=99

$$-\frac{\tan^{-1}\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{2\sqrt{1-\sqrt{a+4}}}-\frac{\tan^{-1}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{2\sqrt{\sqrt{a+4}+1}}+\frac{\tanh^{-1}\left(\frac{(x-1)^2+1}{\sqrt{a+4}}\right)}{\sqrt{a+4}}$$

```
[Out] -ArcTan[(-1 + x)/Sqrt[1 - Sqrt[4 + a]]]/(2*Sqrt[1 - Sqrt[4 + a]]) - ArcTan[
(-1 + x)/Sqrt[1 + Sqrt[4 + a]]]/(2*Sqrt[1 + Sqrt[4 + a]]) + ArcTanh[(1 + (-
1 + x)^2)/Sqrt[4 + a]]/Sqrt[4 + a]
```

Rubi [A] time = 0.0870459, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {1680, 1673, 1166, 204, 12, 1107, 618, 206}

$$-\frac{\tan^{-1}\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{2\sqrt{1-\sqrt{a+4}}}-\frac{\tan^{-1}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{2\sqrt{\sqrt{a+4}+1}}+\frac{\tanh^{-1}\left(\frac{(x-1)^2+1}{\sqrt{a+4}}\right)}{\sqrt{a+4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4),x]
```

```
[Out] -ArcTan[(-1 + x)/Sqrt[1 - Sqrt[4 + a]]]/(2*Sqrt[1 - Sqrt[4 + a]]) - ArcTan[
(-1 + x)/Sqrt[1 + Sqrt[4 + a]]]/(2*Sqrt[1 + Sqrt[4 + a]]) + ArcTanh[(1 + (-
1 + x)^2)/Sqrt[4 + a]]/Sqrt[4 + a]
```

Rule 1680

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -(d/(4*e)) + x)*(a + d^4/(256*e^3) - (
b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /;
 EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq,
x] && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^2}{a+8x-8x^2+4x^3-x^4}\,dx = \text{Subst}\left(\int \frac{(1+x)^2}{3+a-2x^2-x^4}\,dx, x, -1+x\right)$$
$$= \text{Subst}\left(\int \frac{2x}{3+a-2x^2-x^4}\,dx, x, -1+x\right) + \text{Subst}\left(\int \frac{1+x^2}{3+a-2x^2-x^4}\,dx, x, -1+x\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{-1-\sqrt{4+a}-x^2}\,dx, x, -1+x\right) + \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{-1+\sqrt{4+a}-x^2}\,dx, x, -\right.$$
$$= \frac{\tan^{-1}\left(\frac{1-x}{\sqrt{1-\sqrt{4+a}}}\right)}{2\sqrt{1-\sqrt{4+a}}} + \frac{\tan^{-1}\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)}{2\sqrt{1+\sqrt{4+a}}} + \text{Subst}\left(\int \frac{1}{3+a-2x-x^2}\,dx, x, (-1+x)^2\right)$$
$$= \frac{\tan^{-1}\left(\frac{1-x}{\sqrt{1-\sqrt{4+a}}}\right)}{2\sqrt{1-\sqrt{4+a}}} + \frac{\tan^{-1}\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)}{2\sqrt{1+\sqrt{4+a}}} - 2\,\text{Subst}\left(\int \frac{1}{4(4+a)-x^2}\,dx, x, -2\left(1+(-1+\right.\right.$$
$$= \frac{\tan^{-1}\left(\frac{1-x}{\sqrt{1-\sqrt{4+a}}}\right)}{2\sqrt{1-\sqrt{4+a}}} + \frac{\tan^{-1}\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)}{2\sqrt{1+\sqrt{4+a}}} + \frac{\tanh^{-1}\left(\frac{1+(-1+x)^2}{\sqrt{4+a}}\right)}{\sqrt{4+a}}$$

Mathematica [C] time = 0.0152123, size = 61, normalized size = 0.62

$$-\frac{1}{4}\text{RootSum}\left[-\#1^4+4\#1^3-8\#1^2+8\#1+a\&, \frac{\#1^2\log(x-\#1)}{\#1^3-3\#1^2+4\#1-2}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4),x]
```

```
[Out] -RootSum[a + 8*#1 - 8*#1^2 + 4*#1^3 - #1^4 & , (Log[x - #1]*#1^2)/(-2 + 4*#
1 - 3*#1^2 + #1^3) & ]/4
```

Maple [C] time = 0.003, size = 52, normalized size = 0.5

$$-\frac{1}{4}\sum_{\_R=\text{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)}\frac{\_R^2\ln(x-\_R)}{\_R^3-3\_R^2+4\_R-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-x^4+4*x^3-8*x^2+a+8*x),x)
```

```
[Out] -1/4*sum(_R^2/(_R^3-3*_R^2+4*_R-2)*ln(x-_R),_R=RootOf(_Z^4-4*_Z^3+8*_Z^2-8*
_Z-a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2}{x^4 - 4\,x^3 + 8\,x^2 - a - 8\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="maxima")
```

```
[Out] -integrate(x^2/(x^4 - 4*x^3 + 8*x^2 - a - 8*x), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [B] time = 3.88377, size = 172, normalized size = 1.74

$$-\operatorname{RootSum}\left(t^4\left(256a^3+2816a^2+10240a+12288\right)+t^2\left(-160a^2-1152a-2048\right)+t\left(-32a^2-256a-512\right)-a^2,\left(t\mapsto\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-x**4+4*x**3-8*x**2+a+8*x),x)
```

```
[Out] -RootSum(_t**4*(256*a**3 + 2816*a**2 + 10240*a + 12288) + _t**2*(-160*a**2
- 1152*a - 2048) + _t*(-32*a**2 - 256*a - 512) - a**2, Lambda(_t, _t*log(x
```

```
+ (-64*_t**3*a**4 - 448*_t**3*a**3 - 256*_t**3*a**2 + 3584*_t**3*a + 6144*_
t**3 - 224*_t**2*a**3 - 2208*_t**2*a**2 - 7168*_t**2*a - 7680*_t**2 + 56*_t
*a**3 + 400*_t*a**2 + 864*_t*a + 512*_t + 5*a**3 + 34*a**2 + 56*a)/(a**3 +
60*a**2 + 320*a + 448))))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.135 $\int \frac{x^2}{\left(a+8x-8x^2+4x^3-x^4\right)^2}\,dx$

Optimal. Leaf size=225

$$\frac{(a+4)\left((x-1)^2+2\right)(x-1)}{4\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)}+\frac{(x-1)^2+1}{2(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)}-\frac{\left(a+\sqrt{a+4}+4\right)\tan^{-1}\left(\frac{x-1}{\sqrt{1-\sqrt{a}}}\right)}{8(a+3)(a+4)\sqrt{1-\sqrt{a+4}}}$$

```
[Out] (1 + (-1 + x)^2)/(2*(4 + a)*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)) + ((4 + a)
*(2 + (-1 + x)^2)*(-1 + x))/(4*(12 + 7*a + a^2)*(3 + a - 2*(-1 + x)^2 - (-1
 + x)^4)) - ((4 + a + Sqrt[4 + a])*ArcTan[(-1 + x)/Sqrt[1 - Sqrt[4 + a]]])/
(8*(3 + a)*(4 + a)*Sqrt[1 - Sqrt[4 + a]]) - ((4 + a - Sqrt[4 + a])*ArcTan[(
-1 + x)/Sqrt[1 + Sqrt[4 + a]]])/(8*(3 + a)*(4 + a)*Sqrt[1 + Sqrt[4 + a]]) +
 ArcTanh[(1 + (-1 + x)^2)/Sqrt[4 + a]]/(2*(4 + a)^(3/2))
```

Rubi [A] time = 0.212609, antiderivative size = 225, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1680, 1673, 1178, 1166, 204, 12, 1107, 614, 618, 206}

$$\frac{(a+4)\left((x-1)^2+2\right)(x-1)}{4\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)}+\frac{(x-1)^2+1}{2(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)}-\frac{\left(a+\sqrt{a+4}+4\right)\tan^{-1}\left(\frac{x-1}{\sqrt{1-\sqrt{a}}}\right)}{8(a+3)(a+4)\sqrt{1-\sqrt{a+4}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]
```



```
[Out] (1 + (-1 + x)^2)/(2*(4 + a)*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)) + ((4 + a)
*(2 + (-1 + x)^2)*(-1 + x))/(4*(12 + 7*a + a^2)*(3 + a - 2*(-1 + x)^2 - (-1
 + x)^4)) - ((4 + a + Sqrt[4 + a])*ArcTan[(-1 + x)/Sqrt[1 - Sqrt[4 + a]]])/
(8*(3 + a)*(4 + a)*Sqrt[1 - Sqrt[4 + a]]) - ((4 + a - Sqrt[4 + a])*ArcTan[(
-1 + x)/Sqrt[1 + Sqrt[4 + a]]])/(8*(3 + a)*(4 + a)*Sqrt[1 + Sqrt[4 + a]]) +
 ArcTanh[(1 + (-1 + x)^2)/Sqrt[4 + a]]/(2*(4 + a)^(3/2))
```


Rule 1680

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
```

```
st[Int[SimplifyIntegrand[(Pq /. x -> -(d/(4*e)) + x)*(a + d^4/(256*e^3) - (
b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /;
 EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq,
x] && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^2}{\left(a+8x-8x^2+4x^3-x^4\right)^2}\,dx = \text{Subst}\left(\int \frac{(1+x)^2}{\left(3+a-2x^2-x^4\right)^2}\,dx, x, -1+x\right)$$

$$= \text{Subst}\left(\int \frac{2x}{\left(3+a-2x^2-x^4\right)^2}\,dx, x, -1+x\right) + \text{Subst}\left(\int \frac{1+x^2}{\left(3+a-2x^2-x^4\right)^2}\,dx, x\right.$$

$$= \frac{(4+a)\left(2+(-1+x)^2\right)(-1+x)}{4\left(12+7a+a^2\right)\left(3+a-2(-1+x)^2-(-1+x)^4\right)} + 2\,\text{Subst}\left(\int \frac{x}{\left(3+a-2x^2-x^4\right)}\right.$$

$$= \frac{(4+a)\left(2+(-1+x)^2\right)(-1+x)}{4\left(12+7a+a^2\right)\left(3+a-2(-1+x)^2-(-1+x)^4\right)} + \frac{\left(4+a-\sqrt{4+a}\right)\text{Subst}\left(\int \frac{}{-1-}\right.}{8\left(12+7a+\right.}$$

$$= \frac{1+(-1+x)^2}{2(4+a)\left(3+a-2(-1+x)^2-(-1+x)^4\right)} + \frac{(4+a)\left(2+(-1+x)^2\right)(-1+x}{4\left(12+7a+a^2\right)\left(3+a-2(-1+x)^2-\right.}$$

$$= \frac{1+(-1+x)^2}{2(4+a)\left(3+a-2(-1+x)^2-(-1+x)^4\right)} + \frac{(4+a)\left(2+(-1+x)^2\right)(-1+x}{4\left(12+7a+a^2\right)\left(3+a-2(-1+x)^2-\right.}$$

$$= \frac{1+(-1+x)^2}{2(4+a)\left(3+a-2(-1+x)^2-(-1+x)^4\right)} + \frac{(4+a)\left(2+(-1+x)^2\right)(-1+x}{4\left(12+7a+a^2\right)\left(3+a-2(-1+x)^2-\right.}$$

Mathematica [C] time = 0.0612668, size = 182, normalized size = 0.81

$$\frac{a\left(x^3-x^2+x+1\right)+2x\left(2x^2-3x+4\right)}{4(a+3)(a+4)\left(a-x\left(x^3-4x^2+8x-8\right)\right)} - \frac{\text{RootSum}\left[-\#1^4+4\#1^3-8\#1^2+8\#1+a\&, \frac{\#1^2 a\log(x-\#1)+4\#1^2\log(x-\#1)+2\#}{\#1^3-3}\right.}{16\left(a^2+7a+12\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]
```

```
[Out] (2*x*(4 - 3*x + 2*x^2) + a*(1 + x - x^2 + x^3))/(4*(3 + a)*(4 + a)*(a - x*(
-8 + 8*x - 4*x^2 + x^3))) - RootSum[a + 8*#1 - 8*#1^2 + 4*#1^3 - #1^4 & , (
-(a*Log[x - #1]) + 4*Log[x - #1]*#1 + 2*a*Log[x - #1]*#1 + 4*Log[x - #1]*#1
^2 + a*Log[x - #1]*#1^2)/(-2 + 4*#1 - 3*#1^2 + #1^3) & ]/(16*(12 + 7*a + a^
```

2))

Maple [C] time = 0.01, size = 160, normalized size = 0.7

$$\frac{1}{x^{4}-4x^{3}+8x^{2}-a-8x}\left(-\frac{x^{3}}{12+4a}+\frac{(6+a)\,x^{2}}{(12+4a)\,(4+a)}-\frac{(8+a)\,x}{(12+4a)\,(4+a)}-\frac{a}{(12+4a)\,(4+a)}\right)+\frac{1}{(48+16a)\,(4+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^2,x)
```

```
[Out] (-1/4/(3+a)*x^3+1/4*(6+a)/(3+a)/(4+a)*x^2-1/4*(8+a)/(3+a)/(4+a)*x-1/4*a/(3+
a)/(4+a))/(x^4-4*x^3+8*x^2-a-8*x)+1/16/(3+a)/(4+a)*sum(((-a-4)*_R^2+2*(-a-2
)*_R+a)/(_R^3-3*_R^2+4*_R-2)*ln(x-_R),_R=RootOf(_Z^4-4*_Z^3+8*_Z^2-8*_Z-a))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: AttributeError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [B] time = 18.0023, size = 559, normalized size = 2.48

$$-\frac{a+x^3(a+4)+x^2(-a-6)+x(a+8)}{-4a^3-28a^2-48a+x^4\left(4a^2+28a+48\right)+x^3\left(-16a^2-112a-192\right)+x^2\left(32a^2+224a+384\right)+x\left(-32a^2-224a-3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-x**4+4*x**3-8*x**2+a+8*x)**2,x)
```

```
[Out] -(a + x**3*(a + 4) + x**2*(-a - 6) + x*(a + 8))/(-4*a**3 - 28*a**2 - 48*a +
 x**4*(4*a**2 + 28*a + 48) + x**3*(-16*a**2 - 112*a - 192) + x**2*(32*a**2
+ 224*a + 384) + x*(-32*a**2 - 224*a - 384)) + RootSum(_t**4*(65536*a**9 +
2162688*a**8 + 31653888*a**7 + 269680640*a**6 + 1473773568*a**5 + 535717478
4*a**4 + 12952010752*a**3 + 20082327552*a**2 + 18119393280*a + 7247757312)
+ _t**2*(-9728*a**6 - 209408*a**5 - 1878016*a**4 - 8986624*a**3 - 24215552*
a**2 - 34865152*a - 20971520) + _t*(256*a**5 + 5888*a**4 + 53248*a**3 + 237
568*a**2 + 524288*a + 458752) - a**4 + 144*a**3 + 1024*a**2 + 1792*a, Lambd
a(_t, _t*log(x + (4096*_t**3*a**12 - 61440*_t**3*a**11 - 5480448*_t**3*a**1
0 - 111403008*_t**3*a**9 - 1227173888*_t**3*a**8 - 8682876928*_t**3*a**7 -
42187440128*_t**3*a**6 - 144630284288*_t**3*a**5 - 350972280832*_t**3*a**4
- 591750234112*_t**3*a**3 - 660716126208*_t**3*a**2 - 439848271872*_t**3*a
- 132271570944*_t**3 - 28672*_t**2*a**10 - 993280*_t**2*a**9 - 15400960*_t*
*2*a**8 - 140742656*_t**2*a**7 - 839462912*_t**2*a**6 - 3414427648*_t**2*a*
*5 - 9590087680*_t**2*a**4 - 18363547648*_t**2*a**3 - 22938255360*_t**2*a**
2 - 16873684992*_t**2*a - 5549064192*_t**2 - 848*_t*a**9 - 6096*_t*a**8 + 1
74608*_t*a**7 + 3323792*_t*a**6 + 26276224*_t*a**5 + 119009280*_t*a**4 + 33
2017664*_t*a**3 + 566497280*_t*a**2 + 544112640*_t*a + 225837056*_t + 11*a*
*8 + 958*a**7 + 17419*a**6 + 142964*a**5 + 632632*a**4 + 1567552*a**3 + 204
9792*a**2 + 1100800*a)/(a**8 + 870*a**7 + 18289*a**6 + 165176*a**5 + 824560
*a**4 + 2452288*a**3 + 4340224*a**2 + 4229120*a + 1748992))))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.136 $\int \frac{x^4}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx$

Optimal. Leaf size=545

$$-\frac{\sqrt[3]{-1}\left(3\sqrt[3]{a}c^{2/3}+2\sqrt[3]{-1}b\right)\tan^{-1}\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{3\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2a^{5/6}b^2c^{2/3}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}-\frac{\left(2b-3\sqrt[3]{a}c^{2/3}\right)\tan^{-1}\left(\frac{3a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}\right)}{9\sqrt{3}a^{5/6}b^2c^{2/3}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}-\frac{(-1)^{2/3}\left(3\sqrt[3]{-1}\sqrt[3]{a}c\right.}{3\sqrt{3}\left(1-\sqrt[3]{-1}\right)\left(1+\right.}$$

```
[Out] -((-1)^(1/3)*(2*(-1)^(1/3)*b + 3*a^(1/3)*c^(2/3))*ArcTan[(3*(-1)^(1/3)*a^(2
/3)*c^(1/3) - 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/
3)])])/(3*Sqrt[3]*(1 + (-1)^(1/3))^2*a^(5/6)*b^2*Sqrt[4*b - 3*(-1)^(2/3)*a^
(1/3)*c^(2/3)]*c^(2/3)) - ((2*b - 3*a^(1/3)*c^(2/3))*ArcTan[(3*a^(2/3)*c^(1
/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*a^(1/3)*c^(2/3)])])/(9*Sqrt[3]*a
^(5/6)*b^2*Sqrt[4*b - 3*a^(1/3)*c^(2/3)]*c^(2/3)) - ((-1)^(2/3)*(2*b + 3*(-
1)^(1/3)*a^(1/3)*c^(2/3))*ArcTan[(3*(-1)^(2/3)*a^(2/3)*c^(1/3) + 2*b*x)/(Sq
rt[3]*Sqrt[a]*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)])])/(3*Sqrt[3]*(1 - (
-1)^(1/3))*(1 + (-1)^(1/3))^2*a^(5/6)*b^2*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3)*c
^(2/3)]*c^(2/3)) - Log[3*a + 3*a^(2/3)*c^(1/3)*x + b*x^2]/(18*a^(2/3)*b^2*c
^(1/3)) + Log[3*a - 3*(-1)^(1/3)*a^(2/3)*c^(1/3)*x + b*x^2]/(6*(1 + (-1)^(1
/3))^2*a^(2/3)*b^2*c^(1/3)) + ((-1)^(1/3)*Log[3*a + 3*(-1)^(2/3)*a^(2/3)*c^
(1/3)*x + b*x^2])/(18*a^(2/3)*b^2*c^(1/3))
```

Rubi [A] time = 1.47645, antiderivative size = 545, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 5, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.109, Rules used = {2097, 634, 618, 204, 628}

$$-\frac{\sqrt[3]{-1}\left(3\sqrt[3]{a}c^{2/3}+2\sqrt[3]{-1}b\right)\tan^{-1}\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{3\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2a^{5/6}b^2c^{2/3}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}-\frac{\left(2b-3\sqrt[3]{a}c^{2/3}\right)\tan^{-1}\left(\frac{3a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}\right)}{9\sqrt{3}a^{5/6}b^2c^{2/3}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}-\frac{(-1)^{2/3}\left(3\sqrt[3]{-1}\sqrt[3]{a}c\right.}{3\sqrt{3}\left(1-\sqrt[3]{-1}\right)\left(1+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6),x]
```



```
[Out] -((-1)^(1/3)*(2*(-1)^(1/3)*b + 3*a^(1/3)*c^(2/3))*ArcTan[(3*(-1)^(1/3)*a^(2
/3)*c^(1/3) - 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/
3)])])/(3*Sqrt[3]*(1 + (-1)^(1/3))^2*a^(5/6)*b^2*Sqrt[4*b - 3*(-1)^(2/3)*a^
(1/3)*c^(2/3)]*c^(2/3)) - ((2*b - 3*a^(1/3)*c^(2/3))*ArcTan[(3*a^(2/3)*c^(1
/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*a^(1/3)*c^(2/3)])])/(9*Sqrt[3]*a
```

```
^(5/6)*b^2*Sqrt[4*b - 3*a^(1/3)*c^(2/3)]*c^(2/3)) - ((-1)^(2/3)*(2*b + 3*(-
1)^(1/3)*a^(1/3)*c^(2/3))*ArcTan[(3*(-1)^(2/3)*a^(2/3)*c^(1/3) + 2*b*x)/(Sq
rt[3]*Sqrt[a]*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)])])/(3*Sqrt[3]*(1 - (
-1)^(1/3))*(1 + (-1)^(1/3))^2*a^(5/6)*b^2*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3)*c
^(2/3)]*c^(2/3)) - Log[3*a + 3*a^(2/3)*c^(1/3)*x + b*x^2]/(18*a^(2/3)*b^2*c
^(1/3)) + Log[3*a - 3*(-1)^(1/3)*a^(2/3)*c^(1/3)*x + b*x^2]/(6*(1 + (-1)^(1
/3))^2*a^(2/3)*b^2*c^(1/3)) + ((-1)^(1/3)*Log[3*a + 3*(-1)^(2/3)*a^(2/3)*c^
(1/3)*x + b*x^2])/(18*a^(2/3)*b^2*c^(1/3))
```

Rule 2097

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^4}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx = \left(19683a^6\right)\int\left(\frac{-(-1)^{2/3}\sqrt[3]{a}-\sqrt[3]{c}x}{59049\left(1+\sqrt[3]{-1}\right)^2 a^{20/3}bc^{2/3}\left(-3a+3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x-b\right.}\right.$$

$$= \frac{\int \frac{-\sqrt[3]{a}-\sqrt[3]{c}x}{3a+3a^{2/3}\sqrt[3]{c}x+bx^2}\,dx}{9a^{2/3}bc^{2/3}} - \frac{(-1)^{2/3}\int \frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{c}x}{3a+3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x+bx^2}\,dx}{9a^{2/3}bc^{2/3}} + \frac{\int \frac{-(-1)^{2/}}{-3a+3\sqrt[3]{-}}}{3\left(1+\sqrt[3]{-}\right.}$$

$$= \frac{\left(3-\frac{2b}{\sqrt[3]{a}c^{2/3}}\right)\int \frac{1}{3a+3a^{2/3}\sqrt[3]{c}x+bx^2}\,dx}{18b^2} + \frac{\left(3-\frac{2(-1)^{2/3}b}{\sqrt[3]{a}c^{2/3}}\right)\int \frac{1}{3a+3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x+b}}{18b^2}$$

$$= -\frac{\log\left(3a+3a^{2/3}\sqrt[3]{c}x+bx^2\right)}{18a^{2/3}b^2\sqrt[3]{c}} + \frac{\log\left(3a-3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x+bx^2\right)}{6\left(1+\sqrt[3]{-1}\right)^2 a^{2/3}b^2\sqrt[3]{c}} + \frac{\sqrt[3]{-}}{}$$

$$= -\frac{\left(3ib+\sqrt{3}\left(b+3\sqrt[3]{a}c^{2/3}\right)\right)\tan^{-1}\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{27a^{5/6}b^2\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}\,c^{2/3}} + \frac{\left(3-\frac{2b}{\sqrt[3]{ac}}\right.}{9\sqrt{}}$$

Mathematica [C] time = 0.0616323, size = 99, normalized size = 0.18

$$\frac{1}{3}\text{RootSum}\left[27\#1^2a^2b+27\#1^3a^2c+9\#1^4ab^2+\#1^6b^3+27a^3\&,\frac{\#1^3\log(x-\#1)}{12\#1^2ab^2+2\#1^4b^3+27\#1a^2c+18a^2b}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6
),x]
```

```
[Out] RootSum[27*a^3 + 27*a^2*b*#1^2 + 27*a^2*c*#1^3 + 9*a*b^2*#1^4 + b^3*#1^6 &
, (Log[x - #1]*#1^3)/(18*a^2*b + 27*a^2*c*#1 + 12*a*b^2*#1^2 + 2*b^3*#1^4)
& ]/3
```

Maple [C] time = 0.01, size = 93, normalized size = 0.2

$$\frac{1}{3}\sum_{\_R=\text{RootOf}\left(b^3\_Z^6+9ab^2\_Z^4+27a^2c\_Z^3+27a^2b\_Z^2+27a^3\right)}\frac{\_R^4\ln(x-\_R)}{2\_R^5b^3+12\_R^3ab^2+27\_R^2a^2c+18\_Ra^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x)
```

```
[Out] 1/3*sum(_R^4/(2*_R^5*b^3+12*_R^3*a*b^2+27*_R^2*a^2*c+18*_R*a^2*b)*ln(x-_R),
_R=RootOf(_Z^6*b^3+9*_Z^4*a*b^2+27*_Z^3*a^2*c+27*_Z^2*a^2*b+27*a^3))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{b^3x^6 + 9\,ab^2x^4 + 27\,a^2cx^3 + 27\,a^2bx^2 + 27\,a^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, alg
orithm="maxima")
```

```
[Out] integrate(x^4/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3
), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b**3*x**6+9*a*b**2*x**4+27*a**2*c*x**3+27*a**2*b*x**2+27*a*
*3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{b^3x^6 + 9\,ab^2x^4 + 27\,a^2cx^3 + 27\,a^2bx^2 + 27\,a^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, alg
orithm="giac")
```

```
[Out] integrate(x^4/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3
), x)
```

3.137 $\int \frac{x^3}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx$

Optimal. Leaf size=487

$$\frac{\log\left(3a^{2/3}\sqrt[3]{c}x+3a+bx^2\right)}{54a^{4/3}bc^{2/3}}-\frac{(-1)^{2/3}\log\left(-3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x+3a+bx^2\right)}{18\left(1+\sqrt[3]{-1}\right)^2a^{4/3}bc^{2/3}}+\frac{(-1)^{2/3}\log\left(3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x+3a+bx^2\right)}{54a^{4/3}bc^{2/3}}-\frac{\quad}{3\sqrt{3}}$$

```
[Out] -ArcTan[(3*(-1)^(1/3)*a^(2/3)*c^(1/3) - 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b -
3*(-1)^(2/3)*a^(1/3)*c^(2/3)])]/(3*Sqrt[3]*(1 + (-1)^(1/3))^2*a^(7/6)*b*Sqr
t[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)]*c^(1/3)) - ArcTan[(3*a^(2/3)*c^(1/3)
+ 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*a^(1/3)*c^(2/3)])]/(9*Sqrt[3]*a^(7/6
)*b*Sqrt[4*b - 3*a^(1/3)*c^(2/3)]*c^(1/3)) + ((-1)^(1/3)*ArcTan[(3*(-1)^(2/
3)*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3
)*c^(2/3)])])/(3*Sqrt[3]*(1 - (-1)^(1/3))*(1 + (-1)^(1/3))^2*a^(7/6)*b*Sqrt
[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)]*c^(1/3)) + Log[3*a + 3*a^(2/3)*c^(1/3)
*x + b*x^2]/(54*a^(4/3)*b*c^(2/3)) - ((-1)^(2/3)*Log[3*a - 3*(-1)^(1/3)*a^(
2/3)*c^(1/3)*x + b*x^2])/(18*(1 + (-1)^(1/3))^2*a^(4/3)*b*c^(2/3)) + ((-1)^
(2/3)*Log[3*a + 3*(-1)^(2/3)*a^(2/3)*c^(1/3)*x + b*x^2])/(54*a^(4/3)*b*c^(2
/3))
```

Rubi [A] time = 0.755903, antiderivative size = 487, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 5, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.109, Rules used = {2097, 634, 618, 204, 628}

$$\frac{\log\left(3a^{2/3}\sqrt[3]{c}x+3a+bx^2\right)}{54a^{4/3}bc^{2/3}}-\frac{(-1)^{2/3}\log\left(-3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x+3a+bx^2\right)}{18\left(1+\sqrt[3]{-1}\right)^2a^{4/3}bc^{2/3}}+\frac{(-1)^{2/3}\log\left(3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x+3a+bx^2\right)}{54a^{4/3}bc^{2/3}}-\frac{\quad}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6),x]
```


```
[Out] -ArcTan[(3*(-1)^(1/3)*a^(2/3)*c^(1/3) - 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b -
3*(-1)^(2/3)*a^(1/3)*c^(2/3)])]/(3*Sqrt[3]*(1 + (-1)^(1/3))^2*a^(7/6)*b*Sqr
t[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)]*c^(1/3)) - ArcTan[(3*a^(2/3)*c^(1/3)
+ 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*a^(1/3)*c^(2/3)])]/(9*Sqrt[3]*a^(7/6
)*b*Sqrt[4*b - 3*a^(1/3)*c^(2/3)]*c^(1/3)) + ((-1)^(1/3)*ArcTan[(3*(-1)^(2/
3)*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3
```

```
)*c^(2/3)])])/(3*Sqrt[3]*(1 - (-1)^(1/3))*(1 + (-1)^(1/3))^2*a^(7/6)*b*Sqrt
[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)]*c^(1/3)) + Log[3*a + 3*a^(2/3)*c^(1/3)
*x + b*x^2]/(54*a^(4/3)*b*c^(2/3)) - ((-1)^(2/3)*Log[3*a - 3*(-1)^(1/3)*a^(
2/3)*c^(1/3)*x + b*x^2])/(18*(1 + (-1)^(1/3))^2*a^(4/3)*b*c^(2/3)) + ((-1)^
(2/3)*Log[3*a + 3*(-1)^(2/3)*a^(2/3)*c^(1/3)*x + b*x^2])/(54*a^(4/3)*b*c^(2
/3))
```

Rule 2097

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^3}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx = \left(19683a^6\right)\int\left(\frac{(-1)^{2/3}x}{177147\left(1+\sqrt[3]{-1}\right)^2 a^{22/3}c^{2/3}\left(-3a+3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x-\right.}\right.$$

$$= \frac{\int \frac{x}{3a+3a^{2/3}\sqrt[3]{c}x+bx^2}\,dx}{27a^{4/3}c^{2/3}} + \frac{(-1)^{2/3}\int \frac{x}{3a+3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x+bx^2}\,dx}{27a^{4/3}c^{2/3}} + \frac{(-1)^{2/3}\int}{9\left(1\right.}$$

$$= \frac{\int \frac{3a^{2/3}\sqrt[3]{c}+2bx}{3a+3a^{2/3}\sqrt[3]{c}x+bx^2}\,dx}{54a^{4/3}bc^{2/3}} + \frac{(-1)^{2/3}\int \frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c}+2bx}{3a+3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x+bx^2}\,dx}{54a^{4/3}bc^{2/3}} - \frac{(-1)^{2/3}\int}{18\left(1\right.}$$

$$= \frac{\log\left(3a+3a^{2/3}\sqrt[3]{c}x+bx^2\right)}{54a^{4/3}bc^{2/3}} - \frac{(-1)^{2/3}\log\left(3a-3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x+bx^2\right)}{18\left(1+\sqrt[3]{-1}\right)^2 a^{4/3}bc^{2/3}}$$

$$= -\frac{\tan^{-1}\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{3\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2 a^{7/6}b\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}\sqrt[3]{c}} - \frac{\tan^{-1}\left(\frac{3a^{2/3}}{\sqrt{3}\sqrt{a}\sqrt{4}}\right)}{9\sqrt{3}a^{7/6}b\sqrt{4b-}}$$

Mathematica [C] time = 0.0486502, size = 99, normalized size = 0.2

$$\frac{1}{3}\text{RootSum}\left[27\#1^2a^2b+27\#1^3a^2c+9\#1^4ab^2+\#1^6b^3+27a^3\&,\frac{\#1^2\log(x-\#1)}{12\#1^2ab^2+2\#1^4b^3+27\#1a^2c+18a^2b}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6
),x]
```

```
[Out] RootSum[27*a^3 + 27*a^2*b*#1^2 + 27*a^2*c*#1^3 + 9*a*b^2*#1^4 + b^3*#1^6 &
, (Log[x - #1]*#1^2)/(18*a^2*b + 27*a^2*c*#1 + 12*a*b^2*#1^2 + 2*b^3*#1^4)
& ]/3
```

Maple [C] time = 0.003, size = 93, normalized size = 0.2

$$\frac{1}{3}\sum_{\_R=\text{RootOf}\left(b^3\_Z^6+9ab^2\_Z^4+27a^2c\_Z^3+27a^2b\_Z^2+27a^3\right)}\frac{\_R^3\ln(x-\_R)}{2\_R^5b^3+12\_R^3ab^2+27\_R^2a^2c+18\_Ra^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x)
```

```
[Out] 1/3*sum(_R^3/(2*_R^5*b^3+12*_R^3*a*b^2+27*_R^2*a^2*c+18*_R*a^2*b)*ln(x-_R),
_R=RootOf(_Z^6*b^3+9*_Z^4*a*b^2+27*_Z^3*a^2*c+27*_Z^2*a^2*b+27*a^3))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{b^3x^6 + 9\,ab^2x^4 + 27\,a^2cx^3 + 27\,a^2bx^2 + 27\,a^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, alg
orithm="maxima")
```

```
[Out] integrate(x^3/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3
), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b**3*x**6+9*a*b**2*x**4+27*a**2*c*x**3+27*a**2*b*x**2+27*a*
*3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{b^3x^6 + 9\,ab^2x^4 + 27\,a^2cx^3 + 27\,a^2bx^2 + 27\,a^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, alg
orithm="giac")
```

```
[Out] integrate(x^3/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3
), x)
```

3.138 $\int \frac{x^2}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx$

Optimal. Leaf size=334

$$\frac{2(-1)^{2/3}\tan^{-1}\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{9\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2a^{11/6}c^{2/3}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}+\frac{2\tan^{-1}\left(\frac{3a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}\right)}{27\sqrt{3}a^{11/6}c^{2/3}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}+\frac{2(-1)^{2/3}\tan^{-1}\left(\frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c}}{\sqrt{3}\sqrt{a}\sqrt{3\sqrt[3]{-1}\sqrt[3]{a}}}\right)}{9\sqrt{3}\left(1-\sqrt[3]{-1}\right)\left(1+\sqrt[3]{-1}\right)^2a^{11/6}c^{2/3}\sqrt{3}}$$

```
[Out] (2*(-1)^(2/3)*ArcTan[(3*(-1)^(1/3)*a^(2/3)*c^(1/3) - 2*b*x)/(Sqrt[3]*Sqrt[a
]*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)])])/(9*Sqrt[3]*(1 + (-1)^(1/3))^2
*a^(11/6)*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)]*c^(2/3)) + (2*ArcTan[(3*
a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*a^(1/3)*c^(2/3)])])/
(27*Sqrt[3]*a^(11/6)*Sqrt[4*b - 3*a^(1/3)*c^(2/3)]*c^(2/3)) + (2*(-1)^(2/3)
*ArcTan[(3*(-1)^(2/3)*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b +
3*(-1)^(1/3)*a^(1/3)*c^(2/3)])])/(9*Sqrt[3]*(1 - (-1)^(1/3))*(1 + (-1)^(1/3
))^2*a^(11/6)*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)]*c^(2/3))
```

Rubi [A] time = 0.469659, antiderivative size = 334, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2097, 618, 204}

$$\frac{2(-1)^{2/3}\tan^{-1}\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{9\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2a^{11/6}c^{2/3}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}+\frac{2\tan^{-1}\left(\frac{3a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}\right)}{27\sqrt{3}a^{11/6}c^{2/3}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}+\frac{2(-1)^{2/3}\tan^{-1}\left(\frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c}}{\sqrt{3}\sqrt{a}\sqrt{3\sqrt[3]{-1}\sqrt[3]{a}}}\right)}{9\sqrt{3}\left(1-\sqrt[3]{-1}\right)\left(1+\sqrt[3]{-1}\right)^2a^{11/6}c^{2/3}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6),x]
```

```
[Out] (2*(-1)^(2/3)*ArcTan[(3*(-1)^(1/3)*a^(2/3)*c^(1/3) - 2*b*x)/(Sqrt[3]*Sqrt[a
]*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)])])/(9*Sqrt[3]*(1 + (-1)^(1/3))^2
*a^(11/6)*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)]*c^(2/3)) + (2*ArcTan[(3*
a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*a^(1/3)*c^(2/3)])])/
(27*Sqrt[3]*a^(11/6)*Sqrt[4*b - 3*a^(1/3)*c^(2/3)]*c^(2/3)) + (2*(-1)^(2/3)
*ArcTan[(3*(-1)^(2/3)*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b +
3*(-1)^(1/3)*a^(1/3)*c^(2/3)])])/(9*Sqrt[3]*(1 - (-1)^(1/3))*(1 + (-1)^(1/3
))^2*a^(11/6)*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)]*c^(2/3))
```

Rule 2097

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^2}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx = \left(19683a^6\right)\int\left(\frac{(-1)^{2/3}}{177147\left(1+\sqrt[3]{-1}\right)^2 a^{22/3}c^{2/3}\left(-3a+3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x-\right.}\right.$$

$$= \frac{\int \frac{1}{3a+3a^{2/3}\sqrt[3]{c}x+bx^2}\,dx}{27a^{4/3}c^{2/3}} + \frac{(-1)^{2/3}\int \frac{1}{3a+3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x+bx^2}\,dx}{27a^{4/3}c^{2/3}} + \frac{(-1)^{2/3}\int}{9\left(1\right.}$$

$$= -\frac{2\,\mathrm{Subst}\left(\int \frac{1}{-3a\left(4b-3\sqrt[3]{a}c^{2/3}\right)-x^2}\,dx, x, 3a^{2/3}\sqrt[3]{c}+2bx\right)}{27a^{4/3}c^{2/3}} - \frac{\left(2(-1)^{2/3}\right)\mathrm{S}}{}$$

$$= \frac{2(-1)^{2/3}\tan^{-1}\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{9\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2 a^{11/6}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}\,c^{2/3}} + \frac{2\tan^{-1}\left(\frac{3a^{2/3}}{\sqrt{3}\sqrt{a}\sqrt{}}\right)}{27\sqrt{3}a^{11/6}\sqrt{4b-}}$$

Mathematica [C] time = 0.0413876, size = 97, normalized size = 0.29

$$\frac{1}{3}\mathrm{RootSum}\left[27\#1^2a^2b+27\#1^3a^2c+9\#1^4ab^2+\#1^6b^3+27a^3\&,\frac{\#1\log(x-\#1)}{12\#1^2ab^2+2\#1^4b^3+27\#1a^2c+18a^2b}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6
),x]
```

```
[Out] RootSum[27*a^3 + 27*a^2*b*#1^2 + 27*a^2*c*#1^3 + 9*a*b^2*#1^4 + b^3*#1^6 &
, (Log[x - #1]*#1)/(18*a^2*b + 27*a^2*c*#1 + 12*a*b^2*#1^2 + 2*b^3*#1^4) &
]/3
```

Maple [C] time = 0.003, size = 93, normalized size = 0.3

$$\frac{1}{3}\sum_{\_R=\mathrm{RootOf}\left(b^3\_Z^6+9\,ab^2\_Z^4+27\,a^2c\_Z^3+27\,a^2b\_Z^2+27\,a^3\right)}\frac{\_R^2\ln\left(x-\_R\right)}{2\_R^5b^3+12\_R^3ab^2+27\_R^2a^2c+18\_R\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x)
```

```
[Out] 1/3*sum(_R^2/(2*_R^5*b^3+12*_R^3*a*b^2+27*_R^2*a^2*c+18*_R*a^2*b)*ln(x-_R),
_R=RootOf(_Z^6*b^3+9*_Z^4*a*b^2+27*_Z^3*a^2*c+27*_Z^2*a^2*b+27*a^3))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{b^3x^6+9\,ab^2x^4+27\,a^2cx^3+27\,a^2bx^2+27\,a^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, alg
orithm="maxima")
```

```
[Out] integrate(x^2/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3
), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 28.1814, size = 167, normalized size = 0.5

$$\operatorname{RootSum}\left(t^6\left(282429536481a^{12}c^6-669462604992a^{11}b^3c^4\right)-129140163t^4a^8c^4+19683t^2a^4c^2-1,\left(t\mapsto t\log\left(x+\frac{62}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b**3*x**6+9*a*b**2*x**4+27*a**2*c*x**3+27*a**2*b*x**2+27*a*
*3),x)
```

```
[Out] RootSum(_t**6*(282429536481*a**12*c**6 - 669462604992*a**11*b**3*c**4) - 12
9140163*_t**4*a**8*c**4 + 19683*_t**2*a**4*c**2 - 1, Lambda(_t, _t*log(x +
(62762119218*_t**5*a**11*c**6 - 148769467776*_t**5*a**10*b**3*c**4 - 387420
489*_t**4*a**9*c**5 + 918330048*_t**4*a**8*b**3*c**3 - 23914845*_t**3*a**7*
c**4 - 11337408*_t**3*a**6*b**3*c**2 + 177147*_t**2*a**5*c**3 + 2187*_t*a**
3*c**2 - 18*a*c)/(8*b**2))))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{b^3x^6+9\,ab^2x^4+27\,a^2cx^3+27\,a^2bx^2+27\,a^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, alg
orithm="giac")
```

```
[Out] integrate(x^2/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3
), x)
```

3.139 $\int \frac{x}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx$

Optimal. Leaf size=469

$$-\frac{\log\left(3a^{2/3}\sqrt[3]{c}x+3a+bx^2\right)}{162a^{7/3}c^{2/3}}+\frac{(-1)^{2/3}\log\left(-3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x+3a+bx^2\right)}{54\left(1+\sqrt[3]{-1}\right)^2a^{7/3}c^{2/3}}-\frac{(-1)^{2/3}\log\left(3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x+3a+bx^2\right)}{162a^{7/3}c^{2/3}}-\cdots$$

```
[Out] -ArcTan[(3*(-1)^(1/3)*a^(2/3)*c^(1/3) - 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b -
3*(-1)^(2/3)*a^(1/3)*c^(2/3)])]/(9*Sqrt[3]*(1 + (-1)^(1/3))^2*a^(13/6)*Sqrt
[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)]*c^(1/3)) - ArcTan[(3*a^(2/3)*c^(1/3) +
 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*a^(1/3)*c^(2/3)])]/(27*Sqrt[3]*a^(13/
6)*Sqrt[4*b - 3*a^(1/3)*c^(2/3)]*c^(1/3)) + ((-1)^(1/3)*ArcTan[(3*(-1)^(2/3
)*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3)
*c^(2/3)])])/(9*Sqrt[3]*(1 - (-1)^(1/3))*(1 + (-1)^(1/3))^2*a^(13/6)*Sqrt[4
*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)]*c^(1/3)) - Log[3*a + 3*a^(2/3)*c^(1/3)*x
 + b*x^2]/(162*a^(7/3)*c^(2/3)) + ((-1)^(2/3)*Log[3*a - 3*(-1)^(1/3)*a^(2/3
)*c^(1/3)*x + b*x^2])/(54*(1 + (-1)^(1/3))^2*a^(7/3)*c^(2/3)) - ((-1)^(2/3)
*Log[3*a + 3*(-1)^(2/3)*a^(2/3)*c^(1/3)*x + b*x^2])/(162*a^(7/3)*c^(2/3))
```

Rubi [A] time = 0.684432, antiderivative size = 469, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 5, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2097, 634, 618, 204, 628}

$$-\frac{\log\left(3a^{2/3}\sqrt[3]{c}x+3a+bx^2\right)}{162a^{7/3}c^{2/3}}+\frac{(-1)^{2/3}\log\left(-3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x+3a+bx^2\right)}{54\left(1+\sqrt[3]{-1}\right)^2a^{7/3}c^{2/3}}-\frac{(-1)^{2/3}\log\left(3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x+3a+bx^2\right)}{162a^{7/3}c^{2/3}}-\cdots$$

Antiderivative was successfully verified.

```
[In] Int[x/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6),x]
```

```
[Out] -ArcTan[(3*(-1)^(1/3)*a^(2/3)*c^(1/3) - 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b -
3*(-1)^(2/3)*a^(1/3)*c^(2/3)])]/(9*Sqrt[3]*(1 + (-1)^(1/3))^2*a^(13/6)*Sqrt
[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)]*c^(1/3)) - ArcTan[(3*a^(2/3)*c^(1/3) +
 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*a^(1/3)*c^(2/3)])]/(27*Sqrt[3]*a^(13/
6)*Sqrt[4*b - 3*a^(1/3)*c^(2/3)]*c^(1/3)) + ((-1)^(1/3)*ArcTan[(3*(-1)^(2/3
)*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3)
*c^(2/3)])])/(9*Sqrt[3]*(1 - (-1)^(1/3))*(1 + (-1)^(1/3))^2*a^(13/6)*Sqrt[4
```

```
*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)]*c^(1/3)) - Log[3*a + 3*a^(2/3)*c^(1/3)*x
 + b*x^2]/(162*a^(7/3)*c^(2/3)) + ((-1)^(2/3)*Log[3*a - 3*(-1)^(1/3)*a^(2/3
)*c^(1/3)*x + b*x^2])/(54*(1 + (-1)^(1/3))^2*a^(7/3)*c^(2/3)) - ((-1)^(2/3)
*Log[3*a + 3*(-1)^(2/3)*a^(2/3)*c^(1/3)*x + b*x^2])/(162*a^(7/3)*c^(2/3))
```

Rule 2097

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx = \left(19683a^6\right)\int\left(\frac{-3a^{2/3}\sqrt[3]{c}-(-1)^{2/3}bx}{531441\left(1+\sqrt[3]{-1}\right)^2a^{25/3}c^{2/3}\left(-3a+3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x-b\right.}\right.$$

$$= \frac{\int\frac{-3a^{2/3}\sqrt[3]{c}-bx}{3a+3a^{2/3}\sqrt[3]{c}x+bx^2}\,dx}{81a^{7/3}c^{2/3}} - \frac{(-1)^{2/3}\int\frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c}+bx}{3a+3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x+bx^2}\,dx}{81a^{7/3}c^{2/3}} + \frac{\int\frac{-3a^{2/3}\sqrt[3]{}}{-3a+3\sqrt[3]{-}}}{27\left(1+\sqrt[3]{}\right.}$$

$$= -\frac{\int\frac{3a^{2/3}\sqrt[3]{c}+2bx}{3a+3a^{2/3}\sqrt[3]{c}x+bx^2}\,dx}{162a^{7/3}c^{2/3}} - \frac{(-1)^{2/3}\int\frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c}+2bx}{3a+3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x+bx^2}\,dx}{162a^{7/3}c^{2/3}} + \frac{(-1)^{2/3}\int}{54\left(\right.}$$

$$= -\frac{\log\left(3a+3a^{2/3}\sqrt[3]{c}x+bx^2\right)}{162a^{7/3}c^{2/3}} + \frac{(-1)^{2/3}\log\left(3a-3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x+bx^2\right)}{54\left(1+\sqrt[3]{-1}\right)^2a^{7/3}c^{2/3}}$$

$$= -\frac{\tan^{-1}\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{9\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2a^{13/6}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}\sqrt[3]{c}} - \frac{\tan^{-1}\left(\frac{3a^{2/3}\sqrt[3]{}}{\sqrt{3}\sqrt{a}\sqrt{4b}}\right.}{27\sqrt{3}a^{13/6}\sqrt{4b-}}$$

Mathematica [C] time = 0.0450699, size = 95, normalized size = 0.2

$$\frac{1}{3}\text{RootSum}\left[27\#1^2a^2b+27\#1^3a^2c+9\#1^4ab^2+\#1^6b^3+27a^3\&,\frac{\log(x-\#1)}{12\#1^2ab^2+2\#1^4b^3+27\#1a^2c+18a^2b}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6),
x]
```

```
[Out] RootSum[27*a^3 + 27*a^2*b*#1^2 + 27*a^2*c*#1^3 + 9*a*b^2*#1^4 + b^3*#1^6 &
, Log[x - #1]/(18*a^2*b + 27*a^2*c*#1 + 12*a*b^2*#1^2 + 2*b^3*#1^4) & ]/3
```

Maple [C] time = 0.003, size = 91, normalized size = 0.2

$$\frac{1}{3}\sum_{\_R=\text{RootOf}\left(b^3\_Z^6+9ab^2\_Z^4+27a^2c\_Z^3+27a^2b\_Z^2+27a^3\right)}\frac{\_R\ln(x-\_R)}{2\_R^5b^3+12\_R^3ab^2+27\_R^2a^2c+18\_Ra^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x)
```

```
[Out] 1/3*sum(_R/(2*_R^5*b^3+12*_R^3*a*b^2+27*_R^2*a^2*c+18*_R*a^2*b)*ln(x-_R),_R
=RootOf(_Z^6*b^3+9*_Z^4*a*b^2+27*_Z^3*a^2*c+27*_Z^2*a^2*b+27*a^3))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{b^3x^6 + 9\,ab^2x^4 + 27\,a^2cx^3 + 27\,a^2bx^2 + 27\,a^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, algor
ithm="maxima")
```

```
[Out] integrate(x/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3),
 x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b**3*x**6+9*a*b**2*x**4+27*a**2*c*x**3+27*a**2*b*x**2+27*a**3)
,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{b^3x^6 + 9\,ab^2x^4 + 27\,a^2cx^3 + 27\,a^2bx^2 + 27\,a^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, algor
ithm="giac")
```

```
[Out] integrate(x/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3),
 x)
```

3.140 $\int \frac{1}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx$

Optimal. Leaf size=522

$$-\frac{\sqrt[3]{-1}\left(3\sqrt[3]{a}c^{2/3}+2\sqrt[3]{-1}b\right)\tan^{-1}\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{27\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2a^{17/6}c^{2/3}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}-\frac{\left(2b-3\sqrt[3]{a}c^{2/3}\right)\tan^{-1}\left(\frac{3a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}\right)}{81\sqrt{3}a^{17/6}c^{2/3}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}-\frac{\left(2(-1)^{2/3}b-3\right.}{27\sqrt{3}\left(1-\sqrt[3]{-1}\right)\left(\right.}$$

```
[Out] -((-1)^(1/3)*(2*(-1)^(1/3)*b + 3*a^(1/3)*c^(2/3))*ArcTan[(3*(-1)^(1/3)*a^(2
/3)*c^(1/3) - 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/
3)])])/(27*Sqrt[3]*(1 + (-1)^(1/3))^2*a^(17/6)*Sqrt[4*b - 3*(-1)^(2/3)*a^(1
/3)*c^(2/3)]*c^(2/3)) - ((2*b - 3*a^(1/3)*c^(2/3))*ArcTan[(3*a^(2/3)*c^(1/3
) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*a^(1/3)*c^(2/3)])])/(81*Sqrt[3]*a^
(17/6)*Sqrt[4*b - 3*a^(1/3)*c^(2/3)]*c^(2/3)) - ((2*(-1)^(2/3)*b - 3*a^(1/3
)*c^(2/3))*ArcTan[(3*(-1)^(2/3)*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*S
qrt[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)])])/(27*Sqrt[3]*(1 - (-1)^(1/3))*(1
+ (-1)^(1/3))^2*a^(17/6)*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)]*c^(2/3))
+ Log[3*a + 3*a^(2/3)*c^(1/3)*x + b*x^2]/(162*a^(8/3)*c^(1/3)) - Log[3*a -
3*(-1)^(1/3)*a^(2/3)*c^(1/3)*x + b*x^2]/(54*(1 + (-1)^(1/3))^2*a^(8/3)*c^(1
/3)) - ((-1)^(1/3)*Log[3*a + 3*(-1)^(2/3)*a^(2/3)*c^(1/3)*x + b*x^2])/(162*
a^(8/3)*c^(1/3))
```

Rubi [A] time = 0.860841, antiderivative size = 522, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.119, Rules used = {2070, 634, 618, 204, 628}

$$-\frac{\sqrt[3]{-1}\left(3\sqrt[3]{a}c^{2/3}+2\sqrt[3]{-1}b\right)\tan^{-1}\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{27\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2a^{17/6}c^{2/3}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}-\frac{\left(2b-3\sqrt[3]{a}c^{2/3}\right)\tan^{-1}\left(\frac{3a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}\right)}{81\sqrt{3}a^{17/6}c^{2/3}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}-\frac{\left(2(-1)^{2/3}b-3\right.}{27\sqrt{3}\left(1-\sqrt[3]{-1}\right)\left(\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6)^(-1),x]
```


```
[Out] -((-1)^(1/3)*(2*(-1)^(1/3)*b + 3*a^(1/3)*c^(2/3))*ArcTan[(3*(-1)^(1/3)*a^(2
/3)*c^(1/3) - 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/
3)])])/(27*Sqrt[3]*(1 + (-1)^(1/3))^2*a^(17/6)*Sqrt[4*b - 3*(-1)^(2/3)*a^(1
/3)*c^(2/3)]*c^(2/3)) - ((2*b - 3*a^(1/3)*c^(2/3))*ArcTan[(3*a^(2/3)*c^(1/3
) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*a^(1/3)*c^(2/3)])])/(81*Sqrt[3]*a^
```

```
(17/6)*Sqrt[4*b - 3*a^(1/3)*c^(2/3)]*c^(2/3)) - ((2*(-1)^(2/3)*b - 3*a^(1/3
)*c^(2/3))*ArcTan[(3*(-1)^(2/3)*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*S
qrt[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)])])/(27*Sqrt[3]*(1 - (-1)^(1/3))*(1
+ (-1)^(1/3))^2*a^(17/6)*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)]*c^(2/3))
+ Log[3*a + 3*a^(2/3)*c^(1/3)*x + b*x^2]/(162*a^(8/3)*c^(1/3)) - Log[3*a -
3*(-1)^(1/3)*a^(2/3)*c^(1/3)*x + b*x^2]/(54*(1 + (-1)^(1/3))^2*a^(8/3)*c^(1
/3)) - ((-1)^(1/3)*Log[3*a + 3*(-1)^(2/3)*a^(2/3)*c^(1/3)*x + b*x^2])/(162*
a^(8/3)*c^(1/3))
```

Rule 2070

```
Int[(Q6_)^(p_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6, x, 2]
, c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist[1/(3
^(3*p)*a^(2*p)), Int[ExpandIntegrand[(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2
)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-1)^(2/3
)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0] && Eq
Q[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[Q6, x,
1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx = \left(19683a^6\right)\int\left(\frac{-(-1)^{2/3}\sqrt[3]{a}b-3\sqrt[3]{-1}a^{2/3}c^{2/3}+b\sqrt[3]{c}x}{531441\left(1+\sqrt[3]{-1}\right)^2 a^{26/3}c^{2/3}\left(-3a+3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x-\right.}\right.$$

$$= \frac{\int \frac{-\sqrt[3]{a}b+3a^{2/3}c^{2/3}+b\sqrt[3]{c}x}{3a+3a^{2/3}\sqrt[3]{c}x+bx^2}\,dx}{81a^{8/3}c^{2/3}} - \frac{\int \frac{(-1)^{2/3}\sqrt[3]{a}b-3a^{2/3}c^{2/3}+\sqrt[3]{-1}b\sqrt[3]{c}x}{3a+3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x+bx^2}\,dx}{81a^{8/3}c^{2/3}} + \frac{\int \frac{-(-1}{}}{2}$$

$$= -\frac{\left(2b-3\sqrt[3]{a}c^{2/3}\right)\int \frac{1}{3a+3a^{2/3}\sqrt[3]{c}x+bx^2}\,dx}{162a^{7/3}c^{2/3}} - \frac{\left(2(-1)^{2/3}b-3\sqrt[3]{a}c^{2/3}\right)\int \frac{}{3a+}}{162a^{7/3}c^{2}}$$

$$= \frac{\log\left(3a+3a^{2/3}\sqrt[3]{c}x+bx^2\right)}{162a^{8/3}\sqrt[3]{c}} - \frac{\log\left(3a-3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x+bx^2\right)}{54\left(1+\sqrt[3]{-1}\right)^2 a^{8/3}\sqrt[3]{c}} - \frac{\sqrt[3]{-}}{}$$

$$= -\frac{\left(2(-1)^{2/3}b+3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}\right)\tan^{-1}\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{27\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2 a^{17/6}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}\,c^{2/3}} - \frac{\left(2b-\right.}{8}$$

Mathematica [C] time = 0.0591077, size = 99, normalized size = 0.19

$$\frac{1}{3}\text{RootSum}\left[27\#1^2a^2b+27\#1^3a^2c+9\#1^4ab^2+\#1^6b^3+27a^3\&,\frac{\log(x-\#1)}{27\#1^2a^2c+12\#1^3ab^2+2\#1^5b^3+18\#1a^2b}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6)^(-
1),x]
```

```
[Out] RootSum[27*a^3 + 27*a^2*b*#1^2 + 27*a^2*c*#1^3 + 9*a*b^2*#1^4 + b^3*#1^6 &
, Log[x - #1]/(18*a^2*b*#1 + 27*a^2*c*#1^2 + 12*a*b^2*#1^3 + 2*b^3*#1^5) &
]/3
```

Maple [C] time = 0.003, size = 90, normalized size = 0.2

$$\frac{1}{3}\sum_{\_R=\text{RootOf}\left(b^3\_Z^6+9ab^2\_Z^4+27a^2c\_Z^3+27a^2b\_Z^2+27a^3\right)}\frac{\ln(x-\_R)}{2\_R^5b^3+12\_R^3ab^2+27\_R^2a^2c+18\_Ra^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x)
```

```
[Out] 1/3*sum(1/(2*_R^5*b^3+12*_R^3*a*b^2+27*_R^2*a^2*c+18*_R*a^2*b)*ln(x-_R),_R=
RootOf(_Z^6*b^3+9*_Z^4*a*b^2+27*_Z^3*a^2*c+27*_Z^2*a^2*b+27*a^3))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{b^3x^6 + 9\,ab^2x^4 + 27\,a^2cx^3 + 27\,a^2bx^2 + 27\,a^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, algor
ithm="maxima")
```

```
[Out] integrate(1/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3),
 x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**3*x**6+9*a*b**2*x**4+27*a**2*c*x**3+27*a**2*b*x**2+27*a**3)
,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{b^3x^6 + 9\,ab^2x^4 + 27\,a^2cx^3 + 27\,a^2bx^2 + 27\,a^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, algor
ithm="giac")
```

```
[Out] integrate(1/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3),
 x)
```

3.141 $\int \frac{1}{x\left(27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6\right)}\,dx$

Optimal. Leaf size=563

$$-\frac{\left(3\sqrt[3]{a}-\frac{b}{c^{2/3}}\right)\log\left(3a^{2/3}\sqrt[3]{c}x+3a+bx^2\right)}{486a^{10/3}}-\frac{\left(6\sqrt[3]{a}c^{2/3}+i\sqrt{3}b+b\right)\log\left(-3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x+3a+bx^2\right)}{972a^{10/3}c^{2/3}}-\frac{\left(3\sqrt[3]{a}-\frac{(-1)^{2/3}b}{c^{2/3}}\right)\log}{}$$

```
[Out] ((b - (-1)^(2/3)*a^(1/3)*c^(2/3))*ArcTan[(3*(-1)^(1/3)*a^(2/3)*c^(1/3) - 2*
b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)])])/(9*Sqrt[3
]*(1 + (-1)^(1/3))^2*a^(19/6)*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)]*c^(1
/3)) + ((b - a^(1/3)*c^(2/3))*ArcTan[(3*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*S
qrt[a]*Sqrt[4*b - 3*a^(1/3)*c^(2/3)])])/(27*Sqrt[3]*a^(19/6)*Sqrt[4*b - 3*a
^(1/3)*c^(2/3)]*c^(1/3)) + ((-1)^(2/3)*((-1)^(2/3)*b - a^(1/3)*c^(2/3))*Arc
Tan[(3*(-1)^(2/3)*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b + 3*(-
1)^(1/3)*a^(1/3)*c^(2/3)])])/(9*Sqrt[3]*(1 - (-1)^(1/3))*(1 + (-1)^(1/3))^2
*a^(19/6)*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)]*c^(1/3)) + Log[x]/(27*a^
3) - ((3*a^(1/3) - b/c^(2/3))*Log[3*a + 3*a^(2/3)*c^(1/3)*x + b*x^2])/(486*
a^(10/3)) - ((b + I*Sqrt[3]*b + 6*a^(1/3)*c^(2/3))*Log[3*a - 3*(-1)^(1/3)*a
^(2/3)*c^(1/3)*x + b*x^2])/(972*a^(10/3)*c^(2/3)) - ((3*a^(1/3) - ((-1)^(2/
3)*b)/c^(2/3))*Log[3*a + 3*(-1)^(2/3)*a^(2/3)*c^(1/3)*x + b*x^2])/(486*a^(1
0/3))
```

Rubi [A] time = 1.15653, antiderivative size = 563, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 5, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.109, Rules used = {2097, 634, 618, 204, 628}

$$-\frac{\left(3\sqrt[3]{a}-\frac{b}{c^{2/3}}\right)\log\left(3a^{2/3}\sqrt[3]{c}x+3a+bx^2\right)}{486a^{10/3}}-\frac{\left(6\sqrt[3]{a}c^{2/3}+i\sqrt{3}b+b\right)\log\left(-3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x+3a+bx^2\right)}{972a^{10/3}c^{2/3}}-\frac{\left(3\sqrt[3]{a}-\frac{(-1)^{2/3}b}{c^{2/3}}\right)\log}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6)),x]
```

```
[Out] ((b - (-1)^(2/3)*a^(1/3)*c^(2/3))*ArcTan[(3*(-1)^(1/3)*a^(2/3)*c^(1/3) - 2*
b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)])])/(9*Sqrt[3
]*(1 + (-1)^(1/3))^2*a^(19/6)*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)]*c^(1
/3)) + ((b - a^(1/3)*c^(2/3))*ArcTan[(3*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*S
```

```
qrt[a]*Sqrt[4*b - 3*a^(1/3)*c^(2/3)])])/(27*Sqrt[3]*a^(19/6)*Sqrt[4*b - 3*a
^(1/3)*c^(2/3)]*c^(1/3)) + ((-1)^(2/3)*((-1)^(2/3)*b - a^(1/3)*c^(2/3))*Arc
Tan[(3*(-1)^(2/3)*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b + 3*(-
1)^(1/3)*a^(1/3)*c^(2/3)])])/(9*Sqrt[3]*(1 - (-1)^(1/3))*(1 + (-1)^(1/3))^2
*a^(19/6)*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)]*c^(1/3)) + Log[x]/(27*a^
3) - ((3*a^(1/3) - b/c^(2/3))*Log[3*a + 3*a^(2/3)*c^(1/3)*x + b*x^2])/(486*
a^(10/3)) - ((b + I*Sqrt[3]*b + 6*a^(1/3)*c^(2/3))*Log[3*a - 3*(-1)^(1/3)*a
^(2/3)*c^(1/3)*x + b*x^2])/(972*a^(10/3)*c^(2/3)) - ((3*a^(1/3) - ((-1)^(2/
3)*b)/c^(2/3))*Log[3*a + 3*(-1)^(2/3)*a^(2/3)*c^(1/3)*x + b*x^2])/(486*a^(1
0/3))
```

Rule 2097

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

e}, x] && EqQ[2*c*d - b*e, 0]

Rubi steps

$$\int \frac{1}{x\left(27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6\right)}\,dx = \left(19683a^6\right)\int\left(\frac{1}{531441a^9x}+\frac{3a^{2/3}\left(2b-3\sqrt[3]{a}c^{2/3}\right)\sqrt[3]{c}+b\left(b-3\sqrt[3]{\right.}}{4782969a^{28/3}c^{2/3}\left(3a+3a^{2/3}\sqrt[3]{c}x+\right.}\right.$$

$$=\frac{\log(x)}{27a^3}+\frac{\int\frac{3a^{2/3}\left(2b-3\sqrt[3]{a}c^{2/3}\right)\sqrt[3]{c}+b\left(b-3\sqrt[3]{a}c^{2/3}\right)x}{3a+3a^{2/3}\sqrt[3]{c}x+bx^2}\,dx}{243a^{10/3}c^{2/3}}+\frac{(-1)^{2/3}\int\frac{3a^{2/3}(2(-}{}}{}$$

$$=\frac{\log(x)}{27a^3}-\frac{\left(3\sqrt[3]{a}-\frac{b}{c^{2/3}}\right)\int\frac{3a^{2/3}\sqrt[3]{c}+2bx}{3a+3a^{2/3}\sqrt[3]{c}x+bx^2}\,dx}{486a^{10/3}}-\frac{\left(3\sqrt[3]{a}-\frac{(-1)^{2/3}b}{c^{2/3}}\right)\int\frac{}{3}}{486}$$

$$=\frac{\log(x)}{27a^3}-\frac{\left(3\sqrt[3]{a}-\frac{b}{c^{2/3}}\right)\log\left(3a+3a^{2/3}\sqrt[3]{c}x+bx^2\right)}{486a^{10/3}}-\frac{\left(b+i\sqrt{3}b+\right.}{}$$

$$=\frac{\left(b-(-1)^{2/3}\sqrt[3]{a}c^{2/3}\right)\tan^{-1}\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{9\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2a^{19/6}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}\sqrt[3]{c}}+\frac{\left(b-\sqrt[3]{a}c^{2/3}\right)}{27\sqrt{3}a^{1}}$$

Mathematica [C] time = 0.0962968, size = 157, normalized size = 0.28

$$-\frac{\text{RootSum}\left[27\#1^2a^2b+27\#1^3a^2c+9\#1^4ab^2+\#1^6b^3+27a^3\&,\frac{9\#1^2ab^2\log(x-\#1)+\#1^4b^3\log(x-\#1)+27a^2b\log(x-\#1)+27\#1a^2c\log(x-\#}{12\#1^2ab^2+2\#1^4b^3+27\#1a^2c+18a^2b}\right.}{81a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^
6)),x]
```

```
[Out] -(-3*Log[x] + RootSum[27*a^3 + 27*a^2*b*#1^2 + 27*a^2*c*#1^3 + 9*a*b^2*#1^4
 + b^3*#1^6 & , (27*a^2*b*Log[x - #1] + 27*a^2*c*Log[x - #1]*#1 + 9*a*b^2*L
og[x - #1]*#1^2 + b^3*Log[x - #1]*#1^4)/(18*a^2*b + 27*a^2*c*#1 + 12*a*b^2*
#1^2 + 2*b^3*#1^4) & ])/(81*a^3)
```

Maple [C] time = 0.008, size = 134, normalized size = 0.2

$$-\frac{1}{81 a^3} \sum_{\_R=\mathrm{RootOf}\left(b^3\_Z^6+9 a b^2\_Z^4+27 a^2 c\_Z^3+27 a^2 b\_Z^2+27 a^3\right)} \frac{\left(\_R^5 b^3+9\_R^3 a b^2+27\_R^2 a^2 c+27\_R a^2 b\right)\ln\left(x-\_R\right)}{2\_R^5 b^3+12\_R^3 a b^2+27\_R^2 a^2 c+18\_R a^2 b}+\frac{\ln(}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x)
```

```
[Out] -1/81/a^3*sum((_R^5*b^3+9*_R^3*a*b^2+27*_R^2*a^2*c+27*_R*a^2*b)/(2*_R^5*b^3
+12*_R^3*a*b^2+27*_R^2*a^2*c+18*_R*a^2*b)*ln(x-_R),_R=RootOf(_Z^6*b^3+9*_Z^
4*a*b^2+27*_Z^3*a^2*c+27*_Z^2*a^2*b+27*a^3))+1/27*ln(x)/a^3
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, alg
orithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b**3*x**6+9*a*b**2*x**4+27*a**2*c*x**3+27*a**2*b*x**2+27*a**
3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b^{3}x^{6}+9\,ab^{2}x^{4}+27\,a^{2}cx^{3}+27\,a^{2}bx^{2}+27\,a^{3}\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, alg
orithm="giac")
```

```
[Out] integrate(1/((b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3)
*x), x)
```

3.142 $\int \frac{1}{x^2(27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6)}\,dx$

Optimal. Leaf size=645

$$\frac{\left(9a^{2/3}c^{4/3}+12\sqrt[3]{-1}\sqrt[3]{a}bc^{2/3}+2(-1)^{2/3}b^2\right)\tan^{-1}\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{81\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2a^{23/6}c^{2/3}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}+\frac{\left(9a^{2/3}c^{4/3}-12\sqrt[3]{a}bc^{2/3}+2b^2\right)\tan^{-1}\left(\frac{3a^{2/3}\sqrt[3]{}}{\sqrt{3}\sqrt{a}\sqrt{4b}}\right)}{243\sqrt{3}a^{23/6}c^{2/3}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}$$

```
[Out] -1/(27*a^3*x) + ((2*(-1)^(2/3)*b^2 + 12*(-1)^(1/3)*a^(1/3)*b*c^(2/3) + 9*a^
(2/3)*c^(4/3))*ArcTan[(3*(-1)^(1/3)*a^(2/3)*c^(1/3) - 2*b*x)/(Sqrt[3]*Sqrt[
a]*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)]]])/(81*Sqrt[3]*(1 + (-1)^(1/3))
^2*a^(23/6)*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)]*c^(2/3)) + ((2*b^2 - 1
2*a^(1/3)*b*c^(2/3) + 9*a^(2/3)*c^(4/3))*ArcTan[(3*a^(2/3)*c^(1/3) + 2*b*x)
/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*a^(1/3)*c^(2/3)]]])/(243*Sqrt[3]*a^(23/6)*Sq
rt[4*b - 3*a^(1/3)*c^(2/3)]*c^(2/3)) + ((-1)^(2/3)*(2*b^2 + 12*(-1)^(1/3)*a
^(1/3)*b*c^(2/3) + 9*(-1)^(2/3)*a^(2/3)*c^(4/3))*ArcTan[(3*(-1)^(2/3)*a^(2/
3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3
)]]])/(81*Sqrt[3]*(1 - (-1)^(1/3))*(1 + (-1)^(1/3))^2*a^(23/6)*Sqrt[4*b + 3
*(-1)^(1/3)*a^(1/3)*c^(2/3)]*c^(2/3)) - ((2*b - 3*a^(1/3)*c^(2/3))*Log[3*a
+ 3*a^(2/3)*c^(1/3)*x + b*x^2])/(486*a^(11/3)*c^(1/3)) + ((2*b - 3*(-1)^(2/
3)*a^(1/3)*c^(2/3))*Log[3*a - 3*(-1)^(1/3)*a^(2/3)*c^(1/3)*x + b*x^2])/(162
*(1 + (-1)^(1/3))^2*a^(11/3)*c^(1/3)) + ((-1)^(1/3)*(2*b + 3*(-1)^(1/3)*a^(
1/3)*c^(2/3))*Log[3*a + 3*(-1)^(2/3)*a^(2/3)*c^(1/3)*x + b*x^2])/(486*a^(11
/3)*c^(1/3))
```

Rubi [A] time = 1.37957, antiderivative size = 640, normalized size of antiderivative = 0.99, number of steps used = 14, number of rules used = 5, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.109, Rules used = {2097, 634, 618, 204, 628}

$$\frac{\left(9a^{2/3}c^{4/3}+12\sqrt[3]{-1}\sqrt[3]{a}bc^{2/3}+2(-1)^{2/3}b^2\right)\tan^{-1}\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{81\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2a^{23/6}c^{2/3}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}+\frac{\left(9a^{2/3}c^{4/3}-12\sqrt[3]{a}bc^{2/3}+2b^2\right)\tan^{-1}\left(\frac{3a^{2/3}\sqrt[3]{}}{\sqrt{3}\sqrt{a}\sqrt{4b}}\right)}{243\sqrt{3}a^{23/6}c^{2/3}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6)),
x]
```

```
[Out] -1/(27*a^3*x) + ((2*(-1)^(2/3)*b^2 + 12*(-1)^(1/3)*a^(1/3)*b*c^(2/3) + 9*a^
(2/3)*c^(4/3))*ArcTan[(3*(-1)^(1/3)*a^(2/3)*c^(1/3) - 2*b*x)/(Sqrt[3]*Sqrt[
```

```
a]*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)])])/(81*Sqrt[3]*(1 + (-1)^(1/3))
^2*a^(23/6)*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)]*c^(2/3)) + ((2*b^2 - 1
2*a^(1/3)*b*c^(2/3) + 9*a^(2/3)*c^(4/3))*ArcTan[(3*a^(2/3)*c^(1/3) + 2*b*x)
/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*a^(1/3)*c^(2/3)])])/(243*Sqrt[3]*a^(23/6)*Sq
rt[4*b - 3*a^(1/3)*c^(2/3)]*c^(2/3)) + ((2*(-1)^(2/3)*b^2 - 12*a^(1/3)*b*c^
(2/3) - 9*(-1)^(1/3)*a^(2/3)*c^(4/3))*ArcTan[(3*(-1)^(2/3)*a^(2/3)*c^(1/3)
+ 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)])])/(81*S
qrt[3]*(1 - (-1)^(1/3))*(1 + (-1)^(1/3))^2*a^(23/6)*Sqrt[4*b + 3*(-1)^(1/3)
*a^(1/3)*c^(2/3)]*c^(2/3)) - ((2*b - 3*a^(1/3)*c^(2/3))*Log[3*a + 3*a^(2/3)
*c^(1/3)*x + b*x^2])/(486*a^(11/3)*c^(1/3)) + ((2*b - 3*(-1)^(2/3)*a^(1/3)*
c^(2/3))*Log[3*a - 3*(-1)^(1/3)*a^(2/3)*c^(1/3)*x + b*x^2])/(162*(1 + (-1)^
(1/3))^2*a^(11/3)*c^(1/3)) + ((-1)^(1/3)*(2*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3
))*Log[3*a + 3*(-1)^(2/3)*a^(2/3)*c^(1/3)*x + b*x^2])/(486*a^(11/3)*c^(1/3)
)
```

<u>Rule 2097</u>

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

<u>Rule 634</u>

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

<u>Rule 618</u>

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

<u>Rule 204</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{x^2\left(27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6\right)}\,dx = \left(19683a^6\right)\int\left(\frac{1}{531441a^9x^2}+\frac{\sqrt[3]{a}\left(b^2-9\sqrt[3]{a}bc^{2/3}+9a^{2/3}c^{4/}\right.}{1594323\left(1-\sqrt[3]{-1}\right)\left(1+\sqrt[3]{-1}\right)^2 a}\right.$$

$$= -\frac{1}{27a^3x}+\frac{\int\frac{\sqrt[3]{a}\left(b^2-9\sqrt[3]{a}bc^{2/3}+9a^{2/3}c^{4/3}\right)-b\left(2b-3\sqrt[3]{a}c^{2/3}\right)\sqrt[3]{c}x}{3a+3a^{2/3}\sqrt[3]{c}x+bx^2}\,dx}{243a^{11/3}c^{2/3}}+\frac{\int\frac{\sqrt[3]{a}}{}}{}$$

$$= -\frac{1}{27a^3x}-\frac{\left(2b-3\sqrt[3]{a}c^{2/3}\right)\int\frac{3a^{2/3}\sqrt[3]{c}+2bx}{3a+3a^{2/3}\sqrt[3]{c}x+bx^2}\,dx}{486a^{11/3}\sqrt[3]{c}}+\frac{\left(\sqrt[3]{-1}\left(2b+3\right.\right.}{}$$

$$= -\frac{1}{27a^3x}-\frac{\left(2b-3\sqrt[3]{a}c^{2/3}\right)\log\left(3a+3a^{2/3}\sqrt[3]{c}x+bx^2\right)}{486a^{11/3}\sqrt[3]{c}}+\frac{\left(2b-\right.}{}$$

$$= -\frac{1}{27a^3x}+\frac{\left(2(-1)^{2/3}b^2+12\sqrt[3]{-1}\sqrt[3]{a}bc^{2/3}+9a^{2/3}c^{4/3}\right)\tan^{-1}\left(\frac{}{\sqrt{3}}\right.}{81\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2 a^{23/6}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{}}}$$

Mathematica [C] time = 0.119681, size = 163, normalized size = 0.25

$$-\frac{x\text{RootSum}\left[27\#1^2a^2b+27\#1^3a^2c+9\#1^4ab^2+\#1^6b^3+27a^3\&,\frac{9\#1^2ab^2\log(x-\#1)+\#1^4b^3\log(x-\#1)+27a^2b\log(x-\#1)+27\#1a^2c\log(x}{27\#1^2a^2c+12\#1^3ab^2+2\#1^5b^3+18\#1a^2b}\right.}{81a^3x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*
x^6)),x]
```

```
[Out] -(3 + x*RootSum[27*a^3 + 27*a^2*b*#1^2 + 27*a^2*c*#1^3 + 9*a*b^2*#1^4 + b^3
*#1^6 & , (27*a^2*b*Log[x - #1] + 27*a^2*c*Log[x - #1]*#1 + 9*a*b^2*Log[x -
 #1]*#1^2 + b^3*Log[x - #1]*#1^4)/(18*a^2*b*#1 + 27*a^2*c*#1^2 + 12*a*b^2*#
```

```
1^3 + 2*b^3*#1^5) & ])/(81*a^3*x)
```

Maple [C] time = 0.007, size = 133, normalized size = 0.2

$$\frac{1}{81\,a^{3}}\sum_{\_R=\mathrm{RootOf}\left(b^{3}\_Z^{6}+9\,ab^{2}\_Z^{4}+27\,a^{2}c\_Z^{3}+27\,a^{2}b\_Z^{2}+27\,a^{3}\right)}\frac{\left(-\_R^{4}b^{3}-9\,\_R^{2}ab^{2}-27\,\_R\,a^{2}c-27\,a^{2}b\right)\ln\left(x-\_R\right)}{2\,\_R^{5}b^{3}+12\,\_R^{3}ab^{2}+27\,\_R^{2}a^{2}c+18\,\_R\,a^{2}b}-\frac{1}{27\,a^{3}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x)
```

```
[Out] 1/81/a^3*sum((-_R^4*b^3-9*_R^2*a*b^2-27*_R*a^2*c-27*a^2*b)/(2*_R^5*b^3+12*_
R^3*a*b^2+27*_R^2*a^2*c+18*_R*a^2*b)*ln(x-_R),_R=RootOf(_Z^6*b^3+9*_Z^4*a*b
^2+27*_Z^3*a^2*c+27*_Z^2*a^2*b+27*a^3))-1/27/a^3/x
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, a
lgorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, a
lgorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b**3*x**6+9*a*b**2*x**4+27*a**2*c*x**3+27*a**2*b*x**2+27*
a**3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b^3x^6 + 9\,ab^2x^4 + 27\,a^2cx^3 + 27\,a^2bx^2 + 27\,a^3\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, a
lgorithm="giac")
```

```
[Out] integrate(1/((b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3)
*x^2), x)
```

3.143 $\int \frac{x^5}{216+108x^2+324x^3+18x^4+x^6}\,dx$

Optimal. Leaf size=395

$$\frac{1}{216}\left(36+2^{2/3}\sqrt[3]{3}\left(1+i\sqrt{3}\right)\right)\log\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)+\frac{1}{108}\left(18-(-2)^{2/3}\sqrt[3]{3}\right)\log\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)+\frac{1}{108}\left(18-2\right.$$

```
[Out] -(((-2)^(1/3)*(1 + (-2)^(1/3)*3^(2/3))*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/
Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(3^(5/6)*Sqrt[8 + (9*I)*2^(1/3)*3^(1/6
) + 3*2^(1/3)*3^(2/3)])) + ((3/2)^(1/6)*(1 - (-3)^(2/3)*2^(1/3))*ArcTan[(2^
(1/6)*(3*(-3)^(1/3) - 2^(1/3)*x))/Sqrt[3*(4 - 3*(-3)^(2/3)*2^(1/3))]])/((1
+ (-1)^(1/3))^2*Sqrt[4 - 3*(-3)^(2/3)*2^(1/3)]) - ((1 - 2^(1/3)*3^(2/3))*Ar
cTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]])/
(2^(1/6)*3^(5/6)*Sqrt[-4 + 3*2^(1/3)*3^(2/3)]) + ((36 + 2^(2/3)*3^(1/3)*(1
+ I*Sqrt[3]))*Log[6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2])/216 + ((18 - (-2)^(2/3
)*3^(1/3))*Log[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2])/108 + ((18 - 2^(2/3)*3^(1
/3))*Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2])/108
```

Rubi [A] time = 1.43847, antiderivative size = 395, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2097, 634, 618, 204, 628, 206}

$$\frac{1}{216}\left(36+2^{2/3}\sqrt[3]{3}\left(1+i\sqrt{3}\right)\right)\log\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)+\frac{1}{108}\left(18-(-2)^{2/3}\sqrt[3]{3}\right)\log\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)+\frac{1}{108}\left(18-2\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
```

```
[Out] -(((-2)^(1/3)*(1 + (-2)^(1/3)*3^(2/3))*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/
Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(3^(5/6)*Sqrt[8 + (9*I)*2^(1/3)*3^(1/6
) + 3*2^(1/3)*3^(2/3)])) + ((3/2)^(1/6)*(1 - (-3)^(2/3)*2^(1/3))*ArcTan[(2^
(1/6)*(3*(-3)^(1/3) - 2^(1/3)*x))/Sqrt[3*(4 - 3*(-3)^(2/3)*2^(1/3))]])/((1
+ (-1)^(1/3))^2*Sqrt[4 - 3*(-3)^(2/3)*2^(1/3)]) - ((1 - 2^(1/3)*3^(2/3))*Ar
cTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]])/
(2^(1/6)*3^(5/6)*Sqrt[-4 + 3*2^(1/3)*3^(2/3)]) + ((36 + 2^(2/3)*3^(1/3)*(1
```

```
+ I*Sqrt[3]))*Log[6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2])/216 + ((18 - (-2)^(2/3
)*3^(1/3))*Log[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2])/108 + ((18 - 2^(2/3)*3^(1
/3))*Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2])/108
```

Rule 2097

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^5}{216+108x^2+324x^3+18x^4+x^6}\,dx = 1259712\int\left(\frac{(-1)^{2/3}\left(3\sqrt[3]{-3}2^{2/3}+\left(1-3(-3)^{2/3}\sqrt[3]{2}\right)x\right)}{3779136\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^2\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}+\frac{(\cdots}{3779136\sqrt[3]{2}\cdots}\right.$$

$$=\frac{1}{54}\int\frac{6\sqrt[3]{2}3^{2/3}+\left(18-2^{2/3}\sqrt[3]{3}\right)x}{6+3\,2^{2/3}\sqrt[3]{3}x+x^2}\,dx+\frac{(-1)^{2/3}\int\frac{3(-2)^{2/3}\sqrt[3]{3}-\left(1+3\sqrt[3]{-2}3^{2/3}\right)x}{6+3(-2)^{2/3}\sqrt[3]{3}x+x^2}\,dx}{9\sqrt[3]{2}3^{2/3}}+\cdots$$

$$=\frac{\left((-1)^{2/3}\left(1-3(-3)^{2/3}\sqrt[3]{2}\right)\right)\int\frac{-3\sqrt[3]{-3}2^{2/3}+2x}{6-3\sqrt[3]{-3}2^{2/3}x+x^2}\,dx}{6\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^2}+\frac{\left((-1)^{2/3}\sqrt[3]{\frac{3}{2}}\left(6+\sqrt[3]{-3}2^{2/3}\right)\right)\int\cdots}{2\left(1+\sqrt[3]{-1}\right)\cdots}$$

$$=\frac{1}{216}\left(36+2^{2/3}\sqrt[3]{3}+i2^{2/3}3^{5/6}\right)\log\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)+\frac{1}{108}\left(18-(-2)^{2/3}\sqrt[3]{\cdots}\right.$$

$$=\frac{(-1)^{2/3}\left((-2)^{2/3}-2\,3^{2/3}\right)\tan^{-1}\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{6^{5/6}\sqrt{4+3\sqrt[3]{-2}3^{2/3}}}-\frac{(-1)^{2/3}\left(\sqrt[3]{-3}+3\sqrt[3]{2}\right)\tan^{-1}\cdots}{\sqrt[6]{6}\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-\cdots}}$$

Mathematica [C] time = 0.0155941, size = 61, normalized size = 0.15

$$\frac{1}{6}\text{RootSum}\left[\#1^6+18\#1^4+324\#1^3+108\#1^2+216\&,\frac{\#1^4\log(x-\#1)}{\#1^4+12\#1^2+162\#1+36}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
```

```
[Out] RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 & , (Log[x - #1]*#1^4)/(
36 + 162*#1 + 12*#1^2 + #1^4) & ]/6
```

Maple [C] time = 0.007, size = 56, normalized size = 0.1

$$\frac{1}{6}\sum_{\_R=\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\_R^5\ln(x-\_R)}{\_R^5+12\_R^3+162\_R^2+36\_R}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^6+18*x^4+324*x^3+108*x^2+216),x)
```

```
[Out] 1/6*sum(_R^5/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(x-_R),_R=RootOf(_Z^6+18*_Z^4+
324*_Z^3+108*_Z^2+216))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{x^6+18\,x^4+324\,x^3+108\,x^2+216}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="maxima")
```

```
[Out] integrate(x^5/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 0.218052, size = 70, normalized size = 0.18

$$\operatorname{RootSum}\left(72662865048t^6 - 72662865048t^5 + 24163559388t^4 - 2646786132t^3 - 6626610t^2 - 4374t - 1, \left(t \mapsto t\log\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(x**6+18*x**4+324*x**3+108*x**2+216),x)
```

```
[Out] RootSum(72662865048*_t**6 - 72662865048*_t**5 + 24163559388*_t**4 - 2646786
132*_t**3 - 6626610*_t**2 - 4374*_t - 1, Lambda(_t, _t*log(-892364171310473
76*_t**5/833243797 + 89301949532998128*_t**4/833243797 - 29740560281805852*
_t**3/833243797 + 192466080408420*_t**2/49014341 + 5867255361684*_t/8332437
97 + x + 5365044886/2499731391)))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{x^6 + 18\,x^4 + 324\,x^3 + 108\,x^2 + 216}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="giac")
```

```
[Out] integrate(x^5/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

3.144 $\int \frac{x^4}{216+108x^2+324x^3+18x^4+x^6}\,dx$

Optimal. Leaf size=377

$$\frac{\log\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)}{6\ 2^{2/3}\sqrt[3]{3}\left(1+\sqrt[3]{-1}\right)^2}+\frac{\sqrt[3]{-\frac{1}{3}}\log\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)}{18\ 2^{2/3}}-\frac{\log\left(x^2+3\ 2^{2/3}\sqrt[3]{3}x+6\right)}{18\ 2^{2/3}\sqrt[3]{3}}+\frac{(-1)^{2/3}\left(3(-3)^{2/3}-2^{2/3}\right)\tan}{9\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^2\sqrt{2}}$$

```
[Out] ((-1)^(2/3)*(3*(-3)^(2/3) - 2^(2/3))*ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sq
rt[6*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(9*3^(1/6)*(1 + (-1)^(1/3))^2*Sqrt[2*(4
- 3*(-3)^(2/3)*2^(1/3))]) + ((9 - (-2)^(2/3)*3^(1/3))*ArcTan[(3*(-2)^(2/3)*
3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(27*Sqrt[3*(8 + (9*I)*2
^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3))]) - ((9 - 2^(2/3)*3^(1/3))*ArcTanh[(2^(
1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]])/(27*Sqrt[6
*(-4 + 3*2^(1/3)*3^(2/3))]) + Log[6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2]/(6*2^(2
/3)*3^(1/3)*(1 + (-1)^(1/3))^2) + ((-1/3)^(1/3)*Log[6 + 3*(-2)^(2/3)*3^(1/3
)*x + x^2])/(18*2^(2/3)) - Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2]/(18*2^(2/3)*3
^(1/3))
```

Rubi [A] time = 0.90978, antiderivative size = 377, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2097, 634, 618, 204, 628, 206}

$$\frac{\log\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)}{6\ 2^{2/3}\sqrt[3]{3}\left(1+\sqrt[3]{-1}\right)^2}+\frac{\sqrt[3]{-\frac{1}{3}}\log\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)}{18\ 2^{2/3}}-\frac{\log\left(x^2+3\ 2^{2/3}\sqrt[3]{3}x+6\right)}{18\ 2^{2/3}\sqrt[3]{3}}+\frac{(-1)^{2/3}\left(3(-3)^{2/3}-2^{2/3}\right)\tan}{9\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
```


```
[Out] ((-1)^(2/3)*(3*(-3)^(2/3) - 2^(2/3))*ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sq
rt[6*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(9*3^(1/6)*(1 + (-1)^(1/3))^2*Sqrt[2*(4
- 3*(-3)^(2/3)*2^(1/3))]) + ((9 - (-2)^(2/3)*3^(1/3))*ArcTan[(3*(-2)^(2/3)*
3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(27*Sqrt[3*(8 + (9*I)*2
^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3))]) - ((9 - 2^(2/3)*3^(1/3))*ArcTanh[(2^(
1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]])/(27*Sqrt[6
*(-4 + 3*2^(1/3)*3^(2/3))]) + Log[6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2]/(6*2^(2
```

```
/3)*3^(1/3)*(1 + (-1)^(1/3))^2) + ((-1/3)^(1/3)*Log[6 + 3*(-2)^(2/3)*3^(1/3
)*x + x^2])/(18*2^(2/3)) - Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2]/(18*2^(2/3)*3
^(1/3))
```

Rule 2097

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^4}{216+108x^2+324x^3+18x^4+x^6}\,dx = 1259712\int\left(\frac{(-1)^{2/3}\left(-2+\sqrt[3]{-3}2^{2/3}x\right)}{7558272\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^2\left(-6+3\sqrt[3]{-3}2^{2/3}x-x^2\right)}+\frac{}{7558272}\right.$$

$$= -\frac{(-1)^{2/3}\int\frac{2+(-2)^{2/3}\sqrt[3]{3}x}{6+3(-2)^{2/3}\sqrt[3]{3}x+x^2}\,dx}{18\sqrt[3]{2}3^{2/3}} - \frac{\int\frac{\sqrt[3]{2}+\sqrt[3]{3}x}{6+3\ 2^{2/3}\sqrt[3]{3}x+x^2}\,dx}{9\ 6^{2/3}} + \frac{(-1)^{2/3}\int\frac{-2+\sqrt[3]{-3}2^{2/3}x}{-6+3\sqrt[3]{-3}2^{2/3}x-}}{6\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)}$$

$$= \frac{\sqrt[3]{-\frac{1}{3}}\int\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{6+3(-2)^{2/3}\sqrt[3]{3}x+x^2}\,dx}{18\ 2^{2/3}} - \frac{\int\frac{3\ 2^{2/3}\sqrt[3]{3}+2x}{6+3\ 2^{2/3}\sqrt[3]{3}x+x^2}\,dx}{18\ 2^{2/3}\sqrt[3]{3}} + \frac{\int\frac{3\sqrt[3]{-3}2^{2/3}-2x}{-6+3\sqrt[3]{-3}2^{2/3}x-x^2}\,dx}{6\ 2^{2/3}\sqrt[3]{3}\left(1+\sqrt[3]{-1}\right)^2} - \cdots$$

$$= \frac{\log\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}{6\ 2^{2/3}\sqrt[3]{3}\left(1+\sqrt[3]{-1}\right)^2} + \frac{\sqrt[3]{-\frac{1}{3}}\log\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}{18\ 2^{2/3}} - \frac{\log\left(6+3\ \cdots\right.}{18\ \cdots}$$

$$= -\frac{\sqrt[3]{-1}\left(9+\sqrt[3]{-3}2^{2/3}\right)\tan^{-1}\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{9\left(1+\sqrt[3]{-1}\right)^2\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}} - \frac{\left((-2)^{2/3}-3\ 3^{2/3}\right)\tan^{-1}\left(\frac{3(\cdots}{\sqrt{6\cdots}}\right.}{27\sqrt[6]{3}\sqrt{2\left(4+3\sqrt[3]{-2}\cdots\right.}}$$

Mathematica [C] time = 0.0131222, size = 61, normalized size = 0.16

$$\frac{1}{6}\text{RootSum}\left[\#1^6+18\#1^4+324\#1^3+108\#1^2+216\&,\frac{\#1^3\log(x-\#1)}{\#1^4+12\#1^2+162\#1+36}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
```

```
[Out] RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 & , (Log[x - #1]*#1^3)/(
36 + 162*#1 + 12*#1^2 + #1^4) & ]/6
```

Maple [C] time = 0.006, size = 56, normalized size = 0.2

$$\frac{1}{6}\sum_{\_R=\text{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\_R^4\ln(x-\_R)}{\_R^5+12\_R^3+162\_R^2+36\_R}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^6+18*x^4+324*x^3+108*x^2+216),x)
```

```
[Out] 1/6*sum(_R^4/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(x-_R),_R=RootOf(_Z^6+18*_Z^4+
324*_Z^3+108*_Z^2+216))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{x^6 + 18\,x^4 + 324\,x^3 + 108\,x^2 + 216}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 0.23073, size = 65, normalized size = 0.17

$$\operatorname{RootSum}\left(156951788503683t^6 - 2066242608t^4 + 1845163152t^3 - 1180980t^2 - 1944t - 1, \left(t \mapsto t\log\left(\frac{614714526178551}{571212951}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(x**6+18*x**4+324*x**3+108*x**2+216),x)
```

```
[Out] RootSum(15695178850368*_t**6 - 2066242608*_t**4 + 1845163152*_t**3 - 118098
0*_t**2 - 1944*_t - 1, Lambda(_t, _t*log(6147145261785517462O8*_t**5/571212
95165 - 1270857362386176*_t**4/57121295165 - 80483053187684376*_t**3/571212
95165 + 72431318325103884*_t**2/57121295165 - 45358602689088*_t/57121295165
 + x - 44532180783/57121295165)))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{x^6 + 18\,x^4 + 324\,x^3 + 108\,x^2 + 216}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="giac")
```

```
[Out] integrate(x^4/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

3.145 $\int \frac{x^3}{216+108x^2+324x^3+18x^4+x^6} \, dx$

Optimal. Leaf size=361

$$-\frac{(-1)^{2/3}\log\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)}{36\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^2}+\frac{(-1)^{2/3}\log\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)}{108\sqrt[3]{2}3^{2/3}}+\frac{\log\left(x^2+3\ 2^{2/3}\sqrt[3]{3}x+6\right)}{108\sqrt[3]{2}3^{2/3}}-\frac{\tan^{-1}\left(\frac{3}{\sqrt{6}}\right.}{6\sqrt[6]{2}3^{5/6}\left(1+\sqrt[3]{-1}\right.}$$

```
[Out] -ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4 - 3*(-3)^(2/3)*2^(1/3))]]/(6
*2^(1/6)*3^(5/6)*(1 + (-1)^(1/3))^2*Sqrt[4 - 3*(-3)^(2/3)*2^(1/3)]) + ((-1)
^(1/3)*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3)
)]])/(9*2^(2/3)*3^(5/6)*Sqrt[8 + (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3)]
) + ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3
))]]/(18*2^(1/6)*3^(5/6)*Sqrt[-4 + 3*2^(1/3)*3^(2/3)]) - ((-1)^(2/3)*Log[6
- 3*(-3)^(1/3)*2^(2/3)*x + x^2])/(36*2^(1/3)*3^(2/3)*(1 + (-1)^(1/3))^2) +
((-1)^(2/3)*Log[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2])/(108*2^(1/3)*3^(2/3)) +
Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2]/(108*2^(1/3)*3^(2/3))
```

Rubi [A] time = 0.514918, antiderivative size = 361, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2097, 634, 618, 204, 628, 206}

$$-\frac{(-1)^{2/3}\log\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)}{36\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^2}+\frac{(-1)^{2/3}\log\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)}{108\sqrt[3]{2}3^{2/3}}+\frac{\log\left(x^2+3\ 2^{2/3}\sqrt[3]{3}x+6\right)}{108\sqrt[3]{2}3^{2/3}}-\frac{\tan^{-1}\left(\frac{3}{\sqrt{6}}\right.}{6\sqrt[6]{2}3^{5/6}\left(1+\sqrt[3]{-1}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
```



```
[Out] -ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4 - 3*(-3)^(2/3)*2^(1/3))]]/(6
*2^(1/6)*3^(5/6)*(1 + (-1)^(1/3))^2*Sqrt[4 - 3*(-3)^(2/3)*2^(1/3)]) + ((-1)
^(1/3)*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3)
)]])/(9*2^(2/3)*3^(5/6)*Sqrt[8 + (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3)]
) + ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3
))]]/(18*2^(1/6)*3^(5/6)*Sqrt[-4 + 3*2^(1/3)*3^(2/3)]) - ((-1)^(2/3)*Log[6
- 3*(-3)^(1/3)*2^(2/3)*x + x^2])/(36*2^(1/3)*3^(2/3)*(1 + (-1)^(1/3))^2) +
((-1)^(2/3)*Log[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2])/(108*2^(1/3)*3^(2/3)) +
```

```
Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2]/(108*2^(1/3)*3^(2/3))
```

Rule 2097

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^3}{216+108x^2+324x^3+18x^4+x^6}\,dx = 1259712\int\left(\frac{(-1)^{2/3}x}{22674816\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^2\left(-6+3\sqrt[3]{-3}2^{2/3}x-x^2\right)} - \frac{}{22674816}\right.$$

$$= \frac{\int \frac{x}{6+3\ 2^{2/3}\sqrt[3]{3}x+x^2}\,dx}{54\sqrt[3]{2}3^{2/3}} + \frac{(-1)^{2/3}\int \frac{x}{6+3(-2)^{2/3}\sqrt[3]{3}x+x^2}\,dx}{54\sqrt[3]{2}3^{2/3}} + \frac{(-1)^{2/3}\int \frac{x}{-6+3\sqrt[3]{-3}2^{2/3}x-x^2}}{18\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^2}$$

$$= \frac{\sqrt[3]{-\frac{1}{3}}\int \frac{1}{6+3(-2)^{2/3}\sqrt[3]{3}x+x^2}\,dx}{18\ 2^{2/3}} + \frac{\int \frac{3\ 2^{2/3}\sqrt[3]{3}+2x}{6+3\ 2^{2/3}\sqrt[3]{3}x+x^2}\,dx}{108\sqrt[3]{2}3^{2/3}} + \frac{(-1)^{2/3}\int \frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{6+3(-2)^{2/3}\sqrt[3]{3}x+x^2}\,dx}{108\sqrt[3]{2}3^{2/3}}$$

$$= -\frac{(-1)^{2/3}\log\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}{36\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^2} + \frac{(-1)^{2/3}\log\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}{108\sqrt[3]{2}3^{2/3}} + \frac{\log}{}$$

$$= -\frac{\tan^{-1}\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{6\sqrt[6]{2}3^{5/6}\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}} + \frac{\sqrt[3]{-1}\tan^{-1}\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{18\sqrt[6]{2}3^{5/6}\sqrt{4+3\sqrt[3]{-2}3^{2/3}}} + \frac{\tan}{18\sqrt[6]{}}$$

Mathematica [C] time = 0.0133283, size = 61, normalized size = 0.17

$$\frac{1}{6}\text{RootSum}\left[\#1^6+18\#1^4+324\#1^3+108\#1^2+216\&,\frac{\#1^2\log(x-\#1)}{\#1^4+12\#1^2+162\#1+36}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
```

```
[Out] RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 & , (Log[x - #1]*#1^2)/(
36 + 162*#1 + 12*#1^2 + #1^4) & ]/6
```

Maple [C] time = 0.006, size = 56, normalized size = 0.2

$$\frac{1}{6}\sum_{\_R=\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\_R^3\ln(x-\_R)}{\_R^5+12\_R^3+162\_R^2+36\_R}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^6+18*x^4+324*x^3+108*x^2+216),x)
```

```
[Out] 1/6*sum(_R^3/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(x-_R),_R=RootOf(_Z^6+18*_Z^4+
324*_Z^3+108*_Z^2+216))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{x^6 + 18\,x^4 + 324\,x^3 + 108\,x^2 + 216}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 0.219544, size = 61, normalized size = 0.17

$$\operatorname{RootSum}\left(3390158631679488t^6 - 74384733888t^4 - 1332145440t^3 - 1417176t^2 - 1, \left(t \mapsto t\log\left(-\frac{8482372214243328}{415817}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**6+18*x**4+324*x**3+108*x**2+216),x)
```

```
[Out] RootSum(3390158631679488*_t**6 - 74384733888*_t**4 - 1332145440*_t**3 - 141
7176*_t**2 - 1, Lambda(_t, _t*log(-8482372214243328*_t**5/415817 + 22160559
```

```
10930560*_t**4/415817 - 2062546612992*_t**3/415817 - 57027208896*_t**2/4158
17 - 416583756*_t/415817 + x - 89938/415817)))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{x^6 + 18\,x^4 + 324\,x^3 + 108\,x^2 + 216}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="giac")
```

```
[Out] integrate(x^3/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

3.146 $\int \frac{x^2}{216+108x^2+324x^3+18x^4+x^6}\,dx$

Optimal. Leaf size=248

$$\frac{(-1)^{2/3}\tan^{-1}\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{27\ 2^{5/6}\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}}+\frac{(-1)^{2/3}\tan^{-1}\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{81\sqrt[3]{2}\sqrt[6]{3}\sqrt{8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}}}-\frac{\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}x+3\sqrt[3]{3}\right)}{\sqrt{3\left(3\sqrt[3]{2}3^{2/3}-4\right)}}\right)}{81\ 2^{5/6}\sqrt[6]{3}\sqrt{3\sqrt[3]{2}3^{2/3}-4}}$$

```
[Out] ((-1)^(2/3)*ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4 - 3*(-3)^(2/3)*2^
(1/3))]])/(27*2^(5/6)*3^(1/6)*(1 + (-1)^(1/3))^2*Sqrt[4 - 3*(-3)^(2/3)*2^(1
/3)]) + ((-1)^(2/3)*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^
(1/3)*3^(2/3))]])/(81*2^(1/3)*3^(1/6)*Sqrt[8 + (9*I)*2^(1/3)*3^(1/6) + 3*2^
(1/3)*3^(2/3)]) - ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*
2^(1/3)*3^(2/3))]]/(81*2^(5/6)*3^(1/6)*Sqrt[-4 + 3*2^(1/3)*3^(2/3)])
```

Rubi [A] time = 0.322549, antiderivative size = 248, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2097, 618, 204, 206}

$$\frac{(-1)^{2/3}\tan^{-1}\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{27\ 2^{5/6}\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}}+\frac{(-1)^{2/3}\tan^{-1}\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{81\sqrt[3]{2}\sqrt[6]{3}\sqrt{8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}}}-\frac{\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}x+3\sqrt[3]{3}\right)}{\sqrt{3\left(3\sqrt[3]{2}3^{2/3}-4\right)}}\right)}{81\ 2^{5/6}\sqrt[6]{3}\sqrt{3\sqrt[3]{2}3^{2/3}-4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
```


```
[Out] ((-1)^(2/3)*ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4 - 3*(-3)^(2/3)*2^
(1/3))]])/(27*2^(5/6)*3^(1/6)*(1 + (-1)^(1/3))^2*Sqrt[4 - 3*(-3)^(2/3)*2^(1
/3)]) + ((-1)^(2/3)*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^
(1/3)*3^(2/3))]])/(81*2^(1/3)*3^(1/6)*Sqrt[8 + (9*I)*2^(1/3)*3^(1/6) + 3*2^
(1/3)*3^(2/3)]) - ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*
2^(1/3)*3^(2/3))]]/(81*2^(5/6)*3^(1/6)*Sqrt[-4 + 3*2^(1/3)*3^(2/3)])
```


Rule 2097

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
```

```
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^2}{216+108x^2+324x^3+18x^4+x^6}\,dx = 1259712\int\left(\frac{(-1)^{2/3}}{22674816\sqrt[3]{2}\,3^{2/3}\left(1+\sqrt[3]{-1}\right)^2\left(-6+3\sqrt[3]{-3}\,2^{2/3}x-x^2\right)}-\frac{\cdots}{22674816\cdots}\cdots\right.$$

$$=\frac{\int\frac{1}{6+3\,2^{2/3}\sqrt[3]{3}x+x^2}\,dx}{54\sqrt[3]{2}\,3^{2/3}}+\frac{(-1)^{2/3}\int\frac{1}{6+3(-2)^{2/3}\sqrt[3]{3}x+x^2}\,dx}{54\sqrt[3]{2}\,3^{2/3}}+\frac{(-1)^{2/3}\int\frac{1}{-6+3\sqrt[3]{-3}\,2^{2/3}x-x^2}\,d\cdots}{18\sqrt[3]{2}\,3^{2/3}\left(1+\sqrt[3]{-1}\right)^2}\cdots$$

$$=-\frac{\text{Subst}\left(\int\frac{1}{-6\left(4-3\sqrt[3]{2}\,3^{2/3}\right)-x^2}\,dx,x,3\,2^{2/3}\sqrt[3]{3}+2x\right)}{27\sqrt[3]{2}\,3^{2/3}}-\frac{(-1)^{2/3}\,\text{Subst}\left(\int\frac{1}{-6\left(4+3\sqrt[3]{-\cdots}\right)\cdots}\cdots\right)}{27\cdots}\cdots$$

$$=\frac{(-1)^{2/3}\tan^{-1}\left(\frac{3\sqrt[3]{-3}\,2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{27\,2^{5/6}\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}}+\frac{(-1)^{2/3}\tan^{-1}\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}\,3^{2/3}\right)}}\right)}{81\,2^{5/6}\sqrt[6]{3}\sqrt{4+3\sqrt[3]{-2}\,3^{2/3}}}-\frac{\text{t}\cdots}{81\cdots}\cdots$$

Mathematica [C] time = 0.0124029, size = 59, normalized size = 0.24

$$\frac{1}{6}\text{RootSum}\left[\#1^6+18\#1^4+324\#1^3+108\#1^2+216\&,\frac{\#1\log(x-\#1)}{\#1^4+12\#1^2+162\#1+36}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
```

```
[Out] RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 & , (Log[x - #1]*#1)/(36
 + 162*#1 + 12*#1^2 + #1^4) & ]/6
```

Maple [C] time = 0.005, size = 56, normalized size = 0.2

$$\frac{1}{6}\sum_{\_R=\text{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\_R^2\ln(x-\_R)}{\_R^5+12\_R^3+162\_R^2+36\_R}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^6+18*x^4+324*x^3+108*x^2+216),x)
```

```
[Out] 1/6*sum(_R^2/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(x-_R),_R=RootOf(_Z^6+18*_Z^4+
324*_Z^3+108*_Z^2+216))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{x^6+18x^4+324x^3+108x^2+216}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

Fricas [B] time = 8.11088, size = 4757, normalized size = 19.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="fricas")
```

```
[Out] 1/324*sqrt(1/633)*sqrt(6*18^(2/3) + 8*18^(1/3) + 81)*log(1/211*sqrt(1/633)*
(3*(6*18^(2/3) + 8*18^(1/3) + 81)^2 - 3741*18^(2/3) - 4988*18^(1/3) - 24867
)*sqrt(6*18^(2/3) + 8*18^(1/3) + 81) - 1/422*(6*18^(2/3) + 8*18^(1/3) + 81)
^2 + 2*x + 729/211*18^(2/3) + 972/211*18^(1/3) + 8289/422) - 1/324*sqrt(1/6
33)*sqrt(6*18^(2/3) + 8*18^(1/3) + 81)*log(-1/211*sqrt(1/633)*(3*(6*18^(2/3
) + 8*18^(1/3) + 81)^2 - 3741*18^(2/3) - 4988*18^(1/3) - 24867)*sqrt(6*18^(
2/3) + 8*18^(1/3) + 81) - 1/422*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 2*x + 72
9/211*18^(2/3) + 972/211*18^(1/3) + 8289/422) - 1/136728*sqrt(1266)*sqrt(-2
/3*18^(2/3) + sqrt(-1/27*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 36*18^(2/3) + 4
8*18^(1/3) + 371) - 8/9*18^(1/3) + 18)*log(2*(6*18^(2/3) + 8*18^(1/3) + 81)
^2 + 18*sqrt(-1/27*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 36*18^(2/3) + 48*18^(
1/3) + 371)*(6*18^(2/3) + 8*18^(1/3) + 81) + 1/211*(6*sqrt(1266)*(6*18^(2/3
) + 8*18^(1/3) + 81)^2 + 9*sqrt(-1/27*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 36
*18^(2/3) + 48*18^(1/3) + 371)*(6*sqrt(1266)*(6*18^(2/3) + 8*18^(1/3) + 81)
 - 211*sqrt(1266)) - 1247*sqrt(1266)*(6*18^(2/3) + 8*18^(1/3) + 81) + 51273
*sqrt(1266))*sqrt(-2/3*18^(2/3) + sqrt(-1/27*(6*18^(2/3) + 8*18^(1/3) + 81)
^2 + 36*18^(2/3) + 48*18^(1/3) + 371) - 8/9*18^(1/3) + 18) + 3376*x - 2916*
18^(2/3) - 3888*18^(1/3) - 16578) + 1/136728*sqrt(1266)*sqrt(-2/3*18^(2/3)
+ sqrt(-1/27*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 36*18^(2/3) + 48*18^(1/3) +
 371) - 8/9*18^(1/3) + 18)*log(2*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 18*sqrt
(-1/27*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 36*18^(2/3) + 48*18^(1/3) + 371)*
(6*18^(2/3) + 8*18^(1/3) + 81) - 1/211*(6*sqrt(1266)*(6*18^(2/3) + 8*18^(1/
3) + 81)^2 + 9*sqrt(-1/27*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 36*18^(2/3) +
48*18^(1/3) + 371)*(6*sqrt(1266)*(6*18^(2/3) + 8*18^(1/3) + 81) - 211*sqrt(
1266)) - 1247*sqrt(1266)*(6*18^(2/3) + 8*18^(1/3) + 81) + 51273*sqrt(1266))
*sqrt(-2/3*18^(2/3) + sqrt(-1/27*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 36*18^(
2/3) + 48*18^(1/3) + 371) - 8/9*18^(1/3) + 18) + 3376*x - 2916*18^(2/3) - 3
888*18^(1/3) - 16578) - 1/136728*sqrt(1266)*sqrt(-2/3*18^(2/3) - sqrt(-1/27
*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 36*18^(2/3) + 48*18^(1/3) + 371) - 8/9*
18^(1/3) + 18)*log(2*(6*18^(2/3) + 8*18^(1/3) + 81)^2 - 18*sqrt(-1/27*(6*18
^(2/3) + 8*18^(1/3) + 81)^2 + 36*18^(2/3) + 48*18^(1/3) + 371)*(6*18^(2/3)
+ 8*18^(1/3) + 81) + 1/211*(6*sqrt(1266)*(6*18^(2/3) + 8*18^(1/3) + 81)^2 -
 9*sqrt(-1/27*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 36*18^(2/3) + 48*18^(1/3)
+ 371)*(6*sqrt(1266)*(6*18^(2/3) + 8*18^(1/3) + 81) - 211*sqrt(1266)) - 124
7*sqrt(1266)*(6*18^(2/3) + 8*18^(1/3) + 81) + 51273*sqrt(1266))*sqrt(-2/3*1
8^(2/3) - sqrt(-1/27*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 36*18^(2/3) + 48*18
^(1/3) + 371) - 8/9*18^(1/3) + 18) + 3376*x - 2916*18^(2/3) - 3888*18^(1/3)
 - 16578) + 1/136728*sqrt(1266)*sqrt(-2/3*18^(2/3) - sqrt(-1/27*(6*18^(2/3)
 + 8*18^(1/3) + 81)^2 + 36*18^(2/3) + 48*18^(1/3) + 371) - 8/9*18^(1/3) + 1
8)*log(2*(6*18^(2/3) + 8*18^(1/3) + 81)^2 - 18*sqrt(-1/27*(6*18^(2/3) + 8*1
```

```
8^(1/3) + 81)^2 + 36*18^(2/3) + 48*18^(1/3) + 371)*(6*18^(2/3) + 8*18^(1/3)
 + 81) - 1/211*(6*sqrt(1266)*(6*18^(2/3) + 8*18^(1/3) + 81)^2 - 9*sqrt(-1/2
7*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 36*18^(2/3) + 48*18^(1/3) + 371)*(6*sq
rt(1266)*(6*18^(2/3) + 8*18^(1/3) + 81) - 211*sqrt(1266)) - 1247*sqrt(1266)
*(6*18^(2/3) + 8*18^(1/3) + 81) + 51273*sqrt(1266))*sqrt(-2/3*18^(2/3) - sq
rt(-1/27*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 36*18^(2/3) + 48*18^(1/3) + 371
) - 8/9*18^(1/3) + 18) + 3376*x - 2916*18^(2/3) - 3888*18^(1/3) - 16578)
```

Sympy [A] time = 0.173886, size = 48, normalized size = 0.19

$$\operatorname{RootSum}\left(732274264442769408t^6 - 2677850419968t^4 + 2834352t^2 - 1, \left(t \mapsto t\log\left(10170475895038464t^5 - 523172\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**6+18*x**4+324*x**3+108*x**2+216),x)
```

```
[Out] RootSum(732274264442769408*_t**6 - 2677850419968*_t**4 + 2834352*_t**2 - 1,
 Lambda(_t, _t*log(10170475895038464*_t**5 - 5231726283456*_t**4 - 31809932
496*_t**3 + 19131876*_t**2 + 19683*_t + x - 27/2)))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{x^6 + 18\,x^4 + 324\,x^3 + 108\,x^2 + 216}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="giac")
```

```
[Out] integrate(x^2/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

3.147 $\int \frac{x}{216+108x^2+324x^3+18x^4+x^6}\,dx$

Optimal. Leaf size=361

$$\frac{(-1)^{2/3}\log\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)}{216\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^2}-\frac{(-1)^{2/3}\log\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)}{648\sqrt[3]{2}3^{2/3}}-\frac{\log\left(x^2+3\ 2^{2/3}\sqrt[3]{3}x+6\right)}{648\sqrt[3]{2}3^{2/3}}-\frac{\tan^{-1}\left(\frac{3\ldots}{\sqrt{6(\ldots}}\right)}{36\sqrt[6]{2}3^{5/6}\left(1+\sqrt[3]{-1}\ldots\right.}$$

```
[Out] -ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4 - 3*(-3)^(2/3)*2^(1/3))]]/(3
6*2^(1/6)*3^(5/6)*(1 + (-1)^(1/3))^2*Sqrt[4 - 3*(-3)^(2/3)*2^(1/3)]) + ((-1
)^(1/3)*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3
))]])/(54*2^(2/3)*3^(5/6)*Sqrt[8 + (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3
)]) + ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2
/3))]]/(108*2^(1/6)*3^(5/6)*Sqrt[-4 + 3*2^(1/3)*3^(2/3)]) + ((-1)^(2/3)*Log
[6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2])/(216*2^(1/3)*3^(2/3)*(1 + (-1)^(1/3))^2
) - ((-1)^(2/3)*Log[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2])/(648*2^(1/3)*3^(2/3)
) - Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2]/(648*2^(1/3)*3^(2/3))
```

Rubi [A] time = 0.547554, antiderivative size = 361, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2097, 634, 618, 204, 628, 206}

$$\frac{(-1)^{2/3}\log\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)}{216\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^2}-\frac{(-1)^{2/3}\log\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)}{648\sqrt[3]{2}3^{2/3}}-\frac{\log\left(x^2+3\ 2^{2/3}\sqrt[3]{3}x+6\right)}{648\sqrt[3]{2}3^{2/3}}-\frac{\tan^{-1}\left(\frac{3\ldots}{\sqrt{6(\ldots}}\right)}{36\sqrt[6]{2}3^{5/6}\left(1+\sqrt[3]{-1}\ldots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
```



```
[Out] -ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4 - 3*(-3)^(2/3)*2^(1/3))]]/(3
6*2^(1/6)*3^(5/6)*(1 + (-1)^(1/3))^2*Sqrt[4 - 3*(-3)^(2/3)*2^(1/3)]) + ((-1
)^(1/3)*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3
))]])/(54*2^(2/3)*3^(5/6)*Sqrt[8 + (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3
)]) + ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2
/3))]]/(108*2^(1/6)*3^(5/6)*Sqrt[-4 + 3*2^(1/3)*3^(2/3)]) + ((-1)^(2/3)*Log
[6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2])/(216*2^(1/3)*3^(2/3)*(1 + (-1)^(1/3))^2
) - ((-1)^(2/3)*Log[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2])/(648*2^(1/3)*3^(2/3)
```

```
) - Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2]/(648*2^(1/3)*3^(2/3))
```

Rule 2097

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x}{216+108x^2+324x^3+18x^4+x^6}\,dx = 1259712\int\left(\frac{(-1)^{2/3}\left(3\sqrt[3]{-3}2^{2/3}-x\right)}{136048896\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^2\left(-6+3\sqrt[3]{-3}2^{2/3}x-x^2\right)}+\frac{}{136048\ldots}\right.$$

$$= -\frac{(-1)^{2/3}\int\frac{3(-2)^{2/3}\sqrt[3]{3}+x}{6+3(-2)^{2/3}\sqrt[3]{3}x+x^2}dx}{324\sqrt[3]{2}3^{2/3}} - \frac{\int\frac{6\sqrt[3]{3}+\sqrt[3]{2}x}{6+3\,2^{2/3}\sqrt[3]{3}x+x^2}dx}{324\,6^{2/3}} + \frac{(-1)^{2/3}\int\frac{3\sqrt[3]{-3}2^{2/3}-x}{-6+3\sqrt[3]{-3}2^{2/3}x-x^2}}{108\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)}$$

$$= \frac{\sqrt[3]{-\frac{1}{3}}\int\frac{1}{6+3(-2)^{2/3}\sqrt[3]{3}x+x^2}dx}{108\,2^{2/3}} - \frac{\int\frac{3\,2^{2/3}\sqrt[3]{3}+2x}{6+3\,2^{2/3}\sqrt[3]{3}x+x^2}dx}{648\sqrt[3]{2}3^{2/3}} - \frac{(-1)^{2/3}\int\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{6+3(-2)^{2/3}\sqrt[3]{3}x+x^2}dx}{648\sqrt[3]{2}3^{2/3}}$$

$$= \frac{(-1)^{2/3}\log\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}{216\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^2} - \frac{(-1)^{2/3}\log\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}{648\sqrt[3]{2}3^{2/3}} - \frac{\log(\ldots}{}$$

$$= -\frac{\tan^{-1}\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{36\sqrt[6]{2}3^{5/6}\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}} + \frac{\sqrt[3]{-1}\tan^{-1}\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{108\sqrt[6]{2}3^{5/6}\sqrt{4+3\sqrt[3]{-2}3^{2/3}}} + \frac{\mathrm{ta}\ldots}{108\ldots}$$

Mathematica [C] time = 0.0116398, size = 57, normalized size = 0.16

$$\frac{1}{6}\text{RootSum}\left[\#1^6+18\#1^4+324\#1^3+108\#1^2+216\&,\frac{\log(x-\#1)}{\#1^4+12\#1^2+162\#1+36}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
```

```
[Out] RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 & , Log[x - #1]/(36 + 16
2*#1 + 12*#1^2 + #1^4) & ]/6
```

Maple [C] time = 0.005, size = 54, normalized size = 0.2

$$\frac{1}{6}\sum_{\_R=\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\_R\ln(x-\_R)}{\_R^5+12\_R^3+162\_R^2+36\_R}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^6+18*x^4+324*x^3+108*x^2+216),x)
```

```
[Out] 1/6*sum(_R/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(x-_R),_R=RootOf(_Z^6+18*_Z^4+32
4*_Z^3+108*_Z^2+216))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{x^6+18\,x^4+324\,x^3+108\,x^2+216}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="maxima")
```

```
[Out] integrate(x/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 0.215503, size = 61, normalized size = 0.17

$$\operatorname{RootSum}\left(158171241119638192128t^6 - 96402615118848t^4 + 287743415040t^3 - 51018336t^2 - 1, \left(t \mapsto t\log\left(\frac{6541839\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**6+18*x**4+324*x**3+108*x**2+216),x)
```

```
[Out] RootSum(158171241119638192128*_t**6 - 96402615118848*_t**4 + 287743415040*_
t**3 - 51018336*_t**2 - 1, Lambda(_t, _t*log(654183994457211409612 80*_t**5/
```

```
415817 + 2480926457425102848*_t**4/415817 - 39451802929737984*_t**3/415817
+ 118071997444800*_t**2/415817 - 16745884920*_t/415817 + x - 268790/415817)
))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{x^6 + 18\,x^4 + 324\,x^3 + 108\,x^2 + 216}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="giac")
```

```
[Out] integrate(x/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

3.148 $\int \frac{1}{216+108x^2+324x^3+18x^4+x^6}\,dx$

Optimal. Leaf size=377

$$-\frac{\log\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)}{216\ 2^{2/3}\sqrt[3]{3}\left(1+\sqrt[3]{-1}\right)^2}-\frac{\sqrt[3]{-\frac{1}{3}}\log\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)}{648\ 2^{2/3}}+\frac{\log\left(x^2+3\ 2^{2/3}\sqrt[3]{3}x+6\right)}{648\ 2^{2/3}\sqrt[3]{3}}+\frac{(-1)^{2/3}\left(3(-3)^{2/3}-2^{2/3}\right)\cdots}{324\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^2\cdots}$$

```
[Out] ((-1)^(2/3)*(3*(-3)^(2/3) - 2^(2/3))*ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sq
rt[6*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(324*3^(1/6)*(1 + (-1)^(1/3))^2*Sqrt[2*(
4 - 3*(-3)^(2/3)*2^(1/3))]) + ((9 - (-2)^(2/3)*3^(1/3))*ArcTan[(3*(-2)^(2/3
)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(972*Sqrt[3*(8 + (9*I
)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3))]) - ((9 - 2^(2/3)*3^(1/3))*ArcTanh[(
2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]])/(972*Sq
rt[6*(-4 + 3*2^(1/3)*3^(2/3))]) - Log[6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2]/(21
6*2^(2/3)*3^(1/3)*(1 + (-1)^(1/3))^2) - ((-1/3)^(1/3)*Log[6 + 3*(-2)^(2/3)*
3^(1/3)*x + x^2])/(648*2^(2/3)) + Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2]/(648*2
^(2/3)*3^(1/3))
```

Rubi [A] time = 0.720951, antiderivative size = 377, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2070, 634, 618, 204, 628, 206}

$$-\frac{\log\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)}{216\ 2^{2/3}\sqrt[3]{3}\left(1+\sqrt[3]{-1}\right)^2}-\frac{\sqrt[3]{-\frac{1}{3}}\log\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)}{648\ 2^{2/3}}+\frac{\log\left(x^2+3\ 2^{2/3}\sqrt[3]{3}x+6\right)}{648\ 2^{2/3}\sqrt[3]{3}}+\frac{(-1)^{2/3}\left(3(-3)^{2/3}-2^{2/3}\right)\cdots}{324\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^2\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^(-1),x]
```


```
[Out] ((-1)^(2/3)*(3*(-3)^(2/3) - 2^(2/3))*ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sq
rt[6*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(324*3^(1/6)*(1 + (-1)^(1/3))^2*Sqrt[2*(
4 - 3*(-3)^(2/3)*2^(1/3))]) + ((9 - (-2)^(2/3)*3^(1/3))*ArcTan[(3*(-2)^(2/3
)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(972*Sqrt[3*(8 + (9*I
)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3))]) - ((9 - 2^(2/3)*3^(1/3))*ArcTanh[(
2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]])/(972*Sq
rt[6*(-4 + 3*2^(1/3)*3^(2/3))]) - Log[6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2]/(21
```

```
6*2^(2/3)*3^(1/3)*(1 + (-1)^(1/3))^2) - ((-1/3)^(1/3)*Log[6 + 3*(-2)^(2/3)*
3^(1/3)*x + x^2])/(648*2^(2/3)) + Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2]/(648*2
^(2/3)*3^(1/3))
```

Rule 2070

```
Int[(Q6_)^(p_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6, x, 2]
, c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist[1/(3
^(3*p)*a^(2*p)), Int[ExpandIntegrand[(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2
)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-1)^(2/3
)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0] && Eq
Q[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[Q6, x,
1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{216+108x^2+324x^3+18x^4+x^6}\,dx = 1259712\int\left(\frac{(-1)^{2/3}\left(-2+6(-3)^{2/3}\sqrt[3]{2}-\sqrt[3]{-3}2^{2/3}x\right)}{272097792\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^2\left(-6+3\sqrt[3]{-3}2^{2/3}x-x^2\right)}+\frac{}{27209}\right.$$

$$=\frac{\int\frac{18-2^{2/3}\sqrt[3]{3}+\sqrt[3]{2}3^{2/3}x}{6+3\ 2^{2/3}\sqrt[3]{3}x+x^2}\,dx}{1944}-\frac{\int\frac{2(-1)^{2/3}-6\sqrt[3]{2}3^{2/3}+\sqrt[3]{-3}2^{2/3}x}{6+3(-2)^{2/3}\sqrt[3]{3}x+x^2}\,dx}{648\sqrt[3]{2}3^{2/3}}+\frac{(-1)^{2/3}\int\frac{-2+6(-3)^{2/3}\sqrt[3]{}}{-6+3\sqrt[3]{-3}}}{216\sqrt[3]{2}3^{2/3}\left(1+\right.}$$

$$=-\frac{\sqrt[3]{-\frac{1}{3}}\int\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{6+3(-2)^{2/3}\sqrt[3]{3}x+x^2}\,dx}{648\ 2^{2/3}}+\frac{\int\frac{3\ 2^{2/3}\sqrt[3]{3}+2x}{6+3\ 2^{2/3}\sqrt[3]{3}x+x^2}\,dx}{648\ 2^{2/3}\sqrt[3]{3}}-\frac{\int\frac{3\sqrt[3]{-3}2^{2/3}-2x}{-6+3\sqrt[3]{-3}2^{2/3}x-x^2}\,dx}{216\ 2^{2/3}\sqrt[3]{3}\left(1+\sqrt[3]{-1}\right)^2}$$

$$=-\frac{\log\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}{216\ 2^{2/3}\sqrt[3]{3}\left(1+\sqrt[3]{-1}\right)^2}-\frac{\sqrt[3]{-\frac{1}{3}}\log\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}{648\ 2^{2/3}}+\frac{\log\left(6+3\right.}{648}$$

$$=-\frac{\sqrt[3]{-1}\left(9+\sqrt[3]{-3}2^{2/3}\right)\tan^{-1}\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{324\left(1+\sqrt[3]{-1}\right)^2\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}-\frac{\left((-2)^{2/3}-3\ 3^{2/3}\right)\tan^{-1}\left(\frac{3(}{\sqrt{6}}\right.}{972\sqrt[6]{3}\sqrt{2\left(4+3\sqrt[3]{-}\right.}}$$

Mathematica [C] time = 0.011869, size = 62, normalized size = 0.16

$$\frac{1}{6}\text{RootSum}\left[\#1^6+18\#1^4+324\#1^3+108\#1^2+216\&,\frac{\log(x-\#1)}{\#1^5+12\#1^3+162\#1^2+36\#1}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^(-1),x]
```

```
[Out] RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 & , Log[x - #1]/(36*#1 +
 162*#1^2 + 12*#1^3 + #1^5) & ]/6
```

Maple [C] time = 0.004, size = 53, normalized size = 0.1

$$\frac{1}{6}\sum_{\_R=\text{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\ln(x-\_R)}{\_R^5+12\_R^3+162\_R^2+36\_R}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6+18*x^4+324*x^3+108*x^2+216),x)
```

```
[Out] 1/6*sum(1/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(x-_R),_R=RootOf(_Z^6+18*_Z^4+324
*_Z^3+108*_Z^2+216))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^6 + 18\,x^4 + 324\,x^3 + 108\,x^2 + 216}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 0.224718, size = 65, normalized size = 0.17

$$\operatorname{RootSum}\left(341649880818418494996 48t^6 - 3470494144278528t^4 - 86087932019712t^3 - 1530550080t^2 + 69984t - 1, \left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**6+18*x**4+324*x**3+108*x**2+216),x)
```

```
[Out] RootSum(34164988081841849499648*_t**6 - 3470494144278528*_t**4 - 8608793201
9712*_t**3 - 1530550080*_t**2 + 69984*_t - 1, Lambda(_t, _t*log(18590444669
9109611410573787136*_t**5/57121295165 + 6377301253267917382766592*_t**4/571
21295165 - 18904636002388564311552*_t**3/57121295165 - 46908055291518172396
8*_t**2/57121295165 - 24358640509989936*_t/57121295165 + x + 152427895956/5
7121295165)))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^6+18x^4+324x^3+108x^2+216}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="giac")
```

```
[Out] integrate(1/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

3.149 $\int \frac{1}{x(216+108x^2+324x^3+18x^4+x^6)}\,dx$

Optimal. Leaf size=415

$$-\frac{\left(36+2^{2/3}\sqrt[3]{3}\left(1+i\sqrt{3}\right)\right)\log\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)}{46656}-\frac{\left(18-(-2)^{2/3}\sqrt[3]{3}\right)\log\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)}{23328}-\frac{\left(18-2^{2/3}\sqrt[3]{3}\right)\log}{2}$$

```
[Out] ((-1)^(2/3)*((-2)^(2/3) - 2*3^(2/3))*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sq
rt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(216*2^(1/3)*3^(5/6)*Sqrt[8 + (9*I)*2^(1
/3)*3^(1/6) + 3*2^(1/3)*3^(2/3)]) - ((-1)^(2/3)*((-3)^(1/3) + 3*2^(1/3))*Ar
cTan[(2^(1/6)*(3*(-3)^(1/3) - 2^(1/3)*x))/Sqrt[3*(4 - 3*(-3)^(2/3)*2^(1/3))
]])/(216*6^(1/6)*(1 + (-1)^(1/3))^2*Sqrt[4 - 3*(-3)^(2/3)*2^(1/3)]) - ((1 -
 2^(1/3)*3^(2/3))*ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*
2^(1/3)*3^(2/3))]])/(216*2^(1/6)*3^(5/6)*Sqrt[-4 + 3*2^(1/3)*3^(2/3)]) + Lo
g[x]/216 - ((36 + 2^(2/3)*3^(1/3)*(1 + I*Sqrt[3]))*Log[6 - 3*(-3)^(1/3)*2^(
2/3)*x + x^2])/46656 - ((18 - (-2)^(2/3)*3^(1/3))*Log[6 + 3*(-2)^(2/3)*3^(1
/3)*x + x^2])/23328 - ((18 - 2^(2/3)*3^(1/3))*Log[6 + 3*2^(2/3)*3^(1/3)*x +
 x^2])/23328
```

Rubi [A] time = 0.901607, antiderivative size = 415, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2097, 634, 618, 204, 628, 206}

$$-\frac{\left(36+2^{2/3}\sqrt[3]{3}\left(1+i\sqrt{3}\right)\right)\log\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)}{46656}-\frac{\left(18-(-2)^{2/3}\sqrt[3]{3}\right)\log\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)}{23328}-\frac{\left(18-2^{2/3}\sqrt[3]{3}\right)\log}{2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)),x]
```

```
[Out] ((-1)^(2/3)*((-2)^(2/3) - 2*3^(2/3))*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sq
rt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(216*2^(1/3)*3^(5/6)*Sqrt[8 + (9*I)*2^(1
/3)*3^(1/6) + 3*2^(1/3)*3^(2/3)]) - ((-1)^(2/3)*((-3)^(1/3) + 3*2^(1/3))*Ar
cTan[(2^(1/6)*(3*(-3)^(1/3) - 2^(1/3)*x))/Sqrt[3*(4 - 3*(-3)^(2/3)*2^(1/3))
]])/(216*6^(1/6)*(1 + (-1)^(1/3))^2*Sqrt[4 - 3*(-3)^(2/3)*2^(1/3)]) - ((1 -
```

```
 2^(1/3)*3^(2/3))*ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*
2^(1/3)*3^(2/3))]])/(216*2^(1/6)*3^(5/6)*Sqrt[-4 + 3*2^(1/3)*3^(2/3)]) + Lo
g[x]/216 - ((36 + 2^(2/3)*3^(1/3)*(1 + I*Sqrt[3]))*Log[6 - 3*(-3)^(1/3)*2^(
2/3)*x + x^2])/46656 - ((18 - (-2)^(2/3)*3^(1/3))*Log[6 + 3*(-2)^(2/3)*3^(1
/3)*x + x^2])/23328 - ((18 - 2^(2/3)*3^(1/3))*Log[6 + 3*2^(2/3)*3^(1/3)*x +
 x^2])/23328
```

Rule 2097

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x\left(216+108x^2+324x^3+18x^4+x^6\right)}\,dx = 1259712\int\left(\frac{1}{272097792x}+\frac{(-1)^{2/3}\left(6\left(9+\sqrt[3]{-3}2^{2/3}\right)-\left(1-3(-3)^{2/3}\sqrt[3]{2}\right.\right.}{816293376\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^2\left(6-3\sqrt[3]{-3}2^{2/3}x\right.}\right.$$

$$= \frac{\log(x)}{216}+\frac{\int\frac{-6\sqrt[3]{6}\left(9\sqrt[3]{2}-2\sqrt[3]{3}\right)-\left(18-2^{2/3}\sqrt[3]{3}\right)x}{6+3\ 2^{2/3}\sqrt[3]{3}x+x^2}\,dx}{11664}+\frac{(-1)^{2/3}\int\frac{-6\left(9-(-2)^{2/3}\sqrt[3]{3}\right)+\left(1+3\sqrt[3]{\right.}}{6+3(-2)^{2/3}\sqrt[3]{3}x+x^2}}{1944\sqrt[3]{2}3^{2/3}}$$

$$= \frac{\log(x)}{216}+\frac{\left(\left(-\frac{1}{6}\right)^{2/3}\left(\sqrt[3]{-3}+3\sqrt[3]{2}\right)\right)\int\frac{1}{6-3\sqrt[3]{-3}2^{2/3}x+x^2}\,dx}{72\left(1+\sqrt[3]{-1}\right)^2}+\frac{\left((-1)^{2/3}\left(-1+3(-\right.\right.}{1296\sqrt[3]{}}$$

$$= \frac{\log(x)}{216}-\frac{(-1)^{2/3}\left(1-3(-3)^{2/3}\sqrt[3]{2}\right)\log\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}{1296\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^2}-\frac{\left(18-(-2)^{2/}\right.}{}$$

$$= \frac{(-1)^{2/3}\left((-2)^{2/3}-2\ 3^{2/3}\right)\tan^{-1}\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{216\ 6^{5/6}\sqrt{4+3\sqrt[3]{-2}3^{2/3}}}-\frac{(-1)^{2/3}\left(\sqrt[3]{-3}+3\sqrt[3]{2}\right)\text{ta}}{216\sqrt[6]{6}\left(1+\sqrt[3]{-1}\right)^2}$$

Mathematica [C] time = 0.0180972, size = 103, normalized size = 0.25

$$\frac{\log(x)}{216}-\frac{\text{RootSum}\left[\#1^6+18\#1^4+324\#1^3+108\#1^2+216\&,\frac{\#1^4\log(x-\#1)+18\#1^2\log(x-\#1)+324\#1\log(x-\#1)+108\log(x-\#1)}{\#1^4+12\#1^2+162\#1+36}\&\right]}{1296}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)),x]
```

```
[Out] Log[x]/216 - RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 & , (108*Lo
g[x - #1] + 324*Log[x - #1]*#1 + 18*Log[x - #1]*#1^2 + Log[x - #1]*#1^4)/(3
6 + 162*#1 + 12*#1^2 + #1^4) & ]/1296
```

Maple [C] time = 0.007, size = 75, normalized size = 0.2

$$-\frac{1}{1296}\sum_{\_R=\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\left(\_R^5+18\_R^3+324\_R^2+108\_R\right)\ln\left(x-\_R\right)}{\_R^5+12\_R^3+162\_R^2+36\_R}+\frac{\ln\left(x\right)}{216}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^6+18*x^4+324*x^3+108*x^2+216),x)
```

```
[Out] -1/1296*sum((_R^5+18*_R^3+324*_R^2+108*_R)/(_R^5+12*_R^3+162*_R^2+36*_R)*ln
(x-_R),_R=RootOf(_Z^6+18*_Z^4+324*_Z^3+108*_Z^2+216))+1/216*ln(x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{216}\int\frac{x^5+18\,x^3+324\,x^2+108\,x}{x^6+18\,x^4+324\,x^3+108\,x^2+216}\,dx+\frac{1}{216}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="maxima")
```

```
[Out] -1/216*integrate((x^5 + 18*x^3 + 324*x^2 + 108*x)/(x^6 + 18*x^4 + 324*x^3 +
 108*x^2 + 216), x) + 1/216*log(x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 0.366844, size = 82, normalized size = 0.2

$$\frac{\log(x)}{216} + \mathrm{RootSum}\Big(737963742567783949192396 8t^6 + 341649880818418494996 48t^5 + 525988092506853703 68t^4 + 2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**6+18*x**4+324*x**3+108*x**2+216),x)
```

```
[Out] log(x)/216 + RootSum(7379637425677839491923968*_t**6 + 34164988081841849499
648*_t**5 + 525988092506853703 68*_t**4 + 26673506015311872*_t**3 - 30917111
6160*_t**2 + 944784*_t - 1, Lambda(_t, _t*log(81455700996688179367833621151
19297360560128*_t**6/143425799309052440063 + 977068766770806381087358257564
745728*_t**5/143425799309052440063 - 116529526608851264288400971539061538 81
6*_t**4/143425799309052440063 - 239359794985242202542501440710766592*_t**3/
143425799309052440063 - 136678312638137094439887341418240*_t**2/14342579930
9052440063 + 1563115569067663795735413696*_t/143425799309052440063 + x - 31
644463150752361900 44/143425799309052440063)))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^6 + 18\,x^4 + 324\,x^3 + 108\,x^2 + 216\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="giac")
```

```
[Out] integrate(1/((x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)*x), x)
```

3.150 $\int \frac{1}{x^2(216+108x^2+324x^3+18x^4+x^6)}\,dx$

Optimal. Leaf size=448

$$-\frac{(-1)^{2/3}\left(9+\sqrt[3]{-3}2^{2/3}\right)\log\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)}{1296\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^2}+\frac{\left(3(-6)^{2/3}+2\sqrt[3]{-2}\right)\log\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)}{7776\sqrt[3]{3}}-\frac{\left(2^{2/3}-3\,3^{2/3}\right)\log}{3}$$

```
[Out] -1/(216*x) - ((27*(-6)^(1/3) - (-2)^(2/3) + 12*3^(2/3))*ArcTan[(3*(-2)^(2/3
)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(5832*3^(1/6)*Sqrt[8
+ (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3)]) - ((-1)^(2/3)*(6*(-6)^(2/3) +
 27*(-3)^(1/3) - 2^(1/3))*ArcTan[(2^(1/6)*(3*(-3)^(1/3) - 2^(1/3)*x))/Sqrt[
3*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(1944*6^(1/6)*(1 + (-1)^(1/3))^2*Sqrt[4 - 3
*(-3)^(2/3)*2^(1/3)]) - ((2^(1/3) + 27*3^(1/3) - 6*6^(2/3))*ArcTanh[(2^(1/6
)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]])/(5832*6^(1/6)
*Sqrt[-4 + 3*2^(1/3)*3^(2/3)]) - ((-1)^(2/3)*(9 + (-3)^(1/3)*2^(2/3))*Log[6
 - 3*(-3)^(1/3)*2^(2/3)*x + x^2])/(1296*2^(1/3)*3^(2/3)*(1 + (-1)^(1/3))^2)
 + ((3*(-6)^(2/3) + 2*(-2)^(1/3))*Log[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2])/(7
776*3^(1/3)) - ((2^(2/3) - 3*3^(2/3))*Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2])/(
3888*6^(1/3))
```

Rubi [A] time = 1.10187, antiderivative size = 448, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2097, 634, 618, 204, 628, 206}

$$-\frac{(-1)^{2/3}\left(9+\sqrt[3]{-3}2^{2/3}\right)\log\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)}{1296\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^2}+\frac{\left(3(-6)^{2/3}+2\sqrt[3]{-2}\right)\log\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)}{7776\sqrt[3]{3}}-\frac{\left(2^{2/3}-3\,3^{2/3}\right)\log}{3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)),x]
```

```
[Out] -1/(216*x) - ((27*(-6)^(1/3) - (-2)^(2/3) + 12*3^(2/3))*ArcTan[(3*(-2)^(2/3
)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(5832*3^(1/6)*Sqrt[8
+ (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3)]) - ((-1)^(2/3)*(6*(-6)^(2/3) +
 27*(-3)^(1/3) - 2^(1/3))*ArcTan[(2^(1/6)*(3*(-3)^(1/3) - 2^(1/3)*x))/Sqrt[
```

```
3*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(1944*6^(1/6)*(1 + (-1)^(1/3))^2*Sqrt[4 - 3
*(-3)^(2/3)*2^(1/3)]) - ((2^(1/3) + 27*3^(1/3) - 6*6^(2/3))*ArcTanh[(2^(1/6
)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]])/(5832*6^(1/6)
*Sqrt[-4 + 3*2^(1/3)*3^(2/3)]) - ((-1)^(2/3)*(9 + (-3)^(1/3)*2^(2/3))*Log[6
 - 3*(-3)^(1/3)*2^(2/3)*x + x^2])/(1296*2^(1/3)*3^(2/3)*(1 + (-1)^(1/3))^2)
 + ((3*(-6)^(2/3) + 2*(-2)^(1/3))*Log[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2])/(7
776*3^(1/3)) - ((2^(2/3) - 3*3^(2/3))*Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2])/(
3888*6^(1/3))
```

Rule 2097

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x^2\left(216+108x^2+324x^3+18x^4+x^6\right)}\,dx = 1259712\int\left(\frac{1}{272097792x^2}+\frac{(-1)^{2/3}\left(-1+9(-3)^{2/3}\sqrt[3]{2}+27\sqrt[3]{-3}2^{2/3}-\left(\right.\right.}{816293376\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^2\left(6-3\sqrt[3]{-}\right.}\right.$$

$$=-\frac{1}{216x}+\frac{\int\frac{-54+2^{2/3}\sqrt[3]{3}+54\sqrt[3]{2}3^{2/3}-6^{2/3}\left(2^{2/3}-3\ 3^{2/3}\right)x}{6+3\ 2^{2/3}\sqrt[3]{3}x+x^2}\,dx}{11664}+\frac{(-1)^{2/3}\int\frac{1+27(-2)^{2/3}\sqrt[3]{}}{6}}{19}$$

$$=-\frac{1}{216x}-\frac{\left((-1)^{2/3}\left(9+\sqrt[3]{-3}2^{2/3}\right)\right)\int\frac{-3\sqrt[3]{-3}2^{2/3}+2x}{6-3\sqrt[3]{-3}2^{2/3}x+x^2}\,dx}{1296\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^2}-\frac{\left((-1)^{2/3}\left((-2)^{2/3}\right.\right.}{}$$

$$=-\frac{1}{216x}-\frac{(-1)^{2/3}\left(9+\sqrt[3]{-3}2^{2/3}\right)\log\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}{1296\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^2}-\frac{(-1)^{2/3}\left((-2\right.}{}$$

$$=-\frac{1}{216x}+\frac{(-1)^{2/3}\left(2+27(-2)^{2/3}\sqrt[3]{3}+12\sqrt[3]{-2}3^{2/3}\right)\tan^{-1}\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{5832\ 2^{5/6}\sqrt[6]{3}\sqrt{4+3\sqrt[3]{-2}3^{2/3}}}$$

Mathematica [C] time = 0.0188605, size = 109, normalized size = 0.24

$$-\frac{\text{RootSum}\left[\#1^6+18\#1^4+324\#1^3+108\#1^2+216\&,\frac{\#1^4\log(x-\#1)+18\#1^2\log(x-\#1)+324\#1\log(x-\#1)+108\log(x-\#1)}{\#1^5+12\#1^3+162\#1^2+36\#1}\&\right]}{1296}-\frac{1}{216x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)),x]
```

```
[Out] -1/(216*x) - RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 & , (108*Lo
g[x - #1] + 324*Log[x - #1]*#1 + 18*Log[x - #1]*#1^2 + Log[x - #1]*#1^4)/(3
6*#1 + 162*#1^2 + 12*#1^3 + #1^5) & ]/1296
```

Maple [C] time = 0.007, size = 74, normalized size = 0.2

$$\frac{1}{1296} \sum_{\_R=\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)} \frac{\left(-\_R^4-18\_R^2-324\_R-108\right)\ln\left(x-\_R\right)}{\_R^5+12\_R^3+162\_R^2+36\_R} - \frac{1}{216x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(x^6+18*x^4+324*x^3+108*x^2+216),x)
```

```
[Out] 1/1296*sum((-_R^4-18*_R^2-324*_R-108)/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(x-_R
),_R=RootOf(_Z^6+18*_Z^4+324*_Z^3+108*_Z^2+216))-1/216/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{216x} - \frac{1}{216}\int \frac{x^4+18x^2+324x+108}{x^6+18x^4+324x^3+108x^2+216}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="maxima")
```

```
[Out] -1/216/x - 1/216*integrate((x^4 + 18*x^2 + 324*x + 108)/(x^6 + 18*x^4 + 324
*x^3 + 108*x^2 + 216), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 0.269043, size = 70, normalized size = 0.16

$$\mathrm{RootSum}\left(15940016839464133302555770 88t^6 + 37916120264603316387 84t^4 - 864367269958950912 0t^3 - 1094282\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(x**6+18*x**4+324*x**3+108*x**2+216),x)
```

```
[Out] RootSum(15940016839464133302555577088*_t**6 + 37916120264603316387 84*_t**4 -
 8643672699589509120*_t**3 - 10942820851968*_t**2 - 839808*_t - 1, Lambda(_
t, _t*log(-4987553276190249600329356123691446802841 6*_t**5/1235044978470399
1795 + 126254898724316203880059752004976 64*_t**4/12350449784703991795 - 118
63769260757377123855079885264486 4*_t**3/12350449784703991795 + 270486324927
832147818193778754816*_t**2/12350449784703991795 + 273914194897479402961199
352*_t/12350449784703991795 + x - 12798926329353908292/12350449784703991795
))) - 1/(216*x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^6 + 18\,x^4 + 324\,x^3 + 108\,x^2 + 216\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="giac")
```

```
[Out] integrate(1/((x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)*x^2), x)
```

3.151 $\int \frac{x^8}{\left(216+108x^2+324x^3+18x^4+x^6\right)^2}\,dx$

Optimal. Leaf size=1064

result too large to display

```
[Out] -((-1/3)^(1/3)*(9*(6 + (-3)^(1/3)*2^(2/3)) + (2 - 2^(2/3)*(6*(-6)^(2/3) + 2
7*(-3)^(1/3)))*x))/(162*2^(2/3)*(1 + (-1)^(1/3))^4*(4 - 3*(-3)^(2/3)*2^(1/3
))*(6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2)) - ((-1/3)^(1/3)*(9*(6 - (-2)^(2/3)*3
^(1/3)) + (2 + 27*(-2)^(2/3)*3^(1/3) + 12*(-2)^(1/3)*3^(2/3))*x))/(729*2^(2
/3)*(8 + (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3))*(6 + 3*(-2)^(2/3)*3^(1/
3)*x + x^2)) + (9*(6 - 2^(2/3)*3^(1/3)) + (2 + 2^(2/3)*(27*3^(1/3) - 6*6^(2
/3)))*x)/(1458*2^(2/3)*3^(1/3)*(4 - 3*2^(1/3)*3^(2/3))*(6 + 3*2^(2/3)*3^(1/
3)*x + x^2)) - ((I/162)*((-2)^(2/3) + 6*3^(2/3))*ArcTan[(3*(-2)^(2/3)*3^(1/
3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(2^(5/6)*3^(1/3)*(1 + (-1)^(
1/3))^5*Sqrt[4 + 3*(-2)^(1/3)*3^(2/3)]) - ((-1)^(1/3)*(2 + 27*(-2)^(2/3)*3^
(1/3) + 12*(-2)^(1/3)*3^(2/3))*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(
4 + 3*(-2)^(1/3)*3^(2/3))]])/(162*2^(1/6)*3^(5/6)*(1 - (-1)^(1/3))^2*(1 + (
-1)^(1/3))^4*(4 + 3*(-2)^(1/3)*3^(2/3))^(3/2)) - ((-1)^(1/3)*(6*(-6)^(2/3)
+ 27*(-3)^(1/3) - 2^(1/3))*ArcTan[(2^(1/6)*(3*(-3)^(1/3) - 2^(1/3)*x))/Sqrt
[3*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(81*Sqrt[2]*3^(5/6)*(1 + (-1)^(1/3))^4*(4
- 3*(-3)^(2/3)*2^(1/3))^(3/2)) + ((I*2^(2/3) - 9*3^(1/6) - (3*I)*3^(2/3))*A
rcTan[(2^(1/6)*(3*(-3)^(1/3) - 2^(1/3)*x))/Sqrt[3*(4 - 3*(-3)^(2/3)*2^(1/3)
)]])/(162*2^(5/6)*3^(1/3)*(1 + (-1)^(1/3))^5*Sqrt[4 - 3*(-3)^(2/3)*2^(1/3)]
) - ((1 + 3*2^(1/3)*3^(2/3))*ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt
[3*(-4 + 3*2^(1/3)*3^(2/3))]])/(1458*2^(1/6)*3^(5/6)*Sqrt[-4 + 3*2^(1/3)*3^
(2/3)]) + ((2^(1/3) + 27*3^(1/3) - 6*6^(2/3))*ArcTanh[(2^(1/6)*(3*3^(1/3) +
 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]])/(81*Sqrt[2]*3^(5/6)*(1 - (-
1)^(1/3))^2*(1 + (-1)^(1/3))^4*(-4 + 3*2^(1/3)*3^(2/3))^(3/2)) - Log[6 - 3*
(-3)^(1/3)*2^(2/3)*x + x^2]/(972*2^(1/3)*3^(2/3)*(1 + (-1)^(1/3))^4) + ((I/
972)*Log[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2])/(2^(1/3)*3^(1/6)*(1 + (-1)^(1/3
))^5) - Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2]/(8748*2^(1/3)*3^(2/3))
```

Rubi [A] time = 2.50381, antiderivative size = 1064, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {2097, 638, 618, 204, 634, 628, 206}

$$-\frac{\sqrt[3]{-\frac{1}{3}}\left(\left(2+27(-2)^{2/3}\sqrt[3]{3}+12\sqrt[3]{-2}3^{2/3}\right)x+9\left(6-(-2)^{2/3}\sqrt[3]{3}\right)\right)}{729\ 2^{2/3}\left(8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)}-\frac{\sqrt[3]{-1}\left(2+27(-2)^{2/3}\sqrt[3]{3}+12\sqrt[3]{-2}3^{2/3}\right)\tan^{-1}\left(\frac{2x+3(-2)^{2/\cdots}}{\sqrt{6\left(4+3\sqrt[3]{-\cdots}\right.}}\right.}{162\sqrt[6]{2}3^{5/6}\left(1-\sqrt[3]{-1}\right)^2\left(1+\sqrt[3]{-1}\right)^4\left(4+3\sqrt[3]{-2}3^{2/3}\right)^{\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

```
[Out] -((-1/3)^(1/3)*(9*(6 + (-3)^(1/3)*2^(2/3)) + (2 - 3*2^(2/3)*(2*(-6)^(2/3) +
 9*(-3)^(1/3)))*x))/(162*2^(2/3)*(1 + (-1)^(1/3))^4*(4 - 3*(-3)^(2/3)*2^(1/
3))*(6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2)) - ((-1/3)^(1/3)*(9*(6 - (-2)^(2/3)*
3^(1/3)) + (2 + 27*(-2)^(2/3)*3^(1/3) + 12*(-2)^(1/3)*3^(2/3))*x))/(729*2^(
2/3)*(8 + (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3))*(6 + 3*(-2)^(2/3)*3^(1
/3)*x + x^2)) + (9*(6 - 2^(2/3)*3^(1/3)) + (2 + 2^(2/3)*(27*3^(1/3) - 6*6^(
2/3)))*x)/(1458*2^(2/3)*3^(1/3)*(4 - 3*2^(1/3)*3^(2/3))*(6 + 3*2^(2/3)*3^(1
/3)*x + x^2)) - ((I/162)*((-2)^(2/3) + 6*3^(2/3))*ArcTan[(3*(-2)^(2/3)*3^(1
/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(2^(5/6)*3^(1/3)*(1 + (-1)^
(1/3))^5*Sqrt[4 + 3*(-2)^(1/3)*3^(2/3)]) - ((-1)^(1/3)*(2 + 27*(-2)^(2/3)*3
^(1/3) + 12*(-2)^(1/3)*3^(2/3))*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*
(4 + 3*(-2)^(1/3)*3^(2/3))]])/(162*2^(1/6)*3^(5/6)*(1 - (-1)^(1/3))^2*(1 +
(-1)^(1/3))^4*(4 + 3*(-2)^(1/3)*3^(2/3))^(3/2)) - ((-1)^(1/3)*(6*(-6)^(2/3)
 + 27*(-3)^(1/3) - 2^(1/3))*ArcTan[(2^(1/6)*(3*(-3)^(1/3) - 2^(1/3)*x))/Sqr
t[3*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(81*Sqrt[2]*3^(5/6)*(1 + (-1)^(1/3))^4*(4
 - 3*(-3)^(2/3)*2^(1/3))^(3/2)) + ((I*2^(2/3) - 9*3^(1/6) - (3*I)*3^(2/3))*
ArcTan[(2^(1/6)*(3*(-3)^(1/3) - 2^(1/3)*x))/Sqrt[3*(4 - 3*(-3)^(2/3)*2^(1/3
))]])/(162*2^(5/6)*3^(1/3)*(1 + (-1)^(1/3))^5*Sqrt[4 - 3*(-3)^(2/3)*2^(1/3)
]) - ((1 + 3*2^(1/3)*3^(2/3))*ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqr
t[3*(-4 + 3*2^(1/3)*3^(2/3))]])/(1458*2^(1/6)*3^(5/6)*Sqrt[-4 + 3*2^(1/3)*3
^(2/3)]) + ((2^(1/3) + 27*3^(1/3) - 6*6^(2/3))*ArcTanh[(2^(1/6)*(3*3^(1/3)
+ 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]])/(81*Sqrt[2]*3^(5/6)*(1 - (
-1)^(1/3))^2*(1 + (-1)^(1/3))^4*(-4 + 3*2^(1/3)*3^(2/3))^(3/2)) - Log[6 - 3
*(-3)^(1/3)*2^(2/3)*x + x^2]/(972*2^(1/3)*3^(2/3)*(1 + (-1)^(1/3))^4) + ((I
/972)*Log[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2])/(2^(1/3)*3^(1/6)*(1 + (-1)^(1/
3))^5) - Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2]/(8748*2^(1/3)*3^(2/3))
```

<u>Rule 2097</u>

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

<u>Rule 638</u>

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
```

1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\int \frac{x^8}{\left(216+108x^2+324x^3+18x^4+x^6\right)^2}\,dx = 1586874322944 \int \left(\frac{\sqrt[3]{-\frac{1}{3}}\left(-1+3(-3)^{2/3}\sqrt[3]{2}+\left(9+\sqrt[3]{-3}2^{2/3}\right)x\right)}{42845606719488\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(6-3\sqrt[3]{-3}2^{2/3}x+ \right.}\right.$$

$$= \frac{\sqrt[3]{-\frac{1}{3}}\int \frac{-1-3\sqrt[3]{-2}3^{2/3}+\left(9-(-2)^{2/3}\sqrt[3]{3}\right)x}{\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)^2}dx}{243\ 2^{2/3}} - \frac{\int \frac{-27+x}{6+3\ 2^{2/3}\sqrt[3]{3}x+x^2}dx}{4374\sqrt[3]{2}3^{2/3}} + \frac{\int \frac{1-3\sqrt[3]{2}3^{2/3}-(}{\left(6+3\ 2^{2/3}\right.}}{243\ 2}$$

$$= -\frac{\sqrt[3]{-\frac{1}{3}}\left(9\left(6+\sqrt[3]{-3}2^{2/3}\right)+\left(2-3\ 2^{2/3}\left(2(-6)^{2/3}+9\sqrt[3]{-3}\right)\right)x\right)}{162\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)} - \frac{\sqrt[3]{-\frac{1}{3}}\left(9\right.}{14}$$

$$= -\frac{\sqrt[3]{-\frac{1}{3}}\left(9\left(6+\sqrt[3]{-3}2^{2/3}\right)+\left(2-3\ 2^{2/3}\left(2(-6)^{2/3}+9\sqrt[3]{-3}\right)\right)x\right)}{162\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)} - \frac{\sqrt[3]{-\frac{1}{3}}\left(9\right.}{14}$$

$$= -\frac{\sqrt[3]{-\frac{1}{3}}\left(9\left(6+\sqrt[3]{-3}2^{2/3}\right)+\left(2-3\ 2^{2/3}\left(2(-6)^{2/3}+9\sqrt[3]{-3}\right)\right)x\right)}{162\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)} - \frac{\sqrt[3]{-\frac{1}{3}}\left(9\right.}{14}$$

Mathematica [C] time = 0.0414178, size = 167, normalized size = 0.16

$$\frac{-9x^5-203x^4-11610x^3-3990x^2+324x-7884}{34182\left(x^6+18x^4+324x^3+108x^2+216\right)} - \frac{\text{RootSum}\left[\#1^6+18\#1^4+324\#1^3+108\#1^2+216\&, \frac{9\#1^4\log(x-\#1)+}{}\right.}{205092}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^8/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

```
[Out] (-7884 + 324*x - 3990*x^2 - 11610*x^3 - 203*x^4 - 9*x^5)/(34182*(216 + 108*
x^2 + 324*x^3 + 18*x^4 + x^6)) - RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^
4 + #1^6 & , (324*Log[x - #1] - 96*Log[x - #1]*#1 + 324*Log[x - #1]*#1^2 +
406*Log[x - #1]*#1^3 + 9*Log[x - #1]*#1^4)/(36*#1 + 162*#1^2 + 12*#1^3 + #1
^5) & ]/205092
```

Maple [C] time = 0.01, size = 122, normalized size = 0.1

$$\frac{1}{x^6+18\,x^4+324\,x^3+108\,x^2+216}\left(-\frac{x^5}{3798}-\frac{203\,x^4}{34182}-\frac{215\,x^3}{633}-\frac{665\,x^2}{5697}+\frac{2\,x}{211}-\frac{146}{633}\right)+\frac{1}{205092}\sum_{\_R=\mathrm{RootOf}\left(\_Z^6+18\_Z^4+32\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x)
```

```
[Out] (-1/3798*x^5-203/34182*x^4-215/633*x^3-665/5697*x^2+2/211*x-146/633)/(x^6+1
8*x^4+324*x^3+108*x^2+216)+1/205092*sum((-9*_R^4-406*_R^3-324*_R^2+96*_R-32
4)/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(x-_R),_R=RootOf(_Z^6+18*_Z^4+324*_Z^3+1
08*_Z^2+216))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{9\,x^5+203\,x^4+11610\,x^3+3990\,x^2-324\,x+7884}{34182\left(x^6+18\,x^4+324\,x^3+108\,x^2+216\right)}-\frac{1}{34182}\int\frac{9\,x^4+406\,x^3+324\,x^2-96\,x+324}{x^6+18\,x^4+324\,x^3+108\,x^2+216}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="maxima")
```

```
[Out] -1/34182*(9*x^5 + 203*x^4 + 11610*x^3 + 3990*x^2 - 324*x + 7884)/(x^6 + 18*
x^4 + 324*x^3 + 108*x^2 + 216) - 1/34182*integrate((9*x^4 + 406*x^3 + 324*x
^2 - 96*x + 324)/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 0.372375, size = 112, normalized size = 0.11

$$\text{RootSum}\left(852560170529641874151233606645 76t^6 + 5010519153338543456870 4t^4 + 48885748051277486016t^3 + 8\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(x**6+18*x**4+324*x**3+108*x**2+216)**2,x)
```

```
[Out] RootSum(852560170529641874151233606645 76*_t**6 + 50105191533385434568704*_t
**4 + 48885748051277486016*_t**3 + 865447782603408*_t**2 + 3220532460*_t +
4513, Lambda(_t, _t*log(354920362040841744041191931354834874665907640 32*_t*
*5/356900697070792948475845 - 1947416006721883708682680963101702230822 4*_t*
*4/713801394141585896951 69 + 2077996307654513223389458276490339654 4*_t**3/3
56900697070792948475845 + 20265219154367004972162198012037344*_t**2/3569006
97070792948475845 + 275192468949210532049075145372*_t/356900697070792948475
845 + x + 129028519129217728962 2012/107070209121237884542 7535))) - (9*x**5
+ 203*x**4 + 11610*x**3 + 3990*x**2 - 324*x + 7884)/(34182*x**6 + 615276*x*
*4 + 11074968*x**3 + 3691656*x**2 + 7383312)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^8}{\left(x^6 + 18\,x^4 + 324\,x^3 + 108\,x^2 + 216\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="giac")
```

```
[Out] integrate(x^8/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)^2, x)
```

3.152 $\int \frac{x^7}{\left(216+108x^2+324x^3+18x^4+x^6\right)^2}\,dx$

Optimal. Leaf size=1005

result too large to display

```
[Out] -(2*(2*(-1)^(1/3)*3^(2/3) + 9*6^(1/3)) - 9*((-2)^(2/3) + 2*(-1)^(1/3)*3^(2/
3))*x)/(972*2^(2/3)*(1 + (-1)^(1/3))^4*(4 - 3*(-3)^(2/3)*2^(1/3))*(6 - 3*(-
3)^(1/3)*2^(2/3)*x + x^2)) - ((-6)^(1/3)*(9*(-2)^(1/3) + 2*3^(1/3)) - 9*(1
+ (-2)^(1/3)*3^(2/3))*x)/(4374*(8 + (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/
3))*(6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2)) + (2*(2 - 3*2^(1/3)*3^(2/3)) - 3*(6
 - 2^(2/3)*3^(1/3))*x)/(2916*2^(2/3)*3^(1/3)*(4 - 3*2^(1/3)*3^(2/3))*(6 + 3
*2^(2/3)*3^(1/3)*x + x^2)) + ((9*I + 3^(1/3)*((2*I)*2^(2/3) - 9*3^(1/6) + 2
*2^(2/3)*Sqrt[3]))*ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4 - 3*(-3)^(
2/3)*2^(1/3))]])/(5832*(1 + (-1)^(1/3))^5*Sqrt[2*(4 - 3*(-3)^(2/3)*2^(1/3))
]) + ((1 + (-2)^(1/3)*3^(2/3))*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(
4 + 3*(-2)^(1/3)*3^(2/3))]])/(54*Sqrt[6]*(1 - (-1)^(1/3))^2*(1 + (-1)^(1/3)
)^4*(4 + 3*(-2)^(1/3)*3^(2/3))^(3/2)) + ((9*3^(1/6) + I*(4*2^(2/3) - 3*3^(2
/3)))*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))
]])/(1944*3^(2/3)*(1 + (-1)^(1/3))^5*Sqrt[2*(4 + 3*(-2)^(1/3)*3^(2/3))]) -
((-1)^(1/3)*((-3)^(1/3) + 3*2^(1/3))*ArcTan[(2^(1/6)*(3*(-3)^(1/3) - 2^(1/3
)*x))/Sqrt[3*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(54*Sqrt[2]*3^(5/6)*(1 + (-1)^(1
/3))^4*(4 - 3*(-3)^(2/3)*2^(1/3))^(3/2)) + ((1 - 2^(1/3)*3^(2/3))*ArcTanh[(
2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]])/(54*Sqr
t[6]*(1 - (-1)^(1/3))^2*(1 + (-1)^(1/3))^4*(-4 + 3*2^(1/3)*3^(2/3))^(3/2))
+ ((2*2^(2/3) + 3*3^(2/3))*ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3
*(-4 + 3*2^(1/3)*3^(2/3))]])/(26244*3^(1/6)*Sqrt[2*(-4 + 3*2^(1/3)*3^(2/3))
]) + ((I/648)*Log[6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2])/(2^(2/3)*3^(5/6)*(1 +
(-1)^(1/3))^5) - ((I + Sqrt[3])*Log[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2])/(129
6*2^(2/3)*3^(5/6)*(1 + (-1)^(1/3))^5) - Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2]/
(17496*2^(2/3)*3^(1/3))
```

Rubi [A] time = 2.40257, antiderivative size = 1005, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {2097, 634, 618, 204, 628, 638, 206}

$$\frac{2\left(2-3\sqrt[3]{2}3^{2/3}\right)-3\left(6-2^{2/3}\sqrt[3]{3}\right)x}{2916\ 2^{2/3}\sqrt[3]{3}\left(4-3\sqrt[3]{2}3^{2/3}\right)\left(x^2+3\ 2^{2/3}\sqrt[3]{3}x+6\right)}+\frac{\left(9i+\sqrt[3]{3}\left(2i2^{2/3}-9\sqrt[6]{3}+2\ 2^{2/3}\sqrt{3}\right)\right)\tan^{-1}\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{5832\left(1+\sqrt[3]{-1}\right)^5\sqrt{2\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

```
[Out] -(2*(2*(-1)^(1/3)*3^(2/3) + 9*6^(1/3)) - 9*((-2)^(2/3) + 2*(-1)^(1/3)*3^(2/
3))*x)/(972*2^(2/3)*(1 + (-1)^(1/3))^4*(4 - 3*(-3)^(2/3)*2^(1/3))*(6 - 3*(-
3)^(1/3)*2^(2/3)*x + x^2)) - ((-6)^(1/3)*(9*(-2)^(1/3) + 2*3^(1/3)) - 9*(1
+ (-2)^(1/3)*3^(2/3))*x)/(4374*(8 + (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/
3))*(6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2)) + (2*(2 - 3*2^(1/3)*3^(2/3)) - 3*(6
 - 2^(2/3)*3^(1/3))*x)/(2916*2^(2/3)*3^(1/3)*(4 - 3*2^(1/3)*3^(2/3))*(6 + 3
*2^(2/3)*3^(1/3)*x + x^2)) + ((9*I + 3^(1/3)*((2*I)*2^(2/3) - 9*3^(1/6) + 2
*2^(2/3)*Sqrt[3]))*ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4 - 3*(-3)^(
2/3)*2^(1/3))]])/(5832*(1 + (-1)^(1/3))^5*Sqrt[2*(4 - 3*(-3)^(2/3)*2^(1/3))
]) + ((1 + (-2)^(1/3)*3^(2/3))*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(
4 + 3*(-2)^(1/3)*3^(2/3))]])/(54*Sqrt[6]*(1 - (-1)^(1/3))^2*(1 + (-1)^(1/3)
)^4*(4 + 3*(-2)^(1/3)*3^(2/3))^(3/2)) + ((9*3^(1/6) + I*(4*2^(2/3) - 3*3^(2
/3)))*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))
]])/(1944*3^(2/3)*(1 + (-1)^(1/3))^5*Sqrt[2*(4 + 3*(-2)^(1/3)*3^(2/3))]) -
((-1)^(1/3)*((-3)^(1/3) + 3*2^(1/3))*ArcTan[(2^(1/6)*(3*(-3)^(1/3) - 2^(1/3
)*x))/Sqrt[3*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(54*Sqrt[2]*3^(5/6)*(1 + (-1)^(1
/3))^4*(4 - 3*(-3)^(2/3)*2^(1/3))^(3/2)) + ((1 - 2^(1/3)*3^(2/3))*ArcTanh[(
2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]])/(54*Sqr
t[6]*(1 - (-1)^(1/3))^2*(1 + (-1)^(1/3))^4*(-4 + 3*2^(1/3)*3^(2/3))^(3/2))
+ ((2*2^(2/3) + 3*3^(2/3))*ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3
*(-4 + 3*2^(1/3)*3^(2/3))]])/(26244*3^(1/6)*Sqrt[2*(-4 + 3*2^(1/3)*3^(2/3))
]) + ((I/648)*Log[6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2])/(2^(2/3)*3^(5/6)*(1 +
(-1)^(1/3))^5) - ((I + Sqrt[3])*Log[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2])/(129
6*2^(2/3)*3^(5/6)*(1 + (-1)^(1/3))^5) - Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2]/
(17496*2^(2/3)*3^(1/3))
```

Rule 2097

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^7}{\left(216+108x^2+324x^3+18x^4+x^6\right)^2}\,dx = 1586874322944\int\left(\frac{-27+3\ 2^{2/3}\sqrt[3]{3}+9i\sqrt{3}+i2^{2/3}3^{5/6}-3i\sqrt[3]{2}\sqrt[6]{3}x}{9254651051409408\left(1+\sqrt[3]{-1}\right)^5\left(-6+3\sqrt[3]{-3}2^{2/3}x-x\right.}\right.$$

$$= \frac{\int \frac{-18-2\ 2^{2/3}\sqrt[3]{3}-\sqrt[3]{2}3^{2/3}x}{6+3\ 2^{2/3}\sqrt[3]{3}x+x^2}dx}{52488} + \frac{\int \frac{9\ 2^{2/3}+\sqrt[3]{-1}3^{2/3}\left(1+3\sqrt[3]{-2}3^{2/3}\right)x}{\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)^2}dx}{4374\ 2^{2/3}} + \frac{\int \frac{3\ 2^{2/3}\sqrt[3]{3}-\left(1\right.}{\left(6+3\ 2^{2/3}\right.}}{1458\ 2}$$

$$= -\frac{2\left(2\sqrt[3]{-1}3^{2/3}+9\sqrt[3]{6}\right)-9\left((-2)^{2/3}+2\sqrt[3]{-1}3^{2/3}\right)x}{972\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)} - \frac{\sqrt[3]{-6}\left(\right.}{8748\left(4\right.}$$

$$= -\frac{2\left(2\sqrt[3]{-1}3^{2/3}+9\sqrt[3]{6}\right)-9\left((-2)^{2/3}+2\sqrt[3]{-1}3^{2/3}\right)x}{972\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)} - \frac{\sqrt[3]{-6}\left(\right.}{8748\left(4\right.}$$

$$= -\frac{2\left(2\sqrt[3]{-1}3^{2/3}+9\sqrt[3]{6}\right)-9\left((-2)^{2/3}+2\sqrt[3]{-1}3^{2/3}\right)x}{972\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)} - \frac{\sqrt[3]{-6}\left(\right.}{8748\left(4\right.}$$

Mathematica [C] time = 0.0298419, size = 167, normalized size = 0.17

$$\frac{\text{RootSum}\left[\#1^6+18\#1^4+324\#1^3+108\#1^2+216\&,\frac{73\#1^4\log(x-\#1)-36\#1^3\log(x-\#1)+96\#1^2\log(x-\#1)-216\#1\log(x-\#1)+96\log(x-\#1)}{\#1^5+12\#1^3+162\#1^2+36\#1}\right.}{410184}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^7/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

```
[Out] (648 - 96*x + 432*x^2 + 908*x^3 - 18*x^4 + 73*x^5)/(68364*(216 + 108*x^2 +
324*x^3 + 18*x^4 + x^6)) + RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1
^6 & , (96*Log[x - #1] - 216*Log[x - #1]*#1 + 96*Log[x - #1]*#1^2 - 36*Log[
x - #1]*#1^3 + 73*Log[x - #1]*#1^4)/(36*#1 + 162*#1^2 + 12*#1^3 + #1^5) & ]
/410184
```

Maple [C] time = 0.01, size = 122, normalized size = 0.1

$$\frac{1}{x^6+18x^4+324x^3+108x^2+216}\left(\frac{73x^5}{68364}-\frac{x^4}{3798}+\frac{227x^3}{17091}+\frac{4x^2}{633}-\frac{8x}{5697}+\frac{2}{211}\right)+\frac{1}{410184}\sum_{\_R=\mathrm{RootOf}(\_Z^6+18\_Z^4+324\_}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x)
```

```
[Out] (73/68364*x^5-1/3798*x^4+227/17091*x^3+4/633*x^2-8/5697*x+2/211)/(x^6+18*x^
4+324*x^3+108*x^2+216)+1/410184*sum((73*_R^4-36*_R^3+96*_R^2-216*_R+96)/(_R
^5+12*_R^3+162*_R^2+36*_R)*ln(x-_R),_R=RootOf(_Z^6+18*_Z^4+324*_Z^3+108*_Z^
2+216))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{73x^5-18x^4+908x^3+432x^2-96x+648}{68364\left(x^6+18x^4+324x^3+108x^2+216\right)}+\frac{1}{68364}\int\frac{73x^4-36x^3+96x^2-216x+96}{x^6+18x^4+324x^3+108x^2+216}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="maxima")
```

```
[Out] 1/68364*(73*x^5 - 18*x^4 + 908*x^3 + 432*x^2 - 96*x + 648)/(x^6 + 18*x^4 +
324*x^3 + 108*x^2 + 216) + 1/68364*integrate((73*x^4 - 36*x^3 + 96*x^2 - 21
6*x + 96)/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 0.381625, size = 112, normalized size = 0.11

$$\operatorname{RootSum}\left(58928958987008846341333266891354931 2t^6 - 539640290266075248405737472t^4 + 92182638168509682\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(x**6+18*x**4+324*x**3+108*x**2+216)**2,x)
```

```
[Out] RootSum(589289589870088463413332668913549312*_t**6 - 5396402902660752484057
37472*_t**4 + 92182638168509682392064*_t**3 - 553241442069170496*_t**2 - 37
59837842016*_t - 7197829, Lambda(_t, _t*log(429960276397274477140037433051б
07461110184385010259993231 36*_t**5/1542060097910520444906943803032375 21 - 4
25847662595081946846897154744222514051572098358476 80*_t**4/1542060097910520
444906943803032375 21 - 375124461288495881501083694493237540783173410821 12*_
t**3/1542060097910520444906943803032375 21 + 71520375940216752676388907155 31
672481920222144*_t**2/1542060097910520444906943803032375 21 - 44227546998835
29772383029179497431052 4032*_t/1542060097910520444906943803032375 21 + x - 1
74573349036676047734132569583024855/1542060097910520444906943803032375 21)))
 + (73*x**5 - 18*x**4 + 908*x**3 + 432*x**2 - 96*x + 648)/(68364*x**6 + 123
0552*x**4 + 22149936*x**3 + 7383312*x**2 + 14766624)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^7}{\left(x^6 + 18\,x^4 + 324\,x^3 + 108\,x^2 + 216\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="giac")
```

```
[Out] integrate(x^7/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)^2, x)
```

3.153 $\int \frac{x^6}{\left(216+108x^2+324x^3+18x^4+x^6\right)^2}\,dx$

Optimal. Leaf size=677

$$\frac{\sqrt[3]{6}\left(9+\sqrt[3]{-3}2^{2/3}\right)x+9(-2)^{2/3}}{2916\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)}+\frac{\sqrt[3]{-1}3^{2/3}\left(2+3\sqrt[3]{-2}3^{2/3}\right)x+9\ 2^{2/3}}{13122\ 2^{2/3}\left(8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x-\right.}$$

```
[Out] (9*(-2)^(2/3) + 6^(1/3)*(9 + (-3)^(1/3)*2^(2/3))*x)/(2916*2^(2/3)*(1 + (-1)
^(1/3))^4*(4 - 3*(-3)^(2/3)*2^(1/3))*(6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2)) +
(9*2^(2/3) + (-1)^(1/3)*3^(2/3)*(2 + 3*(-2)^(1/3)*3^(2/3))*x)/(13122*2^(2/3
)*(8 + (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3))*(6 + 3*(-2)^(2/3)*3^(1/3)
*x + x^2)) + (3*2^(2/3)*3^(1/3) - (2 - 3*2^(1/3)*3^(2/3))*x)/(8748*2^(2/3)*
3^(1/3)*(4 - 3*2^(1/3)*3^(2/3))*(6 + 3*2^(2/3)*3^(1/3)*x + x^2)) + ((-1)^(1
/3)*(3*(-3)^(2/3) - 2^(2/3))*ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4
- 3*(-3)^(2/3)*2^(1/3))]])/(486*6^(5/6)*(1 + (-1)^(1/3))^4*(4 - 3*(-3)^(2/3
)*2^(1/3))^(3/2)) + ((3*(-3)^(2/3) + (-1)^(1/3)*2^(2/3))*ArcTan[(3*(-2)^(2/
3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(486*6^(5/6)*(1 - (-
1)^(1/3))^2*(1 + (-1)^(1/3))^4*(4 + 3*(-2)^(1/3)*3^(2/3))^(3/2)) - ((2^(2/3
) - 3*3^(2/3))*ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(
1/3)*3^(2/3))]])/(486*6^(5/6)*(1 - (-1)^(1/3))^2*(1 + (-1)^(1/3))^4*(-4 + 3
*2^(1/3)*3^(2/3))^(3/2)) + ((-1/3)^(1/6)*Log[6 - 3*(-3)^(1/3)*2^(2/3)*x + x
^2])/(5832*2^(1/3)*(1 + (-1)^(1/3))^5) - ((I/5832)*Log[6 + 3*(-2)^(2/3)*3^(
1/3)*x + x^2])/(2^(1/3)*3^(1/6)*(1 + (-1)^(1/3))^5) + Log[6 + 3*2^(2/3)*3^(
1/3)*x + x^2]/(52488*2^(1/3)*3^(2/3))
```

Rubi [A] time = 1.55326, antiderivative size = 677, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2097, 638, 618, 204, 628, 206}

$$\frac{\sqrt[3]{6}\left(9+\sqrt[3]{-3}2^{2/3}\right)x+9(-2)^{2/3}}{2916\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)}+\frac{\sqrt[3]{-1}3^{2/3}\left(2+3\sqrt[3]{-2}3^{2/3}\right)x+9\ 2^{2/3}}{13122\ 2^{2/3}\left(8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

```
[Out] (9*(-2)^(2/3) + 6^(1/3)*(9 + (-3)^(1/3)*2^(2/3))*x)/(2916*2^(2/3)*(1 + (-1)
^(1/3))^4*(4 - 3*(-3)^(2/3)*2^(1/3))*(6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2)) +
(9*2^(2/3) + (-1)^(1/3)*3^(2/3)*(2 + 3*(-2)^(1/3)*3^(2/3))*x)/(13122*2^(2/3
)*(8 + (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3))*(6 + 3*(-2)^(2/3)*3^(1/3)
*x + x^2)) + (3*2^(2/3)*3^(1/3) - (2 - 3*2^(1/3)*3^(2/3))*x)/(8748*2^(2/3)*
3^(1/3)*(4 - 3*2^(1/3)*3^(2/3))*(6 + 3*2^(2/3)*3^(1/3)*x + x^2)) + ((-1)^(1
/3)*(3*(-3)^(2/3) - 2^(2/3))*ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4
- 3*(-3)^(2/3)*2^(1/3))]])/(486*6^(5/6)*(1 + (-1)^(1/3))^4*(4 - 3*(-3)^(2/3
)*2^(1/3))^(3/2)) + ((3*(-3)^(2/3) + (-1)^(1/3)*2^(2/3))*ArcTan[(3*(-2)^(2/
3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(486*6^(5/6)*(1 - (-
1)^(1/3))^2*(1 + (-1)^(1/3))^4*(4 + 3*(-2)^(1/3)*3^(2/3))^(3/2)) - ((2^(2/3
) - 3*3^(2/3))*ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(
1/3)*3^(2/3))]])/(486*6^(5/6)*(1 - (-1)^(1/3))^2*(1 + (-1)^(1/3))^4*(-4 + 3
*2^(1/3)*3^(2/3))^(3/2)) + ((-1/3)^(1/6)*Log[6 - 3*(-3)^(1/3)*2^(2/3)*x + x
^2])/(5832*2^(1/3)*(1 + (-1)^(1/3))^5) - ((I/5832)*Log[6 + 3*(-2)^(2/3)*3^(
1/3)*x + x^2])/(2^(1/3)*3^(1/6)*(1 + (-1)^(1/3))^5) + Log[6 + 3*2^(2/3)*3^(
1/3)*x + x^2]/(52488*2^(1/3)*3^(2/3))
```

Rule 2097

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
```

```
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^6}{\left(216+108x^2+324x^3+18x^4+x^6\right)^2}\,dx = 1586874322944\int\left(-\frac{-2\sqrt[3]{-1}3^{2/3}+3(-2)^{2/3}x}{1542441841901568\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(-6+3\sqrt[3]{-3}2^{2/3}x\right.}\right.$$

$$= -\frac{\int \frac{-2\sqrt[3]{-1}3^{2/3}+3\ 2^{2/3}x}{\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)^2}\,dx}{8748\ 2^{2/3}} - \frac{\int \frac{2+2^{2/3}\sqrt[3]{3}x}{\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)^2}\,dx}{2916\ 2^{2/3}\sqrt[3]{3}} + \frac{\int \frac{3\sqrt[3]{3}+\sqrt[3]{2}x}{6+3\ 2^{2/3}\sqrt[3]{3}x+x^2}\,dx}{26244\ 6^{2/3}} + \ldots$$

$$= \frac{9(-2)^{2/3}+\sqrt[3]{6}\left(9+\sqrt[3]{-3}2^{2/3}\right)x}{2916\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)} + \frac{9\ \ldots}{26244\ 2^{2/3}\ldots}$$

$$= \frac{9(-2)^{2/3}+\sqrt[3]{6}\left(9+\sqrt[3]{-3}2^{2/3}\right)x}{2916\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)} + \frac{9\ \ldots}{26244\ 2^{2/3}\ldots}$$

$$= \frac{9(-2)^{2/3}+\sqrt[3]{6}\left(9+\sqrt[3]{-3}2^{2/3}\right)x}{2916\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)} + \frac{9\ \ldots}{26244\ 2^{2/3}\ldots}$$

Mathematica [C] time = 0.0453387, size = 167, normalized size = 0.25

$$\frac{-3x^5+73x^4-72x^3-64x^2+108x-96}{68364\left(x^6+18x^4+324x^3+108x^2+216\right)} - \frac{\text{RootSum}\left[\#1^6+18\#1^4+324\#1^3+108\#1^2+216\&, \frac{3\#1^4\log(x-\#1)-146\#1^3\log(\ldots}{\ldots}\right.}{410184}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^6/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

```
[Out] (-96 + 108*x - 64*x^2 - 72*x^3 + 73*x^4 - 3*x^5)/(68364*(216 + 108*x^2 + 32
4*x^3 + 18*x^4 + x^6)) - RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6
 & , (108*Log[x - #1] - 32*Log[x - #1]*#1 + 108*Log[x - #1]*#1^2 - 146*Log[
x - #1]*#1^3 + 3*Log[x - #1]*#1^4)/(36*#1 + 162*#1^2 + 12*#1^3 + #1^5) & ]/
410184
```

Maple [C] time = 0.01, size = 122, normalized size = 0.2

$$\frac{1}{x^6+18x^4+324x^3+108x^2+216}\left(-\frac{x^5}{22788}+\frac{73x^4}{68364}-\frac{2x^3}{1899}-\frac{16x^2}{17091}+\frac{x}{633}-\frac{8}{5697}\right)+\frac{1}{410184}\sum_{\_R=\mathrm{RootOf}\left(\_Z^6+18\_Z^4\cdot\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x)
```

```
[Out] (-1/22788*x^5+73/68364*x^4-2/1899*x^3-16/17091*x^2+1/633*x-8/5697)/(x^6+18*
x^4+324*x^3+108*x^2+216)+1/410184*sum((-3*_R^4+146*_R^3-108*_R^2+32*_R-108)
/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(x-_R),_R=RootOf(_Z^6+18*_Z^4+324*_Z^3+108
*_Z^2+216))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{3x^5-73x^4+72x^3+64x^2-108x+96}{68364\left(x^6+18x^4+324x^3+108x^2+216\right)}-\frac{1}{68364}\int\frac{3x^4-146x^3+108x^2-32x+108}{x^6+18x^4+324x^3+108x^2+216}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="maxima")
```

```
[Out] -1/68364*(3*x^5 - 73*x^4 + 72*x^3 + 64*x^2 - 108*x + 96)/(x^6 + 18*x^4 + 32
4*x^3 + 108*x^2 + 216) - 1/68364*integrate((3*x^4 - 146*x^3 + 108*x^2 - 32*
x + 108)/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 0.357636, size = 112, normalized size = 0.17

$$\mathrm{RootSum}\Big(397770473162309712803999551516645785 6t^6 - 1010314319415295961050951680t^4 - 201682244770939 5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(x**6+18*x**4+324*x**3+108*x**2+216)**2,x)
```

```
[Out] RootSum(3977704731623097128039995515166457856*_t**6 - 101031431941529596105
0951680*_t**4 - 20168224477093957151232*_t**3 - 112582856818899648*_t**2 -
50648453064*_t - 880007, Lambda(_t, _t*log(-2736555670900189915706499414143
95560986199688040644608*_t**5/49797855396139900267573395695 + 1183700847019
6046085308646230764354292805044570112*_t**4/49797855396139900267573395695 -
 10570581900446717266374077482873315047787008*_t**3/49797855396139900267573
395695 - 1552547411569469872387563218792789323392*_t**2/4979785539613990026
7573395695 - 12542923791159140826909003250295928*_t/49797855396139900267573
395695 + x - 23066533870320322410834348296/49797855396139900267573395695)))
 - (3*x**5 - 73*x**4 + 72*x**3 + 64*x**2 - 108*x + 96)/(68364*x**6 + 123055
2*x**4 + 22149936*x**3 + 7383312*x**2 + 14766624)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\left(x^6+18\,x^4+324\,x^3+108\,x^2+216\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="giac")
```

```
[Out] integrate(x^6/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)^2, x)
```

3.154 $\int \frac{x^5}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx$

Optimal. Leaf size=682

$$\frac{\sqrt[3]{-\frac{1}{3}}\left(4-\sqrt[3]{-3}2^{2/3}x\right)}{1944\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)}+\frac{\sqrt[3]{-\frac{1}{3}}\left((-2)^{2/3}\sqrt[3]{3}x+4\right)}{8748\ 2^{2/3}\left(8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+\right.}$$

```
[Out] ((-1/3)^(1/3)*(4 - (-3)^(1/3)*2^(2/3)*x))/(1944*2^(2/3)*(1 + (-1)^(1/3))^4*
(4 - 3*(-3)^(2/3)*2^(1/3))*(6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2)) + ((-1/3)^(1
/3)*(4 + (-2)^(2/3)*3^(1/3)*x))/(8748*2^(2/3)*(8 + (9*I)*2^(1/3)*3^(1/6) +
3*2^(1/3)*3^(2/3))*(6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2)) - (4 + 2^(2/3)*3^(1/
3)*x)/(17496*2^(2/3)*3^(1/3)*(4 - 3*2^(1/3)*3^(2/3))*(6 + 3*2^(2/3)*3^(1/3)
*x + x^2)) - ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4 - 3*(-3)^(2/3)*2
^(1/3))]]/(4374*2^(5/6)*3^(1/6)*(1 + (-1)^(1/3))^4*Sqrt[4 - 3*(-3)^(2/3)*2^
(1/3)]) + ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4 - 3*(-3)^(2/3)*2^(1
/3))]]/(4374*Sqrt[3]*(8 - (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3))^(3/2))
 - ((I/1458)*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3
^(2/3))]])/(2^(5/6)*3^(2/3)*(1 + (-1)^(1/3))^5*Sqrt[4 + 3*(-2)^(1/3)*3^(2/3
)]) - ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))
]]/(4374*Sqrt[3]*(8 + (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3))^(3/2)) - A
rcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]]/
(8748*Sqrt[6]*(-4 + 3*2^(1/3)*3^(2/3))^(3/2)) - ArcTanh[(2^(1/6)*(3*3^(1/3)
 + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]]/(39366*2^(5/6)*3^(1/6)*Sqr
t[-4 + 3*2^(1/3)*3^(2/3)])
```

Rubi [A] time = 1.23518, antiderivative size = 682, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2097, 638, 618, 204, 206}

$$\frac{\sqrt[3]{-\frac{1}{3}}\left(4-\sqrt[3]{-3}2^{2/3}x\right)}{1944\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)}+\frac{\sqrt[3]{-\frac{1}{3}}\left((-2)^{2/3}\sqrt[3]{3}x+4\right)}{8748\ 2^{2/3}\left(8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

```
[Out] ((-1/3)^(1/3)*(4 - (-3)^(1/3)*2^(2/3)*x))/(1944*2^(2/3)*(1 + (-1)^(1/3))^4*
(4 - 3*(-3)^(2/3)*2^(1/3))*(6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2)) + ((-1/3)^(1
/3)*(4 + (-2)^(2/3)*3^(1/3)*x))/(8748*2^(2/3)*(8 + (9*I)*2^(1/3)*3^(1/6) +
3*2^(1/3)*3^(2/3))*(6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2)) - (4 + 2^(2/3)*3^(1/
3)*x)/(17496*2^(2/3)*3^(1/3)*(4 - 3*2^(1/3)*3^(2/3))*(6 + 3*2^(2/3)*3^(1/3)
*x + x^2)) - ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4 - 3*(-3)^(2/3)*2
^(1/3))]]/(4374*2^(5/6)*3^(1/6)*(1 + (-1)^(1/3))^4*Sqrt[4 - 3*(-3)^(2/3)*2^
(1/3)]) + ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4 - 3*(-3)^(2/3)*2^(1
/3))]]/(4374*Sqrt[3]*(8 - (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3))^(3/2))
 - ((I/1458)*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3
^(2/3))]])/(2^(5/6)*3^(2/3)*(1 + (-1)^(1/3))^5*Sqrt[4 + 3*(-2)^(1/3)*3^(2/3
)]) - ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))
]]/(4374*Sqrt[3]*(8 + (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3))^(3/2)) - A
rcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]]/
(8748*Sqrt[6]*(-4 + 3*2^(1/3)*3^(2/3))^(3/2)) - ArcTanh[(2^(1/6)*(3*3^(1/3)
 + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]]/(39366*2^(5/6)*3^(1/6)*Sqr
t[-4 + 3*2^(1/3)*3^(2/3)])
```

Rule 2097

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
```

```
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^5}{\left(216+108x^2+324x^3+18x^4+x^6\right)^2}\,dx = 1586874322944 \int \left(-\frac{\sqrt[3]{-\frac{1}{3}}x}{1542441841901568\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(-6+3\sqrt[3]{-3}2^{2/3}x\right.}\right.$$

$$= -\frac{\sqrt[3]{-\frac{1}{3}}\int \frac{x}{\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)^2}\,dx}{8748\ 2^{2/3}} + \frac{\int \frac{1}{6+3\ 2^{2/3}\sqrt[3]{3}x+x^2}\,dx}{26244\sqrt[3]{2}3^{2/3}} + \frac{\int \frac{x}{\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)^2}\,d}{8748\ 2^{2/3}\sqrt[3]{3}}$$

$$= \frac{\sqrt[3]{-\frac{1}{3}}\left(4-\sqrt[3]{-3}2^{2/3}x\right)}{1944\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)} + \frac{}{17496\ 2^{2/3}}$$

$$= \frac{\sqrt[3]{-\frac{1}{3}}\left(4-\sqrt[3]{-3}2^{2/3}x\right)}{1944\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)} + \frac{}{17496\ 2^{2/3}}$$

$$= \frac{\sqrt[3]{-\frac{1}{3}}\left(4-\sqrt[3]{-3}2^{2/3}x\right)}{1944\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)} + \frac{}{17496\ 2^{2/3}}$$

Mathematica [C] time = 0.0247175, size = 167, normalized size = 0.24

$$\frac{\text{RootSum}\left[\#1^6+18\#1^4+324\#1^3+108\#1^2+216\&, \frac{4\#1^4\log(x-\#1)-54\#1^3\log(x-\#1)+2043\#1^2\log(x-\#1)-324\#1\log(x-\#1)+144\log(x-\#1)}{\#1^5+12\#1^3+162\#1^2+36\#1}\right.}{3691656}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^5/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

```
[Out] (972 - 144*x + 648*x^2 + 729*x^3 - 27*x^4 + 4*x^5)/(615276*(216 + 108*x^2 +
 324*x^3 + 18*x^4 + x^6)) + RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #
1^6 & , (144*Log[x - #1] - 324*Log[x - #1]*#1 + 2043*Log[x - #1]*#1^2 - 54*
Log[x - #1]*#1^3 + 4*Log[x - #1]*#1^4)/(36*#1 + 162*#1^2 + 12*#1^3 + #1^5)
& ]/3691656
```

Maple [C] time = 0.008, size = 122, normalized size = 0.2

$$\frac{1}{x^6+18x^4+324x^3+108x^2+216}\left(\frac{x^5}{153819}-\frac{x^4}{22788}+\frac{x^3}{844}+\frac{2x^2}{1899}-\frac{4x}{17091}+\frac{1}{633}\right)+\frac{1}{3691656}\sum_{\_R=\mathrm{RootOf}\left(\_Z^6+18\_Z^4+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x)
```

```
[Out] (1/153819*x^5-1/22788*x^4+1/844*x^3+2/1899*x^2-4/17091*x+1/633)/(x^6+18*x^4
+324*x^3+108*x^2+216)+1/3691656*sum((4*_R^4-54*_R^3+2043*_R^2-324*_R+144)/(
_R^5+12*_R^3+162*_R^2+36*_R)*ln(x-_R),_R=RootOf(_Z^6+18*_Z^4+324*_Z^3+108*_
Z^2+216))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{4x^5-27x^4+729x^3+648x^2-144x+972}{615276\left(x^6+18x^4+324x^3+108x^2+216\right)}+\frac{1}{615276}\int\frac{4x^4-54x^3+2043x^2-324x+144}{x^6+18x^4+324x^3+108x^2+216}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="maxima")
```

```
[Out] 1/615276*(4*x^5 - 27*x^4 + 729*x^3 + 648*x^2 - 144*x + 972)/(x^6 + 18*x^4 +
 324*x^3 + 108*x^2 + 216) + 1/615276*integrate((4*x^4 - 54*x^3 + 2043*x^2 -
 324*x + 144)/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

Fricas [B] time = 9.74688, size = 8841, normalized size = 12.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="fricas")
```

```
[Out] 1/28041818976*(182304*x^5 - 1230552*x^4 + 33224904*x^3 + 422*sqrt(1/633)*(x
^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)*sqrt(5034474*18^(2/3) + 9367856*18^(
1/3) + 44687457)*log(2/1982119441*sqrt(1/633)*(7238020557*(5034474*18^(2/3)
 + 9367856*18^(1/3) + 44687457)^2 - 447902374840040617697967 3*18^(2/3) - 83
343065225076612586451 12*18^(1/3) - 26862559811422885347120477)*sqrt(5034474
*18^(2/3) + 9367856*18^(1/3) + 44687457) - 7383041510/9393931*(5034474*18^(
2/3) + 9367856*18^(1/3) + 44687457)^2 + 247458158879850620*x + 513225545496
0803463351330/9393931*18^(2/3) + 95498020363770460407535 20/9393931*18^(1/3)
 + 27278928233033940032425830/9393931) - 422*sqrt(1/633)*(x^6 + 18*x^4 + 32
4*x^3 + 108*x^2 + 216)*sqrt(5034474*18^(2/3) + 9367856*18^(1/3) + 44687457)
*log(-2/1982119441*sqrt(1/633)*(7238020557*(5034474*18^(2/3) + 9367856*18^(
1/3) + 44687457)^2 - 4479023748400406176979673*18^(2/3) - 83343065225076612
58645112*18^(1/3) - 26862559811422885347120477)*sqrt(5034474*18^(2/3) + 936
7856*18^(1/3) + 44687457) - 7383041510/9393931*(5034474*18^(2/3) + 9367856*
18^(1/3) + 44687457)^2 + 247458158879850620*x + 5132255454960803463351330/9
393931*18^(2/3) + 9549802036377046040753520/9393931*18^(1/3) + 272789282330
33940032425830/9393931) - 9*sqrt(422)*(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 2
16)*sqrt(-20718*18^(2/3) + sqrt(-1/19683*(5034474*18^(2/3) + 9367856*18^(1/
3) + 44687457)^2 + 22860116892*18^(2/3) + 3445478701088/81*18^(1/3) + 27397
4962699) - 9367856/243*18^(1/3) + 367798)*log(14766083020/211*(5034474*18^(
2/3) + 9367856*18^(1/3) + 44687457)^2 + 3064230/211*sqrt(-1/19683*(5034474*
18^(2/3) + 9367856*18^(1/3) + 44687457)^2 + 22860116892*18^(2/3) + 34454787
01088/81*18^(1/3) + 273974962699)*(5895278433468*18^(2/3) + 10969590754592*
18^(1/3) + 57028339027521) + 9/9393931*(14476041114*sqrt(422)*(5034474*18^(
2/3) + 9367856*18^(1/3) + 44687457)^2 + 243*sqrt(-1/19683*(5034474*18^(2/3)
 + 9367856*18^(1/3) + 44687457)^2 + 22860116892*18^(2/3) + 3445478701088/81
*18^(1/3) + 273974962699)*(14476041114*sqrt(422)*(5034474*18^(2/3) + 936785
6*18^(1/3) + 44687457) - 161351097450615865*sqrt(422)) - 177934129698570542
9*sqrt(422)*(5034474*18^(2/3) + 9367856*18^(1/3) + 44687457) + 265058558805
69051992480475*sqrt(422))*sqrt(-20718*18^(2/3) + sqrt(-1/19683*(5034474*18^
(2/3) + 9367856*18^(1/3) + 44687457)^2 + 22860116892*18^(2/3) + 34454787010
88/81*18^(1/3) + 273974962699) - 9367856/243*18^(1/3) + 367798) + 440683387
65959317812080*x - 10264510909921606926702660/211*18^(2/3) - 19099604072754
092081507040/211*18^(1/3) - 5455785646606788006485166 0/211) + 9*sqrt(422)*(
x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)*sqrt(-20718*18^(2/3) + sqrt(-1/1968
3*(5034474*18^(2/3) + 9367856*18^(1/3) + 44687457)^2 + 22860116892*18^(2/3)
```

+ 3445478701088/81*18^(1/3) + 273974962699) - 9367856/243*18^(1/3) + 36779
8)*log(14766083020/211*(5034474*18^(2/3) + 9367856*18^(1/3) + 44687457)^2 +
3064230/211*sqrt(-1/19683*(5034474*18^(2/3) + 9367856*18^(1/3) + 44687457)
^2 + 22860116892*18^(2/3) + 3445478701088/81*18^(1/3) + 273974962699)*(5895
278433468*18^(2/3) + 10969590754592*18^(1/3) + 57028339027521) - 9/9393931*
(14476041114*sqrt(422)*(5034474*18^(2/3) + 9367856*18^(1/3) + 44687457)^2 +
243*sqrt(-1/19683*(5034474*18^(2/3) + 9367856*18^(1/3) + 44687457)^2 + 228
60116892*18^(2/3) + 3445478701088/81*18^(1/3) + 273974962699)*(14476041114*
sqrt(422)*(5034474*18^(2/3) + 9367856*18^(1/3) + 44687457) - 16135109745061
5865*sqrt(422)) - 1779341296985705429*sqrt(422)*(5034474*18^(2/3) + 9367856
*18^(1/3) + 44687457) + 26505855880569051992480475*sqrt(422))*sqrt(-20718*1
8^(2/3) + sqrt(-1/19683*(5034474*18^(2/3) + 9367856*18^(1/3) + 44687457)^2
+ 22860116892*18^(2/3) + 3445478701088/81*18^(1/3) + 273974962699) - 936785
6/243*18^(1/3) + 367798) + 44068338765959317812080*x - 10264510909921606926
702660/211*18^(2/3) - 19099604072754092081507040/211*18^(1/3) - 54557856466
067880064851660/211) - 9*sqrt(422)*(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)
*sqrt(-20718*18^(2/3) - sqrt(-1/19683*(5034474*18^(2/3) + 9367856*18^(1/3)
+ 44687457)^2 + 22860116892*18^(2/3) + 3445478701088/81*18^(1/3) + 27397496
2699) - 9367856/243*18^(1/3) + 367798)*log(14766083020/211*(5034474*18^(2/3
) + 9367856*18^(1/3) + 44687457)^2 - 3064230/211*sqrt(-1/19683*(5034474*18^
(2/3) + 9367856*18^(1/3) + 44687457)^2 + 22860116892*18^(2/3) + 34454787010
88/81*18^(1/3) + 273974962699)*(5895278433468*18^(2/3) + 10969590754592*18^
(1/3) + 57028339027521) + 9/9393931*(14476041114*sqrt(422)*(5034474*18^(2/3
) + 9367856*18^(1/3) + 44687457)^2 - 243*sqrt(-1/19683*(5034474*18^(2/3) +
9367856*18^(1/3) + 44687457)^2 + 22860116892*18^(2/3) + 3445478701088/81*18
^(1/3) + 273974962699)*(14476041114*sqrt(422)*(5034474*18^(2/3) + 9367856*1
8^(1/3) + 44687457) - 161351097450615865*sqrt(422)) - 1779341296985705429*s
qrt(422)*(5034474*18^(2/3) + 9367856*18^(1/3) + 44687457) + 265058558805690
51992480475*sqrt(422))*sqrt(-20718*18^(2/3) - sqrt(-1/19683*(5034474*18^(2/
3) + 9367856*18^(1/3) + 44687457)^2 + 22860116892*18^(2/3) + 3445478701088/
81*18^(1/3) + 273974962699) - 9367856/243*18^(1/3) + 367798) + 440683387659
59317812080*x - 10264510909921606926702660/211*18^(2/3) - 19099604072754092
081507040/211*18^(1/3) - 54557856466067880064851660/211) + 9*sqrt(422)*(x^6
+ 18*x^4 + 324*x^3 + 108*x^2 + 216)*sqrt(-20718*18^(2/3) - sqrt(-1/19683*(
5034474*18^(2/3) + 9367856*18^(1/3) + 44687457)^2 + 22860116892*18^(2/3) +
3445478701088/81*18^(1/3) + 273974962699) - 9367856/243*18^(1/3) + 367798)*
log(14766083020/211*(5034474*18^(2/3) + 9367856*18^(1/3) + 44687457)^2 - 30
64230/211*sqrt(-1/19683*(5034474*18^(2/3) + 9367856*18^(1/3) + 44687457)^2
+ 22860116892*18^(2/3) + 3445478701088/81*18^(1/3) + 273974962699)*(5895278
433468*18^(2/3) + 10969590754592*18^(1/3) + 57028339027521) - 9/9393931*(14
476041114*sqrt(422)*(5034474*18^(2/3) + 9367856*18^(1/3) + 44687457)^2 - 24
3*sqrt(-1/19683*(5034474*18^(2/3) + 9367856*18^(1/3) + 44687457)^2 + 228601
16892*18^(2/3) + 3445478701088/81*18^(1/3) + 273974962699)*(14476041114*sqr
t(422)*(5034474*18^(2/3) + 9367856*18^(1/3) + 44687457) - 16135109745061586
5*sqrt(422)) - 1779341296985705429*sqrt(422)*(5034474*18^(2/3) + 9367856*18

```
^(1/3) + 44687457) + 265058558805690519924804475*sqrt(422))*sqrt(-20718*18^(
2/3) - sqrt(-1/19683*(5034474*18^(2/3) + 9367856*18^(1/3) + 44687457)^2 + 2
2860116892*18^(2/3) + 3445478701088/81*18^(1/3) + 273974962699) - 9367856/2
43*18^(1/3) + 367798) + 440683387659593178120800*x - 1026451090992160692670 2
660/211*18^(2/3) - 190996040727540920815070 40/211*18^(1/3) - 54557856466067
880064851660/211) + 29533248*x^2 - 6562944*x + 44299872)/(x^6 + 18*x^4 + 32
4*x^3 + 108*x^2 + 216)
```

Sympy [A] time = 0.281599, size = 104, normalized size = 0.15

$$\text{RootSum}\Big(274938951049788473490124490008305567006 72t^6 - 13187181892269500888629831925 76t^4 + 121209177$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(x**6+18*x**4+324*x**3+108*x**2+216)**2,x)
```

```
[Out] RootSum(27493895104978847349012449000830556700672*_t**6 - 13187181892269500
88862983192576*_t**4 + 1212091770477677 6448*_t**2 - 39753025, Lambda(_t, _t
*log(9478422590012887239098320545502099502420459 52*_t**5/61864539719962655
- 24345864681777560763965488948081459 2*_t**4/9811980923071 - 41682556475067
500431787310779667456*_t**3/61864539719962655 + 1202687744266432861646 2272*
_t**2/9811980923071 + 216142618488859793668428*_t/61864539719962655 + x - 3
08574300024117/39247923692284))) + (4*x**5 - 27*x**4 + 729*x**3 + 648*x**2
- 144*x + 972)/(615276*x**6 + 11074968*x**4 + 199349424*x**3 + 66449808*x**
2 + 132899616)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.155 $\int \frac{x^4}{\left(216+108x^2+324x^3+18x^4+x^6\right)^2}\,dx$

Optimal. Leaf size=850

result too large to display

```
[Out] ((-1/3)^(1/3)*(3*(-3)^(1/3)*2^(2/3) - 2*x))/(5832*2^(2/3)*(1 + (-1)^(1/3))^
4*(4 - 3*(-3)^(2/3)*2^(1/3))*(6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2)) - ((-1/3)^
(1/3)*(3*(-2)^(2/3)*3^(1/3) + 2*x))/(26244*2^(2/3)*(8 + (9*I)*2^(1/3)*3^(1/
6) + 3*2^(1/3)*3^(2/3))*(6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2)) - (3*3^(1/3) +
2^(1/3)*x)/(52488*(9*2^(1/3) - 4*3^(1/3))*(6 + 3*2^(2/3)*3^(1/3)*x + x^2))
 + ((-1)^(1/3)*ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4 - 3*(-3)^(2/3)*
2^(1/3))]])/(729*2^(2/3)*3^(5/6)*(1 + (-1)^(1/3))^4*(8 - (9*I)*2^(1/3)*3^(1
/6) + 3*2^(1/3)*3^(2/3))^(3/2)) - ((-1)^(1/3)*ArcTan[(3*(-2)^(2/3)*3^(1/3)
 + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(2916*2^(1/6)*3^(5/6)*(1 - (-1)
^(1/3))^2*(1 + (-1)^(1/3))^4*(4 + 3*(-2)^(1/3)*3^(2/3))^(3/2)) - ((I + Sqrt
[3])*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]
])/(11664*2^(1/6)*3^(1/3)*(1 + (-1)^(1/3))^5*Sqrt[4 + 3*(-2)^(1/3)*3^(2/3)]
) - ((I/5832)*ArcTan[(2^(1/6)*(3*(-3)^(1/3) - 2^(1/3)*x))/Sqrt[3*(4 - 3*(-3
)^(2/3)*2^(1/3))]])/(2^(1/6)*3^(1/3)*(1 + (-1)^(1/3))^5*Sqrt[4 - 3*(-3)^(2/
3)*2^(1/3)]) + ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(
1/3)*3^(2/3))]]/(26244*2^(1/6)*3^(5/6)*(-4 + 3*2^(1/3)*3^(2/3))^(3/2)) + Ar
cTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]]/(
52488*2^(1/6)*3^(5/6)*Sqrt[-4 + 3*2^(1/3)*3^(2/3)]) - Log[6 - 3*(-3)^(1/3)*
2^(2/3)*x + x^2]/(34992*2^(1/3)*3^(2/3)*(1 + (-1)^(1/3))^4) + ((I/34992)*Lo
g[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2])/(2^(1/3)*3^(1/6)*(1 + (-1)^(1/3))^5) -
 Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2]/(314928*2^(1/3)*3^(2/3))
```

Rubi [A] time = 1.46965, antiderivative size = 850, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {2097, 614, 618, 204, 634, 628, 206}

$$\frac{\sqrt[3]{-\frac{1}{3}}\left(3\sqrt[3]{-3}2^{2/3}-2x\right)}{5832\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)}+\frac{\sqrt[3]{-1}\tan^{-1}\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{729\ 2^{2/3}3^{5/6}\left(1+\sqrt[3]{-1}\right)^4\left(8-9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)^{3/2}}-$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

```
[Out] ((-1/3)^(1/3)*(3*(-3)^(1/3)*2^(2/3) - 2*x))/(5832*2^(2/3)*(1 + (-1)^(1/3))^
4*(4 - 3*(-3)^(2/3)*2^(1/3))*(6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2)) - ((-1/3)^
(1/3)*(3*(-2)^(2/3)*3^(1/3) + 2*x))/(26244*2^(2/3)*(8 + (9*I)*2^(1/3)*3^(1/
6) + 3*2^(1/3)*3^(2/3))*(6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2)) - (3*3^(1/3) +
2^(1/3)*x)/(52488*(9*2^(1/3) - 4*3^(1/3))*(6 + 3*2^(2/3)*3^(1/3)*x + x^2))
+ ((-1)^(1/3)*ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4 - 3*(-3)^(2/3)*
2^(1/3))]])/(729*2^(2/3)*3^(5/6)*(1 + (-1)^(1/3))^4*(8 - (9*I)*2^(1/3)*3^(1
/6) + 3*2^(1/3)*3^(2/3))^(3/2)) - ((-1)^(1/3)*ArcTan[(3*(-2)^(2/3)*3^(1/3)
+ 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(2916*2^(1/6)*3^(5/6)*(1 - (-1)
^(1/3))^2*(1 + (-1)^(1/3))^4*(4 + 3*(-2)^(1/3)*3^(2/3))^(3/2)) - ((I + Sqrt
[3])*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]
])/(11664*2^(1/6)*3^(1/3)*(1 + (-1)^(1/3))^5*Sqrt[4 + 3*(-2)^(1/3)*3^(2/3)]
) - ((I/5832)*ArcTan[(2^(1/6)*(3*(-3)^(1/3) - 2^(1/3)*x))/Sqrt[3*(4 - 3*(-3
)^(2/3)*2^(1/3))]])/(2^(1/6)*3^(1/3)*(1 + (-1)^(1/3))^5*Sqrt[4 - 3*(-3)^(2/
3)*2^(1/3)]) + ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(
1/3)*3^(2/3))]]/(26244*2^(1/6)*3^(5/6)*(-4 + 3*2^(1/3)*3^(2/3))^(3/2)) + Ar
cTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]]/(
52488*2^(1/6)*3^(5/6)*Sqrt[-4 + 3*2^(1/3)*3^(2/3)]) - Log[6 - 3*(-3)^(1/3)*
2^(2/3)*x + x^2]/(34992*2^(1/3)*3^(2/3)*(1 + (-1)^(1/3))^4) + ((I/34992)*Lo
g[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2])/(2^(1/3)*3^(1/6)*(1 + (-1)^(1/3))^5) -
 Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2]/(314928*2^(1/3)*3^(2/3))
```

Rule 2097

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^4}{\left(216+108x^2+324x^3+18x^4+x^6\right)^2}\,dx = 1586874322944 \int \left(-\frac{\sqrt[3]{-\frac{1}{3}}}{1542441841901568\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(-6+3\sqrt[3]{-3}2^{2/3}x\right.}\right.$$

$$= -\frac{\sqrt[3]{-\frac{1}{3}}\int \frac{1}{\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)^2}\,dx}{8748\ 2^{2/3}} - \frac{\int \frac{3\ 2^{2/3}\sqrt[3]{3}+x}{6+3\ 2^{2/3}\sqrt[3]{3}x+x^2}\,dx}{157464\sqrt[3]{2}3^{2/3}} + \frac{\int \frac{1}{\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)^2}\,d}{8748\ 2^{2/3}\sqrt[3]{3}}$$

$$= \frac{\sqrt[3]{-\frac{1}{3}}\left(3\sqrt[3]{-3}2^{2/3}-2x\right)}{5832\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)} - \frac{}{52488\ 2^{2/3}}$$

$$= \frac{\sqrt[3]{-\frac{1}{3}}\left(3\sqrt[3]{-3}2^{2/3}-2x\right)}{5832\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)} - \frac{}{52488\ 2^{2/3}}$$

$$= \frac{\sqrt[3]{-\frac{1}{3}}\left(3\sqrt[3]{-3}2^{2/3}-2x\right)}{5832\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)} - \frac{}{52488\ 2^{2/3}}$$

Mathematica [C] time = 0.035202, size = 167, normalized size = 0.2

$$\frac{-9x^5+8x^4-216x^3-1458x^2+324x-288}{1230552\left(x^6+18x^4+324x^3+108x^2+216\right)} - \frac{\text{RootSum}\left[\#1^6+18\#1^4+324\#1^3+108\#1^2+216\&, \frac{9\#1^4\log(x-\#1)-16\#1^3\text{lo}}{}\right.}{7383312}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

```
[Out] (-288 + 324*x - 1458*x^2 - 216*x^3 + 8*x^4 - 9*x^5)/(1230552*(216 + 108*x^2
 + 324*x^3 + 18*x^4 + x^6)) - RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 +
 #1^6 & , (324*Log[x - #1] - 2628*Log[x - #1]*#1 + 324*Log[x - #1]*#1^2 - 1
6*Log[x - #1]*#1^3 + 9*Log[x - #1]*#1^4)/(36*#1 + 162*#1^2 + 12*#1^3 + #1^5
) & ]/7383312
```

Maple [C] time = 0.009, size = 122, normalized size = 0.1

$$\frac{1}{x^6+18x^4+324x^3+108x^2+216}\left(-\frac{x^5}{136728}+\frac{x^4}{153819}-\frac{x^3}{5697}-\frac{x^2}{844}+\frac{x}{3798}-\frac{4}{17091}\right)+\frac{1}{7383312}\ {}_{\_R=\mathrm{RootOf}\left(\_Z^6+18\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x)
```

```
[Out] (-1/136728*x^5+1/153819*x^4-1/5697*x^3-1/844*x^2+1/3798*x-4/17091)/(x^6+18*
x^4+324*x^3+108*x^2+216)+1/7383312*sum((-9*_R^4+16*_R^3-324*_R^2+2628*_R-32
4)/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(x-_R),_R=RootOf(_Z^6+18*_Z^4+324*_Z^3+1
08*_Z^2+216))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{9x^5-8x^4+216x^3+1458x^2-324x+288}{1230552\left(x^6+18x^4+324x^3+108x^2+216\right)}-\frac{1}{1230552}\int\frac{9x^4-16x^3+324x^2-2628x+324}{x^6+18x^4+324x^3+108x^2+216}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="maxima")
```

```
[Out] -1/1230552*(9*x^5 - 8*x^4 + 216*x^3 + 1458*x^2 - 324*x + 288)/(x^6 + 18*x^4
 + 324*x^3 + 108*x^2 + 216) - 1/1230552*integrate((9*x^4 - 16*x^3 + 324*x^2
 - 2628*x + 324)/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 0.36885, size = 112, normalized size = 0.13

$$\mathrm{RootSum}\Big(1855837919586072196058340307556062577295366t^6 - 1309367357962223565522033377280t^4 + 4356336\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(x**6+18*x**4+324*x**3+108*x**2+216)**2,x)
```

```
[Out] RootSum(1855837919586072196058340307556062577295366*_t**6 - 1309367357962223
565522033377280*_t**4 + 435633648705229474466611 2*_t**3 - 40529828454803873
28*_t**2 + 303890718384*_t - 880007, Lambda(_t, _t*log(3908346265795559347 6
8410447073335659764129527592806346915 84*_t**5/497978553961399002675733956 95
 + 8836979346223785538912817601414711102396804462575616*_t**4/4979785539613
9900267573395695 - 264930581348308532588844249597134695706805067776*_t**3/4
979785539613990026757339 5695 + 886135333547363185201515109826158376250624*_
t**2/497978553961399002675733956 95 - 682321479574909906511394635855601936*_
t/49797855396139900267573395695 + x - 2137556077084648622429151 9568/4979785
5396139900267573395695))) - (9*x**5 - 8*x**4 + 216*x**3 + 1458*x**2 - 324*x
 + 288)/(1230552*x**6 + 22149936*x**4 + 398698848*x**3 + 132899616*x**2 + 2
65799232)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(x^6 + 18\,x^4 + 324\,x^3 + 108\,x^2 + 216\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="giac")
```

```
[Out] integrate(x^4/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)^2, x)
```

3.156 $\int \frac{x^3}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx$

Optimal. Leaf size=873

result too large to display

```
[Out] ((-6)^(1/3)*(2*(-3)^(1/3) + 9*2^(1/3)) - 3*x)/(157464*(8 - (9*I)*2^(1/3)*3^
(1/6) + 3*2^(1/3)*3^(2/3))*(6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2)) - ((-6)^(1/3
)*(9*(-2)^(1/3) + 2*3^(1/3)) + 3*x)/(157464*(8 + (9*I)*2^(1/3)*3^(1/6) + 3*
2^(1/3)*3^(2/3))*(6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2)) - (2*2^(1/3) - 3*6^(2/
3) - 3^(1/3)*x)/(104976*(9*2^(1/3) - 4*3^(1/3))*(6 + 3*2^(2/3)*3^(1/3)*x +
x^2)) + ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4 - 3*(-3)^(2/3)*2^(1/3
))]]/(26244*Sqrt[3]*(8 - (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3))^(3/2))
- ((9*I - 3^(1/3)*((2*I)*2^(2/3) + 9*3^(1/6) + 2*2^(2/3)*Sqrt[3]))*ArcTan[(
3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(209952*(1
 + (-1)^(1/3))^5*Sqrt[2*(4 - 3*(-3)^(2/3)*2^(1/3))]) - ArcTan[(3*(-2)^(2/3)
*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]]/(26244*Sqrt[3]*(8 + (9*
I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3))^(3/2)) + ((9*I + 3^(1/3)*((4*I)*2^(
2/3) - 9*3^(1/6)))*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(
1/3)*3^(2/3))]])/(209952*(1 + (-1)^(1/3))^5*Sqrt[2*(4 + 3*(-2)^(1/3)*3^(2/3
))]) - ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(
2/3))]]/(52488*Sqrt[6]*(-4 + 3*2^(1/3)*3^(2/3))^(3/2)) + ((2*2^(2/3) - 3*3^
(2/3))*ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(
2/3))]])/(944784*3^(1/6)*Sqrt[2*(-4 + 3*2^(1/3)*3^(2/3))]) - ((I/23328)*Log
[6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2])/(2^(2/3)*3^(5/6)*(1 + (-1)^(1/3))^5) +
((I + Sqrt[3])*Log[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2])/(46656*2^(2/3)*3^(5/6
)*(1 + (-1)^(1/3))^5) + Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2]/(629856*2^(2/3)*
3^(1/3))
```

Rubi [A] time = 1.91551, antiderivative size = 873, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {2097, 638, 618, 204, 634, 628, 206}

$$\frac{\sqrt[3]{-6}\left(2\sqrt[3]{-3}+9\sqrt[3]{2}\right)-3x}{157464\left(8-9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)}-\frac{\left(9i-\sqrt[3]{3}\left(2i2^{2/3}+9\sqrt[6]{3}+2\,2^{2/3}\sqrt{3}\right)\right)\tan^{-1}\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{209952\left(1+\sqrt[3]{-1}\right)^5\sqrt{2\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

```
[Out] ((-6)^(1/3)*(2*(-3)^(1/3) + 9*2^(1/3)) - 3*x)/(157464*(8 - (9*I)*2^(1/3)*3^
(1/6) + 3*2^(1/3)*3^(2/3))*(6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2)) - ((-6)^(1/3
)*(9*(-2)^(1/3) + 2*3^(1/3)) + 3*x)/(157464*(8 + (9*I)*2^(1/3)*3^(1/6) + 3*
2^(1/3)*3^(2/3))*(6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2)) - (2*2^(1/3) - 3*6^(2/
3) - 3^(1/3)*x)/(104976*(9*2^(1/3) - 4*3^(1/3))*(6 + 3*2^(2/3)*3^(1/3)*x +
x^2)) + ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4 - 3*(-3)^(2/3)*2^(1/3
))]]/(26244*Sqrt[3]*(8 - (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3))^(3/2))
- ((9*I - 3^(1/3)*((2*I)*2^(2/3) + 9*3^(1/6) + 2*2^(2/3)*Sqrt[3]))*ArcTan[(
3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(209952*(1
 + (-1)^(1/3))^5*Sqrt[2*(4 - 3*(-3)^(2/3)*2^(1/3))]) - ArcTan[(3*(-2)^(2/3)
*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]]/(26244*Sqrt[3]*(8 + (9*
I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3))^(3/2)) + ((9*I + 3^(1/3)*((4*I)*2^(
2/3) - 9*3^(1/6)))*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(
1/3)*3^(2/3))]])/(209952*(1 + (-1)^(1/3))^5*Sqrt[2*(4 + 3*(-2)^(1/3)*3^(2/3
))]) - ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(
2/3))]]/(52488*Sqrt[6]*(-4 + 3*2^(1/3)*3^(2/3))^(3/2)) + ((2*2^(2/3) - 3*3^
(2/3))*ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(
2/3))]])/(944784*3^(1/6)*Sqrt[2*(-4 + 3*2^(1/3)*3^(2/3))]) - ((I/23328)*Log
[6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2])/(2^(2/3)*3^(5/6)*(1 + (-1)^(1/3))^5) +
((I + Sqrt[3])*Log[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2])/(46656*2^(2/3)*3^(5/6
)*(1 + (-1)^(1/3))^5) + Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2]/(629856*2^(2/3)*
3^(1/3))
```

<u>Rule 2097</u>

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

<u>Rule 638</u>

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

<u>Rule 618</u>

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^3}{\left(216+108x^2+324x^3+18x^4+x^6\right)^2}\,dx = 1586874322944 \int \Bigg(-\frac{9(-2)^{2/3}-\sqrt[3]{-1}3^{2/3}x}{27763953154228224\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(-6+3\sqrt[3]{-3}2^{2/\cdots}\right.}$$

$$= \frac{\int \frac{18-2\ 2^{2/3}\sqrt[3]{3}+\sqrt[3]{2}3^{2/3}x}{6+3\ 2^{2/3}\sqrt[3]{3}x+x^2}\,dx}{1889568} - \frac{\int \frac{9\ 2^{2/3}-\sqrt[3]{-1}3^{2/3}x}{\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)^2}\,dx}{157464\ 2^{2/3}} - \frac{\int \frac{3\ 2^{2/3}\sqrt[3]{3}+x}{\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)^2}\,dx}{52488\ 2^{2/3}\sqrt[3]{3}} \cdots$$

$$= \frac{\sqrt[3]{-6}\left(2\sqrt[3]{-3}+9\sqrt[3]{2}\right)-3x}{157464\left(8-9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)} - \frac{\sqrt[3]{-6}\,(\cdots}{314928\left(4+3\sqrt[3]{\cdots}\right.}$$

$$= \frac{\sqrt[3]{-6}\left(2\sqrt[3]{-3}+9\sqrt[3]{2}\right)-3x}{157464\left(8-9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)} - \frac{\sqrt[3]{-6}\,(\cdots}{314928\left(4+3\sqrt[3]{\cdots}\right.}$$

$$= \frac{\sqrt[3]{-6}\left(2\sqrt[3]{-3}+9\sqrt[3]{2}\right)-3x}{157464\left(8-9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)} - \frac{\sqrt[3]{-6}\,(\cdots}{314928\left(4+3\sqrt[3]{\cdots}\right.}$$

Mathematica [C] time = 0.0273065, size = 167, normalized size = 0.19

$$\frac{\text{RootSum}\left[\#1^6+18\#1^4+324\#1^3+108\#1^2+216\&,\frac{2\#1^4\log(x-\#1)-27\#1^3\log(x-\#1)+72\#1^2\log(x-\#1)-162\#1\log(x-\#1)+1971\log(x-\#1)}{\#1^5+12\#1^3+162\#1^2+36\#1}\&\cdots\right.}{11074968}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

```
[Out] (972 - 3942*x + 648*x^2 + 96*x^3 - 27*x^4 + 4*x^5)/(3691656*(216 + 108*x^2
+ 324*x^3 + 18*x^4 + x^6)) + RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 +
#1^6 & , (1971*Log[x - #1] - 162*Log[x - #1]*#1 + 72*Log[x - #1]*#1^2 - 27*
Log[x - #1]*#1^3 + 2*Log[x - #1]*#1^4)/(36*#1 + 162*#1^2 + 12*#1^3 + #1^5)
& ]/11074968
```

Maple [C] time = 0.009, size = 122, normalized size = 0.1

$$\frac{1}{x^6+18x^4+324x^3+108x^2+216}\left(\frac{x^5}{922914}-\frac{x^4}{136728}+\frac{4x^3}{153819}+\frac{x^2}{5697}-\frac{73x}{68364}+\frac{1}{3798}\right)+\frac{1}{11074968}\ {}_{\_R=\mathrm{RootOf}(\_Z}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x)
```

```
[Out] (1/922914*x^5-1/136728*x^4+4/153819*x^3+1/5697*x^2-73/68364*x+1/3798)/(x^6+
18*x^4+324*x^3+108*x^2+216)+1/11074968*sum((2*_R^4-27*_R^3+72*_R^2-162*_R+1
971)/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(x-_R),_R=RootOf(_Z^6+18*_Z^4+324*_Z^3
+108*_Z^2+216))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{4x^5-27x^4+96x^3+648x^2-3942x+972}{3691656\left(x^6+18x^4+324x^3+108x^2+216\right)}+\frac{1}{1845828}\int\frac{2x^4-27x^3+72x^2-162x+1971}{x^6+18x^4+324x^3+108x^2+216}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="maxima")
```

```
[Out] 1/3691656*(4*x^5 - 27*x^4 + 96*x^3 + 648*x^2 - 3942*x + 972)/(x^6 + 18*x^4
+ 324*x^3 + 108*x^2 + 216) + 1/1845828*integrate((2*x^4 - 27*x^3 + 72*x^2 -
 162*x + 1971)/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 0.371063, size = 112, normalized size = 0.13

$$\operatorname{RootSum}\left(1282755170017893101915524820582750453426552832t^6 - 906388465775544244426251149770752t^4 - 43\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**6+18*x**4+324*x**3+108*x**2+216)**2,x)
```

```
[Out] RootSum(1282755170017893101915524820582750453426552832*_t**6 - 906388465775
544244426251149770752*_t**4 - 430087316638998774168413798 4*_t**3 - 71700090
8921644962816*_t**2 + 135354162312576*_t - 7197829, Lambda(_t, _t*log(17257
935592810449901409556597891882995604001083339368041361480613888*_t**5/15420
6009791052044490694380303237521 + 23896074006209855243763588535726522071819
5632456058768405 2992*_t**4/154206009791052044490694380303237521 - 122860721
6088328393071171594887826007899699219348838 8096*_t**3/154206009791052044490
694380303237521 - 5949055357395917316112549601352790975415655841 0752*_t**2/
154206009791052044490694380303237521 - 175201496798366911123670641977137530
04827200*_t/154206009791052044490694380303237521 + x + 766422988707229615 05
5855287040887332/154206009791052044490694380303237521))) + (4*x**5 - 27*x**
4 + 96*x**3 + 648*x**2 - 3942*x + 972)/(3691656*x**6 + 66449808*x**4 + 1196
096544*x**3 + 398698848*x**2 + 797397696)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(x^6+18\,x^4+324\,x^3+108\,x^2+216\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)^2, x)
```

3.157 $\int \frac{x^2}{\left(216+108x^2+324x^3+18x^4+x^6\right)^2}\,dx$

Optimal. Leaf size=986

result too large to display

```
[Out] -(27*((-2)^(2/3) + 2*(-1)^(1/3)*3^(2/3)) - 6^(1/3)*(9 + (-3)^(1/3)*2^(2/3))
*x)/(104976*2^(2/3)*(1 + (-1)^(1/3))^4*(4 - 3*(-3)^(2/3)*2^(1/3))*(6 - 3*(-
3)^(1/3)*2^(2/3)*x + x^2)) - (27*2^(2/3)*(1 + (-2)^(1/3)*3^(2/3)) - (-1)^(1
/3)*3^(2/3)*(2 + 3*(-2)^(1/3)*3^(2/3))*x)/(472392*2^(2/3)*(8 + (9*I)*2^(1/3
)*3^(1/6) + 3*2^(1/3)*3^(2/3))*(6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2)) + (9*(6
- 2^(2/3)*3^(1/3)) - (2 - 3*2^(1/3)*3^(2/3))*x)/(314928*2^(2/3)*3^(1/3)*(4
- 3*2^(1/3)*3^(2/3))*(6 + 3*2^(2/3)*3^(1/3)*x + x^2)) - ((1 + I*Sqrt[3] + 3
*2^(1/3)*3^(2/3))*ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4 - 3*(-3)^(2
/3)*2^(1/3))]])/(8748*2^(2/3)*3^(5/6)*(1 + (-1)^(1/3))^4*(8 - (9*I)*2^(1/3)
*3^(1/6) + 3*2^(1/3)*3^(2/3))^(3/2)) + ((3*(-3)^(2/3) + (-1)^(1/3)*2^(2/3))
*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(
17496*6^(5/6)*(1 - (-1)^(1/3))^2*(1 + (-1)^(1/3))^4*(4 + 3*(-2)^(1/3)*3^(2/
3))^(3/2)) + ((I + Sqrt[3])*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 +
 3*(-2)^(1/3)*3^(2/3))]])/(34992*2^(1/6)*3^(1/3)*(1 + (-1)^(1/3))^5*Sqrt[4
+ 3*(-2)^(1/3)*3^(2/3)]) + ((I/17496)*ArcTan[(2^(1/6)*(3*(-3)^(1/3) - 2^(1/
3)*x))/Sqrt[3*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(2^(1/6)*3^(1/3)*(1 + (-1)^(1/3
))^5*Sqrt[4 - 3*(-3)^(2/3)*2^(1/3)]) - ((2^(2/3) - 3*3^(2/3))*ArcTanh[(2^(1
/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]])/(17496*6^(5
/6)*(1 - (-1)^(1/3))^2*(1 + (-1)^(1/3))^4*(-4 + 3*2^(1/3)*3^(2/3))^(3/2)) -
 ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]
]/(157464*2^(1/6)*3^(5/6)*Sqrt[-4 + 3*2^(1/3)*3^(2/3)]) + ((I + Sqrt[3])*Lo
g[6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2])/(419904*2^(1/3)*3^(1/6)*(1 + (-1)^(1/3
))^5) - ((I/209952)*Log[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2])/(2^(1/3)*3^(1/6)
*(1 + (-1)^(1/3))^5) + Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2]/(1889568*2^(1/3)*
3^(2/3))
```

Rubi [A] time = 1.92698, antiderivative size = 986, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {2097, 638, 618, 204, 634, 628, 206}

$$-\frac{27\left((-2)^{2/3}+2\sqrt[3]{-1}3^{2/3}\right)-\sqrt[3]{6}\left(9+\sqrt[3]{-3}2^{2/3}\right)x}{104976\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)}-\frac{\left(1+i\sqrt{3}+3\sqrt[3]{2}3^{2/3}\right)\tan^{-1}\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{8748\ 2^{2/3}3^{5/6}\left(1+\sqrt[3]{-1}\right)^4\left(8-9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

```
[Out] -(27*((-2)^(2/3) + 2*(-1)^(1/3)*3^(2/3)) - 6^(1/3)*(9 + (-3)^(1/3)*2^(2/3))
*x)/(104976*2^(2/3)*(1 + (-1)^(1/3))^4*(4 - 3*(-3)^(2/3)*2^(1/3))*(6 - 3*(-
3)^(1/3)*2^(2/3)*x + x^2)) - (27*2^(2/3)*(1 + (-2)^(1/3)*3^(2/3)) - (-1)^(1
/3)*3^(2/3)*(2 + 3*(-2)^(1/3)*3^(2/3))*x)/(472392*2^(2/3)*(8 + (9*I)*2^(1/3
)*3^(1/6) + 3*2^(1/3)*3^(2/3))*(6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2)) + (9*(6
- 2^(2/3)*3^(1/3)) - (2 - 3*2^(1/3)*3^(2/3))*x)/(314928*2^(2/3)*3^(1/3)*(4
- 3*2^(1/3)*3^(2/3))*(6 + 3*2^(2/3)*3^(1/3)*x + x^2)) - ((1 + I*Sqrt[3] + 3
*2^(1/3)*3^(2/3))*ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4 - 3*(-3)^(2
/3)*2^(1/3))]])/(8748*2^(2/3)*3^(5/6)*(1 + (-1)^(1/3))^4*(8 - (9*I)*2^(1/3)
*3^(1/6) + 3*2^(1/3)*3^(2/3))^(3/2)) + ((3*(-3)^(2/3) + (-1)^(1/3)*2^(2/3))
*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(
17496*6^(5/6)*(1 - (-1)^(1/3))^2*(1 + (-1)^(1/3))^4*(4 + 3*(-2)^(1/3)*3^(2/
3))^(3/2)) + ((I + Sqrt[3])*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 +
 3*(-2)^(1/3)*3^(2/3))]])/(34992*2^(1/6)*3^(1/3)*(1 + (-1)^(1/3))^5*Sqrt[4
+ 3*(-2)^(1/3)*3^(2/3)]) + ((I/17496)*ArcTan[(2^(1/6)*(3*(-3)^(1/3) - 2^(1/
3)*x))/Sqrt[3*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(2^(1/6)*3^(1/3)*(1 + (-1)^(1/3
))^5*Sqrt[4 - 3*(-3)^(2/3)*2^(1/3)]) - ((2^(2/3) - 3*3^(2/3))*ArcTanh[(2^(1
/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]])/(17496*6^(5
/6)*(1 - (-1)^(1/3))^2*(1 + (-1)^(1/3))^4*(-4 + 3*2^(1/3)*3^(2/3))^(3/2)) -
 ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]
]/(157464*2^(1/6)*3^(5/6)*Sqrt[-4 + 3*2^(1/3)*3^(2/3)]) + ((I + Sqrt[3])*Lo
g[6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2])/(419904*2^(1/3)*3^(1/6)*(1 + (-1)^(1/3
))^5) - ((I/209952)*Log[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2])/(2^(1/3)*3^(1/6)
*(1 + (-1)^(1/3))^5) + Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2]/(1889568*2^(1/3)*
3^(2/3))
```

Rule 2097

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
```

```
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^2}{\left(216+108x^2+324x^3+18x^4+x^6\right)^2}\,dx = 1586874322944 \int \left(\frac{2\sqrt[3]{-1}3^{2/3}+18\sqrt[3]{6}+3(-2)^{2/3}x}{55527906308456448\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(-6+3\sqrt[3]{-3}2^{2/3}\right.}\right.$$

$$= \frac{\int \frac{2\sqrt[3]{-1}3^{2/3}+18(-1)^{2/3}\sqrt[3]{6}+3\ 2^{2/3}x}{\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)^2}\,dx}{314928\ 2^{2/3}} + \frac{\int \frac{-2+6\sqrt[3]{2}3^{2/3}+2^{2/3}\sqrt[3]{3}x}{\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)^2}\,dx}{104976\ 2^{2/3}\sqrt[3]{3}} + \frac{\int \frac{9\sqrt[3]{3}+\sqrt[3]{2}x}{6+3\ 2^{2/3}\sqrt[3]{3}x+x}}{944784\ 6^{2/3}}$$

$$= -\frac{27\left((-2)^{2/3}+2\sqrt[3]{-1}3^{2/3}\right)-\sqrt[3]{6}\left(9+\sqrt[3]{-3}2^{2/3}\right)x}{104976\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)} - \frac{27\ 2^{2/}}{94478}$$

$$= -\frac{27\left((-2)^{2/3}+2\sqrt[3]{-1}3^{2/3}\right)-\sqrt[3]{6}\left(9+\sqrt[3]{-3}2^{2/3}\right)x}{104976\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)} - \frac{27\ 2^{2/}}{94478}$$

$$= -\frac{27\left((-2)^{2/3}+2\sqrt[3]{-1}3^{2/3}\right)-\sqrt[3]{6}\left(9+\sqrt[3]{-3}2^{2/3}\right)x}{104976\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)} - \frac{27\ 2^{2/}}{94478}$$

Mathematica [C] time = 0.0340482, size = 167, normalized size = 0.17

$$\frac{-9x^5+8x^4-216x^3-2724x^2+324x-7884}{7383312\left(x^6+18x^4+324x^3+108x^2+216\right)} - \frac{\text{RootSum}\left[\#1^6+18\#1^4+324\#1^3+108\#1^2+216\&,\frac{9\#1^4\log(x-\#1)-16\#1^3\log}{}\right.}{44299872}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

```
[Out] (-7884 + 324*x - 2724*x^2 - 216*x^3 + 8*x^4 - 9*x^5)/(7383312*(216 + 108*x^
2 + 324*x^3 + 18*x^4 + x^6)) - RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4
+ #1^6 & , (324*Log[x - #1] + 2436*Log[x - #1]*#1 + 324*Log[x - #1]*#1^2 -
16*Log[x - #1]*#1^3 + 9*Log[x - #1]*#1^4)/(36*#1 + 162*#1^2 + 12*#1^3 + #1^
5) & ]/44299872
```

Maple [C] time = 0.009, size = 122, normalized size = 0.1

$$\frac{1}{x^6+18x^4+324x^3+108x^2+216}\left(-\frac{x^5}{820368}+\frac{x^4}{922914}-\frac{x^3}{34182}-\frac{227x^2}{615276}+\frac{x}{22788}-\frac{73}{68364}\right)+\frac{1}{44299872}\ _{\_R=RootC}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x)
```

```
[Out] (-1/820368*x^5+1/922914*x^4-1/34182*x^3-227/615276*x^2+1/22788*x-73/68364)/
(x^6+18*x^4+324*x^3+108*x^2+216)+1/44299872*sum((-9*_R^4+16*_R^3-324*_R^2-2
436*_R-324)/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(x-_R),_R=RootOf(_Z^6+18*_Z^4+3
24*_Z^3+108*_Z^2+216))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{9x^5-8x^4+216x^3+2724x^2-324x+7884}{7383312\left(x^6+18x^4+324x^3+108x^2+216\right)}-\frac{1}{7383312}\int\frac{9x^4-16x^3+324x^2+2436x+324}{x^6+18x^4+324x^3+108x^2+216}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="maxima")
```

```
[Out] -1/7383312*(9*x^5 - 8*x^4 + 216*x^3 + 2724*x^2 - 324*x + 7884)/(x^6 + 18*x^
4 + 324*x^3 + 108*x^2 + 216) - 1/7383312*integrate((9*x^4 - 16*x^3 + 324*x^
2 + 2436*x + 324)/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 0.362038, size = 112, normalized size = 0.11

$$\operatorname{RootSum}\left(865859739762077843792979253893356556062923 1616t^6 + 1090680958717701682488386456125 44t^4 - 49\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**6+18*x**4+324*x**3+108*x**2+216)**2,x)
```

```
[Out] RootSum(8658597397620778437929792538933565560629231616*_t**6 + 109068095871
770168248838645612544*_t**4 - 49265570759336691571349913 6*_t**3 + 403783317
45144603648*_t**2 - 695635011360*_t + 4513, Lambda(_t, _t*log(1014425315618
04181113161287039859349851881619653631712165888*_t**5/3569006970707929484 75
845 - 14979655008235933511270943497197508896705021005004 8*_t**4/35690069707
0792948475845 + 12224097544582728185058987777686707836172369 92*_t**3/356900
697070792948475845 - 57750555242515957230229019385582614538 24*_t**2/3569006
97070792948475845 + 96165242200260265765603930470432*_t/7138013941415858969
5169 + x - 17059152341129698120545584/107070209121237884542753 5))) - (9*x**
5 - 8*x**4 + 216*x**3 + 2724*x**2 - 324*x + 7884)/(7383312*x**6 + 132899616
*x**4 + 2392193088*x**3 + 797397696*x**2 + 1594795392)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(x^6 + 18\,x^4 + 324\,x^3 + 108\,x^2 + 216\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)^2, x)
```

3.158 $\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{c+dx} \, dx$

Optimal. Leaf size=25

$$a^2x + \frac{2}{3}abx^3 + \frac{b^2x^5}{5}$$

```
[Out] a^2*x + (2*a*b*x^3)/3 + (b^2*x^5)/5
```

Rubi [A] time = 0.0553536, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 52, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.019, Rules used = {1586}

$$a^2x + \frac{2}{3}abx^3 + \frac{b^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*x^5)/(
c + d*x),x]
```

```
[Out] a^2*x + (2*a*b*x^3)/3 + (b^2*x^5)/5
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a^2c + a^2dx + 2abcx^2 + 2abdx^3 + b^2cx^4 + b^2dx^5}{c + dx} \, dx &= \int \left(a^2 + 2abx^2 + b^2x^4\right) dx \\ &= a^2x + \frac{2}{3}abx^3 + \frac{b^2x^5}{5}\end{aligned}$$

Mathematica [A] time = 0.0023163, size = 25, normalized size = 1.

$$a^2x + \frac{2}{3}abx^3 + \frac{b^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*
x^5)/(c + d*x),x]
```

```
[Out] a^2*x + (2*a*b*x^3)/3 + (b^2*x^5)/5
```

Maple [A] time = 0., size = 22, normalized size = 0.9

$$xa^2 + \frac{2\,abx^3}{3} + \frac{b^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(d*x+c),x)
```

```
[Out] x*a^2+2/3*a*b*x^3+1/5*b^2*x^5
```

Maxima [A] time = 1.09901, size = 28, normalized size = 1.12

$$\frac{1}{5}\,b^2x^5 + \frac{2}{3}\,abx^3 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(d*x+
c),x, algorithm="maxima")
```

```
[Out] 1/5*b^2*x^5 + 2/3*a*b*x^3 + a^2*x
```

Fricas [A] time = 1.25756, size = 47, normalized size = 1.88

$$\frac{1}{5}\,b^2x^5 + \frac{2}{3}\,abx^3 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(d*x+
c),x, algorithm="fricas")
```

```
[Out] 1/5*b^2*x^5 + 2/3*a*b*x^3 + a^2*x
```

Sympy [A] time = 0.081872, size = 22, normalized size = 0.88

$$a^2x + \frac{2abx^3}{3} + \frac{b^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*d*x**5+b**2*c*x**4+2*a*b*d*x**3+2*a*b*c*x**2+a**2*d*x+a**2*
c)/(d*x+c),x)
```

```
[Out] a**2*x + 2*a*b*x**3/3 + b**2*x**5/5
```

Giac [A] time = 1.19591, size = 28, normalized size = 1.12

$$\frac{1}{5}\,b^2x^5 + \frac{2}{3}\,abx^3 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(d*x+
c),x, algorithm="giac")
```

```
[Out] 1/5*b^2*x^5 + 2/3*a*b*x^3 + a^2*x
```

3.159 $\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{(c+dx)^2}\,dx$

Optimal. Leaf size=94

$$\frac{bx^2\left(2ad^2+bc^2\right)}{2d^3}-\frac{bcx\left(2ad^2+bc^2\right)}{d^4}+\frac{\left(ad^2+bc^2\right)^2\log(c+dx)}{d^5}-\frac{b^2cx^3}{3d^2}+\frac{b^2x^4}{4d}$$

```
[Out] -((b*c*(b*c^2 + 2*a*d^2)*x)/d^4) + (b*(b*c^2 + 2*a*d^2)*x^2)/(2*d^3) - (b^2
*c*x^3)/(3*d^2) + (b^2*x^4)/(4*d) + ((b*c^2 + a*d^2)^2*Log[c + d*x])/d^5
```

Rubi [A] time = 0.127105, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 52, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.058, Rules used = {1586, 28, 697}

$$\frac{bx^2\left(2ad^2+bc^2\right)}{2d^3}-\frac{bcx\left(2ad^2+bc^2\right)}{d^4}+\frac{\left(ad^2+bc^2\right)^2\log(c+dx)}{d^5}-\frac{b^2cx^3}{3d^2}+\frac{b^2x^4}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*x^5)/(
c + d*x)^2,x]
```

```
[Out] -((b*c*(b*c^2 + 2*a*d^2)*x)/d^4) + (b*(b*c^2 + 2*a*d^2)*x^2)/(2*d^3) - (b^2
*c*x^3)/(3*d^2) + (b^2*x^4)/(4*d) + ((b*c^2 + a*d^2)^2*Log[c + d*x])/d^5
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
```

```
x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{a^2c + a^2dx + 2abcx^2 + 2abdx^3 + b^2cx^4 + b^2dx^5}{(c+dx)^2}\,dx = \int \frac{a^2 + 2abx^2 + b^2x^4}{c+dx}\,dx$$

$$= \frac{\int \frac{\left(ab+b^2x^2\right)^2}{c+dx}\,dx}{b^2}$$

$$= \frac{\int \left(-\frac{b^3c\left(bc^2+2ad^2\right)}{d^4} + \frac{b^3\left(bc^2+2ad^2\right)x}{d^3} - \frac{b^4cx^2}{d^2} + \frac{b^4x^3}{d} + \frac{b^2\left(bc^2+ad^2\right)^2}{d^4(c+dx)}\right) dx}{b^2}$$

$$= -\frac{bc\left(bc^2+2ad^2\right)x}{d^4} + \frac{b\left(bc^2+2ad^2\right)x^2}{2d^3} - \frac{b^2cx^3}{3d^2} + \frac{b^2x^4}{4d} + \frac{\left(bc^2\right.}{}$$

Mathematica [A] time = 0.0394355, size = 79, normalized size = 0.84

$$\frac{bdx\left(12ad^2(dx-2c) + b\left(6c^2dx - 12c^3 - 4cd^2x^2 + 3d^3x^3\right)\right) + 12\left(ad^2+bc^2\right)^2\log(c+dx)}{12d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*
x^5)/(c + d*x)^2,x]
```

```
[Out] (b*d*x*(12*a*d^2*(-2*c + d*x) + b*(-12*c^3 + 6*c^2*d*x - 4*c*d^2*x^2 + 3*d^
3*x^3)) + 12*(b*c^2 + a*d^2)^2*Log[c + d*x])/(12*d^5)
```

Maple [A] time = 0.004, size = 114, normalized size = 1.2

$$\frac{b^2x^4}{4\,d} - \frac{b^2cx^3}{3\,d^2} + \frac{bx^2a}{d} + \frac{b^2x^2c^2}{2\,d^3} - 2\,\frac{abcx}{d^2} - \frac{b^2c^3x}{d^4} + \frac{\ln(dx+c)\,a^2}{d} + 2\,\frac{\ln(dx+c)\,abc^2}{d^3} + \frac{\ln(dx+c)\,b^2c^4}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(d*x+c)^2,x
)
```

```
[Out] 1/4*b^2*x^4/d-1/3*b^2*c*x^3/d^2+b/d*x^2*a+1/2*b^2/d^3*x^2*c^2-2*b/d^2*a*c*x
-b^2/d^4*c^3*x+1/d*ln(d*x+c)*a^2+2/d^3*ln(d*x+c)*a*b*c^2+1/d^5*ln(d*x+c)*b^
2*c^4
```

Maxima [A] time = 1.0549, size = 142, normalized size = 1.51

$$\frac{3\,b^2d^3x^4 - 4\,b^2cd^2x^3 + 6\left(b^2c^2d + 2\,abd^3\right)x^2 - 12\left(b^2c^3 + 2\,abcd^2\right)x}{12\,d^4} + \frac{\left(b^2c^4 + 2\,abc^2d^2 + a^2d^4\right)\log\left(dx + c\right)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(d*x+
c)^2,x, algorithm="maxima")
```

```
[Out] 1/12*(3*b^2*d^3*x^4 - 4*b^2*c*d^2*x^3 + 6*(b^2*c^2*d + 2*a*b*d^3)*x^2 - 12*
(b^2*c^3 + 2*a*b*c*d^2)*x)/d^4 + (b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)*log(d*
x + c)/d^5
```

Fricas [A] time = 1.24347, size = 223, normalized size = 2.37

$$\frac{3\,b^2d^4x^4 - 4\,b^2cd^3x^3 + 6\left(b^2c^2d^2 + 2\,abd^4\right)x^2 - 12\left(b^2c^3d + 2\,abcd^3\right)x + 12\left(b^2c^4 + 2\,abc^2d^2 + a^2d^4\right)\log\left(dx + c\right)}{12\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(d*x+
c)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^2*d^4*x^4 - 4*b^2*c*d^3*x^3 + 6*(b^2*c^2*d^2 + 2*a*b*d^4)*x^2 - 1
2*(b^2*c^3*d + 2*a*b*c*d^3)*x + 12*(b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)*log(
d*x + c))/d^5
```

Sympy [A] time = 0.449617, size = 90, normalized size = 0.96

$$-\frac{b^2cx^3}{3d^2} + \frac{b^2x^4}{4d} + \frac{x^2\left(2abd^2 + b^2c^2\right)}{2d^3} - \frac{x\left(2abcd^2 + b^2c^3\right)}{d^4} + \frac{\left(ad^2 + bc^2\right)^2\log\left(c + dx\right)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*d*x**5+b**2*c*x**4+2*a*b*d*x**3+2*a*b*c*x**2+a**2*d*x+a**2*
c)/(d*x+c)**2,x)
```

```
[Out] -b**2*c*x**3/(3*d**2) + b**2*x**4/(4*d) + x**2*(2*a*b*d**2 + b**2*c**2)/(2*
d**3) - x*(2*a*b*c*d**2 + b**2*c**3)/d**4 + (a*d**2 + b*c**2)**2*log(c + d*
x)/d**5
```

Giac [B] time = 1.48736, size = 493, normalized size = 5.24

$$-\frac{1}{12}\,b^2d\left(\frac{(dx+c)^4\left(\frac{20\,c}{dx+c}-\frac{60\,c^2}{(dx+c)^2}+\frac{120\,c^3}{(dx+c)^3}-3\right)}{d^6}+\frac{60\,c^4\log\left(\frac{|dx+c|}{(dx+c)^2|d|}\right)}{d^6}-\frac{12\,c^5}{(dx+c)d^6}\right)-\frac{1}{3}\,b^2c\left(\frac{(dx+c)^3\left(\frac{6\,c}{dx+c}-\frac{18\,c^2}{(dx+c)^2}-\right.}{d^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(d*x+
c)^2,x, algorithm="giac")
```

```
[Out] -1/12*b^2*d*((d*x + c)^4*(20*c/(d*x + c) - 60*c^2/(d*x + c)^2 + 120*c^3/(d*
x + c)^3 - 3)/d^6 + 60*c^4*log(abs(d*x + c)/((d*x + c)^2*abs(d)))/d^6 - 12*
c^5/((d*x + c)*d^6)) - 1/3*b^2*c*((d*x + c)^3*(6*c/(d*x + c) - 18*c^2/(d*x
+ c)^2 - 1)/d^5 - 12*c^3*log(abs(d*x + c)/((d*x + c)^2*abs(d)))/d^5 + 3*c^4
/((d*x + c)*d^5)) - a*b*d*((d*x + c)^2*(6*c/(d*x + c) - 1)/d^4 + 6*c^2*log(
abs(d*x + c)/((d*x + c)^2*abs(d)))/d^4 - 2*c^3/((d*x + c)*d^4)) + 2*a*b*c*(
2*c*log(abs(d*x + c)/((d*x + c)^2*abs(d)))/d^3 + (d*x + c)/d^3 - c^2/((d*x
+ c)*d^3)) - a^2*(log(abs(d*x + c)/((d*x + c)^2*abs(d)))/d - c/((d*x + c)*d
)) - a^2*c/((d*x + c)*d)
```

3.160 $\int (b+2cx)\left(bx+cx^2\right)^{13}\,dx$

Optimal. Leaf size=15

$$\frac{1}{14}\left(bx+cx^2\right)^{14}$$

[Out] (b*x + c*x^2)^14/14

Rubi [A] time = 0.0181078, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {629}

$$\frac{1}{14}\left(bx+cx^2\right)^{14}$$

Antiderivative was successfully verified.

[In] Int[(b + 2*c*x)*(b*x + c*x^2)^13,x]

[Out] (b*x + c*x^2)^14/14

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int (b+2cx)\left(bx+cx^2\right)^{13}\,dx=\frac{1}{14}\left(bx+cx^2\right)^{14}$$

Mathematica [B] time = 0.0062455, size = 172, normalized size = 11.47

$$\frac{13}{2}b^2c^{12}x^{26}+26b^3c^{11}x^{25}+\frac{143}{2}b^4c^{10}x^{24}+143b^5c^9x^{23}+\frac{429}{2}b^6c^8x^{22}+\frac{1716}{7}b^7c^7x^{21}+\frac{429}{2}b^8c^6x^{20}+143b^9c^5x^{19}+\frac{143}{2}b^{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x)*(b*x + c*x^2)^13,x]
```

```
[Out] (b^14*x^14)/14 + b^13*c*x^15 + (13*b^12*c^2*x^16)/2 + 26*b^11*c^3*x^17 + (1
43*b^10*c^4*x^18)/2 + 143*b^9*c^5*x^19 + (429*b^8*c^6*x^20)/2 + (1716*b^7*c
^7*x^21)/7 + (429*b^6*c^8*x^22)/2 + 143*b^5*c^9*x^23 + (143*b^4*c^10*x^24)/
2 + 26*b^3*c^11*x^25 + (13*b^2*c^12*x^26)/2 + b*c^13*x^27 + (c^14*x^28)/14
```

Maple [B] time = 0.001, size = 155, normalized size = 10.3

$$\frac{c^{14}x^{28}}{14}+bc^{13}x^{27}+\frac{13\,b^2c^{12}x^{26}}{2}+26\,b^3c^{11}x^{25}+\frac{143\,b^4c^{10}x^{24}}{2}+143\,b^5c^9x^{23}+\frac{429\,b^6c^8x^{22}}{2}+\frac{1716\,b^7c^7x^{21}}{7}+\frac{429\,b^8c^6x^{20}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x+b)*(c*x^2+b*x)^13,x)
```

```
[Out] 1/14*c^14*x^28+b*c^13*x^27+13/2*b^2*c^12*x^26+26*b^3*c^11*x^25+143/2*b^4*c^
10*x^24+143*b^5*c^9*x^23+429/2*b^6*c^8*x^22+1716/7*b^7*c^7*x^21+429/2*b^8*c
^6*x^20+143*b^9*c^5*x^19+143/2*b^10*c^4*x^18+26*b^11*c^3*x^17+13/2*b^12*c^2
*x^16+b^13*c*x^15+1/14*b^14*x^14
```

Maxima [A] time = 0.98839, size = 18, normalized size = 1.2

$$\frac{1}{14}\left(cx^2+bx\right)^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x)^13,x, algorithm="maxima")
```

```
[Out] 1/14*(c*x^2 + b*x)^14
```

Fricas [B] time = 1.21636, size = 387, normalized size = 25.8

$$\frac{1}{14}x^{28}c^{14}+x^{27}c^{13}b+\frac{13}{2}x^{26}c^{12}b^2+26x^{25}c^{11}b^3+\frac{143}{2}x^{24}c^{10}b^4+143x^{23}c^9b^5+\frac{429}{2}x^{22}c^8b^6+\frac{1716}{7}x^{21}c^7b^7+\frac{429}{2}x^{20}c^6b^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x)^13,x, algorithm="fricas")
```

```
[Out] 1/14*x^28*c^14 + x^27*c^13*b + 13/2*x^26*c^12*b^2 + 26*x^25*c^11*b^3 + 143/
2*x^24*c^10*b^4 + 143*x^23*c^9*b^5 + 429/2*x^22*c^8*b^6 + 1716/7*x^21*c^7*b
^7 + 429/2*x^20*c^6*b^8 + 143*x^19*c^5*b^9 + 143/2*x^18*c^4*b^10 + 26*x^17*
c^3*b^11 + 13/2*x^16*c^2*b^12 + x^15*c*b^13 + 1/14*x^14*b^14
```

Sympy [B] time = 0.115511, size = 175, normalized size = 11.67

$$\frac{b^{14}x^{14}}{14} + b^{13}cx^{15} + \frac{13b^{12}c^2x^{16}}{2} + 26b^{11}c^3x^{17} + \frac{143b^{10}c^4x^{18}}{2} + 143b^9c^5x^{19} + \frac{429b^8c^6x^{20}}{2} + \frac{1716b^7c^7x^{21}}{7} + \frac{429b^6c^8x^{22}}{2} + 1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x**2+b*x)**13,x)
```

```
[Out] b**14*x**14/14 + b**13*c*x**15 + 13*b**12*c**2*x**16/2 + 26*b**11*c**3*x**1
7 + 143*b**10*c**4*x**18/2 + 143*b**9*c**5*x**19 + 429*b**8*c**6*x**20/2 +
1716*b**7*c**7*x**21/7 + 429*b**6*c**8*x**22/2 + 143*b**5*c**9*x**23 + 143*
b**4*c**10*x**24/2 + 26*b**3*c**11*x**25 + 13*b**2*c**12*x**26/2 + b*c**13*
x**27 + c**14*x**28/14
```

Giac [B] time = 1.16984, size = 208, normalized size = 13.87

$$\frac{1}{14}c^{14}x^{28} + bc^{13}x^{27} + \frac{13}{2}b^2c^{12}x^{26} + 26\,b^3c^{11}x^{25} + \frac{143}{2}b^4c^{10}x^{24} + 143\,b^5c^9x^{23} + \frac{429}{2}b^6c^8x^{22} + \frac{1716}{7}b^7c^7x^{21} + \frac{429}{2}b^8c^6x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x)^13,x, algorithm="giac")
```

```
[Out] 1/14*c^14*x^28 + b*c^13*x^27 + 13/2*b^2*c^12*x^26 + 26*b^3*c^11*x^25 + 143/
2*b^4*c^10*x^24 + 143*b^5*c^9*x^23 + 429/2*b^6*c^8*x^22 + 1716/7*b^7*c^7*x^
21 + 429/2*b^8*c^6*x^20 + 143*b^9*c^5*x^19 + 143/2*b^10*c^4*x^18 + 26*b^11*
c^3*x^17 + 13/2*b^12*c^2*x^16 + b^13*c*x^15 + 1/14*b^14*x^14
```

3.161 $\int x^{14} \left(b+2 c x^2\right) \left(b x+c x^3\right)^{13} \, dx$

Optimal. Leaf size=16

$$\frac{1}{28} x^{28} \left(b+c x^2\right)^{14}$$

```
[Out] (x^28*(b + c*x^2)^14)/28
```

Rubi [A] time = 0.0548421, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {1584, 446, 74}

$$\frac{1}{28} x^{28} \left(b+c x^2\right)^{14}$$

Antiderivative was successfully verified.

```
[In] Int[x^14*(b + 2*c*x^2)*(b*x + c*x^3)^13,x]
```

```
[Out] (x^28*(b + c*x^2)^14)/28
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rubi steps

$$\int x^{14}\left(b+2cx^2\right)\left(bx+cx^3\right)^{13}\,dx=\int x^{27}\left(b+cx^2\right)^{13}\left(b+2cx^2\right)\,dx$$
$$=\frac{1}{2}\operatorname{Subst}\left(\int x^{13}(b+cx)^{13}(b+2cx)\,dx,x,x^2\right)$$
$$=\frac{1}{28}x^{28}\left(b+cx^2\right)^{14}$$

Mathematica [B] time = 0.0056777, size = 182, normalized size = 11.38

$$\frac{13}{4}b^2c^{12}x^{52}+13b^3c^{11}x^{50}+\frac{143}{4}b^4c^{10}x^{48}+\frac{143}{2}b^5c^9x^{46}+\frac{429}{4}b^6c^8x^{44}+\frac{858}{7}b^7c^7x^{42}+\frac{429}{4}b^8c^6x^{40}+\frac{143}{2}b^9c^5x^{38}+\frac{143}{4}b^{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^14*(b + 2*c*x^2)*(b*x + c*x^3)^13,x]
```

```
[Out] (b^14*x^28)/28 + (b^13*c*x^30)/2 + (13*b^12*c^2*x^32)/4 + 13*b^11*c^3*x^34
+ (143*b^10*c^4*x^36)/4 + (143*b^9*c^5*x^38)/2 + (429*b^8*c^6*x^40)/4 + (85
8*b^7*c^7*x^42)/7 + (429*b^6*c^8*x^44)/4 + (143*b^5*c^9*x^46)/2 + (143*b^4*
c^10*x^48)/4 + 13*b^3*c^11*x^50 + (13*b^2*c^12*x^52)/4 + (b*c^13*x^54)/2 +
(c^14*x^56)/28
```

Maple [B] time = 0.002, size = 157, normalized size = 9.8

$$\frac{c^{14}x^{56}}{28}+\frac{bc^{13}x^{54}}{2}+\frac{13\,b^2c^{12}x^{52}}{4}+13\,b^3c^{11}x^{50}+\frac{143\,b^4c^{10}x^{48}}{4}+\frac{143\,b^5c^9x^{46}}{2}+\frac{429\,b^6c^8x^{44}}{4}+\frac{858\,b^7c^7x^{42}}{7}+\frac{429\,b^8c^6x^{40}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14*(2*c*x^2+b)*(c*x^3+b*x)^13,x)
```

```
[Out] 1/28*c^14*x^56+1/2*b*c^13*x^54+13/4*b^2*c^12*x^52+13*b^3*c^11*x^50+143/4*b^
4*c^10*x^48+143/2*b^5*c^9*x^46+429/4*b^6*c^8*x^44+858/7*b^7*c^7*x^42+429/4*
b^8*c^6*x^40+143/2*b^9*c^5*x^38+143/4*b^10*c^4*x^36+13*b^11*c^3*x^34+13/4*b
^12*c^2*x^32+1/2*b^13*c*x^30+1/28*b^14*x^28
```

Maxima [B] time = 1.02844, size = 211, normalized size = 13.19

$$\frac{1}{28}\,c^{14}x^{56} + \frac{1}{2}\,bc^{13}x^{54} + \frac{13}{4}\,b^2c^{12}x^{52} + 13\,b^3c^{11}x^{50} + \frac{143}{4}\,b^4c^{10}x^{48} + \frac{143}{2}\,b^5c^9x^{46} + \frac{429}{4}\,b^6c^8x^{44} + \frac{858}{7}\,b^7c^7x^{42} + \frac{429}{4}\,b^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14*(2*c*x^2+b)*(c*x^3+b*x)^13,x, algorithm="maxima")
```

```
[Out] 1/28*c^14*x^56 + 1/2*b*c^13*x^54 + 13/4*b^2*c^12*x^52 + 13*b^3*c^11*x^50 +
143/4*b^4*c^10*x^48 + 143/2*b^5*c^9*x^46 + 429/4*b^6*c^8*x^44 + 858/7*b^7*c
^7*x^42 + 429/4*b^8*c^6*x^40 + 143/2*b^9*c^5*x^38 + 143/4*b^10*c^4*x^36 + 1
3*b^11*c^3*x^34 + 13/4*b^12*c^2*x^32 + 1/2*b^13*c*x^30 + 1/28*b^14*x^28
```

Fricas [B] time = 1.09083, size = 402, normalized size = 25.12

$$\frac{1}{28}x^{56}c^{14} + \frac{1}{2}x^{54}c^{13}b + \frac{13}{4}x^{52}c^{12}b^2 + 13x^{50}c^{11}b^3 + \frac{143}{4}x^{48}c^{10}b^4 + \frac{143}{2}x^{46}c^9b^5 + \frac{429}{4}x^{44}c^8b^6 + \frac{858}{7}x^{42}c^7b^7 + \frac{429}{4}x^{40}c^6b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14*(2*c*x^2+b)*(c*x^3+b*x)^13,x, algorithm="fricas")
```

```
[Out] 1/28*x^56*c^14 + 1/2*x^54*c^13*b + 13/4*x^52*c^12*b^2 + 13*x^50*c^11*b^3 +
143/4*x^48*c^10*b^4 + 143/2*x^46*c^9*b^5 + 429/4*x^44*c^8*b^6 + 858/7*x^42*
c^7*b^7 + 429/4*x^40*c^6*b^8 + 143/2*x^38*c^5*b^9 + 143/4*x^36*c^4*b^10 + 1
3*x^34*c^3*b^11 + 13/4*x^32*c^2*b^12 + 1/2*x^30*c*b^13 + 1/28*x^28*b^14
```

Sympy [B] time = 0.121753, size = 182, normalized size = 11.38

$$\frac{b^{14}x^{28}}{28} + \frac{b^{13}cx^{30}}{2} + \frac{13b^{12}c^2x^{32}}{4} + 13b^{11}c^3x^{34} + \frac{143b^{10}c^4x^{36}}{4} + \frac{143b^9c^5x^{38}}{2} + \frac{429b^8c^6x^{40}}{4} + \frac{858b^7c^7x^{42}}{7} + \frac{429b^6c^8x^{44}}{4} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14*(2*c*x**2+b)*(c*x**3+b*x)**13,x)
```

```
[Out] b**14*x**28/28 + b**13*c*x**30/2 + 13*b**12*c**2*x**32/4 + 13*b**11*c**3*x*
*34 + 143*b**10*c**4*x**36/4 + 143*b**9*c**5*x**38/2 + 429*b**8*c**6*x**40/
4 + 858*b**7*c**7*x**42/7 + 429*b**6*c**8*x**44/4 + 143*b**5*c**9*x**46/2 +
```

```
 143*b**4*c**10*x**48/4 + 13*b**3*c**11*x**50 + 13*b**2*c**12*x**52/4 + b*c
**13*x**54/2 + c**14*x**56/28
```

Giac [B] time = 1.21142, size = 211, normalized size = 13.19

$$\frac{1}{28}\,c^{14}x^{56} + \frac{1}{2}\,bc^{13}x^{54} + \frac{13}{4}\,b^2c^{12}x^{52} + 13\,b^3c^{11}x^{50} + \frac{143}{4}\,b^4c^{10}x^{48} + \frac{143}{2}\,b^5c^9x^{46} + \frac{429}{4}\,b^6c^8x^{44} + \frac{858}{7}\,b^7c^7x^{42} + \frac{429}{4}\,b^8c^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14*(2*c*x^2+b)*(c*x^3+b*x)^13,x, algorithm="giac")
```

```
[Out] 1/28*c^14*x^56 + 1/2*b*c^13*x^54 + 13/4*b^2*c^12*x^52 + 13*b^3*c^11*x^50 +
143/4*b^4*c^10*x^48 + 143/2*b^5*c^9*x^46 + 429/4*b^6*c^8*x^44 + 858/7*b^7*c
^7*x^42 + 429/4*b^8*c^6*x^40 + 143/2*b^9*c^5*x^38 + 143/4*b^10*c^4*x^36 + 1
3*b^11*c^3*x^34 + 13/4*b^12*c^2*x^32 + 1/2*b^13*c*x^30 + 1/28*b^14*x^28
```

3.162 $\int x^{28} \left(b+2cx^3\right) \left(bx+cx^4\right)^{13} \, dx$

Optimal. Leaf size=16

$$\frac{1}{42}x^{42} \left(b+cx^3\right)^{14}$$

```
[Out] (x^42*(b + c*x^3)^14)/42
```

Rubi [A] time = 0.0497209, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {1584, 446, 74}

$$\frac{1}{42}x^{42} \left(b+cx^3\right)^{14}$$

Antiderivative was successfully verified.

```
[In] Int[x^28*(b + 2*c*x^3)*(b*x + c*x^4)^13,x]
```

```
[Out] (x^42*(b + c*x^3)^14)/42
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rubi steps

$$\int x^{28}\left(b+2cx^3\right)\left(bx+cx^4\right)^{13}\,dx = \int x^{41}\left(b+cx^3\right)^{13}\left(b+2cx^3\right)\,dx$$
$$= \frac{1}{3}\,\text{Subst}\left(\int x^{13}(b+cx)^{13}(b+2cx)\,dx, x, x^3\right)$$
$$= \frac{1}{42}x^{42}\left(b+cx^3\right)^{14}$$

Mathematica [B] time = 0.0060816, size = 186, normalized size = 11.62

$$\frac{13}{6}b^2c^{12}x^{78}+\frac{26}{3}b^3c^{11}x^{75}+\frac{143}{6}b^4c^{10}x^{72}+\frac{143}{3}b^5c^9x^{69}+\frac{143}{2}b^6c^8x^{66}+\frac{572}{7}b^7c^7x^{63}+\frac{143}{2}b^8c^6x^{60}+\frac{143}{3}b^9c^5x^{57}+\frac{143}{6}b^1$$

Antiderivative was successfully verified.

```
[In] Integrate[x^28*(b + 2*c*x^3)*(b*x + c*x^4)^13,x]
```

```
[Out] (b^14*x^42)/42 + (b^13*c*x^45)/3 + (13*b^12*c^2*x^48)/6 + (26*b^11*c^3*x^51
)/3 + (143*b^10*c^4*x^54)/6 + (143*b^9*c^5*x^57)/3 + (143*b^8*c^6*x^60)/2 +
 (572*b^7*c^7*x^63)/7 + (143*b^6*c^8*x^66)/2 + (143*b^5*c^9*x^69)/3 + (143*
b^4*c^10*x^72)/6 + (26*b^3*c^11*x^75)/3 + (13*b^2*c^12*x^78)/6 + (b*c^13*x^
81)/3 + (c^14*x^84)/42
```

Maple [B] time = 0.001, size = 157, normalized size = 9.8

$$\frac{c^{14}x^{84}}{42}+\frac{bc^{13}x^{81}}{3}+\frac{13\,b^2c^{12}x^{78}}{6}+\frac{26\,b^3c^{11}x^{75}}{3}+\frac{143\,b^4c^{10}x^{72}}{6}+\frac{143\,b^5c^9x^{69}}{3}+\frac{143\,b^6c^8x^{66}}{2}+\frac{572\,b^7c^7x^{63}}{7}+\frac{143\,b^8c^6x^{60}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^28*(2*c*x^3+b)*(c*x^4+b*x)^13,x)
```

```
[Out] 1/42*c^14*x^84+1/3*b*c^13*x^81+13/6*b^2*c^12*x^78+26/3*b^3*c^11*x^75+143/6*
b^4*c^10*x^72+143/3*b^5*c^9*x^69+143/2*b^6*c^8*x^66+572/7*b^7*c^7*x^63+143/
2*b^8*c^6*x^60+143/3*b^9*c^5*x^57+143/6*b^10*c^4*x^54+26/3*b^11*c^3*x^51+13
/6*b^12*c^2*x^48+1/3*b^13*c*x^45+1/42*b^14*x^42
```

Maxima [B] time = 1.0192, size = 211, normalized size = 13.19

$$\frac{1}{42}c^{14}x^{84} + \frac{1}{3}bc^{13}x^{81} + \frac{13}{6}b^2c^{12}x^{78} + \frac{26}{3}b^3c^{11}x^{75} + \frac{143}{6}b^4c^{10}x^{72} + \frac{143}{3}b^5c^9x^{69} + \frac{143}{2}b^6c^8x^{66} + \frac{572}{7}b^7c^7x^{63} + \frac{143}{2}b^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^28*(2*c*x^3+b)*(c*x^4+b*x)^13,x, algorithm="maxima")
```

```
[Out] 1/42*c^14*x^84 + 1/3*b*c^13*x^81 + 13/6*b^2*c^12*x^78 + 26/3*b^3*c^11*x^75
+ 143/6*b^4*c^10*x^72 + 143/3*b^5*c^9*x^69 + 143/2*b^6*c^8*x^66 + 572/7*b^7
*c^7*x^63 + 143/2*b^8*c^6*x^60 + 143/3*b^9*c^5*x^57 + 143/6*b^10*c^4*x^54 +
 26/3*b^11*c^3*x^51 + 13/6*b^12*c^2*x^48 + 1/3*b^13*c*x^45 + 1/42*b^14*x^42
```

Fricas [B] time = 1.18778, size = 408, normalized size = 25.5

$$\frac{1}{42}x^{84}c^{14} + \frac{1}{3}x^{81}c^{13}b + \frac{13}{6}x^{78}c^{12}b^2 + \frac{26}{3}x^{75}c^{11}b^3 + \frac{143}{6}x^{72}c^{10}b^4 + \frac{143}{3}x^{69}c^9b^5 + \frac{143}{2}x^{66}c^8b^6 + \frac{572}{7}x^{63}c^7b^7 + \frac{143}{2}x^{60}c^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^28*(2*c*x^3+b)*(c*x^4+b*x)^13,x, algorithm="fricas")
```

```
[Out] 1/42*x^84*c^14 + 1/3*x^81*c^13*b + 13/6*x^78*c^12*b^2 + 26/3*x^75*c^11*b^3
+ 143/6*x^72*c^10*b^4 + 143/3*x^69*c^9*b^5 + 143/2*x^66*c^8*b^6 + 572/7*x^6
3*c^7*b^7 + 143/2*x^60*c^6*b^8 + 143/3*x^57*c^5*b^9 + 143/6*x^54*c^4*b^10 +
 26/3*x^51*c^3*b^11 + 13/6*x^48*c^2*b^12 + 1/3*x^45*c*b^13 + 1/42*x^42*b^14
```

Sympy [B] time = 0.125615, size = 185, normalized size = 11.56

$$\frac{b^{14}x^{42}}{42} + \frac{b^{13}cx^{45}}{3} + \frac{13b^{12}c^2x^{48}}{6} + \frac{26b^{11}c^3x^{51}}{3} + \frac{143b^{10}c^4x^{54}}{6} + \frac{143b^9c^5x^{57}}{3} + \frac{143b^8c^6x^{60}}{2} + \frac{572b^7c^7x^{63}}{7} + \frac{143b^6c^8x^{66}}{2} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**28*(2*c*x**3+b)*(c*x**4+b*x)**13,x)
```

```
[Out] b**14*x**42/42 + b**13*c*x**45/3 + 13*b**12*c**2*x**48/6 + 26*b**11*c**3*x*
*51/3 + 143*b**10*c**4*x**54/6 + 143*b**9*c**5*x**57/3 + 143*b**8*c**6*x**6
0/2 + 572*b**7*c**7*x**63/7 + 143*b**6*c**8*x**66/2 + 143*b**5*c**9*x**69/3
```

```
+ 143*b**4*c**10*x**72/6 + 26*b**3*c**11*x**75/3 + 13*b**2*c**12*x**78/6 +
b*c**13*x**81/3 + c**14*x**84/42
```

Giac [B] time = 1.15298, size = 211, normalized size = 13.19

$$\frac{1}{42}c^{14}x^{84} + \frac{1}{3}bc^{13}x^{81} + \frac{13}{6}b^2c^{12}x^{78} + \frac{26}{3}b^3c^{11}x^{75} + \frac{143}{6}b^4c^{10}x^{72} + \frac{143}{3}b^5c^9x^{69} + \frac{143}{2}b^6c^8x^{66} + \frac{572}{7}b^7c^7x^{63} + \frac{143}{2}b^8c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^28*(2*c*x^3+b)*(c*x^4+b*x)^13,x, algorithm="giac")
```

```
[Out] 1/42*c^14*x^84 + 1/3*b*c^13*x^81 + 13/6*b^2*c^12*x^78 + 26/3*b^3*c^11*x^75
+ 143/6*b^4*c^10*x^72 + 143/3*b^5*c^9*x^69 + 143/2*b^6*c^8*x^66 + 572/7*b^7
*c^7*x^63 + 143/2*b^8*c^6*x^60 + 143/3*b^9*c^5*x^57 + 143/6*b^10*c^4*x^54 +
 26/3*b^11*c^3*x^51 + 13/6*b^12*c^2*x^48 + 1/3*b^13*c*x^45 + 1/42*b^14*x^42
```

3.163 $\int x^{14(-1+n)}(b+2cx^n)\left(bx+cx^{1+n}\right)^{13}\,dx$

Optimal. Leaf size=21

$$\frac{x^{14n}(b+cx^n)^{14}}{14n}$$

```
[Out] (x^(14*n)*(b + c*x^n)^14)/(14*n)
```

Rubi [A] time = 0.0313831, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1584, 446, 74}

$$\frac{x^{14n}(b+cx^n)^{14}}{14n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(14*(-1 + n))*(b + 2*c*x^n)*(b*x + c*x^(1 + n))^13,x]
```

```
[Out] (x^(14*n)*(b + c*x^n)^14)/(14*n)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rubi steps

$$\int x^{14(-1+n)}(b+2cx^n)\left(bx+cx^{1+n}\right)^{13}\,dx=\int x^{13+14(-1+n)}(b+cx^n)^{13}(b+2cx^n)\,dx$$
$$=\frac{\text{Subst}\left(\int x^{13}(b+cx)^{13}(b+2cx)\,dx,x,x^n\right)}{n}$$
$$=\frac{x^{14n}(b+cx^n)^{14}}{14n}$$

Mathematica [A] time = 0.115378, size = 21, normalized size = 1.

$$\frac{x^{14n}(b+cx^n)^{14}}{14n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(14*(-1 + n))*(b + 2*c*x^n)*(b*x + c*x^(1 + n))^13,x]
```

```
[Out] (x^(14*n)*(b + c*x^n)^14)/(14*n)
```

Maple [B] time = 0.038, size = 230, normalized size = 11.

$$\frac{c^{14}(x^n)^{28}}{14\,n}+\frac{bc^{13}(x^n)^{27}}{n}+\frac{13\,c^{12}(x^n)^{26}b^2}{2\,n}+26\,\frac{b^3c^{11}(x^n)^{25}}{n}+\frac{143\,c^{10}(x^n)^{24}b^4}{2\,n}+143\,\frac{b^5c^9(x^n)^{23}}{n}+\frac{429\,c^8(x^n)^{22}b^6}{2\,n}+171$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-14+14*n)*(b+2*c*x^n)*(b*x+c*x^(1+n))^13,x)
```

```
[Out] 1/14*c^14/n*(x^n)^28+b*c^13/n*(x^n)^27+13/2*c^12/n*(x^n)^26*b^2+26*b^3*c^11
/n*(x^n)^25+143/2*c^10/n*(x^n)^24*b^4+143*b^5*c^9/n*(x^n)^23+429/2*c^8/n*(x
^n)^22*b^6+1716/7*b^7*c^7/n*(x^n)^21+429/2*c^6/n*(x^n)^20*b^8+143*b^9*c^5/n
*(x^n)^19+143/2*c^4/n*(x^n)^18*b^10+26*b^11*c^3/n*(x^n)^17+13/2*c^2/n*(x^n)
^16*b^12+b^13*c/n*(x^n)^15+1/14/n*(x^n)^14*b^14
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-14+14*n)*(b+2*c*x^n)*(b*x+c*x^(1+n))^13,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.61473, size = 657, normalized size = 31.29

$$\frac{b^{14}x^{14}x^{14\,n+14}+14\,b^{13}cx^{13}x^{15\,n+15}+91\,b^{12}c^{2}x^{12}x^{16\,n+16}+364\,b^{11}c^{3}x^{11}x^{17\,n+17}+1001\,b^{10}c^{4}x^{10}x^{18\,n+18}+2002\,b^{9}c^{5}x^{9}x^{1}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-14+14*n)*(b+2*c*x^n)*(b*x+c*x^(1+n))^13,x, algorithm="fricas"
)
```

```
[Out] 1/14*(b^14*x^14*x^(14*n + 14) + 14*b^13*c*x^13*x^(15*n + 15) + 91*b^12*c^2*
x^12*x^(16*n + 16) + 364*b^11*c^3*x^11*x^(17*n + 17) + 1001*b^10*c^4*x^10*x
^(18*n + 18) + 2002*b^9*c^5*x^9*x^(19*n + 19) + 3003*b^8*c^6*x^8*x^(20*n +
20) + 3432*b^7*c^7*x^7*x^(21*n + 21) + 3003*b^6*c^8*x^6*x^(22*n + 22) + 200
2*b^5*c^9*x^5*x^(23*n + 23) + 1001*b^4*c^10*x^4*x^(24*n + 24) + 364*b^3*c^1
1*x^3*x^(25*n + 25) + 91*b^2*c^12*x^2*x^(26*n + 26) + 14*b*c^13*x*x^(27*n +
 27) + c^14*x^(28*n + 28))/(n*x^28)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-14+14*n)*(b+2*c*x**n)*(b*x+c*x**(1+n))**13,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.86753, size = 255, normalized size = 12.14

$$\frac{c^{14}x^{28\,n}+14\,bc^{13}x^{27\,n}+91\,b^{2}c^{12}x^{26\,n}+364\,b^{3}c^{11}x^{25\,n}+1001\,b^{4}c^{10}x^{24\,n}+2002\,b^{5}c^{9}x^{23\,n}+3003\,b^{6}c^{8}x^{22\,n}+3432\,b^{7}c^{7}x^{2}}{14\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-14+14*n)*(b+2*c*x^n)*(b*x+c*x^(1+n))^13,x, algorithm="giac")
```

```
[Out] 1/14*(c^14*x^(28*n) + 14*b*c^13*x^(27*n) + 91*b^2*c^12*x^(26*n) + 364*b^3*c
^11*x^(25*n) + 1001*b^4*c^10*x^(24*n) + 2002*b^5*c^9*x^(23*n) + 3003*b^6*c^
8*x^(22*n) + 3432*b^7*c^7*x^(21*n) + 3003*b^8*c^6*x^(20*n) + 2002*b^9*c^5*x
^(19*n) + 1001*b^10*c^4*x^(18*n) + 364*b^11*c^3*x^(17*n) + 91*b^12*c^2*x^(1
6*n) + 14*b^13*c*x^(15*n) + b^14*x^(14*n))/n
```

3.164 $\int \frac{b+2cx}{bx+cx^2} dx$

Optimal. Leaf size=10

$$\log \left(bx+cx^2\right)$$

```
[Out] Log[b*x + c*x^2]
```

Rubi [A] time = 0.0039648, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {628}

$$\log \left(bx+cx^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x)/(b*x + c*x^2),x]
```

```
[Out] Log[b*x + c*x^2]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{b+2cx}{bx+cx^2} \, dx=\log \left(bx+cx^2\right)$$

Mathematica [A] time = 0.0037198, size = 9, normalized size = 0.9

$$\log (b+cx)+\log (x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x)/(b*x + c*x^2),x]
```

```
[Out] Log[x] + Log[b + c*x]
```

Maple [A] time = 0., size = 9, normalized size = 0.9

$$\ln(x(cx+b))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x+b)/(c*x^2+b*x),x)
```

```
[Out] ln(x*(c*x+b))
```

Maxima [A] time = 1.01807, size = 14, normalized size = 1.4

$$\log\left(cx^2+bx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x),x, algorithm="maxima")
```

```
[Out] log(c*x^2 + b*x)
```

Fricas [A] time = 1.30286, size = 24, normalized size = 2.4

$$\log\left(cx^2+bx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x),x, algorithm="fricas")
```

```
[Out] log(c*x^2 + b*x)
```

Sympy [A] time = 0.284718, size = 8, normalized size = 0.8

$$\log\left(bx+cx^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x**2+b*x),x)
```

```
[Out] log(b*x + c*x**2)
```

Giac [A] time = 1.21233, size = 15, normalized size = 1.5

$$\log(|cx + b|) + \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x),x, algorithm="giac")
```

```
[Out] log(abs(c*x + b)) + log(abs(x))
```

3.165 $\int \frac{b+2cx^2}{bx+cx^3}\,dx$

Optimal. Leaf size=15

$$\frac{1}{2}\log\left(b+cx^2\right)+\log(x)$$

```
[Out] Log[x] + Log[b + c*x^2]/2
```

Rubi [A] time = 0.0264676, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {1593, 446, 72}

$$\frac{1}{2}\log\left(b+cx^2\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x^2)/(b*x + c*x^3),x]
```

```
[Out] Log[x] + Log[b + c*x^2]/2
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \frac{b+2cx^2}{bx+cx^3}\,dx = \int \frac{b+2cx^2}{x\left(b+cx^2\right)}\,dx$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \frac{b+2cx}{x(b+cx)}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{1}{x}+\frac{c}{b+cx}\right)dx, x, x^2\right)$$
$$= \log(x)+\frac{1}{2}\log\left(b+cx^2\right)$$

Mathematica [A] time = 0.0054783, size = 15, normalized size = 1.

$$\frac{1}{2}\log\left(b+cx^2\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x^2)/(b*x + c*x^3),x]
```

```
[Out] Log[x] + Log[b + c*x^2]/2
```

Maple [A] time = 0.004, size = 14, normalized size = 0.9

$$\ln(x)+\frac{\ln\left(cx^2+b\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x^2+b)/(c*x^3+b*x),x)
```

```
[Out] ln(x)+1/2*ln(c*x^2+b)
```

Maxima [A] time = 1.03648, size = 18, normalized size = 1.2

$$\frac{1}{2}\log\left(cx^2+b\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^2+b)/(c*x^3+b*x),x, algorithm="maxima")
```

```
[Out] 1/2*log(c*x^2 + b) + log(x)
```

Fricas [A] time = 1.2574, size = 39, normalized size = 2.6

$$\frac{1}{2}\log\left(cx^2+b\right)+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^2+b)/(c*x^3+b*x),x, algorithm="fricas")
```

```
[Out] 1/2*log(c*x^2 + b) + log(x)
```

Sympy [A] time = 0.321459, size = 12, normalized size = 0.8

$$\log\left(x\right)+\frac{\log\left(\frac{b}{c}+x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x**2+b)/(c*x**3+b*x),x)
```

```
[Out] log(x) + log(b/c + x**2)/2
```

Giac [A] time = 1.22189, size = 24, normalized size = 1.6

$$\frac{1}{2}\log\left(x^2\right)+\frac{1}{2}\log\left(\left|cx^2+b\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^2+b)/(c*x^3+b*x),x, algorithm="giac")
```

```
[Out] 1/2*log(x^2) + 1/2*log(abs(c*x^2 + b))
```

3.166 $\int \frac{b+2cx^3}{bx+cx^4}\,dx$

Optimal. Leaf size=15

$$\frac{1}{3}\log\left(b+cx^3\right)+\log(x)$$

```
[Out] Log[x] + Log[b + c*x^3]/3
```

Rubi [A] time = 0.0265704, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {1593, 446, 72}

$$\frac{1}{3}\log\left(b+cx^3\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x^3)/(b*x + c*x^4),x]
```

```
[Out] Log[x] + Log[b + c*x^3]/3
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{b+2cx^3}{bx+cx^4}\,dx &= \int \frac{b+2cx^3}{x\left(b+cx^3\right)}\,dx \\
&= \frac{1}{3}\,\text{Subst}\left(\int \frac{b+2cx}{x(b+cx)}\,dx, x, x^3\right) \\
&= \frac{1}{3}\,\text{Subst}\left(\int \left(\frac{1}{x}+\frac{c}{b+cx}\right)dx, x, x^3\right) \\
&= \log(x)+\frac{1}{3}\log\left(b+cx^3\right)
\end{aligned}
$$

Mathematica [A] time = 0.0058888, size = 15, normalized size = 1.

$$\frac{1}{3}\log\left(b+cx^3\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x^3)/(b*x + c*x^4),x]
```

```
[Out] Log[x] + Log[b + c*x^3]/3
```

Maple [A] time = 0.005, size = 14, normalized size = 0.9

$$\ln(x)+\frac{\ln\left(cx^3+b\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x^3+b)/(c*x^4+b*x),x)
```

```
[Out] ln(x)+1/3*ln(c*x^3+b)
```

Maxima [A] time = 1.04453, size = 18, normalized size = 1.2

$$\frac{1}{3}\log\left(cx^3+b\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^3+b)/(c*x^4+b*x),x, algorithm="maxima")
```

```
[Out] 1/3*log(c*x^3 + b) + log(x)
```

Fricas [A] time = 1.36152, size = 39, normalized size = 2.6

$$\frac{1}{3}\log\left(cx^3+b\right)+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^3+b)/(c*x^4+b*x),x, algorithm="fricas")
```

```
[Out] 1/3*log(c*x^3 + b) + log(x)
```

Sympy [A] time = 0.334277, size = 12, normalized size = 0.8

$$\log\left(x\right)+\frac{\log\left(\frac{b}{c}+x^3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x**3+b)/(c*x**4+b*x),x)
```

```
[Out] log(x) + log(b/c + x**3)/3
```

Giac [A] time = 1.19742, size = 20, normalized size = 1.33

$$\frac{1}{3}\log\left(\left|cx^3+b\right|\right)+\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^3+b)/(c*x^4+b*x),x, algorithm="giac")
```

```
[Out] 1/3*log(abs(c*x^3 + b)) + log(abs(x))
```

3.167 $\int \frac{b+2cx^n}{bx+cx^{1+n}}\,dx$

Optimal. Leaf size=15

$$\frac{\log(b+cx^n)}{n}+\log(x)$$

```
[Out] Log[x] + Log[b + c*x^n]/n
```

Rubi [A] time = 0.0253765, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1593, 446, 72}

$$\frac{\log(b+cx^n)}{n}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x^n)/(b*x + c*x^(1 + n)),x]
```

```
[Out] Log[x] + Log[b + c*x^n]/n
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \frac{b+2cx^n}{bx+cx^{1+n}}\,dx = \int \frac{b+2cx^n}{x(b+cx^n)}\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{b+2cx}{x(b+cx)}\,dx, x, x^n\right)}{n}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{x}+\frac{c}{b+cx}\right)dx, x, x^n\right)}{n}$$
$$= \log(x) + \frac{\log(b+cx^n)}{n}$$

Mathematica [A] time = 0.0117224, size = 15, normalized size = 1.

$$\frac{\log(b+cx^n)}{n} + \log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x^n)/(b*x + c*x^(1 + n)),x]
```

```
[Out] Log[x] + Log[b + c*x^n]/n
```

Maple [A] time = 0.013, size = 18, normalized size = 1.2

$$\ln(x) + \frac{\ln\left(c\mathrm{e}^{n\ln(x)}+b\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b+2*c*x^n)/(b*x+c*x^(1+n)),x)
```

```
[Out] ln(x)+1/n*ln(c*exp(n*ln(x))+b)
```

Maxima [B] time = 1.05882, size = 63, normalized size = 4.2

$$b\left(\frac{\log(x)}{b} - \frac{\log\left(\frac{cx^n+b}{c}\right)}{bn}\right) + \frac{2\log\left(\frac{cx^n+b}{c}\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+2*c*x^n)/(b*x+c*x^(1+n)),x, algorithm="maxima")
```

```
[Out] b*(log(x)/b - log((c*x^n + b)/c)/(b*n)) + 2*log((c*x^n + b)/c)/n
```

Fricas [A] time = 1.54709, size = 61, normalized size = 4.07

$$\frac{(n-1)\log(x)+\log\left(bx+cx^{n+1}\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+2*c*x^n)/(b*x+c*x^(1+n)),x, algorithm="fricas")
```

```
[Out] ((n - 1)*log(x) + log(b*x + c*x^(n + 1)))/n
```

Sympy [A] time = 1.41776, size = 29, normalized size = 1.93

$$\begin{cases} \log(x) & \text{for } c=0 \wedge n=0 \\ \frac{(b+2c)\log(x)}{b+c} & \text{for } n=0 \\ \log(x) & \text{for } c=0 \\ \log(x)+\frac{\log\left(\frac{b}{c}+x^n\right)}{n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+2*c*x**n)/(b*x+c*x**(1+n)),x)
```

```
[Out] Piecewise((log(x), Eq(c, 0) & Eq(n, 0)), ((b + 2*c)*log(x)/(b + c), Eq(n, 0
)), (log(x), Eq(c, 0)), (log(x) + log(b/c + x**n)/n, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2\,cx^n+b}{bx+cx^{n+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+2*c*x^n)/(b*x+c*x^(1+n)),x, algorithm="giac")
```

```
[Out] integrate((2*c*x^n + b)/(b*x + c*x^(n + 1)), x)
```

3.168 $\int \frac{b+2cx}{(bx+cx^2)^8} dx$

Optimal. Leaf size=15

$$-\frac{1}{7(bx+cx^2)^7}$$

```
[Out] -1/(7*(b*x + c*x^2)^7)
```

Rubi [A] time = 0.0039839, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {629}

$$-\frac{1}{7(bx+cx^2)^7}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x)/(b*x + c*x^2)^8,x]
```

```
[Out] -1/(7*(b*x + c*x^2)^7)
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{b+2cx}{(bx+cx^2)^8} dx = -\frac{1}{7(bx+cx^2)^7}$$

Mathematica [A] time = 0.021585, size = 14, normalized size = 0.93

$$-\frac{1}{7x^7(b+cx)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x)/(b*x + c*x^2)^8,x]
```

```
[Out] -1/(7*x^7*(b + c*x)^7)
```

Maple [B] time = 0., size = 177, normalized size = 11.8

$$-\frac{1}{7\,b^7x^7}-132\,\frac{c^6}{b^{13}x}+66\,\frac{c^5}{b^{12}x^2}-30\,\frac{c^4}{b^{11}x^3}+12\,\frac{c^3}{b^{10}x^4}-4\,\frac{c^2}{b^9x^5}+\frac{c}{b^8x^6}+132\,\frac{c^7}{b^{13}\,(cx+b)}+66\,\frac{c^7}{b^{12}\,(cx+b)^2}+30\,\frac{}{b^{11}\,(c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x+b)/(c*x^2+b*x)^8,x)
```

```
[Out] -1/7/b^7/x^7-132/b^13*c^6/x+66/b^12*c^5/x^2-30/b^11*c^4/x^3+12/b^10*c^3/x^4
-4/b^9*c^2/x^5+1/b^8*c/x^6+132/b^13*c^7/(c*x+b)+66/b^12*c^7/(c*x+b)^2+30/b^
11*c^7/(c*x+b)^3+12/b^10*c^7/(c*x+b)^4+4/b^9*c^7/(c*x+b)^5+c^7/b^8/(c*x+b)^
6+1/7*c^7/b^7/(c*x+b)^7
```

Maxima [A] time = 1.07506, size = 18, normalized size = 1.2

$$-\frac{1}{7\,\left(cx^2+bx\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x)^8,x, algorithm="maxima")
```

```
[Out] -1/7/(c*x^2 + b*x)^7
```

Fricas [B] time = 1.35239, size = 171, normalized size = 11.4

$$-\frac{1}{7\,\left(c^7x^{14}+7\,bc^6x^{13}+21\,b^2c^5x^{12}+35\,b^3c^4x^{11}+35\,b^4c^3x^{10}+21\,b^5c^2x^9+7\,b^6cx^8+b^7x^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x)^8,x, algorithm="fricas")
```

```
[Out] -1/7/(c^7*x^14 + 7*b*c^6*x^13 + 21*b^2*c^5*x^12 + 35*b^3*c^4*x^11 + 35*b^4*
c^3*x^10 + 21*b^5*c^2*x^9 + 7*b^6*c*x^8 + b^7*x^7)
```

Sympy [B] time = 3.97716, size = 87, normalized size = 5.8

$$-\frac{1}{7b^7x^7 + 49b^6cx^8 + 147b^5c^2x^9 + 245b^4c^3x^{10} + 245b^3c^4x^{11} + 147b^2c^5x^{12} + 49bc^6x^{13} + 7c^7x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x**2+b*x)**8,x)
```

```
[Out] -1/(7*b**7*x**7 + 49*b**6*c*x**8 + 147*b**5*c**2*x**9 + 245*b**4*c**3*x**10
 + 245*b**3*c**4*x**11 + 147*b**2*c**5*x**12 + 49*b*c**6*x**13 + 7*c**7*x**
14)
```

Giac [A] time = 1.23788, size = 18, normalized size = 1.2

$$-\frac{1}{7\left(cx^2 + bx\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x)^8,x, algorithm="giac")
```

```
[Out] -1/7/(c*x^2 + b*x)^7
```

3.169 $\int \frac{b+2cx^2}{x^7(bx+cx^3)^8} dx$

Optimal. Leaf size=16

$$-\frac{1}{14x^{14}\left(b+cx^2\right)^7}$$

```
[Out] -1/(14*x^14*(b + c*x^2)^7)
```

Rubi [A] time = 0.023114, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {1584, 446, 74}

$$-\frac{1}{14x^{14}\left(b+cx^2\right)^7}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x^2)/(x^7*(b*x + c*x^3)^8),x]
```

```
[Out] -1/(14*x^14*(b + c*x^2)^7)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
```

```
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{b+2cx^2}{x^7\left(bx+cx^3\right)^8}\,dx &= \int \frac{b+2cx^2}{x^{15}\left(b+cx^2\right)^8}\,dx\\ &= \frac{1}{2}\,\text{Subst}\left(\int \frac{b+2cx}{x^8(b+cx)^8}\,dx, x, x^2\right)\\ &= -\frac{1}{14x^{14}\left(b+cx^2\right)^7}\end{aligned}$$

Mathematica [A] time = 0.028989, size = 16, normalized size = 1.

$$-\frac{1}{14x^{14}\left(b+cx^2\right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x^2)/(x^7*(b*x + c*x^3)^8),x]
```

```
[Out] -1/(14*x^14*(b + c*x^2)^7)
```

Maple [B] time = 0.018, size = 197, normalized size = 12.3

$$-\frac{1}{14\,b^7x^{14}}-66\,\frac{c^6}{b^{13}x^2}+33\,\frac{c^5}{b^{12}x^4}-15\,\frac{c^4}{b^{11}x^6}+6\,\frac{c^3}{b^{10}x^8}-2\,\frac{c^2}{b^9x^{10}}+\frac{c}{2\,b^8x^{12}}-\frac{c^8}{2\,b^{13}}\left(-\frac{b^6}{7\,c\left(cx^2+b\right)^7}-132\,\frac{1}{c\left(cx^2+b\right)}-\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x^2+b)/x^7/(c*x^3+b*x)^8,x)
```

```
[Out] -1/14/b^7/x^14-66/b^13*c^6/x^2+33/b^12*c^5/x^4-15/b^11*c^4/x^6+6/b^10*c^3/x
^8-2/b^9*c^2/x^10+1/2/b^8*c/x^12-1/2*c^8/b^13*(-1/7/c*b^6/(c*x^2+b)^7-132/c
/(c*x^2+b)-4/c*b^4/(c*x^2+b)^5-12/c*b^3/(c*x^2+b)^4-30*b^2/c/(c*x^2+b)^3-66
/c*b/(c*x^2+b)^2-1/c*b^5/(c*x^2+b)^6)
```

Maxima [B] time = 1.10015, size = 109, normalized size = 6.81

$$-\frac{1}{14\left(c^{7}x^{28}+7\,bc^{6}x^{26}+21\,b^{2}c^{5}x^{24}+35\,b^{3}c^{4}x^{22}+35\,b^{4}c^{3}x^{20}+21\,b^{5}c^{2}x^{18}+7\,b^{6}cx^{16}+b^{7}x^{14}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^2+b)/x^7/(c*x^3+b*x)^8,x, algorithm="maxima")
```

```
[Out] -1/14/(c^7*x^28 + 7*b*c^6*x^26 + 21*b^2*c^5*x^24 + 35*b^3*c^4*x^22 + 35*b^4
*c^3*x^20 + 21*b^5*c^2*x^18 + 7*b^6*c*x^16 + b^7*x^14)
```

Fricas [B] time = 1.405, size = 177, normalized size = 11.06

$$-\frac{1}{14\left(c^{7}x^{28}+7\,bc^{6}x^{26}+21\,b^{2}c^{5}x^{24}+35\,b^{3}c^{4}x^{22}+35\,b^{4}c^{3}x^{20}+21\,b^{5}c^{2}x^{18}+7\,b^{6}cx^{16}+b^{7}x^{14}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^2+b)/x^7/(c*x^3+b*x)^8,x, algorithm="fricas")
```

```
[Out] -1/14/(c^7*x^28 + 7*b*c^6*x^26 + 21*b^2*c^5*x^24 + 35*b^3*c^4*x^22 + 35*b^4
*c^3*x^20 + 21*b^5*c^2*x^18 + 7*b^6*c*x^16 + b^7*x^14)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x**2+b)/x**7/(c*x**3+b*x)**8,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.3487, size = 20, normalized size = 1.25

$$-\frac{1}{14\left(cx^{4}+bx^{2}\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^2+b)/x^7/(c*x^3+b*x)^8,x, algorithm="giac")
```

```
[Out] -1/14/(c*x^4 + b*x^2)^7
```

3.170 $\int \frac{b+2cx^3}{x^{14}\left(bx+cx^4\right)^8}\,dx$

Optimal. Leaf size=16

$$-\frac{1}{21x^{21}\left(b+cx^3\right)^7}$$

[Out] -1/(21*x^21*(b + c*x^3)^7)

Rubi [A] time = 0.0232854, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {1584, 446, 74}

$$-\frac{1}{21x^{21}\left(b+cx^3\right)^7}$$

Antiderivative was successfully verified.

[In] Int[(b + 2*c*x^3)/(x^14*(b*x + c*x^4)^8),x]

[Out] -1/(21*x^21*(b + c*x^3)^7)

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
```

```
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{b+2cx^3}{x^{14}\left(bx+cx^4\right)^8}\,dx &= \int \frac{b+2cx^3}{x^{22}\left(b+cx^3\right)^8}\,dx\\ &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{b+2cx}{x^8(b+cx)^8}\,dx,x,x^3\right)\\ &= -\frac{1}{21x^{21}\left(b+cx^3\right)^7}\end{aligned}$$

Mathematica [A] time = 0.035783, size = 16, normalized size = 1.

$$-\frac{1}{21x^{21}\left(b+cx^3\right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x^3)/(x^14*(b*x + c*x^4)^8),x]
```

```
[Out] -1/(21*x^21*(b + c*x^3)^7)
```

Maple [B] time = 0.013, size = 197, normalized size = 12.3

$$-\frac{c^8}{3\,b^{13}}\left(-132\,\frac{1}{c\left(cx^3+b\right)}-\frac{b^5}{c\left(cx^3+b\right)^6}-4\,\frac{b^4}{c\left(cx^3+b\right)^5}-12\,\frac{b^3}{c\left(cx^3+b\right)^4}-30\,\frac{b^2}{c\left(cx^3+b\right)^3}-\frac{b^6}{7\,c\left(cx^3+b\right)^7}-66\,\frac{b}{c\left(cx^3+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x^3+b)/x^14/(c*x^4+b*x)^8,x)
```

```
[Out] -1/3*c^8/b^13*(-132/c/(c*x^3+b)-1/c*b^5/(c*x^3+b)^6-4/c*b^4/(c*x^3+b)^5-12/
c*b^3/(c*x^3+b)^4-30*b^2/c/(c*x^3+b)^3-1/7/c*b^6/(c*x^3+b)^7-66/c*b/(c*x^3+
b)^2)-1/21/b^7/x^21-44/b^13*c^6/x^3+22/b^12*c^5/x^6-10/b^11*c^4/x^9+4/b^10*
c^3/x^12-4/3/b^9*c^2/x^15+1/3/b^8*c/x^18
```

Maxima [B] time = 1.12978, size = 109, normalized size = 6.81

$$-\frac{1}{21\left(c^7x^{42}+7\,bc^6x^{39}+21\,b^2c^5x^{36}+35\,b^3c^4x^{33}+35\,b^4c^3x^{30}+21\,b^5c^2x^{27}+7\,b^6cx^{24}+b^7x^{21}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^3+b)/x^14/(c*x^4+b*x)^8,x, algorithm="maxima")
```

```
[Out] -1/21/(c^7*x^42 + 7*b*c^6*x^39 + 21*b^2*c^5*x^36 + 35*b^3*c^4*x^33 + 35*b^4
*c^3*x^30 + 21*b^5*c^2*x^27 + 7*b^6*c*x^24 + b^7*x^21)
```

Fricas [B] time = 1.31839, size = 177, normalized size = 11.06

$$-\frac{1}{21\left(c^7x^{42}+7\,bc^6x^{39}+21\,b^2c^5x^{36}+35\,b^3c^4x^{33}+35\,b^4c^3x^{30}+21\,b^5c^2x^{27}+7\,b^6cx^{24}+b^7x^{21}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^3+b)/x^14/(c*x^4+b*x)^8,x, algorithm="fricas")
```

```
[Out] -1/21/(c^7*x^42 + 7*b*c^6*x^39 + 21*b^2*c^5*x^36 + 35*b^3*c^4*x^33 + 35*b^4
*c^3*x^30 + 21*b^5*c^2*x^27 + 7*b^6*c*x^24 + b^7*x^21)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x**3+b)/x**14/(c*x**4+b*x)**8,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.43324, size = 20, normalized size = 1.25

$$-\frac{1}{21\left(cx^6+bx^3\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^3+b)/x^14/(c*x^4+b*x)^8,x, algorithm="giac")
```

```
[Out] -1/21/(c*x^6 + b*x^3)^7
```

3.171 $\int \frac{x^{-7(-1+n)}(b+2cx^n)}{\left(bx+cx^{1+n}\right)^8}\,dx$

Optimal. Leaf size=21

$$-\frac{x^{-7n}}{7n\,(b+cx^n)^7}$$

```
[Out] -1/(7*n*x^(7*n)*(b + c*x^n)^7)
```

Rubi [A] time = 0.0323436, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1584, 446, 74}

$$-\frac{x^{-7n}}{7n\,(b+cx^n)^7}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x^n)/(x^(7*(-1 + n))*(b*x + c*x^(1 + n))^8),x]
```

```
[Out] -1/(7*n*x^(7*n)*(b + c*x^n)^7)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
```

```
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rubi steps

$$\int \frac{x^{-7(-1+n)}(b+2cx^n)}{\left(bx+cx^{1+n}\right)^8}\,dx = \int \frac{x^{-8-7(-1+n)}(b+2cx^n)}{(b+cx^n)^8}\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{b+2cx}{x^8(b+cx)^8}\,dx, x, x^n\right)}{n}$$
$$= -\frac{x^{-7n}}{7n\,(b+cx^n)^7}$$

Mathematica [A] time = 0.172435, size = 21, normalized size = 1.

$$-\frac{x^{-7n}}{7n\,(b+cx^n)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x^n)/(x^(7*(-1 + n))*(b*x + c*x^(1 + n))^8),x]
```

```
[Out] -1/(7*n*x^(7*n)*(b + c*x^n)^7)
```

Maple [B] time = 0.049, size = 203, normalized size = 9.7

$$-132\,\frac{c^6}{b^{13}nx^n}+66\,\frac{c^5}{b^{12}n\,(x^n)^2}-30\,\frac{c^4}{b^{11}n\,(x^n)^3}+12\,\frac{c^3}{b^{10}n\,(x^n)^4}-4\,\frac{c^2}{b^9n\,(x^n)^5}+\frac{c}{b^8n\,(x^n)^6}-\frac{1}{7\,b^7n\,(x^n)^7}+\frac{c^7\left(924\,(x^n)^6\,c^6+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b+2*c*x^n)/(x^(-7+7*n))/(b*x+c*x^(1+n))^8,x)
```

```
[Out] -132/b^13*c^6/n/(x^n)+66/b^12*c^5/n/(x^n)^2-30/b^11*c^4/n/(x^n)^3+12/b^10*c
^3/n/(x^n)^4-4/b^9*c^2/n/(x^n)^5+1/b^8*c/n/(x^n)^6-1/7/b^7/n/(x^n)^7+1/7*c^
7*(924*(x^n)^6*c^6+6006*b*c^5*(x^n)^5+16380*b^2*c^4*(x^n)^4+24024*b^3*c^3*(
x^n)^3+20020*b^4*c^2*(x^n)^2+9009*b^5*c*x^n+1716*b^6)/b^13/n/(b+c*x^n)^7
```

Maxima [B] time = 1.23946, size = 826, normalized size = 39.33

$$-\frac{1}{105}\,b\left(\frac{360360\,c^{13}x^{13\,n}+2342340\,bc^{12}x^{12\,n}+6426420\,b^{2}c^{11}x^{11\,n}+9579570\,b^{3}c^{10}x^{10\,n}+8270262\,b^{4}c^{9}x^{9\,n}+4018014\,b}{b^{14}c^{7}nx^{14\,n}+7\,b^{15}c^{6}nx^{13\,n}+21\,b^{16}c^{5}nx^{12\,n}+35\,b^{17}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+2*c*x^n)/(x^(-7+7*n))/(b*x+c*x^(1+n))^8,x, algorithm="maxima")
```

```
[Out] -1/105*b*((360360*c^13*x^(13*n) + 2342340*b*c^12*x^(12*n) + 6426420*b^2*c^1
1*x^(11*n) + 9579570*b^3*c^10*x^(10*n) + 8270262*b^4*c^9*x^(9*n) + 4018014*
b^5*c^8*x^(8*n) + 934362*b^6*c^7*x^(7*n) + 45045*b^7*c^6*x^(6*n) - 5005*b^8
*c^5*x^(5*n) + 1001*b^9*c^4*x^(4*n) - 273*b^10*c^3*x^(3*n) + 91*b^11*c^2*x^
(2*n) - 35*b^12*c*x^n + 15*b^13)/(b^14*c^7*n*x^(14*n) + 7*b^15*c^6*n*x^(13*
n) + 21*b^16*c^5*n*x^(12*n) + 35*b^17*c^4*n*x^(11*n) + 35*b^18*c^3*n*x^(10*
n) + 21*b^19*c^2*n*x^(9*n) + 7*b^20*c*n*x^(8*n) + b^21*n*x^(7*n)) + 360360*
c^7*log(x)/b^15 - 360360*c^7*log((c*x^n + b)/c)/(b^15*n)) + 1/105*c*((36036
0*c^12*x^(12*n) + 2342340*b*c^11*x^(11*n) + 6426420*b^2*c^10*x^(10*n) + 957
9570*b^3*c^9*x^(9*n) + 8270262*b^4*c^8*x^(8*n) + 4018014*b^5*c^7*x^(7*n) +
934362*b^6*c^6*x^(6*n) + 45045*b^7*c^5*x^(5*n) - 5005*b^8*c^4*x^(4*n) + 100
1*b^9*c^3*x^(3*n) - 273*b^10*c^2*x^(2*n) + 91*b^11*c*x^n - 35*b^12)/(b^13*c
^7*n*x^(13*n) + 7*b^14*c^6*n*x^(12*n) + 21*b^15*c^5*n*x^(11*n) + 35*b^16*c^
4*n*x^(10*n) + 35*b^17*c^3*n*x^(9*n) + 21*b^18*c^2*n*x^(8*n) + 7*b^19*c*n*x
^(7*n) + b^20*n*x^(6*n)) + 360360*c^6*log(x)/b^14 - 360360*c^6*log((c*x^n +
 b)/c)/(b^14*n))
```

Fricas [B] time = 2.46904, size = 328, normalized size = 15.62

$$-\frac{x^{14}}{7\left(b^{7}nx^{7}x^{7\,n+7}+7\,b^{6}cnx^{6}x^{8\,n+8}+21\,b^{5}c^{2}nx^{5}x^{9\,n+9}+35\,b^{4}c^{3}nx^{4}x^{10\,n+10}+35\,b^{3}c^{4}nx^{3}x^{11\,n+11}+21\,b^{2}c^{5}nx^{2}x^{12\,n+12}+7\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+2*c*x^n)/(x^(-7+7*n))/(b*x+c*x^(1+n))^8,x, algorithm="fricas")
```

```
[Out] -1/7*x^14/(b^7*n*x^7*x^(7*n + 7) + 7*b^6*c*n*x^6*x^(8*n + 8) + 21*b^5*c^2*n
*x^5*x^(9*n + 9) + 35*b^4*c^3*n*x^4*x^(10*n + 10) + 35*b^3*c^4*n*x^3*x^(11*
n + 11) + 21*b^2*c^5*n*x^2*x^(12*n + 12) + 7*b*c^6*n*x*x^(13*n + 13) + c^7*
```

```
n*x^(14*n + 14))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+2*c*x**n)/(x**(-7+7*n))/(b*x+c*x**(1+n))**8,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2\,cx^{n}+b}{\left(bx+cx^{n+1}\right)^{8}x^{7\,n-7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+2*c*x^n)/(x^(-7+7*n))/(b*x+c*x^(1+n))^8,x, algorithm="giac")
```

```
[Out] integrate((2*c*x^n + b)/((b*x + c*x^(n + 1))^8*x^(7*n - 7)), x)
```

3.172 $\int (b+2cx)\left(bx+cx^2\right)^p\,dx$

Optimal. Leaf size=19

$$\frac{\left(bx+cx^2\right)^{p+1}}{p+1}$$

```
[Out] (b*x + c*x^2)^(1 + p)/(1 + p)
```

Rubi [A] time = 0.0054707, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {629}

$$\frac{\left(bx+cx^2\right)^{p+1}}{p+1}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x)*(b*x + c*x^2)^p,x]
```

```
[Out] (b*x + c*x^2)^(1 + p)/(1 + p)
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int (b+2cx)\left(bx+cx^2\right)^p\,dx=\frac{\left(bx+cx^2\right)^{1+p}}{1+p}$$

Mathematica [A] time = 0.0090834, size = 17, normalized size = 0.89

$$\frac{(x(b+cx))^{p+1}}{p+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x)*(b*x + c*x^2)^p,x]
```

```
[Out] (x*(b + c*x))^(1 + p)/(1 + p)
```

Maple [A] time = 0., size = 24, normalized size = 1.3

$$\frac{x(cx+b)\left(cx^2+bx\right)^p}{1+p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x+b)*(c*x^2+b*x)^p,x)
```

```
[Out] x*(c*x+b)/(1+p)*(c*x^2+b*x)^p
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x)^p,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.33617, size = 53, normalized size = 2.79

$$\frac{\left(cx^2+bx\right)\left(cx^2+bx\right)^p}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x)^p,x, algorithm="fricas")
```

```
[Out] (c*x^2 + b*x)*(c*x^2 + b*x)^p/(p + 1)
```

Sympy [A] time = 0.560271, size = 46, normalized size = 2.42

$$\begin{cases} \frac{bx(bx+cx^2)^p}{p+1} + \frac{cx^2(bx+cx^2)^p}{p+1} & \text{for } p \neq -1 \\ \log(x) + \log\left(\frac{b}{c} + x\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x**2+b*x)**p,x)
```

```
[Out] Piecewise((b*x*(b*x + c*x**2)**p/(p + 1) + c*x**2*(b*x + c*x**2)**p/(p + 1)
, Ne(p, -1)), (log(x) + log(b/c + x), True))
```

Giac [A] time = 1.25932, size = 50, normalized size = 2.63

$$\frac{(cx^2 + bx)^p cx^2 + (cx^2 + bx)^p bx}{p + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x)^p,x, algorithm="giac")
```

```
[Out] ((c*x^2 + b*x)^p*c*x^2 + (c*x^2 + b*x)^p*b*x)/(p + 1)
```

3.173 $\int x^{1+p}\left(b+2cx^2\right)\left(bx+cx^3\right)^p\,dx$

Optimal. Leaf size=27

$$\frac{x^{p+1}\left(bx+cx^3\right)^{p+1}}{2(p+1)}$$

```
[Out] (x^(1 + p)*(b*x + c*x^3)^(1 + p))/(2*(1 + p))
```

Rubi [A] time = 0.0230009, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {1590}

$$\frac{x^{p+1}\left(bx+cx^3\right)^{p+1}}{2(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(1 + p)*(b + 2*c*x^2)*(b*x + c*x^3)^p,x]
```

```
[Out] (x^(1 + p)*(b*x + c*x^3)^(1 + p))/(2*(1 + p))
```

Rule 1590

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q - r + 1)*Q
q^(m + 1)*Rr^(n + 1))/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r])
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\int x^{1+p}\left(b+2cx^2\right)\left(bx+cx^3\right)^p\,dx=\frac{x^{1+p}\left(bx+cx^3\right)^{1+p}}{2(1+p)}$$

Mathematica [C] time = 0.0753241, size = 97, normalized size = 3.59

$$\frac{x^{p+2}\left(x\left(b+cx^2\right)\right)^p\left(\frac{cx^2}{b}+1\right)^{-p}\left(2c(p+1)x^2\,{}_2F_1\left(-p,p+2;p+3;-\frac{cx^2}{b}\right)+b(p+2)\,{}_2F_1\left(-p,p+1;p+2;-\frac{cx^2}{b}\right)\right)}{2(p+1)(p+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(1 + p)*(b + 2*c*x^2)*(b*x + c*x^3)^p,x]
```

```
[Out] (x^(2 + p)*(x*(b + c*x^2))^p*(b*(2 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p,
 -((c*x^2)/b)] + 2*c*(1 + p)*x^2*Hypergeometric2F1[-p, 2 + p, 3 + p, -((c*x
^2)/b)]))/(2*(1 + p)*(2 + p)*(1 + (c*x^2)/b)^p)
```

Maple [A] time = 0.004, size = 31, normalized size = 1.2

$$\frac{x^{2+p}\left(cx^2+b\right)\left(cx^3+bx\right)^p}{2+2p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1+p)*(2*c*x^2+b)*(c*x^3+b*x)^p,x)
```

```
[Out] 1/2*x^(2+p)*(c*x^2+b)/(1+p)*(c*x^3+b*x)^p
```

Maxima [A] time = 1.2988, size = 47, normalized size = 1.74

$$\frac{\left(cx^4+bx^2\right)e^{\left(p\log\left(cx^2+b\right)+2p\log(x)\right)}}{2\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+p)*(2*c*x^2+b)*(c*x^3+b*x)^p,x, algorithm="maxima")
```

```
[Out] 1/2*(c*x^4 + b*x^2)*e^(p*log(c*x^2 + b) + 2*p*log(x))/(p + 1)
```

Fricas [A] time = 1.42504, size = 72, normalized size = 2.67

$$\frac{\left(cx^3+bx\right)\left(cx^3+bx\right)^p x^{p+1}}{2\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+p)*(2*c*x^2+b)*(c*x^3+b*x)^p,x, algorithm="fricas")
```

```
[Out] 1/2*(c*x^3 + b*x)*(c*x^3 + b*x)^p*x^(p + 1)/(p + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1+p)*(2*c*x**2+b)*(c*x**3+b*x)**p,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.18203, size = 73, normalized size = 2.7

$$\frac{cx^3 e^{\left(p\log\left(cx^2+b\right)+2\,p\log\left(x\right)+\log\left(x\right)\right)}+bxe^{\left(p\log\left(cx^2+b\right)+2\,p\log\left(x\right)+\log\left(x\right)\right)}}{2\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+p)*(2*c*x^2+b)*(c*x^3+b*x)^p,x, algorithm="giac")
```

```
[Out] 1/2*(c*x^3*e^(p*log(c*x^2 + b) + 2*p*log(x) + log(x)) + b*x*e^(p*log(c*x^2
+ b) + 2*p*log(x) + log(x)))/(p + 1)
```

3.174 $\int \left(bx^{1+p}\left(bx+cx^3\right)^p+2cx^{3+p}\left(bx+cx^3\right)^p\right)dx$

Optimal. Leaf size=27

$$\frac{x^{p+1}\left(bx+cx^3\right)^{p+1}}{2(p+1)}$$

```
[Out] (x^(1 + p)*(b*x + c*x^3)^(1 + p))/(2*(1 + p))
```

Rubi [C] time = 0.101833, antiderivative size = 116, normalized size of antiderivative = 4.3, number of steps used = 7, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {2032, 365, 364}

$$\frac{bx^{p+2}\left(bx+cx^3\right)^p\left(\frac{cx^2}{b}+1\right)^{-p}{}_2F_1\left(-p,p+1;p+2;-\frac{cx^2}{b}\right)}{2(p+1)}+\frac{cx^{p+4}\left(bx+cx^3\right)^p\left(\frac{cx^2}{b}+1\right)^{-p}{}_2F_1\left(-p,p+2;p+3;-\frac{cx^2}{b}\right)}{p+2}$$

Antiderivative was successfully verified.

```
[In] Int[b*x^(1 + p)*(b*x + c*x^3)^p + 2*c*x^(3 + p)*(b*x + c*x^3)^p,x]
```

```
[Out] (b*x^(2 + p)*(b*x + c*x^3)^p*Hypergeometric2F1[-p, 1 + p, 2 + p, -((c*x^2)/
b)])/(2*(1 + p)*(1 + (c*x^2)/b)^p) + (c*x^(4 + p)*(b*x + c*x^3)^p*Hypergeom
etric2F1[-p, 2 + p, 3 + p, -((c*x^2)/b)])/((2 + p)*(1 + (c*x^2)/b)^p)
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \left(bx^{1+p}\left(bx+cx^3\right)^p+2cx^{3+p}\left(bx+cx^3\right)^p\right)dx = b\int x^{1+p}\left(bx+cx^3\right)^p\,dx+(2c)\int x^{3+p}\left(bx+cx^3\right)^p\,dx$$
$$=\left(bx^{-p}\left(b+cx^2\right)^{-p}\left(bx+cx^3\right)^p\right)\int x^{1+2p}\left(b+cx^2\right)^p\,dx+\left(2cx^{-p}\left(b+cx^2\right.\right.$$
$$=\left(bx^{-p}\left(1+\frac{cx^2}{b}\right)^{-p}\left(bx+cx^3\right)^p\right)\int x^{1+2p}\left(1+\frac{cx^2}{b}\right)^p\,dx+\left(2cx^{-p}\left(1+\frac{c}{b}\right.\right.$$
$$=\frac{bx^{2+p}\left(1+\frac{cx^2}{b}\right)^{-p}\left(bx+cx^3\right)^p\,{}_2F_1\left(-p,1+p;2+p;-\frac{cx^2}{b}\right)}{2(1+p)}+\frac{cx^{4+p}\left(1+\right.}{}$$

Mathematica [C] time = 0.0266186, size = 97, normalized size = 3.59

$$\frac{x^{p+2}\left(x\left(b+cx^2\right)\right)^p\left(\frac{cx^2}{b}+1\right)^{-p}\left(2c(p+1)x^2\,{}_2F_1\left(-p,p+2;p+3;-\frac{cx^2}{b}\right)+b(p+2)\,{}_2F_1\left(-p,p+1;p+2;-\frac{cx^2}{b}\right)\right)}{2(p+1)(p+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[b*x^(1 + p)*(b*x + c*x^3)^p + 2*c*x^(3 + p)*(b*x + c*x^3)^p,x]
```

```
[Out] (x^(2 + p)*(x*(b + c*x^2))^p*(b*(2 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p,
 -((c*x^2)/b)] + 2*c*(1 + p)*x^2*Hypergeometric2F1[-p, 2 + p, 3 + p, -((c*x
^2)/b)]))/(2*(1 + p)*(2 + p)*(1 + (c*x^2)/b)^p)
```

Maple [C] time = 0.114, size = 142, normalized size = 5.3

$$\frac{x\left(cx^2+b\right)x^{1+p}}{2+2p}e^{-\frac{p\left(i\pi\left(\mathrm{csgn}\left(ix\left(cx^2+b\right)\right)\right)^3-i\pi\left(\mathrm{csgn}\left(ix\left(cx^2+b\right)\right)\right)^2\mathrm{csgn}(ix)-i\pi\left(\mathrm{csgn}\left(ix\left(cx^2+b\right)\right)\right)^2\mathrm{csgn}\left(i\left(cx^2+b\right)\right)+i\pi\,\mathrm{csgn}\left(ix\left(cx^2+b\right)\right)\mathrm{csgn}(ix)\mathrm{csgn}\left(i\left(cx^2+b\right)\right)-2\ln(x)-2\ln(c\right.}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b*x^(1+p)*(c*x^3+b*x)^p+2*c*x^(3+p)*(c*x^3+b*x)^p,x)
```

```
[Out] 1/2*(c*x^2+b)*x*x^(1+p)/(1+p)*exp(-1/2*p*(I*Pi*csgn(I*x*(c*x^2+b))^3-I*Pi*c
sgn(I*x*(c*x^2+b))^2*csgn(I*x)-I*Pi*csgn(I*x*(c*x^2+b))^2*csgn(I*(c*x^2+b))
+I*Pi*csgn(I*x*(c*x^2+b))*csgn(I*x)*csgn(I*(c*x^2+b))-2*ln(x)-2*ln(c*x^2+b)
))
```

Maxima [A] time = 1.21373, size = 47, normalized size = 1.74

$$\frac{\left(cx^4+bx^2\right)e^{\left(p\log\left(cx^2+b\right)+2\,p\log(x)\right)}}{2\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x^(1+p)*(c*x^3+b*x)^p+2*c*x^(3+p)*(c*x^3+b*x)^p,x, algorithm="m
axima")
```

```
[Out] 1/2*(c*x^4 + b*x^2)*e^(p*log(c*x^2 + b) + 2*p*log(x))/(p + 1)
```

Fricas [A] time = 1.31456, size = 74, normalized size = 2.74

$$\frac{\left(cx^2+b\right)\left(cx^3+bx\right)^p x^{p+3}}{2\left(p+1\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x^(1+p)*(c*x^3+b*x)^p+2*c*x^(3+p)*(c*x^3+b*x)^p,x, algorithm="f
ricas")
```

```
[Out] 1/2*(c*x^2 + b)*(c*x^3 + b*x)^p*x^(p + 3)/((p + 1)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x**(1+p)*(c*x**3+b*x)**p+2*c*x**(3+p)*(c*x**3+b*x)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int 2\left(cx^3+bx\right)^p cx^{p+3}+\left(cx^3+bx\right)^p bx^{p+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x^(1+p)*(c*x^3+b*x)^p+2*c*x^(3+p)*(c*x^3+b*x)^p,x, algorithm="g
iac")
```

```
[Out] integrate(2*(c*x^3 + b*x)^p*c*x^(p + 3) + (c*x^3 + b*x)^p*b*x^(p + 1), x)
```

3.175 $\int x^{2(1+p)}\left(b+2cx^3\right)\left(bx+cx^4\right)^p\,dx$

Optimal. Leaf size=29

$$\frac{x^{2(p+1)}\left(bx+cx^4\right)^{p+1}}{3(p+1)}$$

```
[Out] (x^(2*(1 + p))*(b*x + c*x^4)^(1 + p))/(3*(1 + p))
```

Rubi [A] time = 0.0238577, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {1590}

$$\frac{x^{2(p+1)}\left(bx+cx^4\right)^{p+1}}{3(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(2*(1 + p))*(b + 2*c*x^3)*(b*x + c*x^4)^p,x]
```

```
[Out] (x^(2*(1 + p))*(b*x + c*x^4)^(1 + p))/(3*(1 + p))
```

Rule 1590

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q - r + 1)*Q
q^(m + 1)*Rr^(n + 1))/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r])
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\int x^{2(1+p)}\left(b+2cx^3\right)\left(bx+cx^4\right)^p\,dx=\frac{x^{2(1+p)}\left(bx+cx^4\right)^{1+p}}{3(1+p)}$$

Mathematica [C] time = 0.0764508, size = 99, normalized size = 3.41

$$\frac{x^{2p+3}\left(x\left(b+cx^3\right)\right)^p\left(\frac{cx^3}{b}+1\right)^{-p}\left(2c(p+1)x^3\,{}_2F_1\left(-p,p+2;p+3;-\frac{cx^3}{b}\right)+b(p+2)\,{}_2F_1\left(-p,p+1;p+2;-\frac{cx^3}{b}\right)\right)}{3(p+1)(p+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(2*(1 + p))*(b + 2*c*x^3)*(b*x + c*x^4)^p,x]
```

```
[Out] (x^(3 + 2*p)*(x*(b + c*x^3))^p*(b*(2 + p)*Hypergeometric2F1[-p, 1 + p, 2 +
p, -((c*x^3)/b)] + 2*c*(1 + p)*x^3*Hypergeometric2F1[-p, 2 + p, 3 + p, -((c
*x^3)/b)]))/(3*(1 + p)*(2 + p)*(1 + (c*x^3)/b)^p)
```

Maple [A] time = 0.003, size = 33, normalized size = 1.1

$$\frac{x^{3+2p}\left(cx^3+b\right)\left(cx^4+bx\right)^p}{3+3p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2+2*p)*(2*c*x^3+b)*(c*x^4+b*x)^p,x)
```

```
[Out] 1/3*x^(3+2*p)*(c*x^3+b)/(1+p)*(c*x^4+b*x)^p
```

Maxima [A] time = 1.59304, size = 47, normalized size = 1.62

$$\frac{\left(cx^6+bx^3\right)e^{\left(p\log\left(cx^3+b\right)+3p\log(x)\right)}}{3\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+2*p)*(2*c*x^3+b)*(c*x^4+b*x)^p,x, algorithm="maxima")
```

```
[Out] 1/3*(c*x^6 + b*x^3)*e^(p*log(c*x^3 + b) + 3*p*log(x))/(p + 1)
```

Fricas [A] time = 1.39162, size = 74, normalized size = 2.55

$$\frac{\left(cx^4 + bx\right)\left(cx^4 + bx\right)^p x^{2p+2}}{3\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+2*p)*(2*c*x^3+b)*(c*x^4+b*x)^p,x, algorithm="fricas")
```

```
[Out] 1/3*(c*x^4 + b*x)*(c*x^4 + b*x)^p*x^(2*p + 2)/(p + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(2+2*p)*(2*c*x**3+b)*(c*x**4+b*x)**p,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.17391, size = 78, normalized size = 2.69

$$\frac{cx^4 e^{\left(p\log\left(cx^3+b\right)+3\,p\log\left(x\right)+2\,\log\left(x\right)\right)} + bxe^{\left(p\log\left(cx^3+b\right)+3\,p\log\left(x\right)+2\,\log\left(x\right)\right)}}{3\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+2*p)*(2*c*x^3+b)*(c*x^4+b*x)^p,x, algorithm="giac")
```

```
[Out] 1/3*(c*x^4*e^(p*log(c*x^3 + b) + 3*p*log(x) + 2*log(x)) + b*x*e^(p*log(c*x^
3 + b) + 3*p*log(x) + 2*log(x)))/(p + 1)
```

3.176 $\int x^{(-1+n)(1+p)}\left(b+2cx^n\right)\left(bx+cx^{1+n}\right)^p\,dx$

Optimal. Leaf size=36

$$\frac{x^{-(1-n)(p+1)}\left(bx+cx^{n+1}\right)^{p+1}}{n(p+1)}$$

```
[Out] (b*x + c*x^(1 + n))^(1 + p)/(n*(1 + p)*x^((1 - n)*(1 + p)))
```

Rubi [A] time = 0.085884, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.032, Rules used = {2036}

$$\frac{x^{-(1-n)(p+1)}\left(bx+cx^{n+1}\right)^{p+1}}{n(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^((-1 + n)*(1 + p))*(b + 2*c*x^n)*(b*x + c*x^(1 + n))^p,x]
```

```
[Out] (b*x + c*x^(1 + n))^(1 + p)/(n*(1 + p)*x^((1 - n)*(1 + p)))
```

Rule 2036

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(a*(m + j*p + 1)), x] /; FreeQ[{a, b, c, d, e, j, m
, n, p}, x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && EqQ[
a*d*(m + j*p + 1) - b*c*(m + n + p*(j + n) + 1), 0] && (GtQ[e, 0] || Intege
rsQ[j]) && NeQ[m + j*p + 1, 0]
```

Rubi steps

$$\int x^{(-1+n)(1+p)}\left(b+2cx^n\right)\left(bx+cx^{1+n}\right)^p\,dx=\frac{x^{-(1-n)(1+p)}\left(bx+cx^{1+n}\right)^{1+p}}{n(1+p)}$$

Mathematica [C] time = 0.147862, size = 108, normalized size = 3.

$$\frac{x^{-p}\left(x\left(b+cx^n\right)\right)^p\left(\frac{cx^n}{b}+1\right)^{-p}\left(b(p+2)x^{n(p+1)}\,{}_2F_1\left(-p,p+1;p+2;-\frac{cx^n}{b}\right)+2c(p+1)x^{n(p+2)}\,{}_2F_1\left(-p,p+2;p+3;-\frac{cx^n}{b}\right)\right)}{n(p+1)(p+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^((-1 + n)*(1 + p))*(b + 2*c*x^n)*(b*x + c*x^(1 + n))^p,x]
```

```
[Out] ((x*(b + c*x^n))^p*(b*(2 + p)*x^(n*(1 + p))*Hypergeometric2F1[-p, 1 + p, 2
+ p, -((c*x^n)/b)] + 2*c*(1 + p)*x^(n*(2 + p))*Hypergeometric2F1[-p, 2 + p,
 3 + p, -((c*x^n)/b)]))/(n*(1 + p)*(2 + p)*x^p*(1 + (c*x^n)/b)^p)
```

Maple [F] time = 0.07, size = 0, normalized size = 0.

$$\int x^{(-1+n)(1+p)}\left(b+2\,cx^n\right)\left(bx+cx^{1+n}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^((-1+n)*(1+p))*(b+2*c*x^n)*(b*x+c*x^(1+n))^p,x)
```

```
[Out] int(x^((-1+n)*(1+p))*(b+2*c*x^n)*(b*x+c*x^(1+n))^p,x)
```

Maxima [A] time = 1.36925, size = 53, normalized size = 1.47

$$\frac{\left(cx^{2\,n}+bx^n\right)e^{\left(np\log(x)+p\log(cx^n+b)\right)}}{n\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^((-1+n)*(1+p))*(b+2*c*x^n)*(b*x+c*x^(1+n))^p,x, algorithm="maxi
ma")
```

```
[Out] (c*x^(2*n) + b*x^n)*e^(n*p*log(x) + p*log(c*x^n + b))/(n*(p + 1))
```

Fricas [A] time = 1.46847, size = 101, normalized size = 2.81

$$\frac{\left(bx + cx^{n+1}\right)\left(bx + cx^{n+1}\right)^{p} x^{(n-1)p+n-1}}{np + n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^((-1+n)*(1+p))*(b+2*c*x^n)*(b*x+c*x^(1+n))^p,x, algorithm="fric
as")
```

```
[Out] (b*x + c*x^(n + 1))*(b*x + c*x^(n + 1))^p*x^((n - 1)*p + n - 1)/(n*p + n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**((-1+n)*(1+p))*(b+2*c*x**n)*(b*x+c*x**(1+n))**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (2\,cx^{n} + b)\left(bx + cx^{n+1}\right)^{p} x^{(n-1)(p+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^((-1+n)*(1+p))*(b+2*c*x^n)*(b*x+c*x^(1+n))^p,x, algorithm="giac
")
```

```
[Out] integrate((2*c*x^n + b)*(b*x + c*x^(n + 1))^p*x^((n - 1)*(p + 1)), x)
```

3.177 $\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{a+bx^2}\,dx$

Optimal. Leaf size=32

$$acx+\frac{1}{2}adx^2+\frac{1}{3}bcx^3+\frac{1}{4}bdx^4$$

```
[Out] a*c*x + (a*d*x^2)/2 + (b*c*x^3)/3 + (b*d*x^4)/4
```

Rubi [A] time = 0.0340129, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 54, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.019, Rules used = {1586}

$$acx+\frac{1}{2}adx^2+\frac{1}{3}bcx^3+\frac{1}{4}bdx^4$$

Antiderivative was successfully verified.

```
[In] Int[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*x^5)/(
a + b*x^2),x]
```

```
[Out] a*c*x + (a*d*x^2)/2 + (b*c*x^3)/3 + (b*d*x^4)/4
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{a+bx^2}\,dx &= \int \left(ac+adx+bcx^2+bdx^3\right)dx\\ &= acx+\frac{1}{2}adx^2+\frac{1}{3}bcx^3+\frac{1}{4}bdx^4\end{aligned}$$

Mathematica [A] time = 0.0018921, size = 32, normalized size = 1.

$$acx+\frac{1}{2}adx^2+\frac{1}{3}bcx^3+\frac{1}{4}bdx^4$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*
x^5)/(a + b*x^2),x]
```

```
[Out] a*c*x + (a*d*x^2)/2 + (b*c*x^3)/3 + (b*d*x^4)/4
```

Maple [A] time = 0., size = 27, normalized size = 0.8

$$acx + \frac{adx^2}{2} + \frac{bcx^3}{3} + \frac{bdx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^2+a),x
)
```

```
[Out] a*c*x+1/2*a*d*x^2+1/3*b*c*x^3+1/4*b*d*x^4
```

Maxima [A] time = 1.02178, size = 35, normalized size = 1.09

$$\frac{1}{4}\,bdx^4 + \frac{1}{3}\,bcx^3 + \frac{1}{2}\,adx^2 + acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^
2+a),x, algorithm="maxima")
```

```
[Out] 1/4*b*d*x^4 + 1/3*b*c*x^3 + 1/2*a*d*x^2 + a*c*x
```

Fricas [A] time = 1.25276, size = 66, normalized size = 2.06

$$\frac{1}{4}\,bdx^4 + \frac{1}{3}\,bcx^3 + \frac{1}{2}\,adx^2 + acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^
2+a),x, algorithm="fricas")
```

```
[Out] 1/4*b*d*x^4 + 1/3*b*c*x^3 + 1/2*a*d*x^2 + a*c*x
```

Sympy [A] time = 0.080349, size = 29, normalized size = 0.91

$$acx + \frac{adx^2}{2} + \frac{bcx^3}{3} + \frac{bdx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*d*x**5+b**2*c*x**4+2*a*b*d*x**3+2*a*b*c*x**2+a**2*d*x+a**2*
c)/(b*x**2+a),x)
```

```
[Out] a*c*x + a*d*x**2/2 + b*c*x**3/3 + b*d*x**4/4
```

Giac [A] time = 1.13044, size = 35, normalized size = 1.09

$$\frac{1}{4}\,bdx^4 + \frac{1}{3}\,bcx^3 + \frac{1}{2}\,adx^2 + acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^
2+a),x, algorithm="giac")
```

```
[Out] 1/4*b*d*x^4 + 1/3*b*c*x^3 + 1/2*a*d*x^2 + a*c*x
```

3.178 $\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{(a+bx^2)^2} dx$

Optimal. Leaf size=12

$$cx + \frac{dx^2}{2}$$

```
[Out] c*x + (d*x^2)/2
```

Rubi [A] time = 0.0485936, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 1, integrand size = 54, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.019, Rules used = {1586}

$$cx + \frac{dx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*x^5)/(
a + b*x^2)^2,x]
```

```
[Out] c*x + (d*x^2)/2
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{a^2c + a^2dx + 2abcx^2 + 2abdx^3 + b^2cx^4 + b^2dx^5}{\left(a + bx^2\right)^2} dx &= \int \frac{ac + adx + bcx^2 + bdx^3}{a + bx^2} dx \\
&= \int (c + dx) dx \\
&= cx + \frac{dx^2}{2}
\end{aligned}$$

Mathematica [A] time = 0.00072, size = 12, normalized size = 1.

$$cx + \frac{dx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*
x^5)/(a + b*x^2)^2,x]
```

```
[Out] c*x + (d*x^2)/2
```

Maple [A] time = 0., size = 11, normalized size = 0.9

$$cx + \frac{dx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^2+a)^2
,x)
```

```
[Out] c*x+1/2*d*x^2
```

Maxima [A] time = 1.01943, size = 14, normalized size = 1.17

$$\frac{1}{2}\,dx^2 + cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^
2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*d*x^2 + c*x
```

Fricas [A] time = 1.24135, size = 23, normalized size = 1.92

$$\frac{1}{2}\,dx^2 + cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^
2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*d*x^2 + c*x
```

Sympy [A] time = 0.081643, size = 8, normalized size = 0.67

$$cx + \frac{dx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*d*x**5+b**2*c*x**4+2*a*b*d*x**3+2*a*b*c*x**2+a**2*d*x+a**2*
c)/(b*x**2+a)**2,x)
```

```
[Out] c*x + d*x**2/2
```

Giac [A] time = 1.24447, size = 14, normalized size = 1.17

$$\frac{1}{2}\,dx^2 + cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^
2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*d*x^2 + c*x
```

3.179 $\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=42

$$\frac{c\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}+\frac{d\log\left(a+bx^2\right)}{2b}$$

```
[Out] (c*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]) + (d*Log[a + b*x^2])/(2*b
)
```

Rubi [A] time = 0.0573733, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 54, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1586, 635, 205, 260}

$$\frac{c\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}+\frac{d\log\left(a+bx^2\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*x^5)/(
a + b*x^2)^3,x]
```

```
[Out] (c*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]) + (d*Log[a + b*x^2])/(2*b
)
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{\left(a+bx^2\right)^3}\,dx &= \int \frac{ac+adx+bcx^2+bdx^3}{\left(a+bx^2\right)^2}\,dx\\
&= \int \frac{c+dx}{a+bx^2}\,dx\\
&= c\int \frac{1}{a+bx^2}\,dx+d\int \frac{x}{a+bx^2}\,dx\\
&= \frac{c\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}+\frac{d\log\left(a+bx^2\right)}{2b}
\end{aligned}$$

Mathematica [A] time = 0.0143338, size = 42, normalized size = 1.

$$\frac{c\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}+\frac{d\log\left(a+bx^2\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*
x^5)/(a + b*x^2)^3,x]
```

```
[Out] (c*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]) + (d*Log[a + b*x^2])/(2*b
)
```

Maple [A] time = 0.004, size = 32, normalized size = 0.8

$$\frac{d\ln\left(bx^2+a\right)}{2\,b}+c\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^2+a)^3
,x)
```

```
[Out] 1/2*d*ln(b*x^2+a)/b+c/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^
2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.33557, size = 225, normalized size = 5.36

$$\left[\frac{ad\log\left(bx^2+a\right)-\sqrt{-ab}c\log\left(\frac{bx^2-2\sqrt{-ab}x-a}{bx^2+a}\right)}{2ab}, \frac{ad\log\left(bx^2+a\right)+2\sqrt{ab}c\arctan\left(\frac{\sqrt{ab}x}{a}\right)}{2ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^
2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/2*(a*d*log(b*x^2 + a) - sqrt(-a*b)*c*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b
*x^2 + a)))/(a*b), 1/2*(a*d*log(b*x^2 + a) + 2*sqrt(a*b)*c*arctan(sqrt(a*b)
*x/a))/(a*b)]
```

Sympy [B] time = 0.287324, size = 124, normalized size = 2.95

$$\left(\frac{d}{2b}-\frac{c\sqrt{-ab^3}}{2ab^2}\right)\log\left(x+\frac{2ab\left(\frac{d}{2b}-\frac{c\sqrt{-ab^3}}{2ab^2}\right)-ad}{bc}\right)+\left(\frac{d}{2b}+\frac{c\sqrt{-ab^3}}{2ab^2}\right)\log\left(x+\frac{2ab\left(\frac{d}{2b}+\frac{c\sqrt{-ab^3}}{2ab^2}\right)-ad}{bc}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*d*x**5+b**2*c*x**4+2*a*b*d*x**3+2*a*b*c*x**2+a**2*d*x+a**2*
c)/(b*x**2+a)**3,x)
```

```
[Out] (d/(2*b) - c*sqrt(-a*b**3)/(2*a*b**2))*log(x + (2*a*b*(d/(2*b) - c*sqrt(-a*
b**3)/(2*a*b**2)) - a*d)/(b*c)) + (d/(2*b) + c*sqrt(-a*b**3)/(2*a*b**2))*lo
g(x + (2*a*b*(d/(2*b) + c*sqrt(-a*b**3)/(2*a*b**2)) - a*d)/(b*c))
```

Giac [A] time = 1.22604, size = 42, normalized size = 1.

$$\frac{c\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}+\frac{d\log\left(bx^2+a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^
2+a)^3,x, algorithm="giac")
```

```
[Out] c*arctan(b*x/sqrt(a*b))/sqrt(a*b) + 1/2*d*log(b*x^2 + a)/b
```

3.180 $\int \left(b+2cx+3dx^2\right)\left(a+bx+cx^2+dx^3\right)^n\,dx$

Optimal. Leaf size=25

$$\frac{\left(a+bx+cx^2+dx^3\right)^{n+1}}{n+1}$$

```
[Out] (a + b*x + c*x^2 + d*x^3)^(1 + n)/(1 + n)
```

Rubi [A] time = 0.0239676, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {1588}

$$\frac{\left(a+bx+cx^2+dx^3\right)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x + 3*d*x^2)*(a + b*x + c*x^2 + d*x^3)^n,x]
```

```
[Out] (a + b*x + c*x^2 + d*x^3)^(1 + n)/(1 + n)
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \left(b+2cx+3dx^2\right)\left(a+bx+cx^2+dx^3\right)^n\,dx=\frac{\left(a+bx+cx^2+dx^3\right)^{1+n}}{1+n}$$

Mathematica [A] time = 0.0143151, size = 23, normalized size = 0.92

$$\frac{(a+x(b+x(c+dx)))^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x + 3*d*x^2)*(a + b*x + c*x^2 + d*x^3)^n,x]
```

```
[Out] (a + x*(b + x*(c + d*x)))^(1 + n)/(1 + n)
```

Maple [A] time = 0.004, size = 26, normalized size = 1.

$$\frac{\left(dx^3 + cx^2 + bx + a\right)^{1+n}}{1+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x+a)^n,x)
```

```
[Out] (d*x^3+c*x^2+b*x+a)^(1+n)/(1+n)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x+a)^n,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.39835, size = 85, normalized size = 3.4

$$\frac{\left(dx^3 + cx^2 + bx + a\right)\left(dx^3 + cx^2 + bx + a\right)^n}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x+a)^n,x, algorithm="fricas")
```

```
[Out] (d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^n/(n + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x**2+2*c*x+b)*(d*x**3+c*x**2+b*x+a)**n,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.29716, size = 122, normalized size = 4.88

$$\frac{\left(dx^3+cx^2+bx+a\right)^n dx^3+\left(dx^3+cx^2+bx+a\right)^n cx^2+\left(dx^3+cx^2+bx+a\right)^n bx+\left(dx^3+cx^2+bx+a\right)^n a}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x+a)^n,x, algorithm="giac")
```

```
[Out] ((d*x^3 + c*x^2 + b*x + a)^n*d*x^3 + (d*x^3 + c*x^2 + b*x + a)^n*c*x^2 + (d
*x^3 + c*x^2 + b*x + a)^n*b*x + (d*x^3 + c*x^2 + b*x + a)^n*a)/(n + 1)
```

3.181 $\int \left(b+2cx+3dx^2\right)\left(bx+cx^2+dx^3\right)^n\,dx$

Optimal. Leaf size=24

$$\frac{\left(bx+cx^2+dx^3\right)^{n+1}}{n+1}$$

```
[Out] (b*x + c*x^2 + d*x^3)^(1 + n)/(1 + n)
```

Rubi [A] time = 0.0175243, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {1588}

$$\frac{\left(bx+cx^2+dx^3\right)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x + 3*d*x^2)*(b*x + c*x^2 + d*x^3)^n,x]
```

```
[Out] (b*x + c*x^2 + d*x^3)^(1 + n)/(1 + n)
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \left(b+2cx+3dx^2\right)\left(bx+cx^2+dx^3\right)^n\,dx=\frac{\left(bx+cx^2+dx^3\right)^{1+n}}{1+n}$$

Mathematica [A] time = 0.0466691, size = 21, normalized size = 0.88

$$\frac{(x(b+x(c+dx)))^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x + 3*d*x^2)*(b*x + c*x^2 + d*x^3)^n,x]
```

```
[Out] (x*(b + x*(c + d*x)))^(1 + n)/(1 + n)
```

Maple [A] time = 0.005, size = 34, normalized size = 1.4

$$\frac{x\left(dx^2+cx+b\right)\left(dx^3+cx^2+bx\right)^n}{1+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x)^n,x)
```

```
[Out] x*(d*x^2+c*x+b)/(1+n)*(d*x^3+c*x^2+b*x)^n
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x)^n,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.49556, size = 74, normalized size = 3.08

$$\frac{\left(dx^3+cx^2+bx\right)\left(dx^3+cx^2+bx\right)^n}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x)^n,x, algorithm="fricas")
```

```
[Out] (d*x^3 + c*x^2 + b*x)*(d*x^3 + c*x^2 + b*x)^n/(n + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x**2+2*c*x+b)*(d*x**3+c*x**2+b*x)**n,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.16479, size = 92, normalized size = 3.83

$$\frac{\left(dx^3+cx^2+bx\right)^n dx^3+\left(dx^3+cx^2+bx\right)^n cx^2+\left(dx^3+cx^2+bx\right)^n bx}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x)^n,x, algorithm="giac")
```

```
[Out] ((d*x^3 + c*x^2 + b*x)^n*d*x^3 + (d*x^3 + c*x^2 + b*x)^n*c*x^2 + (d*x^3 + c
*x^2 + b*x)^n*b*x)/(n + 1)
```

3.182 $\int x^n \left(b+cx+dx^2\right)^n \left(b+2cx+3dx^2\right) dx$

Optimal. Leaf size=25

$$\frac{x^{n+1}\left(b+cx+dx^2\right)^{n+1}}{n+1}$$

```
[Out] (x^(1 + n)*(b + c*x + d*x^2)^(1 + n))/(1 + n)
```

Rubi [A] time = 0.0212676, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {1590}

$$\frac{x^{n+1}\left(b+cx+dx^2\right)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^n*(b + c*x + d*x^2)^n*(b + 2*c*x + 3*d*x^2),x]
```

```
[Out] (x^(1 + n)*(b + c*x + d*x^2)^(1 + n))/(1 + n)
```

Rule 1590

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q - r + 1)*Q
q^(m + 1)*Rr^(n + 1))/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r])
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\int x^n \left(b+cx+dx^2\right)^n \left(b+2cx+3dx^2\right) dx = \frac{x^{1+n}\left(b+cx+dx^2\right)^{1+n}}{1+n}$$

Mathematica [A] time = 0.0204268, size = 24, normalized size = 0.96

$$\frac{x^{n+1}(b+x(c+dx))^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^n*(b + c*x + d*x^2)^n*(b + 2*c*x + 3*d*x^2),x]
```

```
[Out] (x^(1 + n)*(b + x*(c + d*x))^(1 + n))/(1 + n)
```

Maple [A] time = 0.003, size = 26, normalized size = 1.

$$\frac{x^{1+n}\left(dx^2+cx+b\right)^{1+n}}{1+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^n*(d*x^2+c*x+b)^n*(3*d*x^2+2*c*x+b),x)
```

```
[Out] x^(1+n)*(d*x^2+c*x+b)^(1+n)/(1+n)
```

Maxima [A] time = 1.20091, size = 53, normalized size = 2.12

$$\frac{\left(dx^3+cx^2+bx\right)e^{\left(n\log\left(dx^2+cx+b\right)+n\log(x)\right)}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n*(d*x^2+c*x+b)^n*(3*d*x^2+2*c*x+b),x, algorithm="maxima")
```

```
[Out] (d*x^3 + c*x^2 + b*x)*e^(n*log(d*x^2 + c*x + b) + n*log(x))/(n + 1)
```

Fricas [A] time = 1.35457, size = 74, normalized size = 2.96

$$\frac{\left(dx^3+cx^2+bx\right)\left(dx^2+cx+b\right)^n x^n}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n*(d*x^2+c*x+b)^n*(3*d*x^2+2*c*x+b),x, algorithm="fricas")
```

```
[Out] (d*x^3 + c*x^2 + b*x)*(d*x^2 + c*x + b)^n*x^n/(n + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**n*(d*x**2+c*x+b)**n*(3*d*x**2+2*c*x+b),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.19264, size = 88, normalized size = 3.52

$$\frac{\left(dx^2+cx+b\right)^n dx^3x^n+\left(dx^2+cx+b\right)^n cx^2x^n+\left(dx^2+cx+b\right)^n bxx^n}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n*(d*x^2+c*x+b)^n*(3*d*x^2+2*c*x+b),x, algorithm="giac")
```

```
[Out] ((d*x^2 + c*x + b)^n*d*x^3*x^n + (d*x^2 + c*x + b)^n*c*x^2*x^n + (d*x^2 + c
*x + b)^n*b*x*x^n)/(n + 1)
```

3.183 $\int \left(b+3dx^2\right)\left(a+bx+dx^3\right)^n\,dx$

Optimal. Leaf size=20

$$\frac{\left(a+bx+dx^3\right)^{n+1}}{n+1}$$

```
[Out] (a + b*x + d*x^3)^(1 + n)/(1 + n)
```

Rubi [A] time = 0.0099577, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {1588}

$$\frac{\left(a+bx+dx^3\right)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 3*d*x^2)*(a + b*x + d*x^3)^n,x]
```

```
[Out] (a + b*x + d*x^3)^(1 + n)/(1 + n)
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \left(b+3dx^2\right)\left(a+bx+dx^3\right)^n\,dx=\frac{\left(a+bx+dx^3\right)^{1+n}}{1+n}$$

Mathematica [A] time = 0.0085864, size = 20, normalized size = 1.

$$\frac{\left(a+bx+dx^3\right)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 3*d*x^2)*(a + b*x + d*x^3)^n,x]
```

```
[Out] (a + b*x + d*x^3)^(1 + n)/(1 + n)
```

Maple [A] time = 0.003, size = 21, normalized size = 1.1

$$\frac{\left(dx^3+bx+a\right)^{1+n}}{1+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*d*x^2+b)*(d*x^3+b*x+a)^n,x)
```

```
[Out] (d*x^3+b*x+a)^(1+n)/(1+n)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+b)*(d*x^3+b*x+a)^n,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.46924, size = 63, normalized size = 3.15

$$\frac{\left(dx^3+bx+a\right)\left(dx^3+bx+a\right)^n}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+b)*(d*x^3+b*x+a)^n,x, algorithm="fricas")
```

```
[Out] (d*x^3 + b*x + a)*(d*x^3 + b*x + a)^n/(n + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x**2+b)*(d*x**3+b*x+a)**n,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.18805, size = 72, normalized size = 3.6

$$\frac{\left(dx^3+bx+a\right)^n dx^3+\left(dx^3+bx+a\right)^n bx+\left(dx^3+bx+a\right)^n a}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+b)*(d*x^3+b*x+a)^n,x, algorithm="giac")
```

```
[Out] ((d*x^3 + b*x + a)^n*d*x^3 + (d*x^3 + b*x + a)^n*b*x + (d*x^3 + b*x + a)^n*
a)/(n + 1)
```

3.184 $\int \left(b+3dx^2\right)\left(bx+dx^3\right)^n\,dx$

Optimal. Leaf size=19

$$\frac{\left(bx+dx^3\right)^{n+1}}{n+1}$$

```
[Out] (b*x + d*x^3)^(1 + n)/(1 + n)
```

Rubi [A] time = 0.0089349, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {1588}

$$\frac{\left(bx+dx^3\right)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 3*d*x^2)*(b*x + d*x^3)^n,x]
```

```
[Out] (b*x + d*x^3)^(1 + n)/(1 + n)
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \left(b+3dx^2\right)\left(bx+dx^3\right)^n\,dx=\frac{\left(bx+dx^3\right)^{1+n}}{1+n}$$

Mathematica [C] time = 0.0645414, size = 106, normalized size = 5.58

$$\frac{x\left(x\left(b+dx^2\right)\right)^n\left(\frac{dx^2}{b}+1\right)^{-n}\left(3d(n+1)x^2\,{}_2F_1\left(-n,\frac{n+3}{2};\frac{n+5}{2};-\frac{dx^2}{b}\right)+b(n+3)\,{}_2F_1\left(-n,\frac{n+1}{2};\frac{n+3}{2};-\frac{dx^2}{b}\right)\right)}{(n+1)(n+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 3*d*x^2)*(b*x + d*x^3)^n,x]
```

```
[Out] (x*(x*(b + d*x^2))^n*(b*(3 + n)*Hypergeometric2F1[-n, (1 + n)/2, (3 + n)/2,
 -((d*x^2)/b)] + 3*d*(1 + n)*x^2*Hypergeometric2F1[-n, (3 + n)/2, (5 + n)/2
, -((d*x^2)/b)]))/((1 + n)*(3 + n)*(1 + (d*x^2)/b)^n)
```

Maple [A] time = 0.003, size = 26, normalized size = 1.4

$$\frac{x\left(dx^2+b\right)\left(dx^3+bx\right)^n}{1+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*d*x^2+b)*(d*x^3+b*x)^n,x)
```

```
[Out] x*(d*x^2+b)/(1+n)*(d*x^3+b*x)^n
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+b)*(d*x^3+b*x)^n,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.64072, size = 53, normalized size = 2.79

$$\frac{\left(dx^3+bx\right)\left(dx^3+bx\right)^n}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+b)*(d*x^3+b*x)^n,x, algorithm="fricas")
```

```
[Out] (d*x^3 + b*x)*(d*x^3 + b*x)^n/(n + 1)
```

Sympy [B] time = 12.4504, size = 73, normalized size = 3.84

$$\begin{cases} \frac{bx(bx+dx^3)^n}{n+1} + \frac{dx^3(bx+dx^3)^n}{n+1} & \text{for } n \neq -1 \\ \log(x) + \log\left(-i\sqrt{b}\sqrt{\frac{1}{d}} + x\right) + \log\left(i\sqrt{b}\sqrt{\frac{1}{d}} + x\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x**2+b)*(d*x**3+b*x)**n,x)
```

```
[Out] Piecewise((b*x*(b*x + d*x**3)**n/(n + 1) + d*x**3*(b*x + d*x**3)**n/(n + 1)
, Ne(n, -1)), (log(x) + log(-I*sqrt(b)*sqrt(1/d) + x) + log(I*sqrt(b)*sqrt(
1/d) + x), True))
```

Giac [A] time = 1.20395, size = 50, normalized size = 2.63

$$\frac{\left(dx^3 + bx\right)^n dx^3 + \left(dx^3 + bx\right)^n bx}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+b)*(d*x^3+b*x)^n,x, algorithm="giac")
```

```
[Out] ((d*x^3 + b*x)^n*d*x^3 + (d*x^3 + b*x)^n*b*x)/(n + 1)
```

3.185 $\int x^n \left(b+dx^2\right)^n \left(b+3dx^2\right) \, dx$

Optimal. Leaf size=22

$$\frac{x^{n+1} \left(b+dx^2\right)^{n+1}}{n+1}$$

```
[Out] (x^(1 + n)*(b + d*x^2)^(1 + n))/(1 + n)
```

Rubi [A] time = 0.0080456, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {449}

$$\frac{x^{n+1} \left(b+dx^2\right)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^n*(b + d*x^2)^n*(b + 3*d*x^2),x]
```

```
[Out] (x^(1 + n)*(b + d*x^2)^(1 + n))/(1 + n)
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\int x^n \left(b+dx^2\right)^n \left(b+3dx^2\right) \, dx = \frac{x^{1+n} \left(b+dx^2\right)^{1+n}}{1+n}$$

Mathematica [C] time = 0.0354555, size = 108, normalized size = 4.91

$$\frac{x^{n+1} \left(b+dx^2\right)^n \left(\frac{dx^2}{b}+1\right)^{-n} \left(3d(n+1)x^2 \, _2F_1\left(-n,\frac{n+3}{2};\frac{n+5}{2};-\frac{dx^2}{b}\right)+b(n+3) \, _2F_1\left(-n,\frac{n+1}{2};\frac{n+3}{2};-\frac{dx^2}{b}\right)\right)}{(n+1)(n+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^n*(b + d*x^2)^n*(b + 3*d*x^2),x]
```

```
[Out] (x^(1 + n)*(b + d*x^2)^n*(b*(3 + n)*Hypergeometric2F1[-n, (1 + n)/2, (3 + n
)/2, -((d*x^2)/b)] + 3*d*(1 + n)*x^2*Hypergeometric2F1[-n, (3 + n)/2, (5 +
n)/2, -((d*x^2)/b)]))/((1 + n)*(3 + n)*(1 + (d*x^2)/b)^n)
```

Maple [A] time = 0.003, size = 23, normalized size = 1.1

$$\frac{x^{1+n} \left(d x^{2}+b\right)^{1+n}}{1+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^n*(d*x^2+b)^n*(3*d*x^2+b),x)
```

```
[Out] x^(1+n)*(d*x^2+b)^(1+n)/(1+n)
```

Maxima [A] time = 1.19996, size = 42, normalized size = 1.91

$$\frac{\left(d x^{3}+b x\right) e^{\left(n \log\left(d x^{2}+b\right)+n \log\left(x\right)\right)}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n*(d*x^2+b)^n*(3*d*x^2+b),x, algorithm="maxima")
```

```
[Out] (d*x^3 + b*x)*e^(n*log(d*x^2 + b) + n*log(x))/(n + 1)
```

Fricas [A] time = 1.6317, size = 55, normalized size = 2.5

$$\frac{\left(d x^{3}+b x\right) \left(d x^{2}+b\right)^{n} x^{n}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n*(d*x^2+b)^n*(3*d*x^2+b),x, algorithm="fricas")
```

```
[Out] (d*x^3 + b*x)*(d*x^2 + b)^n*x^n/(n + 1)
```

Sympy [B] time = 58.6106, size = 76, normalized size = 3.45

$$\begin{cases} \frac{bxx^n(b+dx^2)^n}{n+1} + \frac{dx^3x^n(b+dx^2)^n}{n+1} & \text{for } n \neq -1 \\ \log(x) + \log\left(-i\sqrt{b}\sqrt{\frac{1}{d}} + x\right) + \log\left(i\sqrt{b}\sqrt{\frac{1}{d}} + x\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**n*(d*x**2+b)**n*(3*d*x**2+b),x)
```

```
[Out] Piecewise((b*x*x**n*(b + d*x**2)**n/(n + 1) + d*x**3*x**n*(b + d*x**2)**n/(
n + 1), Ne(n, -1)), (log(x) + log(-I*sqrt(b)*sqrt(1/d) + x) + log(I*sqrt(b)
*sqrt(1/d) + x), True))
```

Giac [A] time = 1.15311, size = 53, normalized size = 2.41

$$\frac{(dx^2+b)^n dx^3x^n + (dx^2+b)^n bxx^n}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n*(d*x^2+b)^n*(3*d*x^2+b),x, algorithm="giac")
```

```
[Out] ((d*x^2 + b)^n*d*x^3*x^n + (d*x^2 + b)^n*b*x*x^n)/(n + 1)
```

3.186 $\int \left(2cx+3dx^2\right)\left(a+cx^2+dx^3\right)^n\,dx$

Optimal. Leaf size=22

$$\frac{\left(a+cx^2+dx^3\right)^{n+1}}{n+1}$$

```
[Out] (a + c*x^2 + d*x^3)^(1 + n)/(1 + n)
```

Rubi [A] time = 0.0177172, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {1588}

$$\frac{\left(a+cx^2+dx^3\right)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(2*c*x + 3*d*x^2)*(a + c*x^2 + d*x^3)^n,x]
```

```
[Out] (a + c*x^2 + d*x^3)^(1 + n)/(1 + n)
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \left(2cx+3dx^2\right)\left(a+cx^2+dx^3\right)^n\,dx=\frac{\left(a+cx^2+dx^3\right)^{1+n}}{1+n}$$

Mathematica [A] time = 0.011625, size = 21, normalized size = 0.95

$$\frac{\left(a+x^2(c+dx)\right)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*c*x + 3*d*x^2)*(a + c*x^2 + d*x^3)^n,x]
```

```
[Out] (a + x^2*(c + d*x))^(1 + n)/(1 + n)
```

Maple [A] time = 0.003, size = 23, normalized size = 1.1

$$\frac{\left(dx^3 + cx^2 + a\right)^{1+n}}{1+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*d*x^2+2*c*x)*(d*x^3+c*x^2+a)^n,x)
```

```
[Out] (d*x^3+c*x^2+a)^(1+n)/(1+n)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2+a)^n,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.62482, size = 69, normalized size = 3.14

$$\frac{\left(dx^3 + cx^2 + a\right)\left(dx^3 + cx^2 + a\right)^n}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2+a)^n,x, algorithm="fricas")
```

```
[Out] (d*x^3 + c*x^2 + a)*(d*x^3 + c*x^2 + a)^n/(n + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x**2+2*c*x)*(d*x**3+c*x**2+a)**n,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.17653, size = 82, normalized size = 3.73

$$\frac{\left(dx^3+cx^2+a\right)^n dx^3+\left(dx^3+cx^2+a\right)^n cx^2+\left(dx^3+cx^2+a\right)^n a}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2+a)^n,x, algorithm="giac")
```

```
[Out] ((d*x^3 + c*x^2 + a)^n*d*x^3 + (d*x^3 + c*x^2 + a)^n*c*x^2 + (d*x^3 + c*x^2
 + a)^n*a)/(n + 1)
```

3.187 $\int \left(2cx+3dx^2\right)\left(cx^2+dx^3\right)^n\,dx$

Optimal. Leaf size=21

$$\frac{\left(cx^2+dx^3\right)^{n+1}}{n+1}$$

[Out] (c*x^2 + d*x^3)^(1 + n)/(1 + n)

Rubi [A] time = 0.0116747, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {1588}

$$\frac{\left(cx^2+dx^3\right)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

[In] Int[(2*c*x + 3*d*x^2)*(c*x^2 + d*x^3)^n,x]

[Out] (c*x^2 + d*x^3)^(1 + n)/(1 + n)

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \left(2cx+3dx^2\right)\left(cx^2+dx^3\right)^n\,dx=\frac{\left(cx^2+dx^3\right)^{1+n}}{1+n}$$

Mathematica [A] time = 0.0110014, size = 19, normalized size = 0.9

$$\frac{\left(x^2(c+dx)\right)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*c*x + 3*d*x^2)*(c*x^2 + d*x^3)^n,x]
```

```
[Out] (x^2*(c + d*x))^(1 + n)/(1 + n)
```

Maple [A] time = 0.003, size = 28, normalized size = 1.3

$$\frac{\left(dx^3+cx^2\right)^n x^2 (dx+c)}{1+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*d*x^2+2*c*x)*(d*x^3+c*x^2)^n,x)
```

```
[Out] (d*x^3+c*x^2)^n*x^2*(d*x+c)/(1+n)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2)^n,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.29356, size = 58, normalized size = 2.76

$$\frac{\left(dx^3+cx^2\right)\left(dx^3+cx^2\right)^n}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2)^n,x, algorithm="fricas")
```

```
[Out] (d*x^3 + c*x^2)*(d*x^3 + c*x^2)^n/(n + 1)
```

Sympy [A] time = 0.762702, size = 53, normalized size = 2.52

$$\begin{cases} \frac{cx^2(cx^2+dx^3)^n}{n+1} + \frac{dx^3(cx^2+dx^3)^n}{n+1} & \text{for } n \neq -1 \\ 2\log(x) + \log\left(\frac{c}{d} + x\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x**2+2*c*x)*(d*x**3+c*x**2)**n,x)
```

```
[Out] Piecewise(((c*x**2*(c*x**2 + d*x**3)**n/(n + 1) + d*x**3*(c*x**2 + d*x**3)**
n/(n + 1), Ne(n, -1)), (2*log(x) + log(c/d + x), True))
```

Giac [B] time = 1.28531, size = 58, normalized size = 2.76

$$\frac{(dx^3 + cx^2)^n dx^3 + (dx^3 + cx^2)^n cx^2}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2)^n,x, algorithm="giac")
```

```
[Out] ((d*x^3 + c*x^2)^n*d*x^3 + (d*x^3 + c*x^2)^n*c*x^2)/(n + 1)
```

3.188 $\int x^n \left(cx+dx^2\right)^n \left(2cx+3dx^2\right) dx$

Optimal. Leaf size=24

$$\frac{x^{n+1}\left(cx+dx^2\right)^{n+1}}{n+1}$$

```
[Out] (x^(1 + n)*(c*x + d*x^2)^(1 + n))/(1 + n)
```

Rubi [A] time = 0.0234698, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1584, 763}

$$\frac{x^{n+1}\left(cx+dx^2\right)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^n*(c*x + d*x^2)^n*(2*c*x + 3*d*x^2),x]
```

```
[Out] (x^(1 + n)*(c*x + d*x^2)^(1 + n))/(1 + n)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 763

```
Int[((e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))*((b_.)*(x_) + (c_.)*(x_)^2)^(p_.
), x_Symbol] :> Simp[(g*(e*x)^m*(b*x + c*x^2)^(p + 1))/(c*(m + 2*p + 2)), x
] /; FreeQ[{b, c, e, f, g, m, p}, x] && EqQ[b*g*(m + p + 1) - c*f*(m + 2*p
+ 2), 0] && NeQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int x^n \left(cx+dx^2\right)^n \left(2cx+3dx^2\right)\,dx = \int x^{1+n}(2c+3dx)\left(cx+dx^2\right)^n\,dx$$
$$= \frac{x^{1+n}\left(cx+dx^2\right)^{1+n}}{1+n}$$

Mathematica [A] time = 0.0134235, size = 22, normalized size = 0.92

$$\frac{x^{n+1}(x(c+dx))^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^n*(c*x + d*x^2)^n*(2*c*x + 3*d*x^2),x]
```

```
[Out] (x^(1 + n)*(x*(c + d*x))^(1 + n))/(1 + n)
```

Maple [A] time = 0.001, size = 28, normalized size = 1.2

$$\frac{\left(dx^2+cx\right)^n x^{2+n}(dx+c)}{1+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^n*(d*x^2+c*x)^n*(3*d*x^2+2*c*x),x)
```

```
[Out] (d*x^2+c*x)^n*x^(2+n)*(d*x+c)/(1+n)
```

Maxima [A] time = 1.17363, size = 43, normalized size = 1.79

$$\frac{\left(dx^3+cx^2\right)e^{(n\log(dx+c)+2\,n\log(x))}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n*(d*x^2+c*x)^n*(3*d*x^2+2*c*x),x, algorithm="maxima")
```

```
[Out] (d*x^3 + c*x^2)*e^(n*log(d*x + c) + 2*n*log(x))/(n + 1)
```

Fricas [A] time = 1.42258, size = 61, normalized size = 2.54

$$\frac{\left(dx^3 + cx^2\right)\left(dx^2 + cx\right)^n x^n}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n*(d*x^2+c*x)^n*(3*d*x^2+2*c*x),x, algorithm="fricas")
```

```
[Out] (d*x^3 + c*x^2)*(d*x^2 + c*x)^n*x^n/(n + 1)
```

Sympy [A] time = 5.1559, size = 56, normalized size = 2.33

$$\begin{cases} \frac{cx^2x^n\left(cx+dx^2\right)^n}{n+1} + \frac{dx^3x^n\left(cx+dx^2\right)^n}{n+1} & \text{for } n \neq -1 \\ 2\log(x) + \log\left(\frac{c}{d} + x\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**n*(d*x**2+c*x)**n*(3*d*x**2+2*c*x),x)
```

```
[Out] Piecewise((c*x**2*x**n*(c*x + d*x**2)**n/(n + 1) + d*x**3*x**n*(c*x + d*x**
2)**n/(n + 1), Ne(n, -1)), (2*log(x) + log(c/d + x), True))
```

Giac [B] time = 1.12181, size = 69, normalized size = 2.88

$$\frac{dx^3x^n e^{(n\log(dx+c)+n\log(x))} + cx^2x^n e^{(n\log(dx+c)+n\log(x))}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n*(d*x^2+c*x)^n*(3*d*x^2+2*c*x),x, algorithm="giac")
```

```
[Out] (d*x^3*x^n*e^(n*log(d*x + c) + n*log(x)) + c*x^2*x^n*e^(n*log(d*x + c) + n*
log(x)))/(n + 1)
```

3.189 $\int x^{2n}(c+dx)^n \left(2cx+3dx^2\right) dx$

Optimal. Leaf size=22

$$\frac{x^{2(n+1)}(c+dx)^{n+1}}{n+1}$$

```
[Out] (x^(2*(1 + n))*(c + d*x)^(1 + n))/(1 + n)
```

Rubi [A] time = 0.009202, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {845}

$$\frac{x^{2(n+1)}(c+dx)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^(2*n)*(c + d*x)^n*(2*c*x + 3*d*x^2),x]
```

```
[Out] (x^(2*(1 + n))*(c + d*x)^(1 + n))/(1 + n)
```

Rule 845

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((b_.)*(x_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[(c*x^(m + 2)*(f + g*x)^(n + 1))/(g*(m + n + 3)), x] /; FreeQ[{
b, c, f, g, m, n}, x] && EqQ[c*f*(m + 2) - b*g*(m + n + 3), 0] && NeQ[m + n
 + 3, 0]
```

Rubi steps

$$\int x^{2n}(c+dx)^n \left(2cx+3dx^2\right) dx = \frac{x^{2(1+n)}(c+dx)^{1+n}}{1+n}$$

Mathematica [A] time = 0.0103197, size = 22, normalized size = 1.

$$\frac{x^{2n+2}(c+dx)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(2*n)*(c + d*x)^n*(2*c*x + 3*d*x^2),x]
```

```
[Out] (x^(2 + 2*n)*(c + d*x)^(1 + n))/(1 + n)
```

Maple [A] time = 0.003, size = 23, normalized size = 1.1

$$\frac{x^{2+2n}(dx+c)^{1+n}}{1+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n)*(d*x+c)^n*(3*d*x^2+2*c*x),x)
```

```
[Out] x^(2+2*n)*(d*x+c)^(1+n)/(1+n)
```

Maxima [A] time = 1.16639, size = 43, normalized size = 1.95

$$\frac{\left(dx^3+cx^2\right)e^{(n\log(dx+c)+2n\log(x))}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2*n)*(d*x+c)^n*(3*d*x^2+2*c*x),x, algorithm="maxima")
```

```
[Out] (d*x^3 + c*x^2)*e^(n*log(d*x + c) + 2*n*log(x))/(n + 1)
```

Fricas [A] time = 1.4309, size = 61, normalized size = 2.77

$$\frac{\left(dx^3+cx^2\right)(dx+c)^n x^{2n}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2*n)*(d*x+c)^n*(3*d*x^2+2*c*x),x, algorithm="fricas")
```

```
[Out] (d*x^3 + c*x^2)*(d*x + c)^n*x^(2*n)/(n + 1)
```

Sympy [A] time = 5.10322, size = 53, normalized size = 2.41

$$\begin{cases} \frac{cx^2x^{2n}(c+dx)^n}{n+1} + \frac{dx^3x^{2n}(c+dx)^n}{n+1} & \text{for } n \neq -1 \\ 2\log(x) + \log\left(\frac{c}{d} + x\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(2*n)*(d*x+c)**n*(3*d*x**2+2*c*x),x)
```

```
[Out] Piecewise((c*x**2*x**(2*n)*(c + d*x)**n/(n + 1) + d*x**3*x**(2*n)*(c + d*x)
**n/(n + 1), Ne(n, -1)), (2*log(x) + log(c/d + x), True))
```

Giac [A] time = 1.15633, size = 55, normalized size = 2.5

$$\frac{(dx+c)^n dx^3 x^{2n} + (dx+c)^n cx^2 x^{2n}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2*n)*(d*x+c)^n*(3*d*x^2+2*c*x),x, algorithm="giac")
```

```
[Out] ((d*x + c)^n*d*x^3*x^(2*n) + (d*x + c)^n*c*x^2*x^(2*n))/(n + 1)
```

3.190 $\int x(2c+3dx)\left(a+cx^2+dx^3\right)^n\,dx$

Optimal. Leaf size=22

$$\frac{\left(a+cx^2+dx^3\right)^{n+1}}{n+1}$$

```
[Out] (a + c*x^2 + d*x^3)^(1 + n)/(1 + n)
```

Rubi [A] time = 0.0133484, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {1588}

$$\frac{\left(a+cx^2+dx^3\right)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[x*(2*c + 3*d*x)*(a + c*x^2 + d*x^3)^n,x]
```

```
[Out] (a + c*x^2 + d*x^3)^(1 + n)/(1 + n)
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int x(2c+3dx)\left(a+cx^2+dx^3\right)^n\,dx=\frac{\left(a+cx^2+dx^3\right)^{1+n}}{1+n}$$

Mathematica [A] time = 0.0062507, size = 21, normalized size = 0.95

$$\frac{\left(a+x^2(c+dx)\right)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(2*c + 3*d*x)*(a + c*x^2 + d*x^3)^n,x]
```

```
[Out] (a + x^2*(c + d*x))^(1 + n)/(1 + n)
```

Maple [A] time = 0.002, size = 23, normalized size = 1.1

$$\frac{\left(dx^3 + cx^2 + a\right)^{1+n}}{1+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(3*d*x+2*c)*(d*x^3+c*x^2+a)^n,x)
```

```
[Out] (d*x^3+c*x^2+a)^(1+n)/(1+n)
```

Maxima [A] time = 1.15229, size = 43, normalized size = 1.95

$$\frac{\left(dx^3 + cx^2 + a\right)\left(dx^3 + cx^2 + a\right)^n}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2+a)^n,x, algorithm="maxima")
```

```
[Out] (d*x^3 + c*x^2 + a)*(d*x^3 + c*x^2 + a)^n/(n + 1)
```

Fricas [A] time = 1.54229, size = 69, normalized size = 3.14

$$\frac{\left(dx^3 + cx^2 + a\right)\left(dx^3 + cx^2 + a\right)^n}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2+a)^n,x, algorithm="fricas")
```

```
[Out] (d*x^3 + c*x^2 + a)*(d*x^3 + c*x^2 + a)^n/(n + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*d*x+2*c)*(d*x**3+c*x**2+a)**n,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.26076, size = 82, normalized size = 3.73

$$\frac{\left(dx^3+cx^2+a\right)^n dx^3+\left(dx^3+cx^2+a\right)^n cx^2+\left(dx^3+cx^2+a\right)^n a}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2+a)^n,x, algorithm="giac")
```

```
[Out] ((d*x^3 + c*x^2 + a)^n*d*x^3 + (d*x^3 + c*x^2 + a)^n*c*x^2 + (d*x^3 + c*x^2
 + a)^n*a)/(n + 1)
```

3.191 $\int x(2c+3dx)\left(cx^2+dx^3\right)^n\,dx$

Optimal. Leaf size=21

$$\frac{\left(cx^2+dx^3\right)^{n+1}}{n+1}$$

```
[Out] (c*x^2 + d*x^3)^(1 + n)/(1 + n)
```

Rubi [A] time = 0.012103, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1588}

$$\frac{\left(cx^2+dx^3\right)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[x*(2*c + 3*d*x)*(c*x^2 + d*x^3)^n,x]
```

```
[Out] (c*x^2 + d*x^3)^(1 + n)/(1 + n)
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int x(2c+3dx)\left(cx^2+dx^3\right)^n\,dx=\frac{\left(cx^2+dx^3\right)^{1+n}}{1+n}$$

Mathematica [A] time = 0.0046984, size = 19, normalized size = 0.9

$$\frac{\left(x^2(c+dx)\right)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(2*c + 3*d*x)*(c*x^2 + d*x^3)^n,x]
```

```
[Out] (x^2*(c + d*x))^(1 + n)/(1 + n)
```

Maple [A] time = 0.002, size = 28, normalized size = 1.3

$$\frac{\left(dx^{3}+cx^{2}\right)^{n} x^{2} \left(dx+c\right)}{1+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(3*d*x+2*c)*(d*x^3+c*x^2)^n,x)
```

```
[Out] (d*x^3+c*x^2)^n*x^2*(d*x+c)/(1+n)
```

Maxima [A] time = 1.17194, size = 43, normalized size = 2.05

$$\frac{\left(dx^{3}+cx^{2}\right)e^{\left(n\log(dx+c)+2\,n\log(x)\right)}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2)^n,x, algorithm="maxima")
```

```
[Out] (d*x^3 + c*x^2)*e^(n*log(d*x + c) + 2*n*log(x))/(n + 1)
```

Fricas [A] time = 1.41657, size = 58, normalized size = 2.76

$$\frac{\left(dx^{3}+cx^{2}\right)\left(dx^{3}+cx^{2}\right)^{n}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2)^n,x, algorithm="fricas")
```

```
[Out] (d*x^3 + c*x^2)*(d*x^3 + c*x^2)^n/(n + 1)
```

Sympy [A] time = 0.773654, size = 53, normalized size = 2.52

$$\begin{cases} \frac{cx^2(cx^2+dx^3)^n}{n+1} + \frac{dx^3(cx^2+dx^3)^n}{n+1} & \text{for } n \neq -1 \\ 2\log(x) + \log\left(\frac{c}{d} + x\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*d*x+2*c)*(d*x**3+c*x**2)**n,x)
```

```
[Out] Piecewise(((c*x**2*(c*x**2 + d*x**3)**n/(n + 1) + d*x**3*(c*x**2 + d*x**3)**
n/(n + 1), Ne(n, -1)), (2*log(x) + log(c/d + x), True))
```

Giac [B] time = 1.29148, size = 58, normalized size = 2.76

$$\frac{(dx^3 + cx^2)^n dx^3 + (dx^3 + cx^2)^n cx^2}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2)^n,x, algorithm="giac")
```

```
[Out] ((d*x^3 + c*x^2)^n*d*x^3 + (d*x^3 + c*x^2)^n*c*x^2)/(n + 1)
```

3.192 $\int \left(b+2cx+3dx^2\right) \left(a+bx+cx^2+dx^3\right)^7 \, dx$

Optimal. Leaf size=21

$$\frac{1}{8} \left(a+bx+cx^2+dx^3\right)^8$$

```
[Out] (a + b*x + c*x^2 + d*x^3)^8/8
```

Rubi [A] time = 0.125167, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {1588}

$$\frac{1}{8} \left(a+bx+cx^2+dx^3\right)^8$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x + 3*d*x^2)*(a + b*x + c*x^2 + d*x^3)^7,x]
```

```
[Out] (a + b*x + c*x^2 + d*x^3)^8/8
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \left(b+2cx+3dx^2\right) \left(a+bx+cx^2+dx^3\right)^7 \, dx = \frac{1}{8} \left(a+bx+cx^2+dx^3\right)^8$$

Mathematica [B] time = 0.146059, size = 143, normalized size = 6.81

$$\frac{1}{8}x(b+x(c+dx)) \left(56a^5x^2(b+x(c+dx))^2+70a^4x^3(b+x(c+dx))^3+56a^3x^4(b+x(c+dx))^4+28a^2x^5(b+x(c+dx))^5\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x + 3*d*x^2)*(a + b*x + c*x^2 + d*x^3)^7,x]
```

```
[Out] (x*(b + x*(c + d*x))*(8*a^7 + 28*a^6*x*(b + x*(c + d*x)) + 56*a^5*x^2*(b +
x*(c + d*x))^2 + 70*a^4*x^3*(b + x*(c + d*x))^3 + 56*a^3*x^4*(b + x*(c + d*
x))^4 + 28*a^2*x^5*(b + x*(c + d*x))^5 + 8*a*x^6*(b + x*(c + d*x))^6 + x^7*
(b + x*(c + d*x))^7))/8
```

Maple [B] time = 0.005, size = 25686, normalized size = 1223.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x+a)^7,x)
```

```
[Out] result too large to display
```

Maxima [A] time = 1.00193, size = 26, normalized size = 1.24

$$\frac{1}{8}\left(dx^3 + cx^2 + bx + a\right)^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x+a)^7,x, algorithm="maxima")
```

```
[Out] 1/8*(d*x^3 + c*x^2 + b*x + a)^8
```

Fricas [B] time = 1.15867, size = 4385, normalized size = 208.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/8*x^24*d^8 + x^23*d^7*c + 7/2*x^22*d^6*c^2 + x^22*d^7*b + 7*x^21*d^5*c^3
+ 7*x^21*d^6*c*b + x^21*d^7*a + 35/4*x^20*d^4*c^4 + 21*x^20*d^5*c^2*b + 7/2
*x^20*d^6*b^2 + 7*x^20*d^6*c*a + 7*x^19*d^3*c^5 + 35*x^19*d^4*c^3*b + 21*x^
19*d^5*c*b^2 + 21*x^19*d^5*c^2*a + 7*x^19*d^6*b*a + 7/2*x^18*d^2*c^6 + 35*x
^18*d^3*c^4*b + 105/2*x^18*d^4*c^2*b^2 + 7*x^18*d^5*b^3 + 35*x^18*d^4*c^3*a
 + 42*x^18*d^5*c*b*a + 7/2*x^18*d^6*a^2 + x^17*d*c^7 + 21*x^17*d^2*c^5*b +
70*x^17*d^3*c^3*b^2 + 35*x^17*d^4*c*b^3 + 35*x^17*d^3*c^4*a + 105*x^17*d^4*
c^2*b*a + 21*x^17*d^5*b^2*a + 21*x^17*d^5*c*a^2 + 1/8*x^16*c^8 + 7*x^16*d*c
^6*b + 105/2*x^16*d^2*c^4*b^2 + 70*x^16*d^3*c^2*b^3 + 35/4*x^16*d^4*b^4 + 2
1*x^16*d^2*c^5*a + 140*x^16*d^3*c^3*b*a + 105*x^16*d^4*c*b^2*a + 105/2*x^16
*d^4*c^2*a^2 + 21*x^16*d^5*b*a^2 + x^15*c^7*b + 21*x^15*d*c^5*b^2 + 70*x^15
*d^2*c^3*b^3 + 35*x^15*d^3*c*b^4 + 7*x^15*d*c^6*a + 105*x^15*d^2*c^4*b*a +
210*x^15*d^3*c^2*b^2*a + 35*x^15*d^4*b^3*a + 70*x^15*d^3*c^3*a^2 + 105*x^15
*d^4*c*b*a^2 + 7*x^15*d^5*a^3 + 7/2*x^14*c^6*b^2 + 35*x^14*d*c^4*b^3 + 105/
2*x^14*d^2*c^2*b^4 + 7*x^14*d^3*b^5 + x^14*c^7*a + 42*x^14*d*c^5*b*a + 210*
x^14*d^2*c^3*b^2*a + 140*x^14*d^3*c*b^3*a + 105/2*x^14*d^2*c^4*a^2 + 210*x^
14*d^3*c^2*b*a^2 + 105/2*x^14*d^4*b^2*a^2 + 35*x^14*d^4*c*a^3 + 7*x^13*c^5*
b^3 + 35*x^13*d*c^3*b^4 + 21*x^13*d^2*c*b^5 + 7*x^13*c^6*b*a + 105*x^13*d*c
^4*b^2*a + 210*x^13*d^2*c^2*b^3*a + 35*x^13*d^3*b^4*a + 21*x^13*d*c^5*a^2 +
 210*x^13*d^2*c^3*b*a^2 + 210*x^13*d^3*c*b^2*a^2 + 70*x^13*d^3*c^2*a^3 + 35
*x^13*d^4*b*a^3 + 35/4*x^12*c^4*b^4 + 21*x^12*d*c^2*b^5 + 7/2*x^12*d^2*b^6
+ 21*x^12*c^5*b^2*a + 140*x^12*d*c^3*b^3*a + 105*x^12*d^2*c*b^4*a + 7/2*x^1
2*c^6*a^2 + 105*x^12*d*c^4*b*a^2 + 315*x^12*d^2*c^2*b^2*a^2 + 70*x^12*d^3*b
^3*a^2 + 70*x^12*d^2*c^3*a^3 + 140*x^12*d^3*c*b*a^3 + 35/4*x^12*d^4*a^4 + 7
*x^11*c^3*b^5 + 7*x^11*d*c*b^6 + 35*x^11*c^4*b^3*a + 105*x^11*d*c^2*b^4*a +
 21*x^11*d^2*b^5*a + 21*x^11*c^5*b*a^2 + 210*x^11*d*c^3*b^2*a^2 + 210*x^11*
d^2*c*b^3*a^2 + 35*x^11*d*c^4*a^3 + 210*x^11*d^2*c^2*b*a^3 + 70*x^11*d^3*b^
2*a^3 + 35*x^11*d^3*c*a^4 + 7/2*x^10*c^2*b^6 + x^10*d*b^7 + 35*x^10*c^3*b^4
*a + 42*x^10*d*c*b^5*a + 105/2*x^10*c^4*b^2*a^2 + 210*x^10*d*c^2*b^3*a^2 +
105/2*x^10*d^2*b^4*a^2 + 7*x^10*c^5*a^3 + 140*x^10*d*c^3*b*a^3 + 210*x^10*d
^2*c*b^2*a^3 + 105/2*x^10*d^2*c^2*a^4 + 35*x^10*d^3*b*a^4 + x^9*c*b^7 + 21*
x^9*c^2*b^5*a + 7*x^9*d*b^6*a + 70*x^9*c^3*b^3*a^2 + 105*x^9*d*c*b^4*a^2 +
35*x^9*c^4*b*a^3 + 210*x^9*d*c^2*b^2*a^3 + 70*x^9*d^2*b^3*a^3 + 35*x^9*d*c^
3*a^4 + 105*x^9*d^2*c*b*a^4 + 7*x^9*d^3*a^5 + 1/8*x^8*b^8 + 7*x^8*c*b^6*a +
 105/2*x^8*c^2*b^4*a^2 + 21*x^8*d*b^5*a^2 + 70*x^8*c^3*b^2*a^3 + 140*x^8*d*
c*b^3*a^3 + 35/4*x^8*c^4*a^4 + 105*x^8*d*c^2*b*a^4 + 105/2*x^8*d^2*b^2*a^4
+ 21*x^8*d^2*c*a^5 + x^7*b^7*a + 21*x^7*c*b^5*a^2 + 70*x^7*c^2*b^3*a^3 + 35
*x^7*d*b^4*a^3 + 35*x^7*c^3*b*a^4 + 105*x^7*d*c*b^2*a^4 + 21*x^7*d*c^2*a^5
+ 21*x^7*d^2*b*a^5 + 7/2*x^6*b^6*a^2 + 35*x^6*c*b^4*a^3 + 105/2*x^6*c^2*b^2
*a^4 + 35*x^6*d*b^3*a^4 + 7*x^6*c^3*a^5 + 42*x^6*d*c*b*a^5 + 7/2*x^6*d^2*a^
6 + 7*x^5*b^5*a^3 + 35*x^5*c*b^3*a^4 + 21*x^5*c^2*b*a^5 + 21*x^5*d*b^2*a^5
+ 7*x^5*d*c*a^6 + 35/4*x^4*b^4*a^4 + 21*x^4*c*b^2*a^5 + 7/2*x^4*c^2*a^6 + 7
*x^4*d*b*a^6 + 7*x^3*b^3*a^5 + 7*x^3*c*b*a^6 + x^3*d*a^7 + 7/2*x^2*b^2*a^6
+ x^2*c*a^7 + x*b*a^7
```

Sympy [B] time = 0.344554, size = 1771, normalized size = 84.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x**2+2*c*x+b)*(d*x**3+c*x**2+b*x+a)**7,x)
```

```
[Out] a**7*b*x + c*d**7*x**23 + d**8*x**24/8 + x**22*(b*d**7 + 7*c**2*d**6/2) + x
**21*(a*d**7 + 7*b*c*d**6 + 7*c**3*d**5) + x**20*(7*a*c*d**6 + 7*b**2*d**6/
2 + 21*b*c**2*d**5 + 35*c**4*d**4/4) + x**19*(7*a*b*d**6 + 21*a*c**2*d**5 +
 21*b**2*c*d**5 + 35*b*c**3*d**4 + 7*c**5*d**3) + x**18*(7*a**2*d**6/2 + 42
*a*b*c*d**5 + 35*a*c**3*d**4 + 7*b**3*d**5 + 105*b**2*c**2*d**4/2 + 35*b*c*
*4*d**3 + 7*c**6*d**2/2) + x**17*(21*a**2*c*d**5 + 21*a*b**2*d**5 + 105*a*b
*c**2*d**4 + 35*a*c**4*d**3 + 35*b**3*c*d**4 + 70*b**2*c**3*d**3 + 21*b*c**
5*d**2 + c**7*d) + x**16*(21*a**2*b*d**5 + 105*a**2*c**2*d**4/2 + 105*a*b**
2*c*d**4 + 140*a*b*c**3*d**3 + 21*a*c**5*d**2 + 35*b**4*d**4/4 + 70*b**3*c*
*2*d**3 + 105*b**2*c**4*d**2/2 + 7*b*c**6*d + c**8/8) + x**15*(7*a**3*d**5
+ 105*a**2*b*c*d**4 + 70*a**2*c**3*d**3 + 35*a*b**3*d**4 + 210*a*b**2*c**2*
d**3 + 105*a*b*c**4*d**2 + 7*a*c**6*d + 35*b**4*c*d**3 + 70*b**3*c**3*d**2
+ 21*b**2*c**5*d + b*c**7) + x**14*(35*a**3*c*d**4 + 105*a**2*b**2*d**4/2 +
 210*a**2*b*c**2*d**3 + 105*a**2*c**4*d**2/2 + 140*a*b**3*c*d**3 + 210*a*b*
*2*c**3*d**2 + 42*a*b*c**5*d + a*c**7 + 7*b**5*d**3 + 105*b**4*c**2*d**2/2
+ 35*b**3*c**4*d + 7*b**2*c**6/2) + x**13*(35*a**3*b*d**4 + 70*a**3*c**2*d*
*3 + 210*a**2*b**2*c*d**3 + 210*a**2*b*c**3*d**2 + 21*a**2*c**5*d + 35*a*b*
*4*d**3 + 210*a*b**3*c**2*d**2 + 105*a*b**2*c**4*d + 7*a*b*c**6 + 21*b**5*c
*d**2 + 35*b**4*c**3*d + 7*b**3*c**5) + x**12*(35*a**4*d**4/4 + 140*a**3*b*
c*d**3 + 70*a**3*c**3*d**2 + 70*a**2*b**3*d**3 + 315*a**2*b**2*c**2*d**2 +
105*a**2*b*c**4*d + 7*a**2*c**6/2 + 105*a*b**4*c*d**2 + 140*a*b**3*c**3*d +
 21*a*b**2*c**5 + 7*b**6*d**2/2 + 21*b**5*c**2*d + 35*b**4*c**4/4) + x**11*
(35*a**4*c*d**3 + 70*a**3*b**2*d**3 + 210*a**3*b*c**2*d**2 + 35*a**3*c**4*d
 + 210*a**2*b**3*c*d**2 + 210*a**2*b**2*c**3*d + 21*a**2*b*c**5 + 21*a*b**5
*d**2 + 105*a*b**4*c**2*d + 35*a*b**3*c**4 + 7*b**6*c*d + 7*b**5*c**3) + x*
*10*(35*a**4*b*d**3 + 105*a**4*c**2*d**2/2 + 210*a**3*b**2*c*d**2 + 140*a**
3*b*c**3*d + 7*a**3*c**5 + 105*a**2*b**4*d**2/2 + 210*a**2*b**3*c**2*d + 10
5*a**2*b**2*c**4/2 + 42*a*b**5*c*d + 35*a*b**4*c**3 + b**7*d + 7*b**6*c**2/
2) + x**9*(7*a**5*d**3 + 105*a**4*b*c*d**2 + 35*a**4*c**3*d + 70*a**3*b**3*
d**2 + 210*a**3*b**2*c**2*d + 35*a**3*b*c**4 + 105*a**2*b**4*c*d + 70*a**2*
b**3*c**3 + 7*a*b**6*d + 21*a*b**5*c**2 + b**7*c) + x**8*(21*a**5*c*d**2 +
105*a**4*b**2*d**2/2 + 105*a**4*b*c**2*d + 35*a**4*c**4/4 + 140*a**3*b**3*c
*d + 70*a**3*b**2*c**3 + 21*a**2*b**5*d + 105*a**2*b**4*c**2/2 + 7*a*b**6*c
 + b**8/8) + x**7*(21*a**5*b*d**2 + 21*a**5*c**2*d + 105*a**4*b**2*c*d + 35
*a**4*b*c**3 + 35*a**3*b**4*d + 70*a**3*b**3*c**2 + 21*a**2*b**5*c + a*b**7
```

```
) + x**6*(7*a**6*d**2/2 + 42*a**5*b*c*d + 7*a**5*c**3 + 35*a**4*b**3*d + 10
5*a**4*b**2*c**2/2 + 35*a**3*b**4*c + 7*a**2*b**6/2) + x**5*(7*a**6*c*d + 2
1*a**5*b**2*d + 21*a**5*b*c**2 + 35*a**4*b**3*c + 7*a**3*b**5) + x**4*(7*a*
*6*b*d + 7*a**6*c**2/2 + 21*a**5*b**2*c + 35*a**4*b**4/4) + x**3*(a**7*d +
7*a**6*b*c + 7*a**5*b**3) + x**2*(a**7*c + 7*a**6*b**2/2)
```

Giac [B] time = 1.22099, size = 2641, normalized size = 125.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x+a)^7,x, algorithm="giac")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + b*d^7*x^22 + 7*c^3*d^5*x^21
+ 7*b*c*d^6*x^21 + a*d^7*x^21 + 35/4*c^4*d^4*x^20 + 21*b*c^2*d^5*x^20 + 7/2
*b^2*d^6*x^20 + 7*a*c*d^6*x^20 + 7*c^5*d^3*x^19 + 35*b*c^3*d^4*x^19 + 21*b^
2*c*d^5*x^19 + 21*a*c^2*d^5*x^19 + 7*a*b*d^6*x^19 + 7/2*c^6*d^2*x^18 + 35*b
*c^4*d^3*x^18 + 105/2*b^2*c^2*d^4*x^18 + 35*a*c^3*d^4*x^18 + 7*b^3*d^5*x^18
 + 42*a*b*c*d^5*x^18 + 7/2*a^2*d^6*x^18 + c^7*d*x^17 + 21*b*c^5*d^2*x^17 +
70*b^2*c^3*d^3*x^17 + 35*a*c^4*d^3*x^17 + 35*b^3*c*d^4*x^17 + 105*a*b*c^2*d
^4*x^17 + 21*a*b^2*d^5*x^17 + 21*a^2*c*d^5*x^17 + 1/8*c^8*x^16 + 7*b*c^6*d*
x^16 + 105/2*b^2*c^4*d^2*x^16 + 21*a*c^5*d^2*x^16 + 70*b^3*c^2*d^3*x^16 + 1
40*a*b*c^3*d^3*x^16 + 35/4*b^4*d^4*x^16 + 105*a*b^2*c*d^4*x^16 + 105/2*a^2*
c^2*d^4*x^16 + 21*a^2*b*d^5*x^16 + b*c^7*x^15 + 21*b^2*c^5*d*x^15 + 7*a*c^6
*d*x^15 + 70*b^3*c^3*d^2*x^15 + 105*a*b*c^4*d^2*x^15 + 35*b^4*c*d^3*x^15 +
210*a*b^2*c^2*d^3*x^15 + 70*a^2*c^3*d^3*x^15 + 35*a*b^3*d^4*x^15 + 105*a^2*
b*c*d^4*x^15 + 7*a^3*d^5*x^15 + 7/2*b^2*c^6*x^14 + a*c^7*x^14 + 35*b^3*c^4*
d*x^14 + 42*a*b*c^5*d*x^14 + 105/2*b^4*c^2*d^2*x^14 + 210*a*b^2*c^3*d^2*x^1
4 + 105/2*a^2*c^4*d^2*x^14 + 7*b^5*d^3*x^14 + 140*a*b^3*c*d^3*x^14 + 210*a^
2*b*c^2*d^3*x^14 + 105/2*a^2*b^2*d^4*x^14 + 35*a^3*c*d^4*x^14 + 7*b^3*c^5*x
^13 + 7*a*b*c^6*x^13 + 35*b^4*c^3*d*x^13 + 105*a*b^2*c^4*d*x^13 + 21*a^2*c^
5*d*x^13 + 21*b^5*c*d^2*x^13 + 210*a*b^3*c^2*d^2*x^13 + 210*a^2*b*c^3*d^2*x
^13 + 35*a*b^4*d^3*x^13 + 210*a^2*b^2*c*d^3*x^13 + 70*a^3*c^2*d^3*x^13 + 35
*a^3*b*d^4*x^13 + 35/4*b^4*c^4*x^12 + 21*a*b^2*c^5*x^12 + 7/2*a^2*c^6*x^12
+ 21*b^5*c^2*d*x^12 + 140*a*b^3*c^3*d*x^12 + 105*a^2*b*c^4*d*x^12 + 7/2*b^6
*d^2*x^12 + 105*a*b^4*c*d^2*x^12 + 315*a^2*b^2*c^2*d^2*x^12 + 70*a^3*c^3*d^
2*x^12 + 70*a^2*b^3*d^3*x^12 + 140*a^3*b*c*d^3*x^12 + 35/4*a^4*d^4*x^12 + 7
*b^5*c^3*x^11 + 35*a*b^3*c^4*x^11 + 21*a^2*b*c^5*x^11 + 7*b^6*c*d*x^11 + 10
5*a*b^4*c^2*d*x^11 + 210*a^2*b^2*c^3*d*x^11 + 35*a^3*c^4*d*x^11 + 21*a*b^5*
d^2*x^11 + 210*a^2*b^3*c*d^2*x^11 + 210*a^3*b*c^2*d^2*x^11 + 70*a^3*b^2*d^3
*x^11 + 35*a^4*c*d^3*x^11 + 7/2*b^6*c^2*x^10 + 35*a*b^4*c^3*x^10 + 105/2*a^
```

```
2*b^2*c^4*x^10 + 7*a^3*c^5*x^10 + b^7*d*x^10 + 42*a*b^5*c*d*x^10 + 210*a^2*
b^3*c^2*d*x^10 + 140*a^3*b*c^3*d*x^10 + 105/2*a^2*b^4*d^2*x^10 + 210*a^3*b^
2*c*d^2*x^10 + 105/2*a^4*c^2*d^2*x^10 + 35*a^4*b*d^3*x^10 + b^7*c*x^9 + 21*
a*b^5*c^2*x^9 + 70*a^2*b^3*c^3*x^9 + 35*a^3*b*c^4*x^9 + 7*a*b^6*d*x^9 + 105
*a^2*b^4*c*d*x^9 + 210*a^3*b^2*c^2*d*x^9 + 35*a^4*c^3*d*x^9 + 70*a^3*b^3*d^
2*x^9 + 105*a^4*b*c*d^2*x^9 + 7*a^5*d^3*x^9 + 1/8*b^8*x^8 + 7*a*b^6*c*x^8 +
 105/2*a^2*b^4*c^2*x^8 + 70*a^3*b^2*c^3*x^8 + 35/4*a^4*c^4*x^8 + 21*a^2*b^5
*d*x^8 + 140*a^3*b^3*c*d*x^8 + 105*a^4*b*c^2*d*x^8 + 105/2*a^4*b^2*d^2*x^8
+ 21*a^5*c*d^2*x^8 + a*b^7*x^7 + 21*a^2*b^5*c*x^7 + 70*a^3*b^3*c^2*x^7 + 35
*a^4*b*c^3*x^7 + 35*a^3*b^4*d*x^7 + 105*a^4*b^2*c*d*x^7 + 21*a^5*c^2*d*x^7
+ 21*a^5*b*d^2*x^7 + 7/2*a^2*b^6*x^6 + 35*a^3*b^4*c*x^6 + 105/2*a^4*b^2*c^2
*x^6 + 7*a^5*c^3*x^6 + 35*a^4*b^3*d*x^6 + 42*a^5*b*c*d*x^6 + 7/2*a^6*d^2*x^
6 + 7*a^3*b^5*x^5 + 35*a^4*b^3*c*x^5 + 21*a^5*b*c^2*x^5 + 21*a^5*b^2*d*x^5
+ 7*a^6*c*d*x^5 + 35/4*a^4*b^4*x^4 + 21*a^5*b^2*c*x^4 + 7/2*a^6*c^2*x^4 + 7
*a^6*b*d*x^4 + 7*a^5*b^3*x^3 + 7*a^6*b*c*x^3 + a^7*d*x^3 + 7/2*a^6*b^2*x^2
+ a^7*c*x^2 + a^7*b*x
```

3.193 $\int \left(b+2cx+3dx^2\right)\left(bx+cx^2+dx^3\right)^7\,dx$

Optimal. Leaf size=20

$$\frac{1}{8}\left(bx+cx^2+dx^3\right)^8$$

```
[Out] (b*x + c*x^2 + d*x^3)^8/8
```

Rubi [A] time = 0.0521532, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {1588}

$$\frac{1}{8}\left(bx+cx^2+dx^3\right)^8$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x + 3*d*x^2)*(b*x + c*x^2 + d*x^3)^7,x]
```

```
[Out] (b*x + c*x^2 + d*x^3)^8/8
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \left(b+2cx+3dx^2\right)\left(bx+cx^2+dx^3\right)^7\,dx=\frac{1}{8}\left(bx+cx^2+dx^3\right)^8$$

Mathematica [A] time = 0.0264014, size = 18, normalized size = 0.9

$$\frac{1}{8}x^8(b+x(c+dx))^8$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x + 3*d*x^2)*(b*x + c*x^2 + d*x^3)^7,x]
```

```
[Out] (x^8*(b + x*(c + d*x))^8)/8
```

Maple [B] time = 0.003, size = 5596, normalized size = 279.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x)^7,x)
```

```
[Out] result too large to display
```

Maxima [A] time = 1.00126, size = 24, normalized size = 1.2

$$\frac{1}{8}\left(dx^3 + cx^2 + bx\right)^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x)^7,x, algorithm="maxima")
```

```
[Out] 1/8*(d*x^3 + c*x^2 + b*x)^8
```

Fricas [B] time = 1.20495, size = 1118, normalized size = 55.9

$$\frac{1}{8}x^{24}d^8 + x^{23}d^7c + \frac{7}{2}x^{22}d^6c^2 + x^{22}d^7b + 7x^{21}d^5c^3 + 7x^{21}d^6cb + \frac{35}{4}x^{20}d^4c^4 + 21x^{20}d^5c^2b + \frac{7}{2}x^{20}d^6b^2 + 7x^{19}d^3c^5 + 35x^{19}d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x)^7,x, algorithm="fricas")
```

```
[Out] 1/8*x^24*d^8 + x^23*d^7*c + 7/2*x^22*d^6*c^2 + x^22*d^7*b + 7*x^21*d^5*c^3
+ 7*x^21*d^6*c*b + 35/4*x^20*d^4*c^4 + 21*x^20*d^5*c^2*b + 7/2*x^20*d^6*b^2
```

```
 + 7*x^19*d^3*c^5 + 35*x^19*d^4*c^3*b + 21*x^19*d^5*c*b^2 + 7/2*x^18*d^2*c^
6 + 35*x^18*d^3*c^4*b + 105/2*x^18*d^4*c^2*b^2 + 7*x^18*d^5*b^3 + x^17*d*c^
7 + 21*x^17*d^2*c^5*b + 70*x^17*d^3*c^3*b^2 + 35*x^17*d^4*c*b^3 + 1/8*x^16*
c^8 + 7*x^16*d*c^6*b + 105/2*x^16*d^2*c^4*b^2 + 70*x^16*d^3*c^2*b^3 + 35/4*
x^16*d^4*b^4 + x^15*c^7*b + 21*x^15*d*c^5*b^2 + 70*x^15*d^2*c^3*b^3 + 35*x^
15*d^3*c*b^4 + 7/2*x^14*c^6*b^2 + 35*x^14*d*c^4*b^3 + 105/2*x^14*d^2*c^2*b^
4 + 7*x^14*d^3*b^5 + 7*x^13*c^5*b^3 + 35*x^13*d*c^3*b^4 + 21*x^13*d^2*c*b^5
 + 35/4*x^12*c^4*b^4 + 21*x^12*d*c^2*b^5 + 7/2*x^12*d^2*b^6 + 7*x^11*c^3*b^
5 + 7*x^11*d*c*b^6 + 7/2*x^10*c^2*b^6 + x^10*d*b^7 + x^9*c*b^7 + 1/8*x^8*b^
8
```

Sympy [B] time = 0.164026, size = 469, normalized size = 23.45

$$\frac{b^8x^8}{8} + b^7cx^9 + cd^7x^{23} + \frac{d^8x^{24}}{8} + x^{22}\left(bd^7 + \frac{7c^2d^6}{2}\right) + x^{21}\left(7bcd^6 + 7c^3d^5\right) + x^{20}\left(\frac{7b^2d^6}{2} + 21bc^2d^5 + \frac{35c^4d^4}{4}\right) + x^{19}\left(21\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x**2+2*c*x+b)*(d*x**3+c*x**2+b*x)**7,x)
```

```
[Out] b**8*x**8/8 + b**7*c*x**9 + c*d**7*x**23 + d**8*x**24/8 + x**22*(b*d**7 + 7
*c**2*d**6/2) + x**21*(7*b*c*d**6 + 7*c**3*d**5) + x**20*(7*b**2*d**6/2 + 2
1*b*c**2*d**5 + 35*c**4*d**4/4) + x**19*(21*b**2*c*d**5 + 35*b*c**3*d**4 +
7*c**5*d**3) + x**18*(7*b**3*d**5 + 105*b**2*c**2*d**4/2 + 35*b*c**4*d**3 +
 7*c**6*d**2/2) + x**17*(35*b**3*c*d**4 + 70*b**2*c**3*d**3 + 21*b*c**5*d**
2 + c**7*d) + x**16*(35*b**4*d**4/4 + 70*b**3*c**2*d**3 + 105*b**2*c**4*d**
2/2 + 7*b*c**6*d + c**8/8) + x**15*(35*b**4*c*d**3 + 70*b**3*c**3*d**2 + 21
*b**2*c**5*d + b*c**7) + x**14*(7*b**5*d**3 + 105*b**4*c**2*d**2/2 + 35*b**
3*c**4*d + 7*b**2*c**6/2) + x**13*(21*b**5*c*d**2 + 35*b**4*c**3*d + 7*b**3
*c**5) + x**12*(7*b**6*d**2/2 + 21*b**5*c**2*d + 35*b**4*c**4/4) + x**11*(7
*b**6*c*d + 7*b**5*c**3) + x**10*(b**7*d + 7*b**6*c**2/2)
```

Giac [B] time = 1.17305, size = 670, normalized size = 33.5

$$\frac{1}{8}d^8x^{24} + cd^7x^{23} + \frac{7}{2}c^2d^6x^{22} + bd^7x^{22} + 7c^3d^5x^{21} + 7bcd^6x^{21} + \frac{35}{4}c^4d^4x^{20} + 21bc^2d^5x^{20} + \frac{7}{2}b^2d^6x^{20} + 7c^5d^3x^{19} + 35$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x)^7,x, algorithm="giac")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + b*d^7*x^22 + 7*c^3*d^5*x^21
+ 7*b*c*d^6*x^21 + 35/4*c^4*d^4*x^20 + 21*b*c^2*d^5*x^20 + 7/2*b^2*d^6*x^20
 + 7*c^5*d^3*x^19 + 35*b*c^3*d^4*x^19 + 21*b^2*c*d^5*x^19 + 7/2*c^6*d^2*x^1
8 + 35*b*c^4*d^3*x^18 + 105/2*b^2*c^2*d^4*x^18 + 7*b^3*d^5*x^18 + c^7*d*x^1
7 + 21*b*c^5*d^2*x^17 + 70*b^2*c^3*d^3*x^17 + 35*b^3*c*d^4*x^17 + 1/8*c^8*x
^16 + 7*b*c^6*d*x^16 + 105/2*b^2*c^4*d^2*x^16 + 70*b^3*c^2*d^3*x^16 + 35/4*
b^4*d^4*x^16 + b*c^7*x^15 + 21*b^2*c^5*d*x^15 + 70*b^3*c^3*d^2*x^15 + 35*b^
4*c*d^3*x^15 + 7/2*b^2*c^6*x^14 + 35*b^3*c^4*d*x^14 + 105/2*b^4*c^2*d^2*x^1
4 + 7*b^5*d^3*x^14 + 7*b^3*c^5*x^13 + 35*b^4*c^3*d*x^13 + 21*b^5*c*d^2*x^13
 + 35/4*b^4*c^4*x^12 + 21*b^5*c^2*d*x^12 + 7/2*b^6*d^2*x^12 + 7*b^5*c^3*x^1
1 + 7*b^6*c*d*x^11 + 7/2*b^6*c^2*x^10 + b^7*d*x^10 + b^7*c*x^9 + 1/8*b^8*x^
8
```

3.194 $\int x^7 \left(b+cx+dx^2\right)^7 \left(b+2cx+3dx^2\right) dx$

Optimal. Leaf size=19

$$\frac{1}{8}x^8\left(b+cx+dx^2\right)^8$$

```
[Out] (x^8*(b + c*x + d*x^2)^8)/8
```

Rubi [A] time = 0.0693817, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {1588}

$$\frac{1}{8}x^8\left(b+cx+dx^2\right)^8$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(b + c*x + d*x^2)^7*(b + 2*c*x + 3*d*x^2),x]
```

```
[Out] (x^8*(b + c*x + d*x^2)^8)/8
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int x^7\left(b+cx+dx^2\right)^7\left(b+2cx+3dx^2\right)\,dx=\frac{1}{8}x^8\left(b+cx+dx^2\right)^8$$

Mathematica [A] time = 0.0112248, size = 18, normalized size = 0.95

$$\frac{1}{8}x^8(b+x(c+dx))^8$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(b + c*x + d*x^2)^7*(b + 2*c*x + 3*d*x^2),x]
```

```
[Out] (x^8*(b + x*(c + d*x))^8)/8
```

Maple [B] time = 0.002, size = 5596, normalized size = 294.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(d*x^2+c*x+b)^7*(3*d*x^2+2*c*x+b),x)
```

```
[Out] result too large to display
```

Maxima [B] time = 0.978812, size = 595, normalized size = 31.32

$$\frac{1}{8}\,d^8x^{24} + cd^7x^{23} + \frac{1}{2}\left(7\,c^2d^6 + 2\,bd^7\right)x^{22} + 7\left(c^3d^5 + bcd^6\right)x^{21} + \frac{7}{4}\left(5\,c^4d^4 + 12\,bc^2d^5 + 2\,b^2d^6\right)x^{20} + 7\left(c^5d^3 + 5\,bc^3d^4 + 3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(d*x^2+c*x+b)^7*(3*d*x^2+2*c*x+b),x, algorithm="maxima")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 1/2*(7*c^2*d^6 + 2*b*d^7)*x^22 + 7*(c^3*d^5 + b
*c*d^6)*x^21 + 7/4*(5*c^4*d^4 + 12*b*c^2*d^5 + 2*b^2*d^6)*x^20 + 7*(c^5*d^3
 + 5*b*c^3*d^4 + 3*b^2*c*d^5)*x^19 + 7/2*(c^6*d^2 + 10*b*c^4*d^3 + 15*b^2*c
^2*d^4 + 2*b^3*d^5)*x^18 + (c^7*d + 21*b*c^5*d^2 + 70*b^2*c^3*d^3 + 35*b^3*
c*d^4)*x^17 + b^7*c*x^9 + 1/8*(c^8 + 56*b*c^6*d + 420*b^2*c^4*d^2 + 560*b^3
*c^2*d^3 + 70*b^4*d^4)*x^16 + 1/8*b^8*x^8 + (b*c^7 + 21*b^2*c^5*d + 70*b^3*
c^3*d^2 + 35*b^4*c*d^3)*x^15 + 7/2*(b^2*c^6 + 10*b^3*c^4*d + 15*b^4*c^2*d^2
 + 2*b^5*d^3)*x^14 + 7*(b^3*c^5 + 5*b^4*c^3*d + 3*b^5*c*d^2)*x^13 + 7/4*(5*
b^4*c^4 + 12*b^5*c^2*d + 2*b^6*d^2)*x^12 + 7*(b^5*c^3 + b^6*c*d)*x^11 + 1/2
*(7*b^6*c^2 + 2*b^7*d)*x^10
```

Fricas [B] time = 1.12426, size = 1118, normalized size = 58.84

$$\frac{1}{8}x^{24}d^8 + x^{23}d^7c + \frac{7}{2}x^{22}d^6c^2 + x^{22}d^7b + 7x^{21}d^5c^3 + 7x^{21}d^6cb + \frac{35}{4}x^{20}d^4c^4 + 21x^{20}d^5c^2b + \frac{7}{2}x^{20}d^6b^2 + 7x^{19}d^3c^5 + 35x^{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(d*x^2+c*x+b)^7*(3*d*x^2+2*c*x+b),x, algorithm="fricas")
```

```
[Out] 1/8*x^24*d^8 + x^23*d^7*c + 7/2*x^22*d^6*c^2 + x^22*d^7*b + 7*x^21*d^5*c^3
+ 7*x^21*d^6*c*b + 35/4*x^20*d^4*c^4 + 21*x^20*d^5*c^2*b + 7/2*x^20*d^6*b^2
 + 7*x^19*d^3*c^5 + 35*x^19*d^4*c^3*b + 21*x^19*d^5*c*b^2 + 7/2*x^18*d^2*c^
6 + 35*x^18*d^3*c^4*b + 105/2*x^18*d^4*c^2*b^2 + 7*x^18*d^5*b^3 + x^17*d*c^
7 + 21*x^17*d^2*c^5*b + 70*x^17*d^3*c^3*b^2 + 35*x^17*d^4*c*b^3 + 1/8*x^16*
c^8 + 7*x^16*d*c^6*b + 105/2*x^16*d^2*c^4*b^2 + 70*x^16*d^3*c^2*b^3 + 35/4*
x^16*d^4*b^4 + x^15*c^7*b + 21*x^15*d*c^5*b^2 + 70*x^15*d^2*c^3*b^3 + 35*x^
15*d^3*c*b^4 + 7/2*x^14*c^6*b^2 + 35*x^14*d*c^4*b^3 + 105/2*x^14*d^2*c^2*b^
4 + 7*x^14*d^3*b^5 + 7*x^13*c^5*b^3 + 35*x^13*d*c^3*b^4 + 21*x^13*d^2*c*b^5
 + 35/4*x^12*c^4*b^4 + 21*x^12*d*c^2*b^5 + 7/2*x^12*d^2*b^6 + 7*x^11*c^3*b^
5 + 7*x^11*d*c*b^6 + 7/2*x^10*c^2*b^6 + x^10*d*b^7 + x^9*c*b^7 + 1/8*x^8*b^
8
```

Sympy [B] time = 0.153872, size = 469, normalized size = 24.68

$$\frac{b^8x^8}{8} + b^7cx^9 + cd^7x^{23} + \frac{d^8x^{24}}{8} + x^{22}\left(bd^7 + \frac{7c^2d^6}{2}\right) + x^{21}\left(7bcd^6 + 7c^3d^5\right) + x^{20}\left(\frac{7b^2d^6}{2} + 21bc^2d^5 + \frac{35c^4d^4}{4}\right) + x^{19}\left(21\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(d*x**2+c*x+b)**7*(3*d*x**2+2*c*x+b),x)
```

```
[Out] b**8*x**8/8 + b**7*c*x**9 + c*d**7*x**23 + d**8*x**24/8 + x**22*(b*d**7 + 7
*c**2*d**6/2) + x**21*(7*b*c*d**6 + 7*c**3*d**5) + x**20*(7*b**2*d**6/2 + 2
1*b*c**2*d**5 + 35*c**4*d**4/4) + x**19*(21*b**2*c*d**5 + 35*b*c**3*d**4 +
7*c**5*d**3) + x**18*(7*b**3*d**5 + 105*b**2*c**2*d**4/2 + 35*b*c**4*d**3 +
 7*c**6*d**2/2) + x**17*(35*b**3*c*d**4 + 70*b**2*c**3*d**3 + 21*b*c**5*d**
2 + c**7*d) + x**16*(35*b**4*d**4/4 + 70*b**3*c**2*d**3 + 105*b**2*c**4*d**
2/2 + 7*b*c**6*d + c**8/8) + x**15*(35*b**4*c*d**3 + 70*b**3*c**3*d**2 + 21
*b**2*c**5*d + b*c**7) + x**14*(7*b**5*d**3 + 105*b**4*c**2*d**2/2 + 35*b**
3*c**4*d + 7*b**2*c**6/2) + x**13*(21*b**5*c*d**2 + 35*b**4*c**3*d + 7*b**3
*c**5) + x**12*(7*b**6*d**2/2 + 21*b**5*c**2*d + 35*b**4*c**4/4) + x**11*(7
```

```
*b**6*c*d + 7*b**5*c**3) + x**10*(b**7*d + 7*b**6*c**2/2)
```

Giac [B] time = 1.14315, size = 670, normalized size = 35.26

$$\frac{1}{8}d^8x^{24} + cd^7x^{23} + \frac{7}{2}c^2d^6x^{22} + bd^7x^{22} + 7c^3d^5x^{21} + 7bcd^6x^{21} + \frac{35}{4}c^4d^4x^{20} + 21bc^2d^5x^{20} + \frac{7}{2}b^2d^6x^{20} + 7c^5d^3x^{19} + 35b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(d*x^2+c*x+b)^7*(3*d*x^2+2*c*x+b),x, algorithm="giac")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + b*d^7*x^22 + 7*c^3*d^5*x^21
+ 7*b*c*d^6*x^21 + 35/4*c^4*d^4*x^20 + 21*b*c^2*d^5*x^20 + 7/2*b^2*d^6*x^20
 + 7*c^5*d^3*x^19 + 35*b*c^3*d^4*x^19 + 21*b^2*c*d^5*x^19 + 7/2*c^6*d^2*x^1
8 + 35*b*c^4*d^3*x^18 + 105/2*b^2*c^2*d^4*x^18 + 7*b^3*d^5*x^18 + c^7*d*x^1
7 + 21*b*c^5*d^2*x^17 + 70*b^2*c^3*d^3*x^17 + 35*b^3*c*d^4*x^17 + 1/8*c^8*x
^16 + 7*b*c^6*d*x^16 + 105/2*b^2*c^4*d^2*x^16 + 70*b^3*c^2*d^3*x^16 + 35/4*
b^4*d^4*x^16 + b*c^7*x^15 + 21*b^2*c^5*d*x^15 + 70*b^3*c^3*d^2*x^15 + 35*b^
4*c*d^3*x^15 + 7/2*b^2*c^6*x^14 + 35*b^3*c^4*d*x^14 + 105/2*b^4*c^2*d^2*x^1
4 + 7*b^5*d^3*x^14 + 7*b^3*c^5*x^13 + 35*b^4*c^3*d*x^13 + 21*b^5*c*d^2*x^13
 + 35/4*b^4*c^4*x^12 + 21*b^5*c^2*d*x^12 + 7/2*b^6*d^2*x^12 + 7*b^5*c^3*x^1
1 + 7*b^6*c*d*x^11 + 7/2*b^6*c^2*x^10 + b^7*d*x^10 + b^7*c*x^9 + 1/8*b^8*x^
8
```

3.195 $\int \left(b+3dx^2\right)\left(a+bx+dx^3\right)^7\,dx$

Optimal. Leaf size=16

$$\frac{1}{8}\left(a+bx+dx^3\right)^8$$

```
[Out] (a + b*x + d*x^3)^8/8
```

Rubi [A] time = 0.0243779, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {1588}

$$\frac{1}{8}\left(a+bx+dx^3\right)^8$$

Antiderivative was successfully verified.

```
[In] Int[(b + 3*d*x^2)*(a + b*x + d*x^3)^7,x]
```

```
[Out] (a + b*x + d*x^3)^8/8
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \left(b+3dx^2\right)\left(a+bx+dx^3\right)^7\,dx=\frac{1}{8}\left(a+bx+dx^3\right)^8$$

Mathematica [B] time = 0.0574379, size = 127, normalized size = 7.94

$$\frac{1}{8}x\left(b+dx^2\right)\left(28a^6x\left(b+dx^2\right)+56a^5x^2\left(b+dx^2\right)^2+70a^4x^3\left(b+dx^2\right)^3+56a^3x^4\left(b+dx^2\right)^4+28a^2x^5\left(b+dx^2\right)^5+8a^7\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 3*d*x^2)*(a + b*x + d*x^3)^7,x]
```

```
[Out] (x*(b + d*x^2)*(8*a^7 + 28*a^6*x*(b + d*x^2) + 56*a^5*x^2*(b + d*x^2)^2 + 7
0*a^4*x^3*(b + d*x^2)^3 + 56*a^3*x^4*(b + d*x^2)^4 + 28*a^2*x^5*(b + d*x^2)
^5 + 8*a*x^6*(b + d*x^2)^6 + x^7*(b + d*x^2)^7))/8
```

Maple [B] time = 0.002, size = 2185, normalized size = 136.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*d*x^2+b)*(d*x^3+b*x+a)^7,x)
```

```
[Out] 1/8*d^8*x^24+b*d^7*x^22+d^7*a*x^21+7/2*b^2*d^6*x^20+7*b*a*d^6*x^19+1/18*(21
*b^3*d^5+3*d*(6*a^2*d^5+15*b^3*d^4+d*(2*(3*a^2*d+b^3)*d^3+18*b^3*d^3+9*a^2*
d^4)))*x^18+21*b^2*a*d^5*x^17+1/16*(b*(6*a^2*d^5+15*b^3*d^4+d*(2*(3*a^2*d+b
^3)*d^3+18*b^3*d^3+9*a^2*d^4))+3*d*(30*a^2*d^4*b+b*(2*(3*a^2*d+b^3)*d^3+18*
b^3*d^3+9*a^2*d^4)+d*(42*a^2*b*d^3+6*(3*a^2*d+b^3)*b*d^2+9*b^4*d^2)))*x^16+
1/15*(105*b^3*a*d^4+3*d*(a*(2*(3*a^2*d+b^3)*d^3+18*b^3*d^3+9*a^2*d^4)+60*b^
3*a*d^3+d*(2*a^3*d^3+54*a*b^3*d^2+6*(3*a^2*d+b^3)*a*d^2)))*x^15+1/14*(b*(30
*a^2*d^4*b+b*(2*(3*a^2*d+b^3)*d^3+18*b^3*d^3+9*a^2*d^4)+d*(42*a^2*b*d^3+6*(
3*a^2*d+b^3)*b*d^2+9*b^4*d^2))+3*d*(60*a^2*b^2*d^3+b*(42*a^2*b*d^3+6*(3*a^2
*d+b^3)*b*d^2+9*b^4*d^2)+d*(72*a^2*b^2*d^2+6*(3*a^2*d+b^3)*b^2*d)))*x^14+1/
13*(b*(a*(2*(3*a^2*d+b^3)*d^3+18*b^3*d^3+9*a^2*d^4)+60*b^3*a*d^3+d*(2*a^3*d
^3+54*a*b^3*d^2+6*(3*a^2*d+b^3)*a*d^2))+3*d*(a*(42*a^2*b*d^3+6*(3*a^2*d+b^3
)*b*d^2+9*b^4*d^2)+b*(2*a^3*d^3+54*a*b^3*d^2+6*(3*a^2*d+b^3)*a*d^2)+d*(24*a
^3*b*d^2+18*a*b^4*d+12*(3*a^2*d+b^3)*d*a*b)))*x^13+1/12*(b*(60*a^2*b^2*d^3+
b*(42*a^2*b*d^3+6*(3*a^2*d+b^3)*b*d^2+9*b^4*d^2)+d*(72*a^2*b^2*d^2+6*(3*a^2
*d+b^3)*b^2*d))+3*d*(a*(2*a^3*d^3+54*a*b^3*d^2+6*(3*a^2*d+b^3)*a*d^2)+b*(72
*a^2*b^2*d^2+6*(3*a^2*d+b^3)*b^2*d)+d*(6*a^4*d^2+54*a^2*b^3*d+(3*a^2*d+b^3)
^2)))*x^12+1/11*(b*(a*(42*a^2*b*d^3+6*(3*a^2*d+b^3)*b*d^2+9*b^4*d^2)+b*(2*a
^3*d^3+54*a*b^3*d^2+6*(3*a^2*d+b^3)*a*d^2)+d*(24*a^3*b*d^2+18*a*b^4*d+12*(3
*a^2*d+b^3)*d*a*b))+3*d*(a*(72*a^2*b^2*d^2+6*(3*a^2*d+b^3)*b^2*d)+b*(24*a^3
*b*d^2+18*a*b^4*d+12*(3*a^2*d+b^3)*d*a*b)+d*(42*a^3*b^2*d+6*a*b^2*(3*a^2*d+
b^3))))*x^11+1/10*(b*(a*(2*a^3*d^3+54*a*b^3*d^2+6*(3*a^2*d+b^3)*a*d^2)+b*(7
2*a^2*b^2*d^2+6*(3*a^2*d+b^3)*b^2*d)+d*(6*a^4*d^2+54*a^2*b^3*d+(3*a^2*d+b^3
)^2))+3*d*(a*(24*a^3*b*d^2+18*a*b^4*d+12*(3*a^2*d+b^3)*d*a*b)+b*(6*a^4*d^2+
54*a^2*b^3*d+(3*a^2*d+b^3)^2)+d*(12*a^4*d*b+6*a^2*b*(3*a^2*d+b^3)+9*a^2*b^4
)))*x^10+1/9*(b*(a*(72*a^2*b^2*d^2+6*(3*a^2*d+b^3)*b^2*d)+b*(24*a^3*b*d^2+1
```

```
8*a*b^4*d+12*(3*a^2*d+b^3)*d*a*b)+d*(42*a^3*b^2*d+6*a*b^2*(3*a^2*d+b^3)))+3
*d*(a*(6*a^4*d^2+54*a^2*b^3*d+(3*a^2*d+b^3)^2)+b*(42*a^3*b^2*d+6*a*b^2*(3*a
^2*d+b^3))+d*(2*a^3*(3*a^2*d+b^3)+18*a^3*b^3)))*x^9+1/8*(b*(a*(24*a^3*b*d^2
+18*a*b^4*d+12*(3*a^2*d+b^3)*d*a*b)+b*(6*a^4*d^2+54*a^2*b^3*d+(3*a^2*d+b^3)
^2)+d*(12*a^4*d*b+6*a^2*b*(3*a^2*d+b^3)+9*a^2*b^4))+3*d*(a*(42*a^3*b^2*d+6*
a*b^2*(3*a^2*d+b^3))+b*(12*a^4*d*b+6*a^2*b*(3*a^2*d+b^3)+9*a^2*b^4)+15*d*a^
4*b^2))*x^8+1/7*(b*(a*(6*a^4*d^2+54*a^2*b^3*d+(3*a^2*d+b^3)^2)+b*(42*a^3*b^
2*d+6*a*b^2*(3*a^2*d+b^3))+d*(2*a^3*(3*a^2*d+b^3)+18*a^3*b^3))+3*d*(a*(12*a
^4*d*b+6*a^2*b*(3*a^2*d+b^3)+9*a^2*b^4)+b*(2*a^3*(3*a^2*d+b^3)+18*a^3*b^3)+
6*d*a^5*b))*x^7+1/6*(b*(a*(42*a^3*b^2*d+6*a*b^2*(3*a^2*d+b^3))+b*(12*a^4*d*
b+6*a^2*b*(3*a^2*d+b^3)+9*a^2*b^4)+15*d*a^4*b^2)+3*d*(a*(2*a^3*(3*a^2*d+b^3
)+18*a^3*b^3)+15*b^3*a^4+d*a^6))*x^6+1/5*(b*(a*(12*a^4*d*b+6*a^2*b*(3*a^2*d
+b^3)+9*a^2*b^4)+b*(2*a^3*(3*a^2*d+b^3)+18*a^3*b^3)+6*d*a^5*b)+63*d*b^2*a^5
)*x^5+1/4*(b*(a*(2*a^3*(3*a^2*d+b^3)+18*a^3*b^3)+15*b^3*a^4+d*a^6)+21*d*a^6
*b)*x^4+1/3*(3*a^7*d+21*a^5*b^3)*x^3+7/2*b^2*a^6*x^2+b*a^7*x
```

Maxima [A] time = 0.986591, size = 19, normalized size = 1.19

$$\frac{1}{8}\left(dx^3 + bx + a\right)^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+b)*(d*x^3+b*x+a)^7,x, algorithm="maxima")
```

```
[Out] 1/8*(d*x^3 + b*x + a)^8
```

Fricas [B] time = 1.17532, size = 1071, normalized size = 66.94

$$\frac{1}{8}x^{24}d^8 + x^{22}d^7b + x^{21}d^7a + \frac{7}{2}x^{20}d^6b^2 + 7x^{19}d^6ba + 7x^{18}d^5b^3 + \frac{7}{2}x^{18}d^6a^2 + 21x^{17}d^5b^2a + \frac{35}{4}x^{16}d^4b^4 + 21x^{16}d^5ba^2 + 35$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+b)*(d*x^3+b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/8*x^24*d^8 + x^22*d^7*b + x^21*d^7*a + 7/2*x^20*d^6*b^2 + 7*x^19*d^6*b*a
+ 7*x^18*d^5*b^3 + 7/2*x^18*d^6*a^2 + 21*x^17*d^5*b^2*a + 35/4*x^16*d^4*b^4
 + 21*x^16*d^5*b*a^2 + 35*x^15*d^4*b^3*a + 7*x^15*d^5*a^3 + 7*x^14*d^3*b^5
+ 105/2*x^14*d^4*b^2*a^2 + 35*x^13*d^3*b^4*a + 35*x^13*d^4*b*a^3 + 7/2*x^12
```

```
*d^2*b^6 + 70*x^12*d^3*b^3*a^2 + 35/4*x^12*d^4*a^4 + 21*x^11*d^2*b^5*a + 70
*x^11*d^3*b^2*a^3 + x^10*d*b^7 + 105/2*x^10*d^2*b^4*a^2 + 35*x^10*d^3*b*a^4
 + 7*x^9*d*b^6*a + 70*x^9*d^2*b^3*a^3 + 7*x^9*d^3*a^5 + 1/8*x^8*b^8 + 21*x^
8*d*b^5*a^2 + 105/2*x^8*d^2*b^2*a^4 + x^7*b^7*a + 35*x^7*d*b^4*a^3 + 21*x^7
*d^2*b*a^5 + 7/2*x^6*b^6*a^2 + 35*x^6*d*b^3*a^4 + 7/2*x^6*d^2*a^6 + 7*x^5*b
^5*a^3 + 21*x^5*d*b^2*a^5 + 35/4*x^4*b^4*a^4 + 7*x^4*d*b*a^6 + 7*x^3*b^3*a^
5 + x^3*d*a^7 + 7/2*x^2*b^2*a^6 + x*b*a^7
```

Sympy [B] time = 0.151129, size = 483, normalized size = 30.19

$$a^7bx + \frac{7a^6b^2x^2}{2} + 21ab^2d^5x^{17} + 7abd^6x^{19} + ad^7x^{21} + \frac{7b^2d^6x^{20}}{2} + bd^7x^{22} + \frac{d^8x^{24}}{8} + x^{18}\left(\frac{7a^2d^6}{2} + 7b^3d^5\right) + x^{16}\left(21a^2bd^5 \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x**2+b)*(d*x**3+b*x+a)**7,x)
```

```
[Out] a**7*b*x + 7*a**6*b**2*x**2/2 + 21*a*b**2*d**5*x**17 + 7*a*b*d**6*x**19 + a
*d**7*x**21 + 7*b**2*d**6*x**20/2 + b*d**7*x**22 + d**8*x**24/8 + x**18*(7*
a**2*d**6/2 + 7*b**3*d**5) + x**16*(21*a**2*b*d**5 + 35*b**4*d**4/4) + x**1
5*(7*a**3*d**5 + 35*a*b**3*d**4) + x**14*(105*a**2*b**2*d**4/2 + 7*b**5*d**
3) + x**13*(35*a**3*b*d**4 + 35*a*b**4*d**3) + x**12*(35*a**4*d**4/4 + 70*a
**2*b**3*d**3 + 7*b**6*d**2/2) + x**11*(70*a**3*b**2*d**3 + 21*a*b**5*d**2)
 + x**10*(35*a**4*b*d**3 + 105*a**2*b**4*d**2/2 + b**7*d) + x**9*(7*a**5*d*
*3 + 70*a**3*b**3*d**2 + 7*a*b**6*d) + x**8*(105*a**4*b**2*d**2/2 + 21*a**2
*b**5*d + b**8/8) + x**7*(21*a**5*b*d**2 + 35*a**3*b**4*d + a*b**7) + x**6*
(7*a**6*d**2/2 + 35*a**4*b**3*d + 7*a**2*b**6/2) + x**5*(21*a**5*b**2*d + 7
*a**3*b**5) + x**4*(7*a**6*b*d + 35*a**4*b**4/4) + x**3*(a**7*d + 7*a**5*b*
*3)
```

Giac [B] time = 1.21341, size = 656, normalized size = 41.

$$\frac{1}{8}\,d^8x^{24} + bd^7x^{22} + ad^7x^{21} + \frac{7}{2}\,b^2d^6x^{20} + 7\,abd^6x^{19} + 7\,b^3d^5x^{18} + \frac{7}{2}\,a^2d^6x^{18} + 21\,ab^2d^5x^{17} + \frac{35}{4}\,b^4d^4x^{16} + 21\,a^2bd^5x^{16} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+b)*(d*x^3+b*x+a)^7,x, algorithm="giac")
```

```
[Out] 1/8*d^8*x^24 + b*d^7*x^22 + a*d^7*x^21 + 7/2*b^2*d^6*x^20 + 7*a*b*d^6*x^19
+ 7*b^3*d^5*x^18 + 7/2*a^2*d^6*x^18 + 21*a*b^2*d^5*x^17 + 35/4*b^4*d^4*x^16
```

```
 + 21*a^2*b*d^5*x^16 + 35*a*b^3*d^4*x^15 + 7*a^3*d^5*x^15 + 7*b^5*d^3*x^14
+ 105/2*a^2*b^2*d^4*x^14 + 35*a*b^4*d^3*x^13 + 35*a^3*b*d^4*x^13 + 7/2*b^6*
d^2*x^12 + 70*a^2*b^3*d^3*x^12 + 35/4*a^4*d^4*x^12 + 21*a*b^5*d^2*x^11 + 70
*a^3*b^2*d^3*x^11 + b^7*d*x^10 + 105/2*a^2*b^4*d^2*x^10 + 35*a^4*b*d^3*x^10
 + 7*a*b^6*d*x^9 + 70*a^3*b^3*d^2*x^9 + 7*a^5*d^3*x^9 + 1/8*b^8*x^8 + 21*a^
2*b^5*d*x^8 + 105/2*a^4*b^2*d^2*x^8 + a*b^7*x^7 + 35*a^3*b^4*d*x^7 + 21*a^5
*b*d^2*x^7 + 7/2*a^2*b^6*x^6 + 35*a^4*b^3*d*x^6 + 7/2*a^6*d^2*x^6 + 7*a^3*b
^5*x^5 + 21*a^5*b^2*d*x^5 + 35/4*a^4*b^4*x^4 + 7*a^6*b*d*x^4 + 7*a^5*b^3*x^
3 + a^7*d*x^3 + 7/2*a^6*b^2*x^2 + a^7*b*x
```

3.196 $\int \left(b+3dx^2\right)\left(bx+dx^3\right)^7\,dx$

Optimal. Leaf size=15

$$\frac{1}{8}\left(bx+dx^3\right)^8$$

```
[Out] (b*x + d*x^3)^8/8
```

Rubi [A] time = 0.0127609, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {1588}

$$\frac{1}{8}\left(bx+dx^3\right)^8$$

Antiderivative was successfully verified.

```
[In] Int[(b + 3*d*x^2)*(b*x + d*x^3)^7,x]
```

```
[Out] (b*x + d*x^3)^8/8
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \left(b+3dx^2\right)\left(bx+dx^3\right)^7\,dx=\frac{1}{8}\left(bx+dx^3\right)^8$$

Mathematica [B] time = 0.0033504, size = 98, normalized size = 6.53

$$\frac{7}{2}b^2d^6x^{20}+7b^3d^5x^{18}+\frac{35}{4}b^4d^4x^{16}+7b^5d^3x^{14}+\frac{7}{2}b^6d^2x^{12}+b^7dx^{10}+\frac{b^8x^8}{8}+bd^7x^{22}+\frac{d^8x^{24}}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 3*d*x^2)*(b*x + d*x^3)^7,x]
```

```
[Out] (b^8*x^8)/8 + b^7*d*x^10 + (7*b^6*d^2*x^12)/2 + 7*b^5*d^3*x^14 + (35*b^4*d^
4*x^16)/4 + 7*b^3*d^5*x^18 + (7*b^2*d^6*x^20)/2 + b*d^7*x^22 + (d^8*x^24)/8
```

Maple [B] time = 0.002, size = 89, normalized size = 5.9

$$\frac{d^8x^{24}}{8}+bd^7x^{22}+\frac{7\,b^2d^6x^{20}}{2}+7\,b^3d^5x^{18}+\frac{35\,b^4d^4x^{16}}{4}+7\,b^5d^3x^{14}+\frac{7\,b^6d^2x^{12}}{2}+db^7x^{10}+\frac{b^8x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*d*x^2+b)*(d*x^3+b*x)^7,x)
```

```
[Out] 1/8*d^8*x^24+b*d^7*x^22+7/2*b^2*d^6*x^20+7*b^3*d^5*x^18+35/4*b^4*d^4*x^16+7
*b^5*d^3*x^14+7/2*b^6*d^2*x^12+d*b^7*x^10+1/8*b^8*x^8
```

Maxima [A] time = 1.01034, size = 18, normalized size = 1.2

$$\frac{1}{8}\left(dx^3+bx\right)^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+b)*(d*x^3+b*x)^7,x, algorithm="maxima")
```

```
[Out] 1/8*(d*x^3 + b*x)^8
```

Fricas [B] time = 1.12616, size = 197, normalized size = 13.13

$$\frac{1}{8}x^{24}d^8+x^{22}d^7b+\frac{7}{2}x^{20}d^6b^2+7x^{18}d^5b^3+\frac{35}{4}x^{16}d^4b^4+7x^{14}d^3b^5+\frac{7}{2}x^{12}d^2b^6+x^{10}db^7+\frac{1}{8}x^8b^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+b)*(d*x^3+b*x)^7,x, algorithm="fricas")
```

```
[Out] 1/8*x^24*d^8 + x^22*d^7*b + 7/2*x^20*d^6*b^2 + 7*x^18*d^5*b^3 + 35/4*x^16*d
^4*b^4 + 7*x^14*d^3*b^5 + 7/2*x^12*d^2*b^6 + x^10*d*b^7 + 1/8*x^8*b^8
```

Sympy [B] time = 0.089477, size = 97, normalized size = 6.47

$$\frac{b^8x^8}{8} + b^7dx^{10} + \frac{7b^6d^2x^{12}}{2} + 7b^5d^3x^{14} + \frac{35b^4d^4x^{16}}{4} + 7b^3d^5x^{18} + \frac{7b^2d^6x^{20}}{2} + bd^7x^{22} + \frac{d^8x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x**2+b)*(d*x**3+b*x)**7,x)
```

```
[Out] b**8*x**8/8 + b**7*d*x**10 + 7*b**6*d**2*x**12/2 + 7*b**5*d**3*x**14 + 35*b
**4*d**4*x**16/4 + 7*b**3*d**5*x**18 + 7*b**2*d**6*x**20/2 + b*d**7*x**22 +
 d**8*x**24/8
```

Giac [B] time = 1.19029, size = 119, normalized size = 7.93

$$\frac{1}{8}\,d^8x^{24} + bd^7x^{22} + \frac{7}{2}\,b^2d^6x^{20} + 7\,b^3d^5x^{18} + \frac{35}{4}\,b^4d^4x^{16} + 7\,b^5d^3x^{14} + \frac{7}{2}\,b^6d^2x^{12} + b^7dx^{10} + \frac{1}{8}\,b^8x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+b)*(d*x^3+b*x)^7,x, algorithm="giac")
```

```
[Out] 1/8*d^8*x^24 + b*d^7*x^22 + 7/2*b^2*d^6*x^20 + 7*b^3*d^5*x^18 + 35/4*b^4*d^
4*x^16 + 7*b^5*d^3*x^14 + 7/2*b^6*d^2*x^12 + b^7*d*x^10 + 1/8*b^8*x^8
```

3.197 $\int x^7 \left(b+dx^2\right)^7 \left(b+3dx^2\right) \, dx$

Optimal. Leaf size=16

$$\frac{1}{8}x^8 \left(b+dx^2\right)^8$$

```
[Out] (x^8*(b + d*x^2)^8)/8
```

Rubi [A] time = 0.0280873, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {446, 74}

$$\frac{1}{8}x^8 \left(b+dx^2\right)^8$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(b + d*x^2)^7*(b + 3*d*x^2),x]
```

```
[Out] (x^8*(b + d*x^2)^8)/8
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rubi steps

$$\int x^7 \left(b+dx^2\right)^7 \left(b+3dx^2\right) dx = \frac{1}{2} \operatorname{Subst}\left(\int x^3(b+dx)^7(b+3dx) \, dx, x, x^2\right)$$
$$= \frac{1}{8} x^8 \left(b+dx^2\right)^8$$

Mathematica [B] time = 0.0023896, size = 98, normalized size = 6.12

$$\frac{7}{2} b^2 d^6 x^{20} + 7 b^3 d^5 x^{18} + \frac{35}{4} b^4 d^4 x^{16} + 7 b^5 d^3 x^{14} + \frac{7}{2} b^6 d^2 x^{12} + b^7 d x^{10} + \frac{b^8 x^8}{8} + b d^7 x^{22} + \frac{d^8 x^{24}}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(b + d*x^2)^7*(b + 3*d*x^2),x]
```

```
[Out] (b^8*x^8)/8 + b^7*d*x^10 + (7*b^6*d^2*x^12)/2 + 7*b^5*d^3*x^14 + (35*b^4*d^
4*x^16)/4 + 7*b^3*d^5*x^18 + (7*b^2*d^6*x^20)/2 + b*d^7*x^22 + (d^8*x^24)/8
```

Maple [B] time = 0., size = 89, normalized size = 5.6

$$\frac{d^8 x^{24}}{8} + b d^7 x^{22} + \frac{7 b^2 d^6 x^{20}}{2} + 7 b^3 d^5 x^{18} + \frac{35 b^4 d^4 x^{16}}{4} + 7 b^5 d^3 x^{14} + \frac{7 b^6 d^2 x^{12}}{2} + d b^7 x^{10} + \frac{b^8 x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(d*x^2+b)^7*(3*d*x^2+b),x)
```

```
[Out] 1/8*d^8*x^24+b*d^7*x^22+7/2*b^2*d^6*x^20+7*b^3*d^5*x^18+35/4*b^4*d^4*x^16+7
*b^5*d^3*x^14+7/2*b^6*d^2*x^12+d*b^7*x^10+1/8*b^8*x^8
```

Maxima [B] time = 0.986427, size = 119, normalized size = 7.44

$$\frac{1}{8} d^8 x^{24} + b d^7 x^{22} + \frac{7}{2} b^2 d^6 x^{20} + 7 b^3 d^5 x^{18} + \frac{35}{4} b^4 d^4 x^{16} + 7 b^5 d^3 x^{14} + \frac{7}{2} b^6 d^2 x^{12} + b^7 d x^{10} + \frac{1}{8} b^8 x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(d*x^2+b)^7*(3*d*x^2+b),x, algorithm="maxima")
```

```
[Out] 1/8*d^8*x^24 + b*d^7*x^22 + 7/2*b^2*d^6*x^20 + 7*b^3*d^5*x^18 + 35/4*b^4*d^
4*x^16 + 7*b^5*d^3*x^14 + 7/2*b^6*d^2*x^12 + b^7*d*x^10 + 1/8*b^8*x^8
```

Fricas [B] time = 1.20792, size = 197, normalized size = 12.31

$$\frac{1}{8}x^{24}d^8 + x^{22}d^7b + \frac{7}{2}x^{20}d^6b^2 + 7x^{18}d^5b^3 + \frac{35}{4}x^{16}d^4b^4 + 7x^{14}d^3b^5 + \frac{7}{2}x^{12}d^2b^6 + x^{10}db^7 + \frac{1}{8}x^8b^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(d*x^2+b)^7*(3*d*x^2+b),x, algorithm="fricas")
```

```
[Out] 1/8*x^24*d^8 + x^22*d^7*b + 7/2*x^20*d^6*b^2 + 7*x^18*d^5*b^3 + 35/4*x^16*d
^4*b^4 + 7*x^14*d^3*b^5 + 7/2*x^12*d^2*b^6 + x^10*d*b^7 + 1/8*x^8*b^8
```

Sympy [B] time = 0.083103, size = 97, normalized size = 6.06

$$\frac{b^8x^8}{8} + b^7dx^{10} + \frac{7b^6d^2x^{12}}{2} + 7b^5d^3x^{14} + \frac{35b^4d^4x^{16}}{4} + 7b^3d^5x^{18} + \frac{7b^2d^6x^{20}}{2} + bd^7x^{22} + \frac{d^8x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(d*x**2+b)**7*(3*d*x**2+b),x)
```

```
[Out] b**8*x**8/8 + b**7*d*x**10 + 7*b**6*d**2*x**12/2 + 7*b**5*d**3*x**14 + 35*b
**4*d**4*x**16/4 + 7*b**3*d**5*x**18 + 7*b**2*d**6*x**20/2 + b*d**7*x**22 +
 d**8*x**24/8
```

Giac [B] time = 1.32276, size = 119, normalized size = 7.44

$$\frac{1}{8}d^8x^{24} + bd^7x^{22} + \frac{7}{2}b^2d^6x^{20} + 7b^3d^5x^{18} + \frac{35}{4}b^4d^4x^{16} + 7b^5d^3x^{14} + \frac{7}{2}b^6d^2x^{12} + b^7dx^{10} + \frac{1}{8}b^8x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(d*x^2+b)^7*(3*d*x^2+b),x, algorithm="giac")
```

```
[Out] 1/8*d^8*x^24 + b*d^7*x^22 + 7/2*b^2*d^6*x^20 + 7*b^3*d^5*x^18 + 35/4*b^4*d^
4*x^16 + 7*b^5*d^3*x^14 + 7/2*b^6*d^2*x^12 + b^7*d*x^10 + 1/8*b^8*x^8
```

3.198 $\int \left(2cx+3dx^2\right)\left(a+cx^2+dx^3\right)^7 \, dx$

Optimal. Leaf size=18

$$\frac{1}{8}\left(a+cx^2+dx^3\right)^8$$

```
[Out] (a + c*x^2 + d*x^3)^8/8
```

Rubi [A] time = 0.0441323, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {1588}

$$\frac{1}{8}\left(a+cx^2+dx^3\right)^8$$

Antiderivative was successfully verified.

```
[In] Int[(2*c*x + 3*d*x^2)*(a + c*x^2 + d*x^3)^7,x]
```

```
[Out] (a + c*x^2 + d*x^3)^8/8
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \left(2cx+3dx^2\right)\left(a+cx^2+dx^3\right)^7 \, dx=\frac{1}{8}\left(a+cx^2+dx^3\right)^8$$

Mathematica [B] time = 0.0532344, size = 115, normalized size = 6.39

$$\frac{1}{8}x^2(c+dx)\left(28a^2x^{10}(c+dx)^5+56a^3x^8(c+dx)^4+70a^4x^6(c+dx)^3+56a^5x^4(c+dx)^2+28a^6x^2(c+dx)+8a^7+8ax^{12}(c\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*c*x + 3*d*x^2)*(a + c*x^2 + d*x^3)^7,x]
```

```
[Out] (x^2*(c + d*x)*(8*a^7 + 28*a^6*x^2*(c + d*x) + 56*a^5*x^4*(c + d*x)^2 + 70*
a^4*x^6*(c + d*x)^3 + 56*a^3*x^8*(c + d*x)^4 + 28*a^2*x^10*(c + d*x)^5 + 8*
a*x^12*(c + d*x)^6 + x^14*(c + d*x)^7))/8
```

Maple [B] time = 0.002, size = 2205, normalized size = 122.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*d*x^2+2*c*x)*(d*x^3+c*x^2+a)^7,x)
```

```
[Out] 1/8*d^8*x^24+c*d^7*x^23+7/2*c^2*d^6*x^22+1/21*(42*c^3*d^5+3*d*(a*d^6+15*c^3
*d^4+d*(2*(3*a*d^2+c^3)*d^3+18*c^3*d^3)))*x^21+1/20*(2*c*(a*d^6+15*c^3*d^4+
d*(2*(3*a*d^2+c^3)*d^3+18*c^3*d^3))+3*d*(6*a*c*d^5+c*(2*(3*a*d^2+c^3)*d^3+1
8*c^3*d^3)+d*(12*a*c*d^4+6*(3*a*d^2+c^3)*c*d^2+9*c^4*d^2)))*x^20+1/19*(2*c*
(6*a*c*d^5+c*(2*(3*a*d^2+c^3)*d^3+18*c^3*d^3)+d*(12*a*c*d^4+6*(3*a*d^2+c^3)
*c*d^2+9*c^4*d^2))+3*d*(15*a*c^2*d^4+c*(12*a*c*d^4+6*(3*a*d^2+c^3)*c*d^2+9*
c^4*d^2)+d*(42*a*c^2*d^3+6*(3*a*d^2+c^3)*c^2*d)))*x^19+1/18*(2*c*(15*a*c^2*
d^4+c*(12*a*c*d^4+6*(3*a*d^2+c^3)*c*d^2+9*c^4*d^2)+d*(42*a*c^2*d^3+6*(3*a*d
^2+c^3)*c^2*d))+3*d*(a*(2*(3*a*d^2+c^3)*d^3+18*c^3*d^3)+c*(42*a*c^2*d^3+6*(
3*a*d^2+c^3)*c^2*d)+d*(6*a^2*d^4+54*a*c^3*d^2+(3*a*d^2+c^3)^2)))*x^18+1/17*
(2*c*(a*(2*(3*a*d^2+c^3)*d^3+18*c^3*d^3)+c*(42*a*c^2*d^3+6*(3*a*d^2+c^3)*c^
2*d)+d*(6*a^2*d^4+54*a*c^3*d^2+(3*a*d^2+c^3)^2))+3*d*(a*(12*a*c*d^4+6*(3*a*
d^2+c^3)*c*d^2+9*c^4*d^2)+c*(6*a^2*d^4+54*a*c^3*d^2+(3*a*d^2+c^3)^2)+d*(24*
a^2*c*d^3+18*c^4*a*d+12*a*c*d*(3*a*d^2+c^3))))*x^17+1/16*(2*c*(a*(12*a*c*d^
4+6*(3*a*d^2+c^3)*c*d^2+9*c^4*d^2)+c*(6*a^2*d^4+54*a*c^3*d^2+(3*a*d^2+c^3)^
2)+d*(24*a^2*c*d^3+18*c^4*a*d+12*a*c*d*(3*a*d^2+c^3)))+3*d*(a*(42*a*c^2*d^3
+6*(3*a*d^2+c^3)*c^2*d)+c*(24*a^2*c*d^3+18*c^4*a*d+12*a*c*d*(3*a*d^2+c^3))+
d*(72*a^2*c^2*d^2+6*c^2*a*(3*a*d^2+c^3))))*x^16+1/15*(2*c*(a*(42*a*c^2*d^3+
6*(3*a*d^2+c^3)*c^2*d)+c*(24*a^2*c*d^3+18*c^4*a*d+12*a*c*d*(3*a*d^2+c^3))+d
*(72*a^2*c^2*d^2+6*c^2*a*(3*a*d^2+c^3)))+3*d*(a*(6*a^2*d^4+54*a*c^3*d^2+(3*
a*d^2+c^3)^2)+c*(72*a^2*c^2*d^2+6*c^2*a*(3*a*d^2+c^3))+d*(2*a^3*d^3+54*a^2*
c^3*d+6*a^2*d*(3*a*d^2+c^3))))*x^15+1/14*(2*c*(a*(6*a^2*d^4+54*a*c^3*d^2+(3
*a*d^2+c^3)^2)+c*(72*a^2*c^2*d^2+6*c^2*a*(3*a*d^2+c^3))+d*(2*a^3*d^3+54*a^2
*c^3*d+6*a^2*d*(3*a*d^2+c^3)))+3*d*(a*(24*a^2*c*d^3+18*c^4*a*d+12*a*c*d*(3*
a*d^2+c^3))+c*(2*a^3*d^3+54*a^2*c^3*d+6*a^2*d*(3*a*d^2+c^3))+d*(42*a^3*c*d^
2+6*a^2*c*(3*a*d^2+c^3)+9*c^4*a^2)))*x^14+1/13*(2*c*(a*(24*a^2*c*d^3+18*c^4
```

```
*a*d+12*a*c*d*(3*a*d^2+c^3))+c*(2*a^3*d^3+54*a^2*c^3*d+6*a^2*d*(3*a*d^2+c^3
))+d*(42*a^3*c*d^2+6*a^2*c*(3*a*d^2+c^3)+9*c^4*a^2))+3*d*(a*(72*a^2*c^2*d^2
+6*c^2*a*(3*a*d^2+c^3))+c*(42*a^3*c*d^2+6*a^2*c*(3*a*d^2+c^3)+9*c^4*a^2)+60
*a^3*c^2*d^2))*x^13+1/12*(2*c*(a*(72*a^2*c^2*d^2+6*c^2*a*(3*a*d^2+c^3))+c*(
42*a^3*c*d^2+6*a^2*c*(3*a*d^2+c^3)+9*c^4*a^2)+60*a^3*c^2*d^2)+3*d*(a*(2*a^3
*d^3+54*a^2*c^3*d+6*a^2*d*(3*a*d^2+c^3))+60*a^3*c^3*d+d*(2*a^3*(3*a*d^2+c^3
)+18*a^3*c^3+9*a^4*d^2)))*x^12+1/11*(2*c*(a*(2*a^3*d^3+54*a^2*c^3*d+6*a^2*d
*(3*a*d^2+c^3))+60*a^3*c^3*d+d*(2*a^3*(3*a*d^2+c^3)+18*a^3*c^3+9*a^4*d^2))+
3*d*(a*(42*a^3*c*d^2+6*a^2*c*(3*a*d^2+c^3)+9*c^4*a^2)+c*(2*a^3*(3*a*d^2+c^3
)+18*a^3*c^3+9*a^4*d^2)+30*d^2*a^4*c))*x^11+1/10*(2*c*(a*(42*a^3*c*d^2+6*a^
2*c*(3*a*d^2+c^3)+9*c^4*a^2)+c*(2*a^3*(3*a*d^2+c^3)+18*a^3*c^3+9*a^4*d^2)+3
0*d^2*a^4*c)+315*d^2*a^4*c^2)*x^10+1/9*(210*c^3*a^4*d+3*d*(a*(2*a^3*(3*a*d^
2+c^3)+18*a^3*c^3+9*a^4*d^2)+15*c^3*a^4+6*d^2*a^5))*x^9+1/8*(2*c*(a*(2*a^3*
(3*a*d^2+c^3)+18*a^3*c^3+9*a^4*d^2)+15*c^3*a^4+6*d^2*a^5)+126*d^2*a^5*c)*x^
8+21*c^2*a^5*d*x^7+1/6*(21*a^6*d^2+42*a^5*c^3)*x^6+7*c*d*a^6*x^5+7/2*c^2*a^
6*x^4+d*a^7*x^3+c*a^7*x^2
```

Maxima [A] time = 0.984774, size = 22, normalized size = 1.22

$$\frac{1}{8}\left(dx^3 + cx^2 + a\right)^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2+a)^7,x, algorithm="maxima")
```

```
[Out] 1/8*(d*x^3 + c*x^2 + a)^8
```

Fricas [B] time = 1.13598, size = 1085, normalized size = 60.28

$$\frac{1}{8}x^{24}d^8 + x^{23}d^7c + \frac{7}{2}x^{22}d^6c^2 + 7x^{21}d^5c^3 + x^{21}d^7a + \frac{35}{4}x^{20}d^4c^4 + 7x^{20}d^6ca + 7x^{19}d^3c^5 + 21x^{19}d^5c^2a + \frac{7}{2}x^{18}d^2c^6 + 35x^{18}d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2+a)^7,x, algorithm="fricas")
```

```
[Out] 1/8*x^24*d^8 + x^23*d^7*c + 7/2*x^22*d^6*c^2 + 7*x^21*d^5*c^3 + x^21*d^7*a
+ 35/4*x^20*d^4*c^4 + 7*x^20*d^6*c*a + 7*x^19*d^3*c^5 + 21*x^19*d^5*c^2*a +
 7/2*x^18*d^2*c^6 + 35*x^18*d^4*c^3*a + 7/2*x^18*d^6*a^2 + x^17*d*c^7 + 35*
```

```
x^17*d^3*c^4*a + 21*x^17*d^5*c*a^2 + 1/8*x^16*c^8 + 21*x^16*d^2*c^5*a + 105
/2*x^16*d^4*c^2*a^2 + 7*x^15*d*c^6*a + 70*x^15*d^3*c^3*a^2 + 7*x^15*d^5*a^3
 + x^14*c^7*a + 105/2*x^14*d^2*c^4*a^2 + 35*x^14*d^4*c*a^3 + 21*x^13*d*c^5*
a^2 + 70*x^13*d^3*c^2*a^3 + 7/2*x^12*c^6*a^2 + 70*x^12*d^2*c^3*a^3 + 35/4*x
^12*d^4*a^4 + 35*x^11*d*c^4*a^3 + 35*x^11*d^3*c*a^4 + 7*x^10*c^5*a^3 + 105/
2*x^10*d^2*c^2*a^4 + 35*x^9*d*c^3*a^4 + 7*x^9*d^3*a^5 + 35/4*x^8*c^4*a^4 +
21*x^8*d^2*c*a^5 + 21*x^7*d*c^2*a^5 + 7*x^6*c^3*a^5 + 7/2*x^6*d^2*a^6 + 7*x
^5*d*c*a^6 + 7/2*x^4*c^2*a^6 + x^3*d*a^7 + x^2*c*a^7
```

Sympy [B] time = 0.152089, size = 484, normalized size = 26.89

$$a^7cx^2 + a^7dx^3 + \frac{7a^6c^2x^4}{2} + 7a^6cdx^5 + 21a^5c^2dx^7 + \frac{7c^2d^6x^{22}}{2} + cd^7x^{23} + \frac{d^8x^{24}}{8} + x^{21}\left(ad^7 + 7c^3d^5\right) + x^{20}\left(7acd^6 + \frac{35c^4}{4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x**2+2*c*x)*(d*x**3+c*x**2+a)**7,x)
```

```
[Out] a**7*c*x**2 + a**7*d*x**3 + 7*a**6*c**2*x**4/2 + 7*a**6*c*d*x**5 + 21*a**5*
c**2*d*x**7 + 7*c**2*d**6*x**22/2 + c*d**7*x**23 + d**8*x**24/8 + x**21*(a*
d**7 + 7*c**3*d**5) + x**20*(7*a*c*d**6 + 35*c**4*d**4/4) + x**19*(21*a*c**
2*d**5 + 7*c**5*d**3) + x**18*(7*a**2*d**6/2 + 35*a*c**3*d**4 + 7*c**6*d**2
/2) + x**17*(21*a**2*c*d**5 + 35*a*c**4*d**3 + c**7*d) + x**16*(105*a**2*c*
*2*d**4/2 + 21*a*c**5*d**2 + c**8/8) + x**15*(7*a**3*d**5 + 70*a**2*c**3*d*
*3 + 7*a*c**6*d) + x**14*(35*a**3*c*d**4 + 105*a**2*c**4*d**2/2 + a*c**7) +
 x**13*(70*a**3*c**2*d**3 + 21*a**2*c**5*d) + x**12*(35*a**4*d**4/4 + 70*a*
*3*c**3*d**2 + 7*a**2*c**6/2) + x**11*(35*a**4*c*d**3 + 35*a**3*c**4*d) + x
**10*(105*a**4*c**2*d**2/2 + 7*a**3*c**5) + x**9*(7*a**5*d**3 + 35*a**4*c**
3*d) + x**8*(21*a**5*c*d**2 + 35*a**4*c**4/4) + x**6*(7*a**6*d**2/2 + 7*a**
5*c**3)
```

Giac [B] time = 1.21118, size = 659, normalized size = 36.61

$$\frac{1}{8}d^8x^{24} + cd^7x^{23} + \frac{7}{2}c^2d^6x^{22} + 7c^3d^5x^{21} + ad^7x^{21} + \frac{35}{4}c^4d^4x^{20} + 7acd^6x^{20} + 7c^5d^3x^{19} + 21ac^2d^5x^{19} + \frac{7}{2}c^6d^2x^{18} + 35$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2+a)^7,x, algorithm="giac")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + a*d^7*x^21
+ 35/4*c^4*d^4*x^20 + 7*a*c*d^6*x^20 + 7*c^5*d^3*x^19 + 21*a*c^2*d^5*x^19 +
 7/2*c^6*d^2*x^18 + 35*a*c^3*d^4*x^18 + 7/2*a^2*d^6*x^18 + c^7*d*x^17 + 35*
a*c^4*d^3*x^17 + 21*a^2*c*d^5*x^17 + 1/8*c^8*x^16 + 21*a*c^5*d^2*x^16 + 105
/2*a^2*c^2*d^4*x^16 + 7*a*c^6*d*x^15 + 70*a^2*c^3*d^3*x^15 + 7*a^3*d^5*x^15
 + a*c^7*x^14 + 105/2*a^2*c^4*d^2*x^14 + 35*a^3*c*d^4*x^14 + 21*a^2*c^5*d*x
^13 + 70*a^3*c^2*d^3*x^13 + 7/2*a^2*c^6*x^12 + 70*a^3*c^3*d^2*x^12 + 35/4*a
^4*d^4*x^12 + 35*a^3*c^4*d*x^11 + 35*a^4*c*d^3*x^11 + 7*a^3*c^5*x^10 + 105/
2*a^4*c^2*d^2*x^10 + 35*a^4*c^3*d*x^9 + 7*a^5*d^3*x^9 + 35/4*a^4*c^4*x^8 +
21*a^5*c*d^2*x^8 + 21*a^5*c^2*d*x^7 + 7*a^5*c^3*x^6 + 7/2*a^6*d^2*x^6 + 7*a
^6*c*d*x^5 + 7/2*a^6*c^2*x^4 + a^7*d*x^3 + a^7*c*x^2
```

3.199 $\int \left(2cx+3dx^2\right)\left(cx^2+dx^3\right)^7\,dx$

Optimal. Leaf size=17

$$\frac{1}{8}\left(cx^2+dx^3\right)^8$$

```
[Out] (c*x^2 + d*x^3)^8/8
```

Rubi [A] time = 0.0251481, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {1588}

$$\frac{1}{8}\left(cx^2+dx^3\right)^8$$

Antiderivative was successfully verified.

```
[In] Int[(2*c*x + 3*d*x^2)*(c*x^2 + d*x^3)^7,x]
```

```
[Out] (c*x^2 + d*x^3)^8/8
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \left(2cx+3dx^2\right)\left(cx^2+dx^3\right)^7\,dx=\frac{1}{8}\left(cx^2+dx^3\right)^8$$

Mathematica [B] time = 0.0030102, size = 98, normalized size = 5.76

$$\frac{7}{2}c^2d^6x^{22}+7c^3d^5x^{21}+\frac{35}{4}c^4d^4x^{20}+7c^5d^3x^{19}+\frac{7}{2}c^6d^2x^{18}+c^7dx^{17}+\frac{c^8x^{16}}{8}+cd^7x^{23}+\frac{d^8x^{24}}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*c*x + 3*d*x^2)*(c*x^2 + d*x^3)^7,x]
```

```
[Out] (c^8*x^16)/8 + c^7*d*x^17 + (7*c^6*d^2*x^18)/2 + 7*c^5*d^3*x^19 + (35*c^4*d
^4*x^20)/4 + 7*c^3*d^5*x^21 + (7*c^2*d^6*x^22)/2 + c*d^7*x^23 + (d^8*x^24)/
8
```

Maple [B] time = 0.002, size = 89, normalized size = 5.2

$$\frac{d^8x^{24}}{8} + cd^7x^{23} + \frac{7\,c^2d^6x^{22}}{2} + 7\,c^3d^5x^{21} + \frac{35\,c^4d^4x^{20}}{4} + 7\,c^5d^3x^{19} + \frac{7\,c^6d^2x^{18}}{2} + c^7dx^{17} + \frac{c^8x^{16}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*d*x^2+2*c*x)*(d*x^3+c*x^2)^7,x)
```

```
[Out] 1/8*d^8*x^24+c*d^7*x^23+7/2*c^2*d^6*x^22+7*c^3*d^5*x^21+35/4*c^4*d^4*x^20+7
*c^5*d^3*x^19+7/2*c^6*d^2*x^18+c^7*d*x^17+1/8*c^8*x^16
```

Maxima [A] time = 0.977805, size = 20, normalized size = 1.18

$$\frac{1}{8}\left(dx^3 + cx^2\right)^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2)^7,x, algorithm="maxima")
```

```
[Out] 1/8*(d*x^3 + c*x^2)^8
```

Fricas [B] time = 1.13133, size = 198, normalized size = 11.65

$$\frac{1}{8}x^{24}d^8 + x^{23}d^7c + \frac{7}{2}x^{22}d^6c^2 + 7x^{21}d^5c^3 + \frac{35}{4}x^{20}d^4c^4 + 7x^{19}d^3c^5 + \frac{7}{2}x^{18}d^2c^6 + x^{17}dc^7 + \frac{1}{8}x^{16}c^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2)^7,x, algorithm="fricas")
```

```
[Out] 1/8*x^24*d^8 + x^23*d^7*c + 7/2*x^22*d^6*c^2 + 7*x^21*d^5*c^3 + 35/4*x^20*d
^4*c^4 + 7*x^19*d^3*c^5 + 7/2*x^18*d^2*c^6 + x^17*d*c^7 + 1/8*x^16*c^8
```

Sympy [B] time = 0.090522, size = 97, normalized size = 5.71

$$\frac{c^8x^{16}}{8} + c^7dx^{17} + \frac{7c^6d^2x^{18}}{2} + 7c^5d^3x^{19} + \frac{35c^4d^4x^{20}}{4} + 7c^3d^5x^{21} + \frac{7c^2d^6x^{22}}{2} + cd^7x^{23} + \frac{d^8x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x**2+2*c*x)*(d*x**3+c*x**2)**7,x)
```

```
[Out] c**8*x**16/8 + c**7*d*x**17 + 7*c**6*d**2*x**18/2 + 7*c**5*d**3*x**19 + 35*
c**4*d**4*x**20/4 + 7*c**3*d**5*x**21 + 7*c**2*d**6*x**22/2 + c*d**7*x**23
+ d**8*x**24/8
```

Giac [B] time = 1.31019, size = 119, normalized size = 7.

$$\frac{1}{8}\,d^8x^{24} + cd^7x^{23} + \frac{7}{2}\,c^2d^6x^{22} + 7\,c^3d^5x^{21} + \frac{35}{4}\,c^4d^4x^{20} + 7\,c^5d^3x^{19} + \frac{7}{2}\,c^6d^2x^{18} + c^7dx^{17} + \frac{1}{8}\,c^8x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2)^7,x, algorithm="giac")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + 35/4*c^4*d^
4*x^20 + 7*c^5*d^3*x^19 + 7/2*c^6*d^2*x^18 + c^7*d*x^17 + 1/8*c^8*x^16
```

3.200 $\int x^7 \left(cx+dx^2\right)^7 \left(2cx+3dx^2\right) dx$

Optimal. Leaf size=14

$$\frac{1}{8}x^{16}(c+dx)^8$$

```
[Out] (x^16*(c + d*x)^8)/8
```

Rubi [A] time = 0.227122, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1584, 845}

$$\frac{1}{8}x^{16}(c+dx)^8$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(c*x + d*x^2)^7*(2*c*x + 3*d*x^2),x]
```

```
[Out] (x^16*(c + d*x)^8)/8
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 845

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((b_.)*(x_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[(c*x^(m + 2)*(f + g*x)^(n + 1))/(g*(m + n + 3)), x] /; FreeQ[{
b, c, f, g, m, n}, x] && EqQ[c*f*(m + 2) - b*g*(m + n + 3), 0] && NeQ[m + n
 + 3, 0]
```

Rubi steps

$$\begin{aligned}\int x^7 \left(cx+dx^2\right)^7 \left(2cx+3dx^2\right) dx &= \int x^{14}(c+dx)^7 \left(2cx+3dx^2\right) dx\\ &= \frac{1}{8}x^{16}(c+dx)^8\end{aligned}$$

Mathematica [B] time = 0.0028447, size = 98, normalized size = 7.

$$\frac{7}{2}c^2d^6x^{22}+7c^3d^5x^{21}+\frac{35}{4}c^4d^4x^{20}+7c^5d^3x^{19}+\frac{7}{2}c^6d^2x^{18}+c^7dx^{17}+\frac{c^8x^{16}}{8}+cd^7x^{23}+\frac{d^8x^{24}}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(c*x + d*x^2)^7*(2*c*x + 3*d*x^2),x]
```

```
[Out] (c^8*x^16)/8 + c^7*d*x^17 + (7*c^6*d^2*x^18)/2 + 7*c^5*d^3*x^19 + (35*c^4*d
^4*x^20)/4 + 7*c^3*d^5*x^21 + (7*c^2*d^6*x^22)/2 + c*d^7*x^23 + (d^8*x^24)/
8
```

Maple [B] time = 0.003, size = 89, normalized size = 6.4

$$\frac{d^8x^{24}}{8}+cd^7x^{23}+\frac{7\,c^2d^6x^{22}}{2}+7\,c^3d^5x^{21}+\frac{35\,c^4d^4x^{20}}{4}+7\,c^5d^3x^{19}+\frac{7\,c^6d^2x^{18}}{2}+c^7dx^{17}+\frac{c^8x^{16}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(d*x^2+c*x)^7*(3*d*x^2+2*c*x),x)
```

```
[Out] 1/8*d^8*x^24+c*d^7*x^23+7/2*c^2*d^6*x^22+7*c^3*d^5*x^21+35/4*c^4*d^4*x^20+7
*c^5*d^3*x^19+7/2*c^6*d^2*x^18+c^7*d*x^17+1/8*c^8*x^16
```

Maxima [B] time = 0.973676, size = 119, normalized size = 8.5

$$\frac{1}{8}\,d^8x^{24}+cd^7x^{23}+\frac{7}{2}\,c^2d^6x^{22}+7\,c^3d^5x^{21}+\frac{35}{4}\,c^4d^4x^{20}+7\,c^5d^3x^{19}+\frac{7}{2}\,c^6d^2x^{18}+c^7dx^{17}+\frac{1}{8}\,c^8x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(d*x^2+c*x)^7*(3*d*x^2+2*c*x),x, algorithm="maxima")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + 35/4*c^4*d^
4*x^20 + 7*c^5*d^3*x^19 + 7/2*c^6*d^2*x^18 + c^7*d*x^17 + 1/8*c^8*x^16
```

Fricas [B] time = 1.07747, size = 198, normalized size = 14.14

$$\frac{1}{8}x^{24}d^8 + x^{23}d^7c + \frac{7}{2}x^{22}d^6c^2 + 7x^{21}d^5c^3 + \frac{35}{4}x^{20}d^4c^4 + 7x^{19}d^3c^5 + \frac{7}{2}x^{18}d^2c^6 + x^{17}dc^7 + \frac{1}{8}x^{16}c^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(d*x^2+c*x)^7*(3*d*x^2+2*c*x),x, algorithm="fricas")
```

```
[Out] 1/8*x^24*d^8 + x^23*d^7*c + 7/2*x^22*d^6*c^2 + 7*x^21*d^5*c^3 + 35/4*x^20*d
^4*c^4 + 7*x^19*d^3*c^5 + 7/2*x^18*d^2*c^6 + x^17*d*c^7 + 1/8*x^16*c^8
```

Sympy [B] time = 0.091965, size = 97, normalized size = 6.93

$$\frac{c^8x^{16}}{8} + c^7dx^{17} + \frac{7c^6d^2x^{18}}{2} + 7c^5d^3x^{19} + \frac{35c^4d^4x^{20}}{4} + 7c^3d^5x^{21} + \frac{7c^2d^6x^{22}}{2} + cd^7x^{23} + \frac{d^8x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(d*x**2+c*x)**7*(3*d*x**2+2*c*x),x)
```

```
[Out] c**8*x**16/8 + c**7*d*x**17 + 7*c**6*d**2*x**18/2 + 7*c**5*d**3*x**19 + 35*
c**4*d**4*x**20/4 + 7*c**3*d**5*x**21 + 7*c**2*d**6*x**22/2 + c*d**7*x**23
+ d**8*x**24/8
```

Giac [B] time = 1.27447, size = 119, normalized size = 8.5

$$\frac{1}{8}d^8x^{24} + cd^7x^{23} + \frac{7}{2}c^2d^6x^{22} + 7c^3d^5x^{21} + \frac{35}{4}c^4d^4x^{20} + 7c^5d^3x^{19} + \frac{7}{2}c^6d^2x^{18} + c^7dx^{17} + \frac{1}{8}c^8x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(d*x^2+c*x)^7*(3*d*x^2+2*c*x),x, algorithm="giac")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + 35/4*c^4*d^
4*x^20 + 7*c^5*d^3*x^19 + 7/2*c^6*d^2*x^18 + c^7*d*x^17 + 1/8*c^8*x^16
```

3.201 $\int x^{14}(c+dx)^7\left(2cx+3dx^2\right)\,dx$

Optimal. Leaf size=14

$$\frac{1}{8}x^{16}(c+dx)^8$$

```
[Out] (x^16*(c + d*x)^8)/8
```

Rubi [A] time = 0.0105136, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {845}

$$\frac{1}{8}x^{16}(c+dx)^8$$

Antiderivative was successfully verified.

```
[In] Int[x^14*(c + d*x)^7*(2*c*x + 3*d*x^2),x]
```

```
[Out] (x^16*(c + d*x)^8)/8
```

Rule 845

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((b_.)*(x_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[(c*x^(m + 2)*(f + g*x)^(n + 1))/(g*(m + n + 3)), x] /; FreeQ[{
b, c, f, g, m, n}, x] && EqQ[c*f*(m + 2) - b*g*(m + n + 3), 0] && NeQ[m + n
 + 3, 0]
```

Rubi steps

$$\int x^{14}(c+dx)^7\left(2cx+3dx^2\right)\,dx=\frac{1}{8}x^{16}(c+dx)^8$$

Mathematica [B] time = 0.002502, size = 98, normalized size = 7.

$$\frac{7}{2}c^2d^6x^{22}+7c^3d^5x^{21}+\frac{35}{4}c^4d^4x^{20}+7c^5d^3x^{19}+\frac{7}{2}c^6d^2x^{18}+c^7dx^{17}+\frac{c^8x^{16}}{8}+cd^7x^{23}+\frac{d^8x^{24}}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^14*(c + d*x)^7*(2*c*x + 3*d*x^2),x]
```

```
[Out] (c^8*x^16)/8 + c^7*d*x^17 + (7*c^6*d^2*x^18)/2 + 7*c^5*d^3*x^19 + (35*c^4*d
^4*x^20)/4 + 7*c^3*d^5*x^21 + (7*c^2*d^6*x^22)/2 + c*d^7*x^23 + (d^8*x^24)/
8
```

Maple [B] time = 0.003, size = 89, normalized size = 6.4

$$\frac{d^8x^{24}}{8}+cd^7x^{23}+\frac{7\,c^2d^6x^{22}}{2}+7\,c^3d^5x^{21}+\frac{35\,c^4d^4x^{20}}{4}+7\,c^5d^3x^{19}+\frac{7\,c^6d^2x^{18}}{2}+c^7dx^{17}+\frac{c^8x^{16}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14*(d*x+c)^7*(3*d*x^2+2*c*x),x)
```

```
[Out] 1/8*d^8*x^24+c*d^7*x^23+7/2*c^2*d^6*x^22+7*c^3*d^5*x^21+35/4*c^4*d^4*x^20+7
*c^5*d^3*x^19+7/2*c^6*d^2*x^18+c^7*d*x^17+1/8*c^8*x^16
```

Maxima [B] time = 0.981793, size = 119, normalized size = 8.5

$$\frac{1}{8}\,d^8x^{24}+cd^7x^{23}+\frac{7}{2}\,c^2d^6x^{22}+7\,c^3d^5x^{21}+\frac{35}{4}\,c^4d^4x^{20}+7\,c^5d^3x^{19}+\frac{7}{2}\,c^6d^2x^{18}+c^7dx^{17}+\frac{1}{8}\,c^8x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14*(d*x+c)^7*(3*d*x^2+2*c*x),x, algorithm="maxima")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + 35/4*c^4*d^
4*x^20 + 7*c^5*d^3*x^19 + 7/2*c^6*d^2*x^18 + c^7*d*x^17 + 1/8*c^8*x^16
```

Fricas [B] time = 1.06935, size = 198, normalized size = 14.14

$$\frac{1}{8}x^{24}d^8+x^{23}d^7c+\frac{7}{2}x^{22}d^6c^2+7x^{21}d^5c^3+\frac{35}{4}x^{20}d^4c^4+7x^{19}d^3c^5+\frac{7}{2}x^{18}d^2c^6+x^{17}dc^7+\frac{1}{8}x^{16}c^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14*(d*x+c)^7*(3*d*x^2+2*c*x),x, algorithm="fricas")
```

```
[Out] 1/8*x^24*d^8 + x^23*d^7*c + 7/2*x^22*d^6*c^2 + 7*x^21*d^5*c^3 + 35/4*x^20*d
^4*c^4 + 7*x^19*d^3*c^5 + 7/2*x^18*d^2*c^6 + x^17*d*c^7 + 1/8*x^16*c^8
```

Sympy [B] time = 0.093671, size = 97, normalized size = 6.93

$$\frac{c^8x^{16}}{8}+c^7dx^{17}+\frac{7c^6d^2x^{18}}{2}+7c^5d^3x^{19}+\frac{35c^4d^4x^{20}}{4}+7c^3d^5x^{21}+\frac{7c^2d^6x^{22}}{2}+cd^7x^{23}+\frac{d^8x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14*(d*x+c)**7*(3*d*x**2+2*c*x),x)
```

```
[Out] c**8*x**16/8 + c**7*d*x**17 + 7*c**6*d**2*x**18/2 + 7*c**5*d**3*x**19 + 35*
c**4*d**4*x**20/4 + 7*c**3*d**5*x**21 + 7*c**2*d**6*x**22/2 + c*d**7*x**23
+ d**8*x**24/8
```

Giac [B] time = 1.28035, size = 119, normalized size = 8.5

$$\frac{1}{8}\,d^8x^{24}+cd^7x^{23}+\frac{7}{2}\,c^2d^6x^{22}+7\,c^3d^5x^{21}+\frac{35}{4}\,c^4d^4x^{20}+7\,c^5d^3x^{19}+\frac{7}{2}\,c^6d^2x^{18}+c^7dx^{17}+\frac{1}{8}\,c^8x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14*(d*x+c)^7*(3*d*x^2+2*c*x),x, algorithm="giac")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + 35/4*c^4*d^
4*x^20 + 7*c^5*d^3*x^19 + 7/2*c^6*d^2*x^18 + c^7*d*x^17 + 1/8*c^8*x^16
```

3.202 $\int x(2c+3dx)\left(a+cx^2+dx^3\right)^7\,dx$

Optimal. Leaf size=18

$$\frac{1}{8}\left(a+cx^2+dx^3\right)^8$$

```
[Out] (a + c*x^2 + d*x^3)^8/8
```

Rubi [A] time = 0.0596462, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {1588}

$$\frac{1}{8}\left(a+cx^2+dx^3\right)^8$$

Antiderivative was successfully verified.

```
[In] Int[x*(2*c + 3*d*x)*(a + c*x^2 + d*x^3)^7,x]
```

```
[Out] (a + c*x^2 + d*x^3)^8/8
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int x(2c+3dx)\left(a+cx^2+dx^3\right)^7\,dx=\frac{1}{8}\left(a+cx^2+dx^3\right)^8$$

Mathematica [B] time = 0.0107206, size = 115, normalized size = 6.39

$$\frac{1}{8}x^2(c+dx)\left(28a^2x^{10}(c+dx)^5+56a^3x^8(c+dx)^4+70a^4x^6(c+dx)^3+56a^5x^4(c+dx)^2+28a^6x^2(c+dx)+8a^7+8ax^{12}(c\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(2*c + 3*d*x)*(a + c*x^2 + d*x^3)^7,x]
```

```
[Out] (x^2*(c + d*x)*(8*a^7 + 28*a^6*x^2*(c + d*x) + 56*a^5*x^4*(c + d*x)^2 + 70*
a^4*x^6*(c + d*x)^3 + 56*a^3*x^8*(c + d*x)^4 + 28*a^2*x^10*(c + d*x)^5 + 8*
a*x^12*(c + d*x)^6 + x^14*(c + d*x)^7))/8
```

Maple [B] time = 0.002, size = 2205, normalized size = 122.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(3*d*x+2*c)*(d*x^3+c*x^2+a)^7,x)
```

```
[Out] 1/8*d^8*x^24+c*d^7*x^23+7/2*c^2*d^6*x^22+1/21*(42*c^3*d^5+3*d*(a*d^6+15*c^3
*d^4+d*(2*(3*a*d^2+c^3)*d^3+18*c^3*d^3)))*x^21+1/20*(2*c*(a*d^6+15*c^3*d^4+
d*(2*(3*a*d^2+c^3)*d^3+18*c^3*d^3))+3*d*(6*a*c*d^5+c*(2*(3*a*d^2+c^3)*d^3+1
8*c^3*d^3)+d*(12*a*c*d^4+6*(3*a*d^2+c^3)*c*d^2+9*c^4*d^2)))*x^20+1/19*(2*c*
(6*a*c*d^5+c*(2*(3*a*d^2+c^3)*d^3+18*c^3*d^3)+d*(12*a*c*d^4+6*(3*a*d^2+c^3)
*c*d^2+9*c^4*d^2))+3*d*(15*a*c^2*d^4+c*(12*a*c*d^4+6*(3*a*d^2+c^3)*c*d^2+9*
c^4*d^2)+d*(42*a*c^2*d^3+6*(3*a*d^2+c^3)*c^2*d)))*x^19+1/18*(2*c*(15*a*c^2*
d^4+c*(12*a*c*d^4+6*(3*a*d^2+c^3)*c*d^2+9*c^4*d^2)+d*(42*a*c^2*d^3+6*(3*a*d
^2+c^3)*c^2*d))+3*d*(a*(2*(3*a*d^2+c^3)*d^3+18*c^3*d^3)+c*(42*a*c^2*d^3+6*(
3*a*d^2+c^3)*c^2*d)+d*(6*a^2*d^4+54*a*c^3*d^2+(3*a*d^2+c^3)^2)))*x^18+1/17*
(2*c*(a*(2*(3*a*d^2+c^3)*d^3+18*c^3*d^3)+c*(42*a*c^2*d^3+6*(3*a*d^2+c^3)*c^
2*d)+d*(6*a^2*d^4+54*a*c^3*d^2+(3*a*d^2+c^3)^2))+3*d*(a*(12*a*c*d^4+6*(3*a*
d^2+c^3)*c*d^2+9*c^4*d^2)+c*(6*a^2*d^4+54*a*c^3*d^2+(3*a*d^2+c^3)^2)+d*(24*
a^2*c*d^3+18*c^4*a*d+12*a*c*d*(3*a*d^2+c^3))))*x^17+1/16*(2*c*(a*(12*a*c*d^
4+6*(3*a*d^2+c^3)*c*d^2+9*c^4*d^2)+c*(6*a^2*d^4+54*a*c^3*d^2+(3*a*d^2+c^3)^
2)+d*(24*a^2*c*d^3+18*c^4*a*d+12*a*c*d*(3*a*d^2+c^3)))+3*d*(a*(42*a*c^2*d^3
+6*(3*a*d^2+c^3)*c^2*d)+c*(24*a^2*c*d^3+18*c^4*a*d+12*a*c*d*(3*a*d^2+c^3))+
d*(72*a^2*c^2*d^2+6*c^2*a*(3*a*d^2+c^3))))*x^16+1/15*(2*c*(a*(42*a*c^2*d^3+
6*(3*a*d^2+c^3)*c^2*d)+c*(24*a^2*c*d^3+18*c^4*a*d+12*a*c*d*(3*a*d^2+c^3))+d
*(72*a^2*c^2*d^2+6*c^2*a*(3*a*d^2+c^3)))+3*d*(a*(6*a^2*d^4+54*a*c^3*d^2+(3*
a*d^2+c^3)^2)+c*(72*a^2*c^2*d^2+6*c^2*a*(3*a*d^2+c^3))+d*(2*a^3*d^3+54*a^2*
c^3*d+6*a^2*d*(3*a*d^2+c^3))))*x^15+1/14*(2*c*(a*(6*a^2*d^4+54*a*c^3*d^2+(3
*a*d^2+c^3)^2)+c*(72*a^2*c^2*d^2+6*c^2*a*(3*a*d^2+c^3))+d*(2*a^3*d^3+54*a^2
*c^3*d+6*a^2*d*(3*a*d^2+c^3)))+3*d*(a*(24*a^2*c*d^3+18*c^4*a*d+12*a*c*d*(3*
a*d^2+c^3))+c*(2*a^3*d^3+54*a^2*c^3*d+6*a^2*d*(3*a*d^2+c^3))+d*(42*a^3*c*d^
2+6*a^2*c*(3*a*d^2+c^3)+9*c^4*a^2)))*x^14+1/13*(2*c*(a*(24*a^2*c*d^3+18*c^4
```

```
*a*d+12*a*c*d*(3*a*d^2+c^3))+c*(2*a^3*d^3+54*a^2*c^3*d+6*a^2*d*(3*a*d^2+c^3
))+d*(42*a^3*c*d^2+6*a^2*c*(3*a*d^2+c^3)+9*c^4*a^2))+3*d*(a*(72*a^2*c^2*d^2
+6*c^2*a*(3*a*d^2+c^3))+c*(42*a^3*c*d^2+6*a^2*c*(3*a*d^2+c^3)+9*c^4*a^2)+60
*a^3*c^2*d^2))*x^13+1/12*(2*c*(a*(72*a^2*c^2*d^2+6*c^2*a*(3*a*d^2+c^3))+c*(
42*a^3*c*d^2+6*a^2*c*(3*a*d^2+c^3)+9*c^4*a^2)+60*a^3*c^2*d^2)+3*d*(a*(2*a^3
*d^3+54*a^2*c^3*d+6*a^2*d*(3*a*d^2+c^3))+60*a^3*c^3*d+d*(2*a^3*(3*a*d^2+c^3
)+18*a^3*c^3+9*a^4*d^2)))*x^12+1/11*(2*c*(a*(2*a^3*d^3+54*a^2*c^3*d+6*a^2*d
*(3*a*d^2+c^3))+60*a^3*c^3*d+d*(2*a^3*(3*a*d^2+c^3)+18*a^3*c^3+9*a^4*d^2))+
3*d*(a*(42*a^3*c*d^2+6*a^2*c*(3*a*d^2+c^3)+9*c^4*a^2)+c*(2*a^3*(3*a*d^2+c^3
)+18*a^3*c^3+9*a^4*d^2)+30*d^2*a^4*c))*x^11+1/10*(2*c*(a*(42*a^3*c*d^2+6*a^
2*c*(3*a*d^2+c^3)+9*c^4*a^2)+c*(2*a^3*(3*a*d^2+c^3)+18*a^3*c^3+9*a^4*d^2)+3
0*d^2*a^4*c)+315*d^2*a^4*c^2)*x^10+1/9*(210*c^3*a^4*d+3*d*(a*(2*a^3*(3*a*d^
2+c^3)+18*a^3*c^3+9*a^4*d^2)+15*c^3*a^4+6*d^2*a^5))*x^9+1/8*(2*c*(a*(2*a^3*
(3*a*d^2+c^3)+18*a^3*c^3+9*a^4*d^2)+15*c^3*a^4+6*d^2*a^5)+126*d^2*a^5*c)*x^
8+21*c^2*a^5*d*x^7+1/6*(21*a^6*d^2+42*a^5*c^3)*x^6+7*c*d*a^6*x^5+7/2*c^2*a^
6*x^4+d*a^7*x^3+c*a^7*x^2
```

Maxima [B] time = 0.995793, size = 618, normalized size = 34.33

$$\frac{1}{8}d^8x^{24} + cd^7x^{23} + \frac{7}{2}c^2d^6x^{22} + \left(7c^3d^5 + ad^7\right)x^{21} + \frac{7}{4}\left(5c^4d^4 + 4acd^6\right)x^{20} + 7\left(c^5d^3 + 3ac^2d^5\right)x^{19} + \frac{7}{2}\left(c^6d^2 + 10ac^3d^4 + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2+a)^7,x, algorithm="maxima")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + (7*c^3*d^5 + a*d^7)*x^21 + 7
/4*(5*c^4*d^4 + 4*a*c*d^6)*x^20 + 7*(c^5*d^3 + 3*a*c^2*d^5)*x^19 + 7/2*(c^6
*d^2 + 10*a*c^3*d^4 + a^2*d^6)*x^18 + (c^7*d + 35*a*c^4*d^3 + 21*a^2*c*d^5)
*x^17 + 1/8*(c^8 + 168*a*c^5*d^2 + 420*a^2*c^2*d^4)*x^16 + 7*(a*c^6*d + 10*
a^2*c^3*d^3 + a^3*d^5)*x^15 + 21*a^5*c^2*d*x^7 + 1/2*(2*a*c^7 + 105*a^2*c^4
*d^2 + 70*a^3*c*d^4)*x^14 + 7*(3*a^2*c^5*d + 10*a^3*c^2*d^3)*x^13 + 7*a^6*c
*d*x^5 + 7/4*(2*a^2*c^6 + 40*a^3*c^3*d^2 + 5*a^4*d^4)*x^12 + 7/2*a^6*c^2*x^
4 + 35*(a^3*c^4*d + a^4*c*d^3)*x^11 + a^7*d*x^3 + 7/2*(2*a^3*c^5 + 15*a^4*c
^2*d^2)*x^10 + a^7*c*x^2 + 7*(5*a^4*c^3*d + a^5*d^3)*x^9 + 7/4*(5*a^4*c^4 +
 12*a^5*c*d^2)*x^8 + 7/2*(2*a^5*c^3 + a^6*d^2)*x^6
```

Fricas [B] time = 1.04234, size = 1085, normalized size = 60.28

$$\frac{1}{8}x^{24}d^8 + x^{23}d^7c + \frac{7}{2}x^{22}d^6c^2 + 7x^{21}d^5c^3 + x^{21}d^7a + \frac{35}{4}x^{20}d^4c^4 + 7x^{20}d^6ca + 7x^{19}d^3c^5 + 21x^{19}d^5c^2a + \frac{7}{2}x^{18}d^2c^6 + 35x^{18}d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2+a)^7,x, algorithm="fricas")
```

```
[Out] 1/8*x^24*d^8 + x^23*d^7*c + 7/2*x^22*d^6*c^2 + 7*x^21*d^5*c^3 + x^21*d^7*a
+ 35/4*x^20*d^4*c^4 + 7*x^20*d^6*c*a + 7*x^19*d^3*c^5 + 21*x^19*d^5*c^2*a +
 7/2*x^18*d^2*c^6 + 35*x^18*d^4*c^3*a + 7/2*x^18*d^6*a^2 + x^17*d*c^7 + 35*
x^17*d^3*c^4*a + 21*x^17*d^5*c*a^2 + 1/8*x^16*c^8 + 21*x^16*d^2*c^5*a + 105
/2*x^16*d^4*c^2*a^2 + 7*x^15*d*c^6*a + 70*x^15*d^3*c^3*a^2 + 7*x^15*d^5*a^3
 + x^14*c^7*a + 105/2*x^14*d^2*c^4*a^2 + 35*x^14*d^4*c*a^3 + 21*x^13*d*c^5*
a^2 + 70*x^13*d^3*c^2*a^3 + 7/2*x^12*c^6*a^2 + 70*x^12*d^2*c^3*a^3 + 35/4*x
^12*d^4*a^4 + 35*x^11*d*c^4*a^3 + 35*x^11*d^3*c*a^4 + 7*x^10*c^5*a^3 + 105/
2*x^10*d^2*c^2*a^4 + 35*x^9*d*c^3*a^4 + 7*x^9*d^3*a^5 + 35/4*x^8*c^4*a^4 +
21*x^8*d^2*c*a^5 + 21*x^7*d*c^2*a^5 + 7*x^6*c^3*a^5 + 7/2*x^6*d^2*a^6 + 7*x
^5*d*c*a^6 + 7/2*x^4*c^2*a^6 + x^3*d*a^7 + x^2*c*a^7
```

Sympy [B] time = 0.151716, size = 484, normalized size = 26.89

$$a^7cx^2 + a^7dx^3 + \frac{7a^6c^2x^4}{2} + 7a^6cdx^5 + 21a^5c^2dx^7 + \frac{7c^2d^6x^{22}}{2} + cd^7x^{23} + \frac{d^8x^{24}}{8} + x^{21}\left(ad^7 + 7c^3d^5\right) + x^{20}\left(7acd^6 + \frac{35c^4}{4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*d*x+2*c)*(d*x**3+c*x**2+a)**7,x)
```

```
[Out] a**7*c*x**2 + a**7*d*x**3 + 7*a**6*c**2*x**4/2 + 7*a**6*c*d*x**5 + 21*a**5*
c**2*d*x**7 + 7*c**2*d**6*x**22/2 + c*d**7*x**23 + d**8*x**24/8 + x**21*(a*
d**7 + 7*c**3*d**5) + x**20*(7*a*c*d**6 + 35*c**4*d**4/4) + x**19*(21*a*c**
2*d**5 + 7*c**5*d**3) + x**18*(7*a**2*d**6/2 + 35*a*c**3*d**4 + 7*c**6*d**2
/2) + x**17*(21*a**2*c*d**5 + 35*a*c**4*d**3 + c**7*d) + x**16*(105*a**2*c*
*2*d**4/2 + 21*a*c**5*d**2 + c**8/8) + x**15*(7*a**3*d**5 + 70*a**2*c**3*d*
*3 + 7*a*c**6*d) + x**14*(35*a**3*c*d**4 + 105*a**2*c**4*d**2/2 + a*c**7) +
 x**13*(70*a**3*c**2*d**3 + 21*a**2*c**5*d) + x**12*(35*a**4*d**4/4 + 70*a*
*3*c**3*d**2 + 7*a**2*c**6/2) + x**11*(35*a**4*c*d**3 + 35*a**3*c**4*d) + x
**10*(105*a**4*c**2*d**2/2 + 7*a**3*c**5) + x**9*(7*a**5*d**3 + 35*a**4*c**
3*d) + x**8*(21*a**5*c*d**2 + 35*a**4*c**4/4) + x**6*(7*a**6*d**2/2 + 7*a**
5*c**3)
```

Giac [B] time = 1.19708, size = 659, normalized size = 36.61

$$\frac{1}{8}d^8x^{24} + cd^7x^{23} + \frac{7}{2}c^2d^6x^{22} + 7c^3d^5x^{21} + ad^7x^{21} + \frac{35}{4}c^4d^4x^{20} + 7acd^6x^{20} + 7c^5d^3x^{19} + 21ac^2d^5x^{19} + \frac{7}{2}c^6d^2x^{18} + 35a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2+a)^7,x, algorithm="giac")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + a*d^7*x^21
+ 35/4*c^4*d^4*x^20 + 7*a*c*d^6*x^20 + 7*c^5*d^3*x^19 + 21*a*c^2*d^5*x^19 +
 7/2*c^6*d^2*x^18 + 35*a*c^3*d^4*x^18 + 7/2*a^2*d^6*x^18 + c^7*d*x^17 + 35*
a*c^4*d^3*x^17 + 21*a^2*c*d^5*x^17 + 1/8*c^8*x^16 + 21*a*c^5*d^2*x^16 + 105
/2*a^2*c^2*d^4*x^16 + 7*a*c^6*d*x^15 + 70*a^2*c^3*d^3*x^15 + 7*a^3*d^5*x^15
 + a*c^7*x^14 + 105/2*a^2*c^4*d^2*x^14 + 35*a^3*c*d^4*x^14 + 21*a^2*c^5*d*x
^13 + 70*a^3*c^2*d^3*x^13 + 7/2*a^2*c^6*x^12 + 70*a^3*c^3*d^2*x^12 + 35/4*a
^4*d^4*x^12 + 35*a^3*c^4*d*x^11 + 35*a^4*c*d^3*x^11 + 7*a^3*c^5*x^10 + 105/
2*a^4*c^2*d^2*x^10 + 35*a^4*c^3*d*x^9 + 7*a^5*d^3*x^9 + 35/4*a^4*c^4*x^8 +
21*a^5*c*d^2*x^8 + 21*a^5*c^2*d*x^7 + 7*a^5*c^3*x^6 + 7/2*a^6*d^2*x^6 + 7*a
^6*c*d*x^5 + 7/2*a^6*c^2*x^4 + a^7*d*x^3 + a^7*c*x^2
```

3.203 $\int x(2c+3dx)\left(cx^2+dx^3\right)^7\,dx$

Optimal. Leaf size=14

$$\frac{1}{8}x^{16}(c+dx)^8$$

```
[Out] (x^16*(c + d*x)^8)/8
```

Rubi [A] time = 0.0219512, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1584, 74}

$$\frac{1}{8}x^{16}(c+dx)^8$$

Antiderivative was successfully verified.

```
[In] Int[x*(2*c + 3*d*x)*(c*x^2 + d*x^3)^7,x]
```

```
[Out] (x^16*(c + d*x)^8)/8
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rubi steps

$$\begin{aligned}\int x(2c+3dx)\left(cx^2+dx^3\right)^7\,dx &= \int x^{15}(c+dx)^7(2c+3dx)\,dx\\ &= \frac{1}{8}x^{16}(c+dx)^8\end{aligned}$$

Mathematica [B] time = 0.0027242, size = 98, normalized size = 7.

$$\frac{7}{2}c^2d^6x^{22}+7c^3d^5x^{21}+\frac{35}{4}c^4d^4x^{20}+7c^5d^3x^{19}+\frac{7}{2}c^6d^2x^{18}+c^7dx^{17}+\frac{c^8x^{16}}{8}+cd^7x^{23}+\frac{d^8x^{24}}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(2*c + 3*d*x)*(c*x^2 + d*x^3)^7,x]
```

```
[Out] (c^8*x^16)/8 + c^7*d*x^17 + (7*c^6*d^2*x^18)/2 + 7*c^5*d^3*x^19 + (35*c^4*d
^4*x^20)/4 + 7*c^3*d^5*x^21 + (7*c^2*d^6*x^22)/2 + c*d^7*x^23 + (d^8*x^24)/
8
```

Maple [B] time = 0.001, size = 89, normalized size = 6.4

$$\frac{d^8x^{24}}{8}+cd^7x^{23}+\frac{7c^2d^6x^{22}}{2}+7c^3d^5x^{21}+\frac{35c^4d^4x^{20}}{4}+7c^5d^3x^{19}+\frac{7c^6d^2x^{18}}{2}+c^7dx^{17}+\frac{c^8x^{16}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(3*d*x+2*c)*(d*x^3+c*x^2)^7,x)
```

```
[Out] 1/8*d^8*x^24+c*d^7*x^23+7/2*c^2*d^6*x^22+7*c^3*d^5*x^21+35/4*c^4*d^4*x^20+7
*c^5*d^3*x^19+7/2*c^6*d^2*x^18+c^7*d*x^17+1/8*c^8*x^16
```

Maxima [B] time = 1.15751, size = 119, normalized size = 8.5

$$\frac{1}{8}d^8x^{24}+cd^7x^{23}+\frac{7}{2}c^2d^6x^{22}+7c^3d^5x^{21}+\frac{35}{4}c^4d^4x^{20}+7c^5d^3x^{19}+\frac{7}{2}c^6d^2x^{18}+c^7dx^{17}+\frac{1}{8}c^8x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2)^7,x, algorithm="maxima")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + 35/4*c^4*d^
4*x^20 + 7*c^5*d^3*x^19 + 7/2*c^6*d^2*x^18 + c^7*d*x^17 + 1/8*c^8*x^16
```

Fricas [B] time = 1.07967, size = 198, normalized size = 14.14

$$\frac{1}{8}x^{24}d^8 + x^{23}d^7c + \frac{7}{2}x^{22}d^6c^2 + 7x^{21}d^5c^3 + \frac{35}{4}x^{20}d^4c^4 + 7x^{19}d^3c^5 + \frac{7}{2}x^{18}d^2c^6 + x^{17}dc^7 + \frac{1}{8}x^{16}c^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2)^7,x, algorithm="fricas")
```

```
[Out] 1/8*x^24*d^8 + x^23*d^7*c + 7/2*x^22*d^6*c^2 + 7*x^21*d^5*c^3 + 35/4*x^20*d
^4*c^4 + 7*x^19*d^3*c^5 + 7/2*x^18*d^2*c^6 + x^17*d*c^7 + 1/8*x^16*c^8
```

Sympy [B] time = 0.089397, size = 97, normalized size = 6.93

$$\frac{c^8x^{16}}{8} + c^7dx^{17} + \frac{7c^6d^2x^{18}}{2} + 7c^5d^3x^{19} + \frac{35c^4d^4x^{20}}{4} + 7c^3d^5x^{21} + \frac{7c^2d^6x^{22}}{2} + cd^7x^{23} + \frac{d^8x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*d*x+2*c)*(d*x**3+c*x**2)**7,x)
```

```
[Out] c**8*x**16/8 + c**7*d*x**17 + 7*c**6*d**2*x**18/2 + 7*c**5*d**3*x**19 + 35*
c**4*d**4*x**20/4 + 7*c**3*d**5*x**21 + 7*c**2*d**6*x**22/2 + c*d**7*x**23
+ d**8*x**24/8
```

Giac [B] time = 1.18284, size = 119, normalized size = 8.5

$$\frac{1}{8}d^8x^{24} + cd^7x^{23} + \frac{7}{2}c^2d^6x^{22} + 7c^3d^5x^{21} + \frac{35}{4}c^4d^4x^{20} + 7c^5d^3x^{19} + \frac{7}{2}c^6d^2x^{18} + c^7dx^{17} + \frac{1}{8}c^8x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2)^7,x, algorithm="giac")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + 35/4*c^4*d^
4*x^20 + 7*c^5*d^3*x^19 + 7/2*c^6*d^2*x^18 + c^7*d*x^17 + 1/8*c^8*x^16
```

3.204 $\int x^8(2c+3dx)\left(cx+dx^2\right)^7\,dx$

Optimal. Leaf size=18

$$\frac{1}{8}x^8\left(cx+dx^2\right)^8$$

```
[Out] (x^8*(c*x + d*x^2)^8)/8
```

Rubi [A] time = 0.0190387, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {763}

$$\frac{1}{8}x^8\left(cx+dx^2\right)^8$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(2*c + 3*d*x)*(c*x + d*x^2)^7,x]
```

```
[Out] (x^8*(c*x + d*x^2)^8)/8
```

Rule 763

```
Int[((e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))*((b_.)*(x_) + (c_.)*(x_)^2)^(p_.
), x_Symbol] :> Simp[(g*(e*x)^m*(b*x + c*x^2)^(p + 1))/(c*(m + 2*p + 2)), x
] /; FreeQ[{b, c, e, f, g, m, p}, x] && EqQ[b*g*(m + p + 1) - c*f*(m + 2*p
+ 2), 0] && NeQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int x^8(2c+3dx)\left(cx+dx^2\right)^7\,dx=\frac{1}{8}x^8\left(cx+dx^2\right)^8$$

Mathematica [B] time = 0.0026391, size = 98, normalized size = 5.44

$$\frac{7}{2}c^2d^6x^{22}+7c^3d^5x^{21}+\frac{35}{4}c^4d^4x^{20}+7c^5d^3x^{19}+\frac{7}{2}c^6d^2x^{18}+c^7dx^{17}+\frac{c^8x^{16}}{8}+cd^7x^{23}+\frac{d^8x^{24}}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*(2*c + 3*d*x)*(c*x + d*x^2)^7,x]
```

```
[Out] (c^8*x^16)/8 + c^7*d*x^17 + (7*c^6*d^2*x^18)/2 + 7*c^5*d^3*x^19 + (35*c^4*d
^4*x^20)/4 + 7*c^3*d^5*x^21 + (7*c^2*d^6*x^22)/2 + c*d^7*x^23 + (d^8*x^24)/
8
```

Maple [B] time = 0.003, size = 89, normalized size = 4.9

$$\frac{d^8x^{24}}{8}+cd^7x^{23}+\frac{7\,c^2d^6x^{22}}{2}+7\,c^3d^5x^{21}+\frac{35\,c^4d^4x^{20}}{4}+7\,c^5d^3x^{19}+\frac{7\,c^6d^2x^{18}}{2}+c^7dx^{17}+\frac{c^8x^{16}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(3*d*x+2*c)*(d*x^2+c*x)^7,x)
```

```
[Out] 1/8*d^8*x^24+c*d^7*x^23+7/2*c^2*d^6*x^22+7*c^3*d^5*x^21+35/4*c^4*d^4*x^20+7
*c^5*d^3*x^19+7/2*c^6*d^2*x^18+c^7*d*x^17+1/8*c^8*x^16
```

Maxima [B] time = 1.01185, size = 119, normalized size = 6.61

$$\frac{1}{8}\,d^8x^{24}+cd^7x^{23}+\frac{7}{2}\,c^2d^6x^{22}+7\,c^3d^5x^{21}+\frac{35}{4}\,c^4d^4x^{20}+7\,c^5d^3x^{19}+\frac{7}{2}\,c^6d^2x^{18}+c^7dx^{17}+\frac{1}{8}\,c^8x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(3*d*x+2*c)*(d*x^2+c*x)^7,x, algorithm="maxima")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + 35/4*c^4*d^
4*x^20 + 7*c^5*d^3*x^19 + 7/2*c^6*d^2*x^18 + c^7*d*x^17 + 1/8*c^8*x^16
```

Fricas [B] time = 1.03941, size = 198, normalized size = 11.

$$\frac{1}{8}x^{24}d^8+x^{23}d^7c+\frac{7}{2}x^{22}d^6c^2+7x^{21}d^5c^3+\frac{35}{4}x^{20}d^4c^4+7x^{19}d^3c^5+\frac{7}{2}x^{18}d^2c^6+x^{17}dc^7+\frac{1}{8}x^{16}c^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(3*d*x+2*c)*(d*x^2+c*x)^7,x, algorithm="fricas")
```

```
[Out] 1/8*x^24*d^8 + x^23*d^7*c + 7/2*x^22*d^6*c^2 + 7*x^21*d^5*c^3 + 35/4*x^20*d
^4*c^4 + 7*x^19*d^3*c^5 + 7/2*x^18*d^2*c^6 + x^17*d*c^7 + 1/8*x^16*c^8
```

Sympy [B] time = 0.088219, size = 97, normalized size = 5.39

$$\frac{c^8x^{16}}{8}+c^7dx^{17}+\frac{7c^6d^2x^{18}}{2}+7c^5d^3x^{19}+\frac{35c^4d^4x^{20}}{4}+7c^3d^5x^{21}+\frac{7c^2d^6x^{22}}{2}+cd^7x^{23}+\frac{d^8x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(3*d*x+2*c)*(d*x**2+c*x)**7,x)
```

```
[Out] c**8*x**16/8 + c**7*d*x**17 + 7*c**6*d**2*x**18/2 + 7*c**5*d**3*x**19 + 35*
c**4*d**4*x**20/4 + 7*c**3*d**5*x**21 + 7*c**2*d**6*x**22/2 + c*d**7*x**23
+ d**8*x**24/8
```

Giac [B] time = 1.19678, size = 119, normalized size = 6.61

$$\frac{1}{8}d^8x^{24}+cd^7x^{23}+\frac{7}{2}c^2d^6x^{22}+7c^3d^5x^{21}+\frac{35}{4}c^4d^4x^{20}+7c^5d^3x^{19}+\frac{7}{2}c^6d^2x^{18}+c^7dx^{17}+\frac{1}{8}c^8x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(3*d*x+2*c)*(d*x^2+c*x)^7,x, algorithm="giac")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + 35/4*c^4*d^
4*x^20 + 7*c^5*d^3*x^19 + 7/2*c^6*d^2*x^18 + c^7*d*x^17 + 1/8*c^8*x^16
```

3.205 $\int x^{15}(c+dx)^7(2c+3dx)\,dx$

Optimal. Leaf size=14

$$\frac{1}{8}x^{16}(c+dx)^8$$

```
[Out] (x^16*(c + d*x)^8)/8
```

Rubi [A] time = 0.0019092, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {74}

$$\frac{1}{8}x^{16}(c+dx)^8$$

Antiderivative was successfully verified.

```
[In] Int[x^15*(c + d*x)^7*(2*c + 3*d*x),x]
```

```
[Out] (x^16*(c + d*x)^8)/8
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rubi steps

$$\int x^{15}(c+dx)^7(2c+3dx)\,dx = \frac{1}{8}x^{16}(c+dx)^8$$

Mathematica [B] time = 0.0023895, size = 98, normalized size = 7.

$$\frac{7}{2}c^2d^6x^{22}+7c^3d^5x^{21}+\frac{35}{4}c^4d^4x^{20}+7c^5d^3x^{19}+\frac{7}{2}c^6d^2x^{18}+c^7dx^{17}+\frac{c^8x^{16}}{8}+cd^7x^{23}+\frac{d^8x^{24}}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^15*(c + d*x)^7*(2*c + 3*d*x),x]
```

```
[Out] (c^8*x^16)/8 + c^7*d*x^17 + (7*c^6*d^2*x^18)/2 + 7*c^5*d^3*x^19 + (35*c^4*d
^4*x^20)/4 + 7*c^3*d^5*x^21 + (7*c^2*d^6*x^22)/2 + c*d^7*x^23 + (d^8*x^24)/
8
```

Maple [B] time = 0.002, size = 89, normalized size = 6.4

$$\frac{d^8x^{24}}{8}+cd^7x^{23}+\frac{7\,c^2d^6x^{22}}{2}+7\,c^3d^5x^{21}+\frac{35\,c^4d^4x^{20}}{4}+7\,c^5d^3x^{19}+\frac{7\,c^6d^2x^{18}}{2}+c^7dx^{17}+\frac{c^8x^{16}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^15*(d*x+c)^7*(3*d*x+2*c),x)
```

```
[Out] 1/8*d^8*x^24+c*d^7*x^23+7/2*c^2*d^6*x^22+7*c^3*d^5*x^21+35/4*c^4*d^4*x^20+7
*c^5*d^3*x^19+7/2*c^6*d^2*x^18+c^7*d*x^17+1/8*c^8*x^16
```

Maxima [B] time = 1.03789, size = 119, normalized size = 8.5

$$\frac{1}{8}\,d^8x^{24}+cd^7x^{23}+\frac{7}{2}\,c^2d^6x^{22}+7\,c^3d^5x^{21}+\frac{35}{4}\,c^4d^4x^{20}+7\,c^5d^3x^{19}+\frac{7}{2}\,c^6d^2x^{18}+c^7dx^{17}+\frac{1}{8}\,c^8x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15*(d*x+c)^7*(3*d*x+2*c),x, algorithm="maxima")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + 35/4*c^4*d^
4*x^20 + 7*c^5*d^3*x^19 + 7/2*c^6*d^2*x^18 + c^7*d*x^17 + 1/8*c^8*x^16
```

Fricas [B] time = 1.06846, size = 198, normalized size = 14.14

$$\frac{1}{8}x^{24}d^8+x^{23}d^7c+\frac{7}{2}x^{22}d^6c^2+7x^{21}d^5c^3+\frac{35}{4}x^{20}d^4c^4+7x^{19}d^3c^5+\frac{7}{2}x^{18}d^2c^6+x^{17}dc^7+\frac{1}{8}x^{16}c^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15*(d*x+c)^7*(3*d*x+2*c),x, algorithm="fricas")
```

```
[Out] 1/8*x^24*d^8 + x^23*d^7*c + 7/2*x^22*d^6*c^2 + 7*x^21*d^5*c^3 + 35/4*x^20*d
^4*c^4 + 7*x^19*d^3*c^5 + 7/2*x^18*d^2*c^6 + x^17*d*c^7 + 1/8*x^16*c^8
```

Sympy [B] time = 0.081833, size = 97, normalized size = 6.93

$$\frac{c^8x^{16}}{8}+c^7dx^{17}+\frac{7c^6d^2x^{18}}{2}+7c^5d^3x^{19}+\frac{35c^4d^4x^{20}}{4}+7c^3d^5x^{21}+\frac{7c^2d^6x^{22}}{2}+cd^7x^{23}+\frac{d^8x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**15*(d*x+c)**7*(3*d*x+2*c),x)
```

```
[Out] c**8*x**16/8 + c**7*d*x**17 + 7*c**6*d**2*x**18/2 + 7*c**5*d**3*x**19 + 35*
c**4*d**4*x**20/4 + 7*c**3*d**5*x**21 + 7*c**2*d**6*x**22/2 + c*d**7*x**23
+ d**8*x**24/8
```

Giac [B] time = 1.21004, size = 119, normalized size = 8.5

$$\frac{1}{8}d^8x^{24}+cd^7x^{23}+\frac{7}{2}c^2d^6x^{22}+7c^3d^5x^{21}+\frac{35}{4}c^4d^4x^{20}+7c^5d^3x^{19}+\frac{7}{2}c^6d^2x^{18}+c^7dx^{17}+\frac{1}{8}c^8x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15*(d*x+c)^7*(3*d*x+2*c),x, algorithm="giac")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + 35/4*c^4*d^
4*x^20 + 7*c^5*d^3*x^19 + 7/2*c^6*d^2*x^18 + c^7*d*x^17 + 1/8*c^8*x^16
```

3.206 $\int (a+bx)\left(1+\left(ax+\frac{bx^2}{2}\right)^4\right)dx$

Optimal. Leaf size=28

$$\frac{1}{160}x^5(2a+bx)^5+ax+\frac{bx^2}{2}$$

```
[Out] a*x + (b*x^2)/2 + (x^5*(2*a + b*x)^5)/160
```

Rubi [A] time = 0.0270042, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1591}

$$\frac{1}{160}x^5(2a+bx)^5+ax+\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(1 + (a*x + (b*x^2)/2)^4),x]
```

```
[Out] a*x + (b*x^2)/2 + (x^5*(2*a + b*x)^5)/160
```

Rule 1591

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\begin{aligned}\int (a+bx)\left(1+\left(ax+\frac{bx^2}{2}\right)^4\right)dx &= \text{Subst}\left(\int \left(1+x^4\right)dx, x, ax+\frac{bx^2}{2}\right)\\ &= ax+\frac{bx^2}{2}+\frac{1}{160}x^5(2a+bx)^5\end{aligned}$$

Mathematica [B] time = 0.0051486, size = 80, normalized size = 2.86

$$\frac{1}{4}a^2b^3x^8 + \frac{1}{2}a^3b^2x^7 + \frac{1}{2}a^4bx^6 + \frac{a^5x^5}{5} + \frac{1}{16}ab^4x^9 + ax + \frac{b^5x^{10}}{160} + \frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(1 + (a*x + (b*x^2)/2)^4),x]
```

```
[Out] a*x + (b*x^2)/2 + (a^5*x^5)/5 + (a^4*b*x^6)/2 + (a^3*b^2*x^7)/2 + (a^2*b^3*
x^8)/4 + (a*b^4*x^9)/16 + (b^5*x^10)/160
```

Maple [B] time = 0.002, size = 67, normalized size = 2.4

$$\frac{b^5x^{10}}{160} + \frac{ab^4x^9}{16} + \frac{a^2b^3x^8}{4} + \frac{a^3b^2x^7}{2} + \frac{a^4bx^6}{2} + \frac{a^5x^5}{5} + \frac{bx^2}{2} + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(1+(a*x+1/2*b*x^2)^4),x)
```

```
[Out] 1/160*b^5*x^10+1/16*a*b^4*x^9+1/4*a^2*b^3*x^8+1/2*a^3*b^2*x^7+1/2*a^4*b*x^6
+1/5*a^5*x^5+1/2*b*x^2+a*x
```

Maxima [B] time = 1.00854, size = 89, normalized size = 3.18

$$\frac{1}{160}\,b^5x^{10} + \frac{1}{16}\,ab^4x^9 + \frac{1}{4}\,a^2b^3x^8 + \frac{1}{2}\,a^3b^2x^7 + \frac{1}{2}\,a^4bx^6 + \frac{1}{5}\,a^5x^5 + \frac{1}{2}\,bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(1+(a*x+1/2*b*x^2)^4),x, algorithm="maxima")
```

```
[Out] 1/160*b^5*x^10 + 1/16*a*b^4*x^9 + 1/4*a^2*b^3*x^8 + 1/2*a^3*b^2*x^7 + 1/2*a
^4*b*x^6 + 1/5*a^5*x^5 + 1/2*b*x^2 + a*x
```

Fricas [B] time = 1.11006, size = 158, normalized size = 5.64

$$\frac{1}{160}x^{10}b^5 + \frac{1}{16}x^9b^4a + \frac{1}{4}x^8b^3a^2 + \frac{1}{2}x^7b^2a^3 + \frac{1}{2}x^6ba^4 + \frac{1}{5}x^5a^5 + \frac{1}{2}x^2b + xa$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(1+(a*x+1/2*b*x^2)^4),x, algorithm="fricas")
```

```
[Out] 1/160*x^10*b^5 + 1/16*x^9*b^4*a + 1/4*x^8*b^3*a^2 + 1/2*x^7*b^2*a^3 + 1/2*x
^6*b*a^4 + 1/5*x^5*a^5 + 1/2*x^2*b + x*a
```

Sympy [B] time = 0.080792, size = 70, normalized size = 2.5

$$\frac{a^5x^5}{5} + \frac{a^4bx^6}{2} + \frac{a^3b^2x^7}{2} + \frac{a^2b^3x^8}{4} + \frac{ab^4x^9}{16} + ax + \frac{b^5x^{10}}{160} + \frac{bx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(1+(a*x+1/2*b*x**2)**4),x)
```

```
[Out] a**5*x**5/5 + a**4*b*x**6/2 + a**3*b**2*x**7/2 + a**2*b**3*x**8/4 + a*b**4*
x**9/16 + a*x + b**5*x**10/160 + b*x**2/2
```

Giac [B] time = 1.23564, size = 89, normalized size = 3.18

$$\frac{1}{160}\,b^5x^{10} + \frac{1}{16}\,ab^4x^9 + \frac{1}{4}\,a^2b^3x^8 + \frac{1}{2}\,a^3b^2x^7 + \frac{1}{2}\,a^4bx^6 + \frac{1}{5}\,a^5x^5 + \frac{1}{2}\,bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(1+(a*x+1/2*b*x^2)^4),x, algorithm="giac")
```

```
[Out] 1/160*b^5*x^10 + 1/16*a*b^4*x^9 + 1/4*a^2*b^3*x^8 + 1/2*a^3*b^2*x^7 + 1/2*a
^4*b*x^6 + 1/5*a^5*x^5 + 1/2*b*x^2 + a*x
```

3.207 $\int (a+bx)\left(1+\left(c+ax+\frac{bx^2}{2}\right)^4\right)dx$

Optimal. Leaf size=31

$$\frac{1}{5}\left(ax+\frac{bx^2}{2}+c\right)^5+ax+\frac{bx^2}{2}$$

```
[Out] a*x + (b*x^2)/2 + (c + a*x + (b*x^2)/2)^5/5
```

Rubi [A] time = 0.0311218, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1591}

$$\frac{1}{5}\left(ax+\frac{bx^2}{2}+c\right)^5+ax+\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(1 + (c + a*x + (b*x^2)/2)^4),x]
```

```
[Out] a*x + (b*x^2)/2 + (c + a*x + (b*x^2)/2)^5/5
```

Rule 1591

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\begin{aligned}\int (a+bx)\left(1+\left(c+ax+\frac{bx^2}{2}\right)^4\right)dx &= \text{Subst}\left(\int \left(1+x^4\right)dx, x, c+ax+\frac{bx^2}{2}\right)\\ &= ax+\frac{bx^2}{2}+\frac{1}{5}\left(c+ax+\frac{bx^2}{2}\right)^5\end{aligned}$$

Mathematica [B] time = 0.03429, size = 108, normalized size = 3.48

$$\frac{1}{160}x(2a+bx)\left(24a^2b^2x^6+32a^3bx^5+16a^4x^4+8ab^3x^7+40c^2x^2(2a+bx)^2+80c^3x(2a+bx)+10cx^3(2a+bx)^3+b^4x^8+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(1 + (c + a*x + (b*x^2)/2)^4),x]
```

```
[Out] (x*(2*a + b*x)*(80 + 80*c^4 + 16*a^4*x^4 + 32*a^3*b*x^5 + 24*a^2*b^2*x^6 +
8*a*b^3*x^7 + b^4*x^8 + 80*c^3*x*(2*a + b*x) + 40*c^2*x^2*(2*a + b*x)^2 + 1
0*c*x^3*(2*a + b*x)^3))/160
```

Maple [B] time = 0.002, size = 325, normalized size = 10.5

$$\frac{b^5x^{10}}{160}+\frac{ab^4x^9}{16}+\frac{x^8}{8}\left(\frac{a^2b^3}{2}+b\left(\frac{(a^2+bc)\,b^2}{2}+b^2a^2\right)\right)+\frac{x^7}{7}\left(a\left(\frac{(a^2+bc)\,b^2}{2}+b^2a^2\right)+b\left(ab^2c+2\,(a^2+bc)\,ab\right)\right)+\frac{x^6}{6}\left(a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(1+(c+a*x+1/2*b*x^2)^4),x)
```

```
[Out] 1/160*b^5*x^10+1/16*a*b^4*x^9+1/8*(1/2*a^2*b^3+b*(1/2*(a^2+b*c)*b^2+b^2*a^2
))*x^8+1/7*(a*(1/2*(a^2+b*c)*b^2+b^2*a^2)+b*(a*b^2*c+2*(a^2+b*c)*a*b))*x^7+
1/6*(a*(a*b^2*c+2*(a^2+b*c)*a*b)+b*(1/2*c^2*b^2+4*a^2*b*c+(a^2+b*c)^2))*x^6
+1/5*(a*(1/2*c^2*b^2+4*a^2*b*c+(a^2+b*c)^2)+b*(2*c^2*a*b+4*a*c*(a^2+b*c)))*
x^5+1/4*(a*(2*c^2*a*b+4*a*c*(a^2+b*c))+b*(2*c^2*(a^2+b*c)+4*a^2*c^2))*x^4+1
/3*(a*(2*c^2*(a^2+b*c)+4*a^2*c^2)+4*a*b*c^3)*x^3+1/2*(4*a^2*c^3+b*(c^4+1))*
x^2+a*(c^4+1)*x
```

Maxima [B] time = 1.02107, size = 252, normalized size = 8.13

$$\frac{1}{160}\,b^5x^{10}+\frac{1}{16}\,ab^4x^9+\frac{1}{16}\left(4\,a^2b^3+b^4c\right)x^8+\frac{1}{2}\left(a^3b^2+ab^3c\right)x^7+\frac{1}{4}\left(2\,a^4b+6\,a^2b^2c+b^3c^2\right)x^6+\frac{1}{10}\left(2\,a^5+20\,a^3bc+1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(1+(c+a*x+1/2*b*x^2)^4),x, algorithm="maxima")
```

```
[Out] 1/160*b^5*x^10 + 1/16*a*b^4*x^9 + 1/16*(4*a^2*b^3 + b^4*c)*x^8 + 1/2*(a^3*b
^2 + a*b^3*c)*x^7 + 1/4*(2*a^4*b + 6*a^2*b^2*c + b^3*c^2)*x^6 + 1/10*(2*a^5
 + 20*a^3*b*c + 15*a*b^2*c^2)*x^5 + 1/2*(2*a^4*c + 6*a^2*b*c^2 + b^2*c^3)*x
^4 + 2*(a^3*c^2 + a*b*c^3)*x^3 + 1/2*(4*a^2*c^3 + b*c^4 + b)*x^2 + (a*c^4 +
 a)*x
```

Fricas [B] time = 1.13951, size = 470, normalized size = 15.16

$$\frac{1}{160}x^{10}b^5 + \frac{1}{16}x^9b^4a + \frac{1}{16}x^8cb^4 + \frac{1}{4}x^8b^3a^2 + \frac{1}{2}x^7cb^3a + \frac{1}{2}x^7b^2a^3 + \frac{1}{4}x^6c^2b^3 + \frac{3}{2}x^6cb^2a^2 + \frac{1}{2}x^6ba^4 + \frac{3}{2}x^5c^2b^2a + 2x^5cb$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(1+(c+a*x+1/2*b*x^2)^4),x, algorithm="fricas")
```

```
[Out] 1/160*x^10*b^5 + 1/16*x^9*b^4*a + 1/16*x^8*c*b^4 + 1/4*x^8*b^3*a^2 + 1/2*x^
7*c*b^3*a + 1/2*x^7*b^2*a^3 + 1/4*x^6*c^2*b^3 + 3/2*x^6*c*b^2*a^2 + 1/2*x^6
*b*a^4 + 3/2*x^5*c^2*b^2*a + 2*x^5*c*b*a^3 + 1/5*x^5*a^5 + 1/2*x^4*c^3*b^2
+ 3*x^4*c^2*b*a^2 + x^4*c*a^4 + 2*x^3*c^3*b*a + 2*x^3*c^2*a^3 + 1/2*x^2*c^4
*b + 2*x^2*c^3*a^2 + x*c^4*a + 1/2*x^2*b + x*a
```

Sympy [B] time = 0.107408, size = 194, normalized size = 6.26

$$\frac{ab^4x^9}{16} + \frac{b^5x^{10}}{160} + x^8\left(\frac{a^2b^3}{4} + \frac{b^4c}{16}\right) + x^7\left(\frac{a^3b^2}{2} + \frac{ab^3c}{2}\right) + x^6\left(\frac{a^4b}{2} + \frac{3a^2b^2c}{2} + \frac{b^3c^2}{4}\right) + x^5\left(\frac{a^5}{5} + 2a^3bc + \frac{3ab^2c^2}{2}\right) + x^4\left(a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(1+(c+a*x+1/2*b*x**2)**4),x)
```

```
[Out] a*b**4*x**9/16 + b**5*x**10/160 + x**8*(a**2*b**3/4 + b**4*c/16) + x**7*(a*
*3*b**2/2 + a*b**3*c/2) + x**6*(a**4*b/2 + 3*a**2*b**2*c/2 + b**3*c**2/4) +
 x**5*(a**5/5 + 2*a**3*b*c + 3*a*b**2*c**2/2) + x**4*(a**4*c + 3*a**2*b*c**
2 + b**2*c**3/2) + x**3*(2*a**3*c**2 + 2*a*b*c**3) + x**2*(2*a**2*c**3 + b*
c**4/2 + b/2) + x*(a*c**4 + a)
```

Giac [B] time = 1.17276, size = 281, normalized size = 9.06

$$\frac{1}{160}b^5x^{10} + \frac{1}{16}ab^4x^9 + \frac{1}{4}a^2b^3x^8 + \frac{1}{16}b^4cx^8 + \frac{1}{2}a^3b^2x^7 + \frac{1}{2}ab^3cx^7 + \frac{1}{2}a^4bx^6 + \frac{3}{2}a^2b^2cx^6 + \frac{1}{4}b^3c^2x^6 + \frac{1}{5}a^5x^5 + 2a^3b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(1+(c+a*x+1/2*b*x^2)^4),x, algorithm="giac")
```

```
[Out] 1/160*b^5*x^10 + 1/16*a*b^4*x^9 + 1/4*a^2*b^3*x^8 + 1/16*b^4*c*x^8 + 1/2*a^
3*b^2*x^7 + 1/2*a*b^3*c*x^7 + 1/2*a^4*b*x^6 + 3/2*a^2*b^2*c*x^6 + 1/4*b^3*c
^2*x^6 + 1/5*a^5*x^5 + 2*a^3*b*c*x^5 + 3/2*a*b^2*c^2*x^5 + a^4*c*x^4 + 3*a^
2*b*c^2*x^4 + 1/2*b^2*c^3*x^4 + 2*a^3*c^2*x^3 + 2*a*b*c^3*x^3 + 2*a^2*c^3*x
^2 + 1/2*b*c^4*x^2 + a*c^4*x + 1/2*b*x^2 + a*x
```

3.208 $\int (a+bx)\left(1+\left(ax+\frac{bx^2}{2}\right)^n\right) dx$

Optimal. Leaf size=34

$$\frac{\left(ax+\frac{bx^2}{2}\right)^{n+1}}{n+1}+ax+\frac{bx^2}{2}$$

```
[Out] a*x + (b*x^2)/2 + (a*x + (b*x^2)/2)^(1 + n)/(1 + n)
```

Rubi [A] time = 0.0084399, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1591}

$$\frac{\left(ax+\frac{bx^2}{2}\right)^{n+1}}{n+1}+ax+\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(1 + (a*x + (b*x^2)/2)^n),x]
```

```
[Out] a*x + (b*x^2)/2 + (a*x + (b*x^2)/2)^(1 + n)/(1 + n)
```

Rule 1591

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\begin{aligned}\int (a+bx)\left(1+\left(ax+\frac{bx^2}{2}\right)^n\right) dx &= \text{Subst}\left(\int (1+x^n)\, dx, x, ax+\frac{bx^2}{2}\right)\\ &= ax+\frac{bx^2}{2}+\frac{\left(ax+\frac{bx^2}{2}\right)^{1+n}}{1+n}\end{aligned}$$

Mathematica [A] time = 0.0547979, size = 34, normalized size = 1.

$$\frac{x(2a+bx)\left(\left(ax+\frac{bx^2}{2}\right)^n+n+1\right)}{2(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(1 + (a*x + (b*x^2)/2)^n),x]
```

```
[Out] (x*(2*a + b*x)*(1 + n + (a*x + (b*x^2)/2)^n))/(2*(1 + n))
```

Maple [A] time = 0.003, size = 31, normalized size = 0.9

$$ax+\frac{bx^2}{2}+\frac{1}{1+n}\left(ax+\frac{bx^2}{2}\right)^{1+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(1+(a*x+1/2*b*x^2)^n),x)
```

```
[Out] a*x+1/2*b*x^2+(a*x+1/2*b*x^2)^(1+n)/(1+n)
```

Maxima [A] time = 1.69603, size = 70, normalized size = 2.06

$$\frac{1}{2}\,bx^2+ax+\frac{\left(bx^2+2\,ax\right)e^{(n\log(bx+2\,a)+n\log(x))}}{2^{n+1}n+2^{n+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(1+(a*x+1/2*b*x^2)^n),x, algorithm="maxima")
```

```
[Out] 1/2*b*x^2 + a*x + (b*x^2 + 2*a*x)*e^(n*log(b*x + 2*a) + n*log(x))/(2^(n + 1
)*n + 2^(n + 1))
```

Fricas [A] time = 1.39302, size = 112, normalized size = 3.29

$$\frac{(bn+b)x^2+\left(bx^2+2\,ax\right)\left(\frac{1}{2}\,bx^2+ax\right)^n+2\,(an+a)x}{2\,(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(1+(a*x+1/2*b*x^2)^n),x, algorithm="fricas")
```

```
[Out] 1/2*((b*n + b)*x^2 + (b*x^2 + 2*a*x)*(1/2*b*x^2 + a*x)^n + 2*(a*n + a)*x)/(
n + 1)
```

Sympy [A] time = 87.1078, size = 230, normalized size = 6.76

$$\begin{cases} a\left(x+\frac{\log(x)}{a}\right) & \text{for}\ b=0\wedge n=-1\\ a\left(\frac{a^n x x^n}{n+1}+\frac{nx}{n+1}+\frac{x}{n+1}\right) & \text{for}\ b=0\\ ax+\frac{bx^2}{2}+\log(x)+\log\left(\frac{2a}{b}+x\right) & \text{for}\ n=-1\\ \frac{2\cdot 2^n abnx}{2\cdot 2^n bn+2\cdot 2^n b}+\frac{2\cdot 2^n abx}{2\cdot 2^n bn+2\cdot 2^n b}+\frac{2^n b^2 nx^2}{2\cdot 2^n bn+2\cdot 2^n b}+\frac{2^n b^2 x^2}{2\cdot 2^n bn+2\cdot 2^n b}+\frac{2abx\left(2ax+bx^2\right)^n}{2\cdot 2^n bn+2\cdot 2^n b}+\frac{b^2x^2\left(2ax+bx^2\right)^n}{2\cdot 2^n bn+2\cdot 2^n b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(1+(a*x+1/2*b*x**2)**n),x)
```

```
[Out] Piecewise((a*(x + log(x)/a), Eq(b, 0) & Eq(n, -1)), (a*(a**n*x*x**n/(n + 1)
 + n*x/(n + 1) + x/(n + 1)), Eq(b, 0)), (a*x + b*x**2/2 + log(x) + log(2*a/
b + x), Eq(n, -1)), (2*2**n*a*b*n*x/(2*2**n*b*n + 2*2**n*b) + 2*2**n*a*b*x/
(2*2**n*b*n + 2*2**n*b) + 2**n*b**2*n*x**2/(2*2**n*b*n + 2*2**n*b) + 2**n*b
**2*x**2/(2*2**n*b*n + 2*2**n*b) + 2*a*b*x*(2*a*x + b*x**2)**n/(2*2**n*b*n
+ 2*2**n*b) + b**2*x**2*(2*a*x + b*x**2)**n/(2*2**n*b*n + 2*2**n*b), True))
```

Giac [A] time = 1.12755, size = 41, normalized size = 1.21

$$\frac{1}{2}\,bx^2+ax+\frac{\left(\frac{1}{2}\,bx^2+ax\right)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(1+(a*x+1/2*b*x^2)^n),x, algorithm="giac")
```

```
[Out] 1/2*b*x^2 + a*x + (1/2*b*x^2 + a*x)^(n + 1)/(n + 1)
```

3.209 $\int (a+bx)\left(1+\left(c+ax+\frac{bx^2}{2}\right)^n\right)\,dx$

Optimal. Leaf size=35

$$\frac{\left(ax+\frac{bx^2}{2}+c\right)^{n+1}}{n+1}+ax+\frac{bx^2}{2}$$

```
[Out] a*x + (b*x^2)/2 + (c + a*x + (b*x^2)/2)^(1 + n)/(1 + n)
```

Rubi [A] time = 0.0090755, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1591}

$$\frac{\left(ax+\frac{bx^2}{2}+c\right)^{n+1}}{n+1}+ax+\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(1 + (c + a*x + (b*x^2)/2)^n),x]
```

```
[Out] a*x + (b*x^2)/2 + (c + a*x + (b*x^2)/2)^(1 + n)/(1 + n)
```

Rule 1591

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\begin{aligned}\int (a+bx)\left(1+\left(c+ax+\frac{bx^2}{2}\right)^n\right)\,dx &= \text{Subst}\left(\int (1+x^n)\,dx, x, c+ax+\frac{bx^2}{2}\right)\\ &= ax+\frac{bx^2}{2}+\frac{\left(c+ax+\frac{bx^2}{2}\right)^{1+n}}{1+n}\end{aligned}$$

Mathematica [A] time = 0.0540426, size = 35, normalized size = 1.

$$\frac{\left(ax+\frac{bx^2}{2}+c\right)^{n+1}}{n+1}+ax+\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(1 + (c + a*x + (b*x^2)/2)^n),x]
```

```
[Out] a*x + (b*x^2)/2 + (c + a*x + (b*x^2)/2)^(1 + n)/(1 + n)
```

Maple [A] time = 0.003, size = 33, normalized size = 0.9

$$c+ax+\frac{bx^2}{2}+\frac{1}{1+n}\left(c+ax+\frac{bx^2}{2}\right)^{1+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(1+(c+a*x+1/2*b*x^2)^n),x)
```

```
[Out] c+a*x+1/2*b*x^2+(c+a*x+1/2*b*x^2)^(1+n)/(1+n)
```

Maxima [A] time = 1.66477, size = 73, normalized size = 2.09

$$\frac{1}{2}\,bx^2+ax+\frac{\left(bx^2+2\,ax+2\,c\right)\left(bx^2+2\,ax+2\,c\right)^n}{2^{n+1}n+2^{n+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(1+(c+a*x+1/2*b*x^2)^n),x, algorithm="maxima")
```

```
[Out] 1/2*b*x^2 + a*x + (b*x^2 + 2*a*x + 2*c)*(b*x^2 + 2*a*x + 2*c)^n/(2^(n + 1)*
n + 2^(n + 1))
```

Fricas [A] time = 1.37815, size = 126, normalized size = 3.6

$$\frac{(bn+b)x^2 + \left(bx^2 + 2\,ax + 2\,c\right)\left(\frac{1}{2}\,bx^2 + ax + c\right)^n + 2\,(an+a)x}{2\,(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(1+(c+a*x+1/2*b*x^2)^n),x, algorithm="fricas")
```

```
[Out] 1/2*((b*n + b)*x^2 + (b*x^2 + 2*a*x + 2*c)*(1/2*b*x^2 + a*x + c)^n + 2*(a*n
 + a)*x)/(n + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(1+(c+a*x+1/2*b*x**2)**n),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.21129, size = 107, normalized size = 3.06

$$\frac{\left(\frac{1}{2}\,bx^2 + ax + c\right)^n bx^2 + bnx^2 + 2\left(\frac{1}{2}\,bx^2 + ax + c\right)^n ax + 2\,anx + bx^2 + 2\left(\frac{1}{2}\,bx^2 + ax + c\right)^n c + 2\,ax}{2\,(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(1+(c+a*x+1/2*b*x^2)^n),x, algorithm="giac")
```

```
[Out] 1/2*((1/2*b*x^2 + a*x + c)^n*b*x^2 + b*n*x^2 + 2*(1/2*b*x^2 + a*x + c)^n*a*
x + 2*a*n*x + b*x^2 + 2*(1/2*b*x^2 + a*x + c)^n*c + 2*a*x)/(n + 1)
```

3.210 $\int \left(a+cx^2\right)\left(1+\left(ax+\frac{cx^3}{3}\right)^5\right)\,dx$

Optimal. Leaf size=30

$$\frac{1}{6}\left(ax+\frac{cx^3}{3}\right)^6+ax+\frac{cx^3}{3}$$

```
[Out] a*x + (c*x^3)/3 + (a*x + (c*x^3)/3)^6/6
```

Rubi [A] time = 0.0198891, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {1591}

$$\frac{1}{6}\left(ax+\frac{cx^3}{3}\right)^6+ax+\frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)*(1 + (a*x + (c*x^3)/3)^5),x]
```

```
[Out] a*x + (c*x^3)/3 + (a*x + (c*x^3)/3)^6/6
```

Rule 1591

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\begin{aligned}\int \left(a+cx^2\right)\left(1+\left(ax+\frac{cx^3}{3}\right)^5\right)\,dx &= \text{Subst}\left(\int \left(1+x^5\right)\,dx,x,ax+\frac{cx^3}{3}\right)\\ &= ax+\frac{cx^3}{3}+\frac{1}{6}\left(ax+\frac{cx^3}{3}\right)^6\end{aligned}$$

Mathematica [B] time = 0.0057631, size = 93, normalized size = 3.1

$$\frac{5}{162}a^2c^4x^{14}+\frac{10}{81}a^3c^3x^{12}+\frac{5}{18}a^4c^2x^{10}+\frac{1}{3}a^5cx^8+\frac{a^6x^6}{6}+\frac{1}{243}ac^5x^{16}+ax+\frac{c^6x^{18}}{4374}+\frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)*(1 + (a*x + (c*x^3)/3)^5),x]
```

```
[Out] a*x + (c*x^3)/3 + (a^6*x^6)/6 + (a^5*c*x^8)/3 + (5*a^4*c^2*x^10)/18 + (10*a
^3*c^3*x^12)/81 + (5*a^2*c^4*x^14)/162 + (a*c^5*x^16)/243 + (c^6*x^18)/4374
```

Maple [B] time = 0.002, size = 78, normalized size = 2.6

$$\frac{c^6x^{18}}{4374}+\frac{c^5ax^{16}}{243}+\frac{5\,c^4a^2x^{14}}{162}+\frac{10\,a^3c^3x^{12}}{81}+\frac{5\,a^4c^2x^{10}}{18}+\frac{a^5cx^8}{3}+\frac{a^6x^6}{6}+\frac{cx^3}{3}+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)*(1+(a*x+1/3*c*x^3)^5),x)
```

```
[Out] 1/4374*c^6*x^18+1/243*c^5*a*x^16+5/162*c^4*a^2*x^14+10/81*a^3*c^3*x^12+5/18
*a^4*c^2*x^10+1/3*a^5*c*x^8+1/6*a^6*x^6+1/3*c*x^3+a*x
```

Maxima [B] time = 1.00994, size = 104, normalized size = 3.47

$$\frac{1}{4374}\,c^6x^{18}+\frac{1}{243}\,ac^5x^{16}+\frac{5}{162}\,a^2c^4x^{14}+\frac{10}{81}\,a^3c^3x^{12}+\frac{5}{18}\,a^4c^2x^{10}+\frac{1}{3}\,a^5cx^8+\frac{1}{6}\,a^6x^6+\frac{1}{3}\,cx^3+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(1+(a*x+1/3*c*x^3)^5),x, algorithm="maxima")
```

```
[Out] 1/4374*c^6*x^18 + 1/243*a*c^5*x^16 + 5/162*a^2*c^4*x^14 + 10/81*a^3*c^3*x^1
2 + 5/18*a^4*c^2*x^10 + 1/3*a^5*c*x^8 + 1/6*a^6*x^6 + 1/3*c*x^3 + a*x
```

Fricas [B] time = 1.11426, size = 197, normalized size = 6.57

$$\frac{1}{4374}x^{18}c^6+\frac{1}{243}x^{16}c^5a+\frac{5}{162}x^{14}c^4a^2+\frac{10}{81}x^{12}c^3a^3+\frac{5}{18}x^{10}c^2a^4+\frac{1}{3}x^8ca^5+\frac{1}{6}x^6a^6+\frac{1}{3}x^3c+xa$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(1+(a*x+1/3*c*x^3)^5),x, algorithm="fricas")
```

```
[Out] 1/4374*x^18*c^6 + 1/243*x^16*c^5*a + 5/162*x^14*c^4*a^2 + 10/81*x^12*c^3*a^
3 + 5/18*x^10*c^2*a^4 + 1/3*x^8*c*a^5 + 1/6*x^6*a^6 + 1/3*x^3*c + x*a
```

Sympy [B] time = 0.090498, size = 87, normalized size = 2.9

$$\frac{a^6x^6}{6} + \frac{a^5cx^8}{3} + \frac{5a^4c^2x^{10}}{18} + \frac{10a^3c^3x^{12}}{81} + \frac{5a^2c^4x^{14}}{162} + \frac{ac^5x^{16}}{243} + ax + \frac{c^6x^{18}}{4374} + \frac{cx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)*(1+(a*x+1/3*c*x**3)**5),x)
```

```
[Out] a**6*x**6/6 + a**5*c*x**8/3 + 5*a**4*c**2*x**10/18 + 10*a**3*c**3*x**12/81
+ 5*a**2*c**4*x**14/162 + a*c**5*x**16/243 + a*x + c**6*x**18/4374 + c*x**3
/3
```

Giac [B] time = 1.22165, size = 104, normalized size = 3.47

$$\frac{1}{4374}c^6x^{18} + \frac{1}{243}ac^5x^{16} + \frac{5}{162}a^2c^4x^{14} + \frac{10}{81}a^3c^3x^{12} + \frac{5}{18}a^4c^2x^{10} + \frac{1}{3}a^5cx^8 + \frac{1}{6}a^6x^6 + \frac{1}{3}cx^3 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(1+(a*x+1/3*c*x^3)^5),x, algorithm="giac")
```

```
[Out] 1/4374*c^6*x^18 + 1/243*a*c^5*x^16 + 5/162*a^2*c^4*x^14 + 10/81*a^3*c^3*x^1
2 + 5/18*a^4*c^2*x^10 + 1/3*a^5*c*x^8 + 1/6*a^6*x^6 + 1/3*c*x^3 + a*x
```

3.211 $\int \left(a+cx^2\right)\left(1+\left(d+ax+\frac{cx^3}{3}\right)^5\right)dx$

Optimal. Leaf size=31

$$\frac{1}{6}\left(ax+\frac{cx^3}{3}+d\right)^6+ax+\frac{cx^3}{3}$$

```
[Out] a*x + (c*x^3)/3 + (d + a*x + (c*x^3)/3)^6/6
```

Rubi [A] time = 0.0384825, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {1591}

$$\frac{1}{6}\left(ax+\frac{cx^3}{3}+d\right)^6+ax+\frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)*(1 + (d + a*x + (c*x^3)/3)^5),x]
```

```
[Out] a*x + (c*x^3)/3 + (d + a*x + (c*x^3)/3)^6/6
```

Rule 1591

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\begin{aligned}\int \left(a+cx^2\right)\left(1+\left(d+ax+\frac{cx^3}{3}\right)^5\right)dx &= \text{Subst}\left(\int \left(1+x^5\right)dx, x, d+ax+\frac{cx^3}{3}\right)\\ &= ax+\frac{cx^3}{3}+\frac{1}{6}\left(d+ax+\frac{cx^3}{3}\right)^6\end{aligned}$$

Mathematica [B] time = 0.0460005, size = 140, normalized size = 4.52

$$\frac{x\left(3a+cx^2\right)\left(90a^2c^3x^{11}+270a^3c^2x^9+405a^4cx^7+243a^5x^5+15ac^4x^{13}+135d^2\left(3ax+cx^3\right)^3+540d^3\left(3ax+cx^3\right)^2+121\right.}{4374}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)*(1 + (d + a*x + (c*x^3)/3)^5),x]
```

```
[Out] (x*(3*a + c*x^2)*(1458 + 1458*d^5 + 243*a^5*x^5 + 405*a^4*c*x^7 + 270*a^3*c
^2*x^9 + 90*a^2*c^3*x^11 + 15*a*c^4*x^13 + c^5*x^15 + 1215*d^4*(3*a*x + c*x
^3) + 540*d^3*(3*a*x + c*x^3)^2 + 135*d^2*(3*a*x + c*x^3)^3 + 18*d*(3*a*x +
 c*x^3)^4))/4374
```

Maple [B] time = 0.002, size = 618, normalized size = 19.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)*(1+(d+a*x+1/3*c*x^3)^5),x)
```

```
[Out] 1/4374*c^6*x^18+1/243*c^5*a*x^16+1/243*c^5*d*x^15+5/162*c^4*a^2*x^14+5/81*c
^4*a*d*x^13+1/12*(10/27*a^3*c^3+c*(4/27*d^2*c^3+2/3*a^3*c^2+1/3*c*(2/3*d^2*
c^2+4/3*a^3*c)))*x^12+10/27*a^2*c^3*d*x^11+1/10*(a*(4/27*d^2*c^3+2/3*a^3*c^
2+1/3*c*(2/3*d^2*c^2+4/3*a^3*c))+c*(4/3*a*c^2*d^2+a*(2/3*d^2*c^2+4/3*a^3*c)
+1/3*c*(a^4+4*a*c*d^2)))*x^10+1/9*(10/3*a^3*c^2*d+c*(d*(2/3*d^2*c^2+4/3*a^3
*c)+4*a^3*d*c+1/3*c*(4/3*c*d^3+4*a^3*d)))*x^9+1/8*(a*(4/3*a*c^2*d^2+a*(2/3*
d^2*c^2+4/3*a^3*c)+1/3*c*(a^4+4*a*c*d^2))+c*(6*d^2*a^2*c+a*(a^4+4*a*c*d^2))
)*x^8+1/7*(a*(d*(2/3*d^2*c^2+4/3*a^3*c)+4*a^3*d*c+1/3*c*(4/3*c*d^3+4*a^3*d)
)+c*(d*(a^4+4*a*c*d^2)+a*(4/3*c*d^3+4*a^3*d)+4/3*a*c*d^3))*x^7+1/6*(a*(6*d^
2*a^2*c+a*(a^4+4*a*c*d^2))+c*(d*(4/3*c*d^3+4*a^3*d)+6*a^3*d^2+1/3*c*d^4))*x
^6+1/5*(a*(d*(a^4+4*a*c*d^2)+a*(4/3*c*d^3+4*a^3*d)+4/3*a*c*d^3)+10*a^2*c*d^
3)*x^5+1/4*(a*(d*(4/3*c*d^3+4*a^3*d)+6*a^3*d^2+1/3*c*d^4)+5*a*c*d^4)*x^4+1/
3*(10*a^3*d^3+c*(d^5+1))*x^3+5/2*a^2*d^4*x^2+a*(d^5+1)*x
```

Maxima [B] time = 1.06855, size = 378, normalized size = 12.19

$$\frac{1}{4374}c^6x^{18}+\frac{1}{243}ac^5x^{16}+\frac{1}{243}c^5dx^{15}+\frac{5}{162}a^2c^4x^{14}+\frac{5}{81}ac^4dx^{13}+\frac{10}{27}a^2c^3dx^{11}+\frac{5}{162}\left(4\,a^3c^3+c^4d^2\right)x^{12}+\frac{5}{54}\left(3\,a^4c^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(1+(d+a*x+1/3*c*x^3)^5),x, algorithm="maxima")
```

```
[Out] 1/4374*c^6*x^18 + 1/243*a*c^5*x^16 + 1/243*c^5*d*x^15 + 5/162*a^2*c^4*x^14
+ 5/81*a*c^4*d*x^13 + 10/27*a^2*c^3*d*x^11 + 5/162*(4*a^3*c^3 + c^4*d^2)*x^
12 + 5/54*(3*a^4*c^2 + 4*a*c^3*d^2)*x^10 + 10/81*(9*a^3*c^2*d + c^3*d^3)*x^
9 + 1/3*(a^5*c + 5*a^2*c^2*d^2)*x^8 + 5/2*a^2*d^4*x^2 + 5/9*(3*a^4*c*d + 2*
a*c^2*d^3)*x^7 + 1/18*(3*a^6 + 60*a^3*c*d^2 + 5*c^2*d^4)*x^6 + 1/3*(3*a^5*d
 + 10*a^2*c*d^3)*x^5 + 5/6*(3*a^4*d^2 + 2*a*c*d^4)*x^4 + 1/3*(10*a^3*d^3 +
c*d^5 + c)*x^3 + (a*d^5 + a)*x
```

Fricas [B] time = 1.16685, size = 710, normalized size = 22.9

$$\frac{1}{4374}x^{18}c^6 + \frac{1}{243}x^{16}c^5a + \frac{1}{243}x^{15}dc^5 + \frac{5}{162}x^{14}c^4a^2 + \frac{5}{81}x^{13}dc^4a + \frac{5}{162}x^{12}d^2c^4 + \frac{10}{81}x^{12}c^3a^3 + \frac{10}{27}x^{11}dc^3a^2 + \frac{10}{27}x^{10}d^2c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(1+(d+a*x+1/3*c*x^3)^5),x, algorithm="fricas")
```

```
[Out] 1/4374*x^18*c^6 + 1/243*x^16*c^5*a + 1/243*x^15*d*c^5 + 5/162*x^14*c^4*a^2
+ 5/81*x^13*d*c^4*a + 5/162*x^12*d^2*c^4 + 10/81*x^12*c^3*a^3 + 10/27*x^11*
d*c^3*a^2 + 10/27*x^10*d^2*c^3*a + 5/18*x^10*c^2*a^4 + 10/81*x^9*d^3*c^3 +
10/9*x^9*d*c^2*a^3 + 5/3*x^8*d^2*c^2*a^2 + 1/3*x^8*c*a^5 + 10/9*x^7*d^3*c^2
*a + 5/3*x^7*d*c*a^4 + 5/18*x^6*d^4*c^2 + 10/3*x^6*d^2*c*a^3 + 1/6*x^6*a^6
+ 10/3*x^5*d^3*c*a^2 + x^5*d*a^5 + 5/3*x^4*d^4*c*a + 5/2*x^4*d^2*a^4 + 1/3*
x^3*d^5*c + 10/3*x^3*d^3*a^3 + 5/2*x^2*d^4*a^2 + x*d^5*a + 1/3*x^3*c + x*a
```

Sympy [B] time = 0.130175, size = 314, normalized size = 10.13

$$\frac{5a^2c^4x^{14}}{162} + \frac{10a^2c^3dx^{11}}{27} + \frac{5a^2d^4x^2}{2} + \frac{ac^5x^{16}}{243} + \frac{5ac^4dx^{13}}{81} + \frac{c^6x^{18}}{4374} + \frac{c^5dx^{15}}{243} + x^{12}\left(\frac{10a^3c^3}{81} + \frac{5c^4d^2}{162}\right) + x^{10}\left(\frac{5a^4c^2}{18} + \frac{10a}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)*(1+(d+a*x+1/3*c*x**3)**5),x)
```

```
[Out] 5*a**2*c**4*x**14/162 + 10*a**2*c**3*d*x**11/27 + 5*a**2*d**4*x**2/2 + a*c*
*5*x**16/243 + 5*a*c**4*d*x**13/81 + c**6*x**18/4374 + c**5*d*x**15/243 + x
```

```
**12*(10*a**3*c**3/81 + 5*c**4*d**2/162) + x**10*(5*a**4*c**2/18 + 10*a*c**
3*d**2/27) + x**9*(10*a**3*c**2*d/9 + 10*c**3*d**3/81) + x**8*(a**5*c/3 + 5
*a**2*c**2*d**2/3) + x**7*(5*a**4*c*d/3 + 10*a*c**2*d**3/9) + x**6*(a**6/6
+ 10*a**3*c*d**2/3 + 5*c**2*d**4/18) + x**5*(a**5*d + 10*a**2*c*d**3/3) + x
**4*(5*a**4*d**2/2 + 5*a*c*d**4/3) + x**3*(10*a**3*d**3/3 + c*d**5/3 + c/3)
 + x*(a*d**5 + a)
```

Giac [B] time = 1.23805, size = 393, normalized size = 12.68

$$\frac{1}{4374}c^6x^{18} + \frac{1}{243}ac^5x^{16} + \frac{1}{243}c^5dx^{15} + \frac{5}{162}a^2c^4x^{14} + \frac{5}{81}ac^4dx^{13} + \frac{10}{81}a^3c^3x^{12} + \frac{5}{162}c^4d^2x^{12} + \frac{10}{27}a^2c^3dx^{11} + \frac{5}{18}a^4c^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(1+(d+a*x+1/3*c*x^3)^5),x, algorithm="giac")
```

```
[Out] 1/4374*c^6*x^18 + 1/243*a*c^5*x^16 + 1/243*c^5*d*x^15 + 5/162*a^2*c^4*x^14
+ 5/81*a*c^4*d*x^13 + 10/81*a^3*c^3*x^12 + 5/162*c^4*d^2*x^12 + 10/27*a^2*c
^3*d*x^11 + 5/18*a^4*c^2*x^10 + 10/27*a*c^3*d^2*x^10 + 10/9*a^3*c^2*d*x^9 +
 10/81*c^3*d^3*x^9 + 1/3*a^5*c*x^8 + 5/3*a^2*c^2*d^2*x^8 + 5/3*a^4*c*d*x^7
+ 10/9*a*c^2*d^3*x^7 + 1/6*a^6*x^6 + 10/3*a^3*c*d^2*x^6 + 5/18*c^2*d^4*x^6
+ a^5*d*x^5 + 10/3*a^2*c*d^3*x^5 + 5/2*a^4*d^2*x^4 + 5/3*a*c*d^4*x^4 + 10/3
*a^3*d^3*x^3 + 1/3*c*d^5*x^3 + 5/2*a^2*d^4*x^2 + a*d^5*x + 1/3*c*x^3 + a*x
```

3.212 $\int \left(bx+cx^2\right)\left(1+\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)\,dx$

Optimal. Leaf size=34

$$\frac{x^{12}(3b+2cx)^6}{279936}+\frac{bx^2}{2}+\frac{cx^3}{3}$$

```
[Out] (b*x^2)/2 + (c*x^3)/3 + (x^12*(3*b + 2*c*x)^6)/279936
```

Rubi [A] time = 0.0329058, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.032, Rules used = {1591}

$$\frac{x^{12}(3b+2cx)^6}{279936}+\frac{bx^2}{2}+\frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)*(1 + ((b*x^2)/2 + (c*x^3)/3)^5),x]
```

```
[Out] (b*x^2)/2 + (c*x^3)/3 + (x^12*(3*b + 2*c*x)^6)/279936
```

Rule 1591

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\begin{aligned}\int \left(bx+cx^2\right)\left(1+\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)dx &= \text{Subst}\left(\int \left(1+x^5\right)dx, x, \frac{bx^2}{2}+\frac{cx^3}{3}\right)\\ &= \frac{bx^2}{2}+\frac{cx^3}{3}+\frac{x^{12}(3b+2cx)^6}{279936}\end{aligned}$$

Mathematica [B] time = 0.0077667, size = 98, normalized size = 2.88

$$\frac{5}{648}b^2c^4x^{16} + \frac{5}{324}b^3c^3x^{15} + \frac{5}{288}b^4c^2x^{14} + \frac{1}{96}b^5cx^{13} + \frac{b^6x^{12}}{384} + \frac{1}{486}bc^5x^{17} + \frac{bx^2}{2} + \frac{c^6x^{18}}{4374} + \frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)*(1 + ((b*x^2)/2 + (c*x^3)/3)^5),x]
```

```
[Out] (b*x^2)/2 + (c*x^3)/3 + (b^6*x^12)/384 + (b^5*c*x^13)/96 + (5*b^4*c^2*x^14)
/288 + (5*b^3*c^3*x^15)/324 + (5*b^2*c^4*x^16)/648 + (b*c^5*x^17)/486 + (c^
6*x^18)/4374
```

Maple [B] time = 0.003, size = 81, normalized size = 2.4

$$\frac{c^6x^{18}}{4374} + \frac{bc^5x^{17}}{486} + \frac{5\,b^2c^4x^{16}}{648} + \frac{5\,b^3c^3x^{15}}{324} + \frac{5\,b^4c^2x^{14}}{288} + \frac{b^5cx^{13}}{96} + \frac{b^6x^{12}}{384} + \frac{cx^3}{3} + \frac{bx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)*(1+(1/2*b*x^2+1/3*c*x^3)^5),x)
```

```
[Out] 1/4374*c^6*x^18+1/486*b*c^5*x^17+5/648*b^2*c^4*x^16+5/324*b^3*c^3*x^15+5/28
8*b^4*c^2*x^14+1/96*b^5*c*x^13+1/384*b^6*x^12+1/3*c*x^3+1/2*b*x^2
```

Maxima [B] time = 0.975224, size = 108, normalized size = 3.18

$$\frac{1}{4374}\,c^6x^{18} + \frac{1}{486}\,bc^5x^{17} + \frac{5}{648}\,b^2c^4x^{16} + \frac{5}{324}\,b^3c^3x^{15} + \frac{5}{288}\,b^4c^2x^{14} + \frac{1}{96}\,b^5cx^{13} + \frac{1}{384}\,b^6x^{12} + \frac{1}{3}\,cx^3 + \frac{1}{2}\,bx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)*(1+(1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="maxima")
```

```
[Out] 1/4374*c^6*x^18 + 1/486*b*c^5*x^17 + 5/648*b^2*c^4*x^16 + 5/324*b^3*c^3*x^1
5 + 5/288*b^4*c^2*x^14 + 1/96*b^5*c*x^13 + 1/384*b^6*x^12 + 1/3*c*x^3 + 1/2
*b*x^2
```

Fricas [B] time = 1.22939, size = 213, normalized size = 6.26

$$\frac{1}{4374}x^{18}c^6 + \frac{1}{486}x^{17}c^5b + \frac{5}{648}x^{16}c^4b^2 + \frac{5}{324}x^{15}c^3b^3 + \frac{5}{288}x^{14}c^2b^4 + \frac{1}{96}x^{13}cb^5 + \frac{1}{384}x^{12}b^6 + \frac{1}{3}x^3c + \frac{1}{2}x^2b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)*(1+(1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="fricas")
```

```
[Out] 1/4374*x^18*c^6 + 1/486*x^17*c^5*b + 5/648*x^16*c^4*b^2 + 5/324*x^15*c^3*b^
3 + 5/288*x^14*c^2*b^4 + 1/96*x^13*c*b^5 + 1/384*x^12*b^6 + 1/3*x^3*c + 1/2
*x^2*b
```

Sympy [B] time = 0.094561, size = 90, normalized size = 2.65

$$\frac{b^6x^{12}}{384} + \frac{b^5cx^{13}}{96} + \frac{5b^4c^2x^{14}}{288} + \frac{5b^3c^3x^{15}}{324} + \frac{5b^2c^4x^{16}}{648} + \frac{bc^5x^{17}}{486} + \frac{bx^2}{2} + \frac{c^6x^{18}}{4374} + \frac{cx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)*(1+(1/2*b*x**2+1/3*c*x**3)**5),x)
```

```
[Out] b**6*x**12/384 + b**5*c*x**13/96 + 5*b**4*c**2*x**14/288 + 5*b**3*c**3*x**1
5/324 + 5*b**2*c**4*x**16/648 + b*c**5*x**17/486 + b*x**2/2 + c**6*x**18/43
74 + c*x**3/3
```

Giac [B] time = 1.17155, size = 108, normalized size = 3.18

$$\frac{1}{4374}\,c^6x^{18} + \frac{1}{486}\,bc^5x^{17} + \frac{5}{648}\,b^2c^4x^{16} + \frac{5}{324}\,b^3c^3x^{15} + \frac{5}{288}\,b^4c^2x^{14} + \frac{1}{96}\,b^5cx^{13} + \frac{1}{384}\,b^6x^{12} + \frac{1}{3}\,cx^3 + \frac{1}{2}\,bx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)*(1+(1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="giac")
```

```
[Out] 1/4374*c^6*x^18 + 1/486*b*c^5*x^17 + 5/648*b^2*c^4*x^16 + 5/324*b^3*c^3*x^1
5 + 5/288*b^4*c^2*x^14 + 1/96*b^5*c*x^13 + 1/384*b^6*x^12 + 1/3*c*x^3 + 1/2
*b*x^2
```

3.213 $\int \left(bx+cx^2\right)\left(1+\left(d+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)\,dx$

Optimal. Leaf size=41

$$\frac{1}{6}\left(\frac{bx^2}{2}+\frac{cx^3}{3}+d\right)^6+\frac{bx^2}{2}+\frac{cx^3}{3}$$

```
[Out] (b*x^2)/2 + (c*x^3)/3 + (d + (b*x^2)/2 + (c*x^3)/3)^6/6
```

Rubi [A] time = 0.0452143, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.031, Rules used = {1591}

$$\frac{1}{6}\left(\frac{bx^2}{2}+\frac{cx^3}{3}+d\right)^6+\frac{bx^2}{2}+\frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)*(1 + (d + (b*x^2)/2 + (c*x^3)/3)^5),x]
```

```
[Out] (b*x^2)/2 + (c*x^3)/3 + (d + (b*x^2)/2 + (c*x^3)/3)^6/6
```

Rule 1591

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\begin{aligned}\int \left(bx+cx^2\right)\left(1+\left(d+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)\,dx &= \text{Subst}\left(\int \left(1+x^5\right)\,dx, x, d+\frac{bx^2}{2}+\frac{cx^3}{3}\right)\\ &= \frac{bx^2}{2}+\frac{cx^3}{3}+\frac{1}{6}\left(d+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^6\end{aligned}$$

Mathematica [B] time = 0.0499823, size = 146, normalized size = 3.56

$$\frac{x^2(3b+2cx)\left(720b^2c^3x^{13}+1080b^3c^2x^{12}+810b^4cx^{11}+243b^5x^{10}+240bc^4x^{14}+540d^2x^6(3b+2cx)^3+4320d^3x^4(3b+2\right.}{279936}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)*(1 + (d + (b*x^2)/2 + (c*x^3)/3)^5),x]
```

```
[Out] (x^2*(3*b + 2*c*x)*(46656 + 46656*d^5 + 243*b^5*x^10 + 810*b^4*c*x^11 + 108
0*b^3*c^2*x^12 + 720*b^2*c^3*x^13 + 240*b*c^4*x^14 + 32*c^5*x^15 + 19440*d^
4*x^2*(3*b + 2*c*x) + 4320*d^3*x^4*(3*b + 2*c*x)^2 + 540*d^2*x^6*(3*b + 2*c
*x)^3 + 36*d*x^8*(3*b + 2*c*x)^4))/279936
```

Maple [B] time = 0.002, size = 646, normalized size = 15.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)*(1+(d+1/2*b*x^2+1/3*c*x^3)^5),x)
```

```
[Out] 1/4374*c^6*x^18+1/486*b*c^5*x^17+5/648*b^2*c^4*x^16+1/15*(5/54*b^3*c^3+c*(1
/81*d*c^4+1/12*b^3*c^2+1/3*c*(4/27*c^3*d+1/6*b^3*c)))*x^15+1/14*(b*(1/81*d*
c^4+1/12*b^3*c^2+1/3*c*(4/27*c^3*d+1/6*b^3*c))+c*(2/27*d*b*c^3+1/2*b*(4/27*
c^3*d+1/6*b^3*c)+1/3*c*(2/3*b*d*c^2+1/16*b^4)))*x^14+1/13*(b*(2/27*d*b*c^3+
1/2*b*(4/27*c^3*d+1/6*b^3*c)+1/3*c*(2/3*b*d*c^2+1/16*b^4))+c*(1/2*d*c^2*b^2
+1/2*b*(2/3*b*d*c^2+1/16*b^4)))*x^13+1/12*(b*(1/2*d*c^2*b^2+1/2*b*(2/3*b*d*
c^2+1/16*b^4))+c*(d*(4/27*c^3*d+1/6*b^3*c)+1/2*b^3*c*d+1/3*c*(2/3*d^2*c^2+1
/2*b^3*d)))*x^12+1/11*(b*(d*(4/27*c^3*d+1/6*b^3*c)+1/2*b^3*c*d+1/3*c*(2/3*d
^2*c^2+1/2*b^3*d))+c*(d*(2/3*b*d*c^2+1/16*b^4)+1/2*b*(2/3*d^2*c^2+1/2*b^3*d
)+2/3*c^2*d^2*b))*x^11+1/10*(b*(d*(2/3*b*d*c^2+1/16*b^4)+1/2*b*(2/3*d^2*c^2
+1/2*b^3*d)+2/3*c^2*d^2*b)+5/2*b^2*c^2*d^2)*x^10+1/9*(5/2*b^3*c*d^2+c*(d*(2
/3*d^2*c^2+1/2*b^3*d)+3/4*b^3*d^2+4/9*c^2*d^3))*x^9+1/8*(b*(d*(2/3*d^2*c^2+
1/2*b^3*d)+3/4*b^3*d^2+4/9*c^2*d^3)+10/3*c^2*b*d^3)*x^8+5/6*b^2*c*d^3*x^7+1
/6*(5/2*b^3*d^3+5/3*c^2*d^4)*x^6+5/6*b*c*d^4*x^5+5/8*b^2*d^4*x^4+1/3*c*(d^5
+1)*x^3+1/2*b*(d^5+1)*x^2
```

Maxima [B] time = 0.987637, size = 390, normalized size = 9.51

$$\frac{1}{4374}c^6x^{18}+\frac{1}{486}bc^5x^{17}+\frac{5}{648}b^2c^4x^{16}+\frac{1}{972}\left(15\,b^3c^3+4\,c^5d\right)x^{15}+\frac{5}{2592}\left(9\,b^4c^2+16\,bc^4d\right)x^{14}+\frac{1}{864}\left(9\,b^5c+80\,b^2c^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)*(1+(d+1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="maxima")
```

```
[Out] 1/4374*c^6*x^18 + 1/486*b*c^5*x^17 + 5/648*b^2*c^4*x^16 + 1/972*(15*b^3*c^3
 + 4*c^5*d)*x^15 + 5/2592*(9*b^4*c^2 + 16*b*c^4*d)*x^14 + 1/864*(9*b^5*c +
80*b^2*c^3*d)*x^13 + 5/6*b^2*c*d^3*x^7 + 1/10368*(27*b^6 + 1440*b^3*c^2*d +
 320*c^4*d^2)*x^12 + 5/432*(9*b^4*c*d + 16*b*c^3*d^2)*x^11 + 5/6*b*c*d^4*x^
5 + 1/96*(3*b^5*d + 40*b^2*c^2*d^2)*x^10 + 5/8*b^2*d^4*x^4 + 5/324*(27*b^3*
c*d^2 + 8*c^3*d^3)*x^9 + 5/288*(9*b^4*d^2 + 32*b*c^2*d^3)*x^8 + 5/36*(3*b^3
*d^3 + 2*c^2*d^4)*x^6 + 1/3*(c*d^5 + c)*x^3 + 1/2*(b*d^5 + b)*x^2
```

Fricas [B] time = 1.12347, size = 747, normalized size = 18.22

$$\frac{1}{4374}x^{18}c^6 + \frac{1}{486}x^{17}c^5b + \frac{5}{648}x^{16}c^4b^2 + \frac{1}{243}x^{15}dc^5 + \frac{5}{324}x^{15}c^3b^3 + \frac{5}{162}x^{14}dc^4b + \frac{5}{288}x^{14}c^2b^4 + \frac{5}{54}x^{13}dc^3b^2 + \frac{1}{96}x^{13}cb$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)*(1+(d+1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="fricas")
```

```
[Out] 1/4374*x^18*c^6 + 1/486*x^17*c^5*b + 5/648*x^16*c^4*b^2 + 1/243*x^15*d*c^5
+ 5/324*x^15*c^3*b^3 + 5/162*x^14*d*c^4*b + 5/288*x^14*c^2*b^4 + 5/54*x^13*
d*c^3*b^2 + 1/96*x^13*c*b^5 + 5/162*x^12*d^2*c^4 + 5/36*x^12*d*c^2*b^3 + 1/
384*x^12*b^6 + 5/27*x^11*d^2*c^3*b + 5/48*x^11*d*c*b^4 + 5/12*x^10*d^2*c^2*
b^2 + 1/32*x^10*d*b^5 + 10/81*x^9*d^3*c^3 + 5/12*x^9*d^2*c*b^3 + 5/9*x^8*d^
3*c^2*b + 5/32*x^8*d^2*b^4 + 5/6*x^7*d^3*c*b^2 + 5/18*x^6*d^4*c^2 + 5/12*x^
6*d^3*b^3 + 5/6*x^5*d^4*c*b + 5/8*x^4*d^4*b^2 + 1/3*x^3*d^5*c + 1/2*x^2*d^5
*b + 1/3*x^3*c + 1/2*x^2*b
```

Sympy [B] time = 0.146376, size = 321, normalized size = 7.83

$$\frac{5b^2c^4x^{16}}{648} + \frac{5b^2cd^3x^7}{6} + \frac{5b^2d^4x^4}{8} + \frac{bc^5x^{17}}{486} + \frac{5bcd^4x^5}{6} + \frac{c^6x^{18}}{4374} + x^{15}\left(\frac{5b^3c^3}{324} + \frac{c^5d}{243}\right) + x^{14}\left(\frac{5b^4c^2}{288} + \frac{5bc^4d}{162}\right) + x^{13}\left(\frac{b^5c}{96} + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)*(1+(d+1/2*b*x**2+1/3*c*x**3)**5),x)
```

```
[Out] 5*b**2*c**4*x**16/648 + 5*b**2*c*d**3*x**7/6 + 5*b**2*d**4*x**4/8 + b*c**5*
x**17/486 + 5*b*c*d**4*x**5/6 + c**6*x**18/4374 + x**15*(5*b**3*c**3/324 +
c**5*d/243) + x**14*(5*b**4*c**2/288 + 5*b*c**4*d/162) + x**13*(b**5*c/96 +
 5*b**2*c**3*d/54) + x**12*(b**6/384 + 5*b**3*c**2*d/36 + 5*c**4*d**2/162)
+ x**11*(5*b**4*c*d/48 + 5*b*c**3*d**2/27) + x**10*(b**5*d/32 + 5*b**2*c**2
*d**2/12) + x**9*(5*b**3*c*d**2/12 + 10*c**3*d**3/81) + x**8*(5*b**4*d**2/3
2 + 5*b*c**2*d**3/9) + x**6*(5*b**3*d**3/12 + 5*c**2*d**4/18) + x**3*(c*d**
5/3 + c/3) + x**2*(b*d**5/2 + b/2)
```

Giac [B] time = 1.20062, size = 402, normalized size = 9.8

$$\frac{1}{4374}c^6x^{18} + \frac{1}{486}bc^5x^{17} + \frac{5}{648}b^2c^4x^{16} + \frac{5}{324}b^3c^3x^{15} + \frac{1}{243}c^5dx^{15} + \frac{5}{288}b^4c^2x^{14} + \frac{5}{162}bc^4dx^{14} + \frac{1}{96}b^5cx^{13} + \frac{5}{54}b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)*(1+(d+1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="giac")
```

```
[Out] 1/4374*c^6*x^18 + 1/486*b*c^5*x^17 + 5/648*b^2*c^4*x^16 + 5/324*b^3*c^3*x^1
5 + 1/243*c^5*d*x^15 + 5/288*b^4*c^2*x^14 + 5/162*b*c^4*d*x^14 + 1/96*b^5*c
*x^13 + 5/54*b^2*c^3*d*x^13 + 1/384*b^6*x^12 + 5/36*b^3*c^2*d*x^12 + 5/162*
c^4*d^2*x^12 + 5/48*b^4*c*d*x^11 + 5/27*b*c^3*d^2*x^11 + 1/32*b^5*d*x^10 +
5/12*b^2*c^2*d^2*x^10 + 5/12*b^3*c*d^2*x^9 + 10/81*c^3*d^3*x^9 + 5/32*b^4*d
^2*x^8 + 5/9*b*c^2*d^3*x^8 + 5/6*b^2*c*d^3*x^7 + 5/12*b^3*d^3*x^6 + 5/18*c^
2*d^4*x^6 + 5/6*b*c*d^4*x^5 + 5/8*b^2*d^4*x^4 + 1/3*c*d^5*x^3 + 1/2*b*d^5*x
^2 + 1/3*c*x^3 + 1/2*b*x^2
```

3.214 $\int \left(a+bx+cx^2\right)\left(1+\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)dx$

Optimal. Leaf size=46

$$\frac{1}{6}\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^6+ax+\frac{bx^2}{2}+\frac{cx^3}{3}$$

```
[Out] a*x + (b*x^2)/2 + (c*x^3)/3 + (a*x + (b*x^2)/2 + (c*x^3)/3)^6/6
```

Rubi [A] time = 0.046813, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {1591}

$$\frac{1}{6}\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^6+ax+\frac{bx^2}{2}+\frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)*(1 + (a*x + (b*x^2)/2 + (c*x^3)/3)^5),x]
```

```
[Out] a*x + (b*x^2)/2 + (c*x^3)/3 + (a*x + (b*x^2)/2 + (c*x^3)/3)^6/6
```

Rule 1591

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx+cx^2\right)\left(1+\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)dx &= \text{Subst}\left(\int \left(1+x^5\right)dx, x, ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)\\ &= ax+\frac{bx^2}{2}+\frac{cx^3}{3}+\frac{1}{6}\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^6\end{aligned}$$

Mathematica [B] time = 0.0619625, size = 244, normalized size = 5.3

$$\frac{5a^2x^{10}(3b+2cx)^4}{2592}+\frac{5}{324}a^3x^9(3b+2cx)^3+\frac{5}{72}a^4x^8(3b+2cx)^2+\frac{1}{6}a^5x^7(3b+2cx)+\frac{a^6x^6}{6}+a\left(\frac{5}{54}b^2c^3x^{14}+\frac{5}{36}b^3c^2x^{13}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)*(1 + (a*x + (b*x^2)/2 + (c*x^3)/3)^5),x]
```

```
[Out] (a^6*x^6)/6 + (a^5*x^7*(3*b + 2*c*x))/6 + (5*a^4*x^8*(3*b + 2*c*x)^2)/72 +
(5*a^3*x^9*(3*b + 2*c*x)^3)/324 + (5*a^2*x^10*(3*b + 2*c*x)^4)/2592 + a*(x
+ (b^5*x^11)/32 + (5*b^4*c*x^12)/48 + (5*b^3*c^2*x^13)/36 + (5*b^2*c^3*x^14
)/54 + (5*b*c^4*x^15)/162 + (c^5*x^16)/243) + (x^2*(729*b^6*x^10 + 2916*b^5
*c*x^11 + 4860*b^4*c^2*x^12 + 4320*b^3*c^3*x^13 + 2160*b^2*c^4*x^14 + 576*b
*(243 + c^5*x^15) + 64*c*x*(1458 + c^5*x^15)))/279936
```

Maple [B] time = 0.002, size = 1523, normalized size = 33.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)*(1+(a*x+1/2*b*x^2+1/3*c*x^3)^5),x)
```

```
[Out] 1/4374*c^6*x^18+1/486*b*c^5*x^17+1/16*(1/243*c^5*a+5/162*b^2*c^4+c*(1/81*a*
c^4+1/27*b^2*c^3+1/3*c*(2/9*(2/3*a*c+1/4*b^2)*c^2+1/9*c^2*b^2)))*x^16+1/15*
(5/162*a*b*c^4+b*(1/81*a*c^4+1/27*b^2*c^3+1/3*c*(2/9*(2/3*a*c+1/4*b^2)*c^2+
1/9*c^2*b^2))+c*(2/27*a*b*c^3+1/2*b*(2/9*(2/3*a*c+1/4*b^2)*c^2+1/9*c^2*b^2)
+1/3*c*(2/9*c^2*a*b+2/3*(2/3*a*c+1/4*b^2)*b*c)))*x^15+1/14*(a*(1/81*a*c^4+1
/27*b^2*c^3+1/3*c*(2/9*(2/3*a*c+1/4*b^2)*c^2+1/9*c^2*b^2))+b*(2/27*a*b*c^3+
1/2*b*(2/9*(2/3*a*c+1/4*b^2)*c^2+1/9*c^2*b^2)+1/3*c*(2/9*c^2*a*b+2/3*(2/3*a
*c+1/4*b^2)*b*c))+c*(a*(2/9*(2/3*a*c+1/4*b^2)*c^2+1/9*c^2*b^2)+1/2*b*(2/9*c
^2*a*b+2/3*(2/3*a*c+1/4*b^2)*b*c)+1/3*c*(2/9*a^2*c^2+2/3*a*b^2*c+(2/3*a*c+1
/4*b^2)^2)))*x^14+1/13*(a*(2/27*a*b*c^3+1/2*b*(2/9*(2/3*a*c+1/4*b^2)*c^2+1/
9*c^2*b^2)+1/3*c*(2/9*c^2*a*b+2/3*(2/3*a*c+1/4*b^2)*b*c))+b*(a*(2/9*(2/3*a*
c+1/4*b^2)*c^2+1/9*c^2*b^2)+1/2*b*(2/9*c^2*a*b+2/3*(2/3*a*c+1/4*b^2)*b*c)+1
/3*c*(2/9*a^2*c^2+2/3*a*b^2*c+(2/3*a*c+1/4*b^2)^2))+c*(a*(2/9*c^2*a*b+2/3*(
2/3*a*c+1/4*b^2)*b*c)+1/2*b*(2/9*a^2*c^2+2/3*a*b^2*c+(2/3*a*c+1/4*b^2)^2)+1
/3*c*(2/3*a^2*b*c+2*a*b*(2/3*a*c+1/4*b^2))))*x^13+1/12*(a*(a*(2/9*(2/3*a*c+
1/4*b^2)*c^2+1/9*c^2*b^2)+1/2*b*(2/9*c^2*a*b+2/3*(2/3*a*c+1/4*b^2)*b*c)+1/3
*c*(2/9*a^2*c^2+2/3*a*b^2*c+(2/3*a*c+1/4*b^2)^2))+b*(a*(2/9*c^2*a*b+2/3*(2/
```

```
3*a*c+1/4*b^2)*b*c)+1/2*b*(2/9*a^2*c^2+2/3*a*b^2*c+(2/3*a*c+1/4*b^2)^2)+1/3
*c*(2/3*a^2*b*c+2*a*b*(2/3*a*c+1/4*b^2)))+c*(a*(2/9*a^2*c^2+2/3*a*b^2*c+(2/
3*a*c+1/4*b^2)^2)+1/2*b*(2/3*a^2*b*c+2*a*b*(2/3*a*c+1/4*b^2))+1/3*c*(2*a^2*
(2/3*a*c+1/4*b^2)+b^2*a^2)))*x^12+1/11*(a*(a*(2/9*c^2*a*b+2/3*(2/3*a*c+1/4*
b^2)*b*c)+1/2*b*(2/9*a^2*c^2+2/3*a*b^2*c+(2/3*a*c+1/4*b^2)^2)+1/3*c*(2/3*a^
2*b*c+2*a*b*(2/3*a*c+1/4*b^2)))+b*(a*(2/9*a^2*c^2+2/3*a*b^2*c+(2/3*a*c+1/4*
b^2)^2)+1/2*b*(2/3*a^2*b*c+2*a*b*(2/3*a*c+1/4*b^2))+1/3*c*(2*a^2*(2/3*a*c+1
/4*b^2)+b^2*a^2))+c*(a*(2/3*a^2*b*c+2*a*b*(2/3*a*c+1/4*b^2))+1/2*b*(2*a^2*(
2/3*a*c+1/4*b^2)+b^2*a^2)+2/3*c*a^3*b))*x^11+1/10*(a*(a*(2/9*a^2*c^2+2/3*a*
b^2*c+(2/3*a*c+1/4*b^2)^2)+1/2*b*(2/3*a^2*b*c+2*a*b*(2/3*a*c+1/4*b^2))+1/3*
c*(2*a^2*(2/3*a*c+1/4*b^2)+b^2*a^2))+b*(a*(2/3*a^2*b*c+2*a*b*(2/3*a*c+1/4*b
^2))+1/2*b*(2*a^2*(2/3*a*c+1/4*b^2)+b^2*a^2)+2/3*c*a^3*b)+c*(a*(2*a^2*(2/3*
a*c+1/4*b^2)+b^2*a^2)+a^3*b^2+1/3*a^4*c))*x^10+1/9*(a*(a*(2/3*a^2*b*c+2*a*b
*(2/3*a*c+1/4*b^2))+1/2*b*(2*a^2*(2/3*a*c+1/4*b^2)+b^2*a^2)+2/3*c*a^3*b)+b*
(a*(2*a^2*(2/3*a*c+1/4*b^2)+b^2*a^2)+a^3*b^2+1/3*a^4*c)+5/2*c*a^4*b)*x^9+1/
8*(a*(a*(2*a^2*(2/3*a*c+1/4*b^2)+b^2*a^2)+a^3*b^2+1/3*a^4*c)+5/2*a^4*b^2+a^
5*c)*x^8+1/2*a^5*b*x^7+1/6*a^6*x^6+1/3*c*x^3+1/2*b*x^2+a*x
```

Maxima [B] time = 1.05579, size = 390, normalized size = 8.48

$$\frac{1}{4374}\,c^6x^{18}+\frac{1}{486}\,bc^5x^{17}+\frac{1}{1944}\left(15\,b^2c^4+8\,ac^5\right)x^{16}+\frac{5}{324}\left(b^3c^3+2\,abc^4\right)x^{15}+\frac{5}{2592}\left(9\,b^4c^2+48\,ab^2c^3+16\,a^2c^4\right)x^{14}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*(1+(a*x+1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="maxim
a")
```

```
[Out] 1/4374*c^6*x^18 + 1/486*b*c^5*x^17 + 1/1944*(15*b^2*c^4 + 8*a*c^5)*x^16 + 5
/324*(b^3*c^3 + 2*a*b*c^4)*x^15 + 5/2592*(9*b^4*c^2 + 48*a*b^2*c^3 + 16*a^2
*c^4)*x^14 + 1/864*(9*b^5*c + 120*a*b^3*c^2 + 160*a^2*b*c^3)*x^13 + 1/2*a^5
*b*x^7 + 1/10368*(27*b^6 + 1080*a*b^4*c + 4320*a^2*b^2*c^2 + 1280*a^3*c^3)*
x^12 + 1/6*a^6*x^6 + 1/288*(9*a*b^5 + 120*a^2*b^3*c + 160*a^3*b*c^2)*x^11 +
 5/288*(9*a^2*b^4 + 48*a^3*b^2*c + 16*a^4*c^2)*x^10 + 5/12*(a^3*b^3 + 2*a^4
*b*c)*x^9 + 1/24*(15*a^4*b^2 + 8*a^5*c)*x^8 + 1/3*c*x^3 + 1/2*b*x^2 + a*x
```

Fricas [B] time = 1.09908, size = 782, normalized size = 17.

$$\frac{1}{4374}x^{18}c^6+\frac{1}{486}x^{17}c^5b+\frac{5}{648}x^{16}c^4b^2+\frac{1}{243}x^{16}c^5a+\frac{5}{324}x^{15}c^3b^3+\frac{5}{162}x^{15}c^4ba+\frac{5}{288}x^{14}c^2b^4+\frac{5}{54}x^{14}c^3b^2a+\frac{5}{162}x^{14}c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*(1+(a*x+1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="frica
s")
```

```
[Out] 1/4374*x^18*c^6 + 1/486*x^17*c^5*b + 5/648*x^16*c^4*b^2 + 1/243*x^16*c^5*a
+ 5/324*x^15*c^3*b^3 + 5/162*x^15*c^4*b*a + 5/288*x^14*c^2*b^4 + 5/54*x^14*
c^3*b^2*a + 5/162*x^14*c^4*a^2 + 1/96*x^13*c*b^5 + 5/36*x^13*c^2*b^3*a + 5/
27*x^13*c^3*b*a^2 + 1/384*x^12*b^6 + 5/48*x^12*c*b^4*a + 5/12*x^12*c^2*b^2*
a^2 + 10/81*x^12*c^3*a^3 + 1/32*x^11*b^5*a + 5/12*x^11*c*b^3*a^2 + 5/9*x^11
*c^2*b*a^3 + 5/32*x^10*b^4*a^2 + 5/6*x^10*c*b^2*a^3 + 5/18*x^10*c^2*a^4 + 5
/12*x^9*b^3*a^3 + 5/6*x^9*c*b*a^4 + 5/8*x^8*b^2*a^4 + 1/3*x^8*c*a^5 + 1/2*x
^7*b*a^5 + 1/6*x^6*a^6 + 1/3*x^3*c + 1/2*x^2*b + x*a
```

Sympy [B] time = 0.145776, size = 323, normalized size = 7.02

$$\frac{a^6x^6}{6}+\frac{a^5bx^7}{2}+ax+\frac{bc^5x^{17}}{486}+\frac{bx^2}{2}+\frac{c^6x^{18}}{4374}+\frac{cx^3}{3}+x^{16}\left(\frac{ac^5}{243}+\frac{5b^2c^4}{648}\right)+x^{15}\left(\frac{5abc^4}{162}+\frac{5b^3c^3}{324}\right)+x^{14}\left(\frac{5a^2c^4}{162}+\frac{5ab^2c^3}{54}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)*(1+(a*x+1/2*b*x**2+1/3*c*x**3)**5),x)
```

```
[Out] a**6*x**6/6 + a**5*b*x**7/2 + a*x + b*c**5*x**17/486 + b*x**2/2 + c**6*x**1
8/4374 + c*x**3/3 + x**16*(a*c**5/243 + 5*b**2*c**4/648) + x**15*(5*a*b*c**
4/162 + 5*b**3*c**3/324) + x**14*(5*a**2*c**4/162 + 5*a*b**2*c**3/54 + 5*b*
*4*c**2/288) + x**13*(5*a**2*b*c**3/27 + 5*a*b**3*c**2/36 + b**5*c/96) + x*
*12*(10*a**3*c**3/81 + 5*a**2*b**2*c**2/12 + 5*a*b**4*c/48 + b**6/384) + x*
*11*(5*a**3*b*c**2/9 + 5*a**2*b**3*c/12 + a*b**5/32) + x**10*(5*a**4*c**2/1
8 + 5*a**3*b**2*c/6 + 5*a**2*b**4/32) + x**9*(5*a**4*b*c/6 + 5*a**3*b**3/12
) + x**8*(a**5*c/3 + 5*a**4*b**2/8)
```

Giac [B] time = 1.18154, size = 417, normalized size = 9.07

$$\frac{1}{4374}\,c^6x^{18}+\frac{1}{486}\,bc^5x^{17}+\frac{5}{648}\,b^2c^4x^{16}+\frac{1}{243}\,ac^5x^{16}+\frac{5}{324}\,b^3c^3x^{15}+\frac{5}{162}\,abc^4x^{15}+\frac{5}{288}\,b^4c^2x^{14}+\frac{5}{54}\,ab^2c^3x^{14}+\frac{5}{162}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*(1+(a*x+1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="giac"
)
```

```
[Out] 1/4374*c^6*x^18 + 1/486*b*c^5*x^17 + 5/648*b^2*c^4*x^16 + 1/243*a*c^5*x^16
+ 5/324*b^3*c^3*x^15 + 5/162*a*b*c^4*x^15 + 5/288*b^4*c^2*x^14 + 5/54*a*b^2
*c^3*x^14 + 5/162*a^2*c^4*x^14 + 1/96*b^5*c*x^13 + 5/36*a*b^3*c^2*x^13 + 5/
27*a^2*b*c^3*x^13 + 1/384*b^6*x^12 + 5/48*a*b^4*c*x^12 + 5/12*a^2*b^2*c^2*x
^12 + 10/81*a^3*c^3*x^12 + 1/32*a*b^5*x^11 + 5/12*a^2*b^3*c*x^11 + 5/9*a^3*
b*c^2*x^11 + 5/32*a^2*b^4*x^10 + 5/6*a^3*b^2*c*x^10 + 5/18*a^4*c^2*x^10 + 5
/12*a^3*b^3*x^9 + 5/6*a^4*b*c*x^9 + 5/8*a^4*b^2*x^8 + 1/3*a^5*c*x^8 + 1/2*a
^5*b*x^7 + 1/6*a^6*x^6 + 1/3*c*x^3 + 1/2*b*x^2 + a*x
```

3.215 $\int \left(a+bx+cx^2\right)\left(1+\left(d+ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right) dx$

Optimal. Leaf size=47

$$\frac{1}{6}\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}+d\right)^6+ax+\frac{bx^2}{2}+\frac{cx^3}{3}$$

```
[Out] a*x + (b*x^2)/2 + (c*x^3)/3 + (d + a*x + (b*x^2)/2 + (c*x^3)/3)^6/6
```

Rubi [A] time = 0.0908815, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {1591}

$$\frac{1}{6}\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}+d\right)^6+ax+\frac{bx^2}{2}+\frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)*(1 + (d + a*x + (b*x^2)/2 + (c*x^3)/3)^5),x]
```

```
[Out] a*x + (b*x^2)/2 + (c*x^3)/3 + (d + a*x + (b*x^2)/2 + (c*x^3)/3)^6/6
```

Rule 1591

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx+cx^2\right)\left(1+\left(d+ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right) dx &= \text{Subst}\left(\int \left(1+x^5\right) dx, x, d+ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)\\ &= ax+\frac{bx^2}{2}+\frac{cx^3}{3}+\frac{1}{6}\left(d+ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^6\end{aligned}$$

Mathematica [B] time = 0.117337, size = 248, normalized size = 5.28

$$\frac{x(6a + x(3b + 2cx)) \left(360a^2x^8(3b + 2cx)^3 + 2160a^3x^7(3b + 2cx)^2 + 6480a^4x^6(3b + 2cx) + 7776a^5x^5 + 540d^2x^3(6a + x(3b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)*(1 + (d + a*x + (b*x^2)/2 + (c*x^3)/3)^5),x]
```

```
[Out] (x*(6*a + x*(3*b + 2*c*x))*(46656 + 46656*d^5 + 7776*a^5*x^5 + 243*b^5*x^10
 + 810*b^4*c*x^11 + 1080*b^3*c^2*x^12 + 720*b^2*c^3*x^13 + 240*b*c^4*x^14 +
 32*c^5*x^15 + 6480*a^4*x^6*(3*b + 2*c*x) + 2160*a^3*x^7*(3*b + 2*c*x)^2 +
360*a^2*x^8*(3*b + 2*c*x)^3 + 30*a*x^9*(3*b + 2*c*x)^4 + 19440*d^4*x*(6*a +
 x*(3*b + 2*c*x)) + 4320*d^3*x^2*(6*a + x*(3*b + 2*c*x))^2 + 540*d^2*x^3*(6
*a + x*(3*b + 2*c*x))^3 + 36*d*x^4*(6*a + x*(3*b + 2*c*x))^4))/279936
```

Maple [B] time = 0.003, size = 4284, normalized size = 91.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)*(1+(d+a*x+1/2*b*x^2+1/3*c*x^3)^5),x)
```

```
[Out] 1/4374*c^6*x^18+1/486*b*c^5*x^17+1/16*(1/243*c^5*a+5/162*b^2*c^4+c*(1/81*a*
c^4+1/27*b^2*c^3+1/3*c*(2/9*(2/3*a*c+1/4*b^2)*c^2+1/9*c^2*b^2)))*x^16+1/15*
(5/162*a*b*c^4+b*(1/81*a*c^4+1/27*b^2*c^3+1/3*c*(2/9*(2/3*a*c+1/4*b^2)*c^2+
1/9*c^2*b^2))+c*(1/81*d*c^4+2/27*a*b*c^3+1/2*b*(2/9*(2/3*a*c+1/4*b^2)*c^2+1
/9*c^2*b^2)+1/3*c*(2/9*(2/3*c*d+a*b)*c^2+2/3*(2/3*a*c+1/4*b^2)*b*c)))*x^15+
1/14*(a*(1/81*a*c^4+1/27*b^2*c^3+1/3*c*(2/9*(2/3*a*c+1/4*b^2)*c^2+1/9*c^2*b
^2))+b*(1/81*d*c^4+2/27*a*b*c^3+1/2*b*(2/9*(2/3*a*c+1/4*b^2)*c^2+1/9*c^2*b^
2)+1/3*c*(2/9*(2/3*c*d+a*b)*c^2+2/3*(2/3*a*c+1/4*b^2)*b*c))+c*(2/27*d*b*c^3
+a*(2/9*(2/3*a*c+1/4*b^2)*c^2+1/9*c^2*b^2)+1/2*b*(2/9*(2/3*c*d+a*b)*c^2+2/3
*(2/3*a*c+1/4*b^2)*b*c)+1/3*c*(2/9*(a^2+b*d)*c^2+2/3*(2/3*c*d+a*b)*b*c+(2/3
*a*c+1/4*b^2)^2)))*x^14+1/13*(a*(1/81*d*c^4+2/27*a*b*c^3+1/2*b*(2/9*(2/3*a*
c+1/4*b^2)*c^2+1/9*c^2*b^2)+1/3*c*(2/9*(2/3*c*d+a*b)*c^2+2/3*(2/3*a*c+1/4*b
^2)*b*c))+b*(2/27*d*b*c^3+a*(2/9*(2/3*a*c+1/4*b^2)*c^2+1/9*c^2*b^2)+1/2*b*(
2/9*(2/3*c*d+a*b)*c^2+2/3*(2/3*a*c+1/4*b^2)*b*c)+1/3*c*(2/9*(a^2+b*d)*c^2+2
/3*(2/3*c*d+a*b)*b*c+(2/3*a*c+1/4*b^2)^2))+c*(d*(2/9*(2/3*a*c+1/4*b^2)*c^2+
1/9*c^2*b^2)+a*(2/9*(2/3*c*d+a*b)*c^2+2/3*(2/3*a*c+1/4*b^2)*b*c)+1/2*b*(2/9
*(a^2+b*d)*c^2+2/3*(2/3*c*d+a*b)*b*c+(2/3*a*c+1/4*b^2)^2)+1/3*c*(4/9*a*c^2*
d+2/3*(a^2+b*d)*b*c+2*(2/3*c*d+a*b)*(2/3*a*c+1/4*b^2))))*x^13+1/12*(a*(2/27
```

```
*d*b*c^3+a*(2/9*(2/3*a*c+1/4*b^2)*c^2+1/9*c^2*b^2)+1/2*b*(2/9*(2/3*c*d+a*b)
*c^2+2/3*(2/3*a*c+1/4*b^2)*b*c)+1/3*c*(2/9*(a^2+b*d)*c^2+2/3*(2/3*c*d+a*b)*
b*c+(2/3*a*c+1/4*b^2)^2))+b*(d*(2/9*(2/3*a*c+1/4*b^2)*c^2+1/9*c^2*b^2)+a*(2
/9*(2/3*c*d+a*b)*c^2+2/3*(2/3*a*c+1/4*b^2)*b*c)+1/2*b*(2/9*(a^2+b*d)*c^2+2/
3*(2/3*c*d+a*b)*b*c+(2/3*a*c+1/4*b^2)^2)+1/3*c*(4/9*a*c^2*d+2/3*(a^2+b*d)*b
*c+2*(2/3*c*d+a*b)*(2/3*a*c+1/4*b^2)))+c*(d*(2/9*(2/3*c*d+a*b)*c^2+2/3*(2/3
*a*c+1/4*b^2)*b*c)+a*(2/9*(a^2+b*d)*c^2+2/3*(2/3*c*d+a*b)*b*c+(2/3*a*c+1/4*
b^2)^2)+1/2*b*(4/9*a*c^2*d+2/3*(a^2+b*d)*b*c+2*(2/3*c*d+a*b)*(2/3*a*c+1/4*b
^2))+1/3*c*(2/9*d^2*c^2+4/3*a*b*c*d+2*(a^2+b*d)*(2/3*a*c+1/4*b^2)+(2/3*c*d+
a*b)^2)))*x^12+1/11*(a*(d*(2/9*(2/3*a*c+1/4*b^2)*c^2+1/9*c^2*b^2)+a*(2/9*(2
/3*c*d+a*b)*c^2+2/3*(2/3*a*c+1/4*b^2)*b*c)+1/2*b*(2/9*(a^2+b*d)*c^2+2/3*(2/
3*c*d+a*b)*b*c+(2/3*a*c+1/4*b^2)^2)+1/3*c*(4/9*a*c^2*d+2/3*(a^2+b*d)*b*c+2*
(2/3*c*d+a*b)*(2/3*a*c+1/4*b^2)))+b*(d*(2/9*(2/3*c*d+a*b)*c^2+2/3*(2/3*a*c+
1/4*b^2)*b*c)+a*(2/9*(a^2+b*d)*c^2+2/3*(2/3*c*d+a*b)*b*c+(2/3*a*c+1/4*b^2)^
2)+1/2*b*(4/9*a*c^2*d+2/3*(a^2+b*d)*b*c+2*(2/3*c*d+a*b)*(2/3*a*c+1/4*b^2))+
1/3*c*(2/9*d^2*c^2+4/3*a*b*c*d+2*(a^2+b*d)*(2/3*a*c+1/4*b^2)+(2/3*c*d+a*b)^
2))+c*(d*(2/9*(a^2+b*d)*c^2+2/3*(2/3*c*d+a*b)*b*c+(2/3*a*c+1/4*b^2)^2)+a*(4
/9*a*c^2*d+2/3*(a^2+b*d)*b*c+2*(2/3*c*d+a*b)*(2/3*a*c+1/4*b^2))+1/2*b*(2/9*
d^2*c^2+4/3*a*b*c*d+2*(a^2+b*d)*(2/3*a*c+1/4*b^2)+(2/3*c*d+a*b)^2)+1/3*c*(2
/3*d^2*b*c+4*a*d*(2/3*a*c+1/4*b^2)+2*(a^2+b*d)*(2/3*c*d+a*b))))*x^11+1/10*(
a*(d*(2/9*(2/3*c*d+a*b)*c^2+2/3*(2/3*a*c+1/4*b^2)*b*c)+a*(2/9*(a^2+b*d)*c^2
+2/3*(2/3*c*d+a*b)*b*c+(2/3*a*c+1/4*b^2)^2)+1/2*b*(4/9*a*c^2*d+2/3*(a^2+b*d
)*b*c+2*(2/3*c*d+a*b)*(2/3*a*c+1/4*b^2))+1/3*c*(2/9*d^2*c^2+4/3*a*b*c*d+2*(
a^2+b*d)*(2/3*a*c+1/4*b^2)+(2/3*c*d+a*b)^2))+b*(d*(2/9*(a^2+b*d)*c^2+2/3*(2
/3*c*d+a*b)*b*c+(2/3*a*c+1/4*b^2)^2)+a*(4/9*a*c^2*d+2/3*(a^2+b*d)*b*c+2*(2/
3*c*d+a*b)*(2/3*a*c+1/4*b^2))+1/2*b*(2/9*d^2*c^2+4/3*a*b*c*d+2*(a^2+b*d)*(2
/3*a*c+1/4*b^2)+(2/3*c*d+a*b)^2)+1/3*c*(2/3*d^2*b*c+4*a*d*(2/3*a*c+1/4*b^2)
+2*(a^2+b*d)*(2/3*c*d+a*b)))+c*(d*(4/9*a*c^2*d+2/3*(a^2+b*d)*b*c+2*(2/3*c*d
+a*b)*(2/3*a*c+1/4*b^2))+a*(2/9*d^2*c^2+4/3*a*b*c*d+2*(a^2+b*d)*(2/3*a*c+1/
4*b^2)+(2/3*c*d+a*b)^2)+1/2*b*(2/3*d^2*b*c+4*a*d*(2/3*a*c+1/4*b^2)+2*(a^2+b
*d)*(2/3*c*d+a*b))+1/3*c*(2*d^2*(2/3*a*c+1/4*b^2)+4*a*d*(2/3*c*d+a*b)+(a^2+
b*d)^2)))*x^10+1/9*(a*(d*(2/9*(a^2+b*d)*c^2+2/3*(2/3*c*d+a*b)*b*c+(2/3*a*c+
1/4*b^2)^2)+a*(4/9*a*c^2*d+2/3*(a^2+b*d)*b*c+2*(2/3*c*d+a*b)*(2/3*a*c+1/4*b
^2))+1/2*b*(2/9*d^2*c^2+4/3*a*b*c*d+2*(a^2+b*d)*(2/3*a*c+1/4*b^2)+(2/3*c*d+
a*b)^2)+1/3*c*(2/3*d^2*b*c+4*a*d*(2/3*a*c+1/4*b^2)+2*(a^2+b*d)*(2/3*c*d+a*b
)))+b*(d*(4/9*a*c^2*d+2/3*(a^2+b*d)*b*c+2*(2/3*c*d+a*b)*(2/3*a*c+1/4*b^2))+
a*(2/9*d^2*c^2+4/3*a*b*c*d+2*(a^2+b*d)*(2/3*a*c+1/4*b^2)+(2/3*c*d+a*b)^2)+1
/2*b*(2/3*d^2*b*c+4*a*d*(2/3*a*c+1/4*b^2)+2*(a^2+b*d)*(2/3*c*d+a*b))+1/3*c*
(2*d^2*(2/3*a*c+1/4*b^2)+4*a*d*(2/3*c*d+a*b)+(a^2+b*d)^2))+c*(d*(2/9*d^2*c^
2+4/3*a*b*c*d+2*(a^2+b*d)*(2/3*a*c+1/4*b^2)+(2/3*c*d+a*b)^2)+a*(2/3*d^2*b*c
+4*a*d*(2/3*a*c+1/4*b^2)+2*(a^2+b*d)*(2/3*c*d+a*b))+1/2*b*(2*d^2*(2/3*a*c+1
/4*b^2)+4*a*d*(2/3*c*d+a*b)+(a^2+b*d)^2)+1/3*c*(2*d^2*(2/3*c*d+a*b)+4*a*d*(
a^2+b*d))))*x^9+1/8*(a*(d*(4/9*a*c^2*d+2/3*(a^2+b*d)*b*c+2*(2/3*c*d+a*b)*(2
/3*a*c+1/4*b^2))+a*(2/9*d^2*c^2+4/3*a*b*c*d+2*(a^2+b*d)*(2/3*a*c+1/4*b^2)+(
2/3*c*d+a*b)^2)+1/2*b*(2/3*d^2*b*c+4*a*d*(2/3*a*c+1/4*b^2)+2*(a^2+b*d)*(2/3
```

```
*c*d+a*b))+1/3*c*(2*d^2*(2/3*a*c+1/4*b^2)+4*a*d*(2/3*c*d+a*b)+(a^2+b*d)^2))
+b*(d*(2/9*d^2*c^2+4/3*a*b*c*d+2*(a^2+b*d)*(2/3*a*c+1/4*b^2)+(2/3*c*d+a*b)^
2)+a*(2/3*d^2*b*c+4*a*d*(2/3*a*c+1/4*b^2)+2*(a^2+b*d)*(2/3*c*d+a*b))+1/2*b*
(2*d^2*(2/3*a*c+1/4*b^2)+4*a*d*(2/3*c*d+a*b)+(a^2+b*d)^2)+1/3*c*(2*d^2*(2/3
*c*d+a*b)+4*a*d*(a^2+b*d)))+c*(d*(2/3*d^2*b*c+4*a*d*(2/3*a*c+1/4*b^2)+2*(a^
2+b*d)*(2/3*c*d+a*b))+a*(2*d^2*(2/3*a*c+1/4*b^2)+4*a*d*(2/3*c*d+a*b)+(a^2+b
*d)^2)+1/2*b*(2*d^2*(2/3*c*d+a*b)+4*a*d*(a^2+b*d))+1/3*c*(2*d^2*(a^2+b*d)+4
*a^2*d^2)))*x^8+1/7*(a*(d*(2/9*d^2*c^2+4/3*a*b*c*d+2*(a^2+b*d)*(2/3*a*c+1/4
*b^2)+(2/3*c*d+a*b)^2)+a*(2/3*d^2*b*c+4*a*d*(2/3*a*c+1/4*b^2)+2*(a^2+b*d)*(
2/3*c*d+a*b))+1/2*b*(2*d^2*(2/3*a*c+1/4*b^2)+4*a*d*(2/3*c*d+a*b)+(a^2+b*d)^
2)+1/3*c*(2*d^2*(2/3*c*d+a*b)+4*a*d*(a^2+b*d)))+b*(d*(2/3*d^2*b*c+4*a*d*(2/
3*a*c+1/4*b^2)+2*(a^2+b*d)*(2/3*c*d+a*b))+a*(2*d^2*(2/3*a*c+1/4*b^2)+4*a*d*
(2/3*c*d+a*b)+(a^2+b*d)^2)+1/2*b*(2*d^2*(2/3*c*d+a*b)+4*a*d*(a^2+b*d))+1/3*
c*(2*d^2*(a^2+b*d)+4*a^2*d^2))+c*(d*(2*d^2*(2/3*a*c+1/4*b^2)+4*a*d*(2/3*c*d
+a*b)+(a^2+b*d)^2)+a*(2*d^2*(2/3*c*d+a*b)+4*a*d*(a^2+b*d))+1/2*b*(2*d^2*(a^
2+b*d)+4*a^2*d^2)+4/3*a*c*d^3))*x^7+1/6*(a*(d*(2/3*d^2*b*c+4*a*d*(2/3*a*c+1
/4*b^2)+2*(a^2+b*d)*(2/3*c*d+a*b))+a*(2*d^2*(2/3*a*c+1/4*b^2)+4*a*d*(2/3*c*
d+a*b)+(a^2+b*d)^2)+1/2*b*(2*d^2*(2/3*c*d+a*b)+4*a*d*(a^2+b*d))+1/3*c*(2*d^
2*(a^2+b*d)+4*a^2*d^2))+b*(d*(2*d^2*(2/3*a*c+1/4*b^2)+4*a*d*(2/3*c*d+a*b)+(
a^2+b*d)^2)+a*(2*d^2*(2/3*c*d+a*b)+4*a*d*(a^2+b*d))+1/2*b*(2*d^2*(a^2+b*d)+
4*a^2*d^2)+4/3*a*c*d^3)+c*(d*(2*d^2*(2/3*c*d+a*b)+4*a*d*(a^2+b*d))+a*(2*d^2
*(a^2+b*d)+4*a^2*d^2)+2*b*d^3*a+1/3*c*d^4))*x^6+1/5*(a*(d*(2*d^2*(2/3*a*c+1
/4*b^2)+4*a*d*(2/3*c*d+a*b)+(a^2+b*d)^2)+a*(2*d^2*(2/3*c*d+a*b)+4*a*d*(a^2+
b*d))+1/2*b*(2*d^2*(a^2+b*d)+4*a^2*d^2)+4/3*a*c*d^3)+b*(d*(2*d^2*(2/3*c*d+a
*b)+4*a*d*(a^2+b*d))+a*(2*d^2*(a^2+b*d)+4*a^2*d^2)+2*b*d^3*a+1/3*c*d^4)+c*(
d*(2*d^2*(a^2+b*d)+4*a^2*d^2)+4*a^2*d^3+1/2*b*d^4))*x^5+1/4*(a*(d*(2*d^2*(2
/3*c*d+a*b)+4*a*d*(a^2+b*d))+a*(2*d^2*(a^2+b*d)+4*a^2*d^2)+2*b*d^3*a+1/3*c*
d^4)+b*(d*(2*d^2*(a^2+b*d)+4*a^2*d^2)+4*a^2*d^3+1/2*b*d^4)+5*a*c*d^4)*x^4+1
/3*(a*(d*(2*d^2*(a^2+b*d)+4*a^2*d^2)+4*a^2*d^3+1/2*b*d^4)+5*b*d^4*a+c*(d^5+
1))*x^3+1/2*(5*a^2*d^4+b*(d^5+1))*x^2+a*(d^5+1)*x
```

Maxima [B] time = 1.01338, size = 1044, normalized size = 22.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*(1+(d+a*x+1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="max
ima")
```

```
[Out] 1/4374*c^6*x^18 + 1/486*b*c^5*x^17 + 1/1944*(15*b^2*c^4 + 8*a*c^5)*x^16 + 1
/972*(15*b^3*c^3 + 30*a*b*c^4 + 4*c^5*d)*x^15 + 5/2592*(9*b^4*c^2 + 48*a*b^
```

```
2*c^3 + 16*a^2*c^4 + 16*b*c^4*d)*x^14 + 1/2592*(27*b^5*c + 360*a*b^3*c^2 +
480*a^2*b*c^3 + 80*(3*b^2*c^3 + 2*a*c^4)*d)*x^13 + 1/10368*(27*b^6 + 1080*a
*b^4*c + 4320*a^2*b^2*c^2 + 1280*a^3*c^3 + 320*c^4*d^2 + 480*(3*b^3*c^2 + 8
*a*b*c^3)*d)*x^12 + 1/864*(27*a*b^5 + 360*a^2*b^3*c + 480*a^3*b*c^2 + 160*b
*c^3*d^2 + 10*(9*b^4*c + 72*a*b^2*c^2 + 32*a^2*c^3)*d)*x^11 + 1/864*(135*a^
2*b^4 + 720*a^3*b^2*c + 240*a^4*c^2 + 40*(9*b^2*c^2 + 8*a*c^3)*d^2 + 9*(3*b
^5 + 80*a*b^3*c + 160*a^2*b*c^2)*d)*x^10 + 5/1296*(108*a^3*b^3 + 216*a^4*b*
c + 32*c^3*d^3 + 108*(b^3*c + 4*a*b*c^2)*d^2 + 9*(9*a*b^4 + 72*a^2*b^2*c +
32*a^3*c^2)*d)*x^9 + 1/288*(180*a^4*b^2 + 96*a^5*c + 160*b*c^2*d^3 + 15*(3*
b^4 + 48*a*b^2*c + 32*a^2*c^2)*d^2 + 120*(3*a^2*b^3 + 8*a^3*b*c)*d)*x^8 + 1
/36*(18*a^5*b + 10*(3*b^2*c + 4*a*c^2)*d^3 + 45*(a*b^3 + 4*a^2*b*c)*d^2 + 3
0*(3*a^3*b^2 + 2*a^4*c)*d)*x^7 + 1/36*(6*a^6 + 90*a^4*b*d + 10*c^2*d^4 + 15
*(b^3 + 8*a*b*c)*d^3 + 15*(9*a^2*b^2 + 8*a^3*c)*d^2)*x^6 + 1/6*(6*a^5*d + 3
0*a^3*b*d^2 + 5*b*c*d^4 + 5*(3*a*b^2 + 4*a^2*c)*d^3)*x^5 + 5/24*(12*a^4*d^2
 + 24*a^2*b*d^3 + (3*b^2 + 8*a*c)*d^4)*x^4 + 1/6*(20*a^3*d^3 + 15*a*b*d^4 +
 2*c*d^5 + 2*c)*x^3 + 1/2*(5*a^2*d^4 + b*d^5 + b)*x^2 + (a*d^5 + a)*x
```

Fricas [B] time = 1.06238, size = 2256, normalized size = 48.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*(1+(d+a*x+1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="fri
cas")
```

```
[Out] 1/4374*x^18*c^6 + 1/486*x^17*c^5*b + 5/648*x^16*c^4*b^2 + 1/243*x^16*c^5*a
+ 1/243*x^15*d*c^5 + 5/324*x^15*c^3*b^3 + 5/162*x^15*c^4*b*a + 5/162*x^14*d
*c^4*b + 5/288*x^14*c^2*b^4 + 5/54*x^14*c^3*b^2*a + 5/162*x^14*c^4*a^2 + 5/
54*x^13*d*c^3*b^2 + 1/96*x^13*c*b^5 + 5/81*x^13*d*c^4*a + 5/36*x^13*c^2*b^3
*a + 5/27*x^13*c^3*b*a^2 + 5/162*x^12*d^2*c^4 + 5/36*x^12*d*c^2*b^3 + 1/384
*x^12*b^6 + 10/27*x^12*d*c^3*b*a + 5/48*x^12*c*b^4*a + 5/12*x^12*c^2*b^2*a^
2 + 10/81*x^12*c^3*a^3 + 5/27*x^11*d^2*c^3*b + 5/48*x^11*d*c*b^4 + 5/6*x^11
*d*c^2*b^2*a + 1/32*x^11*b^5*a + 10/27*x^11*d*c^3*a^2 + 5/12*x^11*c*b^3*a^2
 + 5/9*x^11*c^2*b*a^3 + 5/12*x^10*d^2*c^2*b^2 + 1/32*x^10*d*b^5 + 10/27*x^1
0*d^2*c^3*a + 5/6*x^10*d*c*b^3*a + 5/3*x^10*d*c^2*b*a^2 + 5/32*x^10*b^4*a^2
 + 5/6*x^10*c*b^2*a^3 + 5/18*x^10*c^2*a^4 + 10/81*x^9*d^3*c^3 + 5/12*x^9*d^
2*c*b^3 + 5/3*x^9*d^2*c^2*b*a + 5/16*x^9*d*b^4*a + 5/2*x^9*d*c*b^2*a^2 + 10
/9*x^9*d*c^2*a^3 + 5/12*x^9*b^3*a^3 + 5/6*x^9*c*b*a^4 + 5/9*x^8*d^3*c^2*b +
 5/32*x^8*d^2*b^4 + 5/2*x^8*d^2*c*b^2*a + 5/3*x^8*d^2*c^2*a^2 + 5/4*x^8*d*b
^3*a^2 + 10/3*x^8*d*c*b*a^3 + 5/8*x^8*b^2*a^4 + 1/3*x^8*c*a^5 + 5/6*x^7*d^3
*c*b^2 + 10/9*x^7*d^3*c^2*a + 5/4*x^7*d^2*b^3*a + 5*x^7*d^2*c*b*a^2 + 5/2*x
```

```
^7*d*b^2*a^3 + 5/3*x^7*d*c*a^4 + 1/2*x^7*b*a^5 + 5/18*x^6*d^4*c^2 + 5/12*x^
6*d^3*b^3 + 10/3*x^6*d^3*c*b*a + 15/4*x^6*d^2*b^2*a^2 + 10/3*x^6*d^2*c*a^3
+ 5/2*x^6*d*b*a^4 + 1/6*x^6*a^6 + 5/6*x^5*d^4*c*b + 5/2*x^5*d^3*b^2*a + 10/
3*x^5*d^3*c*a^2 + 5*x^5*d^2*b*a^3 + x^5*d*a^5 + 5/8*x^4*d^4*b^2 + 5/3*x^4*d
^4*c*a + 5*x^4*d^3*b*a^2 + 5/2*x^4*d^2*a^4 + 1/3*x^3*d^5*c + 5/2*x^3*d^4*b*
a + 10/3*x^3*d^3*a^3 + 1/2*x^2*d^5*b + 5/2*x^2*d^4*a^2 + x*d^5*a + 1/3*x^3*
c + 1/2*x^2*b + x*a
```

Sympy [B] time = 0.235683, size = 930, normalized size = 19.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)*(1+(d+a*x+1/2*b*x**2+1/3*c*x**3)**5),x)
```

```
[Out] b*c**5*x**17/486 + c**6*x**18/4374 + x**16*(a*c**5/243 + 5*b**2*c**4/648) +
 x**15*(5*a*b*c**4/162 + 5*b**3*c**3/324 + c**5*d/243) + x**14*(5*a**2*c**4
/162 + 5*a*b**2*c**3/54 + 5*b**4*c**2/288 + 5*b*c**4*d/162) + x**13*(5*a**2
*b*c**3/27 + 5*a*b**3*c**2/36 + 5*a*c**4*d/81 + b**5*c/96 + 5*b**2*c**3*d/5
4) + x**12*(10*a**3*c**3/81 + 5*a**2*b**2*c**2/12 + 5*a*b**4*c/48 + 10*a*b*
c**3*d/27 + b**6/384 + 5*b**3*c**2*d/36 + 5*c**4*d**2/162) + x**11*(5*a**3*
b*c**2/9 + 5*a**2*b**3*c/12 + 10*a**2*c**3*d/27 + a*b**5/32 + 5*a*b**2*c**2
*d/6 + 5*b**4*c*d/48 + 5*b*c**3*d**2/27) + x**10*(5*a**4*c**2/18 + 5*a**3*b
**2*c/6 + 5*a**2*b**4/32 + 5*a**2*b*c**2*d/3 + 5*a*b**3*c*d/6 + 10*a*c**3*d
**2/27 + b**5*d/32 + 5*b**2*c**2*d**2/12) + x**9*(5*a**4*b*c/6 + 5*a**3*b**
3/12 + 10*a**3*c**2*d/9 + 5*a**2*b**2*c*d/2 + 5*a*b**4*d/16 + 5*a*b*c**2*d*
*2/3 + 5*b**3*c*d**2/12 + 10*c**3*d**3/81) + x**8*(a**5*c/3 + 5*a**4*b**2/8
 + 10*a**3*b*c*d/3 + 5*a**2*b**3*d/4 + 5*a**2*c**2*d**2/3 + 5*a*b**2*c*d**2
/2 + 5*b**4*d**2/32 + 5*b*c**2*d**3/9) + x**7*(a**5*b/2 + 5*a**4*c*d/3 + 5*
a**3*b**2*d/2 + 5*a**2*b*c*d**2 + 5*a*b**3*d**2/4 + 10*a*c**2*d**3/9 + 5*b*
*2*c*d**3/6) + x**6*(a**6/6 + 5*a**4*b*d/2 + 10*a**3*c*d**2/3 + 15*a**2*b**
2*d**2/4 + 10*a*b*c*d**3/3 + 5*b**3*d**3/12 + 5*c**2*d**4/18) + x**5*(a**5*
d + 5*a**3*b*d**2 + 10*a**2*c*d**3/3 + 5*a*b**2*d**3/2 + 5*b*c*d**4/6) + x*
*4*(5*a**4*d**2/2 + 5*a**2*b*d**3 + 5*a*c*d**4/3 + 5*b**2*d**4/8) + x**3*(1
0*a**3*d**3/3 + 5*a*b*d**4/2 + c*d**5/3 + c/3) + x**2*(5*a**2*d**4/2 + b*d*
*5/2 + b/2) + x*(a*d**5 + a)
```

Giac [B] time = 1.21755, size = 1253, normalized size = 26.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*(1+(d+a*x+1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="gia
c")
```

```
[Out] 1/4374*c^6*x^18 + 1/486*b*c^5*x^17 + 5/648*b^2*c^4*x^16 + 1/243*a*c^5*x^16
+ 5/324*b^3*c^3*x^15 + 5/162*a*b*c^4*x^15 + 1/243*c^5*d*x^15 + 5/288*b^4*c^
2*x^14 + 5/54*a*b^2*c^3*x^14 + 5/162*a^2*c^4*x^14 + 5/162*b*c^4*d*x^14 + 1/
96*b^5*c*x^13 + 5/36*a*b^3*c^2*x^13 + 5/27*a^2*b*c^3*x^13 + 5/54*b^2*c^3*d*
x^13 + 5/81*a*c^4*d*x^13 + 1/384*b^6*x^12 + 5/48*a*b^4*c*x^12 + 5/12*a^2*b^
2*c^2*x^12 + 10/81*a^3*c^3*x^12 + 5/36*b^3*c^2*d*x^12 + 10/27*a*b*c^3*d*x^1
2 + 5/162*c^4*d^2*x^12 + 1/32*a*b^5*x^11 + 5/12*a^2*b^3*c*x^11 + 5/9*a^3*b*
c^2*x^11 + 5/48*b^4*c*d*x^11 + 5/6*a*b^2*c^2*d*x^11 + 10/27*a^2*c^3*d*x^11
+ 5/27*b*c^3*d^2*x^11 + 5/32*a^2*b^4*x^10 + 5/6*a^3*b^2*c*x^10 + 5/18*a^4*c
^2*x^10 + 1/32*b^5*d*x^10 + 5/6*a*b^3*c*d*x^10 + 5/3*a^2*b*c^2*d*x^10 + 5/1
2*b^2*c^2*d^2*x^10 + 10/27*a*c^3*d^2*x^10 + 5/12*a^3*b^3*x^9 + 5/6*a^4*b*c*
x^9 + 5/16*a*b^4*d*x^9 + 5/2*a^2*b^2*c*d*x^9 + 10/9*a^3*c^2*d*x^9 + 5/12*b^
3*c*d^2*x^9 + 5/3*a*b*c^2*d^2*x^9 + 10/81*c^3*d^3*x^9 + 5/8*a^4*b^2*x^8 + 1
/3*a^5*c*x^8 + 5/4*a^2*b^3*d*x^8 + 10/3*a^3*b*c*d*x^8 + 5/32*b^4*d^2*x^8 +
5/2*a*b^2*c*d^2*x^8 + 5/3*a^2*c^2*d^2*x^8 + 5/9*b*c^2*d^3*x^8 + 1/2*a^5*b*x
^7 + 5/2*a^3*b^2*d*x^7 + 5/3*a^4*c*d*x^7 + 5/4*a*b^3*d^2*x^7 + 5*a^2*b*c*d^
2*x^7 + 5/6*b^2*c*d^3*x^7 + 10/9*a*c^2*d^3*x^7 + 1/6*a^6*x^6 + 5/2*a^4*b*d*
x^6 + 15/4*a^2*b^2*d^2*x^6 + 10/3*a^3*c*d^2*x^6 + 5/12*b^3*d^3*x^6 + 10/3*a
*b*c*d^3*x^6 + 5/18*c^2*d^4*x^6 + a^5*d*x^5 + 5*a^3*b*d^2*x^5 + 5/2*a*b^2*d
^3*x^5 + 10/3*a^2*c*d^3*x^5 + 5/6*b*c*d^4*x^5 + 5/2*a^4*d^2*x^4 + 5*a^2*b*d
^3*x^4 + 5/8*b^2*d^4*x^4 + 5/3*a*c*d^4*x^4 + 10/3*a^3*d^3*x^3 + 5/2*a*b*d^4
*x^3 + 1/3*c*d^5*x^3 + 5/2*a^2*d^4*x^2 + 1/2*b*d^5*x^2 + a*d^5*x + 1/3*c*x^
3 + 1/2*b*x^2 + a*x
```

3.216 $\int \left(a+cx^2\right)\left(1+\left(ax+\frac{cx^3}{3}\right)^n\right)\,dx$

Optimal. Leaf size=34

$$\frac{\left(ax+\frac{cx^3}{3}\right)^{n+1}}{n+1}+ax+\frac{cx^3}{3}$$

```
[Out] a*x + (c*x^3)/3 + (a*x + (c*x^3)/3)^(1 + n)/(1 + n)
```

Rubi [A] time = 0.0090691, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {1591}

$$\frac{\left(ax+\frac{cx^3}{3}\right)^{n+1}}{n+1}+ax+\frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)*(1 + (a*x + (c*x^3)/3)^n),x]
```

```
[Out] a*x + (c*x^3)/3 + (a*x + (c*x^3)/3)^(1 + n)/(1 + n)
```

Rule 1591

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\begin{aligned}\int \left(a+cx^2\right)\left(1+\left(ax+\frac{cx^3}{3}\right)^n\right)\,dx &= \text{Subst}\left(\int \left(1+x^n\right)\,dx, x, ax+\frac{cx^3}{3}\right)\\ &= ax+\frac{cx^3}{3}+\frac{\left(ax+\frac{cx^3}{3}\right)^{1+n}}{1+n}\end{aligned}$$

Mathematica [A] time = 0.0616451, size = 36, normalized size = 1.06

$$\frac{x\left(3a+cx^2\right)\left(\left(ax+\frac{cx^3}{3}\right)^n+n+1\right)}{3(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)*(1 + (a*x + (c*x^3)/3)^n),x]
```

```
[Out] (x*(3*a + c*x^2)*(1 + n + (a*x + (c*x^3)/3)^n))/(3*(1 + n))
```

Maple [A] time = 0.001, size = 31, normalized size = 0.9

$$ax+\frac{cx^3}{3}+\frac{1}{1+n}\left(ax+\frac{cx^3}{3}\right)^{1+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)*(1+(a*x+1/3*c*x^3)^n),x)
```

```
[Out] a*x+1/3*c*x^3+(a*x+1/3*c*x^3)^(1+n)/(1+n)
```

Maxima [A] time = 1.65076, size = 73, normalized size = 2.15

$$\frac{1}{3}\,cx^3+ax+\frac{\left(cx^3+3\,ax\right)e^{\left(n\log\left(cx^2+3\,a\right)+n\log(x)\right)}}{3^{n+1}n+3^{n+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(1+(a*x+1/3*c*x^3)^n),x, algorithm="maxima")
```

```
[Out] 1/3*c*x^3 + a*x + (c*x^3 + 3*a*x)*e^(n*log(c*x^2 + 3*a) + n*log(x))/(3^(n +
 1)*n + 3^(n + 1))
```

Fricas [A] time = 1.35445, size = 112, normalized size = 3.29

$$\frac{(cn+c)x^3+\left(cx^3+3\,ax\right)\left(\frac{1}{3}\,cx^3+ax\right)^n+3\,(an+a)x}{3\,(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(1+(a*x+1/3*c*x^3)^n),x, algorithm="fricas")
```

```
[Out] 1/3*((c*n + c)*x^3 + (c*x^3 + 3*a*x)*(1/3*c*x^3 + a*x)^n + 3*(a*n + a)*x)/(
n + 1)
```

Sympy [B] time = 175.955, size = 201, normalized size = 5.91

$$\begin{cases} \frac{3\cdot 3^n anx}{3\cdot 3^n n+3\cdot 3^n}+\frac{3\cdot 3^n ax}{3\cdot 3^n n+3\cdot 3^n}+\frac{3^n cnx^3}{3\cdot 3^n n+3\cdot 3^n}+\frac{3^n cx^3}{3\cdot 3^n n+3\cdot 3^n}+\frac{3ax\left(3ax+cx^3\right)^n}{3\cdot 3^n n+3\cdot 3^n}+\frac{cx^3\left(3ax+cx^3\right)^n}{3\cdot 3^n n+3\cdot 3^n} & \text{for } n\neq -1 \\ ax+\frac{cx^3}{3}+\log(x)+\log\left(-\sqrt{3}i\sqrt{a}\sqrt{\frac{1}{c}}+x\right)+\log\left(\sqrt{3}i\sqrt{a}\sqrt{\frac{1}{c}}+x\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)*(1+(a*x+1/3*c*x**3)**n),x)
```

```
[Out] Piecewise((3*3**n*a*n*x/(3*3**n*n + 3*3**n) + 3*3**n*a*x/(3*3**n*n + 3*3**n
) + 3**n*c*n*x**3/(3*3**n*n + 3*3**n) + 3**n*c*x**3/(3*3**n*n + 3*3**n) + 3
*a*x*(3*a*x + c*x**3)**n/(3*3**n*n + 3*3**n) + c*x**3*(3*a*x + c*x**3)**n/(
3*3**n*n + 3*3**n), Ne(n, -1)), (a*x + c*x**3/3 + log(x) + log(-sqrt(3)*I*s
qrt(a)*sqrt(1/c) + x) + log(sqrt(3)*I*sqrt(a)*sqrt(1/c) + x), True))
```

Giac [A] time = 1.17752, size = 41, normalized size = 1.21

$$\frac{1}{3}\,cx^3+ax+\frac{\left(\frac{1}{3}\,cx^3+ax\right)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(1+(a*x+1/3*c*x^3)^n),x, algorithm="giac")
```

```
[Out] 1/3*c*x^3 + a*x + (1/3*c*x^3 + a*x)^(n + 1)/(n + 1)
```

3.217 $\int \left(bx+cx^2\right)\left(1+\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^n\right)\,dx$

Optimal. Leaf size=44

$$\frac{\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^{n+1}}{n+1}+\frac{bx^2}{2}+\frac{cx^3}{3}$$

```
[Out] (b*x^2)/2 + (c*x^3)/3 + ((b*x^2)/2 + (c*x^3)/3)^(1 + n)/(1 + n)
```

Rubi [A] time = 0.0095982, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.032, Rules used = {1591}

$$\frac{\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^{n+1}}{n+1}+\frac{bx^2}{2}+\frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)*(1 + ((b*x^2)/2 + (c*x^3)/3)^n),x]
```

```
[Out] (b*x^2)/2 + (c*x^3)/3 + ((b*x^2)/2 + (c*x^3)/3)^(1 + n)/(1 + n)
```

Rule 1591

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\begin{aligned}\int \left(bx+cx^2\right)\left(1+\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^n\right)\,dx &= \text{Subst}\left(\int \left(1+x^n\right)\,dx, x, \frac{bx^2}{2}+\frac{cx^3}{3}\right)\\ &= \frac{bx^2}{2}+\frac{cx^3}{3}+\frac{\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^{1+n}}{1+n}\end{aligned}$$

Mathematica [A] time = 0.0828407, size = 42, normalized size = 0.95

$$\frac{x^2(3b+2cx)\left(\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^n+n+1\right)}{6(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)*(1 + ((b*x^2)/2 + (c*x^3)/3)^n),x]
```

```
[Out] (x^2*(3*b + 2*c*x)*(1 + n + ((b*x^2)/2 + (c*x^3)/3)^n))/(6*(1 + n))
```

Maple [A] time = 0.003, size = 37, normalized size = 0.8

$$\frac{bx^2}{2}+\frac{cx^3}{3}+\frac{1}{1+n}\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^{1+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)*(1+(1/2*b*x^2+1/3*c*x^3)^n),x)
```

```
[Out] 1/2*b*x^2+1/3*c*x^3+(1/2*b*x^2+1/3*c*x^3)^(1+n)/(1+n)
```

Maxima [A] time = 1.70073, size = 96, normalized size = 2.18

$$\frac{1}{3}cx^3+\frac{1}{2}bx^2+\frac{\left(2cx^3+3bx^2\right)e^{(n\log(2cx+3b)+2n\log(x))}}{3^{n+1}2^{n+1}n+3^{n+1}2^{n+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)*(1+(1/2*b*x^2+1/3*c*x^3)^n),x, algorithm="maxima")
```

```
[Out] 1/3*c*x^3 + 1/2*b*x^2 + (2*c*x^3 + 3*b*x^2)*e^(n*log(2*c*x + 3*b) + 2*n*log
(x))/(3^(n + 1)*2^(n + 1)*n + 3^(n + 1)*2^(n + 1))
```

Fricas [A] time = 1.39462, size = 131, normalized size = 2.98

$$\frac{2\,(cn+c)x^3+3\,(bn+b)x^2+\left(2\,cx^3+3\,bx^2\right)\left(\frac{1}{3}\,cx^3+\frac{1}{2}\,bx^2\right)^n}{6\,(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)*(1+(1/2*b*x^2+1/3*c*x^3)^n),x, algorithm="fricas")
```

```
[Out] 1/6*(2*(c*n + c)*x^3 + 3*(b*n + b)*x^2 + (2*c*x^3 + 3*b*x^2)*(1/3*c*x^3 + 1
/2*b*x^2)^n)/(n + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)*(1+(1/2*b*x**2+1/3*c*x**3)**n),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14951, size = 49, normalized size = 1.11

$$\frac{1}{3}\,cx^3+\frac{1}{2}\,bx^2+\frac{\left(\frac{1}{3}\,cx^3+\frac{1}{2}\,bx^2\right)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)*(1+(1/2*b*x^2+1/3*c*x^3)^n),x, algorithm="giac")
```

```
[Out] 1/3*c*x^3 + 1/2*b*x^2 + (1/3*c*x^3 + 1/2*b*x^2)^(n + 1)/(n + 1)
```

3.218 $\int \left(a+bx+cx^2\right)\left(1+\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^n\right)\,dx$

Optimal. Leaf size=50

$$\frac{\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^{n+1}}{n+1}+ax+\frac{bx^2}{2}+\frac{cx^3}{3}$$

```
[Out] a*x + (b*x^2)/2 + (c*x^3)/3 + (a*x + (b*x^2)/2 + (c*x^3)/3)^(1 + n)/(1 + n)
```

Rubi [A] time = 0.0091307, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {1591}

$$\frac{\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^{n+1}}{n+1}+ax+\frac{bx^2}{2}+\frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)*(1 + (a*x + (b*x^2)/2 + (c*x^3)/3)^n),x]
```

```
[Out] a*x + (b*x^2)/2 + (c*x^3)/3 + (a*x + (b*x^2)/2 + (c*x^3)/3)^(1 + n)/(1 + n)
```

Rule 1591

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx+cx^2\right)\left(1+\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^n\right)\,dx &= \text{Subst}\left(\int \left(1+x^n\right)\,dx, x, ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)\\ &= ax+\frac{bx^2}{2}+\frac{cx^3}{3}+\frac{\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^{1+n}}{1+n}\end{aligned}$$

Mathematica [A] time = 0.187863, size = 49, normalized size = 0.98

$$\frac{x(6a + x(3b + 2cx))\left(\left(ax + \frac{bx^2}{2} + \frac{cx^3}{3}\right)^n + n + 1\right)}{6(n + 1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)*(1 + (a*x + (b*x^2)/2 + (c*x^3)/3)^n),x]
```

```
[Out] (x*(6*a + x*(3*b + 2*c*x))*(1 + n + (a*x + (b*x^2)/2 + (c*x^3)/3)^n))/(6*(1
 + n))
```

Maple [A] time = 0.003, size = 43, normalized size = 0.9

$$ax + \frac{bx^2}{2} + \frac{cx^3}{3} + \frac{1}{1+n}\left(ax + \frac{bx^2}{2} + \frac{cx^3}{3}\right)^{1+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)*(1+(a*x+1/2*b*x^2+1/3*c*x^3)^n),x)
```

```
[Out] a*x+1/2*b*x^2+1/3*c*x^3+(a*x+1/2*b*x^2+1/3*c*x^3)^(1+n)/(1+n)
```

Maxima [A] time = 1.7359, size = 112, normalized size = 2.24

$$\frac{1}{3}\,cx^3 + \frac{1}{2}\,bx^2 + ax + \frac{\left(2\,cx^3 + 3\,bx^2 + 6\,ax\right)e^{\left(n\log\left(2\,cx^2+3\,bx+6\,a\right)+n\log(x)\right)}}{3^{n+1}2^{n+1}n + 3^{n+1}2^{n+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*(1+(a*x+1/2*b*x^2+1/3*c*x^3)^n),x, algorithm="maxim
a")
```

```
[Out] 1/3*c*x^3 + 1/2*b*x^2 + a*x + (2*c*x^3 + 3*b*x^2 + 6*a*x)*e^(n*log(2*c*x^2
+ 3*b*x + 6*a) + n*log(x))/(3^(n + 1)*2^(n + 1)*n + 3^(n + 1)*2^(n + 1))
```

Fricas [A] time = 1.37342, size = 171, normalized size = 3.42

$$\frac{2\,(cn+c)x^3+3\,(bn+b)x^2+\left(2\,cx^3+3\,bx^2+6\,ax\right)\left(\frac{1}{3}\,cx^3+\frac{1}{2}\,bx^2+ax\right)^n+6\,(an+a)x}{6\,(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*(1+(a*x+1/2*b*x^2+1/3*c*x^3)^n),x, algorithm="frica
s")
```

```
[Out] 1/6*(2*(c*n + c)*x^3 + 3*(b*n + b)*x^2 + (2*c*x^3 + 3*b*x^2 + 6*a*x)*(1/3*c
*x^3 + 1/2*b*x^2 + a*x)^n + 6*(a*n + a)*x)/(n + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)*(1+(a*x+1/2*b*x**2+1/3*c*x**3)**n),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19638, size = 57, normalized size = 1.14

$$\frac{1}{3}\,cx^3+\frac{1}{2}\,bx^2+ax+\frac{\left(\frac{1}{3}\,cx^3+\frac{1}{2}\,bx^2+ax\right)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*(1+(a*x+1/2*b*x^2+1/3*c*x^3)^n),x, algorithm="giac"
)
```

```
[Out] 1/3*c*x^3 + 1/2*b*x^2 + a*x + (1/3*c*x^3 + 1/2*b*x^2 + a*x)^(n + 1)/(n + 1)
```

3.219 $\int \left(-4+4x+x^2\right)\left(5-12x+6x^2+x^3\right)\,dx$

Optimal. Leaf size=19

$$\frac{1}{6}\left(x^3+6x^2-12x+5\right)^2$$

```
[Out] (5 - 12*x + 6*x^2 + x^3)^2/6
```

Rubi [A] time = 0.0088695, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1588}

$$\frac{1}{6}\left(x^3+6x^2-12x+5\right)^2$$

Antiderivative was successfully verified.

```
[In] Int[(-4 + 4*x + x^2)*(5 - 12*x + 6*x^2 + x^3),x]
```

```
[Out] (5 - 12*x + 6*x^2 + x^3)^2/6
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \left(-4+4x+x^2\right)\left(5-12x+6x^2+x^3\right)\,dx=\frac{1}{6}\left(5-12x+6x^2+x^3\right)^2$$

Mathematica [A] time = 0.0015824, size = 33, normalized size = 1.74

$$\frac{x^6}{6}+2x^5+2x^4-\frac{67x^3}{3}+34x^2-20x$$

Antiderivative was successfully verified.

```
[In] Integrate[(-4 + 4*x + x^2)*(5 - 12*x + 6*x^2 + x^3),x]
```

```
[Out] -20*x + 34*x^2 - (67*x^3)/3 + 2*x^4 + 2*x^5 + x^6/6
```

Maple [A] time = 0.001, size = 30, normalized size = 1.6

$$\frac{x^6}{6} + 2\,x^5 + 2\,x^4 - \frac{67\,x^3}{3} + 34\,x^2 - 20\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+4*x-4)*(x^3+6*x^2-12*x+5),x)
```

```
[Out] 1/6*x^6+2*x^5+2*x^4-67/3*x^3+34*x^2-20*x
```

Maxima [A] time = 1.05212, size = 23, normalized size = 1.21

$$\frac{1}{6}\left(x^3 + 6\,x^2 - 12\,x + 5\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+4*x-4)*(x^3+6*x^2-12*x+5),x, algorithm="maxima")
```

```
[Out] 1/6*(x^3 + 6*x^2 - 12*x + 5)^2
```

Fricas [A] time = 1.11781, size = 70, normalized size = 3.68

$$\frac{1}{6}x^6 + 2x^5 + 2x^4 - \frac{67}{3}x^3 + 34x^2 - 20x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+4*x-4)*(x^3+6*x^2-12*x+5),x, algorithm="fricas")
```

```
[Out] 1/6*x^6 + 2*x^5 + 2*x^4 - 67/3*x^3 + 34*x^2 - 20*x
```

Sympy [A] time = 0.058059, size = 29, normalized size = 1.53

$$\frac{x^6}{6} + 2x^5 + 2x^4 - \frac{67x^3}{3} + 34x^2 - 20x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+4*x-4)*(x**3+6*x**2-12*x+5),x)
```

```
[Out] x**6/6 + 2*x**5 + 2*x**4 - 67*x**3/3 + 34*x**2 - 20*x
```

Giac [A] time = 1.19765, size = 39, normalized size = 2.05

$$\frac{1}{6}\,x^6 + 2\,x^5 + 2\,x^4 - \frac{67}{3}\,x^3 + 34\,x^2 - 20\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+4*x-4)*(x^3+6*x^2-12*x+5),x, algorithm="giac")
```

```
[Out] 1/6*x^6 + 2*x^5 + 2*x^4 - 67/3*x^3 + 34*x^2 - 20*x
```

3.220 $\int \left(2x+x^3\right)\left(1+4x^2+x^4\right)\,dx$

Optimal. Leaf size=16

$$\frac{1}{8}\left(x^4+4x^2+1\right)^2$$

```
[Out] (1 + 4*x^2 + x^4)^2/8
```

Rubi [A] time = 0.0066653, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1588}

$$\frac{1}{8}\left(x^4+4x^2+1\right)^2$$

Antiderivative was successfully verified.

```
[In] Int[(2*x + x^3)*(1 + 4*x^2 + x^4),x]
```

```
[Out] (1 + 4*x^2 + x^4)^2/8
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \left(2x+x^3\right)\left(1+4x^2+x^4\right)\,dx=\frac{1}{8}\left(1+4x^2+x^4\right)^2$$

Mathematica [A] time = 0.0014668, size = 21, normalized size = 1.31

$$\frac{x^8}{8}+x^6+\frac{9x^4}{4}+x^2$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*x + x^3)*(1 + 4*x^2 + x^4),x]
```

```
[Out] x^2 + (9*x^4)/4 + x^6 + x^8/8
```

Maple [A] time = 0.001, size = 18, normalized size = 1.1

$$\frac{x^8}{8} + x^6 + \frac{9\,x^4}{4} + x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+2*x)*(x^4+4*x^2+1),x)
```

```
[Out] 1/8*x^8+x^6+9/4*x^4+x^2
```

Maxima [A] time = 0.985187, size = 19, normalized size = 1.19

$$\frac{1}{8}\left(x^4 + 4\,x^2 + 1\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+2*x)*(x^4+4*x^2+1),x, algorithm="maxima")
```

```
[Out] 1/8*(x^4 + 4*x^2 + 1)^2
```

Fricas [A] time = 1.10848, size = 42, normalized size = 2.62

$$\frac{1}{8}x^8 + x^6 + \frac{9}{4}x^4 + x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+2*x)*(x^4+4*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/8*x^8 + x^6 + 9/4*x^4 + x^2
```

Sympy [A] time = 0.053976, size = 17, normalized size = 1.06

$$\frac{x^8}{8} + x^6 + \frac{9x^4}{4} + x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+2*x)*(x**4+4*x**2+1),x)
```

```
[Out] x**8/8 + x**6 + 9*x**4/4 + x**2
```

Giac [A] time = 1.1962, size = 23, normalized size = 1.44

$$\frac{1}{8}\,x^8 + x^6 + \frac{9}{4}\,x^4 + x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+2*x)*(x^4+4*x^2+1),x, algorithm="giac")
```

```
[Out] 1/8*x^8 + x^6 + 9/4*x^4 + x^2
```

3.221 $\int (1+2x)\left(x+x^2\right)^3\left(-18+7\left(x+x^2\right)^3\right)^2\,dx$

Optimal. Leaf size=33

$$\frac{49}{10}x^{10}(x+1)^{10}-36x^7(x+1)^7+81x^4(x+1)^4$$

```
[Out] 81*x^4*(1 + x)^4 - 36*x^7*(1 + x)^7 + (49*x^10*(1 + x)^10)/10
```

Rubi [B] time = 0.197782, antiderivative size = 96, normalized size of antiderivative = 2.91, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1593, 1612}

$$\frac{49x^{20}}{10}+49x^{19}+\frac{441x^{18}}{2}+588x^{17}+1029x^{16}+\frac{6174x^{15}}{5}+993x^{14}+336x^{13}-\frac{1071x^{12}}{2}-1211x^{11}-\frac{12551x^{10}}{10}-756x^9$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)*(x + x^2)^3*(-18 + 7*(x + x^2)^3)^2,x]
```

```
[Out] 81*x^4 + 324*x^5 + 486*x^6 + 288*x^7 - 171*x^8 - 756*x^9 - (12551*x^10)/10
 - 1211*x^11 - (1071*x^12)/2 + 336*x^13 + 993*x^14 + (6174*x^15)/5 + 1029*x^
16 + 588*x^17 + (441*x^18)/2 + 49*x^19 + (49*x^20)/10
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1612

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^
n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px,
 x] && IntegersQ[m, n]
```

Rubi steps

$$\int (1+2x)\left(x+x^2\right)^3\left(-18+7\left(x+x^2\right)^3\right)^2\,dx = \int x^3(1+x)^3(1+2x)\left(-18+7\left(x+x^2\right)^3\right)^2\,dx$$
$$= \int \left(324x^3+1620x^4+2916x^5+2016x^6-1368x^7-6804x^8-12551x^9-1\right.$$
$$= 81x^4+324x^5+486x^6+288x^7-171x^8-756x^9-\frac{12551x^{10}}{10}-1211x^{11}-$$

Mathematica [B] time = 0.006101, size = 96, normalized size = 2.91

$$\frac{49x^{20}}{10}+49x^{19}+\frac{441x^{18}}{2}+588x^{17}+1029x^{16}+\frac{6174x^{15}}{5}+993x^{14}+336x^{13}-\frac{1071x^{12}}{2}-1211x^{11}-\frac{12551x^{10}}{10}-756x^9-$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)*(x + x^2)^3*(-18 + 7*(x + x^2)^3)^2,x]
```

```
[Out] 81*x^4 + 324*x^5 + 486*x^6 + 288*x^7 - 171*x^8 - 756*x^9 - (12551*x^10)/10
- 1211*x^11 - (1071*x^12)/2 + 336*x^13 + 993*x^14 + (6174*x^15)/5 + 1029*x^
16 + 588*x^17 + (441*x^18)/2 + 49*x^19 + (49*x^20)/10
```

Maple [B] time = 0.002, size = 87, normalized size = 2.6

$$\frac{49\,x^{20}}{10}+49\,x^{19}+\frac{441\,x^{18}}{2}+588\,x^{17}+1029\,x^{16}+\frac{6174\,x^{15}}{5}+993\,x^{14}+336\,x^{13}-\frac{1071\,x^{12}}{2}-1211\,x^{11}-\frac{12551\,x^{10}}{10}-756$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)*(x^2+x)^3*(-18+7*(x^2+x)^3)^2,x)
```

```
[Out] 49/10*x^20+49*x^19+441/2*x^18+588*x^17+1029*x^16+6174/5*x^15+993*x^14+336*x
^13-1071/2*x^12-1211*x^11-12551/10*x^10-756*x^9-171*x^8+288*x^7+486*x^6+324
*x^5+81*x^4
```

Maxima [B] time = 0.992962, size = 116, normalized size = 3.52

$$\frac{49}{10}\,x^{20}+49\,x^{19}+\frac{441}{2}\,x^{18}+588\,x^{17}+1029\,x^{16}+\frac{6174}{5}\,x^{15}+993\,x^{14}+336\,x^{13}-\frac{1071}{2}\,x^{12}-1211\,x^{11}-\frac{12551}{10}\,x^{10}-756$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(x^2+x)^3*(-18+7*(x^2+x)^3)^2,x, algorithm="maxima")
```

```
[Out] 49/10*x^20 + 49*x^19 + 441/2*x^18 + 588*x^17 + 1029*x^16 + 6174/5*x^15 + 99
3*x^14 + 336*x^13 - 1071/2*x^12 - 1211*x^11 - 12551/10*x^10 - 756*x^9 - 171
*x^8 + 288*x^7 + 486*x^6 + 324*x^5 + 81*x^4
```

Fricas [B] time = 1.1462, size = 263, normalized size = 7.97

$$\frac{49}{10}x^{20} + 49x^{19} + \frac{441}{2}x^{18} + 588x^{17} + 1029x^{16} + \frac{6174}{5}x^{15} + 993x^{14} + 336x^{13} - \frac{1071}{2}x^{12} - 1211x^{11} - \frac{12551}{10}x^{10} - 756x^9 -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(x^2+x)^3*(-18+7*(x^2+x)^3)^2,x, algorithm="fricas")
```

```
[Out] 49/10*x^20 + 49*x^19 + 441/2*x^18 + 588*x^17 + 1029*x^16 + 6174/5*x^15 + 99
3*x^14 + 336*x^13 - 1071/2*x^12 - 1211*x^11 - 12551/10*x^10 - 756*x^9 - 171
*x^8 + 288*x^7 + 486*x^6 + 324*x^5 + 81*x^4
```

Sympy [B] time = 0.07515, size = 94, normalized size = 2.85

$$\frac{49x^{20}}{10} + 49x^{19} + \frac{441x^{18}}{2} + 588x^{17} + 1029x^{16} + \frac{6174x^{15}}{5} + 993x^{14} + 336x^{13} - \frac{1071x^{12}}{2} - 1211x^{11} - \frac{12551x^{10}}{10} - 756x^9 -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(x**2+x)**3*(-18+7*(x**2+x)**3)**2,x)
```

```
[Out] 49*x**20/10 + 49*x**19 + 441*x**18/2 + 588*x**17 + 1029*x**16 + 6174*x**15/
5 + 993*x**14 + 336*x**13 - 1071*x**12/2 - 1211*x**11 - 12551*x**10/10 - 75
6*x**9 - 171*x**8 + 288*x**7 + 486*x**6 + 324*x**5 + 81*x**4
```

Giac [B] time = 1.22729, size = 116, normalized size = 3.52

$$\frac{49}{10}x^{20} + 49x^{19} + \frac{441}{2}x^{18} + 588x^{17} + 1029x^{16} + \frac{6174}{5}x^{15} + 993x^{14} + 336x^{13} - \frac{1071}{2}x^{12} - 1211x^{11} - \frac{12551}{10}x^{10} - 75$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(x^2+x)^3*(-18+7*(x^2+x)^3)^2,x, algorithm="giac")
```

```
[Out] 49/10*x^20 + 49*x^19 + 441/2*x^18 + 588*x^17 + 1029*x^16 + 6174/5*x^15 + 99
3*x^14 + 336*x^13 - 1071/2*x^12 - 1211*x^11 - 12551/10*x^10 - 756*x^9 - 171
*x^8 + 288*x^7 + 486*x^6 + 324*x^5 + 81*x^4
```

3.222 $\int x^3(1+x)^3(1+2x)\left(-18+7x^3(1+x)^3\right)^2\,dx$

Optimal. Leaf size=33

$$\frac{49}{10}x^{10}(x+1)^{10}-36x^7(x+1)^7+81x^4(x+1)^4$$

```
[Out] 81*x^4*(1 + x)^4 - 36*x^7*(1 + x)^7 + (49*x^10*(1 + x)^10)/10
```

Rubi [B] time = 0.154192, antiderivative size = 96, normalized size of antiderivative = 2.91, number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {1612}

$$\frac{49x^{20}}{10}+49x^{19}+\frac{441x^{18}}{2}+588x^{17}+1029x^{16}+\frac{6174x^{15}}{5}+993x^{14}+336x^{13}-\frac{1071x^{12}}{2}-1211x^{11}-\frac{12551x^{10}}{10}-756x^9$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(1 + x)^3*(1 + 2*x)*(-18 + 7*x^3*(1 + x)^3)^2,x]
```

```
[Out] 81*x^4 + 324*x^5 + 486*x^6 + 288*x^7 - 171*x^8 - 756*x^9 - (12551*x^10)/10
- 1211*x^11 - (1071*x^12)/2 + 336*x^13 + 993*x^14 + (6174*x^15)/5 + 1029*x^
16 + 588*x^17 + (441*x^18)/2 + 49*x^19 + (49*x^20)/10
```

Rule 1612

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^
n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px,
 x] && IntegersQ[m, n]
```

Rubi steps

$$\int x^3(1+x)^3(1+2x)\left(-18+7x^3(1+x)^3\right)^2\,dx=\int\left(324x^3+1620x^4+2916x^5+2016x^6-1368x^7-6804x^8-12551x^9\right.$$
$$=81x^4+324x^5+486x^6+288x^7-171x^8-756x^9-\frac{12551x^{10}}{10}-1211x^{11}$$

Mathematica [B] time = 0.0050423, size = 96, normalized size = 2.91

$$\frac{49x^{20}}{10}+49x^{19}+\frac{441x^{18}}{2}+588x^{17}+1029x^{16}+\frac{6174x^{15}}{5}+993x^{14}+336x^{13}-\frac{1071x^{12}}{2}-1211x^{11}-\frac{12551x^{10}}{10}-756x^9$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(1 + x)^3*(1 + 2*x)*(-18 + 7*x^3*(1 + x)^3)^2,x]
```

```
[Out] 81*x^4 + 324*x^5 + 486*x^6 + 288*x^7 - 171*x^8 - 756*x^9 - (12551*x^10)/10
- 1211*x^11 - (1071*x^12)/2 + 336*x^13 + 993*x^14 + (6174*x^15)/5 + 1029*x^
16 + 588*x^17 + (441*x^18)/2 + 49*x^19 + (49*x^20)/10
```

Maple [B] time = 0.001, size = 87, normalized size = 2.6

$$\frac{49\,x^{20}}{10}+49\,x^{19}+\frac{441\,x^{18}}{2}+588\,x^{17}+1029\,x^{16}+\frac{6174\,x^{15}}{5}+993\,x^{14}+336\,x^{13}-\frac{1071\,x^{12}}{2}-1211\,x^{11}-\frac{12551\,x^{10}}{10}-756$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(1+x)^3*(1+2*x)*(-18+7*x^3*(1+x)^3)^2,x)
```

```
[Out] 49/10*x^20+49*x^19+441/2*x^18+588*x^17+1029*x^16+6174/5*x^15+993*x^14+336*x
^13-1071/2*x^12-1211*x^11-12551/10*x^10-756*x^9-171*x^8+288*x^7+486*x^6+324
*x^5+81*x^4
```

Maxima [B] time = 0.986058, size = 116, normalized size = 3.52

$$\frac{49}{10}\,x^{20}+49\,x^{19}+\frac{441}{2}\,x^{18}+588\,x^{17}+1029\,x^{16}+\frac{6174}{5}\,x^{15}+993\,x^{14}+336\,x^{13}-\frac{1071}{2}\,x^{12}-1211\,x^{11}-\frac{12551}{10}\,x^{10}-756$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(1+x)^3*(1+2*x)*(-18+7*x^3*(1+x)^3)^2,x, algorithm="maxima")
```

```
[Out] 49/10*x^20 + 49*x^19 + 441/2*x^18 + 588*x^17 + 1029*x^16 + 6174/5*x^15 + 99
3*x^14 + 336*x^13 - 1071/2*x^12 - 1211*x^11 - 12551/10*x^10 - 756*x^9 - 171
*x^8 + 288*x^7 + 486*x^6 + 324*x^5 + 81*x^4
```

Fricas [B] time = 1.12317, size = 263, normalized size = 7.97

$$\frac{49}{10}x^{20}+49x^{19}+\frac{441}{2}x^{18}+588x^{17}+1029x^{16}+\frac{6174}{5}x^{15}+993x^{14}+336x^{13}-\frac{1071}{2}x^{12}-1211x^{11}-\frac{12551}{10}x^{10}-756x^{9}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(1+x)^3*(1+2*x)*(-18+7*x^3*(1+x)^3)^2,x, algorithm="fricas")
```

```
[Out] 49/10*x^20 + 49*x^19 + 441/2*x^18 + 588*x^17 + 1029*x^16 + 6174/5*x^15 + 99
3*x^14 + 336*x^13 - 1071/2*x^12 - 1211*x^11 - 12551/10*x^10 - 756*x^9 - 171
*x^8 + 288*x^7 + 486*x^6 + 324*x^5 + 81*x^4
```

Sympy [B] time = 0.083052, size = 94, normalized size = 2.85

$$\frac{49x^{20}}{10} + 49x^{19} + \frac{441x^{18}}{2} + 588x^{17} + 1029x^{16} + \frac{6174x^{15}}{5} + 993x^{14} + 336x^{13} - \frac{1071x^{12}}{2} - 1211x^{11} - \frac{12551x^{10}}{10} - 756x^9 -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(1+x)**3*(1+2*x)*(-18+7*x**3*(1+x)**3)**2,x)
```

```
[Out] 49*x**20/10 + 49*x**19 + 441*x**18/2 + 588*x**17 + 1029*x**16 + 6174*x**15/
5 + 993*x**14 + 336*x**13 - 1071*x**12/2 - 1211*x**11 - 12551*x**10/10 - 75
6*x**9 - 171*x**8 + 288*x**7 + 486*x**6 + 324*x**5 + 81*x**4
```

Giac [B] time = 1.26181, size = 116, normalized size = 3.52

$$\frac{49}{10}x^{20} + 49x^{19} + \frac{441}{2}x^{18} + 588x^{17} + 1029x^{16} + \frac{6174}{5}x^{15} + 993x^{14} + 336x^{13} - \frac{1071}{2}x^{12} - 1211x^{11} - \frac{12551}{10}x^{10} - 75$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(1+x)^3*(1+2*x)*(-18+7*x^3*(1+x)^3)^2,x, algorithm="giac")
```

```
[Out] 49/10*x^20 + 49*x^19 + 441/2*x^18 + 588*x^17 + 1029*x^16 + 6174/5*x^15 + 99
3*x^14 + 336*x^13 - 1071/2*x^12 - 1211*x^11 - 12551/10*x^10 - 756*x^9 - 171
*x^8 + 288*x^7 + 486*x^6 + 324*x^5 + 81*x^4
```

3.223 $\int \frac{2-x^2}{\left(1-6x+x^3\right)^5} \, dx$

Optimal. Leaf size=14

$$\frac{1}{12\left(x^3-6x+1\right)^4}$$

```
[Out] 1/(12*(1 - 6*x + x^3)^4)
```

Rubi [A] time = 0.0075984, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1588}

$$\frac{1}{12\left(x^3-6x+1\right)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(2 - x^2)/(1 - 6*x + x^3)^5,x]
```

```
[Out] 1/(12*(1 - 6*x + x^3)^4)
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{2-x^2}{\left(1-6x+x^3\right)^5} \, dx = \frac{1}{12\left(1-6x+x^3\right)^4}$$

Mathematica [A] time = 0.0052907, size = 14, normalized size = 1.

$$\frac{1}{12\left(x^3-6x+1\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - x^2)/(1 - 6*x + x^3)^5,x]
```

```
[Out] 1/(12*(1 - 6*x + x^3)^4)
```

Maple [A] time = 0.001, size = 13, normalized size = 0.9

$$\frac{1}{12\left(x^3-6\,x+1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+2)/(x^3-6*x+1)^5,x)
```

```
[Out] 1/12/(x^3-6*x+1)^4
```

Maxima [A] time = 1.02059, size = 16, normalized size = 1.14

$$\frac{1}{12\left(x^3-6\,x+1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2)/(x^3-6*x+1)^5,x, algorithm="maxima")
```

```
[Out] 1/12/(x^3 - 6*x + 1)^4
```

Fricas [B] time = 1.30672, size = 151, normalized size = 10.79

$$\frac{1}{12\left(x^{12}-24\,x^{10}+4\,x^9+216\,x^8-72\,x^7-858\,x^6+432\,x^5+1224\,x^4-860\,x^3+216\,x^2-24\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2)/(x^3-6*x+1)^5,x, algorithm="fricas")
```

```
[Out] 1/12/(x^12 - 24*x^10 + 4*x^9 + 216*x^8 - 72*x^7 - 858*x^6 + 432*x^5 + 1224*
x^4 - 860*x^3 + 216*x^2 - 24*x + 1)
```

Sympy [B] time = 0.208288, size = 56, normalized size = 4.

$$\frac{1}{12x^{12} - 288x^{10} + 48x^9 + 2592x^8 - 864x^7 - 10296x^6 + 5184x^5 + 14688x^4 - 10320x^3 + 2592x^2 - 288x + 12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+2)/(x**3-6*x+1)**5,x)
```

```
[Out] 1/(12*x**12 - 288*x**10 + 48*x**9 + 2592*x**8 - 864*x**7 - 10296*x**6 + 518
4*x**5 + 14688*x**4 - 10320*x**3 + 2592*x**2 - 288*x + 12)
```

Giac [A] time = 1.23666, size = 16, normalized size = 1.14

$$\frac{1}{12\left(x^3 - 6x + 1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2)/(x^3-6*x+1)^5,x, algorithm="giac")
```

```
[Out] 1/12/(x^3 - 6*x + 1)^4
```

3.224 $\int \frac{2x+x^2}{4+3x^2+x^3}\,dx$

Optimal. Leaf size=15

$$\frac{1}{3}\log\left(x^3+3x^2+4\right)$$

```
[Out] Log[4 + 3*x^2 + x^3]/3
```

Rubi [A] time = 0.0092377, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {1587}

$$\frac{1}{3}\log\left(x^3+3x^2+4\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2*x + x^2)/(4 + 3*x^2 + x^3),x]
```

```
[Out] Log[4 + 3*x^2 + x^3]/3
```

Rule 1587

```
Int[(Pp_)/(Qq_), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Si
mp[(Coeff[Pp, x, p]*Log[RemoveContent[Qq, x]])/(q*Coeff[Qq, x, q]), x] /; E
qQ[p, q - 1] && EqQ[Pp, Simplify[(Coeff[Pp, x, p]*D[Qq, x])/(q*Coeff[Qq, x,
 q])]]] /; PolyQ[Pp, x] && PolyQ[Qq, x]
```

Rubi steps

$$\int \frac{2x+x^2}{4+3x^2+x^3}\,dx=\frac{1}{3}\log\left(4+3x^2+x^3\right)$$

Mathematica [A] time = 0.0045035, size = 15, normalized size = 1.

$$\frac{1}{3}\log\left(x^3+3x^2+4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*x + x^2)/(4 + 3*x^2 + x^3),x]
```

```
[Out] Log[4 + 3*x^2 + x^3]/3
```

Maple [A] time = 0.003, size = 14, normalized size = 0.9

$$\frac{\ln\left(x^3+3x^2+4\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+2*x)/(x^3+3*x^2+4),x)
```

```
[Out] 1/3*ln(x^3+3*x^2+4)
```

Maxima [A] time = 1.01126, size = 18, normalized size = 1.2

$$\frac{1}{3}\log\left(x^3+3x^2+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x)/(x^3+3*x^2+4),x, algorithm="maxima")
```

```
[Out] 1/3*log(x^3 + 3*x^2 + 4)
```

Fricas [A] time = 1.25613, size = 35, normalized size = 2.33

$$\frac{1}{3}\log\left(x^3+3x^2+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x)/(x^3+3*x^2+4),x, algorithm="fricas")
```

```
[Out] 1/3*log(x^3 + 3*x^2 + 4)
```

Sympy [A] time = 0.084097, size = 12, normalized size = 0.8

$$\frac{\log\left(x^3+3x^2+4\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+2*x)/(x**3+3*x**2+4),x)
```

```
[Out] log(x**3 + 3*x**2 + 4)/3
```

Giac [A] time = 1.18419, size = 19, normalized size = 1.27

$$\frac{1}{3}\log\left(\left|x^3+3x^2+4\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x)/(x^3+3*x^2+4),x, algorithm="giac")
```

```
[Out] 1/3*log(abs(x^3 + 3*x^2 + 4))
```

3.225 $\int \frac{1+x+x^3}{4x+2x^2+x^4} \, dx$

Optimal. Leaf size=17

$$\frac{1}{4}\log\left(x^4+2x^2+4x\right)$$

```
[Out] Log[4*x + 2*x^2 + x^4]/4
```

Rubi [A] time = 0.0095623, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {1587}

$$\frac{1}{4}\log\left(x^4+2x^2+4x\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + x^3)/(4*x + 2*x^2 + x^4),x]
```

```
[Out] Log[4*x + 2*x^2 + x^4]/4
```

Rule 1587

```
Int[(Pp_)/(Qq_), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Si
mp[(Coeff[Pp, x, p]*Log[RemoveContent[Qq, x]])/(q*Coeff[Qq, x, q]), x] /; E
qQ[p, q - 1] && EqQ[Pp, Simplify[(Coeff[Pp, x, p]*D[Qq, x])/(q*Coeff[Qq, x,
 q])]]] /; PolyQ[Pp, x] && PolyQ[Qq, x]
```

Rubi steps

$$\int \frac{1+x+x^3}{4x+2x^2+x^4} \, dx = \frac{1}{4}\log\left(4x+2x^2+x^4\right)$$

Mathematica [A] time = 0.0056426, size = 20, normalized size = 1.18

$$\frac{1}{4}\log\left(x^3+2x+4\right)+\frac{\log(x)}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + x^3)/(4*x + 2*x^2 + x^4),x]
```

```
[Out] Log[x]/4 + Log[4 + 2*x + x^3]/4
```

Maple [A] time = 0., size = 14, normalized size = 0.8

$$\frac{\ln\left(x\left(x^3+2x+4\right)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x+1)/(x^4+2*x^2+4*x),x)
```

```
[Out] 1/4*ln(x*(x^3+2*x+4))
```

Maxima [A] time = 1.07102, size = 20, normalized size = 1.18

$$\frac{1}{4}\log\left(x^4+2x^2+4x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x+1)/(x^4+2*x^2+4*x),x, algorithm="maxima")
```

```
[Out] 1/4*log(x^4 + 2*x^2 + 4*x)
```

Fricas [A] time = 1.18996, size = 38, normalized size = 2.24

$$\frac{1}{4}\log\left(x^4+2x^2+4x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x+1)/(x^4+2*x^2+4*x),x, algorithm="fricas")
```

```
[Out] 1/4*log(x^4 + 2*x^2 + 4*x)
```

Sympy [A] time = 0.095233, size = 14, normalized size = 0.82

$$\frac{\log\left(x^4 + 2x^2 + 4x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x+1)/(x**4+2*x**2+4*x),x)
```

```
[Out] log(x**4 + 2*x**2 + 4*x)/4
```

Giac [A] time = 1.12469, size = 24, normalized size = 1.41

$$\frac{1}{4}\log\left(\left|x^3 + 2x + 4\right|\right) + \frac{1}{4}\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x+1)/(x^4+2*x^2+4*x),x, algorithm="giac")
```

```
[Out] 1/4*log(abs(x^3 + 2*x + 4)) + 1/4*log(abs(x))
```

3.226 $\int \frac{bc-ad-2aex-bex^2-3afx^2-2bfx^3}{(c+dx+ex^2+fx^3)^2} dx$

Optimal. Leaf size=40

$$\frac{a}{c+dx+ex^2+fx^3}+\frac{bx}{c+dx+ex^2+fx^3}$$

```
[Out] a/(c + d*x + e*x^2 + f*x^3) + (b*x)/(c + d*x + e*x^2 + f*x^3)
```

Rubi [A] time = 0.0968871, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 52, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.058, Rules used = {6, 2102, 1588}

$$\frac{a}{c+dx+ex^2+fx^3}+\frac{bx}{c+dx+ex^2+fx^3}$$

Antiderivative was successfully verified.

```
[In] Int[(b*c - a*d - 2*a*e*x - b*e*x^2 - 3*a*f*x^2 - 2*b*f*x^3)/(c + d*x + e*x^
2 + f*x^3)^2,x]
```

```
[Out] a/(c + d*x + e*x^2 + f*x^3) + (b*x)/(c + d*x + e*x^2 + f*x^3)
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 2102

```
Int[(Pm_)*(Qn_)^(p_.), x_Symbol] :> With[{m = Expon[Pm, x], n = Expon[Qn, x
]}, Simp[(Coeff[Pm, x, m]*x^(m - n + 1)*Qn^(p + 1))/((m + n*p + 1)*Coeff[Qn
, x, n]), x] + Dist[1/((m + n*p + 1)*Coeff[Qn, x, n]), Int[ExpandToSum[(m +
 n*p + 1)*Coeff[Qn, x, n]*Pm - Coeff[Pm, x, m]*x^(m - n)*((m - n + 1)*Qn +
(p + 1)*x*D[Qn, x]), x]*Qn^p, x], x] /; LtQ[1, n, m + 1] && m + n*p + 1 < 0
] /; FreeQ[p, x] && PolyQ[Pm, x] && PolyQ[Qn, x] && LtQ[p, -1]
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
```

```
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{bc-ad-2aex-bex^2-3afx^2-2bfx^3}{\left(c+dx+ex^2+fx^3\right)^2}\,dx=\int \frac{bc-ad-2aex+(-be-3af)x^2-2bfx^3}{\left(c+dx+ex^2+fx^3\right)^2}\,dx$$
$$=\frac{bx}{c+dx+ex^2+fx^3}-\frac{\int \frac{2adf+4aefx+6af^2x^2}{\left(c+dx+ex^2+fx^3\right)^2}\,dx}{2f}$$
$$=\frac{a}{c+dx+ex^2+fx^3}+\frac{bx}{c+dx+ex^2+fx^3}$$

Mathematica [A] time = 0.0619845, size = 23, normalized size = 0.57

$$\frac{a+bx}{c+dx+ex^2+fx^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*c - a*d - 2*a*e*x - b*e*x^2 - 3*a*f*x^2 - 2*b*f*x^3)/(c + d*x
+ e*x^2 + f*x^3)^2,x]
```

```
[Out] (a + b*x)/(c + d*x + e*x^2 + f*x^3)
```

Maple [A] time = 0.009, size = 28, normalized size = 0.7

$$-\frac{-bx-a}{fx^3+ex^2+dx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*b*f*x^3-3*a*f*x^2-b*e*x^2-2*a*e*x-a*d+b*c)/(f*x^3+e*x^2+d*x+c)^2,x)
```

```
[Out] -(-b*x-a)/(f*x^3+e*x^2+d*x+c)
```

Maxima [A] time = 1.05355, size = 31, normalized size = 0.78

$$\frac{bx + a}{fx^3 + ex^2 + dx + c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*b*f*x^3-3*a*f*x^2-b*e*x^2-2*a*e*x-a*d+b*c)/(f*x^3+e*x^2+d*x+c
)^2,x, algorithm="maxima")
```

```
[Out] (b*x + a)/(f*x^3 + e*x^2 + d*x + c)
```

Fricas [A] time = 1.24705, size = 50, normalized size = 1.25

$$\frac{bx + a}{fx^3 + ex^2 + dx + c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*b*f*x^3-3*a*f*x^2-b*e*x^2-2*a*e*x-a*d+b*c)/(f*x^3+e*x^2+d*x+c
)^2,x, algorithm="fricas")
```

```
[Out] (b*x + a)/(f*x^3 + e*x^2 + d*x + c)
```

Sympy [A] time = 30.0342, size = 19, normalized size = 0.48

$$\frac{a + bx}{c + dx + ex^2 + fx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*b*f*x**3-3*a*f*x**2-b*e*x**2-2*a*e*x-a*d+b*c)/(f*x**3+e*x**2+
d*x+c)**2,x)
```

```
[Out] (a + b*x)/(c + d*x + e*x**2 + f*x**3)
```

Giac [A] time = 1.89861, size = 32, normalized size = 0.8

$$\frac{bx + a}{fx^3 + x^2e + dx + c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*b*f*x^3-3*a*f*x^2-b*e*x^2-2*a*e*x-a*d+b*c)/(f*x^3+e*x^2+d*x+c
)^2,x, algorithm="giac")
```

```
[Out] (b*x + a)/(f*x^3 + x^2*e + d*x + c)
```

3.227 $\int \frac{A+Bx+Cx^2+Dx^3}{a+bx+cx^2+bx^3+ax^4}\,dx$

Optimal. Leaf size=605

$$\frac{\tan^{-1}\left(\frac{-\sqrt{8a^2-4ac+b^2}+4ax+b}{\sqrt{2}\sqrt{-b\left(b-\sqrt{8a^2-4ac+b^2}\right)+4a^2+2ac}}\right)\left(-a\left(A\left(b-\sqrt{8a^2-4ac+b^2}\right)-C\sqrt{8a^2-4ac+b^2}+bC+2cD\right)+bD\left(b-\sqrt{8a^2-\cdots\right)\right)}{\sqrt{2}a\sqrt{8a^2-4ac+b^2}\sqrt{-b\left(b-\sqrt{8a^2-4ac+b^2}\right)+4a^2+2ac}}$$

```
[Out] ((4*a^2*B + b*(b - Sqrt[8*a^2 + b^2 - 4*a*c])*D - a*(A*(b - Sqrt[8*a^2 + b^
2 - 4*a*c]) + b*C - Sqrt[8*a^2 + b^2 - 4*a*c]*C + 2*c*D))*ArcTan[(b - Sqrt[
8*a^2 + b^2 - 4*a*c] + 4*a*x)/(Sqrt[2]*Sqrt[4*a^2 + 2*a*c - b*(b - Sqrt[8*a
^2 + b^2 - 4*a*c])])])/(Sqrt[2]*a*Sqrt[8*a^2 + b^2 - 4*a*c]*Sqrt[4*a^2 + 2*
a*c - b*(b - Sqrt[8*a^2 + b^2 - 4*a*c])]) - ((4*a^2*B + b*(b + Sqrt[8*a^2 +
 b^2 - 4*a*c])*D - a*(A*(b + Sqrt[8*a^2 + b^2 - 4*a*c]) + b*C + Sqrt[8*a^2
+ b^2 - 4*a*c]*C + 2*c*D))*ArcTan[(b + Sqrt[8*a^2 + b^2 - 4*a*c] + 4*a*x)/(
Sqrt[2]*Sqrt[4*a^2 + 2*a*c - b*(b + Sqrt[8*a^2 + b^2 - 4*a*c])])])/(Sqrt[2]
*a*Sqrt[8*a^2 + b^2 - 4*a*c]*Sqrt[4*a^2 + 2*a*c - b*(b + Sqrt[8*a^2 + b^2 -
 4*a*c])]) - ((2*a*(A - C) + (b - Sqrt[8*a^2 + b^2 - 4*a*c])*D)*Log[2*a + (
b - Sqrt[8*a^2 + b^2 - 4*a*c])*x + 2*a*x^2])/(4*a*Sqrt[8*a^2 + b^2 - 4*a*c]
) + ((2*a*(A - C) + (b + Sqrt[8*a^2 + b^2 - 4*a*c])*D)*Log[2*a + (b + Sqrt[
8*a^2 + b^2 - 4*a*c])*x + 2*a*x^2])/(4*a*Sqrt[8*a^2 + b^2 - 4*a*c])
```

Rubi [A] time = 4.53531, antiderivative size = 605, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {2086, 634, 618, 204, 628}

$$\frac{\tan^{-1}\left(\frac{-\sqrt{8a^2-4ac+b^2}+4ax+b}{\sqrt{2}\sqrt{-b\left(b-\sqrt{8a^2-4ac+b^2}\right)+4a^2+2ac}}\right)\left(-a\left(A\left(b-\sqrt{8a^2-4ac+b^2}\right)-C\sqrt{8a^2-4ac+b^2}+bC+2cD\right)+bD\left(b-\sqrt{8a^2-\cdots\right)\right)}{\sqrt{2}a\sqrt{8a^2-4ac+b^2}\sqrt{-b\left(b-\sqrt{8a^2-4ac+b^2}\right)+4a^2+2ac}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2 + D*x^3)/(a + b*x + c*x^2 + b*x^3 + a*x^4),x]
```

```
[Out] ((4*a^2*B + b*(b - Sqrt[8*a^2 + b^2 - 4*a*c])*D - a*(A*(b - Sqrt[8*a^2 + b^
2 - 4*a*c]) + b*C - Sqrt[8*a^2 + b^2 - 4*a*c]*C + 2*c*D))*ArcTan[(b - Sqrt[
8*a^2 + b^2 - 4*a*c] + 4*a*x)/(Sqrt[2]*Sqrt[4*a^2 + 2*a*c - b*(b - Sqrt[8*a
^2 + b^2 - 4*a*c])])])/(Sqrt[2]*a*Sqrt[8*a^2 + b^2 - 4*a*c]*Sqrt[4*a^2 + 2*
```

```
a*c - b*(b - Sqrt[8*a^2 + b^2 - 4*a*c])]) - ((4*a^2*B + b*(b + Sqrt[8*a^2 +
 b^2 - 4*a*c])*D - a*(A*(b + Sqrt[8*a^2 + b^2 - 4*a*c]) + b*C + Sqrt[8*a^2
+ b^2 - 4*a*c]*C + 2*c*D))*ArcTan[(b + Sqrt[8*a^2 + b^2 - 4*a*c] + 4*a*x)/(
Sqrt[2]*Sqrt[4*a^2 + 2*a*c - b*(b + Sqrt[8*a^2 + b^2 - 4*a*c])])])/(Sqrt[2]
*a*Sqrt[8*a^2 + b^2 - 4*a*c]*Sqrt[4*a^2 + 2*a*c - b*(b + Sqrt[8*a^2 + b^2 -
 4*a*c])]) - ((2*a*(A - C) + (b - Sqrt[8*a^2 + b^2 - 4*a*c])*D)*Log[2*a + (
b - Sqrt[8*a^2 + b^2 - 4*a*c])*x + 2*a*x^2])/(4*a*Sqrt[8*a^2 + b^2 - 4*a*c]
) + ((2*a*(A - C) + (b + Sqrt[8*a^2 + b^2 - 4*a*c])*D)*Log[2*a + (b + Sqrt[
8*a^2 + b^2 - 4*a*c])*x + 2*a*x^2])/(4*a*Sqrt[8*a^2 + b^2 - 4*a*c])
```

Rule 2086

```
Int[(P3_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (e_.)*(x_)^4),
 x_Symbol] :> With[{q = Sqrt[8*a^2 + b^2 - 4*a*c], A = Coeff[P3, x, 0], B =
 Coeff[P3, x, 1], C = Coeff[P3, x, 2], D = Coeff[P3, x, 3]}, Dist[1/q, Int[
(b*A - 2*a*B + 2*a*D + A*q + (2*a*A - 2*a*C + b*D + D*q)*x)/(2*a + (b + q)*
x + 2*a*x^2), x], x] - Dist[1/q, Int[(b*A - 2*a*B + 2*a*D - A*q + (2*a*A -
2*a*C + b*D - D*q)*x)/(2*a + (b - q)*x + 2*a*x^2), x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[P3, x, 3] && EqQ[a, e] && EqQ[b, d]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2+Dx^3}{a+bx+cx^2+bx^3+ax^4}\,dx = -\frac{\int \frac{Ab-2aB-A\sqrt{8a^2+b^2-4ac}+2aD+\left(2aA-2aC+bD-\sqrt{8a^2+b^2-4ac}D\right)x}{2a+\left(b-\sqrt{8a^2+b^2-4ac}\right)x+2ax^2}\,dx}{\sqrt{8a^2+b^2-4ac}} + \frac{\int \frac{Ab-2aB+A\sqrt{8a^2+b^2-4ac}+2}{2a+\left(b+\sqrt{\right.}}}{\sqrt{8}}$$

$$= -\frac{\left(2a(A-C)+\left(b-\sqrt{8a^2+b^2-4ac}\right)D\right)\int \frac{b-\sqrt{8a^2+b^2-4ac}+4ax}{2a+\left(b-\sqrt{8a^2+b^2-4ac}\right)x+2ax^2}\,dx}{4a\sqrt{8a^2+b^2-4ac}} + \frac{\left(2a(A-C)\right.}{}$$

$$= -\frac{\left(2a(A-C)+\left(b-\sqrt{8a^2+b^2-4ac}\right)D\right)\log\left(2a+\left(b-\sqrt{8a^2+b^2-4ac}\right)x+2ax^2\right)}{4a\sqrt{8a^2+b^2-4ac}}$$

$$= \frac{\left(4a^2B+b\left(b-\sqrt{8a^2+b^2-4ac}\right)D-a\left(A\left(b-\sqrt{8a^2+b^2-4ac}\right)+bC-\sqrt{8a^2+b^2-\right.\right.}{\sqrt{2}a\sqrt{8a^2+b^2-4ac}\sqrt{4a^2+2ac-b\left(b-\sqrt{8a^2}\right.}}$$

Mathematica [C] time = 0.0750757, size = 98, normalized size = 0.16

$$\text{RootSum}\left[\#1^4a+\#1^3b+\#1^2c+\#1b+a\&,\frac{\#1^2C\log(x-\#1)+\#1^3D\log(x-\#1)+A\log(x-\#1)+\#1B\log(x-\#1)}{4\#1^3a+3\#1^2b+2\#1c+b}\&\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2 + D*x^3)/(a + b*x + c*x^2 + b*x^3 + a*x^4),x]
```

```
[Out] RootSum[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , (A*Log[x - #1] + B*Log[x -
#1]*#1 + C*Log[x - #1]*#1^2 + D*Log[x - #1]*#1^3)/(b + 2*c*#1 + 3*b*#1^2 +
4*a*#1^3) & ]
```

Maple [B] time = 0.047, size = 2105, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((D*x^3+C*x^2+B*x+A)/(a*x^4+b*x^3+c*x^2+b*x+a),x)
```

```
[Out] 1/2/(8*a^2-4*a*c+b^2)^(1/2)*ln(2*a*x^2+(8*a^2-4*a*c+b^2)^(1/2)*x+b*x+2*a)*A
-1/2/(8*a^2-4*a*c+b^2)^(1/2)*ln(2*a*x^2+(8*a^2-4*a*c+b^2)^(1/2)*x+b*x+2*a)*
C+1/4/a*ln(2*a*x^2+(8*a^2-4*a*c+b^2)^(1/2)*x+b*x+2*a)*D+1/4/a/(8*a^2-4*a*c+
b^2)^(1/2)*ln(2*a*x^2+(8*a^2-4*a*c+b^2)^(1/2)*x+b*x+2*a)*D*b+1/(8*a^2+4*a*c
-2*b^2-2*b*(8*a^2-4*a*c+b^2)^(1/2))^(1/2)*arctan((b+4*a*x+(8*a^2-4*a*c+b^2)
^(1/2))/(8*a^2+4*a*c-2*b^2-2*b*(8*a^2-4*a*c+b^2)^(1/2))^(1/2))*A+1/(8*a^2-4
*a*c+b^2)^(1/2)/(8*a^2+4*a*c-2*b^2-2*b*(8*a^2-4*a*c+b^2)^(1/2))^(1/2)*arcta
n((b+4*a*x+(8*a^2-4*a*c+b^2)^(1/2))/(8*a^2+4*a*c-2*b^2-2*b*(8*a^2-4*a*c+b^2
)^(1/2))^(1/2))*b*A+1/(8*a^2+4*a*c-2*b^2-2*b*(8*a^2-4*a*c+b^2)^(1/2))^(1/2)
*arctan((b+4*a*x+(8*a^2-4*a*c+b^2)^(1/2))/(8*a^2+4*a*c-2*b^2-2*b*(8*a^2-4*a
*c+b^2)^(1/2))^(1/2))*C+1/(8*a^2-4*a*c+b^2)^(1/2)/(8*a^2+4*a*c-2*b^2-2*b*(8
*a^2-4*a*c+b^2)^(1/2))^(1/2)*arctan((b+4*a*x+(8*a^2-4*a*c+b^2)^(1/2))/(8*a^
2+4*a*c-2*b^2-2*b*(8*a^2-4*a*c+b^2)^(1/2))^(1/2))*b*C+2/(8*a^2-4*a*c+b^2)^(
1/2)/(8*a^2+4*a*c-2*b^2-2*b*(8*a^2-4*a*c+b^2)^(1/2))^(1/2)*arctan((b+4*a*x+
(8*a^2-4*a*c+b^2)^(1/2))/(8*a^2+4*a*c-2*b^2-2*b*(8*a^2-4*a*c+b^2)^(1/2))^(1
/2))*D*c-1/a/(8*a^2-4*a*c+b^2)^(1/2)/(8*a^2+4*a*c-2*b^2-2*b*(8*a^2-4*a*c+b^
2)^(1/2))^(1/2)*arctan((b+4*a*x+(8*a^2-4*a*c+b^2)^(1/2))/(8*a^2+4*a*c-2*b^2
-2*b*(8*a^2-4*a*c+b^2)^(1/2))^(1/2))*D*b^2-1/a/(8*a^2+4*a*c-2*b^2-2*b*(8*a^
2-4*a*c+b^2)^(1/2))^(1/2)*arctan((b+4*a*x+(8*a^2-4*a*c+b^2)^(1/2))/(8*a^2+4
*a*c-2*b^2-2*b*(8*a^2-4*a*c+b^2)^(1/2))^(1/2))*D*b-4*a/(8*a^2-4*a*c+b^2)^(1
/2)/(8*a^2+4*a*c-2*b^2-2*b*(8*a^2-4*a*c+b^2)^(1/2))^(1/2)*arctan((b+4*a*x+(
8*a^2-4*a*c+b^2)^(1/2))/(8*a^2+4*a*c-2*b^2-2*b*(8*a^2-4*a*c+b^2)^(1/2))^(1/
2))*B-1/2/(8*a^2-4*a*c+b^2)^(1/2)*ln(-2*a*x^2+(8*a^2-4*a*c+b^2)^(1/2)*x-b*x
-2*a)*A+1/2/(8*a^2-4*a*c+b^2)^(1/2)*ln(-2*a*x^2+(8*a^2-4*a*c+b^2)^(1/2)*x-b
*x-2*a)*C+1/4/a*ln(-2*a*x^2+(8*a^2-4*a*c+b^2)^(1/2)*x-b*x-2*a)*D-1/4/a/(8*a
^2-4*a*c+b^2)^(1/2)*ln(-2*a*x^2+(8*a^2-4*a*c+b^2)^(1/2)*x-b*x-2*a)*D*b-1/(8
*a^2+4*a*c-2*b^2+2*b*(8*a^2-4*a*c+b^2)^(1/2))^(1/2)*arctan((-4*a*x+(8*a^2-4
*a*c+b^2)^(1/2)-b)/(8*a^2+4*a*c-2*b^2+2*b*(8*a^2-4*a*c+b^2)^(1/2))^(1/2))*A
+1/(8*a^2-4*a*c+b^2)^(1/2)/(8*a^2+4*a*c-2*b^2+2*b*(8*a^2-4*a*c+b^2)^(1/2))^
(1/2)*arctan((-4*a*x+(8*a^2-4*a*c+b^2)^(1/2)-b)/(8*a^2+4*a*c-2*b^2+2*b*(8*a
^2-4*a*c+b^2)^(1/2))^(1/2))*b*A-1/(8*a^2+4*a*c-2*b^2+2*b*(8*a^2-4*a*c+b^2)^
(1/2))^(1/2)*arctan((-4*a*x+(8*a^2-4*a*c+b^2)^(1/2)-b)/(8*a^2+4*a*c-2*b^2+2
*b*(8*a^2-4*a*c+b^2)^(1/2))^(1/2))*C+1/(8*a^2-4*a*c+b^2)^(1/2)/(8*a^2+4*a*c
-2*b^2+2*b*(8*a^2-4*a*c+b^2)^(1/2))^(1/2)*arctan((-4*a*x+(8*a^2-4*a*c+b^2)^
(1/2)-b)/(8*a^2+4*a*c-2*b^2+2*b*(8*a^2-4*a*c+b^2)^(1/2))^(1/2))*b*C+2/(8*a^
2-4*a*c+b^2)^(1/2)/(8*a^2+4*a*c-2*b^2+2*b*(8*a^2-4*a*c+b^2)^(1/2))^(1/2)*ar
ctan((-4*a*x+(8*a^2-4*a*c+b^2)^(1/2)-b)/(8*a^2+4*a*c-2*b^2+2*b*(8*a^2-4*a*c
+b^2)^(1/2))^(1/2))*D*c-1/a/(8*a^2-4*a*c+b^2)^(1/2)/(8*a^2+4*a*c-2*b^2+2*b*
(8*a^2-4*a*c+b^2)^(1/2))^(1/2)*arctan((-4*a*x+(8*a^2-4*a*c+b^2)^(1/2)-b)/(8
*a^2+4*a*c-2*b^2+2*b*(8*a^2-4*a*c+b^2)^(1/2))^(1/2))*D*b^2+1/a/(8*a^2+4*a*c
-2*b^2+2*b*(8*a^2-4*a*c+b^2)^(1/2))^(1/2)*arctan((-4*a*x+(8*a^2-4*a*c+b^2)^
(1/2)-b)/(8*a^2+4*a*c-2*b^2+2*b*(8*a^2-4*a*c+b^2)^(1/2))^(1/2))*D*b-4*a/(8*
a^2-4*a*c+b^2)^(1/2)/(8*a^2+4*a*c-2*b^2+2*b*(8*a^2-4*a*c+b^2)^(1/2))^(1/2)*
arctan((-4*a*x+(8*a^2-4*a*c+b^2)^(1/2)-b)/(8*a^2+4*a*c-2*b^2+2*b*(8*a^2-4*a
*c+b^2)^(1/2))^(1/2))*B
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Dx^3 + Cx^2 + Bx + A}{ax^4 + bx^3 + cx^2 + bx + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/(a*x^4+b*x^3+c*x^2+b*x+a),x, algorithm="maxim
a")
```

```
[Out] integrate((D*x^3 + C*x^2 + B*x + A)/(a*x^4 + b*x^3 + c*x^2 + b*x + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/(a*x^4+b*x^3+c*x^2+b*x+a),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x**3+C*x**2+B*x+A)/(a*x**4+b*x**3+c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Dx^3 + Cx^2 + Bx + A}{ax^4 + bx^3 + cx^2 + bx + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/(a*x^4+b*x^3+c*x^2+b*x+a),x, algorithm="giac"
)
```

```
[Out] integrate((D*x^3 + C*x^2 + B*x + A)/(a*x^4 + b*x^3 + c*x^2 + b*x + a), x)
```

3.228 $\int \frac{2+x-4x^2+2x^3}{1-x+x^2-x^3+x^4} \, dx$

Optimal. Leaf size=63

$$-\frac{2\log\left(2x^2-\left(1-\sqrt{5}\right)x+2\right)}{1-\sqrt{5}}-\frac{2\log\left(2x^2-\left(1+\sqrt{5}\right)x+2\right)}{1+\sqrt{5}}$$

```
[Out] (-2*Log[2 - (1 - Sqrt[5])*x + 2*x^2])/(1 - Sqrt[5]) - (2*Log[2 - (1 + Sqrt[
5])*x + 2*x^2])/(1 + Sqrt[5])
```

Rubi [A] time = 0.0655793, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2086, 628}

$$-\frac{2\log\left(2x^2-\left(1-\sqrt{5}\right)x+2\right)}{1-\sqrt{5}}-\frac{2\log\left(2x^2-\left(1+\sqrt{5}\right)x+2\right)}{1+\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x - 4*x^2 + 2*x^3)/(1 - x + x^2 - x^3 + x^4),x]
```

```
[Out] (-2*Log[2 - (1 - Sqrt[5])*x + 2*x^2])/(1 - Sqrt[5]) - (2*Log[2 - (1 + Sqrt[
5])*x + 2*x^2])/(1 + Sqrt[5])
```

Rule 2086

```
Int[(P3_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (e_.)*(x_)^4),
 x_Symbol] :> With[{q = Sqrt[8*a^2 + b^2 - 4*a*c], A = Coeff[P3, x, 0], B =
 Coeff[P3, x, 1], C = Coeff[P3, x, 2], D = Coeff[P3, x, 3]}, Dist[1/q, Int[
(b*A - 2*a*B + 2*a*D + A*q + (2*a*A - 2*a*C + b*D + D*q)*x)/(2*a + (b + q)*
x + 2*a*x^2), x], x] - Dist[1/q, Int[(b*A - 2*a*B + 2*a*D - A*q + (2*a*A -
2*a*C + b*D - D*q)*x)/(2*a + (b - q)*x + 2*a*x^2), x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[P3, x, 3] && EqQ[a, e] && EqQ[b, d]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{2+x-4x^2+2x^3}{1-x+x^2-x^3+x^4}\,dx = -\frac{\int \frac{-2\sqrt{5}+\left(10-2\sqrt{5}\right)x}{2+\left(-1-\sqrt{5}\right)x+2x^2}\,dx}{\sqrt{5}} + \frac{\int \frac{2\sqrt{5}+\left(10+2\sqrt{5}\right)x}{2+\left(-1+\sqrt{5}\right)x+2x^2}\,dx}{\sqrt{5}}$$
$$= -\frac{2\log\left(2-\left(1-\sqrt{5}\right)x+2x^2\right)}{1-\sqrt{5}} - \frac{2\log\left(2-\left(1+\sqrt{5}\right)x+2x^2\right)}{1+\sqrt{5}}$$

Mathematica [A] time = 0.0273721, size = 55, normalized size = 0.87

$$\frac{1}{2}\left(\left(1+\sqrt{5}\right)\log\left(2x^2+\left(\sqrt{5}-1\right)x+2\right)-\left(\sqrt{5}-1\right)\log\left(-2x^2+\sqrt{5}x+x-2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x - 4*x^2 + 2*x^3)/(1 - x + x^2 - x^3 + x^4),x]
```

```
[Out] (-((-1 + Sqrt[5])*Log[-2 + x + Sqrt[5]*x - 2*x^2]) + (1 + Sqrt[5])*Log[2 +
(-1 + Sqrt[5])*x + 2*x^2])/2
```

Maple [A] time = 0.024, size = 82, normalized size = 1.3

$$\frac{\ln\left(x\sqrt{5}+2\,x^2-x+2\right)\sqrt{5}}{2}+\frac{\ln\left(x\sqrt{5}+2\,x^2-x+2\right)}{2}+\frac{\ln\left(-x\sqrt{5}+2\,x^2-x+2\right)}{2}-\frac{\ln\left(-x\sqrt{5}+2\,x^2-x+2\right)\sqrt{5}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3-4*x^2+x+2)/(x^4-x^3+x^2-x+1),x)
```

```
[Out] 1/2*ln(x*5^(1/2)+2*x^2-x+2)*5^(1/2)+1/2*ln(x*5^(1/2)+2*x^2-x+2)+1/2*ln(-x*5
^(1/2)+2*x^2-x+2)-1/2*ln(-x*5^(1/2)+2*x^2-x+2)*5^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2\,x^3-4\,x^2+x+2}{x^4-x^3+x^2-x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3-4*x^2+x+2)/(x^4-x^3+x^2-x+1),x, algorithm="maxima")
```

```
[Out] integrate((2*x^3 - 4*x^2 + x + 2)/(x^4 - x^3 + x^2 - x + 1), x)
```

Fricas [A] time = 1.4834, size = 193, normalized size = 3.06

$$\frac{1}{2}\sqrt{5}\log\left(\frac{2x^4-2x^3+7x^2+\sqrt{5}\left(2x^3-x^2+2x\right)-2x+2}{x^4-x^3+x^2-x+1}\right)+\frac{1}{2}\log\left(x^4-x^3+x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3-4*x^2+x+2)/(x^4-x^3+x^2-x+1),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(5)*log((2*x^4 - 2*x^3 + 7*x^2 + sqrt(5)*(2*x^3 - x^2 + 2*x) - 2*x
+ 2)/(x^4 - x^3 + x^2 - x + 1)) + 1/2*log(x^4 - x^3 + x^2 - x + 1)
```

Sympy [A] time = 0.113659, size = 58, normalized size = 0.92

$$\left(\frac{1}{2}+\frac{\sqrt{5}}{2}\right)\log\left(x^2+x\left(-\frac{1}{2}+\frac{\sqrt{5}}{2}\right)+1\right)+\left(\frac{1}{2}-\frac{\sqrt{5}}{2}\right)\log\left(x^2+x\left(-\frac{\sqrt{5}}{2}-\frac{1}{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**3-4*x**2+x+2)/(x**4-x**3+x**2-x+1),x)
```

```
[Out] (1/2 + sqrt(5)/2)*log(x**2 + x*(-1/2 + sqrt(5)/2) + 1) + (1/2 - sqrt(5)/2)*
log(x**2 + x*(-sqrt(5)/2 - 1/2) + 1)
```

Giac [A] time = 1.20082, size = 78, normalized size = 1.24

$$-\frac{1}{2}\sqrt{5}\log\left(x^2-\frac{1}{2}x\left(\sqrt{5}+1\right)+1\right)+\frac{1}{2}\sqrt{5}\log\left(x^2+\frac{1}{2}x\left(\sqrt{5}-1\right)+1\right)+\frac{1}{2}\log\left(x^4-x^3+x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3-4*x^2+x+2)/(x^4-x^3+x^2-x+1),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(5)*log(x^2 - 1/2*x*(sqrt(5) + 1) + 1) + 1/2*sqrt(5)*log(x^2 + 1/2
*x*(sqrt(5) - 1) + 1) + 1/2*log(x^4 - x^3 + x^2 - x + 1)
```

3.229 $\int \frac{3x+3x^2+x^3}{1+4x+6x^2+4x^3+x^4}\,dx$

Optimal. Leaf size=14

$$\frac{1}{3(x+1)^3}+\log(x+1)$$

```
[Out] 1/(3*(1 + x)^3) + Log[1 + x]
```

Rubi [A] time = 0.0502709, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1594, 1680, 14}

$$\frac{1}{3(x+1)^3}+\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(3*x + 3*x^2 + x^3)/(1 + 4*x + 6*x^2 + 4*x^3 + x^4),x]
```

```
[Out] 1/(3*(1 + x)^3) + Log[1 + x]
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1680

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -(d/(4*e)) + x)*(a + d^4/(256*e^3) - (
b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /;
 EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq,
x] && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
```

```
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{3x+3x^2+x^3}{1+4x+6x^2+4x^3+x^4}\,dx &= \int \frac{x\left(3+3x+x^2\right)}{1+4x+6x^2+4x^3+x^4}\,dx\\ &= \text{Subst}\left(\int \frac{-1+x^3}{x^4}\,dx,x,1+x\right)\\ &= \text{Subst}\left(\int \left(-\frac{1}{x^4}+\frac{1}{x}\right)dx,x,1+x\right)\\ &= \frac{1}{3(1+x)^3}+\log(1+x)\end{aligned}$$

Mathematica [A] time = 0.0067934, size = 14, normalized size = 1.

$$\frac{1}{3(x+1)^3}+\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3*x + 3*x^2 + x^3)/(1 + 4*x + 6*x^2 + 4*x^3 + x^4),x]
```

```
[Out] 1/(3*(1 + x)^3) + Log[1 + x]
```

Maple [A] time = 0.006, size = 13, normalized size = 0.9

$$\frac{1}{3\,(1+x)^3}+\ln(1+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+3*x^2+3*x)/(x^4+4*x^3+6*x^2+4*x+1),x)
```

```
[Out] 1/3/(1+x)^3+ln(1+x)
```

Maxima [A] time = 1.04856, size = 30, normalized size = 2.14

$$\frac{1}{3\left(x^3+3x^2+3x+1\right)}+\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+3*x^2+3*x)/(x^4+4*x^3+6*x^2+4*x+1),x, algorithm="maxima")
```

```
[Out] 1/3/(x^3 + 3*x^2 + 3*x + 1) + log(x + 1)
```

Fricas [B] time = 1.49361, size = 97, normalized size = 6.93

$$\frac{3\left(x^3+3x^2+3x+1\right)\log\left(x+1\right)+1}{3\left(x^3+3x^2+3x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+3*x^2+3*x)/(x^4+4*x^3+6*x^2+4*x+1),x, algorithm="fricas")
```

```
[Out] 1/3*(3*(x^3 + 3*x^2 + 3*x + 1)*log(x + 1) + 1)/(x^3 + 3*x^2 + 3*x + 1)
```

Sympy [A] time = 0.092009, size = 20, normalized size = 1.43

$$\log\left(x+1\right)+\frac{1}{3x^3+9x^2+9x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+3*x**2+3*x)/(x**4+4*x**3+6*x**2+4*x+1),x)
```

```
[Out] log(x + 1) + 1/(3*x**3 + 9*x**2 + 9*x + 3)
```

Giac [A] time = 1.16372, size = 18, normalized size = 1.29

$$\frac{1}{3\left(x+1\right)^3}+\log\left(\left|x+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+3*x^2+3*x)/(x^4+4*x^3+6*x^2+4*x+1),x, algorithm="giac")
```

```
[Out] 1/3/(x + 1)^3 + log(abs(x + 1))
```

3.230 $\int \frac{-1+3x-3x^2+x^3}{1+4x+6x^2+4x^3+x^4} \, dx$

Optimal. Leaf size=28

$$\frac{6}{x+1} - \frac{6}{(x+1)^2} + \frac{8}{3(x+1)^3} + \log(x+1)$$

```
[Out] 8/(3*(1 + x)^3) - 6/(1 + x)^2 + 6/(1 + x) + Log[1 + x]
```

Rubi [A] time = 0.0283567, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {1680, 43}

$$\frac{6}{x+1} - \frac{6}{(x+1)^2} + \frac{8}{3(x+1)^3} + \log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 3*x - 3*x^2 + x^3)/(1 + 4*x + 6*x^2 + 4*x^3 + x^4),x]
```

```
[Out] 8/(3*(1 + x)^3) - 6/(1 + x)^2 + 6/(1 + x) + Log[1 + x]
```

Rule 1680

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -(d/(4*e)) + x)*(a + d^4/(256*e^3) - (
b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /;
 EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq,
x] && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{-1+3x-3x^2+x^3}{1+4x+6x^2+4x^3+x^4}\,dx = \text{Subst}\left(\int \frac{(-2+x)^3}{x^4}\,dx, x, 1+x\right)$$
$$= \text{Subst}\left(\int \left(-\frac{8}{x^4}+\frac{12}{x^3}-\frac{6}{x^2}+\frac{1}{x}\right)dx, x, 1+x\right)$$
$$= \frac{8}{3(1+x)^3}-\frac{6}{(1+x)^2}+\frac{6}{1+x}+\log(1+x)$$

Mathematica [A] time = 0.0123057, size = 24, normalized size = 0.86

$$\frac{2\left(9x^2+9x+4\right)}{3(x+1)^3}+\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + 3*x - 3*x^2 + x^3)/(1 + 4*x + 6*x^2 + 4*x^3 + x^4),x]
```

```
[Out] (2*(4 + 9*x + 9*x^2))/(3*(1 + x)^3) + Log[1 + x]
```

Maple [A] time = 0.005, size = 27, normalized size = 1.

$$\frac{8}{3\,(1+x)^3}-6\,(1+x)^{-2}+6\,(1+x)^{-1}+\ln(1+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-3*x^2+3*x-1)/(x^4+4*x^3+6*x^2+4*x+1),x)
```

```
[Out] 8/3/(1+x)^3-6/(1+x)^2+6/(1+x)+ln(1+x)
```

Maxima [A] time = 1.00807, size = 43, normalized size = 1.54

$$\frac{2\left(9\,x^2+9\,x+4\right)}{3\left(x^3+3\,x^2+3\,x+1\right)}+\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-3*x^2+3*x-1)/(x^4+4*x^3+6*x^2+4*x+1),x, algorithm="maxima")
```

```
[Out] 2/3*(9*x^2 + 9*x + 4)/(x^3 + 3*x^2 + 3*x + 1) + log(x + 1)
```

Fricas [A] time = 1.49448, size = 119, normalized size = 4.25

$$\frac{18\,x^2 + 3\left(x^3 + 3\,x^2 + 3\,x + 1\right)\log\left(x + 1\right) + 18\,x + 8}{3\left(x^3 + 3\,x^2 + 3\,x + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-3*x^2+3*x-1)/(x^4+4*x^3+6*x^2+4*x+1),x, algorithm="fricas")
```

```
[Out] 1/3*(18*x^2 + 3*(x^3 + 3*x^2 + 3*x + 1)*log(x + 1) + 18*x + 8)/(x^3 + 3*x^2
 + 3*x + 1)
```

Sympy [A] time = 0.104068, size = 29, normalized size = 1.04

$$\frac{18x^2 + 18x + 8}{3x^3 + 9x^2 + 9x + 3} + \log\left(x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-3*x**2+3*x-1)/(x**4+4*x**3+6*x**2+4*x+1),x)
```

```
[Out] (18*x**2 + 18*x + 8)/(3*x**3 + 9*x**2 + 9*x + 3) + log(x + 1)
```

Giac [A] time = 1.28322, size = 31, normalized size = 1.11

$$\frac{2\left(9\,x^2 + 9\,x + 4\right)}{3\left(x + 1\right)^3} + \log\left(|x + 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-3*x^2+3*x-1)/(x^4+4*x^3+6*x^2+4*x+1),x, algorithm="giac")
```

```
[Out] 2/3*(9*x^2 + 9*x + 4)/(x + 1)^3 + log(abs(x + 1))
```

3.231 $\int \frac{9-40x-18x^2+174x^4+24x^5+26x^6-39x^8}{(3+2x^2+x^4)^3} dx$

Optimal. Leaf size=59

$$\frac{13x}{x^4+2x^2+3}-\frac{2(13x^2+18)x}{(x^4+2x^2+3)^2}+\frac{2(1-2x^2)}{(x^4+2x^2+3)^2}$$

```
[Out] (2*(1 - 2*x^2))/(3 + 2*x^2 + x^4)^2 - (2*x*(18 + 13*x^2))/(3 + 2*x^2 + x^4)
^2 + (13*x)/(3 + 2*x^2 + x^4)
```

Rubi [A] time = 0.0906975, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {1673, 1678, 1588, 1663, 1660, 8}

$$\frac{13x}{x^4+2x^2+3}-\frac{2(13x^2+18)x}{(x^4+2x^2+3)^2}+\frac{2(1-2x^2)}{(x^4+2x^2+3)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(9 - 40*x - 18*x^2 + 174*x^4 + 24*x^5 + 26*x^6 - 39*x^8)/(3 + 2*x^2 + x
^4)^3,x]
```

```
[Out] (2*(1 - 2*x^2))/(3 + 2*x^2 + x^4)^2 - (2*x*(18 + 13*x^2))/(3 + 2*x^2 + x^4)
^2 + (13*x)/(3 + 2*x^2 + x^4)
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
```

```
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{9-40x-18x^2+174x^4+24x^5+26x^6-39x^8}{\left(3+2x^2+x^4\right)^3}\,dx = \int \frac{x\left(-40+24x^4\right)}{\left(3+2x^2+x^4\right)^3}\,dx + \int \frac{9-18x^2+174x^4+26x^6-39x^8}{\left(3+2x^2+x^4\right)^3}\,dx$$
$$= -\frac{2x\left(18+13x^2\right)}{\left(3+2x^2+x^4\right)^2} + \frac{1}{96}\int \frac{3744-2496x^2-3744x^4}{\left(3+2x^2+x^4\right)^2}\,dx + \frac{1}{2}\,\text{Subst}$$
$$= \frac{2\left(1-2x^2\right)}{\left(3+2x^2+x^4\right)^2} - \frac{2x\left(18+13x^2\right)}{\left(3+2x^2+x^4\right)^2} + \frac{13x}{3+2x^2+x^4} + \frac{1}{32}\,\text{Subst}\Big($$
$$= \frac{2\left(1-2x^2\right)}{\left(3+2x^2+x^4\right)^2} - \frac{2x\left(18+13x^2\right)}{\left(3+2x^2+x^4\right)^2} + \frac{13x}{3+2x^2+x^4}$$

Mathematica [A] time = 0.0118335, size = 28, normalized size = 0.47

$$\frac{13x^5-4x^2+3x+2}{\left(x^4+2x^2+3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(9 - 40*x - 18*x^2 + 174*x^4 + 24*x^5 + 26*x^6 - 39*x^8)/(3 + 2*x
^2 + x^4)^3,x]
```

```
[Out] (2 + 3*x - 4*x^2 + 13*x^5)/(3 + 2*x^2 + x^4)^2
```

Maple [A] time = 0.006, size = 30, normalized size = 0.5

$$-\frac{-13\,x^5+4\,x^2-3\,x-2}{\left(x^4+2\,x^2+3\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-39*x^8+26*x^6+24*x^5+174*x^4-18*x^2-40*x+9)/(x^4+2*x^2+3)^3,x)
```

```
[Out] -(-13*x^5+4*x^2-3*x-2)/(x^4+2*x^2+3)^2
```

Maxima [A] time = 1.01136, size = 51, normalized size = 0.86

$$\frac{13\,x^5-4\,x^2+3\,x+2}{x^8+4\,x^6+10\,x^4+12\,x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-39*x^8+26*x^6+24*x^5+174*x^4-18*x^2-40*x+9)/(x^4+2*x^2+3)^3,x,
algorithm="maxima")
```

```
[Out] (13*x^5 - 4*x^2 + 3*x + 2)/(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)
```

Fricas [A] time = 1.20708, size = 86, normalized size = 1.46

$$\frac{13\,x^5-4\,x^2+3\,x+2}{x^8+4\,x^6+10\,x^4+12\,x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-39*x^8+26*x^6+24*x^5+174*x^4-18*x^2-40*x+9)/(x^4+2*x^2+3)^3,x,
algorithm="fricas")
```

```
[Out] (13*x^5 - 4*x^2 + 3*x + 2)/(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)
```

Sympy [A] time = 0.181473, size = 34, normalized size = 0.58

$$\frac{13x^5-4x^2+3x+2}{x^8+4x^6+10x^4+12x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-39*x**8+26*x**6+24*x**5+174*x**4-18*x**2-40*x+9)/(x**4+2*x**2+3
)**3,x)
```

```
[Out] (13*x**5 - 4*x**2 + 3*x + 2)/(x**8 + 4*x**6 + 10*x**4 + 12*x**2 + 9)
```

Giac [A] time = 1.19465, size = 38, normalized size = 0.64

$$\frac{13\,x^5-4\,x^2+3\,x+2}{\left(x^4+2\,x^2+3\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-39*x^8+26*x^6+24*x^5+174*x^4-18*x^2-40*x+9)/(x^4+2*x^2+3)^3,x,
algorithm="giac")
```

```
[Out] (13*x^5 - 4*x^2 + 3*x + 2)/(x^4 + 2*x^2 + 3)^2
```

3.232 $\int \frac{-1+4x^5}{\left(1+x+x^5\right)^2} dx$

Optimal. Leaf size=11

$$-\frac{x}{x^5+x+1}$$

```
[Out] -(x/(1 + x + x^5))
```

Rubi [A] time = 0.0068737, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1588}

$$-\frac{x}{x^5+x+1}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 4*x^5)/(1 + x + x^5)^2,x]
```

```
[Out] -(x/(1 + x + x^5))
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{-1+4x^5}{\left(1+x+x^5\right)^2} dx = -\frac{x}{1+x+x^5}$$

Mathematica [A] time = 0.0065788, size = 11, normalized size = 1.

$$-\frac{x}{x^5+x+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + 4*x^5)/(1 + x + x^5)^2,x]
```

```
[Out] -(x/(1 + x + x^5))
```

Maple [B] time = 0.009, size = 41, normalized size = 3.7

$$-\frac{-3\,x^{2}+5\,x-1}{7\,x^{3}-7\,x^{2}+7}+\frac{-3\,x-1}{7\,x^{2}+7\,x+7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^5-1)/(x^5+x+1)^2,x)
```

```
[Out] -1/7*(-3*x^2+5*x-1)/(x^3-x^2+1)+1/7*(-3*x-1)/(x^2+x+1)
```

Maxima [A] time = 1.02207, size = 15, normalized size = 1.36

$$-\frac{x}{x^{5}+x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^5-1)/(x^5+x+1)^2,x, algorithm="maxima")
```

```
[Out] -x/(x^5 + x + 1)
```

Fricas [A] time = 1.35098, size = 24, normalized size = 2.18

$$-\frac{x}{x^{5}+x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^5-1)/(x^5+x+1)^2,x, algorithm="fricas")
```

```
[Out] -x/(x^5 + x + 1)
```

Sympy [A] time = 0.119274, size = 8, normalized size = 0.73

$$-\frac{x}{x^5 + x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**5-1)/(x**5+x+1)**2,x)
```

```
[Out] -x/(x**5 + x + 1)
```

Giac [A] time = 1.24403, size = 15, normalized size = 1.36

$$-\frac{x}{x^5 + x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^5-1)/(x^5+x+1)^2,x, algorithm="giac")
```

```
[Out] -x/(x^5 + x + 1)
```

3.233 $\int \frac{1+x^2}{\left(1-7x^2+7x^4-x^6\right)^2}\,dx$

Optimal. Leaf size=91

$$\frac{x}{16\left(1-x^2\right)}+\frac{\left(29-5x^2\right)x}{32\left(x^4-6x^2+1\right)}+\frac{1}{4}\tanh^{-1}(x)+\frac{1}{64}\left(\left(3-2\sqrt{2}\right)\tanh^{-1}\left(\left(\sqrt{2}-1\right)x\right)-\left(3+2\sqrt{2}\right)\tanh^{-1}\left(\left(1+\sqrt{2}\right)x\right)\right)$$

```
[Out] x/(16*(1 - x^2)) + (x*(29 - 5*x^2))/(32*(1 - 6*x^2 + x^4)) + ArcTanh[x]/4 +
 ((3 - 2*Sqrt[2])*ArcTanh[(-1 + Sqrt[2])*x] - (3 + 2*Sqrt[2])*ArcTanh[(1 +
Sqrt[2])*x])/64
```

Rubi [B] time = 0.148645, antiderivative size = 205, normalized size of antiderivative = 2.25, number of steps used = 15, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {2073, 207, 638, 618, 206, 632, 31}

$$-\frac{12-5x}{64\left(-x^2+2x+1\right)}+\frac{5x+12}{64\left(-x^2-2x+1\right)}+\frac{1}{32(1-x)}-\frac{1}{32(x+1)}-\frac{3}{256}\left(2+3\sqrt{2}\right)\log\left(-x-\sqrt{2}+1\right)-\frac{3}{256}\left(2-3\sqrt{2}\right)\text{l}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/(1 - 7*x^2 + 7*x^4 - x^6)^2,x]
```

```
[Out] 1/(32*(1 - x)) - 1/(32*(1 + x)) + (12 + 5*x)/(64*(1 - 2*x - x^2)) - (12 - 5
*x)/(64*(1 + 2*x - x^2)) - (5*ArcTanh[(1 - x)/Sqrt[2]])/(64*Sqrt[2]) + ArcT
anh[x]/4 + (5*ArcTanh[(1 + x)/Sqrt[2]])/(64*Sqrt[2]) - (3*(2 + 3*Sqrt[2])*L
og[1 - Sqrt[2] - x])/256 - (3*(2 - 3*Sqrt[2])*Log[1 + Sqrt[2] - x])/256 + (
3*(2 + 3*Sqrt[2])*Log[1 - Sqrt[2] + x])/256 + (3*(2 - 3*Sqrt[2])*Log[1 + Sq
rt[2] + x])/256
```

Rule 2073

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P /. x -> Sqrt[x]]
}, Int[ExpandIntegrand[(PP /. x -> x^2)^p*Q^q, x], x] /;  !SumQ[NonfreeFact
ors[PP, x]]] /; FreeQ[q, x] && PolyQ[P, x^2] && PolyQ[Q, x] && ILtQ[p, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1+x^2}{\left(1-7x^2+7x^4-x^6\right)^2}\,dx = \int \left(\frac{1}{32(-1+x)^2}+\frac{1}{32(1+x)^2}-\frac{1}{4\left(-1+x^2\right)}+\frac{17-7x}{32\left(-1-2x+x^2\right)^2}-\frac{3(-4+x)}{64\left(-1-2x+x^2\right)}+\right.$$

$$=\frac{1}{32(1-x)}-\frac{1}{32(1+x)}+\frac{1}{32}\int \frac{17-7x}{\left(-1-2x+x^2\right)^2}\,dx+\frac{1}{32}\int \frac{17+7x}{\left(-1+2x+x^2\right)^2}\,dx-\frac{3}{64}\int$$

$$=\frac{1}{32(1-x)}-\frac{1}{32(1+x)}+\frac{12+5x}{64\left(1-2x-x^2\right)}-\frac{12-5x}{64\left(1+2x-x^2\right)}+\frac{1}{4}\tanh^{-1}(x)-\frac{5}{64}\int$$

$$=\frac{1}{32(1-x)}-\frac{1}{32(1+x)}+\frac{12+5x}{64\left(1-2x-x^2\right)}-\frac{12-5x}{64\left(1+2x-x^2\right)}+\frac{1}{4}\tanh^{-1}(x)-\frac{3}{256}\left(2+3\right.$$

$$=\frac{1}{32(1-x)}-\frac{1}{32(1+x)}+\frac{12+5x}{64\left(1-2x-x^2\right)}-\frac{12-5x}{64\left(1+2x-x^2\right)}-\frac{5\tanh^{-1}\left(\frac{1-x}{\sqrt{2}}\right)}{64\sqrt{2}}+\frac{1}{4}\tanh$$

Mathematica [A] time = 0.0889908, size = 132, normalized size = 1.45

$$\frac{1}{128}\left(-\frac{4x\left(7x^4-46x^2+31\right)}{x^6-7x^4+7x^2-1}-16\log(1-x)+\left(3+2\sqrt{2}\right)\log\left(-x+\sqrt{2}-1\right)+\left(2\sqrt{2}-3\right)\log\left(-x+\sqrt{2}+1\right)+16\log(x+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/(1 - 7*x^2 + 7*x^4 - x^6)^2,x]
```

```
[Out] ((-4*x*(31 - 46*x^2 + 7*x^4))/(-1 + 7*x^2 - 7*x^4 + x^6) - 16*Log[1 - x] +
(3 + 2*Sqrt[2])*Log[-1 + Sqrt[2] - x] + (-3 + 2*Sqrt[2])*Log[1 + Sqrt[2] -
x] + 16*Log[1 + x] - (3 + 2*Sqrt[2])*Log[-1 + Sqrt[2] + x] + (3 - 2*Sqrt[2]
)*Log[1 + Sqrt[2] + x])/128
```

Maple [A] time = 0.016, size = 116, normalized size = 1.3

$$-\frac{-12+5x}{64x^2-128x-64}-\frac{3\ln\left(x^2-2x-1\right)}{128}-\frac{\sqrt{2}}{32}\mathrm{Artanh}\left(\frac{(2x-2)\sqrt{2}}{4}\right)-\frac{1}{32x-32}-\frac{\ln(x-1)}{8}+\frac{-5x-12}{64x^2+128x-64}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(-x^6+7*x^4-7*x^2+1)^2,x)
```

```
[Out] -1/64*(-12+5*x)/(x^2-2*x-1)-3/128*ln(x^2-2*x-1)-1/32*2^(1/2)*arctanh(1/4*(2
*x-2)*2^(1/2))-1/32/(x-1)-1/8*ln(x-1)+1/64*(-5*x-12)/(x^2+2*x-1)+3/128*ln(x
^2+2*x-1)-1/32*2^(1/2)*arctanh(1/4*(2+2*x)*2^(1/2))-1/32/(1+x)+1/8*ln(1+x)
```

Maxima [A] time = 1.5148, size = 154, normalized size = 1.69

$$\frac{1}{64}\sqrt{2}\log\left(\frac{x-\sqrt{2}+1}{x+\sqrt{2}+1}\right)+\frac{1}{64}\sqrt{2}\log\left(\frac{x-\sqrt{2}-1}{x+\sqrt{2}-1}\right)-\frac{7x^5-46x^3+31x}{32\left(x^6-7x^4+7x^2-1\right)}+\frac{3}{128}\log\left(x^2+2x-1\right)-\frac{3}{128}\log\left(x^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(-x^6+7*x^4-7*x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 1/64*sqrt(2)*log((x - sqrt(2) + 1)/(x + sqrt(2) + 1)) + 1/64*sqrt(2)*log((x
 - sqrt(2) - 1)/(x + sqrt(2) - 1)) - 1/32*(7*x^5 - 46*x^3 + 31*x)/(x^6 - 7*
x^4 + 7*x^2 - 1) + 3/128*log(x^2 + 2*x - 1) - 3/128*log(x^2 - 2*x - 1) + 1/
8*log(x + 1) - 1/8*log(x - 1)
```

Fricas [B] time = 1.3181, size = 585, normalized size = 6.43

$$-\frac{28x^5-184x^3-2\sqrt{2}\left(x^6-7x^4+7x^2-1\right)\log\left(\frac{x^2-2\sqrt{2}(x+1)+2x+3}{x^2+2x-1}\right)-2\sqrt{2}\left(x^6-7x^4+7x^2-1\right)\log\left(\frac{x^2-2\sqrt{2}(x-1)-2x+3}{x^2-2x-1}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(-x^6+7*x^4-7*x^2+1)^2,x, algorithm="fricas")
```

```
[Out] -1/128*(28*x^5 - 184*x^3 - 2*sqrt(2)*(x^6 - 7*x^4 + 7*x^2 - 1)*log((x^2 - 2
*sqrt(2)*(x + 1) + 2*x + 3)/(x^2 + 2*x - 1)) - 2*sqrt(2)*(x^6 - 7*x^4 + 7*x
^2 - 1)*log((x^2 - 2*sqrt(2)*(x - 1) - 2*x + 3)/(x^2 - 2*x - 1)) - 3*(x^6 -
 7*x^4 + 7*x^2 - 1)*log(x^2 + 2*x - 1) + 3*(x^6 - 7*x^4 + 7*x^2 - 1)*log(x^
2 - 2*x - 1) - 16*(x^6 - 7*x^4 + 7*x^2 - 1)*log(x + 1) + 16*(x^6 - 7*x^4 +
7*x^2 - 1)*log(x - 1) + 124*x)/(x^6 - 7*x^4 + 7*x^2 - 1)
```

Sympy [B] time = 1.04127, size = 272, normalized size = 2.99

$$-\frac{7x^5-46x^3+31x}{32x^6-224x^4+224x^2-32}-\frac{\log(x-1)}{8}+\frac{\log(x+1)}{8}+\left(-\frac{3}{128}-\frac{\sqrt{2}}{64}\right)\log\left(x-\frac{38423555}{909328}-\frac{38423555\sqrt{2}}{1363992}+\frac{9549859}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(-x**6+7*x**4-7*x**2+1)**2,x)
```

```
[Out] -(7*x**5 - 46*x**3 + 31*x)/(32*x**6 - 224*x**4 + 224*x**2 - 32) - log(x - 1
)/8 + log(x + 1)/8 + (-3/128 - sqrt(2)/64)*log(x - 38423555/909328 - 384235
55*sqrt(2)/1363992 + 9549859782656*(-3/128 - sqrt(2)/64)**5/170499 - 562673
74592*(-3/128 - sqrt(2)/64)**3/56833) + (-3/128 + sqrt(2)/64)*log(x - 38423
555/909328 + 9549859782656*(-3/128 + sqrt(2)/64)**5/170499 - 56267374592*(-
3/128 + sqrt(2)/64)**3/56833 + 38423555*sqrt(2)/1363992) + (3/128 - sqrt(2)
/64)*log(x - 38423555*sqrt(2)/1363992 - 56267374592*(3/128 - sqrt(2)/64)**3
/56833 + 9549859782656*(3/128 - sqrt(2)/64)**5/170499 + 38423555/909328) +
(sqrt(2)/64 + 3/128)*log(x - 56267374592*(sqrt(2)/64 + 3/128)**3/56833 + 95
49859782656*(sqrt(2)/64 + 3/128)**5/170499 + 38423555*sqrt(2)/1363992 + 384
23555/909328)
```

Giac [A] time = 1.23768, size = 181, normalized size = 1.99

$$\frac{1}{64}\sqrt{2}\log\left(\frac{|2x-2\sqrt{2}+2|}{|2x+2\sqrt{2}+2|}\right)+\frac{1}{64}\sqrt{2}\log\left(\frac{|2x-2\sqrt{2}-2|}{|2x+2\sqrt{2}-2|}\right)-\frac{7x^5-46x^3+31x}{32(x^6-7x^4+7x^2-1)}+\frac{3}{128}\log\left(|x^2+2x-1|\right)-\frac{3}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(-x^6+7*x^4-7*x^2+1)^2,x, algorithm="giac")
```

```
[Out] 1/64*sqrt(2)*log(abs(2*x - 2*sqrt(2) + 2)/abs(2*x + 2*sqrt(2) + 2)) + 1/64*
sqrt(2)*log(abs(2*x - 2*sqrt(2) - 2)/abs(2*x + 2*sqrt(2) - 2)) - 1/32*(7*x^
5 - 46*x^3 + 31*x)/(x^6 - 7*x^4 + 7*x^2 - 1) + 3/128*log(abs(x^2 + 2*x - 1)
) - 3/128*log(abs(x^2 - 2*x - 1)) + 1/8*log(abs(x + 1)) - 1/8*log(abs(x - 1
))
```

3.234 $\int x^m \left(a + bx + cx^2 + dx^3\right)^p (a(1+m)+x(b(2+m+p) + x(c(3 + m + 2p) + d(4 + m + 3p)x))) \, dx$

Optimal. Leaf size=25

$$x^{m+1} \left(a + bx + cx^2 + dx^3\right)^{p+1}$$

```
[Out] x^(1 + m)*(a + b*x + c*x^2 + d*x^3)^(1 + p)
```

Rubi [A] time = 0.0247382, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 56, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.018, Rules used = {1590}

$$x^{m+1} \left(a + bx + cx^2 + dx^3\right)^{p+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x + c*x^2 + d*x^3)^p*(a*(1 + m) + x*(b*(2 + m + p) + x*(c*(3
 + m + 2*p) + d*(4 + m + 3*p)*x))),x]
```

```
[Out] x^(1 + m)*(a + b*x + c*x^2 + d*x^3)^(1 + p)
```

Rule 1590

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q - r + 1)*Q
q^(m + 1)*Rr^(n + 1))/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r])
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\int x^m \left(a + bx + cx^2 + dx^3\right)^p (a(1 + m) + x(b(2 + m + p) + x(c(3 + m + 2p) + d(4 + m + 3p)x))) \, dx = x^{1+m} \left(a + bx + cx^2\right.$$

Mathematica [A] time = 0.336487, size = 23, normalized size = 0.92

$$x^{m+1}(a+x(b+x(c+dx)))^{p+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x + c*x^2 + d*x^3)^p*(a*(1 + m) + x*(b*(2 + m + p) + x
*(c*(3 + m + 2*p) + d*(4 + m + 3*p)*x))),x]
```

```
[Out] x^(1 + m)*(a + x*(b + x*(c + d*x)))^(1 + p)
```

Maple [A] time = 0.007, size = 26, normalized size = 1.

$$x^{1+m}\left(dx^3+cx^2+bx+a\right)^{1+p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(d*x^3+c*x^2+b*x+a)^p*(a*(1+m)+x*(b*(2+m+p)+x*(c*(3+m+2*p)+d*(4+m+3
*p)*x))),x)
```

```
[Out] x^(1+m)*(d*x^3+c*x^2+b*x+a)^(1+p)
```

Maxima [A] time = 1.3811, size = 59, normalized size = 2.36

$$\left(dx^4+cx^3+bx^2+ax\right)e^{\left(p\log\left(dx^3+cx^2+bx+a\right)+m\log(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(d*x^3+c*x^2+b*x+a)^p*(a*(1+m)+x*(b*(2+m+p)+x*(c*(3+m+2*p)+d*
(4+m+3*p)*x))),x, algorithm="maxima")
```

```
[Out] (d*x^4 + c*x^3 + b*x^2 + a*x)*e^(p*log(d*x^3 + c*x^2 + b*x + a) + m*log(x))
```

Fricas [A] time = 3.31732, size = 85, normalized size = 3.4

$$\left(dx^4+cx^3+bx^2+ax\right)\left(dx^3+cx^2+bx+a\right)^p x^m$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(d*x^3+c*x^2+b*x+a)^p*(a*(1+m)+x*(b*(2+m+p)+x*(c*(3+m+2*p)+d*
(4+m+3*p)*x))),x, algorithm="fricas")
```

```
[Out] (d*x^4 + c*x^3 + b*x^2 + a*x)*(d*x^3 + c*x^2 + b*x + a)^p*x^m
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(d*x**3+c*x**2+b*x+a)**p*(a*(1+m)+x*(b*(2+m+p)+x*(c*(3+m+2*p
)+d*(4+m+3*p)*x))),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.61687, size = 134, normalized size = 5.36

$$\left(dx^3 + cx^2 + bx + a\right)^p dx^4 x^m + \left(dx^3 + cx^2 + bx + a\right)^p cx^3 x^m + \left(dx^3 + cx^2 + bx + a\right)^p bx^2 x^m + \left(dx^3 + cx^2 + bx + a\right)^p axx^m$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(d*x^3+c*x^2+b*x+a)^p*(a*(1+m)+x*(b*(2+m+p)+x*(c*(3+m+2*p)+d*
(4+m+3*p)*x))),x, algorithm="giac")
```

```
[Out] (d*x^3 + c*x^2 + b*x + a)^p*d*x^4*x^m + (d*x^3 + c*x^2 + b*x + a)^p*c*x^3*x
^m + (d*x^3 + c*x^2 + b*x + a)^p*b*x^2*x^m + (d*x^3 + c*x^2 + b*x + a)^p*a*
x*x^m
```

3.235 $\int x^2 \left(a+bx+cx^2+dx^3\right)^p \left(3a+b(4+p)x+c(5+2p)x^2+\right.$

Optimal. Leaf size=23

$$x^3 \left(a+bx+cx^2+dx^3\right)^{p+1}$$

```
[Out] x^3*(a + b*x + c*x^2 + d*x^3)^(1 + p)
```

Rubi [A] time = 0.0819645, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.02, Rules used = {1588}

$$x^3 \left(a+bx+cx^2+dx^3\right)^{p+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x + c*x^2 + d*x^3)^p*(3*a + b*(4 + p)*x + c*(5 + 2*p)*x^2 +
d*(6 + 3*p)*x^3),x]
```

```
[Out] x^3*(a + b*x + c*x^2 + d*x^3)^(1 + p)
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int x^2 \left(a+bx+cx^2+dx^3\right)^p \left(3a+b(4+p)x+c(5+2p)x^2+d(6+3p)x^3\right)\,dx = x^3 \left(a+bx+cx^2+dx^3\right)^{1+p}$$

Mathematica [A] time = 0.234352, size = 21, normalized size = 0.91

$$x^3(a+x(b+x(c+dx)))^{p+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x + c*x^2 + d*x^3)^p*(3*a + b*(4 + p)*x + c*(5 + 2*p)*
x^2 + d*(6 + 3*p)*x^3),x]
```

```
[Out] x^3*(a + x*(b + x*(c + d*x)))^(1 + p)
```

Maple [A] time = 0.008, size = 24, normalized size = 1.

$$x^{3}\left(d x^{3}+c x^{2}+b x+a\right)^{1+p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d*x^3+c*x^2+b*x+a)^p*(3*a+b*(4+p)*x+c*(5+2*p)*x^2+d*(6+3*p)*x^3),x
)
```

```
[Out] x^3*(d*x^3+c*x^2+b*x+a)^(1+p)
```

Maxima [A] time = 1.3265, size = 53, normalized size = 2.3

$$\left(d x^{6}+c x^{5}+b x^{4}+a x^{3}\right)\left(d x^{3}+c x^{2}+b x+a\right)^{p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c*x^2+b*x+a)^p*(3*a+b*(4+p)*x+c*(5+2*p)*x^2+d*(6+3*p)*
x^3),x, algorithm="maxima")
```

```
[Out] (d*x^6 + c*x^5 + b*x^4 + a*x^3)*(d*x^3 + c*x^2 + b*x + a)^p
```

Fricas [A] time = 1.6841, size = 82, normalized size = 3.57

$$\left(d x^{6}+c x^{5}+b x^{4}+a x^{3}\right)\left(d x^{3}+c x^{2}+b x+a\right)^{p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c*x^2+b*x+a)^p*(3*a+b*(4+p)*x+c*(5+2*p)*x^2+d*(6+3*p)*
x^3),x, algorithm="fricas")
```

```
[Out] (d*x^6 + c*x^5 + b*x^4 + a*x^3)*(d*x^3 + c*x^2 + b*x + a)^p
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x**3+c*x**2+b*x+a)**p*(3*a+b*(4+p)*x+c*(5+2*p)*x**2+d*(6+
3*p)*x**3),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.78756, size = 120, normalized size = 5.22

$$\left(dx^3+cx^2+bx+a\right)^p dx^6+\left(dx^3+cx^2+bx+a\right)^p cx^5+\left(dx^3+cx^2+bx+a\right)^p bx^4+\left(dx^3+cx^2+bx+a\right)^p ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c*x^2+b*x+a)^p*(3*a+b*(4+p)*x+c*(5+2*p)*x^2+d*(6+3*p)*
x^3),x, algorithm="giac")
```

```
[Out] (d*x^3 + c*x^2 + b*x + a)^p*d*x^6 + (d*x^3 + c*x^2 + b*x + a)^p*c*x^5 + (d*
x^3 + c*x^2 + b*x + a)^p*b*x^4 + (d*x^3 + c*x^2 + b*x + a)^p*a*x^3
```

3.236 $\int x\left(a+bx+cx^2+dx^3\right)^p\left(2a+b(3+p)x+c(4+2p)x^2+\right.$

Optimal. Leaf size=23

$$x^2\left(a+bx+cx^2+dx^3\right)^{p+1}$$

```
[Out] x^2*(a + b*x + c*x^2 + d*x^3)^(1 + p)
```

Rubi [A] time = 0.0590147, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.02, Rules used = {1588}

$$x^2\left(a+bx+cx^2+dx^3\right)^{p+1}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x + c*x^2 + d*x^3)^p*(2*a + b*(3 + p)*x + c*(4 + 2*p)*x^2 + d*
(5 + 3*p)*x^3),x]
```

```
[Out] x^2*(a + b*x + c*x^2 + d*x^3)^(1 + p)
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int x\left(a+bx+cx^2+dx^3\right)^p\left(2a+b(3+p)x+c(4+2p)x^2+d(5+3p)x^3\right)\,dx=x^2\left(a+bx+cx^2+dx^3\right)^{1+p}$$

Mathematica [A] time = 0.177702, size = 21, normalized size = 0.91

$$x^2(a+x(b+x(c+dx)))^{p+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x + c*x^2 + d*x^3)^p*(2*a + b*(3 + p)*x + c*(4 + 2*p)*x^
2 + d*(5 + 3*p)*x^3),x]
```

```
[Out] x^2*(a + x*(b + x*(c + d*x)))^(1 + p)
```

Maple [A] time = 0.006, size = 24, normalized size = 1.

$$x^2 \left(dx^3 + cx^2 + bx + a\right)^{1+p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d*x^3+c*x^2+b*x+a)^p*(2*a+b*(3+p)*x+c*(4+2*p)*x^2+d*(5+3*p)*x^3),x)
```

```
[Out] x^2*(d*x^3+c*x^2+b*x+a)^(1+p)
```

Maxima [A] time = 1.20393, size = 53, normalized size = 2.3

$$\left(dx^5 + cx^4 + bx^3 + ax^2\right)\left(dx^3 + cx^2 + bx + a\right)^p$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c*x^2+b*x+a)^p*(2*a+b*(3+p)*x+c*(4+2*p)*x^2+d*(5+3*p)*x^
3),x, algorithm="maxima")
```

```
[Out] (d*x^5 + c*x^4 + b*x^3 + a*x^2)*(d*x^3 + c*x^2 + b*x + a)^p
```

Fricas [A] time = 1.42424, size = 82, normalized size = 3.57

$$\left(dx^5 + cx^4 + bx^3 + ax^2\right)\left(dx^3 + cx^2 + bx + a\right)^p$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c*x^2+b*x+a)^p*(2*a+b*(3+p)*x+c*(4+2*p)*x^2+d*(5+3*p)*x^
3),x, algorithm="fricas")
```

```
[Out] (d*x^5 + c*x^4 + b*x^3 + a*x^2)*(d*x^3 + c*x^2 + b*x + a)^p
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x**3+c*x**2+b*x+a)**p*(2*a+b*(3+p)*x+c*(4+2*p)*x**2+d*(5+3*p
)*x**3),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.36567, size = 120, normalized size = 5.22

$$\left(dx^3+cx^2+bx+a\right)^p dx^5+\left(dx^3+cx^2+bx+a\right)^p cx^4+\left(dx^3+cx^2+bx+a\right)^p bx^3+\left(dx^3+cx^2+bx+a\right)^p ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c*x^2+b*x+a)^p*(2*a+b*(3+p)*x+c*(4+2*p)*x^2+d*(5+3*p)*x^
3),x, algorithm="giac")
```

```
[Out] (d*x^3 + c*x^2 + b*x + a)^p*d*x^5 + (d*x^3 + c*x^2 + b*x + a)^p*c*x^4 + (d*
x^3 + c*x^2 + b*x + a)^p*b*x^3 + (d*x^3 + c*x^2 + b*x + a)^p*a*x^2
```

3.237 $\int \left(a+bx+cx^2+dx^3\right)^p \left(a+b(2+p)x+c(3+2p)x^2+d(4$

Optimal. Leaf size=21

$$x\left(a+bx+cx^2+dx^3\right)^{p+1}$$

[Out] x*(a + b*x + c*x^2 + d*x^3)^(1 + p)

Rubi [A] time = 0.0436484, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.022, Rules used = {1588}

$$x\left(a+bx+cx^2+dx^3\right)^{p+1}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x + c*x^2 + d*x^3)^p*(a + b*(2 + p)*x + c*(3 + 2*p)*x^2 + d*(4 + 3*p)*x^3),x]

[Out] x*(a + b*x + c*x^2 + d*x^3)^(1 + p)

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \left(a+bx+cx^2+dx^3\right)^p \left(a+b(2+p)x+c(3+2p)x^2+d(4+3p)x^3\right) dx = x\left(a+bx+cx^2+dx^3\right)^{1+p}$$

Mathematica [A] time = 0.119521, size = 19, normalized size = 0.9

$$x(a+x(b+x(c+dx)))^{p+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2 + d*x^3)^p*(a + b*(2 + p)*x + c*(3 + 2*p)*x^2 +
d*(4 + 3*p)*x^3),x]
```

```
[Out] x*(a + x*(b + x*(c + d*x)))^(1 + p)
```

Maple [A] time = 0.006, size = 22, normalized size = 1.1

$$x\left(dx^3+cx^2+bx+a\right)^{1+p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c*x^2+b*x+a)^p*(a+b*(2+p)*x+c*(3+2*p)*x^2+d*(4+3*p)*x^3),x)
```

```
[Out] x*(d*x^3+c*x^2+b*x+a)^(1+p)
```

Maxima [A] time = 1.21642, size = 50, normalized size = 2.38

$$\left(dx^4+cx^3+bx^2+ax\right)\left(dx^3+cx^2+bx+a\right)^p$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x+a)^p*(a+b*(2+p)*x+c*(3+2*p)*x^2+d*(4+3*p)*x^3),x
, algorithm="maxima")
```

```
[Out] (d*x^4 + c*x^3 + b*x^2 + a*x)*(d*x^3 + c*x^2 + b*x + a)^p
```

Fricas [A] time = 1.46482, size = 80, normalized size = 3.81

$$\left(dx^4+cx^3+bx^2+ax\right)\left(dx^3+cx^2+bx+a\right)^p$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x+a)^p*(a+b*(2+p)*x+c*(3+2*p)*x^2+d*(4+3*p)*x^3),x
, algorithm="fricas")
```

```
[Out] (d*x^4 + c*x^3 + b*x^2 + a*x)*(d*x^3 + c*x^2 + b*x + a)^p
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c*x**2+b*x+a)**p*(a+b*(2+p)*x+c*(3+2*p)*x**2+d*(4+3*p)*x**3),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.28569, size = 117, normalized size = 5.57

$$\left(dx^3+cx^2+bx+a\right)^p dx^4+\left(dx^3+cx^2+bx+a\right)^p cx^3+\left(dx^3+cx^2+bx+a\right)^p bx^2+\left(dx^3+cx^2+bx+a\right)^p ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x+a)^p*(a+b*(2+p)*x+c*(3+2*p)*x^2+d*(4+3*p)*x^3),x
, algorithm="giac")
```

```
[Out] (d*x^3 + c*x^2 + b*x + a)^p*d*x^4 + (d*x^3 + c*x^2 + b*x + a)^p*c*x^3 + (d*
x^3 + c*x^2 + b*x + a)^p*b*x^2 + (d*x^3 + c*x^2 + b*x + a)^p*a*x
```

3.238 $\int \frac{\left(a+bx+cx^2+dx^3\right)^p\left(b(1+p)x+c(2+2p)x^2+d(3+3p)x^3\right)}{x}\,dx$

Optimal. Leaf size=19

$$\left(a+bx+cx^2+dx^3\right)^{p+1}$$

```
[Out] (a + b*x + c*x^2 + d*x^3)^(1 + p)
```

Rubi [A] time = 0.0427075, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {1585, 1588}

$$\left(a+bx+cx^2+dx^3\right)^{p+1}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x + c*x^2 + d*x^3)^p*(b*(1 + p)*x + c*(2 + 2*p)*x^2 + d*(3 + 3*
p)*x^3))/x,x]
```

```
[Out] (a + b*x + c*x^2 + d*x^3)^(1 + p)
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2+dx^3\right)^p \left(b(1+p)x+c(2+2p)x^2+d(3+3p)x^3\right)}{x}\,dx = \int \left(b(1+p)+c(2+2p)x+d(3+3p)x^2\right)\left(a+bx-\right.$$
$$= \left(a+bx+cx^2+dx^3\right)^{1+p}$$

Mathematica [A] time = 0.00958, size = 17, normalized size = 0.89

$$(a+x(b+x(c+dx)))^{p+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x + c*x^2 + d*x^3)^p*(b*(1 + p)*x + c*(2 + 2*p)*x^2 + d*(
3 + 3*p)*x^3))/x,x]
```

```
[Out] (a + x*(b + x*(c + d*x)))^(1 + p)
```

Maple [A] time = 0.004, size = 20, normalized size = 1.1

$$\left(dx^3+cx^2+bx+a\right)^{1+p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c*x^2+b*x+a)^p*(b*(1+p)*x+c*(2+2*p)*x^2+d*(3+3*p)*x^3)/x,x)
```

```
[Out] (d*x^3+c*x^2+b*x+a)^(1+p)
```

Maxima [A] time = 1.29372, size = 45, normalized size = 2.37

$$\left(dx^3+cx^2+bx+a\right)\left(dx^3+cx^2+bx+a\right)^p$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x+a)^p*(b*(1+p)*x+c*(2+2*p)*x^2+d*(3+3*p)*x^3)/x,x
, algorithm="maxima")
```

```
[Out] (d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^p
```

Fricas [A] time = 1.41991, size = 74, normalized size = 3.89

$$\left(dx^3 + cx^2 + bx + a\right)\left(dx^3 + cx^2 + bx + a\right)^p$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x+a)^p*(b*(1+p)*x+c*(2+2*p)*x^2+d*(3+3*p)*x^3)/x,x
, algorithm="fricas")
```

```
[Out] (d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^p
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c*x**2+b*x+a)**p*(b*(1+p)*x+c*(2+2*p)*x**2+d*(3+3*p)*x**3
)/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(3\,d\left(p+1\right)x^3 + 2\,c\left(p+1\right)x^2 + b\left(p+1\right)x\right)\left(dx^3 + cx^2 + bx + a\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x+a)^p*(b*(1+p)*x+c*(2+2*p)*x^2+d*(3+3*p)*x^3)/x,x
, algorithm="giac")
```

```
[Out] integrate((3*d*(p + 1)*x^3 + 2*c*(p + 1)*x^2 + b*(p + 1)*x)*(d*x^3 + c*x^2
+ b*x + a)^p/x, x)
```

3.239 $\int \frac{\left(a+bx+cx^2+dx^3\right)^p \left(-a+bpx+c(1+2p)x^2+d(2+3p)x^3\right)}{x^2} \, dx$

Optimal. Leaf size=23

$$\frac{\left(a+bx+cx^2+dx^3\right)^{p+1}}{x}$$

```
[Out] (a + b*x + c*x^2 + d*x^3)^(1 + p)/x
```

Rubi [A] time = 0.033962, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.02, Rules used = {1590}

$$\frac{\left(a+bx+cx^2+dx^3\right)^{p+1}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x + c*x^2 + d*x^3)^p*(-a + b*p*x + c*(1 + 2*p)*x^2 + d*(2 + 3*p
)*x^3))/x^2,x]
```

```
[Out] (a + b*x + c*x^2 + d*x^3)^(1 + p)/x
```

Rule 1590

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q - r + 1)*Q
q^(m + 1)*Rr^(n + 1))/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r])
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2+dx^3\right)^p \left(-a+bpx+c(1+2p)x^2+d(2+3p)x^3\right)}{x^2} \, dx = \frac{\left(a+bx+cx^2+dx^3\right)^{1+p}}{x}$$

Mathematica [A] time = 0.174692, size = 21, normalized size = 0.91

$$\frac{(a + x(b + x(c + dx)))^{p+1}}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x + c*x^2 + d*x^3)^p*(-a + b*p*x + c*(1 + 2*p)*x^2 + d*(2
 + 3*p)*x^3))/x^2,x]
```

```
[Out] (a + x*(b + x*(c + d*x)))^(1 + p)/x
```

Maple [A] time = 0.004, size = 24, normalized size = 1.

$$\frac{\left(dx^3 + cx^2 + bx + a\right)^{1+p}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c*x^2+b*x+a)^p*(-a+b*p*x+c*(1+2*p)*x^2+d*(2+3*p)*x^3)/x^2,x)
```

```
[Out] (d*x^3+c*x^2+b*x+a)^(1+p)/x
```

Maxima [A] time = 1.27626, size = 49, normalized size = 2.13

$$\frac{\left(dx^3 + cx^2 + bx + a\right)\left(dx^3 + cx^2 + bx + a\right)^p}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x+a)^p*(-a+b*p*x+c*(1+2*p)*x^2+d*(2+3*p)*x^3)/x^2,
x, algorithm="maxima")
```

```
[Out] (d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^p/x
```

Fricas [A] time = 1.61028, size = 77, normalized size = 3.35

$$\frac{(dx^3 + cx^2 + bx + a)(dx^3 + cx^2 + bx + a)^p}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x+a)^p*(-a+b*p*x+c*(1+2*p)*x^2+d*(2+3*p)*x^3)/x^2,
x, algorithm="fricas")
```

```
[Out] (d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^p/x
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c*x**2+b*x+a)**p*(-a+b*p*x+c*(1+2*p)*x**2+d*(2+3*p)*x**3)
/x**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d(3p+2)x^3 + c(2p+1)x^2 + bpx - a)(dx^3 + cx^2 + bx + a)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x+a)^p*(-a+b*p*x+c*(1+2*p)*x^2+d*(2+3*p)*x^3)/x^2,
x, algorithm="giac")
```

```
[Out] integrate((d*(3*p + 2)*x^3 + c*(2*p + 1)*x^2 + b*p*x - a)*(d*x^3 + c*x^2 +
b*x + a)^p/x^2, x)
```

3.240 $\int \frac{(a+bx+cx^2+dx^3)^p (-2a+b(-1+p)x+2cpx^2+d(1+3p)x^3)}{x^3} dx$

Optimal. Leaf size=23

$$\frac{(a+bx+cx^2+dx^3)^{p+1}}{x^2}$$

```
[Out] (a + b*x + c*x^2 + d*x^3)^(1 + p)/x^2
```

Rubi [A] time = 0.0353046, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.021, Rules used = {1590}

$$\frac{(a+bx+cx^2+dx^3)^{p+1}}{x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x + c*x^2 + d*x^3)^p*(-2*a + b*(-1 + p)*x + 2*c*p*x^2 + d*(1 +
3*p)*x^3))/x^3,x]
```

```
[Out] (a + b*x + c*x^2 + d*x^3)^(1 + p)/x^2
```

Rule 1590

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q - r + 1)*Q
q^(m + 1)*Rr^(n + 1))/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r])
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{(a+bx+cx^2+dx^3)^p \left(-2a+b(-1+p)x+2cpx^2+d(1+3p)x^3\right)}{x^3} \, dx = \frac{(a+bx+cx^2+dx^3)^{1+p}}{x^2}$$

Mathematica [A] time = 0.182003, size = 21, normalized size = 0.91

$$\frac{(a+x(b+x(c+dx)))^{p+1}}{x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x + c*x^2 + d*x^3)^p*(-2*a + b*(-1 + p)*x + 2*c*p*x^2 + d
*(1 + 3*p)*x^3))/x^3,x]
```

```
[Out] (a + x*(b + x*(c + d*x)))^(1 + p)/x^2
```

Maple [A] time = 0.007, size = 24, normalized size = 1.

$$\frac{\left(dx^3+cx^2+bx+a\right)^{1+p}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c*x^2+b*x+a)^p*(-2*a+b*(-1+p)*x+2*c*p*x^2+d*(1+3*p)*x^3)/x^3,x)
```

```
[Out] (d*x^3+c*x^2+b*x+a)^(1+p)/x^2
```

Maxima [A] time = 1.31428, size = 49, normalized size = 2.13

$$\frac{\left(dx^3+cx^2+bx+a\right)\left(dx^3+cx^2+bx+a\right)^p}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x+a)^p*(-2*a+b*(-1+p)*x+2*c*p*x^2+d*(1+3*p)*x^3)/x
^3,x, algorithm="maxima")
```

```
[Out] (d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^p/x^2
```

Fricas [A] time = 1.82445, size = 80, normalized size = 3.48

$$\frac{\left(dx^3+cx^2+bx+a\right)\left(dx^3+cx^2+bx+a\right)^p}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x+a)^p*(-2*a+b*(-1+p)*x+2*c*p*x^2+d*(1+3*p)*x^3)/x
^3,x, algorithm="fricas")
```

```
[Out] (d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^p/x^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c*x**2+b*x+a)**p*(-2*a+b*(-1+p)*x+2*c*p*x**2+d*(1+3*p)*x*
*3)/x**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d(3p+1)x^3+2\,cpx^2+b(p-1)x-2\,a\right)\left(dx^3+cx^2+bx+a\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x+a)^p*(-2*a+b*(-1+p)*x+2*c*p*x^2+d*(1+3*p)*x^3)/x
^3,x, algorithm="giac")
```

```
[Out] integrate((d*(3*p + 1)*x^3 + 2*c*p*x^2 + b*(p - 1)*x - 2*a)*(d*x^3 + c*x^2
+ b*x + a)^p/x^3, x)
```

3.241 $\int \frac{\left(a+bx+cx^2+dx^3\right)^p\left(-3a+b(-2+p)x+c(-1+2p)x^2+3dpx^3\right)}{x^4}\,dx$

Optimal. Leaf size=23

$$\frac{\left(a+bx+cx^2+dx^3\right)^{p+1}}{x^3}$$

```
[Out] (a + b*x + c*x^2 + d*x^3)^(1 + p)/x^3
```

Rubi [A] time = 0.0358817, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.021, Rules used = {1590}

$$\frac{\left(a+bx+cx^2+dx^3\right)^{p+1}}{x^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x + c*x^2 + d*x^3)^p*(-3*a + b*(-2 + p)*x + c*(-1 + 2*p)*x^2 +
3*d*p*x^3))/x^4,x]
```

```
[Out] (a + b*x + c*x^2 + d*x^3)^(1 + p)/x^3
```

Rule 1590

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q - r + 1)*Q
q^(m + 1)*Rr^(n + 1))/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r])
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2+dx^3\right)^p\left(-3a+b(-2+p)x+c(-1+2p)x^2+3dpx^3\right)}{x^4}\,dx=\frac{\left(a+bx+cx^2+dx^3\right)^{1+p}}{x^3}$$

Mathematica [A] time = 0.212611, size = 21, normalized size = 0.91

$$\frac{(a+x(b+x(c+dx)))^{p+1}}{x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x + c*x^2 + d*x^3)^p*(-3*a + b*(-2 + p)*x + c*(-1 + 2*p)*
x^2 + 3*d*p*x^3))/x^4,x]
```

```
[Out] (a + x*(b + x*(c + d*x)))^(1 + p)/x^3
```

Maple [A] time = 0.005, size = 24, normalized size = 1.

$$\frac{\left(dx^3+cx^2+bx+a\right)^{1+p}}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c*x^2+b*x+a)^p*(-3*a+b*(-2+p)*x+c*(-1+2*p)*x^2+3*d*p*x^3)/x^4,x)
```

```
[Out] (d*x^3+c*x^2+b*x+a)^(1+p)/x^3
```

Maxima [A] time = 1.32212, size = 49, normalized size = 2.13

$$\frac{\left(dx^3+cx^2+bx+a\right)\left(dx^3+cx^2+bx+a\right)^p}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x+a)^p*(-3*a+b*(-2+p)*x+c*(-1+2*p)*x^2+3*d*p*x^3)/
x^4,x, algorithm="maxima")
```

```
[Out] (d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^p/x^3
```

Fricas [A] time = 2.04265, size = 80, normalized size = 3.48

$$\frac{\left(dx^3+cx^2+bx+a\right)\left(dx^3+cx^2+bx+a\right)^p}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x+a)^p*(-3*a+b*(-2+p)*x+c*(-1+2*p)*x^2+3*d*p*x^3)/
x^4,x, algorithm="fricas")
```

```
[Out] (d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^p/x^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c*x**2+b*x+a)**p*(-3*a+b*(-2+p)*x+c*(-1+2*p)*x**2+3*d*p*x
**3)/x**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(3\,dpx^3+c\left(2\,p-1\right)x^2+b\left(p-2\right)x-3\,a\right)\left(dx^3+cx^2+bx+a\right)^p}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x+a)^p*(-3*a+b*(-2+p)*x+c*(-1+2*p)*x^2+3*d*p*x^3)/
x^4,x, algorithm="giac")
```

```
[Out] integrate((3*d*p*x^3 + c*(2*p - 1)*x^2 + b*(p - 2)*x - 3*a)*(d*x^3 + c*x^2
+ b*x + a)^p/x^4, x)
```

3.242 $\int \frac{x^4(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx$

Optimal. Leaf size=97

$$\frac{x^4}{4}+\frac{x^3}{3}-\frac{3x^2}{4}+\frac{1}{3}\log\left(x^2+x+1\right)-\frac{13}{48}\log\left(2x^2-x+2\right)+\frac{5x}{4}+\frac{1}{24}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)-\frac{10\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] (5*x)/4 - (3*x^2)/4 + x^3/3 + x^4/4 + (Sqrt[5/3]*ArcTan[(1 - 4*x)/Sqrt[15]]
)/24 - (10*ArcTan[(1 + 2*x)/Sqrt[3]])/(3*Sqrt[3]) + Log[1 + x + x^2]/3 - (1
3*Log[2 - x + 2*x^2])/48
```

Rubi [A] time = 0.137518, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2075, 634, 618, 204, 628}

$$\frac{x^4}{4}+\frac{x^3}{3}-\frac{3x^2}{4}+\frac{1}{3}\log\left(x^2+x+1\right)-\frac{13}{48}\log\left(2x^2-x+2\right)+\frac{5x}{4}+\frac{1}{24}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)-\frac{10\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x^4),x]
```

```
[Out] (5*x)/4 - (3*x^2)/4 + x^3/3 + x^4/4 + (Sqrt[5/3]*ArcTan[(1 - 4*x)/Sqrt[15]]
)/24 - (10*ArcTan[(1 + 2*x)/Sqrt[3]])/(3*Sqrt[3]) + Log[1 + x + x^2]/3 - (1
3*Log[2 - x + 2*x^2])/48
```

Rule 2075

```
Int[(P_)^(p_)*(Qm_), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandIntegran
d[PP^p*Qm, x], x] /; QuadraticProductQ[PP, x]] /; PolyQ[Qm, x] && PolyQ[P,
x] && ILtQ[p, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^4\left(5+x+3x^2+2x^3\right)}{2+x+3x^2+x^3+2x^4}\,dx = \int \left(\frac{5}{4}-\frac{3x}{2}+x^2+x^3+\frac{2(-2+x)}{3\left(1+x+x^2\right)}+\frac{2-13x}{12\left(2-x+2x^2\right)}\right)dx$$
$$= \frac{5x}{4}-\frac{3x^2}{4}+\frac{x^3}{3}+\frac{x^4}{4}+\frac{1}{12}\int \frac{2-13x}{2-x+2x^2}\,dx+\frac{2}{3}\int \frac{-2+x}{1+x+x^2}\,dx$$
$$= \frac{5x}{4}-\frac{3x^2}{4}+\frac{x^3}{3}+\frac{x^4}{4}-\frac{5}{48}\int \frac{1}{2-x+2x^2}\,dx-\frac{13}{48}\int \frac{-1+4x}{2-x+2x^2}\,dx+\frac{1}{3}\int \frac{1+2x}{1+x+x^2}\,d$$
$$= \frac{5x}{4}-\frac{3x^2}{4}+\frac{x^3}{3}+\frac{x^4}{4}+\frac{1}{3}\log\left(1+x+x^2\right)-\frac{13}{48}\log\left(2-x+2x^2\right)+\frac{5}{24}\,\mathrm{Subst}\left(\int \frac{1}{-15-\right.$$
$$= \frac{5x}{4}-\frac{3x^2}{4}+\frac{x^3}{3}+\frac{x^4}{4}+\frac{1}{24}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)-\frac{10\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{3}\log\left(1+x+x^2\right)-$$

Mathematica [A] time = 0.0368234, size = 83, normalized size = 0.86

$$\frac{1}{144}\left(36x^4+48x^3-108x^2+48\log\left(x^2+x+1\right)-39\log\left(2x^2-x+2\right)+180x-160\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)-2\sqrt{15}\tan^{-1}\left(\frac{4}{}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x^4),x]
```

```
[Out] (180*x - 108*x^2 + 48*x^3 + 36*x^4 - 160*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]]
- 2*Sqrt[15]*ArcTan[(-1 + 4*x)/Sqrt[15]] + 48*Log[1 + x + x^2] - 39*Log[2 -
 x + 2*x^2])/144
```

Maple [A] time = 0.007, size = 74, normalized size = 0.8

$$\frac{x^4}{4} + \frac{x^3}{3} - \frac{3x^2}{4} + \frac{5x}{4} - \frac{13\ln\left(2x^2 - x + 2\right)}{48} - \frac{\sqrt{15}}{72}\arctan\left(\frac{(-1+4x)\sqrt{15}}{15}\right) + \frac{\ln\left(x^2 + x + 1\right)}{3} - \frac{10\sqrt{3}}{9}\arctan\left(\frac{(1+2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x)
```

```
[Out] 1/4*x^4+1/3*x^3-3/4*x^2+5/4*x-13/48*ln(2*x^2-x+2)-1/72*15^(1/2)*arctan(1/15
*(-1+4*x)*15^(1/2))+1/3*ln(x^2+x+1)-10/9*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2
)
```

Maxima [A] time = 1.50038, size = 99, normalized size = 1.02

$$\frac{1}{4}x^4 + \frac{1}{3}x^3 - \frac{3}{4}x^2 - \frac{1}{72}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right) - \frac{10}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{5}{4}x - \frac{13}{48}\log\left(2x^2 - x + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="maxima"
)
```

```
[Out] 1/4*x^4 + 1/3*x^3 - 3/4*x^2 - 1/72*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1))
 - 10/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 5/4*x - 13/48*log(2*x^2 - x
 + 2) + 1/3*log(x^2 + x + 1)
```

Fricas [A] time = 1.2355, size = 262, normalized size = 2.7

$$\frac{1}{4}x^4 + \frac{1}{3}x^3 - \frac{3}{4}x^2 - \frac{1}{72}\sqrt{5}\sqrt{3}\arctan\left(\frac{1}{15}\sqrt{5}\sqrt{3}(4x-1)\right) - \frac{10}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{5}{4}x - \frac{13}{48}\log\left(2x^2 - x +\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="fricas"
)
```

```
[Out] 1/4*x^4 + 1/3*x^3 - 3/4*x^2 - 1/72*sqrt(5)*sqrt(3)*arctan(1/15*sqrt(5)*sqrt
(3)*(4*x - 1)) - 10/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 5/4*x - 13/48
*log(2*x^2 - x + 2) + 1/3*log(x^2 + x + 1)
```

Sympy [A] time = 0.212874, size = 97, normalized size = 1.

$$\frac{x^4}{4} + \frac{x^3}{3} - \frac{3x^2}{4} + \frac{5x}{4} - \frac{13\log\left(x^2 - \frac{x}{2} + 1\right)}{48} + \frac{\log\left(x^2 + x + 1\right)}{3} - \frac{\sqrt{15}\,\mathrm{atan}\left(\frac{4\sqrt{15}x}{15} - \frac{\sqrt{15}}{15}\right)}{72} - \frac{10\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3} + \frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(2*x**3+3*x**2+x+5)/(2*x**4+x**3+3*x**2+x+2),x)
```

```
[Out] x**4/4 + x**3/3 - 3*x**2/4 + 5*x/4 - 13*log(x**2 - x/2 + 1)/48 + log(x**2 +
 x + 1)/3 - sqrt(15)*atan(4*sqrt(15)*x/15 - sqrt(15)/15)/72 - 10*sqrt(3)*at
an(2*sqrt(3)*x/3 + sqrt(3)/3)/9
```

Giac [A] time = 1.13766, size = 99, normalized size = 1.02

$$\frac{1}{4}\,x^4 + \frac{1}{3}\,x^3 - \frac{3}{4}\,x^2 - \frac{1}{72}\,\sqrt{15}\arctan\left(\frac{1}{15}\,\sqrt{15}(4\,x - 1)\right) - \frac{10}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x + 1)\right) + \frac{5}{4}\,x - \frac{13}{48}\,\log\left(2\,x^2 - x + 2\right) +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="giac")
```

```
[Out] 1/4*x^4 + 1/3*x^3 - 3/4*x^2 - 1/72*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1))
 - 10/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 5/4*x - 13/48*log(2*x^2 - x
 + 2) + 1/3*log(x^2 + x + 1)
```

3.243 $\int \frac{x^3(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx$

Optimal. Leaf size=90

$$\frac{x^3}{3}+\frac{x^2}{2}+\frac{2}{3}\log\left(x^2+x+1\right)-\frac{1}{24}\log\left(2x^2-x+2\right)-\frac{3x}{2}+\frac{5}{12}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{8\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] (-3*x)/2 + x^2/2 + x^3/3 + (5*Sqrt[5/3]*ArcTan[(1 - 4*x)/Sqrt[15]])/12 + (8
*ArcTan[(1 + 2*x)/Sqrt[3]])/(3*Sqrt[3]) + (2*Log[1 + x + x^2])/3 - Log[2 -
x + 2*x^2]/24
```

Rubi [A] time = 0.122615, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2075, 634, 618, 204, 628}

$$\frac{x^3}{3}+\frac{x^2}{2}+\frac{2}{3}\log\left(x^2+x+1\right)-\frac{1}{24}\log\left(2x^2-x+2\right)-\frac{3x}{2}+\frac{5}{12}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{8\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x^4),x]
```


```
[Out] (-3*x)/2 + x^2/2 + x^3/3 + (5*Sqrt[5/3]*ArcTan[(1 - 4*x)/Sqrt[15]])/12 + (8
*ArcTan[(1 + 2*x)/Sqrt[3]])/(3*Sqrt[3]) + (2*Log[1 + x + x^2])/3 - Log[2 -
x + 2*x^2]/24
```


Rule 2075

```
Int[(P_)^(p_)*(Qm_), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandIntegran
d[PP^p*Qm, x], x] /; QuadraticProductQ[PP, x]] /; PolyQ[Qm, x] && PolyQ[P,
x] && ILtQ[p, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^3\left(5+x+3x^2+2x^3\right)}{2+x+3x^2+x^3+2x^4}\,dx = \int \left(-\frac{3}{2}+x+x^2+\frac{2(3+2x)}{3\left(1+x+x^2\right)}+\frac{-6-x}{6\left(2-x+2x^2\right)}\right)dx$$
$$= -\frac{3x}{2}+\frac{x^2}{2}+\frac{x^3}{3}+\frac{1}{6}\int \frac{-6-x}{2-x+2x^2}\,dx+\frac{2}{3}\int \frac{3+2x}{1+x+x^2}\,dx$$
$$= -\frac{3x}{2}+\frac{x^2}{2}+\frac{x^3}{3}-\frac{1}{24}\int \frac{-1+4x}{2-x+2x^2}\,dx+\frac{2}{3}\int \frac{1+2x}{1+x+x^2}\,dx-\frac{25}{24}\int \frac{1}{2-x+2x^2}\,dx+\frac{4}{3}$$
$$= -\frac{3x}{2}+\frac{x^2}{2}+\frac{x^3}{3}+\frac{2}{3}\log\left(1+x+x^2\right)-\frac{1}{24}\log\left(2-x+2x^2\right)+\frac{25}{12}\text{Subst}\left(\int \frac{1}{-15-x^2}\,dx\right.$$
$$= -\frac{3x}{2}+\frac{x^2}{2}+\frac{x^3}{3}+\frac{5}{12}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{8\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{2}{3}\log\left(1+x+x^2\right)-\frac{1}{24}\log$$

Mathematica [A] time = 0.0229295, size = 78, normalized size = 0.87

$$\frac{1}{72}\left(24x^3+36x^2+48\log\left(x^2+x+1\right)-3\log\left(2x^2-x+2\right)-108x+64\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)-10\sqrt{15}\tan^{-1}\left(\frac{4x-1}{\sqrt{15}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x^4),x]
```

```
[Out] (-108*x + 36*x^2 + 24*x^3 + 64*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - 10*Sqrt[
15]*ArcTan[(-1 + 4*x)/Sqrt[15]] + 48*Log[1 + x + x^2] - 3*Log[2 - x + 2*x^2
])/72
```

Maple [A] time = 0.006, size = 69, normalized size = 0.8

$$\frac{x^3}{3}+\frac{x^2}{2}-\frac{3x}{2}-\frac{\ln\left(2x^2-x+2\right)}{24}-\frac{5\sqrt{15}}{36}\arctan\left(\frac{(-1+4x)\sqrt{15}}{15}\right)+\frac{2\ln\left(x^2+x+1\right)}{3}+\frac{8\sqrt{3}}{9}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x)
```

```
[Out] 1/3*x^3+1/2*x^2-3/2*x-1/24*ln(2*x^2-x+2)-5/36*15^(1/2)*arctan(1/15*(-1+4*x)
*15^(1/2))+2/3*ln(x^2+x+1)+8/9*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Maxima [A] time = 1.55046, size = 92, normalized size = 1.02

$$\frac{1}{3}x^3+\frac{1}{2}x^2-\frac{5}{36}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right)+\frac{8}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{3}{2}x-\frac{1}{24}\log\left(2x^2-x+2\right)+\frac{2}{3}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="maxima"
)
```

```
[Out] 1/3*x^3 + 1/2*x^2 - 5/36*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) + 8/9*sqr
t(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 3/2*x - 1/24*log(2*x^2 - x + 2) + 2/3*
log(x^2 + x + 1)
```

Fricas [A] time = 1.40235, size = 246, normalized size = 2.73

$$\frac{1}{3}x^3+\frac{1}{2}x^2-\frac{5}{36}\sqrt{5}\sqrt{3}\arctan\left(\frac{1}{15}\sqrt{5}\sqrt{3}(4x-1)\right)+\frac{8}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{3}{2}x-\frac{1}{24}\log\left(2x^2-x+2\right)+\frac{2}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="fricas"
)
```

```
[Out] 1/3*x^3 + 1/2*x^2 - 5/36*sqrt(5)*sqrt(3)*arctan(1/15*sqrt(5)*sqrt(3)*(4*x -
 1)) + 8/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 3/2*x - 1/24*log(2*x^2 -
 x + 2) + 2/3*log(x^2 + x + 1)
```

Sympy [A] time = 0.209759, size = 92, normalized size = 1.02

$$\frac{x^3}{3} + \frac{x^2}{2} - \frac{3x}{2} - \frac{\log\left(x^2 - \frac{x}{2} + 1\right)}{24} + \frac{2\log\left(x^2 + x + 1\right)}{3} - \frac{5\sqrt{15}\operatorname{atan}\left(\frac{4\sqrt{15}x}{15} - \frac{\sqrt{15}}{15}\right)}{36} + \frac{8\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3} + \frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(2*x**3+3*x**2+x+5)/(2*x**4+x**3+3*x**2+x+2),x)
```

```
[Out] x**3/3 + x**2/2 - 3*x/2 - log(x**2 - x/2 + 1)/24 + 2*log(x**2 + x + 1)/3 -
5*sqrt(15)*atan(4*sqrt(15)*x/15 - sqrt(15)/15)/36 + 8*sqrt(3)*atan(2*sqrt(3
)*x/3 + sqrt(3)/3)/9
```

Giac [A] time = 1.21717, size = 92, normalized size = 1.02

$$\frac{1}{3}x^3 + \frac{1}{2}x^2 - \frac{5}{36}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right) + \frac{8}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{3}{2}x - \frac{1}{24}\log\left(2x^2 - x + 2\right) + \frac{2}{3}\log($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="giac")
```

```
[Out] 1/3*x^3 + 1/2*x^2 - 5/36*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) + 8/9*sqr
t(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 3/2*x - 1/24*log(2*x^2 - x + 2) + 2/3*
log(x^2 + x + 1)
```

3.244 $\int \frac{x^2(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx$

Optimal. Leaf size=77

$$\frac{x^2}{2}-\log\left(x^2+x+1\right)+\frac{1}{4}\log\left(2x^2-x+2\right)+x+\frac{1}{6}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{2\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] x + x^2/2 + (Sqrt[5/3]*ArcTan[(1 - 4*x)/Sqrt[15]])/6 + (2*ArcTan[(1 + 2*x)/
Sqrt[3]])/(3*Sqrt[3]) - Log[1 + x + x^2] + Log[2 - x + 2*x^2]/4
```

Rubi [A] time = 0.120702, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2075, 634, 618, 204, 628}

$$\frac{x^2}{2}-\log\left(x^2+x+1\right)+\frac{1}{4}\log\left(2x^2-x+2\right)+x+\frac{1}{6}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{2\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x^4),x]
```

```
[Out] x + x^2/2 + (Sqrt[5/3]*ArcTan[(1 - 4*x)/Sqrt[15]])/6 + (2*ArcTan[(1 + 2*x)/
Sqrt[3]])/(3*Sqrt[3]) - Log[1 + x + x^2] + Log[2 - x + 2*x^2]/4
```

Rule 2075

```
Int[(P_)^(p_)*(Qm_), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandIntegran
d[PP^p*Qm, x], x] /; QuadraticProductQ[PP, x]] /; PolyQ[Qm, x] && PolyQ[P,
x] && ILtQ[p, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2\left(5+x+3x^2+2x^3\right)}{2+x+3x^2+x^3+2x^4}\,dx &= \int \left(1+x-\frac{2(1+3x)}{3\left(1+x+x^2\right)}+\frac{-2+3x}{3\left(2-x+2x^2\right)}\right)dx \\
&= x+\frac{x^2}{2}+\frac{1}{3}\int \frac{-2+3x}{2-x+2x^2}\,dx-\frac{2}{3}\int \frac{1+3x}{1+x+x^2}\,dx \\
&= x+\frac{x^2}{2}+\frac{1}{4}\int \frac{-1+4x}{2-x+2x^2}\,dx+\frac{1}{3}\int \frac{1}{1+x+x^2}\,dx-\frac{5}{12}\int \frac{1}{2-x+2x^2}\,dx-\int \frac{1+2x}{1+x+\ldots} \\
&= x+\frac{x^2}{2}-\log\left(1+x+x^2\right)+\frac{1}{4}\log\left(2-x+2x^2\right)-\frac{2}{3}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2x\right)+\ldots \\
&= x+\frac{x^2}{2}+\frac{1}{6}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{2\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}}-\log\left(1+x+x^2\right)+\frac{1}{4}\log\left(2-x+2x^2\ldots\right.
\end{aligned}$$

Mathematica [A] time = 0.0304992, size = 72, normalized size = 0.94

$$\frac{1}{36}\left(9\left(-4\log\left(x^2+x+1\right)+\log\left(2x^2-x+2\right)+2x(x+2)\right)+8\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)-2\sqrt{15}\tan^{-1}\left(\frac{4x-1}{\sqrt{15}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x^4),x]
```

```
[Out] (8*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - 2*Sqrt[15]*ArcTan[(-1 + 4*x)/Sqrt[15
]] + 9*(2*x*(2 + x) - 4*Log[1 + x + x^2] + Log[2 - x + 2*x^2]))/36
```

Maple [A] time = 0.004, size = 62, normalized size = 0.8

$$\frac{x^2}{2}+x+\frac{\ln\left(2x^2-x+2\right)}{4}-\frac{\sqrt{15}}{18}\arctan\left(\frac{(-1+4x)\sqrt{15}}{15}\right)-\ln\left(x^2+x+1\right)+\frac{2\sqrt{3}}{9}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x)
```

```
[Out] 1/2*x^2+x+1/4*ln(2*x^2-x+2)-1/18*15^(1/2)*arctan(1/15*(-1+4*x)*15^(1/2))-ln
(x^2+x+1)+2/9*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Maxima [A] time = 1.64498, size = 82, normalized size = 1.06

$$\frac{1}{2}x^2-\frac{1}{18}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right)+\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+x+\frac{1}{4}\log\left(2x^2-x+2\right)-\log\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="maxima"
)
```

```
[Out] 1/2*x^2 - 1/18*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) + 2/9*sqrt(3)*arcta
n(1/3*sqrt(3)*(2*x + 1)) + x + 1/4*log(2*x^2 - x + 2) - log(x^2 + x + 1)
```

Fricas [A] time = 1.50663, size = 220, normalized size = 2.86

$$\frac{1}{2}x^2-\frac{1}{18}\sqrt{5}\sqrt{3}\arctan\left(\frac{1}{15}\sqrt{5}\sqrt{3}(4x-1)\right)+\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+x+\frac{1}{4}\log\left(2x^2-x+2\right)-\log\left(x^2+x+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="fricas"
)
```

```
[Out] 1/2*x^2 - 1/18*sqrt(5)*sqrt(3)*arctan(1/15*sqrt(5)*sqrt(3)*(4*x - 1)) + 2/9
*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + x + 1/4*log(2*x^2 - x + 2) - log(x
^2 + x + 1)
```

Sympy [A] time = 0.204281, size = 78, normalized size = 1.01

$$\frac{x^2}{2} + x + \frac{\log\left(x^2 - \frac{x}{2} + 1\right)}{4} - \log\left(x^2 + x + 1\right) - \frac{\sqrt{15}\,\mathrm{atan}\left(\frac{4\sqrt{15}x}{15} - \frac{\sqrt{15}}{15}\right)}{18} + \frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3} + \frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(2*x**3+3*x**2+x+5)/(2*x**4+x**3+3*x**2+x+2),x)
```

```
[Out] x**2/2 + x + log(x**2 - x/2 + 1)/4 - log(x**2 + x + 1) - sqrt(15)*atan(4*sq
rt(15)*x/15 - sqrt(15)/15)/18 + 2*sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/9
```

Giac [A] time = 1.20184, size = 82, normalized size = 1.06

$$\frac{1}{2}\,x^2 - \frac{1}{18}\,\sqrt{15}\arctan\left(\frac{1}{15}\,\sqrt{15}(4\,x - 1)\right) + \frac{2}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x + 1)\right) + x + \frac{1}{4}\,\log\left(2\,x^2 - x + 2\right) - \log\left(x^2 + x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 1/18*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) + 2/9*sqrt(3)*arcta
n(1/3*sqrt(3)*(2*x + 1)) + x + 1/4*log(2*x^2 - x + 2) - log(x^2 + x + 1)
```

3.245 $\int \frac{x(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx$

Optimal. Leaf size=72

$$\frac{1}{3}\log\left(x^2+x+1\right)+\frac{1}{6}\log\left(2x^2-x+2\right)+x-\frac{1}{3}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)-\frac{10\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] x - (Sqrt[5/3]*ArcTan[(1 - 4*x)/Sqrt[15]])/3 - (10*ArcTan[(1 + 2*x)/Sqrt[3]
])/(3*Sqrt[3]) + Log[1 + x + x^2]/3 + Log[2 - x + 2*x^2]/6
```

Rubi [A] time = 0.0926255, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2075, 634, 618, 204, 628}

$$\frac{1}{3}\log\left(x^2+x+1\right)+\frac{1}{6}\log\left(2x^2-x+2\right)+x-\frac{1}{3}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)-\frac{10\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x^4),x]
```

```
[Out] x - (Sqrt[5/3]*ArcTan[(1 - 4*x)/Sqrt[15]])/3 - (10*ArcTan[(1 + 2*x)/Sqrt[3]
])/(3*Sqrt[3]) + Log[1 + x + x^2]/3 + Log[2 - x + 2*x^2]/6
```

Rule 2075

```
Int[(P_)^(p_)*(Qm_), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandIntegran
d[PP^p*Qm, x], x] /; QuadraticProductQ[PP, x]] /; PolyQ[Qm, x] && PolyQ[P,
x] && ILtQ[p, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x\left(5+x+3x^2+2x^3\right)}{2+x+3x^2+x^3+2x^4}\,dx &= \int \left(1+\frac{2(-2+x)}{3\left(1+x+x^2\right)}+\frac{2(1+x)}{3\left(2-x+2x^2\right)}\right)dx\\
&= x+\frac{2}{3}\int \frac{-2+x}{1+x+x^2}\,dx+\frac{2}{3}\int \frac{1+x}{2-x+2x^2}\,dx\\
&= x+\frac{1}{6}\int \frac{-1+4x}{2-x+2x^2}\,dx+\frac{1}{3}\int \frac{1+2x}{1+x+x^2}\,dx+\frac{5}{6}\int \frac{1}{2-x+2x^2}\,dx-\frac{5}{3}\int \frac{1}{1+x+x^2}\,dx\\
&= x+\frac{1}{3}\log\left(1+x+x^2\right)+\frac{1}{6}\log\left(2-x+2x^2\right)-\frac{5}{3}\,\mathrm{Subst}\left(\int \frac{1}{-15-x^2}\,dx,x,-1+4x\right)+\ldots\\
&= x-\frac{1}{3}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)-\frac{10\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{3}\log\left(1+x+x^2\right)+\frac{1}{6}\log\left(2-x+2x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.0217062, size = 69, normalized size = 0.96

$$
\frac{1}{18}\left(3\left(2\log\left(x^2+x+1\right)+\log\left(2x^2-x+2\right)+6x\right)-20\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)+2\sqrt{15}\tan^{-1}\left(\frac{4x-1}{\sqrt{15}}\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x^4),x]
```

```
[Out] (-20*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + 2*Sqrt[15]*ArcTan[(-1 + 4*x)/Sqrt[
15]] + 3*(6*x + 2*Log[1 + x + x^2] + Log[2 - x + 2*x^2]))/18
```

Maple [A] time = 0.005, size = 57, normalized size = 0.8

$$x+\frac{\ln\left(2x^{2}-x+2\right)}{6}+\frac{\sqrt{15}}{9}\arctan\left(\frac{(-1+4x)\sqrt{15}}{15}\right)+\frac{\ln\left(x^{2}+x+1\right)}{3}-\frac{10\sqrt{3}}{9}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x)
```

```
[Out] x+1/6*ln(2*x^2-x+2)+1/9*15^(1/2)*arctan(1/15*(-1+4*x)*15^(1/2))+1/3*ln(x^2+
x+1)-10/9*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Maxima [A] time = 1.62814, size = 76, normalized size = 1.06

$$\frac{1}{9}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right)-\frac{10}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+x+\frac{1}{6}\log\left(2x^{2}-x+2\right)+\frac{1}{3}\log\left(x^{2}+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="maxima")
```

```
[Out] 1/9*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) - 10/9*sqrt(3)*arctan(1/3*sqrt
(3)*(2*x + 1)) + x + 1/6*log(2*x^2 - x + 2) + 1/3*log(x^2 + x + 1)
```

Fricas [A] time = 1.38692, size = 212, normalized size = 2.94

$$\frac{1}{9}\sqrt{5}\sqrt{3}\arctan\left(\frac{1}{15}\sqrt{5}\sqrt{3}(4x-1)\right)-\frac{10}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+x+\frac{1}{6}\log\left(2x^{2}-x+2\right)+\frac{1}{3}\log\left(x^{2}+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="fricas")
```

```
[Out] 1/9*sqrt(5)*sqrt(3)*arctan(1/15*sqrt(5)*sqrt(3)*(4*x - 1)) - 10/9*sqrt(3)*a
rctan(1/3*sqrt(3)*(2*x + 1)) + x + 1/6*log(2*x^2 - x + 2) + 1/3*log(x^2 + x
 + 1)
```

Sympy [A] time = 0.19997, size = 75, normalized size = 1.04

$$x+\frac{\log\left(x^2-\frac{x}{2}+1\right)}{6}+\frac{\log\left(x^2+x+1\right)}{3}+\frac{\sqrt{15}\,\mathrm{atan}\left(\frac{4\sqrt{15}x}{15}-\frac{\sqrt{15}}{15}\right)}{9}-\frac{10\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*x**3+3*x**2+x+5)/(2*x**4+x**3+3*x**2+x+2),x)
```

```
[Out] x + log(x**2 - x/2 + 1)/6 + log(x**2 + x + 1)/3 + sqrt(15)*atan(4*sqrt(15)*
x/15 - sqrt(15)/15)/9 - 10*sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/9
```

Giac [A] time = 1.24324, size = 76, normalized size = 1.06

$$\frac{1}{9}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right)-\frac{10}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+x+\frac{1}{6}\log\left(2x^2-x+2\right)+\frac{1}{3}\log\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="giac")
```

```
[Out] 1/9*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) - 10/9*sqrt(3)*arctan(1/3*sqrt
(3)*(2*x + 1)) + x + 1/6*log(2*x^2 - x + 2) + 1/3*log(x^2 + x + 1)
```

3.246 $\int \frac{5+x+3x^2+2x^3}{2+x+3x^2+x^3+2x^4} \, dx$

Optimal. Leaf size=71

$$\frac{2}{3}\log\left(x^2+x+1\right)-\frac{1}{6}\log\left(2x^2-x+2\right)-\frac{1}{3}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{8\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] -(Sqrt[5/3]*ArcTan[(1 - 4*x)/Sqrt[15]])/3 + (8*ArcTan[(1 + 2*x)/Sqrt[3]])/(
3*Sqrt[3]) + (2*Log[1 + x + x^2])/3 - Log[2 - x + 2*x^2]/6
```

Rubi [A] time = 0.0782529, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {2074, 634, 618, 204, 628}

$$\frac{2}{3}\log\left(x^2+x+1\right)-\frac{1}{6}\log\left(2x^2-x+2\right)-\frac{1}{3}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{8\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + x + 3*x^2 + 2*x^3)/(2 + x + 3*x^2 + x^3 + 2*x^4),x]
```

```
[Out] -(Sqrt[5/3]*ArcTan[(1 - 4*x)/Sqrt[15]])/3 + (8*ArcTan[(1 + 2*x)/Sqrt[3]])/(
3*Sqrt[3]) + (2*Log[1 + x + x^2])/3 - Log[2 - x + 2*x^2]/6
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{5+x+3x^2+2x^3}{2+x+3x^2+x^3+2x^4}\,dx = \int \left(\frac{2(3+2x)}{3\left(1+x+x^2\right)}+\frac{3-2x}{3\left(2-x+2x^2\right)}\right)dx$$
$$= \frac{1}{3}\int \frac{3-2x}{2-x+2x^2}\,dx+\frac{2}{3}\int \frac{3+2x}{1+x+x^2}\,dx$$
$$= -\left(\frac{1}{6}\int \frac{-1+4x}{2-x+2x^2}\,dx\right)+\frac{2}{3}\int \frac{1+2x}{1+x+x^2}\,dx+\frac{5}{6}\int \frac{1}{2-x+2x^2}\,dx+\frac{4}{3}\int \frac{1}{1+x+x^2}\,d$$
$$= \frac{2}{3}\log\left(1+x+x^2\right)-\frac{1}{6}\log\left(2-x+2x^2\right)-\frac{5}{3}\text{Subst}\left(\int \frac{1}{-15-x^2}\,dx,x,-1+4x\right)-\frac{8}{3}\text{Sub}$$
$$= -\frac{1}{3}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{8\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{2}{3}\log\left(1+x+x^2\right)-\frac{1}{6}\log\left(2-x+2x^2\right)$$

Mathematica [A] time = 0.017241, size = 65, normalized size = 0.92

$$\frac{1}{18}\left(12\log\left(x^2+x+1\right)-3\log\left(2x^2-x+2\right)+16\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)+2\sqrt{15}\tan^{-1}\left(\frac{4x-1}{\sqrt{15}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + x + 3*x^2 + 2*x^3)/(2 + x + 3*x^2 + x^3 + 2*x^4),x]
```

```
[Out] (16*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + 2*Sqrt[15]*ArcTan[(-1 + 4*x)/Sqrt[1
5]] + 12*Log[1 + x + x^2] - 3*Log[2 - x + 2*x^2])/18
```

Maple [A] time = 0.005, size = 56, normalized size = 0.8

$$-\frac{\ln\left(2x^2-x+2\right)}{6}+\frac{\sqrt{15}}{9}\arctan\left(\frac{(-1+4x)\sqrt{15}}{15}\right)+\frac{2\ln\left(x^2+x+1\right)}{3}+\frac{8\sqrt{3}}{9}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x)
```

```
[Out] -1/6*ln(2*x^2-x+2)+1/9*15^(1/2)*arctan(1/15*(-1+4*x)*15^(1/2))+2/3*ln(x^2+x
+1)+8/9*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Maxima [A] time = 1.47321, size = 74, normalized size = 1.04

$$\frac{1}{9}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right)+\frac{8}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{6}\log\left(2x^2-x+2\right)+\frac{2}{3}\log\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="maxima")
```

```
[Out] 1/9*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) + 8/9*sqrt(3)*arctan(1/3*sqrt(
3)*(2*x + 1)) - 1/6*log(2*x^2 - x + 2) + 2/3*log(x^2 + x + 1)
```

Fricas [A] time = 1.43082, size = 205, normalized size = 2.89

$$\frac{1}{9}\sqrt{5}\sqrt{3}\arctan\left(\frac{1}{15}\sqrt{5}\sqrt{3}(4x-1)\right)+\frac{8}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{6}\log\left(2x^2-x+2\right)+\frac{2}{3}\log\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="fricas")
```

```
[Out] 1/9*sqrt(5)*sqrt(3)*arctan(1/15*sqrt(5)*sqrt(3)*(4*x - 1)) + 8/9*sqrt(3)*ar
ctan(1/3*sqrt(3)*(2*x + 1)) - 1/6*log(2*x^2 - x + 2) + 2/3*log(x^2 + x + 1)
```

Sympy [A] time = 0.19463, size = 75, normalized size = 1.06

$$-\frac{\log\left(x^2-\frac{x}{2}+1\right)}{6}+\frac{2\log\left(x^2+x+1\right)}{3}+\frac{\sqrt{15}\,\mathrm{atan}\left(\frac{4\sqrt{15}x}{15}-\frac{\sqrt{15}}{15}\right)}{9}+\frac{8\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**3+3*x**2+x+5)/(2*x**4+x**3+3*x**2+x+2),x)
```

```
[Out] -log(x**2 - x/2 + 1)/6 + 2*log(x**2 + x + 1)/3 + sqrt(15)*atan(4*sqrt(15)*x
/15 - sqrt(15)/15)/9 + 8*sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/9
```

Giac [A] time = 1.19205, size = 74, normalized size = 1.04

$$\frac{1}{9}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right)+\frac{8}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{6}\log\left(2x^2-x+2\right)+\frac{2}{3}\log\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="giac")
```

```
[Out] 1/9*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) + 8/9*sqrt(3)*arctan(1/3*sqrt(
3)*(2*x + 1)) - 1/6*log(2*x^2 - x + 2) + 2/3*log(x^2 + x + 1)
```

3.247 $\int \frac{5+x+3x^2+2x^3}{x(2+x+3x^2+x^3+2x^4)}\,dx$

Optimal. Leaf size=75

$$-\log\left(x^2+x+1\right)-\frac{1}{4}\log\left(2x^2-x+2\right)+\frac{5\log(x)}{2}+\frac{1}{6}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{2\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] (Sqrt[5/3]*ArcTan[(1 - 4*x)/Sqrt[15]])/6 + (2*ArcTan[(1 + 2*x)/Sqrt[3]])/(3
*Sqrt[3]) + (5*Log[x])/2 - Log[1 + x + x^2] - Log[2 - x + 2*x^2]/4
```

Rubi [A] time = 0.135082, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2087, 800, 634, 618, 204, 628}

$$-\log\left(x^2+x+1\right)-\frac{1}{4}\log\left(2x^2-x+2\right)+\frac{5\log(x)}{2}+\frac{1}{6}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{2\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + x + 3*x^2 + 2*x^3)/(x*(2 + x + 3*x^2 + x^3 + 2*x^4)),x]
```

```
[Out] (Sqrt[5/3]*ArcTan[(1 - 4*x)/Sqrt[15]])/6 + (2*ArcTan[(1 + 2*x)/Sqrt[3]])/(3
*Sqrt[3]) + (5*Log[x])/2 - Log[1 + x + x^2] - Log[2 - x + 2*x^2]/4
```

Rule 2087

```
Int[((P3_)*(x_)^(m_.))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (
e_.)*(x_)^4), x_Symbol] :> With[{q = Sqrt[8*a^2 + b^2 - 4*a*c], A = Coeff[P
3, x, 0], B = Coeff[P3, x, 1], C = Coeff[P3, x, 2], D = Coeff[P3, x, 3]}, D
ist[1/q, Int[(x^m*(b*A - 2*a*B + 2*a*D + A*q + (2*a*A - 2*a*C + b*D + D*q)*
x))/(2*a + (b + q)*x + 2*a*x^2), x], x] - Dist[1/q, Int[(x^m*(b*A - 2*a*B +
 2*a*D - A*q + (2*a*A - 2*a*C + b*D - D*q)*x))/(2*a + (b - q)*x + 2*a*x^2),
 x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[P3, x, 3] && EqQ[a, e] && EqQ[b
, d]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
```

```
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{5+x+3x^2+2x^3}{x\left(2+x+3x^2+x^3+2x^4\right)}\,dx = -\left(\frac{1}{3}\int \frac{-6+4x}{x\left(4-2x+4x^2\right)}\,dx\right)+\frac{1}{3}\int \frac{24+16x}{x\left(4+4x+4x^2\right)}\,dx$$
$$= \frac{1}{3}\int \left(\frac{6}{x}-\frac{2(1+3x)}{1+x+x^2}\right)dx-\frac{1}{3}\int \left(-\frac{3}{2x}+\frac{1+6x}{2\left(2-x+2x^2\right)}\right)dx$$
$$= \frac{5\log(x)}{2}-\frac{1}{6}\int \frac{1+6x}{2-x+2x^2}\,dx-\frac{2}{3}\int \frac{1+3x}{1+x+x^2}\,dx$$
$$= \frac{5\log(x)}{2}-\frac{1}{4}\int \frac{-1+4x}{2-x+2x^2}\,dx+\frac{1}{3}\int \frac{1}{1+x+x^2}\,dx-\frac{5}{12}\int \frac{1}{2-x+2x^2}\,dx-\int \frac{1}{1\ldots}$$
$$= \frac{5\log(x)}{2}-\log\left(1+x+x^2\right)-\frac{1}{4}\log\left(2-x+2x^2\right)-\frac{2}{3}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2\ldots\right.$$
$$= \frac{1}{6}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{2\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{5\log(x)}{2}-\log\left(1+x+x^2\right)-\frac{1}{4}\log\left(2-\ldots\right.$$

Mathematica [A] time = 0.0198001, size = 69, normalized size = 0.92

$$\frac{1}{36}\left(-36\log\left(x^2+x+1\right)-9\log\left(2x^2-x+2\right)+90\log(x)+8\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)-2\sqrt{15}\tan^{-1}\left(\frac{4x-1}{\sqrt{15}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + x + 3*x^2 + 2*x^3)/(x*(2 + x + 3*x^2 + x^3 + 2*x^4)),x]
```

```
[Out] (8*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - 2*Sqrt[15]*ArcTan[(-1 + 4*x)/Sqrt[15
]] + 90*Log[x] - 36*Log[1 + x + x^2] - 9*Log[2 - x + 2*x^2])/36
```

Maple [A] time = 0.006, size = 60, normalized size = 0.8

$$\frac{5\ln(x)}{2}-\frac{\ln\left(2x^2-x+2\right)}{4}-\frac{\sqrt{15}}{18}\arctan\left(\frac{(-1+4x)\sqrt{15}}{15}\right)-\ln\left(x^2+x+1\right)+\frac{2\sqrt{3}}{9}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3+3*x^2+x+5)/x/(2*x^4+x^3+3*x^2+x+2),x)
```

```
[Out] 5/2*ln(x)-1/4*ln(2*x^2-x+2)-1/18*15^(1/2)*arctan(1/15*(-1+4*x)*15^(1/2))-ln
(x^2+x+1)+2/9*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Maxima [A] time = 1.50376, size = 80, normalized size = 1.07

$$-\frac{1}{18}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right)+\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{4}\log\left(2x^2-x+2\right)-\log\left(x^2+x+1\right)+\frac{5}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/x/(2*x^4+x^3+3*x^2+x+2),x, algorithm="maxima")
```

```
[Out] -1/18*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) + 2/9*sqrt(3)*arctan(1/3*sqr
t(3)*(2*x + 1)) - 1/4*log(2*x^2 - x + 2) - log(x^2 + x + 1) + 5/2*log(x)
```

Fricas [A] time = 1.49135, size = 220, normalized size = 2.93

$$-\frac{1}{18}\sqrt{5}\sqrt{3}\arctan\left(\frac{1}{15}\sqrt{5}\sqrt{3}(4x-1)\right)+\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{4}\log\left(2x^2-x+2\right)-\log\left(x^2+x+1\right)+\frac{5}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/x/(2*x^4+x^3+3*x^2+x+2),x, algorithm="fricas")
```

```
[Out] -1/18*sqrt(5)*sqrt(3)*arctan(1/15*sqrt(5)*sqrt(3)*(4*x - 1)) + 2/9*sqrt(3)*
arctan(1/3*sqrt(3)*(2*x + 1)) - 1/4*log(2*x^2 - x + 2) - log(x^2 + x + 1) +
 5/2*log(x)
```

Sympy [A] time = 0.249895, size = 78, normalized size = 1.04

$$\frac{5\log(x)}{2}-\frac{\log\left(x^2-\frac{x}{2}+1\right)}{4}-\log\left(x^2+x+1\right)-\frac{\sqrt{15}\operatorname{atan}\left(\frac{4\sqrt{15}x}{15}-\frac{\sqrt{15}}{15}\right)}{18}+\frac{2\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**3+3*x**2+x+5)/x/(2*x**4+x**3+3*x**2+x+2),x)
```

```
[Out] 5*log(x)/2 - log(x**2 - x/2 + 1)/4 - log(x**2 + x + 1) - sqrt(15)*atan(4*sq
rt(15)*x/15 - sqrt(15)/15)/18 + 2*sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/9
```

Giac [A] time = 1.23436, size = 81, normalized size = 1.08

$$-\frac{1}{18}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right)+\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{4}\log\left(2x^2-x+2\right)-\log\left(x^2+x+1\right)+\frac{5}{2}\log(|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/x/(2*x^4+x^3+3*x^2+x+2),x, algorithm="giac")
```

```
[Out] -1/18*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) + 2/9*sqrt(3)*arctan(1/3*sqr
t(3)*(2*x + 1)) - 1/4*log(2*x^2 - x + 2) - log(x^2 + x + 1) + 5/2*log(abs(x
))
```

3.248 $\int \frac{5+x+3x^2+2x^3}{x^2(2+x+3x^2+x^3+2x^4)}\,dx$

Optimal. Leaf size=84

$$\frac{1}{3}\log\left(x^2+x+1\right)+\frac{1}{24}\log\left(2x^2-x+2\right)-\frac{5}{2x}-\frac{3\log(x)}{4}+\frac{5}{12}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)-\frac{10\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] -5/(2*x) + (5*Sqrt[5/3]*ArcTan[(1 - 4*x)/Sqrt[15]])/12 - (10*ArcTan[(1 + 2*
x)/Sqrt[3]])/(3*Sqrt[3]) - (3*Log[x])/4 + Log[1 + x + x^2]/3 + Log[2 - x +
2*x^2]/24
```

Rubi [A] time = 0.151957, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2087, 800, 634, 618, 204, 628}

$$\frac{1}{3}\log\left(x^2+x+1\right)+\frac{1}{24}\log\left(2x^2-x+2\right)-\frac{5}{2x}-\frac{3\log(x)}{4}+\frac{5}{12}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)-\frac{10\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + x + 3*x^2 + 2*x^3)/(x^2*(2 + x + 3*x^2 + x^3 + 2*x^4)),x]
```

```
[Out] -5/(2*x) + (5*Sqrt[5/3]*ArcTan[(1 - 4*x)/Sqrt[15]])/12 - (10*ArcTan[(1 + 2*
x)/Sqrt[3]])/(3*Sqrt[3]) - (3*Log[x])/4 + Log[1 + x + x^2]/3 + Log[2 - x +
2*x^2]/24
```

Rule 2087

```
Int[((P3_)*(x_)^(m_.))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (
e_.)*(x_)^4), x_Symbol] :> With[{q = Sqrt[8*a^2 + b^2 - 4*a*c], A = Coeff[P
3, x, 0], B = Coeff[P3, x, 1], C = Coeff[P3, x, 2], D = Coeff[P3, x, 3]}, D
ist[1/q, Int[(x^m*(b*A - 2*a*B + 2*a*D + A*q + (2*a*A - 2*a*C + b*D + D*q)*
x))/(2*a + (b + q)*x + 2*a*x^2), x], x] - Dist[1/q, Int[(x^m*(b*A - 2*a*B +
 2*a*D - A*q + (2*a*A - 2*a*C + b*D - D*q)*x))/(2*a + (b - q)*x + 2*a*x^2),
 x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[P3, x, 3] && EqQ[a, e] && EqQ[b
, d]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{5+x+3x^2+2x^3}{x^2(2+x+3x^2+x^3+2x^4)}\,dx = -\left(\frac{1}{3}\int \frac{-6+4x}{x^2(4-2x+4x^2)}\,dx\right)+\frac{1}{3}\int \frac{24+16x}{x^2(4+4x+4x^2)}\,dx$$
$$= \frac{1}{3}\int\left(\frac{6}{x^2}-\frac{2}{x}+\frac{2(-2+x)}{1+x+x^2}\right)dx-\frac{1}{3}\int\left(-\frac{3}{2x^2}+\frac{1}{4x}+\frac{13-2x}{4(2-x+2x^2)}\right)dx$$
$$= -\frac{5}{2x}-\frac{3\log(x)}{4}-\frac{1}{12}\int\frac{13-2x}{2-x+2x^2}\,dx+\frac{2}{3}\int\frac{-2+x}{1+x+x^2}\,dx$$
$$= -\frac{5}{2x}-\frac{3\log(x)}{4}+\frac{1}{24}\int\frac{-1+4x}{2-x+2x^2}\,dx+\frac{1}{3}\int\frac{1+2x}{1+x+x^2}\,dx-\frac{25}{24}\int\frac{1}{2-x+2x^2}\,d$$
$$= -\frac{5}{2x}-\frac{3\log(x)}{4}+\frac{1}{3}\log(1+x+x^2)+\frac{1}{24}\log(2-x+2x^2)+\frac{25}{12}\text{Subst}\left(\int\frac{1}{-15-}\right.$$
$$= -\frac{5}{2x}+\frac{5}{12}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)-\frac{10\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{3\log(x)}{4}+\frac{1}{3}\log(1+x+x^2)+$$

Mathematica [A] time = 0.0332324, size = 78, normalized size = 0.93

$$-\frac{-24x\log(x^2+x+1)-3x\log(2x^2-x+2)+54x\log(x)+80\sqrt{3}x\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)+10\sqrt{15}x\tan^{-1}\left(\frac{4x-1}{\sqrt{15}}\right)+180}{72x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + x + 3*x^2 + 2*x^3)/(x^2*(2 + x + 3*x^2 + x^3 + 2*x^4)),x]
```

```
[Out] -(180 + 80*Sqrt[3]*x*ArcTan[(1 + 2*x)/Sqrt[3]] + 10*Sqrt[15]*x*ArcTan[(-1 +
 4*x)/Sqrt[15]] + 54*x*Log[x] - 24*x*Log[1 + x + x^2] - 3*x*Log[2 - x + 2*x
^2])/(72*x)
```

Maple [A] time = 0.007, size = 65, normalized size = 0.8

$$-\frac{5}{2x}-\frac{3\ln(x)}{4}+\frac{\ln(2x^2-x+2)}{24}-\frac{5\sqrt{15}}{36}\arctan\left(\frac{(-1+4x)\sqrt{15}}{15}\right)+\frac{\ln(x^2+x+1)}{3}-\frac{10\sqrt{3}}{9}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3+3*x^2+x+5)/x^2/(2*x^4+x^3+3*x^2+x+2),x)
```

```
[Out] -5/2/x-3/4*ln(x)+1/24*ln(2*x^2-x+2)-5/36*15^(1/2)*arctan(1/15*(-1+4*x)*15^(
1/2))+1/3*ln(x^2+x+1)-10/9*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Maxima [A] time = 1.51129, size = 86, normalized size = 1.02

$$-\frac{5}{36}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right)-\frac{10}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{5}{2x}+\frac{1}{24}\log\left(2x^2-x+2\right)+\frac{1}{3}\log(x^2+x+1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/x^2/(2*x^4+x^3+3*x^2+x+2),x, algorithm="maxima"
)
```

```
[Out] -5/36*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) - 10/9*sqrt(3)*arctan(1/3*sq
rt(3)*(2*x + 1)) - 5/2/x + 1/24*log(2*x^2 - x + 2) + 1/3*log(x^2 + x + 1) -
 3/4*log(x)
```

Fricas [A] time = 1.40958, size = 250, normalized size = 2.98

$$-\frac{10\sqrt{5}\sqrt{3}x\arctan\left(\frac{1}{15}\sqrt{5}\sqrt{3}(4x-1)\right)+80\sqrt{3}x\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-3x\log\left(2x^2-x+2\right)-24x\log(x^2+x+1}{72x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/x^2/(2*x^4+x^3+3*x^2+x+2),x, algorithm="fricas"
)
```

```
[Out] -1/72*(10*sqrt(5)*sqrt(3)*x*arctan(1/15*sqrt(5)*sqrt(3)*(4*x - 1)) + 80*sqr
t(3)*x*arctan(1/3*sqrt(3)*(2*x + 1)) - 3*x*log(2*x^2 - x + 2) - 24*x*log(x^
2 + x + 1) + 54*x*log(x) + 180)/x
```

Sympy [A] time = 0.275504, size = 87, normalized size = 1.04

$$-\frac{3\log(x)}{4}+\frac{\log\left(x^2-\frac{x}{2}+1\right)}{24}+\frac{\log\left(x^2+x+1\right)}{3}-\frac{5\sqrt{15}\operatorname{atan}\left(\frac{4\sqrt{15}x}{15}-\frac{\sqrt{15}}{15}\right)}{36}-\frac{10\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{9}-\frac{5}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**3+3*x**2+x+5)/x**2/(2*x**4+x**3+3*x**2+x+2),x)
```

```
[Out] -3*log(x)/4 + log(x**2 - x/2 + 1)/24 + log(x**2 + x + 1)/3 - 5*sqrt(15)*ata
n(4*sqrt(15)*x/15 - sqrt(15)/15)/36 - 10*sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(
3)/3)/9 - 5/(2*x)
```

Giac [A] time = 1.19833, size = 88, normalized size = 1.05

$$-\frac{5}{36}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right)-\frac{10}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{5}{2x}+\frac{1}{24}\log\left(2x^2-x+2\right)+\frac{1}{3}\log\left(x^2+x+1\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/x^2/(2*x^4+x^3+3*x^2+x+2),x, algorithm="giac")
```

```
[Out] -5/36*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) - 10/9*sqrt(3)*arctan(1/3*sq
rt(3)*(2*x + 1)) - 5/2/x + 1/24*log(2*x^2 - x + 2) + 1/3*log(x^2 + x + 1) -
 3/4*log(abs(x))
```

3.249 $\int \frac{5+x+3x^2+2x^3}{x^3(2+x+3x^2+x^3+2x^4)}\,dx$

Optimal. Leaf size=91

$$-\frac{5}{4x^2}+\frac{2}{3}\log\left(x^2+x+1\right)+\frac{13}{48}\log\left(2x^2-x+2\right)+\frac{3}{4x}-\frac{15\log(x)}{8}+\frac{1}{24}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{8\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] -5/(4*x^2) + 3/(4*x) + (Sqrt[5/3]*ArcTan[(1 - 4*x)/Sqrt[15]])/24 + (8*ArcTa
n[(1 + 2*x)/Sqrt[3]])/(3*Sqrt[3]) - (15*Log[x])/8 + (2*Log[1 + x + x^2])/3
+ (13*Log[2 - x + 2*x^2])/48
```

Rubi [A] time = 0.156817, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2087, 800, 634, 618, 204, 628}

$$-\frac{5}{4x^2}+\frac{2}{3}\log\left(x^2+x+1\right)+\frac{13}{48}\log\left(2x^2-x+2\right)+\frac{3}{4x}-\frac{15\log(x)}{8}+\frac{1}{24}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{8\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + x + 3*x^2 + 2*x^3)/(x^3*(2 + x + 3*x^2 + x^3 + 2*x^4)),x]
```


```
[Out] -5/(4*x^2) + 3/(4*x) + (Sqrt[5/3]*ArcTan[(1 - 4*x)/Sqrt[15]])/24 + (8*ArcTa
n[(1 + 2*x)/Sqrt[3]])/(3*Sqrt[3]) - (15*Log[x])/8 + (2*Log[1 + x + x^2])/3
+ (13*Log[2 - x + 2*x^2])/48
```


Rule 2087

```
Int[((P3_)*(x_)^(m_.))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (
e_.)*(x_)^4), x_Symbol] :> With[{q = Sqrt[8*a^2 + b^2 - 4*a*c], A = Coeff[P
3, x, 0], B = Coeff[P3, x, 1], C = Coeff[P3, x, 2], D = Coeff[P3, x, 3]}, D
ist[1/q, Int[(x^m*(b*A - 2*a*B + 2*a*D + A*q + (2*a*A - 2*a*C + b*D + D*q)*
x))/(2*a + (b + q)*x + 2*a*x^2), x], x] - Dist[1/q, Int[(x^m*(b*A - 2*a*B +
 2*a*D - A*q + (2*a*A - 2*a*C + b*D - D*q)*x))/(2*a + (b - q)*x + 2*a*x^2),
 x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[P3, x, 3] && EqQ[a, e] && EqQ[b
, d]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{5+x+3x^2+2x^3}{x^3\left(2+x+3x^2+x^3+2x^4\right)}\,dx = -\left(\frac{1}{3}\int \frac{-6+4x}{x^3\left(4-2x+4x^2\right)}\,dx\right)+\frac{1}{3}\int \frac{24+16x}{x^3\left(4+4x+4x^2\right)}\,dx$$

$$= \frac{1}{3}\int \left(\frac{6}{x^3}-\frac{2}{x^2}-\frac{4}{x}+\frac{2(3+2x)}{1+x+x^2}\right)dx-\frac{1}{3}\int \left(-\frac{3}{2x^3}+\frac{1}{4x^2}+\frac{13}{8x}+\frac{9-26x}{8\left(2-x+2x^2\right)}\right.$$

$$= -\frac{5}{4x^2}+\frac{3}{4x}-\frac{15\log(x)}{8}-\frac{1}{24}\int \frac{9-26x}{2-x+2x^2}\,dx+\frac{2}{3}\int \frac{3+2x}{1+x+x^2}\,dx$$

$$= -\frac{5}{4x^2}+\frac{3}{4x}-\frac{15\log(x)}{8}-\frac{5}{48}\int \frac{1}{2-x+2x^2}\,dx+\frac{13}{48}\int \frac{-1+4x}{2-x+2x^2}\,dx+\frac{2}{3}\int \frac{1}{1+}$$

$$= -\frac{5}{4x^2}+\frac{3}{4x}-\frac{15\log(x)}{8}+\frac{2}{3}\log\left(1+x+x^2\right)+\frac{13}{48}\log\left(2-x+2x^2\right)+\frac{5}{24}\,\mathrm{Subst}\Big($$

$$= -\frac{5}{4x^2}+\frac{3}{4x}+\frac{1}{24}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{8\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{15\log(x)}{8}+\frac{2}{3}\log\left(1+x\right.$$

Mathematica [A] time = 0.0547384, size = 82, normalized size = 0.9

$$\frac{1}{144}\left(3\left(-\frac{60}{x^2}+32\log\left(x^2+x+1\right)+13\log\left(2x^2-x+2\right)+\frac{36}{x}-90\log(x)\right)+128\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)-2\sqrt{15}\tan^{-1}\left(\frac{4x}{}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + x + 3*x^2 + 2*x^3)/(x^3*(2 + x + 3*x^2 + x^3 + 2*x^4)),x]
```

```
[Out] (128*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - 2*Sqrt[15]*ArcTan[(-1 + 4*x)/Sqrt[
15]] + 3*(-60/x^2 + 36/x - 90*Log[x] + 32*Log[1 + x + x^2] + 13*Log[2 - x +
 2*x^2]))/144
```

Maple [A] time = 0.008, size = 70, normalized size = 0.8

$$-\frac{5}{4x^2}+\frac{3}{4x}-\frac{15\ln(x)}{8}+\frac{13\ln\left(2x^2-x+2\right)}{48}-\frac{\sqrt{15}}{72}\arctan\left(\frac{(-1+4x)\sqrt{15}}{15}\right)+\frac{2\ln\left(x^2+x+1\right)}{3}+\frac{8\sqrt{3}}{9}\arctan\left((1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3+3*x^2+x+5)/x^3/(2*x^4+x^3+3*x^2+x+2),x)
```

```
[Out] -5/4/x^2+3/4/x-15/8*ln(x)+13/48*ln(2*x^2-x+2)-1/72*15^(1/2)*arctan(1/15*(-1
+4*x)*15^(1/2))+2/3*ln(x^2+x+1)+8/9*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Maxima [A] time = 1.4885, size = 93, normalized size = 1.02

$$-\frac{1}{72}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right)+\frac{8}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{3x-5}{4x^2}+\frac{13}{48}\log\left(2x^2-x+2\right)+\frac{2}{3}\log\left(x^2+x+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/x^3/(2*x^4+x^3+3*x^2+x+2),x, algorithm="maxima"
)
```

```
[Out] -1/72*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) + 8/9*sqrt(3)*arctan(1/3*sqr
t(3)*(2*x + 1)) + 1/4*(3*x - 5)/x^2 + 13/48*log(2*x^2 - x + 2) + 2/3*log(x^
2 + x + 1) - 15/8*log(x)
```

Fricas [A] time = 1.30494, size = 281, normalized size = 3.09

$$-\frac{2\sqrt{5}\sqrt{3}x^2\arctan\left(\frac{1}{15}\sqrt{5}\sqrt{3}(4x-1)\right)-128\sqrt{3}x^2\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-39x^2\log\left(2x^2-x+2\right)-96x^2\log\left(x^2+x\right.}{144x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/x^3/(2*x^4+x^3+3*x^2+x+2),x, algorithm="fricas"
)
```

```
[Out] -1/144*(2*sqrt(5)*sqrt(3)*x^2*arctan(1/15*sqrt(5)*sqrt(3)*(4*x - 1)) - 128*
sqrt(3)*x^2*arctan(1/3*sqrt(3)*(2*x + 1)) - 39*x^2*log(2*x^2 - x + 2) - 96*
x^2*log(x^2 + x + 1) + 270*x^2*log(x) - 108*x + 180)/x^2
```

Sympy [A] time = 0.28945, size = 94, normalized size = 1.03

$$-\frac{15\log(x)}{8}+\frac{13\log\left(x^2-\frac{x}{2}+1\right)}{48}+\frac{2\log\left(x^2+x+1\right)}{3}-\frac{\sqrt{15}\operatorname{atan}\left(\frac{4\sqrt{15}x}{15}-\frac{\sqrt{15}}{15}\right)}{72}+\frac{8\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{9}+\frac{3x-5}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**3+3*x**2+x+5)/x**3/(2*x**4+x**3+3*x**2+x+2),x)
```

```
[Out] -15*log(x)/8 + 13*log(x**2 - x/2 + 1)/48 + 2*log(x**2 + x + 1)/3 - sqrt(15)
*atan(4*sqrt(15)*x/15 - sqrt(15)/15)/72 + 8*sqrt(3)*atan(2*sqrt(3)*x/3 + sq
rt(3)/3)/9 + (3*x - 5)/(4*x**2)
```

Giac [A] time = 1.13334, size = 95, normalized size = 1.04

$$-\frac{1}{72}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right)+\frac{8}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{3x-5}{4x^2}+\frac{13}{48}\log\left(2x^2-x+2\right)+\frac{2}{3}\log\left(x^2+x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/x^3/(2*x^4+x^3+3*x^2+x+2),x, algorithm="giac")
```

```
[Out] -1/72*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) + 8/9*sqrt(3)*arctan(1/3*sqr
t(3)*(2*x + 1)) + 1/4*(3*x - 5)/x^2 + 13/48*log(2*x^2 - x + 2) + 2/3*log(x^
2 + x + 1) - 15/8*log(abs(x))
```

3.250 $\int \frac{x^3(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx$

Optimal. Leaf size=307

$$\frac{1}{42}\left(7+5i\sqrt{7}\right)x^3+\frac{1}{42}\left(7-5i\sqrt{7}\right)x^3+\frac{1}{28}\left(7+5i\sqrt{7}\right)x^2+\frac{1}{28}\left(7-5i\sqrt{7}\right)x^2+\frac{3}{112}\left(7-11i\sqrt{7}\right)\log\left(4x^2+\left(1-i\sqrt{7}\right)x+\right.$$

```
[Out] -((35 - (9*I)*Sqrt[7])*x)/28 - ((35 + (9*I)*Sqrt[7])*x)/28 + ((7 - (5*I)*Sq
rt[7])*x^2)/28 + ((7 + (5*I)*Sqrt[7])*x^2)/28 + ((7 - (5*I)*Sqrt[7])*x^3)/4
2 + ((7 + (5*I)*Sqrt[7])*x^3)/42 + (11*(9*I + 5*Sqrt[7])*ArcTan[(1 - I*Sqrt
[7] + 8*x)/Sqrt[2*(35 + I*Sqrt[7])]])/(4*Sqrt[14*(35 + I*Sqrt[7])]) - (11*(
9*I - 5*Sqrt[7])*ArcTan[(1 + I*Sqrt[7] + 8*x)/Sqrt[2*(35 - I*Sqrt[7])]])/(4
*Sqrt[14*(35 - I*Sqrt[7])]) + (3*(7 - (11*I)*Sqrt[7])*Log[4 + (1 - I*Sqrt[7
])*x + 4*x^2])/112 + (3*(7 + (11*I)*Sqrt[7])*Log[4 + (1 + I*Sqrt[7])*x + 4*
x^2])/112
```

Rubi [A] time = 0.578327, antiderivative size = 307, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2087, 800, 634, 618, 204, 628}

$$\frac{1}{42}\left(7+5i\sqrt{7}\right)x^3+\frac{1}{42}\left(7-5i\sqrt{7}\right)x^3+\frac{1}{28}\left(7+5i\sqrt{7}\right)x^2+\frac{1}{28}\left(7-5i\sqrt{7}\right)x^2+\frac{3}{112}\left(7-11i\sqrt{7}\right)\log\left(4x^2+\left(1-i\sqrt{7}\right)x+\right.$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 5*x^2 + x^3 + 2*x^4),x]
```


```
[Out] -((35 - (9*I)*Sqrt[7])*x)/28 - ((35 + (9*I)*Sqrt[7])*x)/28 + ((7 - (5*I)*Sq
rt[7])*x^2)/28 + ((7 + (5*I)*Sqrt[7])*x^2)/28 + ((7 - (5*I)*Sqrt[7])*x^3)/4
2 + ((7 + (5*I)*Sqrt[7])*x^3)/42 + (11*(9*I + 5*Sqrt[7])*ArcTan[(1 - I*Sqrt
[7] + 8*x)/Sqrt[2*(35 + I*Sqrt[7])]])/(4*Sqrt[14*(35 + I*Sqrt[7])]) - (11*(
9*I - 5*Sqrt[7])*ArcTan[(1 + I*Sqrt[7] + 8*x)/Sqrt[2*(35 - I*Sqrt[7])]])/(4
*Sqrt[14*(35 - I*Sqrt[7])]) + (3*(7 - (11*I)*Sqrt[7])*Log[4 + (1 - I*Sqrt[7
])*x + 4*x^2])/112 + (3*(7 + (11*I)*Sqrt[7])*Log[4 + (1 + I*Sqrt[7])*x + 4*
x^2])/112
```

Rule 2087

```
Int[((P3_)*(x_)^(m_.))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (
e_.)*(x_)^4), x_Symbol] :> With[{q = Sqrt[8*a^2 + b^2 - 4*a*c], A = Coeff[P
3, x, 0], B = Coeff[P3, x, 1], C = Coeff[P3, x, 2], D = Coeff[P3, x, 3]}, D
ist[1/q, Int[(x^m*(b*A - 2*a*B + 2*a*D + A*q + (2*a*A - 2*a*C + b*D + D*q)*
x))/(2*a + (b + q)*x + 2*a*x^2), x], x] - Dist[1/q, Int[(x^m*(b*A - 2*a*B +
 2*a*D - A*q + (2*a*A - 2*a*C + b*D - D*q)*x))/(2*a + (b - q)*x + 2*a*x^2),
 x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[P3, x, 3] && EqQ[a, e] && EqQ[b
, d]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^3\left(5+x+3x^2+2x^3\right)}{2+x+5x^2+x^3+2x^4}\,dx = \frac{i\int \frac{x^3\left(9-5i\sqrt{7}+\left(10-2i\sqrt{7}\right)x\right)}{4+\left(1-i\sqrt{7}\right)x+4x^2}\,dx}{\sqrt{7}} - \frac{i\int \frac{x^3\left(9+5i\sqrt{7}+\left(10+2i\sqrt{7}\right)x\right)}{4+\left(1+i\sqrt{7}\right)x+4x^2}\,dx}{\sqrt{7}}$$

$$= \frac{i\int \left(\frac{1}{4}\left(-9+5i\sqrt{7}\right)+\frac{1}{2}\left(5-i\sqrt{7}\right)x+\frac{1}{2}\left(5-i\sqrt{7}\right)x^2+\frac{2\left(9-5i\sqrt{7}\right)-3\left(11+i\sqrt{7}\right)x}{2\left(4+\left(1-i\sqrt{7}\right)x+4x^2\right)}\right)dx}{\sqrt{7}} - \frac{i\int\left(\frac{1}{4}\left(\right.\right.}{}$$

$$= -\frac{1}{28}\left(35-9i\sqrt{7}\right)x-\frac{1}{28}\left(35+9i\sqrt{7}\right)x+\frac{1}{28}\left(7-5i\sqrt{7}\right)x^2+\frac{1}{28}\left(7+5i\sqrt{7}\right)x^2+\frac{1}{42}\left(7\right.$$

$$= -\frac{1}{28}\left(35-9i\sqrt{7}\right)x-\frac{1}{28}\left(35+9i\sqrt{7}\right)x+\frac{1}{28}\left(7-5i\sqrt{7}\right)x^2+\frac{1}{28}\left(7+5i\sqrt{7}\right)x^2+\frac{1}{42}\left(7\right.$$

$$= -\frac{1}{28}\left(35-9i\sqrt{7}\right)x-\frac{1}{28}\left(35+9i\sqrt{7}\right)x+\frac{1}{28}\left(7-5i\sqrt{7}\right)x^2+\frac{1}{28}\left(7+5i\sqrt{7}\right)x^2+\frac{1}{42}\left(7\right.$$

$$= -\frac{1}{28}\left(35-9i\sqrt{7}\right)x-\frac{1}{28}\left(35+9i\sqrt{7}\right)x+\frac{1}{28}\left(7-5i\sqrt{7}\right)x^2+\frac{1}{28}\left(7+5i\sqrt{7}\right)x^2+\frac{1}{42}\left(7\right.$$

Mathematica [C] time = 0.0191559, size = 109, normalized size = 0.36

$$\frac{1}{6}\left(3\text{RootSum}\left[2\#1^4+\#1^3+5\#1^2+\#1+2\&,\frac{3\#1^3\log(x-\#1)+19\#1^2\log(x-\#1)+\#1\log(x-\#1)+10\log(x-\#1)}{8\#1^3+3\#1^2+10\#1+1}\&\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 5*x^2 + x^3 + 2*x^4),x]
```

```
[Out] (x*(-15 + 3*x + 2*x^2) + 3*RootSum[2 + #1 + 5*#1^2 + #1^3 + 2*#1^4 & , (10*
Log[x - #1] + Log[x - #1]*#1 + 19*Log[x - #1]*#1^2 + 3*Log[x - #1]*#1^3)/(1
 + 10*#1 + 3*#1^2 + 8*#1^3) & ])/6
```

Maple [C] time = 0.007, size = 74, normalized size = 0.2

$$\frac{x^3}{3}+\frac{x^2}{2}-\frac{5x}{2}+\frac{1}{2}\sum_{\_R=\text{RootOf}\left(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2\right)}\frac{\left(3\_R^3+19\_R^2+\_R+10\right)\ln(x-\_R)}{8\_R^3+3\_R^2+10\_R+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x)
```

```
[Out] 1/3*x^3+1/2*x^2-5/2*x+1/2*sum((3*_R^3+19*_R^2+_R+10)/(8*_R^3+3*_R^2+10*_R+1
)*ln(x-_R),_R=RootOf(2*_Z^4+_Z^3+5*_Z^2+_Z+2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{3}x^3+\frac{1}{2}x^2-\frac{5}{2}x+\frac{1}{2}\int\frac{3x^3+19x^2+x+10}{2x^4+x^3+5x^2+x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="maxima"
)
```

```
[Out] 1/3*x^3 + 1/2*x^2 - 5/2*x + 1/2*integrate((3*x^3 + 19*x^2 + x + 10)/(2*x^4
+ x^3 + 5*x^2 + x + 2), x)
```

Fricas [B] time = 9.70013, size = 5511, normalized size = 17.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="fricas"
)
```

```
[Out] 1/3*x^3 + 1/2*x^2 - 1/112*(-33*I*sqrt(7) + 56*sqrt(2101/1568*I*sqrt(7) - 55
/32) - 21)*log(23324*(33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/
32) + 3/16)^3 - 23765*(33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55
/32) + 3/16)^2 + 7744*x + 19470*I*sqrt(7) - 33040*sqrt(2101/1568*I*sqrt(7)
- 55/32) + 38950) - 1/112*(33*I*sqrt(7) + 56*sqrt(-2101/1568*I*sqrt(7) - 55
/32) - 21)*log(-23324*(33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55
/32) + 3/16)^3 + 49/4*(-33/112*I*sqrt(7) - 1/2*sqrt(-2101/1568*I*sqrt(7) -
55/32) + 3/16)^2*(-561*I*sqrt(7) + 952*sqrt(2101/1568*I*sqrt(7) - 55/32) -
869) + 1/256*(53312*(33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/3
2) + 3/16)^2 - 11781*I*sqrt(7) + 19992*sqrt(2101/1568*I*sqrt(7) - 55/32) -
```

36681)*(33*I*sqrt(7) + 56*sqrt(-2101/1568*I*sqrt(7) - 55/32) - 21) + 17493*
(33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/32) + 3/16)^2 + 7744*
x - 15708*I*sqrt(7) + 26656*sqrt(2101/1568*I*sqrt(7) - 55/32) - 29132) + 1/
112*(2*sqrt(7)*sqrt(-336*(33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) -
55/32) + 3/16)^2 - 336*(-33/112*I*sqrt(7) - 1/2*sqrt(-2101/1568*I*sqrt(7)
- 55/32) + 3/16)^2 - 1/56*(33*I*sqrt(7) + 56*sqrt(-2101/1568*I*sqrt(7) - 55
/32) - 21)*(-33*I*sqrt(7) + 56*sqrt(2101/1568*I*sqrt(7) - 55/32) + 63) + 99
/2*I*sqrt(7) - 84*sqrt(2101/1568*I*sqrt(7) - 55/32) - 1859/2) + 28*sqrt(210
1/1568*I*sqrt(7) - 55/32) + 28*sqrt(-2101/1568*I*sqrt(7) - 55/32) + 21)*log
(-49/4*(-33/112*I*sqrt(7) - 1/2*sqrt(-2101/1568*I*sqrt(7) - 55/32) + 3/16)^
2*(-561*I*sqrt(7) + 952*sqrt(2101/1568*I*sqrt(7) - 55/32) - 869) - 1/256*(5
3312*(33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/32) + 3/16)^2 -
11781*I*sqrt(7) + 19992*sqrt(2101/1568*I*sqrt(7) - 55/32) - 36681)*(33*I*sq
rt(7) + 56*sqrt(-2101/1568*I*sqrt(7) - 55/32) - 21) + 6272*(33/112*I*sqrt(7
) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/32) + 3/16)^2 + 1/256*((17*sqrt(7)*(-
33*I*sqrt(7) + 56*sqrt(2101/1568*I*sqrt(7) - 55/32) - 21) - 512*sqrt(7))*(3
3*I*sqrt(7) + 56*sqrt(-2101/1568*I*sqrt(7) - 55/32) - 21) - 512*sqrt(7)*(-3
3*I*sqrt(7) + 56*sqrt(2101/1568*I*sqrt(7) - 55/32) - 21) + 73728*sqrt(7))*s
qrt(-336*(33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/32) + 3/16)^
2 - 336*(-33/112*I*sqrt(7) - 1/2*sqrt(-2101/1568*I*sqrt(7) - 55/32) + 3/16)
^2 - 1/56*(33*I*sqrt(7) + 56*sqrt(-2101/1568*I*sqrt(7) - 55/32) - 21)*(-33*
I*sqrt(7) + 56*sqrt(2101/1568*I*sqrt(7) - 55/32) + 63) + 99/2*I*sqrt(7) - 8
4*sqrt(2101/1568*I*sqrt(7) - 55/32) - 1859/2) + 15488*x - 3762*I*sqrt(7) +
6384*sqrt(2101/1568*I*sqrt(7) - 55/32) - 5946) - 1/112*(2*sqrt(7)*sqrt(-336
*(33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/32) + 3/16)^2 - 336*
(-33/112*I*sqrt(7) - 1/2*sqrt(-2101/1568*I*sqrt(7) - 55/32) + 3/16)^2 - 1/5
6*(33*I*sqrt(7) + 56*sqrt(-2101/1568*I*sqrt(7) - 55/32) - 21)*(-33*I*sqrt(7
) + 56*sqrt(2101/1568*I*sqrt(7) - 55/32) + 63) + 99/2*I*sqrt(7) - 84*sqrt(2
101/1568*I*sqrt(7) - 55/32) - 1859/2) - 28*sqrt(2101/1568*I*sqrt(7) - 55/32
) - 28*sqrt(-2101/1568*I*sqrt(7) - 55/32) - 21)*log(-49/4*(-33/112*I*sqrt(7
) - 1/2*sqrt(-2101/1568*I*sqrt(7) - 55/32) + 3/16)^2*(-561*I*sqrt(7) + 952*
sqrt(2101/1568*I*sqrt(7) - 55/32) - 869) - 1/256*(53312*(33/112*I*sqrt(7) -
1/2*sqrt(2101/1568*I*sqrt(7) - 55/32) + 3/16)^2 - 11781*I*sqrt(7) + 19992*
sqrt(2101/1568*I*sqrt(7) - 55/32) - 36681)*(33*I*sqrt(7) + 56*sqrt(-2101/15
68*I*sqrt(7) - 55/32) - 21) + 6272*(33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I
sqrt(7) - 55/32) + 3/16)^2 - 1/256((17*sqrt(7)*(-33*I*sqrt(7) + 56*sqrt(2
101/1568*I*sqrt(7) - 55/32) - 21) - 512*sqrt(7))*(33*I*sqrt(7) + 56*sqrt(-2
101/1568*I*sqrt(7) - 55/32) - 21) - 512*sqrt(7)*(-33*I*sqrt(7) + 56*sqrt(21
01/1568*I*sqrt(7) - 55/32) - 21) + 73728*sqrt(7))*sqrt(-336*(33/112*I*sqrt(
7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/32) + 3/16)^2 - 336*(-33/112*I*sqrt(
7) - 1/2*sqrt(-2101/1568*I*sqrt(7) - 55/32) + 3/16)^2 - 1/56*(33*I*sqrt(7)
+ 56*sqrt(-2101/1568*I*sqrt(7) - 55/32) - 21)*(-33*I*sqrt(7) + 56*sqrt(2101
/1568*I*sqrt(7) - 55/32) + 63) + 99/2*I*sqrt(7) - 84*sqrt(2101/1568*I*sqrt(
7) - 55/32) - 1859/2) + 15488*x - 3762*I*sqrt(7) + 6384*sqrt(2101/1568*I*sq
rt(7) - 55/32) - 5946) - 5/2*x

Sympy [A] time = 0.802154, size = 61, normalized size = 0.2

$$\frac{x^3}{3} + \frac{x^2}{2} - \frac{5x}{2} + \text{RootSum}\left(1372t^4 - 1029t^3 + 3136t^2 + 2688t + 512, \left(t \mapsto t\log\left(\frac{5831t^3}{1936} - \frac{23765t^2}{7744} + \frac{2065t}{242} + x + \frac{41}{12}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(2*x**3+3*x**2+x+5)/(2*x**4+x**3+5*x**2+x+2),x)
```

```
[Out] x**3/3 + x**2/2 - 5*x/2 + RootSum(1372*_t**4 - 1029*_t**3 + 3136*_t**2 + 26
88*_t + 512, Lambda(_t, _t*log(5831*_t**3/1936 - 23765*_t**2/7744 + 2065*_t
/242 + x + 415/121)))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2x^3 + 3x^2 + x + 5)x^3}{2x^4 + x^3 + 5x^2 + x + 2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="giac")
```

```
[Out] integrate((2*x^3 + 3*x^2 + x + 5)*x^3/(2*x^4 + x^3 + 5*x^2 + x + 2), x)
```

3.251 $\int \frac{x^2(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx$

Optimal. Leaf size=269

$$\frac{1}{28}\left(7+5i\sqrt{7}\right)x^2+\frac{1}{28}\left(7-5i\sqrt{7}\right)x^2-\frac{1}{56}\left(35+9i\sqrt{7}\right)\log\left(4x^2+\left(1-i\sqrt{7}\right)x+4\right)-\frac{1}{56}\left(35-9i\sqrt{7}\right)\log\left(4x^2+\left(1+i\sqrt{\right.\right.$$

```
[Out] ((7 - (5*I)*Sqrt[7])*x)/14 + ((7 + (5*I)*Sqrt[7])*x)/14 + ((7 - (5*I)*Sqrt[
7])*x^2)/28 + ((7 + (5*I)*Sqrt[7])*x^2)/28 - ((53*I + Sqrt[7])*ArcTan[(1 -
I*Sqrt[7] + 8*x)/Sqrt[2*(35 + I*Sqrt[7])]])/(2*Sqrt[14*(35 + I*Sqrt[7])]) +
 ((53*I - Sqrt[7])*ArcTan[(1 + I*Sqrt[7] + 8*x)/Sqrt[2*(35 - I*Sqrt[7])]])/
(2*Sqrt[14*(35 - I*Sqrt[7])]) - ((35 + (9*I)*Sqrt[7])*Log[4 + (1 - I*Sqrt[7
])*x + 4*x^2])/56 - ((35 - (9*I)*Sqrt[7])*Log[4 + (1 + I*Sqrt[7])*x + 4*x^2
])/56
```

Rubi [A] time = 0.392077, antiderivative size = 269, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2087, 800, 634, 618, 204, 628}

$$\frac{1}{28}\left(7+5i\sqrt{7}\right)x^2+\frac{1}{28}\left(7-5i\sqrt{7}\right)x^2-\frac{1}{56}\left(35+9i\sqrt{7}\right)\log\left(4x^2+\left(1-i\sqrt{7}\right)x+4\right)-\frac{1}{56}\left(35-9i\sqrt{7}\right)\log\left(4x^2+\left(1+i\sqrt{\right.\right.$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 5*x^2 + x^3 + 2*x^4),x]
```

```
[Out] ((7 - (5*I)*Sqrt[7])*x)/14 + ((7 + (5*I)*Sqrt[7])*x)/14 + ((7 - (5*I)*Sqrt[
7])*x^2)/28 + ((7 + (5*I)*Sqrt[7])*x^2)/28 - ((53*I + Sqrt[7])*ArcTan[(1 -
I*Sqrt[7] + 8*x)/Sqrt[2*(35 + I*Sqrt[7])]])/(2*Sqrt[14*(35 + I*Sqrt[7])]) +
 ((53*I - Sqrt[7])*ArcTan[(1 + I*Sqrt[7] + 8*x)/Sqrt[2*(35 - I*Sqrt[7])]])/
(2*Sqrt[14*(35 - I*Sqrt[7])]) - ((35 + (9*I)*Sqrt[7])*Log[4 + (1 - I*Sqrt[7
])*x + 4*x^2])/56 - ((35 - (9*I)*Sqrt[7])*Log[4 + (1 + I*Sqrt[7])*x + 4*x^2
])/56
```

Rule 2087

```
Int[((P3_)*(x_)^(m_.))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (
e_.)*(x_)^4), x_Symbol] :> With[{q = Sqrt[8*a^2 + b^2 - 4*a*c], A = Coeff[P
3, x, 0], B = Coeff[P3, x, 1], C = Coeff[P3, x, 2], D = Coeff[P3, x, 3]}, D
ist[1/q, Int[(x^m*(b*A - 2*a*B + 2*a*D + A*q + (2*a*A - 2*a*C + b*D + D*q)*
x))/(2*a + (b + q)*x + 2*a*x^2), x], x] - Dist[1/q, Int[(x^m*(b*A - 2*a*B +
 2*a*D - A*q + (2*a*A - 2*a*C + b*D - D*q)*x))/(2*a + (b - q)*x + 2*a*x^2),
 x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[P3, x, 3] && EqQ[a, e] && EqQ[b
, d]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^2\left(5+x+3x^2+2x^3\right)}{2+x+5x^2+x^3+2x^4}\,dx = \frac{i\int \frac{x^2\left(9-5i\sqrt{7}+\left(10-2i\sqrt{7}\right)x\right)}{4+\left(1-i\sqrt{7}\right)x+4x^2}\,dx}{\sqrt{7}} - \frac{i\int \frac{x^2\left(9+5i\sqrt{7}+\left(10+2i\sqrt{7}\right)x\right)}{4+\left(1+i\sqrt{7}\right)x+4x^2}\,dx}{\sqrt{7}}$$

$$= \frac{i\int \left(\frac{1}{2}\left(5-i\sqrt{7}\right)+\frac{1}{2}\left(5-i\sqrt{7}\right)x+\frac{i\left(2\left(5i+\sqrt{7}\right)+\left(9i+5\sqrt{7}\right)x\right)}{4+\left(1-i\sqrt{7}\right)x+4x^2}\right)dx}{\sqrt{7}} - \frac{i\int \left(\frac{1}{2}\left(5+i\sqrt{7}\right)+\frac{1}{2}\left(5+i\right.\right.}{}$$

$$= \frac{1}{14}\left(7-5i\sqrt{7}\right)x+\frac{1}{14}\left(7+5i\sqrt{7}\right)x+\frac{1}{28}\left(7-5i\sqrt{7}\right)x^2+\frac{1}{28}\left(7+5i\sqrt{7}\right)x^2-\frac{\int \frac{2\left(5i+\sqrt{7}\right)+}{4+\left(1-i\right.}}{}$$

$$= \frac{1}{14}\left(7-5i\sqrt{7}\right)x+\frac{1}{14}\left(7+5i\sqrt{7}\right)x+\frac{1}{28}\left(7-5i\sqrt{7}\right)x^2+\frac{1}{28}\left(7+5i\sqrt{7}\right)x^2-\frac{1}{56}\left(35-9\right.$$

$$= \frac{1}{14}\left(7-5i\sqrt{7}\right)x+\frac{1}{14}\left(7+5i\sqrt{7}\right)x+\frac{1}{28}\left(7-5i\sqrt{7}\right)x^2+\frac{1}{28}\left(7+5i\sqrt{7}\right)x^2-\frac{1}{56}\left(35+9\right.$$

$$= \frac{1}{14}\left(7-5i\sqrt{7}\right)x+\frac{1}{14}\left(7+5i\sqrt{7}\right)x+\frac{1}{28}\left(7-5i\sqrt{7}\right)x^2+\frac{1}{28}\left(7+5i\sqrt{7}\right)x^2-\frac{\left(53i+\sqrt{7}\right.}{2\sqrt{1}}$$

Mathematica [C] time = 0.0163307, size = 101, normalized size = 0.38

$$-\text{RootSum}\left[2\#1^4+\#1^3+5\#1^2+\#1+2\&,\frac{5\#1^3\log(x-\#1)+\#1^2\log(x-\#1)+3\#1\log(x-\#1)+2\log(x-\#1)}{8\#1^3+3\#1^2+10\#1+1}\&\right]+$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 5*x^2 + x^3 + 2*x^4),x]
```

```
[Out] x + x^2/2 - RootSum[2 + #1 + 5*#1^2 + #1^3 + 2*#1^4 & , (2*Log[x - #1] + 3*
Log[x - #1]*#1 + Log[x - #1]*#1^2 + 5*Log[x - #1]*#1^3)/(1 + 10*#1 + 3*#1^2
 + 8*#1^3) & ]
```

Maple [C] time = 0.006, size = 67, normalized size = 0.3

$$\frac{x^2}{2}+x+\sum_{\_R=\text{RootOf}\left(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2\right)}\frac{\left(-5\_R^3-\_R^2-3\_R-2\right)\ln\left(x-\_R\right)}{8\_R^3+3\_R^2+10\_R+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x)
```

```
[Out] 1/2*x^2+x+sum((-5*_R^3-_R^2-3*_R-2)/(8*_R^3+3*_R^2+10*_R+1)*ln(x-_R),_R=Roo
tOf(2*_Z^4+_Z^3+5*_Z^2+_Z+2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}x^2 + x - \int \frac{5x^3 + x^2 + 3x + 2}{2x^4 + x^3 + 5x^2 + x + 2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="maxima"
)
```

```
[Out] 1/2*x^2 + x - integrate((5*x^3 + x^2 + 3*x + 2)/(2*x^4 + x^3 + 5*x^2 + x +
2), x)
```

Fricas [B] time = 9.30163, size = 5019, normalized size = 18.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="fricas"
)
```

```
[Out] 1/2*x^2 - 1/56*(-9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/56) + 35)*log(
49/4*(135*I*sqrt(7) + 420*sqrt(-37/392*I*sqrt(7) + 79/56) - 1459)*(9/56*I*s
qrt(7) - 1/2*sqrt(37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 10290*(-9/56*I*sqrt(
7) - 1/2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5/8)^3 - 25725*(-9/56*I*sqrt(7)
- 1/2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5/8)^2 + 3/64*(3920*(-9/56*I*sqrt(7
) - 1/2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 1575*I*sqrt(7) - 4900*sq
rt(-37/392*I*sqrt(7) + 79/56) + 5587)*(-9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt
(7) + 79/56) + 35) + 8384*x + 6615/2*I*sqrt(7) + 10290*sqrt(-37/392*I*sqrt(
7) + 79/56) + 13373/2) + 1/8*(2*sqrt(-12*(9/56*I*sqrt(7) - 1/2*sqrt(37/392*
I*sqrt(7) + 79/56) - 5/8)^2 - 12*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt
```

```
(7) + 79/56) - 5/8)^2 - 1/392*(9*I*sqrt(7) + 28*sqrt(-37/392*I*sqrt(7) + 79
/56) - 105)*(-9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/56) + 35) + 45/14
*I*sqrt(7) + 10*sqrt(-37/392*I*sqrt(7) + 79/56) + 11/2) + 2*sqrt(37/392*I*s
qrt(7) + 79/56) + 2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5)*log(-49/4*(135*I*s
qrt(7) + 420*sqrt(-37/392*I*sqrt(7) + 79/56) - 1459)*(9/56*I*sqrt(7) - 1/2*
sqrt(37/392*I*sqrt(7) + 79/56) - 5/8)^2 + 24304*(-9/56*I*sqrt(7) - 1/2*sqrt
(-37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 3/64*(3920*(-9/56*I*sqrt(7) - 1/2*sq
rt(-37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 1575*I*sqrt(7) - 4900*sqrt(-37/392
*I*sqrt(7) + 79/56) + 5587)*(-9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/5
6) + 35) + 7/64*sqrt(-12*(9/56*I*sqrt(7) - 1/2*sqrt(37/392*I*sqrt(7) + 79/5
6) - 5/8)^2 - 12*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5
/8)^2 - 1/392*(9*I*sqrt(7) + 28*sqrt(-37/392*I*sqrt(7) + 79/56) - 105)*(-9*
I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/56) + 35) + 45/14*I*sqrt(7) + 10*
sqrt(-37/392*I*sqrt(7) + 79/56) + 11/2)*((135*I*sqrt(7) + 420*sqrt(-37/392*
I*sqrt(7) + 79/56) - 1459)*(-9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/56
) + 35) - 17856*I*sqrt(7) - 55552*sqrt(-37/392*I*sqrt(7) + 79/56) + 67776)
+ 16768*x - 4941*I*sqrt(7) - 15372*sqrt(-37/392*I*sqrt(7) + 79/56) - 9391)
- 1/8*(2*sqrt(-12*(9/56*I*sqrt(7) - 1/2*sqrt(37/392*I*sqrt(7) + 79/56) - 5/
8)^2 - 12*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5/8)^2 -
 1/392*(9*I*sqrt(7) + 28*sqrt(-37/392*I*sqrt(7) + 79/56) - 105)*(-9*I*sqrt(
7) + 28*sqrt(37/392*I*sqrt(7) + 79/56) + 35) + 45/14*I*sqrt(7) + 10*sqrt(-3
7/392*I*sqrt(7) + 79/56) + 11/2) - 2*sqrt(37/392*I*sqrt(7) + 79/56) - 2*sqr
t(-37/392*I*sqrt(7) + 79/56) + 5)*log(-49/4*(135*I*sqrt(7) + 420*sqrt(-37/3
92*I*sqrt(7) + 79/56) - 1459)*(9/56*I*sqrt(7) - 1/2*sqrt(37/392*I*sqrt(7) +
 79/56) - 5/8)^2 + 24304*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) + 79
/56) - 5/8)^2 - 3/64*(3920*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) +
79/56) - 5/8)^2 - 1575*I*sqrt(7) - 4900*sqrt(-37/392*I*sqrt(7) + 79/56) + 5
587)*(-9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/56) + 35) - 7/64*sqrt(-1
2*(9/56*I*sqrt(7) - 1/2*sqrt(37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 12*(-9/56
*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 1/392*(9*I*sqrt
(7) + 28*sqrt(-37/392*I*sqrt(7) + 79/56) - 105)*(-9*I*sqrt(7) + 28*sqrt(37/
392*I*sqrt(7) + 79/56) + 35) + 45/14*I*sqrt(7) + 10*sqrt(-37/392*I*sqrt(7)
+ 79/56) + 11/2)*((135*I*sqrt(7) + 420*sqrt(-37/392*I*sqrt(7) + 79/56) - 14
59)*(-9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/56) + 35) - 17856*I*sqrt(
7) - 55552*sqrt(-37/392*I*sqrt(7) + 79/56) + 67776) + 16768*x - 4941*I*sqrt
(7) - 15372*sqrt(-37/392*I*sqrt(7) + 79/56) - 9391) - 1/56*(9*I*sqrt(7) + 2
8*sqrt(-37/392*I*sqrt(7) + 79/56) + 35)*log(10290*(-9/56*I*sqrt(7) - 1/2*sq
rt(-37/392*I*sqrt(7) + 79/56) - 5/8)^3 + 1421*(-9/56*I*sqrt(7) - 1/2*sqrt(-
37/392*I*sqrt(7) + 79/56) - 5/8)^2 + 8384*x + 3267/2*I*sqrt(7) + 5082*sqrt(
-37/392*I*sqrt(7) + 79/56) + 13793/2) + x
```

Sympy [A] time = 0.72476, size = 53, normalized size = 0.2

$$\frac{x^2}{2} + x + \operatorname{RootSum}\left(686t^4 + 1715t^3 + 1372t^2 + 448t + 256, \left(t \mapsto t\log\left(\frac{5145t^3}{4192} + \frac{1421t^2}{8384} - \frac{2541t}{2096} + x + \frac{17}{262}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(2*x**3+3*x**2+x+5)/(2*x**4+x**3+5*x**2+x+2),x)
```

```
[Out] x**2/2 + x + RootSum(686*_t**4 + 1715*_t**3 + 1372*_t**2 + 448*_t + 256, La
mbda(_t, _t*log(5145*_t**3/4192 + 1421*_t**2/8384 - 2541*_t/2096 + x + 17/2
62)))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2x^3 + 3x^2 + x + 5)x^2}{2x^4 + x^3 + 5x^2 + x + 2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="giac")
```

```
[Out] integrate((2*x^3 + 3*x^2 + x + 5)*x^2/(2*x^4 + x^3 + 5*x^2 + x + 2), x)
```

3.252 $\int \frac{x(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx$

Optimal. Leaf size=230

$$\frac{1}{28}\left(7+5i\sqrt{7}\right)\log\left(4x^2+\left(1-i\sqrt{7}\right)x+4\right)+\frac{1}{28}\left(7-5i\sqrt{7}\right)\log\left(4x^2+\left(1+i\sqrt{7}\right)x+4\right)+\frac{1}{14}\left(7+5i\sqrt{7}\right)x+\frac{1}{14}\left(7-5i\right.$$

```
[Out] ((7 - (5*I)*Sqrt[7])*x)/14 + ((7 + (5*I)*Sqrt[7])*x)/14 - ((19*I + 7*Sqrt[7
])*ArcTan[(1 - I*Sqrt[7] + 8*x)/Sqrt[2*(35 + I*Sqrt[7])]])/Sqrt[14*(35 + I*
Sqrt[7])] + ((19*I - 7*Sqrt[7])*ArcTan[(1 + I*Sqrt[7] + 8*x)/Sqrt[2*(35 - I
*Sqrt[7])]])/Sqrt[14*(35 - I*Sqrt[7])] + ((7 + (5*I)*Sqrt[7])*Log[4 + (1 -
I*Sqrt[7])*x + 4*x^2])/28 + ((7 - (5*I)*Sqrt[7])*Log[4 + (1 + I*Sqrt[7])*x
+ 4*x^2])/28
```

Rubi [A] time = 0.357268, antiderivative size = 230, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2087, 773, 634, 618, 204, 628}

$$\frac{1}{28}\left(7+5i\sqrt{7}\right)\log\left(4x^2+\left(1-i\sqrt{7}\right)x+4\right)+\frac{1}{28}\left(7-5i\sqrt{7}\right)\log\left(4x^2+\left(1+i\sqrt{7}\right)x+4\right)+\frac{1}{14}\left(7+5i\sqrt{7}\right)x+\frac{1}{14}\left(7-5i\right.$$

Antiderivative was successfully verified.

```
[In] Int[(x*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 5*x^2 + x^3 + 2*x^4),x]
```


```
[Out] ((7 - (5*I)*Sqrt[7])*x)/14 + ((7 + (5*I)*Sqrt[7])*x)/14 - ((19*I + 7*Sqrt[7
])*ArcTan[(1 - I*Sqrt[7] + 8*x)/Sqrt[2*(35 + I*Sqrt[7])]])/Sqrt[14*(35 + I*
Sqrt[7])] + ((19*I - 7*Sqrt[7])*ArcTan[(1 + I*Sqrt[7] + 8*x)/Sqrt[2*(35 - I
*Sqrt[7])]])/Sqrt[14*(35 - I*Sqrt[7])] + ((7 + (5*I)*Sqrt[7])*Log[4 + (1 -
I*Sqrt[7])*x + 4*x^2])/28 + ((7 - (5*I)*Sqrt[7])*Log[4 + (1 + I*Sqrt[7])*x
+ 4*x^2])/28
```


Rule 2087

```
Int[((P3_)*(x_)^(m_.))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (
e_.)*(x_)^4), x_Symbol] :> With[{q = Sqrt[8*a^2 + b^2 - 4*a*c], A = Coeff[P
3, x, 0], B = Coeff[P3, x, 1], C = Coeff[P3, x, 2], D = Coeff[P3, x, 3]}, D
```

```
ist[1/q, Int[(x^m*(b*A - 2*a*B + 2*a*D + A*q + (2*a*A - 2*a*C + b*D + D*q)*
x))/(2*a + (b + q)*x + 2*a*x^2), x], x] - Dist[1/q, Int[(x^m*(b*A - 2*a*B +
 2*a*D - A*q + (2*a*A - 2*a*C + b*D - D*q)*x))/(2*a + (b - q)*x + 2*a*x^2),
 x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[P3, x, 3] && EqQ[a, e] && EqQ[b
, d]
```

Rule 773

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx = \frac{i\int \frac{x(9-5i\sqrt{7}+(10-2i\sqrt{7})x)}{4+(1-i\sqrt{7})x+4x^2}\,dx}{\sqrt{7}} - \frac{i\int \frac{x(9+5i\sqrt{7}+(10+2i\sqrt{7})x)}{4+(1+i\sqrt{7})x+4x^2}\,dx}{\sqrt{7}}$$

$$= \frac{1}{14}\left(7-5i\sqrt{7}\right)x + \frac{1}{14}\left(7+5i\sqrt{7}\right)x + \frac{i\int \frac{-4(10-2i\sqrt{7})+(-(1-i\sqrt{7})(10-2i\sqrt{7})+4(9-5i\sqrt{7}))x}{4+(1-i\sqrt{7})x+4x^2}\,dx}{4\sqrt{7}} - \frac{i\int}{}$$

$$= \frac{1}{14}\left(7-5i\sqrt{7}\right)x + \frac{1}{14}\left(7+5i\sqrt{7}\right)x - \frac{1}{28}\left(-7+5i\sqrt{7}\right)\int \frac{1+i\sqrt{7}+8x}{4+\left(1+i\sqrt{7}\right)x+4x^2}\,dx + \frac{1}{28}$$

$$= \frac{1}{14}\left(7-5i\sqrt{7}\right)x + \frac{1}{14}\left(7+5i\sqrt{7}\right)x + \frac{1}{28}\left(7+5i\sqrt{7}\right)\log\left(4+\left(1-i\sqrt{7}\right)x+4x^2\right) + \frac{1}{28}\left(7\right.$$

$$= \frac{1}{14}\left(7-5i\sqrt{7}\right)x + \frac{1}{14}\left(7+5i\sqrt{7}\right)x - \frac{\left(19i+7\sqrt{7}\right)\tan^{-1}\left(\frac{1-i\sqrt{7}+8x}{\sqrt{2(35+i\sqrt{7})}}\right)}{\sqrt{14\left(35+i\sqrt{7}\right)}} + \frac{\left(19i-7\sqrt{7}\right)\tan}{\sqrt{14\left(35\right.}}$$

Mathematica [C] time = 0.0143922, size = 94, normalized size = 0.41

$$2\text{RootSum}\left[2\#1^4+\#1^3+5\#1^2+\#1+2\&,\frac{\#1^3\log(x-\#1)-2\#1^2\log(x-\#1)+2\#1\log(x-\#1)-\log(x-\#1)}{8\#1^3+3\#1^2+10\#1+1}\&\right]+x$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 5*x^2 + x^3 + 2*x^4),x]
```

```
[Out] x + 2*RootSum[2 + #1 + 5*#1^2 + #1^3 + 2*#1^4 & , (-Log[x - #1] + 2*Log[x -
 #1]*#1 - 2*Log[x - #1]*#1^2 + Log[x - #1]*#1^3)/(1 + 10*#1 + 3*#1^2 + 8*#1
^3) & ]
```

Maple [C] time = 0.004, size = 62, normalized size = 0.3

$$x+2\sum_{\_R=\text{RootOf}\left(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2\right)}\frac{\left(\_R^3-2\_R^2+2\_R-1\right)\ln\left(x-\_R\right)}{8\_R^3+3\_R^2+10\_R+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x)
```

```
[Out] x+2*sum((_R^3-2*_R^2+2*_R-1)/(8*_R^3+3*_R^2+10*_R+1)*ln(x-_R),_R=RootOf(2*_
Z^4+_Z^3+5*_Z^2+_Z+2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$x+2\int \frac{x^3-2x^2+2x-1}{2x^4+x^3+5x^2+x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="maxima")
```

```
[Out] x + 2*integrate((x^3 - 2*x^2 + 2*x - 1)/(2*x^4 + x^3 + 5*x^2 + x + 2), x)
```

Fricas [B] time = 9.29867, size = 4852, normalized size = 21.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="fricas")
```

```
[Out] -1/28*(-5*I*sqrt(7) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) - 7)*log(49/4*(55*I*
sqrt(7) + 154*sqrt(53/98*I*sqrt(7) - 1/14) + 147)*(5/28*I*sqrt(7) - 1/2*sqr
t(-53/98*I*sqrt(7) - 1/14) + 1/4)^2 - 3773*(-5/28*I*sqrt(7) - 1/2*sqrt(53/9
8*I*sqrt(7) - 1/14) + 1/4)^3 + 3773*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqr
t(7) - 1/14) + 1/4)^2 + 11/16*(196*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt
(7) - 1/14) + 1/4)^2 + 35*I*sqrt(7) + 98*sqrt(53/98*I*sqrt(7) - 1/14) + 15)
*(-5*I*sqrt(7) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) - 7) + 304*x + 1155/2*I*s
qrt(7) + 1617*sqrt(53/98*I*sqrt(7) - 1/14) + 1903/2) + 1/28*(2*sqrt(7)*sqrt
(-21*(5/28*I*sqrt(7) - 1/2*sqrt(-53/98*I*sqrt(7) - 1/14) + 1/4)^2 - 21*(-5/
28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 - 1/56*(5*I*sqrt(7
) + 14*sqrt(53/98*I*sqrt(7) - 1/14) + 21)*(-5*I*sqrt(7) + 14*sqrt(-53/98*I*
sqrt(7) - 1/14) - 7) - 5/2*I*sqrt(7) - 7*sqrt(53/98*I*sqrt(7) - 1/14) - 27/
2) + 7*sqrt(53/98*I*sqrt(7) - 1/14) + 7*sqrt(-53/98*I*sqrt(7) - 1/14) + 7)*
log(-49/4*(55*I*sqrt(7) + 154*sqrt(53/98*I*sqrt(7) - 1/14) + 147)*(5/28*I*s
qrt(7) - 1/2*sqrt(-53/98*I*sqrt(7) - 1/14) + 1/4)^2 - 2744*(-5/28*I*sqrt(7)
 - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 - 11/16*(196*(-5/28*I*sqrt(7)
- 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 + 35*I*sqrt(7) + 98*sqrt(53/98*
I*sqrt(7) - 1/14) + 15)*(-5*I*sqrt(7) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) -
```

```
7) + 1/16*sqrt(-21*(5/28*I*sqrt(7) - 1/2*sqrt(-53/98*I*sqrt(7) - 1/14) + 1/
4)^2 - 21*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 - 1/
56*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(7) - 1/14) + 21)*(-5*I*sqrt(7) + 14*
sqrt(-53/98*I*sqrt(7) - 1/14) - 7) - 5/2*I*sqrt(7) - 7*sqrt(53/98*I*sqrt(7)
 - 1/14) - 27/2)*((11*sqrt(7)*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(7) - 1/14
) - 7) + 224*sqrt(7))*(-5*I*sqrt(7) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) - 7)
 + 224*sqrt(7)*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(7) - 1/14) - 7) + 3456*s
qrt(7)) + 608*x - 220*I*sqrt(7) - 616*sqrt(53/98*I*sqrt(7) - 1/14) + 636) -
 1/28*(2*sqrt(7)*sqrt(-21*(5/28*I*sqrt(7) - 1/2*sqrt(-53/98*I*sqrt(7) - 1/1
4) + 1/4)^2 - 21*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)
^2 - 1/56*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(7) - 1/14) + 21)*(-5*I*sqrt(7
) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) - 7) - 5/2*I*sqrt(7) - 7*sqrt(53/98*I*
sqrt(7) - 1/14) - 27/2) - 7*sqrt(53/98*I*sqrt(7) - 1/14) - 7*sqrt(-53/98*I*
sqrt(7) - 1/14) - 7)*log(-49/4*(55*I*sqrt(7) + 154*sqrt(53/98*I*sqrt(7) - 1
/14) + 147)*(5/28*I*sqrt(7) - 1/2*sqrt(-53/98*I*sqrt(7) - 1/14) + 1/4)^2 -
2744*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 - 11/16*(
196*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 + 35*I*sqr
t(7) + 98*sqrt(53/98*I*sqrt(7) - 1/14) + 15)*(-5*I*sqrt(7) + 14*sqrt(-53/98
*I*sqrt(7) - 1/14) - 7) - 1/16*sqrt(-21*(5/28*I*sqrt(7) - 1/2*sqrt(-53/98*I
*sqrt(7) - 1/14) + 1/4)^2 - 21*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7)
- 1/14) + 1/4)^2 - 1/56*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(7) - 1/14) + 21
)*(-5*I*sqrt(7) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) - 7) - 5/2*I*sqrt(7) - 7
*sqrt(53/98*I*sqrt(7) - 1/14) - 27/2)*((11*sqrt(7)*(5*I*sqrt(7) + 14*sqrt(5
3/98*I*sqrt(7) - 1/14) - 7) + 224*sqrt(7))*(-5*I*sqrt(7) + 14*sqrt(-53/98*I
*sqrt(7) - 1/14) - 7) + 224*sqrt(7)*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(7)
- 1/14) - 7) + 3456*sqrt(7)) + 608*x - 220*I*sqrt(7) - 616*sqrt(53/98*I*sqr
t(7) - 1/14) + 636) - 1/28*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(7) - 1/14) -
 7)*log(3773*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^3 -
 1029*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 + 304*x
- 715/2*I*sqrt(7) - 1001*sqrt(53/98*I*sqrt(7) - 1/14) - 2871/2) + x
```

Sympy [A] time = 0.730912, size = 48, normalized size = 0.21

$$x + \operatorname{RootSum}\left(343t^4 - 343t^3 + 294t^2 - 336t + 128, \left(t \mapsto t\log\left(\frac{3773t^3}{304} - \frac{1029t^2}{304} + \frac{1001t}{152} + x - \frac{121}{19}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*x**3+3*x**2+x+5)/(2*x**4+x**3+5*x**2+x+2),x)
```

```
[Out] x + RootSum(343*_t**4 - 343*_t**3 + 294*_t**2 - 336*_t + 128, Lambda(_t, _t
*log(3773*_t**3/304 - 1029*_t**2/304 + 1001*_t/152 + x - 121/19)))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2x^3 + 3x^2 + x + 5)x}{2x^4 + x^3 + 5x^2 + x + 2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="giac")
```

```
[Out] integrate((2*x^3 + 3*x^2 + x + 5)*x/(2*x^4 + x^3 + 5*x^2 + x + 2), x)
```

3.253 $\int \frac{5+x+3x^2+2x^3}{2+x+5x^2+x^3+2x^4} \, dx$

Optimal. Leaf size=198

$$\frac{1}{28}\left(7+5i\sqrt{7}\right)\log\left(4x^2+\left(1-i\sqrt{7}\right)x+4\right)+\frac{1}{28}\left(7-5i\sqrt{7}\right)\log\left(4x^2+\left(1+i\sqrt{7}\right)x+4\right)+\frac{\left(7\sqrt{7}+19i\right)\tan^{-1}\left(\frac{8x-i\sqrt{7}+1}{\sqrt{2\left(35+i\sqrt{7}\right)}}\right)}{\sqrt{14\left(35+i\sqrt{7}\right)}}$$

```
[Out] ((19*I + 7*Sqrt[7])*ArcTan[(1 - I*Sqrt[7] + 8*x)/Sqrt[2*(35 + I*Sqrt[7])]])
/Sqrt[14*(35 + I*Sqrt[7])] - ((19*I - 7*Sqrt[7])*ArcTan[(1 + I*Sqrt[7] + 8*
x)/Sqrt[2*(35 - I*Sqrt[7])]])/Sqrt[14*(35 - I*Sqrt[7])] + ((7 + (5*I)*Sqrt[
7])*Log[4 + (1 - I*Sqrt[7])*x + 4*x^2])/28 + ((7 - (5*I)*Sqrt[7])*Log[4 + (
1 + I*Sqrt[7])*x + 4*x^2])/28
```

Rubi [A] time = 0.194412, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {2086, 634, 618, 204, 628}

$$\frac{1}{28}\left(7+5i\sqrt{7}\right)\log\left(4x^2+\left(1-i\sqrt{7}\right)x+4\right)+\frac{1}{28}\left(7-5i\sqrt{7}\right)\log\left(4x^2+\left(1+i\sqrt{7}\right)x+4\right)+\frac{\left(7\sqrt{7}+19i\right)\tan^{-1}\left(\frac{8x-i\sqrt{7}+1}{\sqrt{2\left(35+i\sqrt{7}\right)}}\right)}{\sqrt{14\left(35+i\sqrt{7}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + x + 3*x^2 + 2*x^3)/(2 + x + 5*x^2 + x^3 + 2*x^4),x]
```

```
[Out] ((19*I + 7*Sqrt[7])*ArcTan[(1 - I*Sqrt[7] + 8*x)/Sqrt[2*(35 + I*Sqrt[7])]])
/Sqrt[14*(35 + I*Sqrt[7])] - ((19*I - 7*Sqrt[7])*ArcTan[(1 + I*Sqrt[7] + 8*
x)/Sqrt[2*(35 - I*Sqrt[7])]])/Sqrt[14*(35 - I*Sqrt[7])] + ((7 + (5*I)*Sqrt[
7])*Log[4 + (1 - I*Sqrt[7])*x + 4*x^2])/28 + ((7 - (5*I)*Sqrt[7])*Log[4 + (
1 + I*Sqrt[7])*x + 4*x^2])/28
```

Rule 2086

```
Int[(P3_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (e_.)*(x_)^4),
 x_Symbol] :> With[{q = Sqrt[8*a^2 + b^2 - 4*a*c], A = Coeff[P3, x, 0], B =
 Coeff[P3, x, 1], C = Coeff[P3, x, 2], D = Coeff[P3, x, 3]}, Dist[1/q, Int[
(b*A - 2*a*B + 2*a*D + A*q + (2*a*A - 2*a*C + b*D + D*q)*x)/(2*a + (b + q)*
x + 2*a*x^2), x], x] - Dist[1/q, Int[(b*A - 2*a*B + 2*a*D - A*q + (2*a*A -
```

```
2*a*C + b*D - D*q)*x)/(2*a + (b - q)*x + 2*a*x^2), x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[P3, x, 3] && EqQ[a, e] && EqQ[b, d]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{5+x+3x^2+2x^3}{2+x+5x^2+x^3+2x^4}\,dx = \frac{i\int \frac{9-5i\sqrt{7}+\left(10-2i\sqrt{7}\right)x}{4+\left(1-i\sqrt{7}\right)x+4x^2}\,dx}{\sqrt{7}} - \frac{i\int \frac{9+5i\sqrt{7}+\left(10+2i\sqrt{7}\right)x}{4+\left(1+i\sqrt{7}\right)x+4x^2}\,dx}{\sqrt{7}}$$

$$= -\left(\frac{1}{28}\left(-7+5i\sqrt{7}\right)\int \frac{1+i\sqrt{7}+8x}{4+\left(1+i\sqrt{7}\right)x+4x^2}\,dx\right) + \frac{1}{28}\left(7+5i\sqrt{7}\right)\int \frac{1-i\sqrt{7}+8x}{4+\left(1-i\sqrt{7}\right)x+4}$$

$$= \frac{1}{28}\left(7+5i\sqrt{7}\right)\log\left(4+\left(1-i\sqrt{7}\right)x+4x^2\right) + \frac{1}{28}\left(7-5i\sqrt{7}\right)\log\left(4+\left(1+i\sqrt{7}\right)x+4x^2\right)$$

$$= \frac{\left(19i+7\sqrt{7}\right)\tan^{-1}\left(\frac{1-i\sqrt{7}+8x}{\sqrt{2\left(35+i\sqrt{7}\right)}}\right)}{\sqrt{14\left(35+i\sqrt{7}\right)}} - \frac{\left(19i-7\sqrt{7}\right)\tan^{-1}\left(\frac{1+i\sqrt{7}+8x}{\sqrt{2\left(35-i\sqrt{7}\right)}}\right)}{\sqrt{14\left(35-i\sqrt{7}\right)}} + \frac{1}{28}\left(7+5i\sqrt{7}\right)\log\left(\right.$$

Mathematica [C] time = 0.0128284, size = 90, normalized size = 0.45

$$\text{RootSum}\left[2\#1^4+\#1^3+5\#1^2+\#1+2\&, \frac{2\#1^3\log(x-\#1)+3\#1^2\log(x-\#1)+\#1\log(x-\#1)+5\log(x-\#1)}{8\#1^3+3\#1^2+10\#1+1}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + x + 3*x^2 + 2*x^3)/(2 + x + 5*x^2 + x^3 + 2*x^4),x]
```

```
[Out] RootSum[2 + #1 + 5*#1^2 + #1^3 + 2*#1^4 & , (5*Log[x - #1] + Log[x - #1]*#1
 + 3*Log[x - #1]*#1^2 + 2*Log[x - #1]*#1^3)/(1 + 10*#1 + 3*#1^2 + 8*#1^3) &
 ]
```

Maple [C] time = 0.005, size = 58, normalized size = 0.3

$$\sum_{\_R=\text{RootOf}\left(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2\right)} \frac{\left(2\_R^3+3\_R^2+\_R+5\right)\ln\left(x-\_R\right)}{8\_R^3+3\_R^2+10\_R+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x)
```

```
[Out] sum((2*_R^3+3*_R^2+_R+5)/(8*_R^3+3*_R^2+10*_R+1)*ln(x-_R),_R=RootOf(2*_Z^4+
_Z^3+5*_Z^2+_Z+2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2x^3 + 3x^2 + x + 5}{2x^4 + x^3 + 5x^2 + x + 2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="maxima")
```

```
[Out] integrate((2*x^3 + 3*x^2 + x + 5)/(2*x^4 + x^3 + 5*x^2 + x + 2), x)
```

Fricas [B] time = 9.75783, size = 4867, normalized size = 24.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="fricas")
```

```
[Out] -1/28*(2*sqrt(7)*sqrt(-21*(5/28*I*sqrt(7) - 1/2*sqrt(-53/98*I*sqrt(7) - 1/1
4) + 1/4)^2 - 21*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)
^2 - 1/56*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(7) - 1/14) + 21)*(-5*I*sqrt(7
) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) - 7) - 5/2*I*sqrt(7) - 7*sqrt(53/98*I*
sqrt(7) - 1/14) - 27/2) - 7*sqrt(53/98*I*sqrt(7) - 1/14) - 7*sqrt(-53/98*I*
sqrt(7) - 1/14) - 7)*log(49/4*(105*I*sqrt(7) + 294*sqrt(53/98*I*sqrt(7) - 1
/14) + 253)*(5/28*I*sqrt(7) - 1/2*sqrt(-53/98*I*sqrt(7) - 1/14) + 1/4)^2 +
4900*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 + 1/16*(4
116*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 + 735*I*sq
rt(7) + 2058*sqrt(53/98*I*sqrt(7) - 1/14) + 11)*(-5*I*sqrt(7) + 14*sqrt(-53
/98*I*sqrt(7) - 1/14) - 7) + 1/16*sqrt(-21*(5/28*I*sqrt(7) - 1/2*sqrt(-53/9
8*I*sqrt(7) - 1/14) + 1/4)^2 - 21*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(
7) - 1/14) + 1/4)^2 - 1/56*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(7) - 1/14) +
 21)*(-5*I*sqrt(7) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) - 7) - 5/2*I*sqrt(7)
- 7*sqrt(53/98*I*sqrt(7) - 1/14) - 27/2)*((21*sqrt(7)*(5*I*sqrt(7) + 14*sqr
t(53/98*I*sqrt(7) - 1/14) - 7) + 400*sqrt(7))*(-5*I*sqrt(7) + 14*sqrt(-53/9
8*I*sqrt(7) - 1/14) - 7) + 400*sqrt(7)*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(
7) - 1/14) - 7) + 7040*sqrt(7)) + 608*x + 325*I*sqrt(7) + 910*sqrt(53/98*I*
sqrt(7) - 1/14) - 1247) + 1/28*(2*sqrt(7)*sqrt(-21*(5/28*I*sqrt(7) - 1/2*sq
rt(-53/98*I*sqrt(7) - 1/14) + 1/4)^2 - 21*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98
*I*sqrt(7) - 1/14) + 1/4)^2 - 1/56*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(7) -
```

```
 1/14) + 21)*(-5*I*sqrt(7) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) - 7) - 5/2*I*
sqrt(7) - 7*sqrt(53/98*I*sqrt(7) - 1/14) - 27/2) + 7*sqrt(53/98*I*sqrt(7) -
 1/14) + 7*sqrt(-53/98*I*sqrt(7) - 1/14) + 7)*log(49/4*(105*I*sqrt(7) + 294
*sqrt(53/98*I*sqrt(7) - 1/14) + 253)*(5/28*I*sqrt(7) - 1/2*sqrt(-53/98*I*sq
rt(7) - 1/14) + 1/4)^2 + 4900*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) -
 1/14) + 1/4)^2 + 1/16*(4116*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) -
1/14) + 1/4)^2 + 735*I*sqrt(7) + 2058*sqrt(53/98*I*sqrt(7) - 1/14) + 11)*(-
5*I*sqrt(7) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) - 7) - 1/16*sqrt(-21*(5/28*I
*sqrt(7) - 1/2*sqrt(-53/98*I*sqrt(7) - 1/14) + 1/4)^2 - 21*(-5/28*I*sqrt(7)
 - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 - 1/56*(5*I*sqrt(7) + 14*sqrt(
53/98*I*sqrt(7) - 1/14) + 21)*(-5*I*sqrt(7) + 14*sqrt(-53/98*I*sqrt(7) - 1/
14) - 7) - 5/2*I*sqrt(7) - 7*sqrt(53/98*I*sqrt(7) - 1/14) - 27/2)*((21*sqrt
(7)*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(7) - 1/14) - 7) + 400*sqrt(7))*(-5*
I*sqrt(7) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) - 7) + 400*sqrt(7)*(5*I*sqrt(7
) + 14*sqrt(53/98*I*sqrt(7) - 1/14) - 7) + 7040*sqrt(7)) + 608*x + 325*I*sq
rt(7) + 910*sqrt(53/98*I*sqrt(7) - 1/14) - 1247) - 1/28*(-5*I*sqrt(7) + 14*
sqrt(-53/98*I*sqrt(7) - 1/14) - 7)*log(-49/4*(105*I*sqrt(7) + 294*sqrt(53/9
8*I*sqrt(7) - 1/14) + 253)*(5/28*I*sqrt(7) - 1/2*sqrt(-53/98*I*sqrt(7) - 1/
14) + 1/4)^2 + 7203*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1
/4)^3 - 7203*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 -
 1/16*(4116*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 +
735*I*sqrt(7) + 2058*sqrt(53/98*I*sqrt(7) - 1/14) + 11)*(-5*I*sqrt(7) + 14*
sqrt(-53/98*I*sqrt(7) - 1/14) - 7) + 304*x - 2205/2*I*sqrt(7) - 3087*sqrt(5
3/98*I*sqrt(7) - 1/14) - 3025/2) - 1/28*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt
(7) - 1/14) - 7)*log(-7203*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/
14) + 1/4)^3 + 2303*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1
/4)^2 + 304*x + 1555/2*I*sqrt(7) + 2177*sqrt(53/98*I*sqrt(7) - 1/14) + 5823
/2)
```

Sympy [A] time = 0.716795, size = 46, normalized size = 0.23

$$\text{RootSum}\left(343t^4 - 343t^3 + 294t^2 - 336t + 128, \left(t \mapsto t\log\left(-\frac{7203t^3}{304} + \frac{2303t^2}{304} - \frac{2177t}{152} + x + \frac{250}{19}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**3+3*x**2+x+5)/(2*x**4+x**3+5*x**2+x+2),x)
```

```
[Out] RootSum(343*_t**4 - 343*_t**3 + 294*_t**2 - 336*_t + 128, Lambda(_t, _t*log
(-7203*_t**3/304 + 2303*_t**2/304 - 2177*_t/152 + x + 250/19)))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2x^3 + 3x^2 + x + 5}{2x^4 + x^3 + 5x^2 + x + 2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="giac")
```

```
[Out] integrate((2*x^3 + 3*x^2 + x + 5)/(2*x^4 + x^3 + 5*x^2 + x + 2), x)
```

3.254 $\int \frac{5+x+3x^2+2x^3}{x(2+x+5x^2+x^3+2x^4)}\,dx$

Optimal. Leaf size=245

$$-\frac{1}{56}\left(35-9i\sqrt{7}\right)\log\left(4ix^2+\left(-\sqrt{7}+i\right)x+4i\right)-\frac{1}{56}\left(35+9i\sqrt{7}\right)\log\left(4ix^2+\left(\sqrt{7}+i\right)x+4i\right)+\frac{1}{28}\left(35+9i\sqrt{7}\right)\log(x)+$$

```
[Out] -((53 + I*Sqrt[7])*ArcTanh[(I - Sqrt[7] + (8*I)*x)/Sqrt[2*(35 - I*Sqrt[7])]
])/(2*Sqrt[14*(35 - I*Sqrt[7])]) + ((53 - I*Sqrt[7])*ArcTanh[(I + Sqrt[7] +
 (8*I)*x)/Sqrt[2*(35 + I*Sqrt[7])]])/(2*Sqrt[14*(35 + I*Sqrt[7])]) + ((35 -
 (9*I)*Sqrt[7])*Log[x])/28 + ((35 + (9*I)*Sqrt[7])*Log[x])/28 - ((35 - (9*I
)*Sqrt[7])*Log[4*I + (I - Sqrt[7])*x + (4*I)*x^2])/56 - ((35 + (9*I)*Sqrt[7
])*Log[4*I + (I + Sqrt[7])*x + (4*I)*x^2])/56
```

Rubi [A] time = 0.473188, antiderivative size = 245, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2087, 800, 634, 618, 206, 628}

$$-\frac{1}{56}\left(35-9i\sqrt{7}\right)\log\left(4ix^2+\left(-\sqrt{7}+i\right)x+4i\right)-\frac{1}{56}\left(35+9i\sqrt{7}\right)\log\left(4ix^2+\left(\sqrt{7}+i\right)x+4i\right)+\frac{1}{28}\left(35+9i\sqrt{7}\right)\log(x)+$$

Antiderivative was successfully verified.

```
[In] Int[(5 + x + 3*x^2 + 2*x^3)/(x*(2 + x + 5*x^2 + x^3 + 2*x^4)),x]
```


```
[Out] -((53 + I*Sqrt[7])*ArcTanh[(I - Sqrt[7] + (8*I)*x)/Sqrt[2*(35 - I*Sqrt[7])]
])/(2*Sqrt[14*(35 - I*Sqrt[7])]) + ((53 - I*Sqrt[7])*ArcTanh[(I + Sqrt[7] +
 (8*I)*x)/Sqrt[2*(35 + I*Sqrt[7])]])/(2*Sqrt[14*(35 + I*Sqrt[7])]) + ((35 -
 (9*I)*Sqrt[7])*Log[x])/28 + ((35 + (9*I)*Sqrt[7])*Log[x])/28 - ((35 - (9*I
)*Sqrt[7])*Log[4*I + (I - Sqrt[7])*x + (4*I)*x^2])/56 - ((35 + (9*I)*Sqrt[7
])*Log[4*I + (I + Sqrt[7])*x + (4*I)*x^2])/56
```


Rule 2087

```
Int[((P3_)*(x_)^(m_.))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (
e_.)*(x_)^4), x_Symbol] :> With[{q = Sqrt[8*a^2 + b^2 - 4*a*c], A = Coeff[P
```

```
3, x, 0], B = Coeff[P3, x, 1], C = Coeff[P3, x, 2], D = Coeff[P3, x, 3]}, D
ist[1/q, Int[(x^m*(b*A - 2*a*B + 2*a*D + A*q + (2*a*A - 2*a*C + b*D + D*q)*
x))/(2*a + (b + q)*x + 2*a*x^2), x], x] - Dist[1/q, Int[(x^m*(b*A - 2*a*B +
 2*a*D - A*q + (2*a*A - 2*a*C + b*D - D*q)*x))/(2*a + (b - q)*x + 2*a*x^2),
 x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[P3, x, 3] && EqQ[a, e] && EqQ[b
, d]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{5+x+3x^2+2x^3}{x\left(2+x+5x^2+x^3+2x^4\right)}\,dx = \frac{i\int \frac{9-5i\sqrt{7}+\left(10-2i\sqrt{7}\right)x}{x\left(4+\left(1-i\sqrt{7}\right)x+4x^2\right)}\,dx}{\sqrt{7}} - \frac{i\int \frac{9+5i\sqrt{7}+\left(10+2i\sqrt{7}\right)x}{x\left(4+\left(1+i\sqrt{7}\right)x+4x^2\right)}\,dx}{\sqrt{7}}$$

$$= -\frac{i\int \left(\frac{9+5i\sqrt{7}}{4x}+\frac{3\left(11i+\sqrt{7}\right)-2\left(9i-5\sqrt{7}\right)x}{2\left(4i+\left(i-\sqrt{7}\right)x+4ix^2\right)}\right)dx}{\sqrt{7}} + \frac{i\int \left(\frac{9-5i\sqrt{7}}{4x}+\frac{3\left(11i-\sqrt{7}\right)-2\left(9i+5\sqrt{7}\right)x}{2\left(4i+\left(i+\sqrt{7}\right)x+4ix^2\right)}\right)dx}{\sqrt{7}}$$

$$= \frac{1}{28}\left(35-9i\sqrt{7}\right)\log(x) + \frac{1}{28}\left(35+9i\sqrt{7}\right)\log(x) - \frac{i\int \frac{3\left(11i+\sqrt{7}\right)-2\left(9i-5\sqrt{7}\right)x}{4i+\left(i-\sqrt{7}\right)x+4ix^2}\,dx}{2\sqrt{7}} + \frac{i\int \frac{3(1}{}}{}$$

$$= \frac{1}{28}\left(35-9i\sqrt{7}\right)\log(x) + \frac{1}{28}\left(35+9i\sqrt{7}\right)\log(x) - \frac{1}{56}\left(35-9i\sqrt{7}\right)\int \frac{i-\sqrt{7}+8}{4i+\left(i-\sqrt{7}\right)x}$$

$$= \frac{1}{28}\left(35-9i\sqrt{7}\right)\log(x) + \frac{1}{28}\left(35+9i\sqrt{7}\right)\log(x) - \frac{1}{56}\left(35-9i\sqrt{7}\right)\log\left(4i+\left(i-\sqrt{7}\right)\right.$$

$$= -\frac{\left(53+i\sqrt{7}\right)\tanh^{-1}\left(\frac{i-\sqrt{7}+8ix}{\sqrt{2\left(35-i\sqrt{7}\right)}}\right)}{2\sqrt{14\left(35-i\sqrt{7}\right)}} + \frac{\left(53-i\sqrt{7}\right)\tanh^{-1}\left(\frac{i+\sqrt{7}+8ix}{\sqrt{2\left(35+i\sqrt{7}\right)}}\right)}{2\sqrt{14\left(35+i\sqrt{7}\right)}} + \frac{1}{28}\left(35-9i\sqrt{\right.$$

Mathematica [C] time = 0.0174082, size = 101, normalized size = 0.41

$$\frac{5\log(x)}{2} - \frac{1}{2}\text{RootSum}\left[2\#1^4+\#1^3+5\#1^2+\#1+2\&,\frac{10\#1^3\log(x-\#1)+\#1^2\log(x-\#1)+19\#1\log(x-\#1)+3\log(}{8\#1^3+3\#1^2+10\#1+1}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + x + 3*x^2 + 2*x^3)/(x*(2 + x + 5*x^2 + x^3 + 2*x^4)),x]
```

```
[Out] (5*Log[x])/2 - RootSum[2 + #1 + 5*#1^2 + #1^3 + 2*#1^4 & , (3*Log[x - #1] +
 19*Log[x - #1]*#1 + Log[x - #1]*#1^2 + 10*Log[x - #1]*#1^3)/(1 + 10*#1 + 3
*#1^2 + 8*#1^3) & ]/2
```

Maple [C] time = 0.008, size = 67, normalized size = 0.3

$$\frac{5\ln(x)}{2} + \frac{1}{2}\sum_{\_R=\text{RootOf}\left(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2\right)} \frac{\left(-10\_R^3-\_R^2-19\_R-3\right)\ln(x-\_R)}{8\_R^3+3\_R^2+10\_R+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3+3*x^2+x+5)/x/(2*x^4+x^3+5*x^2+x+2),x)
```

```
[Out] 5/2*ln(x)+1/2*sum((-10*_R^3-_R^2-19*_R-3)/(8*_R^3+3*_R^2+10*_R+1)*ln(x-_R),
_R=RootOf(2*_Z^4+_Z^3+5*_Z^2+_Z+2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2}\int\frac{10\,x^3+x^2+19\,x+3}{2\,x^4+x^3+5\,x^2+x+2}\,dx+\frac{5}{2}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/x/(2*x^4+x^3+5*x^2+x+2),x, algorithm="maxima")
```

```
[Out] -1/2*integrate((10*x^3 + x^2 + 19*x + 3)/(2*x^4 + x^3 + 5*x^2 + x + 2), x)
+ 5/2*log(x)
```

Fricas [B] time = 9.44186, size = 5003, normalized size = 20.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/x/(2*x^4+x^3+5*x^2+x+2),x, algorithm="fricas")
```

```
[Out] -1/56*(-9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/56) + 35)*log(49/4*(27*
I*sqrt(7) + 84*sqrt(-37/392*I*sqrt(7) + 79/56) + 1385)*(9/56*I*sqrt(7) - 1/
2*sqrt(37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 2058*(-9/56*I*sqrt(7) - 1/2*sqr
t(-37/392*I*sqrt(7) + 79/56) - 5/8)^3 - 5145*(-9/56*I*sqrt(7) - 1/2*sqrt(-3
7/392*I*sqrt(7) + 79/56) - 5/8)^2 + 1/64*(2352*(-9/56*I*sqrt(7) - 1/2*sqrt(
-37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 945*I*sqrt(7) - 2940*sqrt(-37/392*I*s
qrt(7) + 79/56) - 28507)*(-9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/56)
+ 35) + 8384*x + 1323/2*I*sqrt(7) + 2058*sqrt(-37/392*I*sqrt(7) + 79/56) +
16089/2) + 1/8*(2*sqrt(-12*(9/56*I*sqrt(7) - 1/2*sqrt(37/392*I*sqrt(7) + 79
/56) - 5/8)^2 - 12*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) + 79/56) -
 5/8)^2 - 1/392*(9*I*sqrt(7) + 28*sqrt(-37/392*I*sqrt(7) + 79/56) - 105)*(-
9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/56) + 35) + 45/14*I*sqrt(7) + 1
```

```
0*sqrt(-37/392*I*sqrt(7) + 79/56) + 11/2) + 2*sqrt(37/392*I*sqrt(7) + 79/56
) + 2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5)*log(-49/4*(27*I*sqrt(7) + 84*sqr
t(-37/392*I*sqrt(7) + 79/56) + 1385)*(9/56*I*sqrt(7) - 1/2*sqrt(37/392*I*sq
rt(7) + 79/56) - 5/8)^2 - 15680*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(
7) + 79/56) - 5/8)^2 - 1/64*(2352*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqr
t(7) + 79/56) - 5/8)^2 - 945*I*sqrt(7) - 2940*sqrt(-37/392*I*sqrt(7) + 79/5
6) - 28507)*(-9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/56) + 35) + 7/64*
sqrt(-12*(9/56*I*sqrt(7) - 1/2*sqrt(37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 12
*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 1/392*(9
*I*sqrt(7) + 28*sqrt(-37/392*I*sqrt(7) + 79/56) - 105)*(-9*I*sqrt(7) + 28*s
qrt(37/392*I*sqrt(7) + 79/56) + 35) + 45/14*I*sqrt(7) + 10*sqrt(-37/392*I*s
qrt(7) + 79/56) + 11/2)*((27*I*sqrt(7) + 84*sqrt(-37/392*I*sqrt(7) + 79/56)
 + 1385)*(-9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/56) + 35) + 11520*I*
sqrt(7) + 35840*sqrt(-37/392*I*sqrt(7) + 79/56) - 35072) + 16768*x + 3492*I
*sqrt(7) + 10864*sqrt(-37/392*I*sqrt(7) + 79/56) + 5484) - 1/8*(2*sqrt(-12*
(9/56*I*sqrt(7) - 1/2*sqrt(37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 12*(-9/56*I
*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 1/392*(9*I*sqrt(7
) + 28*sqrt(-37/392*I*sqrt(7) + 79/56) - 105)*(-9*I*sqrt(7) + 28*sqrt(37/39
2*I*sqrt(7) + 79/56) + 35) + 45/14*I*sqrt(7) + 10*sqrt(-37/392*I*sqrt(7) +
79/56) + 11/2) - 2*sqrt(37/392*I*sqrt(7) + 79/56) - 2*sqrt(-37/392*I*sqrt(7
) + 79/56) + 5)*log(-49/4*(27*I*sqrt(7) + 84*sqrt(-37/392*I*sqrt(7) + 79/56
) + 1385)*(9/56*I*sqrt(7) - 1/2*sqrt(37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 1
5680*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 1/64
*(2352*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 94
5*I*sqrt(7) - 2940*sqrt(-37/392*I*sqrt(7) + 79/56) - 28507)*(-9*I*sqrt(7) +
 28*sqrt(37/392*I*sqrt(7) + 79/56) + 35) - 7/64*sqrt(-12*(9/56*I*sqrt(7) -
1/2*sqrt(37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 12*(-9/56*I*sqrt(7) - 1/2*sqr
t(-37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 1/392*(9*I*sqrt(7) + 28*sqrt(-37/39
2*I*sqrt(7) + 79/56) - 105)*(-9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/5
6) + 35) + 45/14*I*sqrt(7) + 10*sqrt(-37/392*I*sqrt(7) + 79/56) + 11/2)*((2
7*I*sqrt(7) + 84*sqrt(-37/392*I*sqrt(7) + 79/56) + 1385)*(-9*I*sqrt(7) + 28
*sqrt(37/392*I*sqrt(7) + 79/56) + 35) + 11520*I*sqrt(7) + 35840*sqrt(-37/39
2*I*sqrt(7) + 79/56) - 35072) + 16768*x + 3492*I*sqrt(7) + 10864*sqrt(-37/3
92*I*sqrt(7) + 79/56) + 5484) - 1/56*(9*I*sqrt(7) + 28*sqrt(-37/392*I*sqrt(
7) + 79/56) + 35)*log(2058*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) +
79/56) - 5/8)^3 + 20825*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) + 79/
56) - 5/8)^2 + 8384*x - 8307/2*I*sqrt(7) - 12922*sqrt(-37/392*I*sqrt(7) + 7
9/56) - 18673/2) + 5/2*log(x)
```

Sympy [B] time = 14.6512, size = 6967, normalized size = 28.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**3+3*x**2+x+5)/x/(2*x**4+x**3+5*x**2+x+2),x)
```

```
[Out] 5*log(x)/2 + (-5/8 + sqrt(79/448 + sqrt(77)/49))*log(x**2 + x*(-89525407879
941184*sqrt(77)*sqrt(553 + 64*sqrt(77)) - 721130562835737166*sqrt(553 + 64*
sqrt(77)) - 7740595633446277*sqrt(2)*sqrt(-8000*sqrt(77)*sqrt(553 + 64*sqrt
(77)) + 11893*sqrt(553 + 64*sqrt(77)) + 1821521 + 550752*sqrt(77)) - 728621
860145648*sqrt(154)*sqrt(-8000*sqrt(77)*sqrt(553 + 64*sqrt(77)) + 11893*sqr
t(553 + 64*sqrt(77)) + 1821521 + 550752*sqrt(77)) + 13024847322224*sqrt(154
)*sqrt(553 + 64*sqrt(77))*sqrt(-8000*sqrt(77)*sqrt(553 + 64*sqrt(77)) + 118
93*sqrt(553 + 64*sqrt(77)) + 1821521 + 550752*sqrt(77)) + 24824327780056103
306 + 2903507615591497728*sqrt(77) + 309070452629351*sqrt(2)*sqrt(553 + 64*
sqrt(77))*sqrt(-8000*sqrt(77)*sqrt(553 + 64*sqrt(77)) + 11893*sqrt(553 + 64
*sqrt(77)) + 1821521 + 550752*sqrt(77)))/(-25109003207503712000 - 268297184
0965222400*sqrt(77) + 66351959812096000*sqrt(77)*sqrt(553 + 64*sqrt(77)) +
875038845907744000*sqrt(553 + 64*sqrt(77))) - (-7741976451726931982255988808
86797194142448400*sqrt(77)*sqrt(553 + 64*sqrt(77)) - 679317938705783891587l
5640958030540245996l034*sqrt(553 + 64*sqrt(77)) - 5542699603695679745585696
26711168844846984*sqrt(154)*sqrt(-8000*sqrt(77)*sqrt(553 + 64*sqrt(77)) + 1
1893*sqrt(553 + 64*sqrt(77)) + 1821521 + 550752*sqrt(77)) - 486369568234453
26408164425659218345555543335*sqrt(2)*sqrt(-8000*sqrt(77)*sqrt(553 + 64*sqrt
(77)) + 11893*sqrt(553 + 64*sqrt(77)) + 1821521 + 550752*sqrt(77)) + 145679
7074393920414818264221682145648735573*sqrt(2)*sqrt(553 + 64*sqrt(77))*sqrt(-
8000*sqrt(77)*sqrt(553 + 64*sqrt(77)) + 11893*sqrt(553 + 64*sqrt(77)) + 182
1521 + 550752*sqrt(77)) + 16602386659666403043164759185008785053368*sqrt(15
4)*sqrt(553 + 64*sqrt(77))*sqrt(-8000*sqrt(77)*sqrt(553 + 64*sqrt(77)) + 11
893*sqrt(553 + 64*sqrt(77)) + 1821521 + 550752*sqrt(77)) + 2268005084338581
705124998211074391889879710214 2 + 258463401960275652428650811343584974252 68
99696*sqrt(77))/(-4824953189621314031348159925313598215500800000 - 54985354
555340344709334808850608512581632 0000*sqrt(77) + 1646748561763845873446026 5
356409726566400000*sqrt(77)*sqrt(553 + 64*sqrt(77)) + 144542207147812013201
815342868100902681600000*sqrt(553 + 64*sqrt(77)))) + (-5/8 - sqrt(79/448 +
sqrt(77)/49))*log(x**2 - x*(-85191793547*sqrt(2)*sqrt(553 + 64*sqrt(77))*sq
rt(-11893*sqrt(553 + 64*sqrt(77)) + 1821521 + 8000*sqrt(77)*sqrt(553 + 64*s
qrt(77)) + 550752*sqrt(77)) - 2177378129073*sqrt(2)*sqrt(-11893*sqrt(553 +
64*sqrt(77)) + 1821521 + 8000*sqrt(77)*sqrt(553 + 64*sqrt(77)) + 550752*sqr
t(77)) - 35068018944*sqrt(154)*sqrt(-11893*sqrt(553 + 64*sqrt(77)) + 182152
1 + 8000*sqrt(77)*sqrt(553 + 64*sqrt(77)) + 550752*sqrt(77)) + 1294998880*s
qrt(154)*sqrt(553 + 64*sqrt(77))*sqrt(-11893*sqrt(553 + 64*sqrt(77)) + 1821
521 + 8000*sqrt(77)*sqrt(553 + 64*sqrt(77)) + 550752*sqrt(77)) + 3979983261
152738 + 126856619440278*sqrt(553 + 64*sqrt(77)) + 15503416977696*sqrt(77)*
sqrt(553 + 64*sqrt(77)) + 553763604587424*sqrt(77))/(3409658777600*sqrt(77)
*sqrt(553 + 64*sqrt(77)) + 358014171648000*sqrt(77) + 5377671203296000 + 22
```

5250582995200*sqrt(553 + 64*sqrt(77))) - (-59777622257425319967069265371984
*sqrt(77)*sqrt(553 + 64*sqrt(77)) - 1983957747279380406707542525293104*sqrt
(77) - 17381190623211340499377993035060186 - 517530258435571318509662236609
726*sqrt(553 + 64*sqrt(77)) + 111225224513278287461756729183*sqrt(2)*sqrt(5
53 + 64*sqrt(77))*sqrt(-11893*sqrt(553 + 64*sqrt(77)) + 1821521 + 8000*sqrt
(77)*sqrt(553 + 64*sqrt(77)) + 550752*sqrt(77)) + 3728227234168298192267084
561133*sqrt(2)*sqrt(-11893*sqrt(553 + 64*sqrt(77)) + 1821521 + 8000*sqrt(77
)*sqrt(553 + 64*sqrt(77)) + 550752*sqrt(77)) + 4253577601040018985719582632
40*sqrt(154)*sqrt(-11893*sqrt(553 + 64*sqrt(77)) + 1821521 + 8000*sqrt(77)*
sqrt(553 + 64*sqrt(77)) + 550752*sqrt(77)) + 12791644171388153684809943608*
sqrt(154)*sqrt(553 + 64*sqrt(77))*sqrt(-11893*sqrt(553 + 64*sqrt(77)) + 182
1521 + 8000*sqrt(77)*sqrt(553 + 64*sqrt(77)) + 550752*sqrt(77)))/(121432985
32693017521943920640000*sqrt(77)*sqrt(553 + 64*sqrt(77)) + 4201961579338667
81930811801600000*sqrt(77) + 3714081866362658715034233625600000 + 115134078
271670720864780170240000*sqrt(553 + 64*sqrt(77)))) + 2*sqrt(-sqrt(2)*sqrt(-
11893*sqrt(553 + 64*sqrt(77)) + 1821521 + 8000*sqrt(77)*sqrt(553 + 64*sqrt(
77)) + 550752*sqrt(77))/(140*(sqrt(553 + 64*sqrt(77)) + 35)) + (64288*sqrt(
553 + 64*sqrt(77)) + 17920*sqrt(77)*sqrt(553 + 64*sqrt(77)) + 10453856 + 18
81600*sqrt(77))/(878080*sqrt(553 + 64*sqrt(77)) + 30732800))*atan(x*(852414
69440*sqrt(5)*sqrt(553 + 64*sqrt(77))*sqrt(sqrt(553 + 64*sqrt(77)) + 35) +
2983451430400*sqrt(5)*sqrt(sqrt(553 + 64*sqrt(77)) + 35))/(-7418715322*sqrt
(553 + 64*sqrt(77))*sqrt(-28*sqrt(2)*sqrt(-11893*sqrt(553 + 64*sqrt(77)) +
1821521 + 8000*sqrt(77)*sqrt(553 + 64*sqrt(77)) + 550752*sqrt(77)) + 287*sq
rt(553 + 64*sqrt(77)) + 80*sqrt(77)*sqrt(553 + 64*sqrt(77)) + 46669 + 8400*
sqrt(77)) - 14369482432*sqrt(77)*sqrt(-28*sqrt(2)*sqrt(-11893*sqrt(553 + 64
*sqrt(77)) + 1821521 + 8000*sqrt(77)*sqrt(553 + 64*sqrt(77)) + 550752*sqrt(
77)) + 287*sqrt(553 + 64*sqrt(77)) + 80*sqrt(77)*sqrt(553 + 64*sqrt(77)) +
46669 + 8400*sqrt(77)) - 94356162446*sqrt(-28*sqrt(2)*sqrt(-11893*sqrt(553
+ 64*sqrt(77)) + 1821521 + 8000*sqrt(77)*sqrt(553 + 64*sqrt(77)) + 550752*s
qrt(77)) + 287*sqrt(553 + 64*sqrt(77)) + 80*sqrt(77)*sqrt(553 + 64*sqrt(77)
) + 46669 + 8400*sqrt(77)) - 62993920*sqrt(77)*sqrt(553 + 64*sqrt(77))*sqrt
(-28*sqrt(2)*sqrt(-11893*sqrt(553 + 64*sqrt(77)) + 1821521 + 8000*sqrt(77)*
sqrt(553 + 64*sqrt(77)) + 550752*sqrt(77)) + 287*sqrt(553 + 64*sqrt(77)) +
80*sqrt(77)*sqrt(553 + 64*sqrt(77)) + 46669 + 8400*sqrt(77)) + 5337759*sqrt
(2)*sqrt(-11893*sqrt(553 + 64*sqrt(77)) + 1821521 + 8000*sqrt(77)*sqrt(553
+ 64*sqrt(77)) + 550752*sqrt(77))*sqrt(-28*sqrt(2)*sqrt(-11893*sqrt(553 + 6
4*sqrt(77)) + 1821521 + 8000*sqrt(77)*sqrt(553 + 64*sqrt(77)) + 550752*sqrt
(77)) + 287*sqrt(553 + 64*sqrt(77)) + 80*sqrt(77)*sqrt(553 + 64*sqrt(77)) +
 46669 + 8400*sqrt(77)) + 467477*sqrt(2)*sqrt(553 + 64*sqrt(77))*sqrt(-1189
3*sqrt(553 + 64*sqrt(77)) + 1821521 + 8000*sqrt(77)*sqrt(553 + 64*sqrt(77))
 + 550752*sqrt(77))*sqrt(-28*sqrt(2)*sqrt(-11893*sqrt(553 + 64*sqrt(77)) +
1821521 + 8000*sqrt(77)*sqrt(553 + 64*sqrt(77)) + 550752*sqrt(77)) + 287*sq
rt(553 + 64*sqrt(77)) + 80*sqrt(77)*sqrt(553 + 64*sqrt(77)) + 46669 + 8400*
sqrt(77))) + (-78313085629056*sqrt(385)*sqrt(553 + 64*sqrt(77))*sqrt(sqrt(5
53 + 64*sqrt(77)) + 35) - 2576709956671680*sqrt(385)*sqrt(sqrt(553 + 64*sqr

t(77)) + 35) - 20417997396921818*sqrt(5)*sqrt(sqrt(553 + 64*sqrt(77)) + 35)
 - 601426067254462*sqrt(5)*sqrt(553 + 64*sqrt(77))*sqrt(sqrt(553 + 64*sqrt(
77)) + 35) - 732638704*sqrt(770)*sqrt(553 + 64*sqrt(77))*sqrt(sqrt(553 + 64
*sqrt(77)) + 35)*sqrt(-11893*sqrt(553 + 64*sqrt(77)) + 1821521 + 8000*sqrt(
77)*sqrt(553 + 64*sqrt(77)) + 550752*sqrt(77)) + 425965790672*sqrt(770)*sqr
t(sqrt(553 + 64*sqrt(77)) + 35)*sqrt(-11893*sqrt(553 + 64*sqrt(77)) + 18215
21 + 8000*sqrt(77)*sqrt(553 + 64*sqrt(77)) + 550752*sqrt(77)) + 83526815620
29*sqrt(10)*sqrt(sqrt(553 + 64*sqrt(77)) + 35)*sqrt(-11893*sqrt(553 + 64*sq
rt(77)) + 1821521 + 8000*sqrt(77)*sqrt(553 + 64*sqrt(77)) + 550752*sqrt(77)
) + 368506493087*sqrt(10)*sqrt(553 + 64*sqrt(77))*sqrt(sqrt(553 + 64*sqrt(7
7)) + 35)*sqrt(-11893*sqrt(553 + 64*sqrt(77)) + 1821521 + 8000*sqrt(77)*sqr
t(553 + 64*sqrt(77)) + 550752*sqrt(77)))/(-111713514536970*sqrt(553 + 64*sq
rt(77))*sqrt(-28*sqrt(2)*sqrt(-11893*sqrt(553 + 64*sqrt(77)) + 1821521 + 80
00*sqrt(77)*sqrt(553 + 64*sqrt(77)) + 550752*sqrt(77)) + 287*sqrt(553 + 64*
sqrt(77)) + 80*sqrt(77)*sqrt(553 + 64*sqrt(77)) + 46669 + 8400*sqrt(77)) -
403722322524640*sqrt(77)*sqrt(-28*sqrt(2)*sqrt(-11893*sqrt(553 + 64*sqrt(77
)) + 1821521 + 8000*sqrt(77)*sqrt(553 + 64*sqrt(77)) + 550752*sqrt(77)) + 2
87*sqrt(553 + 64*sqrt(77)) + 80*sqrt(77)*sqrt(553 + 64*sqrt(77)) + 46669 +
8400*sqrt(77)) - 3404395019571710*sqrt(-28*sqrt(2)*sqrt(-11893*sqrt(553 + 6
4*sqrt(77)) + 1821521 + 8000*sqrt(77)*sqrt(553 + 64*sqrt(77)) + 550752*sqrt
(77)) + 287*sqrt(553 + 64*sqrt(77)) + 80*sqrt(77)*sqrt(553 + 64*sqrt(77)) +
 46669 + 8400*sqrt(77)) - 10982619586400*sqrt(77)*sqrt(553 + 64*sqrt(77))*s
qrt(-28*sqrt(2)*sqrt(-11893*sqrt(553 + 64*sqrt(77)) + 1821521 + 8000*sqrt(7
7)*sqrt(553 + 64*sqrt(77)) + 550752*sqrt(77)) + 287*sqrt(553 + 64*sqrt(77))
 + 80*sqrt(77)*sqrt(553 + 64*sqrt(77)) + 46669 + 8400*sqrt(77)) + 310987680
*sqrt(154)*sqrt(553 + 64*sqrt(77))*sqrt(-11893*sqrt(553 + 64*sqrt(77)) + 18
21521 + 8000*sqrt(77)*sqrt(553 + 64*sqrt(77)) + 550752*sqrt(77))*sqrt(-28*s
qrt(2)*sqrt(-11893*sqrt(553 + 64*sqrt(77)) + 1821521 + 8000*sqrt(77)*sqrt(5
53 + 64*sqrt(77)) + 550752*sqrt(77)) + 287*sqrt(553 + 64*sqrt(77)) + 80*sqr
t(77)*sqrt(553 + 64*sqrt(77)) + 46669 + 8400*sqrt(77)) + 15974127040*sqrt(1
54)*sqrt(-11893*sqrt(553 + 64*sqrt(77)) + 1821521 + 8000*sqrt(77)*sqrt(553
+ 64*sqrt(77)) + 550752*sqrt(77))*sqrt(-28*sqrt(2)*sqrt(-11893*sqrt(553 + 6
4*sqrt(77)) + 1821521 + 8000*sqrt(77)*sqrt(553 + 64*sqrt(77)) + 550752*sqrt
(77)) + 287*sqrt(553 + 64*sqrt(77)) + 80*sqrt(77)*sqrt(553 + 64*sqrt(77)) +
 46669 + 8400*sqrt(77)) + 158860035135*sqrt(2)*sqrt(-11893*sqrt(553 + 64*sq
rt(77)) + 1821521 + 8000*sqrt(77)*sqrt(553 + 64*sqrt(77)) + 550752*sqrt(77)
)*sqrt(-28*sqrt(2)*sqrt(-11893*sqrt(553 + 64*sqrt(77)) + 1821521 + 8000*sqr
t(77)*sqrt(553 + 64*sqrt(77)) + 550752*sqrt(77)) + 287*sqrt(553 + 64*sqrt(7
7)) + 80*sqrt(77)*sqrt(553 + 64*sqrt(77)) + 46669 + 8400*sqrt(77)) + 622814
0485*sqrt(2)*sqrt(553 + 64*sqrt(77))*sqrt(-11893*sqrt(553 + 64*sqrt(77)) +
1821521 + 8000*sqrt(77)*sqrt(553 + 64*sqrt(77)) + 550752*sqrt(77))*sqrt(-28
*sqrt(2)*sqrt(-11893*sqrt(553 + 64*sqrt(77)) + 1821521 + 8000*sqrt(77)*sqrt
(553 + 64*sqrt(77)) + 550752*sqrt(77)) + 287*sqrt(553 + 64*sqrt(77)) + 80*s
qrt(77)*sqrt(553 + 64*sqrt(77)) + 46669 + 8400*sqrt(77)))) - 2*sqrt(sqrt(2)
*sqrt(-8000*sqrt(77)*sqrt(553 + 64*sqrt(77)) + 11893*sqrt(553 + 64*sqrt(77)

) + 1821521 + 550752*sqrt(77))/(140*(-35 + sqrt(553 + 64*sqrt(77)))) + (-17
920*sqrt(77)*sqrt(553 + 64*sqrt(77)) - 64288*sqrt(553 + 64*sqrt(77)) + 1045
3856 + 1881600*sqrt(77))/(30732800 - 878080*sqrt(553 + 64*sqrt(77))))*atan(
x*(-4302136962636800*sqrt(5) - 286411337318400*sqrt(385) + 2727727022080*sq
rt(385)*sqrt(553 + 64*sqrt(77)) + 180200466396160*sqrt(5)*sqrt(553 + 64*sqr
t(77)))/(-506282651744*sqrt(77)*sqrt(-11200*sqrt(77)*sqrt(553 + 64*sqrt(77)
) - 980*sqrt(2)*sqrt(-8000*sqrt(77)*sqrt(553 + 64*sqrt(77)) + 11893*sqrt(55
3 + 64*sqrt(77)) + 1821521 + 550752*sqrt(77)) - 56714*sqrt(553 + 64*sqrt(77
)) + 2186366 + 28*sqrt(2)*sqrt(553 + 64*sqrt(77))*sqrt(-8000*sqrt(77)*sqrt(
553 + 64*sqrt(77)) + 11893*sqrt(553 + 64*sqrt(77)) + 1821521 + 550752*sqrt(
77)) + 356608*sqrt(77)) - 3857724648218*sqrt(-11200*sqrt(77)*sqrt(553 + 64*
sqrt(77)) - 980*sqrt(2)*sqrt(-8000*sqrt(77)*sqrt(553 + 64*sqrt(77)) + 11893
*sqrt(553 + 64*sqrt(77)) + 1821521 + 550752*sqrt(77)) - 56714*sqrt(553 + 64
*sqrt(77)) + 2186366 + 28*sqrt(2)*sqrt(553 + 64*sqrt(77))*sqrt(-8000*sqrt(7
7)*sqrt(553 + 64*sqrt(77)) + 11893*sqrt(553 + 64*sqrt(77)) + 1821521 + 5507
52*sqrt(77)) + 356608*sqrt(77)) - 10849727*sqrt(2)*sqrt(553 + 64*sqrt(77))*
sqrt(-8000*sqrt(77)*sqrt(553 + 64*sqrt(77)) + 11893*sqrt(553 + 64*sqrt(77))
 + 1821521 + 550752*sqrt(77))*sqrt(-11200*sqrt(77)*sqrt(553 + 64*sqrt(77))
 - 980*sqrt(2)*sqrt(-8000*sqrt(77)*sqrt(553 + 64*sqrt(77)) + 11893*sqrt(553
 + 64*sqrt(77)) + 1821521 + 550752*sqrt(77)) - 56714*sqrt(553 + 64*sqrt(77))
 + 2186366 + 28*sqrt(2)*sqrt(553 + 64*sqrt(77))*sqrt(-8000*sqrt(77)*sqrt(55
3 + 64*sqrt(77)) + 11893*sqrt(553 + 64*sqrt(77)) + 1821521 + 550752*sqrt(77
)) + 356608*sqrt(77)) + 14959264*sqrt(154)*sqrt(-8000*sqrt(77)*sqrt(553 + 6
4*sqrt(77)) + 11893*sqrt(553 + 64*sqrt(77)) + 1821521 + 550752*sqrt(77))*sq
rt(-11200*sqrt(77)*sqrt(553 + 64*sqrt(77)) - 980*sqrt(2)*sqrt(-8000*sqrt(77
)*sqrt(553 + 64*sqrt(77)) + 11893*sqrt(553 + 64*sqrt(77)) + 1821521 + 55075
2*sqrt(77)) - 56714*sqrt(553 + 64*sqrt(77)) + 2186366 + 28*sqrt(2)*sqrt(553
 + 64*sqrt(77))*sqrt(-8000*sqrt(77)*sqrt(553 + 64*sqrt(77)) + 11893*sqrt(55
3 + 64*sqrt(77)) + 1821521 + 550752*sqrt(77)) + 356608*sqrt(77)) + 22266817
3*sqrt(2)*sqrt(-8000*sqrt(77)*sqrt(553 + 64*sqrt(77)) + 11893*sqrt(553 + 64
*sqrt(77)) + 1821521 + 550752*sqrt(77))*sqrt(-11200*sqrt(77)*sqrt(553 + 64*
sqrt(77)) - 980*sqrt(2)*sqrt(-8000*sqrt(77)*sqrt(553 + 64*sqrt(77)) + 11893
*sqrt(553 + 64*sqrt(77)) + 1821521 + 550752*sqrt(77)) - 56714*sqrt(553 + 64
*sqrt(77)) + 2186366 + 28*sqrt(2)*sqrt(553 + 64*sqrt(77))*sqrt(-8000*sqrt(7
7)*sqrt(553 + 64*sqrt(77)) + 11893*sqrt(553 + 64*sqrt(77)) + 1821521 + 5507
52*sqrt(77)) + 356608*sqrt(77)) + 8287134816*sqrt(77)*sqrt(553 + 64*sqrt(77
))*sqrt(-11200*sqrt(77)*sqrt(553 + 64*sqrt(77)) - 980*sqrt(2)*sqrt(-8000*sq
rt(77)*sqrt(553 + 64*sqrt(77)) + 11893*sqrt(553 + 64*sqrt(77)) + 1821521 +
550752*sqrt(77)) - 56714*sqrt(553 + 64*sqrt(77)) + 2186366 + 28*sqrt(2)*sqr
t(553 + 64*sqrt(77))*sqrt(-8000*sqrt(77)*sqrt(553 + 64*sqrt(77)) + 11893*sq
rt(553 + 64*sqrt(77)) + 1821521 + 550752*sqrt(77)) + 356608*sqrt(77)) + 177
005599358*sqrt(553 + 64*sqrt(77))*sqrt(-11200*sqrt(77)*sqrt(553 + 64*sqrt(7
7)) - 980*sqrt(2)*sqrt(-8000*sqrt(77)*sqrt(553 + 64*sqrt(77)) + 11893*sqrt(
553 + 64*sqrt(77)) + 1821521 + 550752*sqrt(77)) - 56714*sqrt(553 + 64*sqrt(
77)) + 2186366 + 28*sqrt(2)*sqrt(553 + 64*sqrt(77))*sqrt(-8000*sqrt(77)*sqr

t(553 + 64*sqrt(77)) + 11893*sqrt(553 + 64*sqrt(77)) + 1821521 + 550752*sqr
t(77)) + 356608*sqrt(77))) - (-495490700541104640608*sqrt(385)*sqrt(553 + 6
4*sqrt(77)) - 4256798025888360381542*sqrt(5)*sqrt(553 + 64*sqrt(77)) - 4080
9647373380213993*sqrt(10)*sqrt(-8000*sqrt(77)*sqrt(553 + 64*sqrt(77)) + 118
93*sqrt(553 + 64*sqrt(77)) + 1821521 + 550752*sqrt(77)) - 45481361851854585
28*sqrt(770)*sqrt(-8000*sqrt(77)*sqrt(553 + 64*sqrt(77)) + 11893*sqrt(553 +
 64*sqrt(77)) + 1821521 + 550752*sqrt(77)) + 124423087064480192*sqrt(770)*s
qrt(553 + 64*sqrt(77))*sqrt(-8000*sqrt(77)*sqrt(553 + 64*sqrt(77)) + 11893*
sqrt(553 + 64*sqrt(77)) + 1821521 + 550752*sqrt(77)) + 1325982774139765875*
sqrt(10)*sqrt(553 + 64*sqrt(77))*sqrt(-8000*sqrt(77)*sqrt(553 + 64*sqrt(77)
) + 11893*sqrt(553 + 64*sqrt(77)) + 1821521 + 550752*sqrt(77)) + 1434097970
22255611682898*sqrt(5) + 16394776370324068018528*sqrt(385))/(-8106191274243
949040*sqrt(22)*sqrt(553 + 64*sqrt(77))*sqrt(-800*sqrt(77)*sqrt(553 + 64*sq
rt(77)) - 70*sqrt(2)*sqrt(-8000*sqrt(77)*sqrt(553 + 64*sqrt(77)) + 11893*sq
rt(553 + 64*sqrt(77)) + 1821521 + 550752*sqrt(77)) - 4051*sqrt(553 + 64*sqr
t(77)) + 156169 + 2*sqrt(2)*sqrt(553 + 64*sqrt(77))*sqrt(-8000*sqrt(77)*sqr
t(553 + 64*sqrt(77)) + 11893*sqrt(553 + 64*sqrt(77)) + 1821521 + 550752*sqr
t(77)) + 25472*sqrt(77)) - 10154993321875521955*sqrt(14)*sqrt(553 + 64*sqrt
(77))*sqrt(-800*sqrt(77)*sqrt(553 + 64*sqrt(77)) - 70*sqrt(2)*sqrt(-8000*sq
rt(77)*sqrt(553 + 64*sqrt(77)) + 11893*sqrt(553 + 64*sqrt(77)) + 1821521 +
550752*sqrt(77)) - 4051*sqrt(553 + 64*sqrt(77)) + 156169 + 2*sqrt(2)*sqrt(5
53 + 64*sqrt(77))*sqrt(-8000*sqrt(77)*sqrt(553 + 64*sqrt(77)) + 11893*sqrt(
553 + 64*sqrt(77)) + 1821521 + 550752*sqrt(77)) + 25472*sqrt(77)) - 2929360
2188164665*sqrt(7)*sqrt(-8000*sqrt(77)*sqrt(553 + 64*sqrt(77)) + 11893*sqrt
(553 + 64*sqrt(77)) + 1821521 + 550752*sqrt(77))*sqrt(-800*sqrt(77)*sqrt(55
3 + 64*sqrt(77)) - 70*sqrt(2)*sqrt(-8000*sqrt(77)*sqrt(553 + 64*sqrt(77)) +
 11893*sqrt(553 + 64*sqrt(77)) + 1821521 + 550752*sqrt(77)) - 4051*sqrt(553
 + 64*sqrt(77)) + 156169 + 2*sqrt(2)*sqrt(553 + 64*sqrt(77))*sqrt(-8000*sqr
t(77)*sqrt(553 + 64*sqrt(77)) + 11893*sqrt(553 + 64*sqrt(77)) + 1821521 + 5
50752*sqrt(77)) + 25472*sqrt(77)) - 23213365561348480*sqrt(11)*sqrt(-8000*s
qrt(77)*sqrt(553 + 64*sqrt(77)) + 11893*sqrt(553 + 64*sqrt(77)) + 1821521 +
 550752*sqrt(77))*sqrt(-800*sqrt(77)*sqrt(553 + 64*sqrt(77)) - 70*sqrt(2)*s
qrt(-8000*sqrt(77)*sqrt(553 + 64*sqrt(77)) + 11893*sqrt(553 + 64*sqrt(77))
 + 1821521 + 550752*sqrt(77)) - 4051*sqrt(553 + 64*sqrt(77)) + 156169 + 2*sq
rt(2)*sqrt(553 + 64*sqrt(77))*sqrt(-8000*sqrt(77)*sqrt(553 + 64*sqrt(77)) +
 11893*sqrt(553 + 64*sqrt(77)) + 1821521 + 550752*sqrt(77)) + 25472*sqrt(77
)) + 668382644855520*sqrt(11)*sqrt(553 + 64*sqrt(77))*sqrt(-8000*sqrt(77)*s
qrt(553 + 64*sqrt(77)) + 11893*sqrt(553 + 64*sqrt(77)) + 1821521 + 550752*s
qrt(77))*sqrt(-800*sqrt(77)*sqrt(553 + 64*sqrt(77)) - 70*sqrt(2)*sqrt(-8000
*sqrt(77)*sqrt(553 + 64*sqrt(77)) + 11893*sqrt(553 + 64*sqrt(77)) + 1821521
 + 550752*sqrt(77)) - 4051*sqrt(553 + 64*sqrt(77)) + 156169 + 2*sqrt(2)*sqr
t(553 + 64*sqrt(77))*sqrt(-8000*sqrt(77)*sqrt(553 + 64*sqrt(77)) + 11893*sq
rt(553 + 64*sqrt(77)) + 1821521 + 550752*sqrt(77)) + 25472*sqrt(77)) + 9111
88824584675*sqrt(7)*sqrt(553 + 64*sqrt(77))*sqrt(-8000*sqrt(77)*sqrt(553 +
64*sqrt(77)) + 11893*sqrt(553 + 64*sqrt(77)) + 1821521 + 550752*sqrt(77))*s

```
qrt(-800*sqrt(77)*sqrt(553 + 64*sqrt(77)) - 70*sqrt(2)*sqrt(-8000*sqrt(77)*
sqrt(553 + 64*sqrt(77)) + 11893*sqrt(553 + 64*sqrt(77)) + 1821521 + 550752*
sqrt(77)) - 4051*sqrt(553 + 64*sqrt(77)) + 156169 + 2*sqrt(2)*sqrt(553 + 64
*sqrt(77))*sqrt(-8000*sqrt(77)*sqrt(553 + 64*sqrt(77)) + 11893*sqrt(553 + 6
4*sqrt(77)) + 1821521 + 550752*sqrt(77)) + 25472*sqrt(77)) + 33864779590452
6996345*sqrt(14)*sqrt(-800*sqrt(77)*sqrt(553 + 64*sqrt(77)) - 70*sqrt(2)*sq
rt(-8000*sqrt(77)*sqrt(553 + 64*sqrt(77)) + 11893*sqrt(553 + 64*sqrt(77)) +
 1821521 + 550752*sqrt(77)) - 4051*sqrt(553 + 64*sqrt(77)) + 156169 + 2*sqr
t(2)*sqrt(553 + 64*sqrt(77))*sqrt(-8000*sqrt(77)*sqrt(553 + 64*sqrt(77)) +
11893*sqrt(553 + 64*sqrt(77)) + 1821521 + 550752*sqrt(77)) + 25472*sqrt(77)
) + 270935250074981726640*sqrt(22)*sqrt(-800*sqrt(77)*sqrt(553 + 64*sqrt(77
)) - 70*sqrt(2)*sqrt(-8000*sqrt(77)*sqrt(553 + 64*sqrt(77)) + 11893*sqrt(55
3 + 64*sqrt(77)) + 1821521 + 550752*sqrt(77)) - 4051*sqrt(553 + 64*sqrt(77)
) + 156169 + 2*sqrt(2)*sqrt(553 + 64*sqrt(77))*sqrt(-8000*sqrt(77)*sqrt(553
 + 64*sqrt(77)) + 11893*sqrt(553 + 64*sqrt(77)) + 1821521 + 550752*sqrt(77)
) + 25472*sqrt(77))))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2x^3 + 3x^2 + x + 5}{(2x^4 + x^3 + 5x^2 + x + 2)x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/x/(2*x^4+x^3+5*x^2+x+2),x, algorithm="giac")
```

```
[Out] integrate((2*x^3 + 3*x^2 + x + 5)/((2*x^4 + x^3 + 5*x^2 + x + 2)*x), x)
```

3.255 $\int \frac{5+x+3x^2+2x^3}{x^2(2+x+5x^2+x^3+2x^4)}\,dx$

Optimal. Leaf size=281

$$\frac{3}{112}\left(7+11i\sqrt{7}\right)\log\left(4ix^2+\left(-\sqrt{7}+i\right)x+4i\right)+\frac{3}{112}\left(7-11i\sqrt{7}\right)\log\left(4ix^2+\left(\sqrt{7}+i\right)x+4i\right)-\frac{35+9i\sqrt{7}}{28x}-\frac{35-9i\sqrt{}}{28x}$$

```
[Out] -(35 - (9*I)*Sqrt[7])/(28*x) - (35 + (9*I)*Sqrt[7])/(28*x) + (11*(9 + (5*I)
*Sqrt[7])*ArcTanh[(I - Sqrt[7] + (8*I)*x)/Sqrt[2*(35 - I*Sqrt[7])]])/(4*Sqr
t[14*(35 - I*Sqrt[7])]) - (11*(9 - (5*I)*Sqrt[7])*ArcTanh[(I + Sqrt[7] + (8
*I)*x)/Sqrt[2*(35 + I*Sqrt[7])]])/(4*Sqrt[14*(35 + I*Sqrt[7])]) - (3*(7 - (
11*I)*Sqrt[7])*Log[x])/56 - (3*(7 + (11*I)*Sqrt[7])*Log[x])/56 + (3*(7 + (1
1*I)*Sqrt[7])*Log[4*I + (I - Sqrt[7])*x + (4*I)*x^2])/112 + (3*(7 - (11*I)*
Sqrt[7])*Log[4*I + (I + Sqrt[7])*x + (4*I)*x^2])/112
```

Rubi [A] time = 0.467315, antiderivative size = 281, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2087, 800, 634, 618, 206, 628}

$$\frac{3}{112}\left(7+11i\sqrt{7}\right)\log\left(4ix^2+\left(-\sqrt{7}+i\right)x+4i\right)+\frac{3}{112}\left(7-11i\sqrt{7}\right)\log\left(4ix^2+\left(\sqrt{7}+i\right)x+4i\right)-\frac{35+9i\sqrt{7}}{28x}-\frac{35-9i\sqrt{}}{28x}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + x + 3*x^2 + 2*x^3)/(x^2*(2 + x + 5*x^2 + x^3 + 2*x^4)),x]
```


```
[Out] -(35 - (9*I)*Sqrt[7])/(28*x) - (35 + (9*I)*Sqrt[7])/(28*x) + (11*(9 + (5*I)
*Sqrt[7])*ArcTanh[(I - Sqrt[7] + (8*I)*x)/Sqrt[2*(35 - I*Sqrt[7])]])/(4*Sqr
t[14*(35 - I*Sqrt[7])]) - (11*(9 - (5*I)*Sqrt[7])*ArcTanh[(I + Sqrt[7] + (8
*I)*x)/Sqrt[2*(35 + I*Sqrt[7])]])/(4*Sqrt[14*(35 + I*Sqrt[7])]) - (3*(7 - (
11*I)*Sqrt[7])*Log[x])/56 - (3*(7 + (11*I)*Sqrt[7])*Log[x])/56 + (3*(7 + (1
1*I)*Sqrt[7])*Log[4*I + (I - Sqrt[7])*x + (4*I)*x^2])/112 + (3*(7 - (11*I)*
Sqrt[7])*Log[4*I + (I + Sqrt[7])*x + (4*I)*x^2])/112
```


Rule 2087

```
Int[((P3_)*(x_)^(m_.))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (
e_.)*(x_)^4), x_Symbol] :> With[{q = Sqrt[8*a^2 + b^2 - 4*a*c], A = Coeff[P
3, x, 0], B = Coeff[P3, x, 1], C = Coeff[P3, x, 2], D = Coeff[P3, x, 3]}, D
ist[1/q, Int[(x^m*(b*A - 2*a*B + 2*a*D + A*q + (2*a*A - 2*a*C + b*D + D*q)*
x))/(2*a + (b + q)*x + 2*a*x^2), x], x] - Dist[1/q, Int[(x^m*(b*A - 2*a*B +
 2*a*D - A*q + (2*a*A - 2*a*C + b*D - D*q)*x))/(2*a + (b - q)*x + 2*a*x^2),
 x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[P3, x, 3] && EqQ[a, e] && EqQ[b
, d]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{5+x+3x^2+2x^3}{x^2\left(2+x+5x^2+x^3+2x^4\right)}\,dx = \frac{i\int \frac{9-5i\sqrt{7}+\left(10-2i\sqrt{7}\right)x}{x^2\left(4+\left(1-i\sqrt{7}\right)x+4x^2\right)}\,dx}{\sqrt{7}} - \frac{i\int \frac{9+5i\sqrt{7}+\left(10+2i\sqrt{7}\right)x}{x^2\left(4+\left(1+i\sqrt{7}\right)x+4x^2\right)}\,dx}{\sqrt{7}}$$

$$= -\frac{i\int \left(\frac{9+5i\sqrt{7}}{4x^2}+\frac{3\left(11-i\sqrt{7}\right)}{8x}+\frac{-7\left(9i-5\sqrt{7}\right)-6\left(11i+\sqrt{7}\right)x}{4\left(4i+\left(i-\sqrt{7}\right)x+4ix^2\right)}\right)dx}{\sqrt{7}} + \frac{i\int \left(\frac{9-5i\sqrt{7}}{4x^2}+\frac{3\left(11+i\sqrt{7}\right)}{8x}+\frac{-7(\cdots}{4\cdots}\right.}{\sqrt{7}}$$

$$= -\frac{35-9i\sqrt{7}}{28x} - \frac{35+9i\sqrt{7}}{28x} - \frac{3}{56}\left(7-11i\sqrt{7}\right)\log(x) - \frac{3}{56}\left(7+11i\sqrt{7}\right)\log(x) - \frac{i\int \cdots}{\cdots}$$

$$= -\frac{35-9i\sqrt{7}}{28x} - \frac{35+9i\sqrt{7}}{28x} - \frac{3}{56}\left(7-11i\sqrt{7}\right)\log(x) - \frac{3}{56}\left(7+11i\sqrt{7}\right)\log(x) - \frac{1}{56}\left(\cdots\right.$$

$$= -\frac{35-9i\sqrt{7}}{28x} - \frac{35+9i\sqrt{7}}{28x} - \frac{3}{56}\left(7-11i\sqrt{7}\right)\log(x) - \frac{3}{56}\left(7+11i\sqrt{7}\right)\log(x) + \frac{3}{112}\cdots$$

$$= -\frac{35-9i\sqrt{7}}{28x} - \frac{35+9i\sqrt{7}}{28x} + \frac{11\left(9+5i\sqrt{7}\right)\tanh^{-1}\left(\frac{i-\sqrt{7}+8ix}{\sqrt{2\left(35-i\sqrt{7}\right)}}\right)}{4\sqrt{14\left(35-i\sqrt{7}\right)}} - \frac{11\left(9-5i\sqrt{7}\right)\tan\cdots}{4\sqrt{14\left(3\cdots\right.}}$$

Mathematica [C] time = 0.0188368, size = 109, normalized size = 0.39

$$\frac{1}{4}\text{RootSum}\left[2\#1^4+\#1^3+5\#1^2+\#1+2\&,\frac{6\#1^3\log(x-\#1)-17\#1^2\log(x-\#1)+13\#1\log(x-\#1)-35\log(x-\#1)}{8\#1^3+3\#1^2+10\#1+1}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + x + 3*x^2 + 2*x^3)/(x^2*(2 + x + 5*x^2 + x^3 + 2*x^4)),x]
```

```
[Out] -5/(2*x) - (3*Log[x])/4 + RootSum[2 + #1 + 5*#1^2 + #1^3 + 2*#1^4 & , (-35*
Log[x - #1] + 13*Log[x - #1]*#1 - 17*Log[x - #1]*#1^2 + 6*Log[x - #1]*#1^3)
/(1 + 10*#1 + 3*#1^2 + 8*#1^3) & ]/4
```

Maple [C] time = 0.008, size = 72, normalized size = 0.3

$$-\frac{5}{2x}-\frac{3\ln(x)}{4}+\frac{1}{4}\sum_{\_R=\text{RootOf}\left(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2\right)}\frac{\left(6\_R^3-17\_R^2+13\_R-35\right)\ln(x-\_R)}{8\_R^3+3\_R^2+10\_R+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3+3*x^2+x+5)/x^2/(2*x^4+x^3+5*x^2+x+2),x)
```

```
[Out] -5/2/x-3/4*ln(x)+1/4*sum((6*_R^3-17*_R^2+13*_R-35)/(8*_R^3+3*_R^2+10*_R+1)*
ln(x-_R),_R=RootOf(2*_Z^4+_Z^3+5*_Z^2+_Z+2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{5}{2\,x}+\frac{1}{4}\int\frac{6\,x^3-17\,x^2+13\,x-35}{2\,x^4+x^3+5\,x^2+x+2}\,dx-\frac{3}{4}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/x^2/(2*x^4+x^3+5*x^2+x+2),x, algorithm="maxima"
)
```

```
[Out] -5/2/x + 1/4*integrate((6*x^3 - 17*x^2 + 13*x - 35)/(2*x^4 + x^3 + 5*x^2 +
x + 2), x) - 3/4*log(x)
```

Fricas [B] time = 9.74885, size = 5700, normalized size = 20.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/x^2/(2*x^4+x^3+5*x^2+x+2),x, algorithm="fricas"
)
```

```
[Out] -1/224*(2*x*(33*I*sqrt(7) + 56*sqrt(-2101/1568*I*sqrt(7) - 55/32) - 21)*log
(91924*(33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/32) + 3/16)^3
- 49/4*(-33/112*I*sqrt(7) - 1/2*sqrt(-2101/1568*I*sqrt(7) - 55/32) + 3/16)^
2*(-2211*I*sqrt(7) + 3752*sqrt(2101/1568*I*sqrt(7) - 55/32) - 3839) - 1/256
*(210112*(33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/32) + 3/16)^
2 - 46431*I*sqrt(7) + 78792*sqrt(2101/1568*I*sqrt(7) - 55/32) - 117483)*(33
*I*sqrt(7) + 56*sqrt(-2101/1568*I*sqrt(7) - 55/32) - 21) - 68943*(33/112*I*
sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/32) + 3/16)^2 + 15488*x + 61908
*I*sqrt(7) - 105056*sqrt(2101/1568*I*sqrt(7) - 55/32) + 123428) + 2*x*(-33*
I*sqrt(7) + 56*sqrt(2101/1568*I*sqrt(7) - 55/32) - 21)*log(-91924*(33/112*I
```

*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/32) + 3/16)^3 + 98735*(33/112*
I*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/32) + 3/16)^2 + 15488*x - 146
487/2*I*sqrt(7) + 124292*sqrt(2101/1568*I*sqrt(7) - 55/32) - 285347/2) + (4
*sqrt(7)*sqrt(-336*(33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/32
) + 3/16)^2 - 336*(-33/112*I*sqrt(7) - 1/2*sqrt(-2101/1568*I*sqrt(7) - 55/3
2) + 3/16)^2 - 1/56*(33*I*sqrt(7) + 56*sqrt(-2101/1568*I*sqrt(7) - 55/32) -
 21)*(-33*I*sqrt(7) + 56*sqrt(2101/1568*I*sqrt(7) - 55/32) + 63) + 99/2*I*s
qrt(7) - 84*sqrt(2101/1568*I*sqrt(7) - 55/32) - 1859/2)*x - x*(33*I*sqrt(7)
 + 56*sqrt(-2101/1568*I*sqrt(7) - 55/32) - 21) - x*(-33*I*sqrt(7) + 56*sqrt
(2101/1568*I*sqrt(7) - 55/32) - 21) - 84*x)*log(49/4*(-33/112*I*sqrt(7) - 1
/2*sqrt(-2101/1568*I*sqrt(7) - 55/32) + 3/16)^2*(-2211*I*sqrt(7) + 3752*sqr
t(2101/1568*I*sqrt(7) - 55/32) - 3839) + 1/256*(210112*(33/112*I*sqrt(7) -
1/2*sqrt(2101/1568*I*sqrt(7) - 55/32) + 3/16)^2 - 46431*I*sqrt(7) + 78792*s
qrt(2101/1568*I*sqrt(7) - 55/32) - 117483)*(33*I*sqrt(7) + 56*sqrt(-2101/15
68*I*sqrt(7) - 55/32) - 21) - 29792*(33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*
I*sqrt(7) - 55/32) + 3/16)^2 + 1/256*((67*sqrt(7)*(-33*I*sqrt(7) + 56*sqrt(
2101/1568*I*sqrt(7) - 55/32) - 21) - 2432*sqrt(7))*(33*I*sqrt(7) + 56*sqrt(
-2101/1568*I*sqrt(7) - 55/32) - 21) - 2432*sqrt(7)*(-33*I*sqrt(7) + 56*sqrt
(2101/1568*I*sqrt(7) - 55/32) - 21) + 147456*sqrt(7))*sqrt(-336*(33/112*I*s
qrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/32) + 3/16)^2 - 336*(-33/112*I*s
qrt(7) - 1/2*sqrt(-2101/1568*I*sqrt(7) - 55/32) + 3/16)^2 - 1/56*(33*I*sqrt
(7) + 56*sqrt(-2101/1568*I*sqrt(7) - 55/32) - 21)*(-33*I*sqrt(7) + 56*sqrt(
2101/1568*I*sqrt(7) - 55/32) + 63) + 99/2*I*sqrt(7) - 84*sqrt(2101/1568*I*s
qrt(7) - 55/32) - 1859/2) + 30976*x + 22671/2*I*sqrt(7) - 19236*sqrt(2101/1
568*I*sqrt(7) - 55/32) + 53979/2) - (4*sqrt(7)*sqrt(-336*(33/112*I*sqrt(7)
- 1/2*sqrt(2101/1568*I*sqrt(7) - 55/32) + 3/16)^2 - 336*(-33/112*I*sqrt(7)
- 1/2*sqrt(-2101/1568*I*sqrt(7) - 55/32) + 3/16)^2 - 1/56*(33*I*sqrt(7) + 5
6*sqrt(-2101/1568*I*sqrt(7) - 55/32) - 21)*(-33*I*sqrt(7) + 56*sqrt(2101/15
68*I*sqrt(7) - 55/32) + 63) + 99/2*I*sqrt(7) - 84*sqrt(2101/1568*I*sqrt(7)
- 55/32) - 1859/2)*x + x*(33*I*sqrt(7) + 56*sqrt(-2101/1568*I*sqrt(7) - 55/
32) - 21) + x*(-33*I*sqrt(7) + 56*sqrt(2101/1568*I*sqrt(7) - 55/32) - 21) +
 84*x)*log(49/4*(-33/112*I*sqrt(7) - 1/2*sqrt(-2101/1568*I*sqrt(7) - 55/32)
 + 3/16)^2*(-2211*I*sqrt(7) + 3752*sqrt(2101/1568*I*sqrt(7) - 55/32) - 3839
) + 1/256*(210112*(33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/32)
 + 3/16)^2 - 46431*I*sqrt(7) + 78792*sqrt(2101/1568*I*sqrt(7) - 55/32) - 11
7483)*(33*I*sqrt(7) + 56*sqrt(-2101/1568*I*sqrt(7) - 55/32) - 21) - 29792*(
33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/32) + 3/16)^2 - 1/256*
((67*sqrt(7)*(-33*I*sqrt(7) + 56*sqrt(2101/1568*I*sqrt(7) - 55/32) - 21) -
2432*sqrt(7))*(33*I*sqrt(7) + 56*sqrt(-2101/1568*I*sqrt(7) - 55/32) - 21) -
 2432*sqrt(7)*(-33*I*sqrt(7) + 56*sqrt(2101/1568*I*sqrt(7) - 55/32) - 21) +
 147456*sqrt(7))*sqrt(-336*(33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7)
 - 55/32) + 3/16)^2 - 336*(-33/112*I*sqrt(7) - 1/2*sqrt(-2101/1568*I*sqrt(7
) - 55/32) + 3/16)^2 - 1/56*(33*I*sqrt(7) + 56*sqrt(-2101/1568*I*sqrt(7) -
55/32) - 21)*(-33*I*sqrt(7) + 56*sqrt(2101/1568*I*sqrt(7) - 55/32) + 63) +
99/2*I*sqrt(7) - 84*sqrt(2101/1568*I*sqrt(7) - 55/32) - 1859/2) + 30976*x +

```
22671/2*I*sqrt(7) - 19236*sqrt(2101/1568*I*sqrt(7) - 55/32) + 53979/2) + 1
68*x*log(x) + 560)/x
```

Sympy [A] time = 2.41469, size = 65, normalized size = 0.23

$$-\frac{3\log(x)}{4} + \mathrm{RootSum}\left(1372t^4 - 1029t^3 + 3136t^2 + 2688t + 512, \left(t \mapsto t\log\left(-\frac{506797249t^4}{34947704} + \frac{21584647t^3}{4368463} - \frac{14969669}{559163}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**3+3*x**2+x+5)/x**2/(2*x**4+x**3+5*x**2+x+2),x)
```

```
[Out] -3*log(x)/4 + RootSum(1372*_t**4 - 1029*_t**3 + 3136*_t**2 + 2688*_t + 512,
 Lambda(_t, _t*log(-506797249*_t**4/34947704 + 21584647*_t**3/4368463 - 149
69669687*_t**2/559163264 - 282513301*_t/6354128 + x - 101471979/8736926)))
- 5/(2*x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2\,x^3 + 3\,x^2 + x + 5}{\left(2\,x^4 + x^3 + 5\,x^2 + x + 2\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/x^2/(2*x^4+x^3+5*x^2+x+2),x, algorithm="giac")
```

```
[Out] integrate((2*x^3 + 3*x^2 + x + 5)/((2*x^4 + x^3 + 5*x^2 + x + 2)*x^2), x)
```

3.256 $\int \frac{5+x+3x^2+2x^3}{x^3\left(2+x+5x^2+x^3+2x^4\right)}\,dx$

Optimal. Leaf size=317

$$-\frac{35+9i\sqrt{7}}{56x^2}-\frac{35-9i\sqrt{7}}{56x^2}+\frac{1}{32}\left(35-9i\sqrt{7}\right)\log\left(4ix^2+\left(-\sqrt{7}+i\right)x+4i\right)+\frac{1}{32}\left(35+9i\sqrt{7}\right)\log\left(4ix^2+\left(\sqrt{7}+i\right)x+4\right.$$

```
[Out] -(35 - (9*I)*Sqrt[7])/(56*x^2) - (35 + (9*I)*Sqrt[7])/(56*x^2) + (3*(7 - (1
1*I)*Sqrt[7]))/(56*x) + (3*(7 + (11*I)*Sqrt[7]))/(56*x) + ((355 - (73*I)*Sq
rt[7])*ArcTanh[(I - Sqrt[7] + (8*I)*x)/Sqrt[2*(35 - I*Sqrt[7])]])/(8*Sqrt[1
4*(35 - I*Sqrt[7])]) - ((355 + (73*I)*Sqrt[7])*ArcTanh[(I + Sqrt[7] + (8*I)
*x)/Sqrt[2*(35 + I*Sqrt[7])]])/(8*Sqrt[14*(35 + I*Sqrt[7])]) - ((35 - (9*I)
*Sqrt[7])*Log[x])/16 - ((35 + (9*I)*Sqrt[7])*Log[x])/16 + ((35 - (9*I)*Sqrt
[7])*Log[4*I + (I - Sqrt[7])*x + (4*I)*x^2])/32 + ((35 + (9*I)*Sqrt[7])*Log
[4*I + (I + Sqrt[7])*x + (4*I)*x^2])/32
```

Rubi [A] time = 0.539418, antiderivative size = 317, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2087, 800, 634, 618, 206, 628}

$$-\frac{35+9i\sqrt{7}}{56x^2}-\frac{35-9i\sqrt{7}}{56x^2}+\frac{1}{32}\left(35-9i\sqrt{7}\right)\log\left(4ix^2+\left(-\sqrt{7}+i\right)x+4i\right)+\frac{1}{32}\left(35+9i\sqrt{7}\right)\log\left(4ix^2+\left(\sqrt{7}+i\right)x+4\right.$$

Antiderivative was successfully verified.

```
[In] Int[(5 + x + 3*x^2 + 2*x^3)/(x^3*(2 + x + 5*x^2 + x^3 + 2*x^4)),x]
```


```
[Out] -(35 - (9*I)*Sqrt[7])/(56*x^2) - (35 + (9*I)*Sqrt[7])/(56*x^2) + (3*(7 - (1
1*I)*Sqrt[7]))/(56*x) + (3*(7 + (11*I)*Sqrt[7]))/(56*x) + ((355 - (73*I)*Sq
rt[7])*ArcTanh[(I - Sqrt[7] + (8*I)*x)/Sqrt[2*(35 - I*Sqrt[7])]])/(8*Sqrt[1
4*(35 - I*Sqrt[7])]) - ((355 + (73*I)*Sqrt[7])*ArcTanh[(I + Sqrt[7] + (8*I)
*x)/Sqrt[2*(35 + I*Sqrt[7])]])/(8*Sqrt[14*(35 + I*Sqrt[7])]) - ((35 - (9*I)
*Sqrt[7])*Log[x])/16 - ((35 + (9*I)*Sqrt[7])*Log[x])/16 + ((35 - (9*I)*Sqrt
[7])*Log[4*I + (I - Sqrt[7])*x + (4*I)*x^2])/32 + ((35 + (9*I)*Sqrt[7])*Log
[4*I + (I + Sqrt[7])*x + (4*I)*x^2])/32
```

Rule 2087

```
Int[((P3_)*(x_)^(m_.))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (
e_.)*(x_)^4), x_Symbol] :> With[{q = Sqrt[8*a^2 + b^2 - 4*a*c], A = Coeff[P
3, x, 0], B = Coeff[P3, x, 1], C = Coeff[P3, x, 2], D = Coeff[P3, x, 3]}, D
ist[1/q, Int[(x^m*(b*A - 2*a*B + 2*a*D + A*q + (2*a*A - 2*a*C + b*D + D*q)*
x))/(2*a + (b + q)*x + 2*a*x^2), x], x] - Dist[1/q, Int[(x^m*(b*A - 2*a*B +
 2*a*D - A*q + (2*a*A - 2*a*C + b*D - D*q)*x))/(2*a + (b - q)*x + 2*a*x^2),
 x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[P3, x, 3] && EqQ[a, e] && EqQ[b
, d]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{5+x+3x^2+2x^3}{x^3\left(2+x+5x^2+x^3+2x^4\right)}\,dx = \frac{i\int \frac{9-5i\sqrt{7}+\left(10-2i\sqrt{7}\right)x}{x^3\left(4+\left(1-i\sqrt{7}\right)x+4x^2\right)}\,dx}{\sqrt{7}} - \frac{i\int \frac{9+5i\sqrt{7}+\left(10+2i\sqrt{7}\right)x}{x^3\left(4+\left(1+i\sqrt{7}\right)x+4x^2\right)}\,dx}{\sqrt{7}}$$

$$= -\frac{i\int \left(\frac{9+5i\sqrt{7}}{4x^3}+\frac{3\left(11-i\sqrt{7}\right)}{8x^2}-\frac{7i\left(-9i+5\sqrt{7}\right)}{16x}+\frac{-223i-61\sqrt{7}+14\left(9i-5\sqrt{7}\right)x}{8\left(4i+\left(i-\sqrt{7}\right)x+4ix^2\right)}\right)dx}{\sqrt{7}} + \frac{i\int \left(\frac{9-5i\sqrt{7}}{4x^3}+\right.}{}$$

$$= -\frac{35-9i\sqrt{7}}{56x^2} - \frac{35+9i\sqrt{7}}{56x^2} + \frac{3\left(7-11i\sqrt{7}\right)}{56x} + \frac{3\left(7+11i\sqrt{7}\right)}{56x} - \frac{1}{16}\left(35-9i\sqrt{7}\right)\log($$

$$= -\frac{35-9i\sqrt{7}}{56x^2} - \frac{35+9i\sqrt{7}}{56x^2} + \frac{3\left(7-11i\sqrt{7}\right)}{56x} + \frac{3\left(7+11i\sqrt{7}\right)}{56x} - \frac{1}{16}\left(35-9i\sqrt{7}\right)\log($$

$$= -\frac{35-9i\sqrt{7}}{56x^2} - \frac{35+9i\sqrt{7}}{56x^2} + \frac{3\left(7-11i\sqrt{7}\right)}{56x} + \frac{3\left(7+11i\sqrt{7}\right)}{56x} - \frac{1}{16}\left(35-9i\sqrt{7}\right)\log($$

$$= -\frac{35-9i\sqrt{7}}{56x^2} - \frac{35+9i\sqrt{7}}{56x^2} + \frac{3\left(7-11i\sqrt{7}\right)}{56x} + \frac{3\left(7+11i\sqrt{7}\right)}{56x} + \frac{\left(355-73i\sqrt{7}\right)\tanh}{8\sqrt{14\left(35\right.}}$$

Mathematica [C] time = 0.0179647, size = 116, normalized size = 0.37

$$\frac{1}{8}\text{RootSum}\left[2\#1^4+\#1^3+5\#1^2+\#1+2\&,\frac{70\#1^3\log(x-\#1)+47\#1^2\log(x-\#1)+141\#1\log(x-\#1)+61\log(x-\#}{8\#1^3+3\#1^2+10\#1+1}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + x + 3*x^2 + 2*x^3)/(x^3*(2 + x + 5*x^2 + x^3 + 2*x^4)),x]
```

```
[Out] -5/(4*x^2) + 3/(4*x) - (35*Log[x])/8 + RootSum[2 + #1 + 5*#1^2 + #1^3 + 2*#
1^4 & , (61*Log[x - #1] + 141*Log[x - #1]*#1 + 47*Log[x - #1]*#1^2 + 70*Log
[x - #1]*#1^3)/(1 + 10*#1 + 3*#1^2 + 8*#1^3) & ]/8
```

Maple [C] time = 0.008, size = 77, normalized size = 0.2

$$-\frac{5}{4x^2}+\frac{3}{4x}-\frac{35\ln(x)}{8}+\frac{1}{8}\sum_{\_R=\text{RootOf}\left(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2\right)}\frac{\left(70\_R^3+47\_R^2+141\_R+61\right)\ln(x-\_R)}{8\_R^3+3\_R^2+10\_R+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3+3*x^2+x+5)/x^3/(2*x^4+x^3+5*x^2+x+2),x)
```

```
[Out] -5/4/x^2+3/4/x-35/8*ln(x)+1/8*sum((70*_R^3+47*_R^2+141*_R+61)/(8*_R^3+3*_R^
2+10*_R+1)*ln(x-_R),_R=RootOf(2*_Z^4+_Z^3+5*_Z^2+_Z+2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{3\,x-5}{4\,x^2}+\frac{1}{8}\int\frac{70\,x^3+47\,x^2+141\,x+61}{2\,x^4+x^3+5\,x^2+x+2}\,dx-\frac{35}{8}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/x^3/(2*x^4+x^3+5*x^2+x+2),x, algorithm="maxima"
)
```

```
[Out] 1/4*(3*x - 5)/x^2 + 1/8*integrate((70*x^3 + 47*x^2 + 141*x + 61)/(2*x^4 + x
^3 + 5*x^2 + x + 2), x) - 35/8*log(x)
```

Fricas [B] time = 9.77968, size = 6164, normalized size = 19.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/x^3/(2*x^4+x^3+5*x^2+x+2),x, algorithm="fricas"
)
```

```
[Out] -1/448*(14*x^2*(-9*I*sqrt(7) + 16*sqrt(9803/6272*I*sqrt(7) + 2815/896) - 35
)*log(-49/4*(207711*I*sqrt(7) + 369264*sqrt(-9803/6272*I*sqrt(7) + 2815/896
) - 957269)*(9/32*I*sqrt(7) - 1/2*sqrt(9803/6272*I*sqrt(7) + 2815/896) + 35
/32)^2 + 9046968*(-9/32*I*sqrt(7) - 1/2*sqrt(-9803/6272*I*sqrt(7) + 2815/89
6) + 35/32)^3 - 39580485*(-9/32*I*sqrt(7) - 1/2*sqrt(-9803/6272*I*sqrt(7) +
 2815/896) + 35/32)^2 - 21/1024*(13785856*(-9/32*I*sqrt(7) - 1/2*sqrt(-9803
/6272*I*sqrt(7) + 2815/896) + 35/32)^2 + 16963065*I*sqrt(7) + 30156560*sqrt
(-9803/6272*I*sqrt(7) + 2815/896) - 68488563)*(-9*I*sqrt(7) + 16*sqrt(9803/
6272*I*sqrt(7) + 2815/896) - 35) + 9662336*x - 68336919/4*I*sqrt(7) - 30371
964*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 257023549/4) + 14*x^2*(9*I*sqrt
```

(7) + 16*sqrt(-9803/6272*I*sqrt(7) + 2815/896) - 35)*log(-9046968*(-9/32*I*
sqrt(7) - 1/2*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 35/32)^3 + 41411909*(
-9/32*I*sqrt(7) - 1/2*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 35/32)^2 + 96
62336*x + 70198191/4*I*sqrt(7) + 31199196*sqrt(-9803/6272*I*sqrt(7) + 2815/
896) - 240366533/4) + 1960*x^2*log(x) + (4*sqrt(7)*sqrt(-1344*(9/32*I*sqrt(
7) - 1/2*sqrt(9803/6272*I*sqrt(7) + 2815/896) + 35/32)^2 - 1344*(-9/32*I*sq
rt(7) - 1/2*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 35/32)^2 - 7/8*(9*I*sqr
t(7) + 16*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 105)*(-9*I*sqrt(7) + 16*s
qrt(9803/6272*I*sqrt(7) + 2815/896) - 35) - 2205/2*I*sqrt(7) - 1960*sqrt(-9
803/6272*I*sqrt(7) + 2815/896) + 1661/2)*x^2 - 7*x^2*(9*I*sqrt(7) + 16*sqrt
(-9803/6272*I*sqrt(7) + 2815/896) - 35) - 7*x^2*(-9*I*sqrt(7) + 16*sqrt(980
3/6272*I*sqrt(7) + 2815/896) - 35) - 980*x^2)*log(49/4*(207711*I*sqrt(7) +
369264*sqrt(-9803/6272*I*sqrt(7) + 2815/896) - 957269)*(9/32*I*sqrt(7) - 1/
2*sqrt(9803/6272*I*sqrt(7) + 2815/896) + 35/32)^2 - 1831424*(-9/32*I*sqrt(7
) - 1/2*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 35/32)^2 + 21/1024*(1378585
6*(-9/32*I*sqrt(7) - 1/2*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 35/32)^2 +
16963065*I*sqrt(7) + 30156560*sqrt(-9803/6272*I*sqrt(7) + 2815/896) - 6848
8563)*(-9*I*sqrt(7) + 16*sqrt(9803/6272*I*sqrt(7) + 2815/896) - 35) + 1/102
4*sqrt(-1344*(9/32*I*sqrt(7) - 1/2*sqrt(9803/6272*I*sqrt(7) + 2815/896) + 3
5/32)^2 - 1344*(-9/32*I*sqrt(7) - 1/2*sqrt(-9803/6272*I*sqrt(7) + 2815/896)
+ 35/32)^2 - 7/8*(9*I*sqrt(7) + 16*sqrt(-9803/6272*I*sqrt(7) + 2815/896) +
105)*(-9*I*sqrt(7) + 16*sqrt(9803/6272*I*sqrt(7) + 2815/896) - 35) - 2205/
2*I*sqrt(7) - 1960*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 1661/2)*(7*(2307
9*sqrt(7)*(9*I*sqrt(7) + 16*sqrt(-9803/6272*I*sqrt(7) + 2815/896) - 35) - 1
49504*sqrt(7))*(-9*I*sqrt(7) + 16*sqrt(9803/6272*I*sqrt(7) + 2815/896) - 35
) - 1046528*sqrt(7)*(9*I*sqrt(7) + 16*sqrt(-9803/6272*I*sqrt(7) + 2815/896)
- 35) - 116260864*sqrt(7)) + 19324672*x - 465318*I*sqrt(7) - 827232*sqrt(-
9803/6272*I*sqrt(7) + 2815/896) + 666914) - (4*sqrt(7)*sqrt(-1344*(9/32*I*s
qrt(7) - 1/2*sqrt(9803/6272*I*sqrt(7) + 2815/896) + 35/32)^2 - 1344*(-9/32*
I*sqrt(7) - 1/2*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 35/32)^2 - 7/8*(9*I
*sqrt(7) + 16*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 105)*(-9*I*sqrt(7) +
16*sqrt(9803/6272*I*sqrt(7) + 2815/896) - 35) - 2205/2*I*sqrt(7) - 1960*sqr
t(-9803/6272*I*sqrt(7) + 2815/896) + 1661/2)*x^2 + 7*x^2*(9*I*sqrt(7) + 16*
sqrt(-9803/6272*I*sqrt(7) + 2815/896) - 35) + 7*x^2*(-9*I*sqrt(7) + 16*sqrt
(9803/6272*I*sqrt(7) + 2815/896) - 35) + 980*x^2)*log(49/4*(207711*I*sqrt(7
) + 369264*sqrt(-9803/6272*I*sqrt(7) + 2815/896) - 957269)*(9/32*I*sqrt(7)
- 1/2*sqrt(9803/6272*I*sqrt(7) + 2815/896) + 35/32)^2 - 1831424*(-9/32*I*sq
rt(7) - 1/2*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 35/32)^2 + 21/1024*(137
85856*(-9/32*I*sqrt(7) - 1/2*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 35/32)
^2 + 16963065*I*sqrt(7) + 30156560*sqrt(-9803/6272*I*sqrt(7) + 2815/896) -
68488563)*(-9*I*sqrt(7) + 16*sqrt(9803/6272*I*sqrt(7) + 2815/896) - 35) - 1
/1024*sqrt(-1344*(9/32*I*sqrt(7) - 1/2*sqrt(9803/6272*I*sqrt(7) + 2815/896)
+ 35/32)^2 - 1344*(-9/32*I*sqrt(7) - 1/2*sqrt(-9803/6272*I*sqrt(7) + 2815/
896) + 35/32)^2 - 7/8*(9*I*sqrt(7) + 16*sqrt(-9803/6272*I*sqrt(7) + 2815/89
6) + 105)*(-9*I*sqrt(7) + 16*sqrt(9803/6272*I*sqrt(7) + 2815/896) - 35) - 2

```
205/2*I*sqrt(7) - 1960*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 1661/2)*(7*(
23079*sqrt(7)*(9*I*sqrt(7) + 16*sqrt(-9803/6272*I*sqrt(7) + 2815/896) - 35)
 - 149504*sqrt(7))*(-9*I*sqrt(7) + 16*sqrt(9803/6272*I*sqrt(7) + 2815/896)
- 35) - 1046528*sqrt(7)*(9*I*sqrt(7) + 16*sqrt(-9803/6272*I*sqrt(7) + 2815/
896) - 35) - 116260864*sqrt(7)) + 19324672*x - 465318*I*sqrt(7) - 827232*sq
rt(-9803/6272*I*sqrt(7) + 2815/896) + 666914) - 336*x + 560)/x^2
```

Sympy [A] time = 1.67274, size = 70, normalized size = 0.22

$$-\frac{35\log(x)}{8} + \mathrm{RootSum}\left(2744t^4 - 12005t^3 + 18424t^2 - 3136t + 1024, \left(t \mapsto t\log\left(-\frac{20101387287723t^4}{91907904361586} + \frac{94451521449}{45953952180}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**3+3*x**2+x+5)/x**3/(2*x**4+x**3+5*x**2+x+2),x)
```

```
[Out] -35*log(x)/8 + RootSum(2744*_t**4 - 12005*_t**3 + 18424*_t**2 - 3136*_t + 1
024, Lambda(_t, _t*log(-20101387287723*_t**4/91907904361586 + 944515214496*
_t**3/45953952180793 + 16572327093911939*_t**2/5882105879141504 - 456447174
9800865*_t/735263234892688 + x + 70084064010625/91907904361586))) + (3*x -
5)/(4*x**2)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2x^3 + 3x^2 + x + 5}{(2x^4 + x^3 + 5x^2 + x + 2)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/x^3/(2*x^4+x^3+5*x^2+x+2),x, algorithm="giac")
```

```
[Out] integrate((2*x^3 + 3*x^2 + x + 5)/((2*x^4 + x^3 + 5*x^2 + x + 2)*x^3), x)
```

3.257 $\int \frac{x^2(3a+bx^2)}{a^2+2abx^2+b^2x^4+c^2x^6}\,dx$

Optimal. Leaf size=19

$$\frac{\tan^{-1}\left(\frac{cx^3}{a+bx^2}\right)}{c}$$

```
[Out] ArcTan[(c*x^3)/(a + b*x^2)]/c
```

Rubi [A] time = 0.104799, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {2094, 205}

$$\frac{\tan^{-1}\left(\frac{cx^3}{a+bx^2}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(3*a + b*x^2))/(a^2 + 2*a*b*x^2 + b^2*x^4 + c^2*x^6),x]
```

```
[Out] ArcTan[(c*x^3)/(a + b*x^2)]/c
```

Rule 2094

```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(n_.)))/((a_) + (b_.)*(x_)^(k_.) + (c_.)
*(x_)^(n_.) + (d_.)*(x_)^(n2_)), x_Symbol] :> Dist[(A^2*(m - n + 1))/(m + 1
), Subst[Int[1/(a + A^2*b*(m - n + 1)^2*x^2), x], x, x^(m + 1)/(A*(m - n +
1) + B*(m + 1)*x^n)], x] /; FreeQ[{a, b, c, d, A, B, m, n}, x] && EqQ[n2, 2
*n] && EqQ[k, 2*(m + 1)] && EqQ[a*B^2*(m + 1)^2 - A^2*d*(m - n + 1)^2, 0] &
& EqQ[B*c*(m + 1) - 2*A*d*(m - n + 1), 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^2 \left(3a + bx^2\right)}{a^2 + 2abx^2 + b^2x^4 + c^2x^6} \, dx = \left(3a^2\right) \text{Subst}\left(\int \frac{1}{a^2 + 9a^2c^2x^2} \, dx, x, \frac{x^3}{3a + 3bx^2}\right)$$
$$= \frac{\tan^{-1}\left(\frac{cx^3}{a+bx^2}\right)}{c}$$

Mathematica [C] time = 0.0452552, size = 87, normalized size = 4.58

$$\frac{1}{2}\text{RootSum}\left[2\#1^2ab + \#1^4b^2 + \#1^6c^2 + a^2\&, \frac{\#1^3b\log(x - \#1) + 3\#1a\log(x - \#1)}{2\#1^2b^2 + 3\#1^4c^2 + 2ab}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(3*a + b*x^2))/(a^2 + 2*a*b*x^2 + b^2*x^4 + c^2*x^6),x]
```

```
[Out] RootSum[a^2 + 2*a*b*#1^2 + b^2*#1^4 + c^2*#1^6 & , (3*a*Log[x - #1]*#1 + b*
Log[x - #1]*#1^3)/(2*a*b + 2*b^2*#1^2 + 3*c^2*#1^4) & ]/2
```

Maple [C] time = 0.107, size = 75, normalized size = 4.

$$\frac{1}{2}\sum_{\_R=\text{RootOf}\left(c^2\_Z^6+b^2\_Z^4+2ab\_Z^2+a^2\right)}\frac{\left(\_R^4b + 3\_R^2a\right)\ln(x - \_R)}{3\_R^5c^2 + 2\_R^3b^2 + 2\_Rab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+3*a)/(c^2*x^6+b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] 1/2*sum((_R^4*b+3*_R^2*a)/(3*_R^5*c^2+2*_R^3*b^2+2*_R*a*b)*ln(x-_R),_R=Root
Of(_Z^6*c^2+_Z^4*b^2+2*_Z^2*a*b+a^2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2 + 3a\right)x^2}{c^2x^6 + b^2x^4 + 2abx^2 + a^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+3*a)/(c^2*x^6+b^2*x^4+2*a*b*x^2+a^2),x, algorithm="max
ima")
```

```
[Out] integrate((b*x^2 + 3*a)*x^2/(c^2*x^6 + b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Fricas [B] time = 1.41602, size = 171, normalized size = 9.

$$\frac{\arctan\left(\frac{cx}{b}\right) - \arctan\left(\frac{bc^2x^5+ab^2x+\left(b^3-ac^2\right)x^3}{a^2c}\right) + \arctan\left(\frac{bc^2x^3+\left(b^3-ac^2\right)x}{abc}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+3*a)/(c^2*x^6+b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fri
cas")
```

```
[Out] (arctan(c*x/b) - arctan((b*c^2*x^5 + a*b^2*x + (b^3 - a*c^2)*x^3)/(a^2*c))
+ arctan((b*c^2*x^3 + (b^3 - a*c^2)*x)/(a*b*c)))/c
```

Sympy [C] time = 1.04768, size = 44, normalized size = 2.32

$$\frac{-\frac{i\log\left(-\frac{ia}{c}-\frac{ibx^2}{c}+x^3\right)}{2}+\frac{i\log\left(\frac{ia}{c}+\frac{ibx^2}{c}+x^3\right)}{2}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+3*a)/(c**2*x**6+b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] (-I*log(-I*a/c - I*b*x**2/c + x**3)/2 + I*log(I*a/c + I*b*x**2/c + x**3)/2)
/c
```

Giac [B] time = 3.98667, size = 117, normalized size = 6.16

$$\frac{\arctan\left(\frac{cx}{b}\right) + \arctan\left(-\frac{bc^2x^5+b^3x^3-ac^2x^3+ab^2x}{a^2c}\right) - \arctan\left(-\frac{bc^2x^3+b^3x-ac^2x}{abc}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+3*a)/(c^2*x^6+b^2*x^4+2*a*b*x^2+a^2),x, algorithm="gia
c")
```

```
[Out] (arctan(c*x/b) + arctan(-(b*c^2*x^5 + b^3*x^3 - a*c^2*x^3 + a*b^2*x)/(a^2*c
)) - arctan(-(b*c^2*x^3 + b^3*x - a*c^2*x)/(a*b*c)))/c
```

3.258 $\int \frac{1-3x^4}{(-2+x)\left(1+x^2\right)^2}\,dx$

Optimal. Leaf size=43

$$-\frac{1-2x}{5\left(x^2+1\right)}-\frac{14}{25}\log\left(x^2+1\right)-\frac{47}{25}\log(2-x)-\frac{46}{25}\tan^{-1}(x)$$

```
[Out] -(1 - 2*x)/(5*(1 + x^2)) - (46*ArcTan[x])/25 - (47*Log[2 - x])/25 - (14*Log
[1 + x^2])/25
```

Rubi [A] time = 0.0669825, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1647, 1629, 635, 203, 260}

$$-\frac{1-2x}{5\left(x^2+1\right)}-\frac{14}{25}\log\left(x^2+1\right)-\frac{47}{25}\log(2-x)-\frac{46}{25}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 3*x^4)/((-2 + x)*(1 + x^2)^2),x]
```

```
[Out] -(1 - 2*x)/(5*(1 + x^2)) - (46*ArcTan[x])/25 - (47*Log[2 - x])/25 - (14*Log
[1 + x^2])/25
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1-3x^4}{(-2+x)\left(1+x^2\right)^2}\,dx &= -\frac{1-2x}{5\left(1+x^2\right)}-\frac{1}{2}\int \frac{-\frac{18}{5}-\frac{4x}{5}+6x^2}{(-2+x)\left(1+x^2\right)}\,dx\\
&= -\frac{1-2x}{5\left(1+x^2\right)}-\frac{1}{2}\int \left(\frac{94}{25(-2+x)}+\frac{4(23+14x)}{25\left(1+x^2\right)}\right)dx\\
&= -\frac{1-2x}{5\left(1+x^2\right)}-\frac{47}{25}\log(2-x)-\frac{2}{25}\int \frac{23+14x}{1+x^2}\,dx\\
&= -\frac{1-2x}{5\left(1+x^2\right)}-\frac{47}{25}\log(2-x)-\frac{28}{25}\int \frac{x}{1+x^2}\,dx-\frac{46}{25}\int \frac{1}{1+x^2}\,dx\\
&= -\frac{1-2x}{5\left(1+x^2\right)}-\frac{46}{25}\tan^{-1}(x)-\frac{47}{25}\log(2-x)-\frac{14}{25}\log\left(1+x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.0226428, size = 57, normalized size = 1.33

$$\frac{2(x-2)+3}{5\left((x-2)^2+4(x-2)+5\right)}-\frac{14}{25}\log\left((x-2)^2+4(x-2)+5\right)-\frac{47}{25}\log(x-2)-\frac{46}{25}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 3*x^4)/((-2 + x)*(1 + x^2)^2),x]
```

```
[Out] (3 + 2*(-2 + x))/(5*(5 + 4*(-2 + x) + (-2 + x)^2)) - (46*ArcTan[x])/25 - (1
4*Log[5 + 4*(-2 + x) + (-2 + x)^2])/25 - (47*Log[-2 + x])/25
```

Maple [A] time = 0.01, size = 34, normalized size = 0.8

$$-\frac{2}{25x^2+25}\left(-5x+\frac{5}{2}\right)-\frac{14\ln\left(x^2+1\right)}{25}-\frac{46\arctan\left(x\right)}{25}-\frac{47\ln\left(-2+x\right)}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3*x^4+1)/(-2+x)/(x^2+1)^2,x)
```

```
[Out] -2/25*(-5*x+5/2)/(x^2+1)-14/25*ln(x^2+1)-46/25*arctan(x)-47/25*ln(-2+x)
```

Maxima [A] time = 1.53747, size = 45, normalized size = 1.05

$$\frac{2x-1}{5\left(x^2+1\right)}-\frac{46}{25}\arctan\left(x\right)-\frac{14}{25}\log\left(x^2+1\right)-\frac{47}{25}\log\left(x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^4+1)/(-2+x)/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 1/5*(2*x - 1)/(x^2 + 1) - 46/25*arctan(x) - 14/25*log(x^2 + 1) - 47/25*log(
x - 2)
```

Fricas [A] time = 1.4701, size = 144, normalized size = 3.35

$$-\frac{46\left(x^2+1\right)\arctan\left(x\right)+14\left(x^2+1\right)\log\left(x^2+1\right)+47\left(x^2+1\right)\log\left(x-2\right)-10x+5}{25\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^4+1)/(-2+x)/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] -1/25*(46*(x^2 + 1)*arctan(x) + 14*(x^2 + 1)*log(x^2 + 1) + 47*(x^2 + 1)*lo
g(x - 2) - 10*x + 5)/(x^2 + 1)
```

Sympy [A] time = 0.154323, size = 36, normalized size = 0.84

$$\frac{2x-1}{5x^2+5} - \frac{47\log(x-2)}{25} - \frac{14\log(x^2+1)}{25} - \frac{46\,\mathrm{atan}(x)}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x**4+1)/(-2+x)/(x**2+1)**2,x)
```

```
[Out] (2*x - 1)/(5*x**2 + 5) - 47*log(x - 2)/25 - 14*log(x**2 + 1)/25 - 46*atan(x
)/25
```

Giac [A] time = 1.14189, size = 46, normalized size = 1.07

$$\frac{2\,x-1}{5\left(x^2+1\right)} - \frac{46}{25}\arctan(x) - \frac{14}{25}\log\left(x^2+1\right) - \frac{47}{25}\log(|x-2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^4+1)/(-2+x)/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] 1/5*(2*x - 1)/(x^2 + 1) - 46/25*arctan(x) - 14/25*log(x^2 + 1) - 47/25*log(
abs(x - 2))
```

3.259 $\int \frac{-9-9x+2x^2}{-9x+x^3} \, dx$

Optimal. Leaf size=17

$$-\log(3-x)+\log(x)+2\log(x+3)$$

```
[Out] -Log[3 - x] + Log[x] + 2*Log[3 + x]
```

Rubi [A] time = 0.0370496, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {1593, 1802}

$$-\log(3-x)+\log(x)+2\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Int[(-9 - 9*x + 2*x^2)/(-9*x + x^3),x]
```

```
[Out] -Log[3 - x] + Log[x] + 2*Log[3 + x]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{-9-9x+2x^2}{-9x+x^3} \, dx &= \int \frac{-9-9x+2x^2}{x\left(-9+x^2\right)} \, dx \\
&= \int \left(\frac{1}{3-x}+\frac{1}{x}+\frac{2}{3+x}\right) dx \\
&= -\log(3-x)+\log(x)+2\log(3+x)
\end{aligned}$$

Mathematica [A] time = 0.0046504, size = 17, normalized size = 1.

$$-\log(3-x)+\log(x)+2\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-9 - 9*x + 2*x^2)/(-9*x + x^3),x]
```

```
[Out] -Log[3 - x] + Log[x] + 2*Log[3 + x]
```

Maple [A] time = 0.006, size = 16, normalized size = 0.9

$$-\ln(-3+x)+\ln(x)+2\ln(3+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-9*x-9)/(x^3-9*x),x)
```

```
[Out] -ln(-3+x)+ln(x)+2*ln(3+x)
```

Maxima [A] time = 0.974869, size = 20, normalized size = 1.18

$$2\log(x+3)-\log(x-3)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-9*x-9)/(x^3-9*x),x, algorithm="maxima")
```

```
[Out] 2*log(x + 3) - log(x - 3) + log(x)
```

Fricas [A] time = 1.44348, size = 49, normalized size = 2.88

$$2\log(x+3)-\log(x-3)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-9*x-9)/(x^3-9*x),x, algorithm="fricas")
```

```
[Out] 2*log(x + 3) - log(x - 3) + log(x)
```

Sympy [A] time = 0.115618, size = 14, normalized size = 0.82

$$\log(x) - \log(x - 3) + 2\log(x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-9*x-9)/(x**3-9*x),x)
```

```
[Out] log(x) - log(x - 3) + 2*log(x + 3)
```

Giac [A] time = 1.41245, size = 24, normalized size = 1.41

$$2\,\log(|x + 3|) - \log(|x - 3|) + \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-9*x-9)/(x^3-9*x),x, algorithm="giac")
```

```
[Out] 2*log(abs(x + 3)) - log(abs(x - 3)) + log(abs(x))
```

3.260 $\int \frac{1+2x^2+x^5}{-x+x^3}\,dx$

Optimal. Leaf size=25

$$\frac{x^3}{3}+x+2\log(1-x)-\log(x)+\log(x+1)$$

```
[Out] x + x^3/3 + 2*Log[1 - x] - Log[x] + Log[1 + x]
```

Rubi [A] time = 0.0372468, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {1593, 1802}

$$\frac{x^3}{3}+x+2\log(1-x)-\log(x)+\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x^2 + x^5)/(-x + x^3),x]
```

```
[Out] x + x^3/3 + 2*Log[1 - x] - Log[x] + Log[1 + x]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{1+2x^2+x^5}{-x+x^3}\,dx = \int \frac{1+2x^2+x^5}{x\left(-1+x^2\right)}\,dx$$
$$= \int \left(1+\frac{2}{-1+x}-\frac{1}{x}+x^2+\frac{1}{1+x}\right)dx$$
$$= x+\frac{x^3}{3}+2\log(1-x)-\log(x)+\log(1+x)$$

Mathematica [A] time = 0.0053016, size = 25, normalized size = 1.

$$\frac{x^3}{3}+x+2\log(1-x)-\log(x)+\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x^2 + x^5)/(-x + x^3),x]
```

```
[Out] x + x^3/3 + 2*Log[1 - x] - Log[x] + Log[1 + x]
```

Maple [A] time = 0.007, size = 22, normalized size = 0.9

$$\frac{x^3}{3}+x+2\ln(x-1)-\ln(x)+\ln(1+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5+2*x^2+1)/(x^3-x),x)
```

```
[Out] 1/3*x^3+x+2*ln(x-1)-ln(x)+ln(1+x)
```

Maxima [A] time = 0.988588, size = 28, normalized size = 1.12

$$\frac{1}{3}x^3+x+\log(x+1)+2\log(x-1)-\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+2*x^2+1)/(x^3-x),x, algorithm="maxima")
```

```
[Out] 1/3*x^3 + x + log(x + 1) + 2*log(x - 1) - log(x)
```

Fricas [A] time = 1.39819, size = 68, normalized size = 2.72

$$\frac{1}{3}x^3 + x + \log(x+1) + 2\log(x-1) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+2*x^2+1)/(x^3-x),x, algorithm="fricas")
```

```
[Out] 1/3*x^3 + x + log(x + 1) + 2*log(x - 1) - log(x)
```

Sympy [A] time = 0.115942, size = 20, normalized size = 0.8

$$\frac{x^3}{3} + x - \log(x) + 2\log(x-1) + \log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**5+2*x**2+1)/(x**3-x),x)
```

```
[Out] x**3/3 + x - log(x) + 2*log(x - 1) + log(x + 1)
```

Giac [A] time = 1.14471, size = 32, normalized size = 1.28

$$\frac{1}{3}x^3 + x + \log(|x+1|) + 2\log(|x-1|) - \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+2*x^2+1)/(x^3-x),x, algorithm="giac")
```

```
[Out] 1/3*x^3 + x + log(abs(x + 1)) + 2*log(abs(x - 1)) - log(abs(x))
```

3.261 $\int \frac{3+2x^2}{(-1+x)^2x} \, dx$

Optimal. Leaf size=22

$$\frac{5}{1-x} - \log(1-x) + 3\log(x)$$

```
[Out] 5/(1 - x) - Log[1 - x] + 3*Log[x]
```

Rubi [A] time = 0.0133821, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {894}

$$\frac{5}{1-x} - \log(1-x) + 3\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 2*x^2)/((-1 + x)^2*x),x]
```

```
[Out] 5/(1 - x) - Log[1 - x] + 3*Log[x]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{3+2x^2}{(-1+x)^2x} \, dx &= \int \left(\frac{1}{1-x} + \frac{5}{(-1+x)^2} + \frac{3}{x}\right) dx \\
&= \frac{5}{1-x} - \log(1-x) + 3\log(x)
\end{aligned}$$

Mathematica [A] time = 0.0095116, size = 20, normalized size = 0.91

$$-\frac{5}{x-1} - \log(1-x) + 3\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 2*x^2)/((-1 + x)^2*x),x]
```

```
[Out] -5/(-1 + x) - Log[1 - x] + 3*Log[x]
```

Maple [A] time = 0.007, size = 19, normalized size = 0.9

$$-5\,(x-1)^{-1}-\ln(x-1)+3\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+3)/(x-1)^2/x,x)
```

```
[Out] -5/(x-1)-ln(x-1)+3*ln(x)
```

Maxima [A] time = 0.986294, size = 24, normalized size = 1.09

$$-\frac{5}{x-1}-\log(x-1)+3\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+3)/(-1+x)^2/x,x, algorithm="maxima")
```

```
[Out] -5/(x - 1) - log(x - 1) + 3*log(x)
```

Fricas [A] time = 1.2272, size = 73, normalized size = 3.32

$$-\frac{(x-1)\log(x-1)-3\,(x-1)\log(x)+5}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+3)/(-1+x)^2/x,x, algorithm="fricas")
```

```
[Out] -((x - 1)*log(x - 1) - 3*(x - 1)*log(x) + 5)/(x - 1)
```

Sympy [A] time = 0.102487, size = 14, normalized size = 0.64

$$3\log(x) - \log(x-1) - \frac{5}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+3)/(-1+x)**2/x,x)
```

```
[Out] 3*log(x) - log(x - 1) - 5/(x - 1)
```

Giac [A] time = 1.17183, size = 38, normalized size = 1.73

$$-\frac{5}{x-1} + 2\log(|x-1|) + 3\log\left(\left|-\frac{1}{x-1} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+3)/(-1+x)^2/x,x, algorithm="giac")
```

```
[Out] -5/(x - 1) + 2*log(abs(x - 1)) + 3*log(abs(-1/(x - 1) - 1))
```

3.262 $\int \frac{-1+2x^2}{(-1+4x)(1+x^2)} \, dx$

Optimal. Leaf size=27

$$\frac{6}{17}\log\left(x^2+1\right)-\frac{7}{34}\log(1-4x)+\frac{3}{17}\tan^{-1}(x)$$

```
[Out] (3*ArcTan[x])/17 - (7*Log[1 - 4*x])/34 + (6*Log[1 + x^2])/17
```

Rubi [A] time = 0.0389818, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1629, 635, 203, 260}

$$\frac{6}{17}\log\left(x^2+1\right)-\frac{7}{34}\log(1-4x)+\frac{3}{17}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 2*x^2)/((-1 + 4*x)*(1 + x^2)),x]
```

```
[Out] (3*ArcTan[x])/17 - (7*Log[1 - 4*x])/34 + (6*Log[1 + x^2])/17
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+2x^2}{(-1+4x)\left(1+x^2\right)}\,dx &= \int \left(-\frac{14}{17(-1+4x)}+\frac{3(1+4x)}{17\left(1+x^2\right)}\right)dx\\ &= -\frac{7}{34}\log(1-4x)+\frac{3}{17}\int \frac{1+4x}{1+x^2}\,dx\\ &= -\frac{7}{34}\log(1-4x)+\frac{3}{17}\int \frac{1}{1+x^2}\,dx+\frac{12}{17}\int \frac{x}{1+x^2}\,dx\\ &= \frac{3}{17}\tan^{-1}(x)-\frac{7}{34}\log(1-4x)+\frac{6}{17}\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.0107369, size = 38, normalized size = 1.41

$$-\frac{7}{34}\log(4x-1)+\frac{6}{17}\log\left((4x-1)^2+2(4x-1)+17\right)+\frac{3}{17}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + 2*x^2)/((-1 + 4*x)*(1 + x^2)),x]
```

```
[Out] (3*ArcTan[x])/17 - (7*Log[-1 + 4*x])/34 + (6*Log[17 + 2*(-1 + 4*x) + (-1 +
4*x)^2])/17
```

Maple [A] time = 0.006, size = 22, normalized size = 0.8

$$\frac{6\ln\left(x^2+1\right)}{17}+\frac{3\arctan(x)}{17}-\frac{7\ln(-1+4x)}{34}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-1)/(-1+4*x)/(x^2+1),x)
```

```
[Out] 6/17*ln(x^2+1)+3/17*arctan(x)-7/34*ln(-1+4*x)
```

Maxima [A] time = 1.51865, size = 28, normalized size = 1.04

$$\frac{3}{17} \arctan(x) + \frac{6}{17} \log\left(x^2 + 1\right) - \frac{7}{34} \log(4x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-1)/(-1+4*x)/(x^2+1),x, algorithm="maxima")
```

```
[Out] 3/17*arctan(x) + 6/17*log(x^2 + 1) - 7/34*log(4*x - 1)
```

Fricas [A] time = 1.51583, size = 76, normalized size = 2.81

$$\frac{3}{17} \arctan(x) + \frac{6}{17} \log\left(x^2 + 1\right) - \frac{7}{34} \log(4x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-1)/(-1+4*x)/(x^2+1),x, algorithm="fricas")
```

```
[Out] 3/17*arctan(x) + 6/17*log(x^2 + 1) - 7/34*log(4*x - 1)
```

Sympy [A] time = 0.126118, size = 26, normalized size = 0.96

$$-\frac{7 \log\left(x - \frac{1}{4}\right)}{34} + \frac{6 \log\left(x^2 + 1\right)}{17} + \frac{3 \operatorname{atan}(x)}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-1)/(-1+4*x)/(x**2+1),x)
```

```
[Out] -7*log(x - 1/4)/34 + 6*log(x**2 + 1)/17 + 3*atan(x)/17
```

Giac [A] time = 1.1492, size = 30, normalized size = 1.11

$$\frac{3}{17} \arctan(x) + \frac{6}{17} \log\left(x^2 + 1\right) - \frac{7}{34} \log(|4x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-1)/(-1+4*x)/(x^2+1),x, algorithm="giac")
```

```
[Out] 3/17*arctan(x) + 6/17*log(x^2 + 1) - 7/34*log(abs(4*x - 1))
```

3.263 $\int \frac{-3+2x-3x^2+x^3}{1+x^2} dx$

Optimal. Leaf size=21

$$\frac{x^2}{2} + \frac{1}{2}\log\left(x^2+1\right) - 3x$$

```
[Out] -3*x + x^2/2 + Log[1 + x^2]/2
```

Rubi [A] time = 0.0145165, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1810, 260}

$$\frac{x^2}{2} + \frac{1}{2}\log\left(x^2+1\right) - 3x$$

Antiderivative was successfully verified.

```
[In] Int[(-3 + 2*x - 3*x^2 + x^3)/(1 + x^2),x]
```

```
[Out] -3*x + x^2/2 + Log[1 + x^2]/2
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{-3+2x-3x^2+x^3}{1+x^2}\,dx &= \int \left(-3+x+\frac{x}{1+x^2}\right) dx \\
&= -3x+\frac{x^2}{2}+\int \frac{x}{1+x^2}\,dx \\
&= -3x+\frac{x^2}{2}+\frac{1}{2}\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.0044459, size = 21, normalized size = 1.

$$\frac{x^2}{2}+\frac{1}{2}\log\left(x^2+1\right)-3x$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 + 2*x - 3*x^2 + x^3)/(1 + x^2),x]
```

```
[Out] -3*x + x^2/2 + Log[1 + x^2]/2
```

Maple [A] time = 0.001, size = 18, normalized size = 0.9

$$-3\,x+\frac{x^2}{2}+\frac{\ln\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-3*x^2+2*x-3)/(x^2+1),x)
```

```
[Out] -3*x+1/2*x^2+1/2*ln(x^2+1)
```

Maxima [A] time = 1.47398, size = 23, normalized size = 1.1

$$\frac{1}{2}\,x^2-3\,x+\frac{1}{2}\,\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-3*x^2+2*x-3)/(x^2+1),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - 3*x + 1/2*log(x^2 + 1)
```

Fricas [A] time = 1.43708, size = 46, normalized size = 2.19

$$\frac{1}{2}\,x^2-3\,x+\frac{1}{2}\,\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-3*x^2+2*x-3)/(x^2+1),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 - 3*x + 1/2*log(x^2 + 1)
```

Sympy [A] time = 0.076529, size = 15, normalized size = 0.71

$$\frac{x^2}{2} - 3x + \frac{\log\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-3*x**2+2*x-3)/(x**2+1),x)
```

```
[Out] x**2/2 - 3*x + log(x**2 + 1)/2
```

Giac [A] time = 1.09907, size = 23, normalized size = 1.1

$$\frac{1}{2}\,x^2 - 3\,x + \frac{1}{2}\,\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-3*x^2+2*x-3)/(x^2+1),x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 3*x + 1/2*log(x^2 + 1)
```

3.264 $\int \frac{x+10x^2+6x^3+x^4}{10+6x+x^2} \, dx$

Optimal. Leaf size=27

$$\frac{x^3}{3}+\frac{1}{2}\log\left(x^2+6x+10\right)-3\tan^{-1}(x+3)$$

```
[Out] x^3/3 - 3*ArcTan[3 + x] + Log[10 + 6*x + x^2]/2
```

Rubi [A] time = 0.0257398, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1657, 634, 618, 204, 628}

$$\frac{x^3}{3}+\frac{1}{2}\log\left(x^2+6x+10\right)-3\tan^{-1}(x+3)$$

Antiderivative was successfully verified.

```
[In] Int[(x + 10*x^2 + 6*x^3 + x^4)/(10 + 6*x + x^2),x]
```

```
[Out] x^3/3 - 3*ArcTan[3 + x] + Log[10 + 6*x + x^2]/2
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x+10x^2+6x^3+x^4}{10+6x+x^2}\,dx &= \int \left(x^2+\frac{x}{10+6x+x^2}\right)dx\\ &= \frac{x^3}{3}+\int \frac{x}{10+6x+x^2}\,dx\\ &= \frac{x^3}{3}+\frac{1}{2}\int \frac{6+2x}{10+6x+x^2}\,dx-3\int \frac{1}{10+6x+x^2}\,dx\\ &= \frac{x^3}{3}+\frac{1}{2}\log\left(10+6x+x^2\right)+6\,\text{Subst}\left(\int \frac{1}{-4-x^2}\,dx,x,6+2x\right)\\ &= \frac{x^3}{3}-3\tan^{-1}(3+x)+\frac{1}{2}\log\left(10+6x+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.0073381, size = 27, normalized size = 1.

$$\frac{x^3}{3}+\frac{1}{2}\log\left(x^2+6x+10\right)-3\tan^{-1}(x+3)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + 10*x^2 + 6*x^3 + x^4)/(10 + 6*x + x^2),x]
```

```
[Out] x^3/3 - 3*ArcTan[3 + x] + Log[10 + 6*x + x^2]/2
```

Maple [A] time = 0.004, size = 24, normalized size = 0.9

$$\frac{x^3}{3}-3\,\arctan(3+x)+\frac{\ln\left(x^2+6\,x+10\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+6*x^3+10*x^2+x)/(x^2+6*x+10),x)
```

```
[Out] 1/3*x^3-3*arctan(3+x)+1/2*ln(x^2+6*x+10)
```

Maxima [A] time = 1.49499, size = 31, normalized size = 1.15

$$\frac{1}{3}x^3 - 3\arctan(x+3) + \frac{1}{2}\log\left(x^2 + 6x + 10\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+6*x^3+10*x^2+x)/(x^2+6*x+10),x, algorithm="maxima")
```

```
[Out] 1/3*x^3 - 3*arctan(x + 3) + 1/2*log(x^2 + 6*x + 10)
```

Fricas [A] time = 1.40229, size = 72, normalized size = 2.67

$$\frac{1}{3}x^3 - 3\arctan(x+3) + \frac{1}{2}\log\left(x^2 + 6x + 10\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+6*x^3+10*x^2+x)/(x^2+6*x+10),x, algorithm="fricas")
```

```
[Out] 1/3*x^3 - 3*arctan(x + 3) + 1/2*log(x^2 + 6*x + 10)
```

Sympy [A] time = 0.097625, size = 22, normalized size = 0.81

$$\frac{x^3}{3} + \frac{\log\left(x^2 + 6x + 10\right)}{2} - 3\operatorname{atan}(x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+6*x**3+10*x**2+x)/(x**2+6*x+10),x)
```

```
[Out] x**3/3 + log(x**2 + 6*x + 10)/2 - 3*atan(x + 3)
```

Giac [A] time = 1.19331, size = 31, normalized size = 1.15

$$\frac{1}{3}x^3 - 3\arctan(x+3) + \frac{1}{2}\log\left(x^2 + 6x + 10\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+6*x^3+10*x^2+x)/(x^2+6*x+10),x, algorithm="giac")
```

```
[Out] 1/3*x^3 - 3*arctan(x + 3) + 1/2*log(x^2 + 6*x + 10)
```

3.265 $\int \frac{1}{-18+27x-7x^2-3x^3+x^4} \, dx$

Optimal. Leaf size=39

$$\frac{1}{8}\log(1-x)-\frac{1}{5}\log(2-x)+\frac{1}{12}\log(3-x)-\frac{1}{120}\log(x+3)$$

```
[Out] Log[1 - x]/8 - Log[2 - x]/5 + Log[3 - x]/12 - Log[3 + x]/120
```

Rubi [A] time = 0.0199904, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {2058}

$$\frac{1}{8}\log(1-x)-\frac{1}{5}\log(2-x)+\frac{1}{12}\log(3-x)-\frac{1}{120}\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Int[(-18 + 27*x - 7*x^2 - 3*x^3 + x^4)^(-1),x]
```

```
[Out] Log[1 - x]/8 - Log[2 - x]/5 + Log[3 - x]/12 - Log[3 + x]/120
```

Rule 2058

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{-18+27x-7x^2-3x^3+x^4} \, dx &= \int \left(\frac{1}{12(-3+x)}-\frac{1}{5(-2+x)}+\frac{1}{8(-1+x)}-\frac{1}{120(3+x)}\right) dx \\ &= \frac{1}{8}\log(1-x)-\frac{1}{5}\log(2-x)+\frac{1}{12}\log(3-x)-\frac{1}{120}\log(3+x) \end{aligned}$$

Mathematica [A] time = 0.0062116, size = 39, normalized size = 1.

$$\frac{1}{8}\log(1-x)-\frac{1}{5}\log(2-x)+\frac{1}{12}\log(3-x)-\frac{1}{120}\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-18 + 27*x - 7*x^2 - 3*x^3 + x^4)^(-1),x]
```

```
[Out] Log[1 - x]/8 - Log[2 - x]/5 + Log[3 - x]/12 - Log[3 + x]/120
```

Maple [A] time = 0.007, size = 26, normalized size = 0.7

$$\frac{\ln(x-1)}{8}+\frac{\ln(-3+x)}{12}-\frac{\ln(3+x)}{120}-\frac{\ln(-2+x)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4-3*x^3-7*x^2+27*x-18),x)
```

```
[Out] 1/8*ln(x-1)+1/12*ln(-3+x)-1/120*ln(3+x)-1/5*ln(-2+x)
```

Maxima [A] time = 0.974703, size = 34, normalized size = 0.87

$$-\frac{1}{120}\log(x+3)+\frac{1}{8}\log(x-1)-\frac{1}{5}\log(x-2)+\frac{1}{12}\log(x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-3*x^3-7*x^2+27*x-18),x, algorithm="maxima")
```

```
[Out] -1/120*log(x + 3) + 1/8*log(x - 1) - 1/5*log(x - 2) + 1/12*log(x - 3)
```

Fricas [A] time = 1.26296, size = 96, normalized size = 2.46

$$-\frac{1}{120}\log(x+3)+\frac{1}{8}\log(x-1)-\frac{1}{5}\log(x-2)+\frac{1}{12}\log(x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-3*x^3-7*x^2+27*x-18),x, algorithm="fricas")
```

```
[Out] -1/120*log(x + 3) + 1/8*log(x - 1) - 1/5*log(x - 2) + 1/12*log(x - 3)
```

Sympy [A] time = 0.210924, size = 26, normalized size = 0.67

$$\frac{\log\left(x-3\right)}{12}-\frac{\log\left(x-2\right)}{5}+\frac{\log\left(x-1\right)}{8}-\frac{\log\left(x+3\right)}{120}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4-3*x**3-7*x**2+27*x-18),x)
```

```
[Out] log(x - 3)/12 - log(x - 2)/5 + log(x - 1)/8 - log(x + 3)/120
```

Giac [A] time = 1.1608, size = 39, normalized size = 1.

$$-\frac{1}{120}\log\left(|x+3|\right)+\frac{1}{8}\log\left(|x-1|\right)-\frac{1}{5}\log\left(|x-2|\right)+\frac{1}{12}\log\left(|x-3|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-3*x^3-7*x^2+27*x-18),x, algorithm="giac")
```

```
[Out] -1/120*log(abs(x + 3)) + 1/8*log(abs(x - 1)) - 1/5*log(abs(x - 2)) + 1/12*l
og(abs(x - 3))
```

3.266 $\int \frac{1+x^3}{-2+x} dx$

Optimal. Leaf size=22

$$\frac{x^3}{3} + x^2 + 4x + 9\log(2-x)$$

```
[Out] 4*x + x^2 + x^3/3 + 9*Log[2 - x]
```

Rubi [A] time = 0.0145268, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1850}

$$\frac{x^3}{3} + x^2 + 4x + 9\log(2-x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^3)/(-2 + x),x]

[Out] 4*x + x^2 + x^3/3 + 9*Log[2 - x]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned} \int \frac{1+x^3}{-2+x} dx &= \int \left(4 + \frac{9}{-2+x} + 2x + x^2\right) dx \\ &= 4x + x^2 + \frac{x^3}{3} + 9\log(2-x) \end{aligned}$$

Mathematica [A] time = 0.0039672, size = 23, normalized size = 1.05

$$\frac{x^3}{3} + x^2 + 4x + 9\log(x-2) - \frac{44}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^3)/(-2 + x),x]
```

```
[Out] -44/3 + 4*x + x^2 + x^3/3 + 9*Log[-2 + x]
```

Maple [A] time = 0.003, size = 19, normalized size = 0.9

$$\frac{x^3}{3} + x^2 + 4x + 9\ln(-2 + x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+1)/(-2+x),x)
```

```
[Out] 1/3*x^3+x^2+4*x+9*ln(-2+x)
```

Maxima [A] time = 0.984447, size = 24, normalized size = 1.09

$$\frac{1}{3}x^3 + x^2 + 4x + 9\log(x - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/(-2+x),x, algorithm="maxima")
```

```
[Out] 1/3*x^3 + x^2 + 4*x + 9*log(x - 2)
```

Fricas [A] time = 1.36932, size = 49, normalized size = 2.23

$$\frac{1}{3}x^3 + x^2 + 4x + 9\log(x - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/(-2+x),x, algorithm="fricas")
```

```
[Out] 1/3*x^3 + x^2 + 4*x + 9*log(x - 2)
```

Sympy [A] time = 0.070757, size = 17, normalized size = 0.77

$$\frac{x^3}{3} + x^2 + 4x + 9\log(x - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+1)/(-2+x),x)
```

```
[Out] x**3/3 + x**2 + 4*x + 9*log(x - 2)
```

Giac [A] time = 1.1229, size = 26, normalized size = 1.18

$$\frac{1}{3}x^3 + x^2 + 4x + 9\log(|x - 2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/(-2+x),x, algorithm="giac")
```

```
[Out] 1/3*x^3 + x^2 + 4*x + 9*log(abs(x - 2))
```

3.267 $\int \frac{3x-4x^2+3x^3}{1+x^2} \, dx$

Optimal. Leaf size=15

$$\frac{3x^2}{2} - 4x + 4\tan^{-1}(x)$$

```
[Out] -4*x + (3*x^2)/2 + 4*ArcTan[x]
```

Rubi [A] time = 0.0291245, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1594, 1802, 203}

$$\frac{3x^2}{2} - 4x + 4\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(3*x - 4*x^2 + 3*x^3)/(1 + x^2),x]
```

```
[Out] -4*x + (3*x^2)/2 + 4*ArcTan[x]
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{3x-4x^2+3x^3}{1+x^2}\,dx &= \int \frac{x\left(3-4x+3x^2\right)}{1+x^2}\,dx\\
&= \int \left(-4+3x+\frac{4}{1+x^2}\right)dx\\
&= -4x+\frac{3x^2}{2}+4\int \frac{1}{1+x^2}\,dx\\
&= -4x+\frac{3x^2}{2}+4\tan^{-1}(x)
\end{aligned}
$$

Mathematica [A] time = 0.0048286, size = 15, normalized size = 1.

$$\frac{3x^2}{2}-4x+4\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3*x - 4*x^2 + 3*x^3)/(1 + x^2),x]
```

```
[Out] -4*x + (3*x^2)/2 + 4*ArcTan[x]
```

Maple [A] time = 0.003, size = 14, normalized size = 0.9

$$-4\,x+\frac{3\,x^2}{2}+4\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^3-4*x^2+3*x)/(x^2+1),x)
```

```
[Out] -4*x+3/2*x^2+4*arctan(x)
```

Maxima [A] time = 1.52836, size = 18, normalized size = 1.2

$$\frac{3}{2}\,x^2-4\,x+4\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^3-4*x^2+3*x)/(x^2+1),x, algorithm="maxima")
```

```
[Out] 3/2*x^2 - 4*x + 4*arctan(x)
```

Fricas [A] time = 1.50666, size = 39, normalized size = 2.6

$$\frac{3}{2}\,x^2 - 4\,x + 4\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^3-4*x^2+3*x)/(x^2+1),x, algorithm="fricas")
```

```
[Out] 3/2*x^2 - 4*x + 4*arctan(x)
```

Sympy [A] time = 0.084244, size = 14, normalized size = 0.93

$$\frac{3x^2}{2} - 4x + 4\,\mathrm{atan}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**3-4*x**2+3*x)/(x**2+1),x)
```

```
[Out] 3*x**2/2 - 4*x + 4*atan(x)
```

Giac [A] time = 1.15116, size = 18, normalized size = 1.2

$$\frac{3}{2}\,x^2 - 4\,x + 4\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^3-4*x^2+3*x)/(x^2+1),x, algorithm="giac")
```

```
[Out] 3/2*x^2 - 4*x + 4*arctan(x)
```

3.268 $\int \frac{5+3x}{1-x-x^2+x^3} \, dx$

Optimal. Leaf size=12

$$\frac{4}{1-x} + \tanh^{-1}(x)$$

```
[Out] 4/(1 - x) + ArcTanh[x]
```

Rubi [A] time = 0.0207068, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2074, 206}

$$\frac{4}{1-x} + \tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(5 + 3*x)/(1 - x - x^2 + x^3),x]
```

```
[Out] 4/(1 - x) + ArcTanh[x]
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{5+3x}{1-x-x^2+x^3}\,dx = \int \left(\frac{4}{(-1+x)^2}+\frac{1}{1-x^2}\right)dx$$
$$= \frac{4}{1-x}+\int \frac{1}{1-x^2}\,dx$$
$$= \frac{4}{1-x}+\tanh^{-1}(x)$$

Mathematica [A] time = 0.0107631, size = 24, normalized size = 2.

$$-\frac{4}{x-1}-\frac{1}{2}\log(x-1)+\frac{1}{2}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + 3*x)/(1 - x - x^2 + x^3),x]
```

```
[Out] -4/(-1 + x) - Log[-1 + x]/2 + Log[1 + x]/2
```

Maple [A] time = 0.007, size = 21, normalized size = 1.8

$$-4\,(x-1)^{-1}-\frac{\ln(x-1)}{2}+\frac{\ln(1+x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5+3*x)/(x^3-x^2-x+1),x)
```

```
[Out] -4/(x-1)-1/2*ln(x-1)+1/2*ln(1+x)
```

Maxima [A] time = 0.98613, size = 27, normalized size = 2.25

$$-\frac{4}{x-1}+\frac{1}{2}\log(x+1)-\frac{1}{2}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+3*x)/(x^3-x^2-x+1),x, algorithm="maxima")
```

```
[Out] -4/(x - 1) + 1/2*log(x + 1) - 1/2*log(x - 1)
```

Fricas [B] time = 1.22273, size = 80, normalized size = 6.67

$$\frac{(x-1)\log(x+1)-(x-1)\log(x-1)-8}{2\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+3*x)/(x^3-x^2-x+1),x, algorithm="fricas")
```

```
[Out] 1/2*((x - 1)*log(x + 1) - (x - 1)*log(x - 1) - 8)/(x - 1)
```

Sympy [B] time = 0.09273, size = 17, normalized size = 1.42

$$-\frac{\log(x-1)}{2}+\frac{\log(x+1)}{2}-\frac{4}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+3*x)/(x**3-x**2-x+1),x)
```

```
[Out] -log(x - 1)/2 + log(x + 1)/2 - 4/(x - 1)
```

Giac [B] time = 1.11756, size = 30, normalized size = 2.5

$$-\frac{4}{x-1}+\frac{1}{2}\log(|x+1|)-\frac{1}{2}\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+3*x)/(x^3-x^2-x+1),x, algorithm="giac")
```

```
[Out] -4/(x - 1) + 1/2*log(abs(x + 1)) - 1/2*log(abs(x - 1))
```

3.269 $\int \frac{-1-x-x^3+x^4}{-x^2+x^3} \, dx$

Optimal. Leaf size=25

$$\frac{x^2}{2}-\frac{1}{x}-2\log(1-x)+2\log(x)$$

```
[Out] -x^(-1) + x^2/2 - 2*Log[1 - x] + 2*Log[x]
```

Rubi [A] time = 0.0356623, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {1593, 1620}

$$\frac{x^2}{2}-\frac{1}{x}-2\log(1-x)+2\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 - x - x^3 + x^4)/(-x^2 + x^3),x]
```

```
[Out] -x^(-1) + x^2/2 - 2*Log[1 - x] + 2*Log[x]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\int \frac{-1-x-x^3+x^4}{-x^2+x^3}\,dx = \int \frac{-1-x-x^3+x^4}{(-1+x)x^2}\,dx$$
$$= \int \left(-\frac{2}{-1+x}+\frac{1}{x^2}+\frac{2}{x}+x\right)dx$$
$$= -\frac{1}{x}+\frac{x^2}{2}-2\log(1-x)+2\log(x)$$

Mathematica [A] time = 0.0061798, size = 25, normalized size = 1.

$$\frac{x^2}{2}-\frac{1}{x}-2\log(1-x)+2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 - x - x^3 + x^4)/(-x^2 + x^3),x]
```

```
[Out] -x^(-1) + x^2/2 - 2*Log[1 - x] + 2*Log[x]
```

Maple [A] time = 0.007, size = 22, normalized size = 0.9

$$\frac{x^2}{2}-2\ln(x-1)-x^{-1}+2\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-x^3-x-1)/(x^3-x^2),x)
```

```
[Out] 1/2*x^2-2*ln(x-1)-1/x+2*ln(x)
```

Maxima [A] time = 1.02026, size = 28, normalized size = 1.12

$$\frac{1}{2}x^2-\frac{1}{x}-2\log(x-1)+2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^3-x-1)/(x^3-x^2),x, algorithm="maxima")
```

[Out] 1/2*x^2 - 1/x - 2*log(x - 1) + 2*log(x)

Fricas [A] time = 1.26056, size = 63, normalized size = 2.52

$$\frac{x^3 - 4x\log(x-1) + 4x\log(x) - 2}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4-x^3-x-1)/(x^3-x^2),x, algorithm="fricas")

[Out] 1/2*(x^3 - 4*x*log(x - 1) + 4*x*log(x) - 2)/x

Sympy [A] time = 0.094558, size = 19, normalized size = 0.76

$$\frac{x^2}{2} + 2\log(x) - 2\log(x-1) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**4-x**3-x-1)/(x**3-x**2),x)

[Out] x**2/2 + 2*log(x) - 2*log(x - 1) - 1/x

Giac [A] time = 1.20626, size = 31, normalized size = 1.24

$$\frac{1}{2}x^2 - \frac{1}{x} - 2\log(|x-1|) + 2\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4-x^3-x-1)/(x^3-x^2),x, algorithm="giac")

[Out] 1/2*x^2 - 1/x - 2*log(abs(x - 1)) + 2*log(abs(x))

3.270 $\int \frac{2+x+x^2+x^3}{2+3x^2+x^4} dx$

Optimal. Leaf size=13

$$\frac{1}{2}\log\left(x^2+2\right)+\tan^{-1}(x)$$

```
[Out] ArcTan[x] + Log[2 + x^2]/2
```

Rubi [A] time = 0.0283439, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1673, 1149, 203, 1247, 626, 31}

$$\frac{1}{2}\log\left(x^2+2\right)+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + x^2 + x^3)/(2 + 3*x^2 + x^4),x]
```

```
[Out] ArcTan[x] + Log[2 + x^2]/2
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 && !PolyQ[Pq, x^2]
```

Rule 1149

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
&& IntegerQ[p]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{2+x+x^2+x^3}{2+3x^2+x^4}\,dx &= \int \frac{x\left(1+x^2\right)}{2+3x^2+x^4}\,dx+\int \frac{2+x^2}{2+3x^2+x^4}\,dx \\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1+x}{2+3x+x^2}\,dx,x,x^2\right)+\int \frac{1}{1+x^2}\,dx \\
&= \tan^{-1}(x)+\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{2+x}\,dx,x,x^2\right) \\
&= \tan^{-1}(x)+\frac{1}{2}\log\left(2+x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.0074211, size = 13, normalized size = 1.

$$\frac{1}{2}\log\left(x^2+2\right)+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + x^2 + x^3)/(2 + 3*x^2 + x^4),x]
```

```
[Out] ArcTan[x] + Log[2 + x^2]/2
```

Maple [A] time = 0.003, size = 12, normalized size = 0.9

$$\arctan(x)+\frac{\ln(x^2+2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x^2+x+2)/(x^4+3*x^2+2),x)
```

```
[Out] arctan(x)+1/2*ln(x^2+2)
```

Maxima [A] time = 1.5041, size = 15, normalized size = 1.15

$$\arctan(x)+\frac{1}{2}\log(x^2+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+2)/(x^4+3*x^2+2),x, algorithm="maxima")
```

```
[Out] arctan(x) + 1/2*log(x^2 + 2)
```

Fricas [A] time = 1.34474, size = 41, normalized size = 3.15

$$\arctan(x)+\frac{1}{2}\log(x^2+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+2)/(x^4+3*x^2+2),x, algorithm="fricas")
```

```
[Out] arctan(x) + 1/2*log(x^2 + 2)
```

Sympy [A] time = 0.104464, size = 10, normalized size = 0.77

$$\frac{\log(x^2+2)}{2}+\operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x**2+x+2)/(x**4+3*x**2+2),x)
```

```
[Out] log(x**2 + 2)/2 + atan(x)
```

Giac [A] time = 1.11024, size = 15, normalized size = 1.15

$$\arctan(x) + \frac{1}{2}\log\left(x^2 + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+2)/(x^4+3*x^2+2),x, algorithm="giac")
```

```
[Out] arctan(x) + 1/2*log(x^2 + 2)
```

3.271 $\int \frac{-4+8x-4x^2+4x^3-x^4+x^5}{(2+x^2)^3} dx$

Optimal. Leaf size=35

$$-\frac{1}{(x^2+2)^2}+\frac{1}{2}\log(x^2+2)-\frac{\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[Out] -(2 + x^2)^(-2) - ArcTan[x/Sqrt[2]]/Sqrt[2] + Log[2 + x^2]/2
```

Rubi [A] time = 0.0336383, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {1814, 1586, 635, 203, 260}

$$-\frac{1}{(x^2+2)^2}+\frac{1}{2}\log(x^2+2)-\frac{\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-4 + 8*x - 4*x^2 + 4*x^3 - x^4 + x^5)/(2 + x^2)^3,x]
```

```
[Out] -(2 + x^2)^(-2) - ArcTan[x/Sqrt[2]]/Sqrt[2] + Log[2 + x^2]/2
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{-4+8x-4x^2+4x^3-x^4+x^5}{\left(2+x^2\right)^3}\,dx &= -\frac{1}{\left(2+x^2\right)^2}-\frac{1}{8}\int \frac{16-16x+8x^2-8x^3}{\left(2+x^2\right)^2}\,dx \\
&= -\frac{1}{\left(2+x^2\right)^2}-\frac{1}{8}\int \frac{8-8x}{2+x^2}\,dx \\
&= -\frac{1}{\left(2+x^2\right)^2}-\int \frac{1}{2+x^2}\,dx+\int \frac{x}{2+x^2}\,dx \\
&= -\frac{1}{\left(2+x^2\right)^2}-\frac{\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{1}{2}\log\left(2+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.0174349, size = 35, normalized size = 1.

$$-\frac{1}{\left(x^2+2\right)^2}+\frac{1}{2}\log\left(x^2+2\right)-\frac{\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-4 + 8*x - 4*x^2 + 4*x^3 - x^4 + x^5)/(2 + x^2)^3,x]
```

```
[Out] -(2 + x^2)^(-2) - ArcTan[x/Sqrt[2]]/Sqrt[2] + Log[2 + x^2]/2
```

Maple [A] time = 0.008, size = 31, normalized size = 0.9

$$-\left(x^{2}+2\right)^{-2}+\frac{\ln\left(x^{2}+2\right)}{2}-\frac{\sqrt{2}}{2}\arctan\left(\frac{x\sqrt{2}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5-x^4+4*x^3-4*x^2+8*x-4)/(x^2+2)^3,x)
```

```
[Out] -1/(x^2+2)^2+1/2*ln(x^2+2)-1/2*arctan(1/2*x*2^(1/2))*2^(1/2)
```

Maxima [A] time = 1.49491, size = 47, normalized size = 1.34

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)-\frac{1}{x^{4}+4x^{2}+4}+\frac{1}{2}\log\left(x^{2}+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-x^4+4*x^3-4*x^2+8*x-4)/(x^2+2)^3,x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/(x^4 + 4*x^2 + 4) + 1/2*log(x^2 + 2)
```

Fricas [A] time = 1.44246, size = 150, normalized size = 4.29

$$-\frac{\sqrt{2}\left(x^{4}+4x^{2}+4\right)\arctan\left(\frac{1}{2}\sqrt{2}x\right)-\left(x^{4}+4x^{2}+4\right)\log\left(x^{2}+2\right)+2}{2\left(x^{4}+4x^{2}+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-x^4+4*x^3-4*x^2+8*x-4)/(x^2+2)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(2)*(x^4 + 4*x^2 + 4)*arctan(1/2*sqrt(2)*x) - (x^4 + 4*x^2 + 4)*l
og(x^2 + 2) + 2)/(x^4 + 4*x^2 + 4)
```

Sympy [A] time = 0.136255, size = 36, normalized size = 1.03

$$\frac{\log\left(x^2+2\right)}{2}-\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)}{2}-\frac{1}{x^4+4x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**5-x**4+4*x**3-4*x**2+8*x-4)/(x**2+2)**3,x)
```

```
[Out] log(x**2 + 2)/2 - sqrt(2)*atan(sqrt(2)*x/2)/2 - 1/(x**4 + 4*x**2 + 4)
```

Giac [A] time = 1.15804, size = 41, normalized size = 1.17

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)-\frac{1}{\left(x^2+2\right)^2}+\frac{1}{2}\log\left(x^2+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-x^4+4*x^3-4*x^2+8*x-4)/(x^2+2)^3,x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/(x^2 + 2)^2 + 1/2*log(x^2 + 2)
```

3.272 $\int \frac{-1-3x+x^2}{-2x+x^2+x^3} \, dx$

Optimal. Leaf size=23

$$-\log(1-x)+\frac{\log(x)}{2}+\frac{3}{2}\log(x+2)$$

```
[Out] -Log[1 - x] + Log[x]/2 + (3*Log[2 + x])/2
```

Rubi [A] time = 0.0367433, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1594, 1628}

$$-\log(1-x)+\frac{\log(x)}{2}+\frac{3}{2}\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 - 3*x + x^2)/(-2*x + x^2 + x^3),x]
```

```
[Out] -Log[1 - x] + Log[x]/2 + (3*Log[2 + x])/2
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{-1-3x+x^2}{-2x+x^2+x^3}\,dx = \int \frac{-1-3x+x^2}{x\left(-2+x+x^2\right)}\,dx$$
$$= \int \left(\frac{1}{1-x}+\frac{1}{2x}+\frac{3}{2(2+x)}\right)dx$$
$$= -\log(1-x)+\frac{\log(x)}{2}+\frac{3}{2}\log(2+x)$$

Mathematica [A] time = 0.0058489, size = 23, normalized size = 1.

$$-\log(1-x)+\frac{\log(x)}{2}+\frac{3}{2}\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 - 3*x + x^2)/(-2*x + x^2 + x^3),x]
```

```
[Out] -Log[1 - x] + Log[x]/2 + (3*Log[2 + x])/2
```

Maple [A] time = 0.007, size = 18, normalized size = 0.8

$$-\ln\left(x-1\right)+\frac{\ln\left(x\right)}{2}+\frac{3\,\ln\left(2+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-3*x-1)/(x^3+x^2-2*x),x)
```

```
[Out] -ln(x-1)+1/2*ln(x)+3/2*ln(2+x)
```

Maxima [A] time = 0.970412, size = 23, normalized size = 1.

$$\frac{3}{2}\,\log\left(x+2\right)-\log\left(x-1\right)+\frac{1}{2}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3*x-1)/(x^3+x^2-2*x),x, algorithm="maxima")
```

```
[Out] 3/2*log(x + 2) - log(x - 1) + 1/2*log(x)
```

Fricas [A] time = 1.34757, size = 57, normalized size = 2.48

$$\frac{3}{2}\log\left(x+2\right)-\log\left(x-1\right)+\frac{1}{2}\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3*x-1)/(x^3+x^2-2*x),x, algorithm="fricas")
```

```
[Out] 3/2*log(x + 2) - log(x - 1) + 1/2*log(x)
```

Sympy [A] time = 0.123064, size = 17, normalized size = 0.74

$$\frac{\log\left(x\right)}{2}-\log\left(x-1\right)+\frac{3\log\left(x+2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-3*x-1)/(x**3+x**2-2*x),x)
```

```
[Out] log(x)/2 - log(x - 1) + 3*log(x + 2)/2
```

Giac [A] time = 1.1124, size = 27, normalized size = 1.17

$$\frac{3}{2}\log\left(|x+2|\right)-\log\left(|x-1|\right)+\frac{1}{2}\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3*x-1)/(x^3+x^2-2*x),x, algorithm="giac")
```

```
[Out] 3/2*log(abs(x + 2)) - log(abs(x - 1)) + 1/2*log(abs(x))
```

3.273 $\int \frac{3-x+3x^2-2x^3+x^4}{3x-2x^2+x^3} \, dx$

Optimal. Leaf size=23

$$\frac{x^2}{2} - \frac{1}{2}\log\left(x^2-2x+3\right) + \log(x)$$

```
[Out] x^2/2 + Log[x] - Log[3 - 2*x + x^2]/2
```

Rubi [A] time = 0.0540768, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1594, 1628, 628}

$$\frac{x^2}{2} - \frac{1}{2}\log\left(x^2-2x+3\right) + \log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 3*x^2 - 2*x^3 + x^4)/(3*x - 2*x^2 + x^3),x]
```

```
[Out] x^2/2 + Log[x] - Log[3 - 2*x + x^2]/2
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{3-x+3x^2-2x^3+x^4}{3x-2x^2+x^3}\,dx = \int \frac{3-x+3x^2-2x^3+x^4}{x\left(3-2x+x^2\right)}\,dx$$
$$= \int \left(\frac{1}{x}+x+\frac{1-x}{3-2x+x^2}\right)dx$$
$$= \frac{x^2}{2}+\log(x)+\int \frac{1-x}{3-2x+x^2}\,dx$$
$$= \frac{x^2}{2}+\log(x)-\frac{1}{2}\log\left(3-2x+x^2\right)$$

Mathematica [A] time = 0.0074805, size = 23, normalized size = 1.

$$\frac{x^2}{2}-\frac{1}{2}\log\left(x^2-2x+3\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 3*x^2 - 2*x^3 + x^4)/(3*x - 2*x^2 + x^3),x]
```

```
[Out] x^2/2 + Log[x] - Log[3 - 2*x + x^2]/2
```

Maple [A] time = 0.004, size = 20, normalized size = 0.9

$$\frac{x^2}{2}+\ln(x)-\frac{\ln\left(x^2-2x+3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-2*x^3+3*x^2-x+3)/(x^3-2*x^2+3*x),x)
```

```
[Out] 1/2*x^2+ln(x)-1/2*ln(x^2-2*x+3)
```

Maxima [A] time = 0.993679, size = 26, normalized size = 1.13

$$\frac{1}{2}x^2-\frac{1}{2}\log\left(x^2-2x+3\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-2*x^3+3*x^2-x+3)/(x^3-2*x^2+3*x),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - 1/2*log(x^2 - 2*x + 3) + log(x)
```

Fricas [A] time = 1.4448, size = 58, normalized size = 2.52

$$\frac{1}{2}x^2 - \frac{1}{2}\log\left(x^2 - 2x + 3\right) + \log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-2*x^3+3*x^2-x+3)/(x^3-2*x^2+3*x),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 - 1/2*log(x^2 - 2*x + 3) + log(x)
```

Sympy [A] time = 0.098815, size = 19, normalized size = 0.83

$$\frac{x^2}{2} + \log\left(x\right) - \frac{\log\left(x^2 - 2x + 3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-2*x**3+3*x**2-x+3)/(x**3-2*x**2+3*x),x)
```

```
[Out] x**2/2 + log(x) - log(x**2 - 2*x + 3)/2
```

Giac [A] time = 1.38314, size = 27, normalized size = 1.17

$$\frac{1}{2}x^2 - \frac{1}{2}\log\left(x^2 - 2x + 3\right) + \log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-2*x^3+3*x^2-x+3)/(x^3-2*x^2+3*x),x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 1/2*log(x^2 - 2*x + 3) + log(abs(x))
```

3.274 $\int \frac{-1+x+x^3}{\left(1+x^2\right)^2} \, dx$

Optimal. Leaf size=29

$$-\frac{x}{2\left(x^2+1\right)}+\frac{1}{2}\log\left(x^2+1\right)-\frac{1}{2}\tan^{-1}(x)$$

```
[Out] -x/(2*(1 + x^2)) - ArcTan[x]/2 + Log[1 + x^2]/2
```

Rubi [A] time = 0.0137334, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1814, 635, 203, 260}

$$-\frac{x}{2\left(x^2+1\right)}+\frac{1}{2}\log\left(x^2+1\right)-\frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x + x^3)/(1 + x^2)^2,x]
```

```
[Out] -x/(2*(1 + x^2)) - ArcTan[x]/2 + Log[1 + x^2]/2
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+x+x^3}{\left(1+x^2\right)^2}\,dx &= -\frac{x}{2\left(1+x^2\right)}-\frac{1}{2}\int \frac{1-2x}{1+x^2}\,dx\\ &= -\frac{x}{2\left(1+x^2\right)}-\frac{1}{2}\int \frac{1}{1+x^2}\,dx+\int \frac{x}{1+x^2}\,dx\\ &= -\frac{x}{2\left(1+x^2\right)}-\frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.0114483, size = 25, normalized size = 0.86

$$\frac{1}{2}\left(-\frac{x}{x^2+1}+\log\left(x^2+1\right)-\tan^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x + x^3)/(1 + x^2)^2,x]
```

```
[Out] (-(x/(1 + x^2)) - ArcTan[x] + Log[1 + x^2])/2
```

Maple [A] time = 0.006, size = 24, normalized size = 0.8

$$-\frac{x}{2\,x^2+2}-\frac{\arctan(x)}{2}+\frac{\ln\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x-1)/(x^2+1)^2,x)
```

```
[Out] -1/2*x/(x^2+1)-1/2*arctan(x)+1/2*ln(x^2+1)
```

Maxima [A] time = 1.48918, size = 31, normalized size = 1.07

$$-\frac{x}{2\left(x^2+1\right)}-\frac{1}{2}\arctan(x)+\frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x-1)/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] -1/2*x/(x^2 + 1) - 1/2*arctan(x) + 1/2*log(x^2 + 1)
```

Fricas [A] time = 1.45813, size = 90, normalized size = 3.1

$$-\frac{\left(x^2+1\right)\arctan(x)-\left(x^2+1\right)\log\left(x^2+1\right)+x}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x-1)/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] -1/2*((x^2 + 1)*arctan(x) - (x^2 + 1)*log(x^2 + 1) + x)/(x^2 + 1)
```

Sympy [A] time = 0.111065, size = 20, normalized size = 0.69

$$-\frac{x}{2x^2+2}+\frac{\log\left(x^2+1\right)}{2}-\frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x-1)/(x**2+1)**2,x)
```

```
[Out] -x/(2*x**2 + 2) + log(x**2 + 1)/2 - atan(x)/2
```

Giac [A] time = 1.11213, size = 31, normalized size = 1.07

$$-\frac{x}{2\left(x^2+1\right)}-\frac{1}{2}\arctan\left(x\right)+\frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x-1)/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] -1/2*x/(x^2 + 1) - 1/2*arctan(x) + 1/2*log(x^2 + 1)
```

3.275 $\int \frac{1+2x-x^2+8x^3+x^4}{(x+x^2)(1+x^3)} \, dx$

Optimal. Leaf size=44

$$\log\left(x^2-x+1\right)-\frac{3}{x+1}+\log(x)-2\log(x+1)-\frac{2\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] -3/(1 + x) - (2*ArcTan[(1 - 2*x)/Sqrt[3]])/Sqrt[3] + Log[x] - 2*Log[1 + x]
+ Log[1 - x + x^2]
```

Rubi [A] time = 0.242529, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1593, 6725, 634, 618, 204, 628}

$$\log\left(x^2-x+1\right)-\frac{3}{x+1}+\log(x)-2\log(x+1)-\frac{2\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x - x^2 + 8*x^3 + x^4)/((x + x^2)*(1 + x^3)),x]
```

```
[Out] -3/(1 + x) - (2*ArcTan[(1 - 2*x)/Sqrt[3]])/Sqrt[3] + Log[x] - 2*Log[1 + x]
+ Log[1 - x + x^2]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1+2x-x^2+8x^3+x^4}{\left(x+x^2\right)\left(1+x^3\right)}\,dx &= \int \frac{1+2x-x^2+8x^3+x^4}{x(1+x)\left(1+x^3\right)}\,dx \\
&= \int \left(\frac{1}{x}+\frac{3}{(1+x)^2}-\frac{2}{1+x}+\frac{2x}{1-x+x^2}\right)dx \\
&= -\frac{3}{1+x}+\log(x)-2\log(1+x)+2\int \frac{x}{1-x+x^2}\,dx \\
&= -\frac{3}{1+x}+\log(x)-2\log(1+x)+\int \frac{1}{1-x+x^2}\,dx+\int \frac{-1+2x}{1-x+x^2}\,dx \\
&= -\frac{3}{1+x}+\log(x)-2\log(1+x)+\log\left(1-x+x^2\right)-2\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right) \\
&= -\frac{3}{1+x}-\frac{2\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}+\log(x)-2\log(1+x)+\log\left(1-x+x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.0249415, size = 44, normalized size = 1.

$$\log\left(x^2-x+1\right)-\frac{3}{x+1}+\log(x)-2\log(x+1)+\frac{2\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x - x^2 + 8*x^3 + x^4)/((x + x^2)*(1 + x^3)),x]
```

```
[Out] -3/(1 + x) + (2*ArcTan[(-1 + 2*x)/Sqrt[3]])/Sqrt[3] + Log[x] - 2*Log[1 + x]
 + Log[1 - x + x^2]
```

Maple [A] time = 0.008, size = 42, normalized size = 1.

$$\ln(x) + \ln\left(x^2 - x + 1\right) + \frac{2\sqrt{3}}{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right) - 3\,(1+x)^{-1} - 2\,\ln(1+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+8*x^3-x^2+2*x+1)/(x^2+x)/(x^3+1),x)
```

```
[Out] ln(x)+ln(x^2-x+1)+2/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))-3/(1+x)-2*ln(1+x)
```

Maxima [A] time = 1.48511, size = 55, normalized size = 1.25

$$\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{3}{x+1} + \log\left(x^2 - x + 1\right) - 2\,\log(x+1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+8*x^3-x^2+2*x+1)/(x^2+x)/(x^3+1),x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 3/(x + 1) + log(x^2 - x + 1) -
2*log(x + 1) + log(x)
```

Fricas [A] time = 1.41565, size = 186, normalized size = 4.23

$$\frac{2\sqrt{3}(x+1)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + 3\,(x+1)\log\left(x^2 - x + 1\right) - 6\,(x+1)\log(x+1) + 3\,(x+1)\log(x) - 9}{3\,(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+8*x^3-x^2+2*x+1)/(x^2+x)/(x^3+1),x, algorithm="fricas")
```

```
[Out] 1/3*(2*sqrt(3)*(x + 1)*arctan(1/3*sqrt(3)*(2*x - 1)) + 3*(x + 1)*log(x^2 -
x + 1) - 6*(x + 1)*log(x + 1) + 3*(x + 1)*log(x) - 9)/(x + 1)
```

Sympy [A] time = 0.186507, size = 49, normalized size = 1.11

$$\log(x) - 2\log(x+1) + \log\left(x^2 - x + 1\right) + \frac{2\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}x}{3} - \frac{\sqrt{3}}{3}\right)}{3} - \frac{3}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+8*x**3-x**2+2*x+1)/(x**2+x)/(x**3+1),x)
```

```
[Out] log(x) - 2*log(x + 1) + log(x**2 - x + 1) + 2*sqrt(3)*atan(2*sqrt(3)*x/3 -
sqrt(3)/3)/3 - 3/(x + 1)
```

Giac [A] time = 1.20969, size = 58, normalized size = 1.32

$$\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{3}{x+1} + \log\left(x^2 - x + 1\right) - 2\log\left(|x+1|\right) + \log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+8*x^3-x^2+2*x+1)/(x^2+x)/(x^3+1),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 3/(x + 1) + log(x^2 - x + 1) -
2*log(abs(x + 1)) + log(abs(x))
```

3.276 $\int \frac{15-5x+x^2+x^3}{(5+x^2)(3+2x+x^2)}\,dx$

Optimal. Leaf size=46

$$\frac{1}{2}\log\left(x^2+2x+3\right)-\sqrt{5}\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)+\frac{5\tan^{-1}\left(\frac{x+1}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[Out] -(Sqrt[5]*ArcTan[x/Sqrt[5]]) + (5*ArcTan[(1 + x)/Sqrt[2]])/Sqrt[2] + Log[3
+ 2*x + x^2]/2
```

Rubi [A] time = 0.13324, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {6725, 203, 634, 618, 204, 628}

$$\frac{1}{2}\log\left(x^2+2x+3\right)-\sqrt{5}\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)+\frac{5\tan^{-1}\left(\frac{x+1}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(15 - 5*x + x^2 + x^3)/((5 + x^2)*(3 + 2*x + x^2)),x]
```

```
[Out] -(Sqrt[5]*ArcTan[x/Sqrt[5]]) + (5*ArcTan[(1 + x)/Sqrt[2]])/Sqrt[2] + Log[3
+ 2*x + x^2]/2
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{15-5x+x^2+x^3}{\left(5+x^2\right)\left(3+2x+x^2\right)}\,dx &= \int \left(-\frac{5}{5+x^2}+\frac{6+x}{3+2x+x^2}\right) dx \\
&= -\left(5\int \frac{1}{5+x^2}\,dx\right)+\int \frac{6+x}{3+2x+x^2}\,dx \\
&= -\sqrt{5}\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)+\frac{1}{2}\int \frac{2+2x}{3+2x+x^2}\,dx+5\int \frac{1}{3+2x+x^2}\,dx \\
&= -\sqrt{5}\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)+\frac{1}{2}\log\left(3+2x+x^2\right)-10\,\mathrm{Subst}\left(\int \frac{1}{-8-x^2}\,dx,x,2+2x\right) \\
&= -\sqrt{5}\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)+\frac{5\tan^{-1}\left(\frac{1+x}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{1}{2}\log\left(3+2x+x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.0200394, size = 46, normalized size = 1.

$$
\frac{1}{2}\log\left(x^2+2x+3\right)-\sqrt{5}\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)+\frac{5\tan^{-1}\left(\frac{x+1}{\sqrt{2}}\right)}{\sqrt{2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(15 - 5*x + x^2 + x^3)/((5 + x^2)*(3 + 2*x + x^2)),x]
```

```
[Out] -(Sqrt[5]*ArcTan[x/Sqrt[5]]) + (5*ArcTan[(1 + x)/Sqrt[2]])/Sqrt[2] + Log[3
+ 2*x + x^2]/2
```

Maple [A] time = 0.006, size = 41, normalized size = 0.9

$$\frac{\ln\left(x^2+2x+3\right)}{2}+\frac{5\sqrt{2}}{2}\arctan\left(\frac{(2+2x)\sqrt{2}}{4}\right)-\arctan\left(\frac{x\sqrt{5}}{5}\right)\sqrt{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x^2-5*x+15)/(x^2+5)/(x^2+2*x+3),x)
```

```
[Out] 1/2*ln(x^2+2*x+3)+5/2*2^(1/2)*arctan(1/4*(2+2*x)*2^(1/2))-arctan(1/5*x*5^(1
/2))*5^(1/2)
```

Maxima [A] time = 1.51628, size = 51, normalized size = 1.11

$$\frac{5}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x+1)\right)-\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}x\right)+\frac{1}{2}\log\left(x^2+2x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2-5*x+15)/(x^2+5)/(x^2+2*x+3),x, algorithm="maxima")
```

```
[Out] 5/2*sqrt(2)*arctan(1/2*sqrt(2)*(x + 1)) - sqrt(5)*arctan(1/5*sqrt(5)*x) + 1
/2*log(x^2 + 2*x + 3)
```

Fricas [A] time = 1.30083, size = 132, normalized size = 2.87

$$\frac{5}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x+1)\right)-\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}x\right)+\frac{1}{2}\log\left(x^2+2x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2-5*x+15)/(x^2+5)/(x^2+2*x+3),x, algorithm="fricas")
```

```
[Out] 5/2*sqrt(2)*arctan(1/2*sqrt(2)*(x + 1)) - sqrt(5)*arctan(1/5*sqrt(5)*x) + 1
/2*log(x^2 + 2*x + 3)
```

Sympy [A] time = 0.186845, size = 51, normalized size = 1.11

$$\frac{\log\left(x^2 + 2x + 3\right)}{2} - \sqrt{5}\operatorname{atan}\left(\frac{\sqrt{5}x}{5}\right) + \frac{5\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2} + \frac{\sqrt{2}}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x**2-5*x+15)/(x**2+5)/(x**2+2*x+3),x)
```

```
[Out] log(x**2 + 2*x + 3)/2 - sqrt(5)*atan(sqrt(5)*x/5) + 5*sqrt(2)*atan(sqrt(2)*
x/2 + sqrt(2)/2)/2
```

Giac [A] time = 1.13888, size = 51, normalized size = 1.11

$$\frac{5}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x+1)\right) - \sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}x\right) + \frac{1}{2}\log\left(x^2 + 2x + 3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2-5*x+15)/(x^2+5)/(x^2+2*x+3),x, algorithm="giac")
```

```
[Out] 5/2*sqrt(2)*arctan(1/2*sqrt(2)*(x + 1)) - sqrt(5)*arctan(1/5*sqrt(5)*x) + 1
/2*log(x^2 + 2*x + 3)
```

3.277 $\int \frac{-3+25x+23x^2+32x^3+15x^4+7x^5+x^6}{(1+x^2)^2(2+x+x^2)^2}\,dx$

Optimal. Leaf size=33

$$-\frac{3}{x^2+1}+\frac{1}{x^2+x+2}+\log\left(x^2+1\right)-\log\left(x^2+x+2\right)$$

```
[Out] -3/(1 + x^2) + (2 + x + x^2)^(-1) + Log[1 + x^2] - Log[2 + x + x^2]
```

Rubi [A] time = 0.165621, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {6742, 261, 260, 629, 628}

$$-\frac{3}{x^2+1}+\frac{1}{x^2+x+2}+\log\left(x^2+1\right)-\log\left(x^2+x+2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-3 + 25*x + 23*x^2 + 32*x^3 + 15*x^4 + 7*x^5 + x^6)/((1 + x^2)^2*(2 +
x + x^2)^2),x]
```

```
[Out] -3/(1 + x^2) + (2 + x + x^2)^(-1) + Log[1 + x^2] - Log[2 + x + x^2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{-3+25x+23x^2+32x^3+15x^4+7x^5+x^6}{(1+x^2)^2(2+x+x^2)^2}\,dx &= \int \left(\frac{6x}{(1+x^2)^2}+\frac{2x}{1+x^2}+\frac{-1-2x}{(2+x+x^2)^2}+\frac{-1-2x}{2+x+x^2}\right)dx\\ &= 2\int \frac{x}{1+x^2}\,dx+6\int \frac{x}{(1+x^2)^2}\,dx+\int \frac{-1-2x}{(2+x+x^2)^2}\,dx+\int \frac{-1}{2+}\\ &= -\frac{3}{1+x^2}+\frac{1}{2+x+x^2}+\log(1+x^2)-\log(2+x+x^2)\end{aligned}$$

Mathematica [A] time = 0.019287, size = 33, normalized size = 1.

$$-\frac{3}{x^2+1}+\frac{1}{x^2+x+2}+\log(x^2+1)-\log(x^2+x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 + 25*x + 23*x^2 + 32*x^3 + 15*x^4 + 7*x^5 + x^6)/((1 + x^2)^2
*(2 + x + x^2)^2),x]
```

```
[Out] -3/(1 + x^2) + (2 + x + x^2)^(-1) + Log[1 + x^2] - Log[2 + x + x^2]
```

Maple [A] time = 0.01, size = 34, normalized size = 1.

$$-3(x^2+1)^{-1}+(x^2+x+2)^{-1}+\ln(x^2+1)-\ln(x^2+x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6+7*x^5+15*x^4+32*x^3+23*x^2+25*x-3)/(x^2+1)^2/(x^2+x+2)^2,x)
```

```
[Out] -3/(x^2+1)+1/(x^2+x+2)+ln(x^2+1)-ln(x^2+x+2)
```

Maxima [A] time = 1.05439, size = 59, normalized size = 1.79

$$-\frac{2\,x^2+3\,x+5}{x^4+x^3+3\,x^2+x+2}-\log\left(x^2+x+2\right)+\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+7*x^5+15*x^4+32*x^3+23*x^2+25*x-3)/(x^2+1)^2/(x^2+x+2)^2,x,
algorithm="maxima")
```

```
[Out] -(2*x^2 + 3*x + 5)/(x^4 + x^3 + 3*x^2 + x + 2) - log(x^2 + x + 2) + log(x^2
 + 1)
```

Fricas [B] time = 1.37243, size = 186, normalized size = 5.64

$$-\frac{2\,x^2+\left(x^4+x^3+3\,x^2+x+2\right)\log\left(x^2+x+2\right)-\left(x^4+x^3+3\,x^2+x+2\right)\log\left(x^2+1\right)+3\,x+5}{x^4+x^3+3\,x^2+x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+7*x^5+15*x^4+32*x^3+23*x^2+25*x-3)/(x^2+1)^2/(x^2+x+2)^2,x,
algorithm="fricas")
```

```
[Out] -(2*x^2 + (x^4 + x^3 + 3*x^2 + x + 2)*log(x^2 + x + 2) - (x^4 + x^3 + 3*x^2
 + x + 2)*log(x^2 + 1) + 3*x + 5)/(x^4 + x^3 + 3*x^2 + x + 2)
```

Sympy [A] time = 0.176571, size = 39, normalized size = 1.18

$$-\frac{2x^2+3x+5}{x^4+x^3+3x^2+x+2}+\log\left(x^2+1\right)-\log\left(x^2+x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6+7*x**5+15*x**4+32*x**3+23*x**2+25*x-3)/(x**2+1)**2/(x**2+x+
2)**2,x)
```

```
[Out] -(2*x**2 + 3*x + 5)/(x**4 + x**3 + 3*x**2 + x + 2) + log(x**2 + 1) - log(x*
*2 + x + 2)
```

Giac [A] time = 1.1171, size = 59, normalized size = 1.79

$$-\frac{2x^2+3x+5}{x^4+x^3+3x^2+x+2}-\log\left(x^2+x+2\right)+\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+7*x^5+15*x^4+32*x^3+23*x^2+25*x-3)/(x^2+1)^2/(x^2+x+2)^2,x,
algorithm="giac")
```

```
[Out] -(2*x^2 + 3*x + 5)/(x^4 + x^3 + 3*x^2 + x + 2) - log(x^2 + x + 2) + log(x^2
 + 1)
```

3.278 $\int \frac{1}{(1+x^2)(4+x^2)} \, dx$

Optimal. Leaf size=17

$$\frac{1}{3}\tan^{-1}(x) - \frac{1}{6}\tan^{-1}\left(\frac{x}{2}\right)$$

```
[Out] -ArcTan[x/2]/6 + ArcTan[x]/3
```

Rubi [A] time = 0.0065376, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {391, 203}

$$\frac{1}{3}\tan^{-1}(x) - \frac{1}{6}\tan^{-1}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x^2)*(4 + x^2)),x]
```

```
[Out] -ArcTan[x/2]/6 + ArcTan[x]/3
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(1+x^2\right)\left(4+x^2\right)} \, dx &= \frac{1}{3}\int \frac{1}{1+x^2} \, dx - \frac{1}{3}\int \frac{1}{4+x^2} \, dx \\ &= -\frac{1}{6}\tan^{-1}\left(\frac{x}{2}\right) + \frac{1}{3}\tan^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.0061688, size = 17, normalized size = 1.

$$\frac{1}{6}\tan^{-1}\left(\frac{2}{x}\right)+\frac{1}{3}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x^2)*(4 + x^2)),x]
```

```
[Out] ArcTan[2/x]/6 + ArcTan[x]/3
```

Maple [A] time = 0.006, size = 12, normalized size = 0.7

$$-\frac{1}{6}\arctan\left(\frac{x}{2}\right)+\frac{\arctan(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)/(x^2+4),x)
```

```
[Out] -1/6*arctan(1/2*x)+1/3*arctan(x)
```

Maxima [A] time = 1.60655, size = 15, normalized size = 0.88

$$-\frac{1}{6}\arctan\left(\frac{1}{2}x\right)+\frac{1}{3}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/(x^2+4),x, algorithm="maxima")
```

```
[Out] -1/6*arctan(1/2*x) + 1/3*arctan(x)
```

Fricas [A] time = 1.46146, size = 49, normalized size = 2.88

$$-\frac{1}{6}\arctan\left(\frac{1}{2}x\right)+\frac{1}{3}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/(x^2+4),x, algorithm="fricas")
```

```
[Out] -1/6*arctan(1/2*x) + 1/3*arctan(x)
```

Sympy [A] time = 0.123822, size = 10, normalized size = 0.59

$$-\frac{\operatorname{atan}\left(\frac{x}{2}\right)}{6}+\frac{\operatorname{atan}(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)/(x**2+4),x)
```

```
[Out] -atan(x/2)/6 + atan(x)/3
```

Giac [A] time = 1.12569, size = 15, normalized size = 0.88

$$-\frac{1}{6}\arctan\left(\frac{1}{2}x\right)+\frac{1}{3}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/(x^2+4),x, algorithm="giac")
```

```
[Out] -1/6*arctan(1/2*x) + 1/3*arctan(x)
```

3.279 $\int \frac{a+bx^3}{1+x^2} \, dx$

Optimal. Leaf size=24

$$a\tan^{-1}(x)+\frac{bx^2}{2}-\frac{1}{2}b\log\left(x^2+1\right)$$

```
[Out] (b*x^2)/2 + a*ArcTan[x] - (b*Log[1 + x^2])/2
```

Rubi [A] time = 0.0183818, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1810, 635, 203, 260}

$$a\tan^{-1}(x)+\frac{bx^2}{2}-\frac{1}{2}b\log\left(x^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)/(1 + x^2),x]
```

```
[Out] (b*x^2)/2 + a*ArcTan[x] - (b*Log[1 + x^2])/2
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+bx^3}{1+x^2}\,dx &= \int \left(bx+\frac{a-bx}{1+x^2}\right)dx \\
&= \frac{bx^2}{2}+\int \frac{a-bx}{1+x^2}\,dx \\
&= \frac{bx^2}{2}+a\int \frac{1}{1+x^2}\,dx-b\int \frac{x}{1+x^2}\,dx \\
&= \frac{bx^2}{2}+a\tan^{-1}(x)-\frac{1}{2}b\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.0092501, size = 22, normalized size = 0.92

$$a\tan^{-1}(x)+\frac{1}{2}b\left(x^2-\log\left(x^2+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)/(1 + x^2),x]
```

```
[Out] a*ArcTan[x] + (b*(x^2 - Log[1 + x^2]))/2
```

Maple [A] time = 0.003, size = 21, normalized size = 0.9

$$\frac{bx^2}{2}+a\arctan(x)-\frac{b\ln\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)/(x^2+1),x)
```

```
[Out] 1/2*b*x^2+a*arctan(x)-1/2*b*ln(x^2+1)
```

Maxima [A] time = 1.50588, size = 27, normalized size = 1.12

$$\frac{1}{2}\,bx^2 + a\arctan\left(x\right) - \frac{1}{2}\,b\log\left(x^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)/(x^2+1),x, algorithm="maxima")
```

```
[Out] 1/2*b*x^2 + a*arctan(x) - 1/2*b*log(x^2 + 1)
```

Fricas [A] time = 1.51032, size = 62, normalized size = 2.58

$$\frac{1}{2}\,bx^2 + a\arctan\left(x\right) - \frac{1}{2}\,b\log\left(x^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)/(x^2+1),x, algorithm="fricas")
```

```
[Out] 1/2*b*x^2 + a*arctan(x) - 1/2*b*log(x^2 + 1)
```

Sympy [C] time = 0.304777, size = 34, normalized size = 1.42

$$\frac{bx^2}{2} + \left(-\frac{ia}{2} - \frac{b}{2}\right)\log\left(x - i\right) + \left(\frac{ia}{2} - \frac{b}{2}\right)\log\left(x + i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)/(x**2+1),x)
```

```
[Out] b*x**2/2 + (-I*a/2 - b/2)*log(x - I) + (I*a/2 - b/2)*log(x + I)
```

Giac [A] time = 1.29415, size = 27, normalized size = 1.12

$$\frac{1}{2}\,bx^2 + a\arctan\left(x\right) - \frac{1}{2}\,b\log\left(x^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)/(x^2+1),x, algorithm="giac")
```

```
[Out] 1/2*b*x^2 + a*arctan(x) - 1/2*b*log(x^2 + 1)
```

3.280 $\int \frac{x+x^2}{(4+x)(-4+x^2)}\,dx$

Optimal. Leaf size=15

$$\log(x+4)-\frac{1}{2}\tanh^{-1}\left(\frac{x}{2}\right)$$

```
[Out] -ArcTanh[x/2]/2 + Log[4 + x]
```

Rubi [A] time = 0.0604497, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1593, 1629, 207}

$$\log(x+4)-\frac{1}{2}\tanh^{-1}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x + x^2)/((4 + x)*(-4 + x^2)),x]
```

```
[Out] -ArcTanh[x/2]/2 + Log[4 + x]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{x+x^2}{(4+x)\left(-4+x^2\right)}\,dx = \int \frac{x(1+x)}{(4+x)\left(-4+x^2\right)}\,dx$$
$$= \int \left(\frac{1}{4+x}+\frac{1}{-4+x^2}\right)dx$$
$$= \log(4+x)+\int \frac{1}{-4+x^2}\,dx$$
$$= -\frac{1}{2}\tanh^{-1}\left(\frac{x}{2}\right)+\log(4+x)$$

Mathematica [A] time = 0.0057477, size = 23, normalized size = 1.53

$$\frac{1}{4}\log(2-x)-\frac{1}{4}\log(x+2)+\log(x+4)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + x^2)/((4 + x)*(-4 + x^2)),x]
```

```
[Out] Log[2 - x]/4 - Log[2 + x]/4 + Log[4 + x]
```

Maple [A] time = 0.008, size = 18, normalized size = 1.2

$$-\frac{\ln(2+x)}{4}+\ln(4+x)+\frac{\ln(-2+x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+x)/(4+x)/(x^2-4),x)
```

```
[Out] -1/4*ln(2+x)+ln(4+x)+1/4*ln(-2+x)
```

Maxima [A] time = 1.48995, size = 23, normalized size = 1.53

$$\log(x+4)-\frac{1}{4}\log(x+2)+\frac{1}{4}\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x)/(4+x)/(x^2-4),x, algorithm="maxima")
```

```
[Out] log(x + 4) - 1/4*log(x + 2) + 1/4*log(x - 2)
```

Fricas [A] time = 1.66527, size = 62, normalized size = 4.13

$$\log(x+4) - \frac{1}{4}\log(x+2) + \frac{1}{4}\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x)/(4+x)/(x^2-4),x, algorithm="fricas")
```

```
[Out] log(x + 4) - 1/4*log(x + 2) + 1/4*log(x - 2)
```

Sympy [A] time = 0.122952, size = 17, normalized size = 1.13

$$\frac{\log(x-2)}{4} - \frac{\log(x+2)}{4} + \log(x+4)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x)/(4+x)/(x**2-4),x)
```

```
[Out] log(x - 2)/4 - log(x + 2)/4 + log(x + 4)
```

Giac [A] time = 1.26201, size = 27, normalized size = 1.8

$$\log(|x+4|) - \frac{1}{4}\log(|x+2|) + \frac{1}{4}\log(|x-2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x)/(4+x)/(x^2-4),x, algorithm="giac")
```

```
[Out] log(abs(x + 4)) - 1/4*log(abs(x + 2)) + 1/4*log(abs(x - 2))
```

3.281 $\int \frac{4+x^2}{\left(1+x^2\right)\left(2+x^2\right)}\,dx$

Optimal. Leaf size=20

$$3\tan^{-1}(x)-\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

```
[Out] 3*ArcTan[x] - Sqrt[2]*ArcTan[x/Sqrt[2]]
```

Rubi [A] time = 0.0121168, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {522, 203}

$$3\tan^{-1}(x)-\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x^2)/((1 + x^2)*(2 + x^2)),x]
```

```
[Out] 3*ArcTan[x] - Sqrt[2]*ArcTan[x/Sqrt[2]]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{4+x^2}{\left(1+x^2\right)\left(2+x^2\right)}\,dx = -\left(2\int \frac{1}{2+x^2}\,dx\right)+3\int \frac{1}{1+x^2}\,dx$$
$$= 3\tan^{-1}(x)-\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Mathematica [A] time = 0.0089066, size = 20, normalized size = 1.

$$3\tan^{-1}(x)-\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x^2)/((1 + x^2)*(2 + x^2)),x]
```

```
[Out] 3*ArcTan[x] - Sqrt[2]*ArcTan[x/Sqrt[2]]
```

Maple [A] time = 0.006, size = 18, normalized size = 0.9

$$3\arctan(x)-\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+4)/(x^2+1)/(x^2+2),x)
```

```
[Out] 3*arctan(x)-arctan(1/2*x*2^(1/2))*2^(1/2)
```

Maxima [A] time = 1.54356, size = 23, normalized size = 1.15

$$-\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+3\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+4)/(x^2+1)/(x^2+2),x, algorithm="maxima")
```

```
[Out] -sqrt(2)*arctan(1/2*sqrt(2)*x) + 3*arctan(x)
```

Fricas [A] time = 1.67032, size = 62, normalized size = 3.1

$$-\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+3\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+4)/(x^2+1)/(x^2+2),x, algorithm="fricas")
```

```
[Out] -sqrt(2)*arctan(1/2*sqrt(2)*x) + 3*arctan(x)
```

Sympy [A] time = 0.132992, size = 19, normalized size = 0.95

$$3\operatorname{atan}\left(x\right)-\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+4)/(x**2+1)/(x**2+2),x)
```

```
[Out] 3*atan(x) - sqrt(2)*atan(sqrt(2)*x/2)
```

Giac [A] time = 1.0889, size = 23, normalized size = 1.15

$$-\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+3\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+4)/(x^2+1)/(x^2+2),x, algorithm="giac")
```

```
[Out] -sqrt(2)*arctan(1/2*sqrt(2)*x) + 3*arctan(x)
```

3.282 $\int \frac{5-4x+3x^2+x^4}{(-1+x)^2(1+x^2)}\,dx$

Optimal. Leaf size=37

$$\frac{3}{4}\log\left(x^2+1\right)+x+\frac{5}{2(1-x)}+\frac{1}{2}\log(1-x)+2\tan^{-1}(x)$$

```
[Out] 5/(2*(1 - x)) + x + 2*ArcTan[x] + Log[1 - x]/2 + (3*Log[1 + x^2])/4
```

Rubi [A] time = 0.0397911, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1629, 635, 203, 260}

$$\frac{3}{4}\log\left(x^2+1\right)+x+\frac{5}{2(1-x)}+\frac{1}{2}\log(1-x)+2\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(5 - 4*x + 3*x^2 + x^4)/((-1 + x)^2*(1 + x^2)),x]
```

```
[Out] 5/(2*(1 - x)) + x + 2*ArcTan[x] + Log[1 - x]/2 + (3*Log[1 + x^2])/4
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{5-4x+3x^2+x^4}{(-1+x)^2\left(1+x^2\right)}\,dx &= \int \left(1+\frac{5}{2(-1+x)^2}+\frac{1}{2(-1+x)}+\frac{4+3x}{2\left(1+x^2\right)}\right)dx \\
&= \frac{5}{2(1-x)}+x+\frac{1}{2}\log(1-x)+\frac{1}{2}\int \frac{4+3x}{1+x^2}\,dx \\
&= \frac{5}{2(1-x)}+x+\frac{1}{2}\log(1-x)+\frac{3}{2}\int \frac{x}{1+x^2}\,dx+2\int \frac{1}{1+x^2}\,dx \\
&= \frac{5}{2(1-x)}+x+2\tan^{-1}(x)+\frac{1}{2}\log(1-x)+\frac{3}{4}\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.0226373, size = 33, normalized size = 0.89

$$\frac{3}{4}\log\left(x^2+1\right)+x+\frac{5}{2-2x}+\frac{1}{2}\log(x-1)+2\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 - 4*x + 3*x^2 + x^4)/((-1 + x)^2*(1 + x^2)),x]
```

```
[Out] 5/(2 - 2*x) + x + 2*ArcTan[x] + Log[-1 + x]/2 + (3*Log[1 + x^2])/4
```

Maple [A] time = 0.005, size = 28, normalized size = 0.8

$$x+\frac{3\ln\left(x^2+1\right)}{4}+2\arctan(x)-\frac{5}{2x-2}+\frac{\ln(x-1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+3*x^2-4*x+5)/(x-1)^2/(x^2+1),x)
```

```
[Out] x+3/4*ln(x^2+1)+2*arctan(x)-5/2/(x-1)+1/2*ln(x-1)
```

Maxima [A] time = 1.51328, size = 36, normalized size = 0.97

$$x - \frac{5}{2(x-1)} + 2\arctan(x) + \frac{3}{4}\log\left(x^2+1\right) + \frac{1}{2}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2-4*x+5)/(-1+x)^2/(x^2+1),x, algorithm="maxima")
```

```
[Out] x - 5/2/(x - 1) + 2*arctan(x) + 3/4*log(x^2 + 1) + 1/2*log(x - 1)
```

Fricas [A] time = 1.60854, size = 138, normalized size = 3.73

$$\frac{4x^2 + 8(x-1)\arctan(x) + 3(x-1)\log\left(x^2+1\right) + 2(x-1)\log(x-1) - 4x - 10}{4(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2-4*x+5)/(-1+x)^2/(x^2+1),x, algorithm="fricas")
```

```
[Out] 1/4*(4*x^2 + 8*(x - 1)*arctan(x) + 3*(x - 1)*log(x^2 + 1) + 2*(x - 1)*log(x
 - 1) - 4*x - 10)/(x - 1)
```

Sympy [A] time = 0.144112, size = 29, normalized size = 0.78

$$x + \frac{\log(x-1)}{2} + \frac{3\log\left(x^2+1\right)}{4} + 2\operatorname{atan}(x) - \frac{5}{2x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+3*x**2-4*x+5)/(-1+x)**2/(x**2+1),x)
```

```
[Out] x + log(x - 1)/2 + 3*log(x**2 + 1)/4 + 2*atan(x) - 5/(2*x - 2)
```

Giac [B] time = 1.1681, size = 81, normalized size = 2.19

$$\frac{1}{2}\pi - 2\pi\left\lfloor\frac{\pi + 4\arctan(x)}{4\pi} + \frac{1}{2}\right\rfloor + x - \frac{5}{2(x-1)} + 2\arctan(x) + \frac{3}{4}\log\left(\frac{2}{x-1} + \frac{2}{(x-1)^2} + 1\right) + 2\log(|x-1|) - 1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2-4*x+5)/(-1+x)^2/(x^2+1),x, algorithm="giac")
```

```
[Out] 1/2*pi - 2*pi*floor(1/4*(pi + 4*arctan(x))/pi + 1/2) + x - 5/2/(x - 1) + 2*
arctan(x) + 3/4*log(2/(x - 1) + 2/(x - 1)^2 + 1) + 2*log(abs(x - 1)) - 1
```

3.283 $\int \frac{1+x^4}{2+x^2} dx$

Optimal. Leaf size=26

$$\frac{x^3}{3} - 2x + \frac{5\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[Out] -2*x + x^3/3 + (5*ArcTan[x/Sqrt[2]])/Sqrt[2]
```

Rubi [A] time = 0.0107413, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1154, 203}

$$\frac{x^3}{3} - 2x + \frac{5\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^4)/(2 + x^2),x]
```

```
[Out] -2*x + x^3/3 + (5*ArcTan[x/Sqrt[2]])/Sqrt[2]
```

Rule 1154

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1+x^4}{2+x^2}\,dx = \int \left(-2+x^2+\frac{5}{2+x^2}\right)dx$$
$$= -2x+\frac{x^3}{3}+5\int \frac{1}{2+x^2}\,dx$$
$$= -2x+\frac{x^3}{3}+\frac{5\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

Mathematica [A] time = 0.0080328, size = 26, normalized size = 1.

$$\frac{x^3}{3}-2x+\frac{5\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^4)/(2 + x^2),x]
```

```
[Out] -2*x + x^3/3 + (5*ArcTan[x/Sqrt[2]])/Sqrt[2]
```

Maple [A] time = 0.003, size = 22, normalized size = 0.9

$$-2\,x+\frac{x^3}{3}+\frac{5\,\sqrt{2}}{2}\arctan\left(\frac{x\sqrt{2}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)/(x^2+2),x)
```

```
[Out] -2*x+1/3*x^3+5/2*arctan(1/2*x*2^(1/2))*2^(1/2)
```

Maxima [A] time = 1.4592, size = 28, normalized size = 1.08

$$\frac{1}{3}\,x^3+\frac{5}{2}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right)-2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^2+2),x, algorithm="maxima")
```

```
[Out] 1/3*x^3 + 5/2*sqrt(2)*arctan(1/2*sqrt(2)*x) - 2*x
```

Fricas [A] time = 1.63274, size = 69, normalized size = 2.65

$$\frac{1}{3}\,x^{3}+\frac{5}{2}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right)-2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^2+2),x, algorithm="fricas")
```

```
[Out] 1/3*x^3 + 5/2*sqrt(2)*arctan(1/2*sqrt(2)*x) - 2*x
```

Sympy [A] time = 0.086571, size = 26, normalized size = 1.

$$\frac{x^{3}}{3}-2x+\frac{5\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)/(x**2+2),x)
```

```
[Out] x**3/3 - 2*x + 5*sqrt(2)*atan(sqrt(2)*x/2)/2
```

Giac [A] time = 1.22294, size = 28, normalized size = 1.08

$$\frac{1}{3}\,x^{3}+\frac{5}{2}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right)-2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^2+2),x, algorithm="giac")
```

```
[Out] 1/3*x^3 + 5/2*sqrt(2)*arctan(1/2*sqrt(2)*x) - 2*x
```

3.284 $\int \frac{2+2x+x^4}{x^4+x^5} \, dx$

Optimal. Leaf size=12

$$\log(x+1) - \frac{2}{3x^3}$$

```
[Out] -2/(3*x^3) + Log[1 + x]
```

Rubi [A] time = 0.0320389, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1593, 1620}

$$\log(x+1) - \frac{2}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 2*x + x^4)/(x^4 + x^5),x]
```

```
[Out] -2/(3*x^3) + Log[1 + x]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\int \frac{2+2x+x^4}{x^4+x^5}\,dx = \int \frac{2+2x+x^4}{x^4(1+x)}\,dx$$
$$= \int \left(\frac{2}{x^4}+\frac{1}{1+x}\right)dx$$
$$= -\frac{2}{3x^3}+\log(1+x)$$

Mathematica [A] time = 0.0039845, size = 12, normalized size = 1.

$$\log(x+1)-\frac{2}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 2*x + x^4)/(x^4 + x^5),x]
```

```
[Out] -2/(3*x^3) + Log[1 + x]
```

Maple [A] time = 0.004, size = 11, normalized size = 0.9

$$-\frac{2}{3\,x^3}+\ln\left(1+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+2*x+2)/(x^5+x^4),x)
```

```
[Out] -2/3/x^3+ln(1+x)
```

Maxima [A] time = 0.989715, size = 14, normalized size = 1.17

$$-\frac{2}{3\,x^3}+\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+2*x+2)/(x^5+x^4),x, algorithm="maxima")
```

```
[Out] -2/3/x^3 + log(x + 1)
```

Fricas [A] time = 1.51519, size = 43, normalized size = 3.58

$$\frac{3\,x^{3}\log\left(x+1\right)-2}{3\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+2*x+2)/(x^5+x^4),x, algorithm="fricas")
```

```
[Out] 1/3*(3*x^3*log(x + 1) - 2)/x^3
```

Sympy [A] time = 0.090996, size = 10, normalized size = 0.83

$$\log\left(x+1\right)-\frac{2}{3x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+2*x+2)/(x**5+x**4),x)
```

```
[Out] log(x + 1) - 2/(3*x**3)
```

Giac [A] time = 1.13153, size = 15, normalized size = 1.25

$$-\frac{2}{3\,x^{3}}+\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+2*x+2)/(x^5+x^4),x, algorithm="giac")
```

```
[Out] -2/3/x^3 + log(abs(x + 1))
```

3.285 $\int \frac{-1-5x+2x^2}{2-x-2x^2+x^3}\,dx$

Optimal. Leaf size=21

$$2\log(1-x)-\log(2-x)+\log(x+1)$$

```
[Out] 2*Log[1 - x] - Log[2 - x] + Log[1 + x]
```

Rubi [A] time = 0.0298171, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {2074}

$$2\log(1-x)-\log(2-x)+\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 - 5*x + 2*x^2)/(2 - x - 2*x^2 + x^3),x]
```

```
[Out] 2*Log[1 - x] - Log[2 - x] + Log[1 + x]
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\begin{aligned}\int \frac{-1-5x+2x^2}{2-x-2x^2+x^3}\,dx &= \int \left(\frac{1}{2-x}+\frac{2}{-1+x}+\frac{1}{1+x}\right)dx\\ &= 2\log(1-x)-\log(2-x)+\log(1+x)\end{aligned}$$

Mathematica [A] time = 0.0068186, size = 21, normalized size = 1.

$$2\log(1-x)-\log(2-x)+\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 - 5*x + 2*x^2)/(2 - x - 2*x^2 + x^3),x]
```

```
[Out] 2*Log[1 - x] - Log[2 - x] + Log[1 + x]
```

Maple [A] time = 0.007, size = 18, normalized size = 0.9

$$2\ln(x-1)+\ln(1+x)-\ln(-2+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-5*x-1)/(x^3-2*x^2-x+2),x)
```

```
[Out] 2*ln(x-1)+ln(1+x)-ln(-2+x)
```

Maxima [A] time = 0.992899, size = 23, normalized size = 1.1

$$\log(x+1)+2\log(x-1)-\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-5*x-1)/(x^3-2*x^2-x+2),x, algorithm="maxima")
```

```
[Out] log(x + 1) + 2*log(x - 1) - log(x - 2)
```

Fricas [A] time = 1.57002, size = 54, normalized size = 2.57

$$\log(x+1)+2\log(x-1)-\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-5*x-1)/(x^3-2*x^2-x+2),x, algorithm="fricas")
```

```
[Out] log(x + 1) + 2*log(x - 1) - log(x - 2)
```

Sympy [A] time = 0.117912, size = 15, normalized size = 0.71

$$-\log(x-2)+2\log(x-1)+\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-5*x-1)/(x**3-2*x**2-x+2),x)
```

```
[Out] -log(x - 2) + 2*log(x - 1) + log(x + 1)
```

Giac [A] time = 1.31567, size = 27, normalized size = 1.29

$$\log(|x+1|)+2\log(|x-1|)-\log(|x-2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-5*x-1)/(x^3-2*x^2-x+2),x, algorithm="giac")
```

```
[Out] log(abs(x + 1)) + 2*log(abs(x - 1)) - log(abs(x - 2))
```

3.286 $\int \frac{2+x+x^3}{1+2x^2+x^4} dx$

Optimal. Leaf size=22

$$\frac{x}{x^2+1}+\frac{1}{2}\log\left(x^2+1\right)+\tan^{-1}(x)$$

```
[Out] x/(1 + x^2) + ArcTan[x] + Log[1 + x^2]/2
```

Rubi [A] time = 0.0145003, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {28, 1814, 635, 203, 260}

$$\frac{x}{x^2+1}+\frac{1}{2}\log\left(x^2+1\right)+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + x^3)/(1 + 2*x^2 + x^4),x]
```

```
[Out] x/(1 + x^2) + ArcTan[x] + Log[1 + x^2]/2
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
```

```
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{2+x+x^3}{1+2x^2+x^4}\,dx &= \int \frac{2+x+x^3}{\left(1+x^2\right)^2}\,dx\\ &= \frac{x}{1+x^2}-\frac{1}{2}\int \frac{-2-2x}{1+x^2}\,dx\\ &= \frac{x}{1+x^2}+\int \frac{1}{1+x^2}\,dx+\int \frac{x}{1+x^2}\,dx\\ &= \frac{x}{1+x^2}+\tan^{-1}(x)+\frac{1}{2}\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.0081954, size = 22, normalized size = 1.

$$\frac{x}{x^2+1}+\frac{1}{2}\log\left(x^2+1\right)+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + x^3)/(1 + 2*x^2 + x^4),x]
```

```
[Out] x/(1 + x^2) + ArcTan[x] + Log[1 + x^2]/2
```

Maple [A] time = 0.003, size = 21, normalized size = 1.

$$\frac{x}{x^2+1}+\arctan(x)+\frac{\ln\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x+2)/(x^4+2*x^2+1),x)
```

```
[Out] x/(x^2+1)+arctan(x)+1/2*ln(x^2+1)
```

Maxima [A] time = 1.46341, size = 27, normalized size = 1.23

$$\frac{x}{x^2+1} + \arctan(x) + \frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x+2)/(x^4+2*x^2+1),x, algorithm="maxima")
```

```
[Out] x/(x^2 + 1) + arctan(x) + 1/2*log(x^2 + 1)
```

Fricas [A] time = 1.51564, size = 95, normalized size = 4.32

$$\frac{2\left(x^2+1\right)\arctan(x) + \left(x^2+1\right)\log\left(x^2+1\right) + 2x}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x+2)/(x^4+2*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(x^2 + 1)*arctan(x) + (x^2 + 1)*log(x^2 + 1) + 2*x)/(x^2 + 1)
```

Sympy [A] time = 0.103382, size = 17, normalized size = 0.77

$$\frac{x}{x^2+1} + \frac{\log\left(x^2+1\right)}{2} + \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x+2)/(x**4+2*x**2+1),x)
```

```
[Out] x/(x**2 + 1) + log(x**2 + 1)/2 + atan(x)
```

Giac [A] time = 1.20552, size = 27, normalized size = 1.23

$$\frac{x}{x^2+1}+\arctan(x)+\frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x+2)/(x^4+2*x^2+1),x, algorithm="giac")
```

```
[Out] x/(x^2 + 1) + arctan(x) + 1/2*log(x^2 + 1)
```

3.287 $\int \frac{1+2x+x^2+x^3}{1+2x^2+x^4} \, dx$

Optimal. Leaf size=24

$$-\frac{1}{2\left(x^2+1\right)}+\frac{1}{2}\log\left(x^2+1\right)+\tan^{-1}(x)$$

```
[Out] -1/(2*(1 + x^2)) + ArcTan[x] + Log[1 + x^2]/2
```

Rubi [A] time = 0.0159376, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {28, 1814, 635, 203, 260}

$$-\frac{1}{2\left(x^2+1\right)}+\frac{1}{2}\log\left(x^2+1\right)+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x + x^2 + x^3)/(1 + 2*x^2 + x^4),x]
```

```
[Out] -1/(2*(1 + x^2)) + ArcTan[x] + Log[1 + x^2]/2
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
```

```
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{1+2x+x^2+x^3}{1+2x^2+x^4}\,dx &= \int \frac{1+2x+x^2+x^3}{\left(1+x^2\right)^2}\,dx\\ &= -\frac{1}{2\left(1+x^2\right)}-\frac{1}{2}\int \frac{-2-2x}{1+x^2}\,dx\\ &= -\frac{1}{2\left(1+x^2\right)}+\int \frac{1}{1+x^2}\,dx+\int \frac{x}{1+x^2}\,dx\\ &= -\frac{1}{2\left(1+x^2\right)}+\tan^{-1}(x)+\frac{1}{2}\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.0097642, size = 24, normalized size = 1.

$$-\frac{1}{2\left(x^2+1\right)}+\frac{1}{2}\log\left(x^2+1\right)+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x + x^2 + x^3)/(1 + 2*x^2 + x^4),x]
```

```
[Out] -1/(2*(1 + x^2)) + ArcTan[x] + Log[1 + x^2]/2
```

Maple [A] time = 0.005, size = 21, normalized size = 0.9

$$-\frac{1}{2\,x^2+2}+\arctan(x)+\frac{\ln\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x^2+2*x+1)/(x^4+2*x^2+1),x)
```

```
[Out] -1/2/(x^2+1)+arctan(x)+1/2*ln(x^2+1)
```

Maxima [A] time = 1.5107, size = 27, normalized size = 1.12

$$-\frac{1}{2\left(x^2+1\right)}+\arctan\left(x\right)+\frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+2*x+1)/(x^4+2*x^2+1),x, algorithm="maxima")
```

```
[Out] -1/2/(x^2 + 1) + arctan(x) + 1/2*log(x^2 + 1)
```

Fricas [A] time = 1.59358, size = 92, normalized size = 3.83

$$\frac{2\left(x^2+1\right)\arctan\left(x\right)+\left(x^2+1\right)\log\left(x^2+1\right)-1}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+2*x+1)/(x^4+2*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(x^2 + 1)*arctan(x) + (x^2 + 1)*log(x^2 + 1) - 1)/(x^2 + 1)
```

Sympy [A] time = 0.111997, size = 19, normalized size = 0.79

$$\frac{\log\left(x^2+1\right)}{2}+\operatorname{atan}\left(x\right)-\frac{1}{2x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x**2+2*x+1)/(x**4+2*x**2+1),x)
```

```
[Out] log(x**2 + 1)/2 + atan(x) - 1/(2*x**2 + 2)
```

Giac [A] time = 1.13662, size = 27, normalized size = 1.12

$$-\frac{1}{2\left(x^2+1\right)}+\arctan\left(x\right)+\frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+2*x+1)/(x^4+2*x^2+1),x, algorithm="giac")
```

```
[Out] -1/2/(x^2 + 1) + arctan(x) + 1/2*log(x^2 + 1)
```

3.288 $\int \frac{3+4x}{(1+x^2)(2+x^2)} \, dx$

Optimal. Leaf size=36

$$2\log\left(x^2+1\right)-2\log\left(x^2+2\right)+3\tan^{-1}(x)-\frac{3\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[Out] 3*ArcTan[x] - (3*ArcTan[x/Sqrt[2]])/Sqrt[2] + 2*Log[1 + x^2] - 2*Log[2 + x^
2]
```

Rubi [A] time = 0.0261453, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1010, 391, 203, 444, 36, 31}

$$2\log\left(x^2+1\right)-2\log\left(x^2+2\right)+3\tan^{-1}(x)-\frac{3\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 4*x)/((1 + x^2)*(2 + x^2)),x]
```

```
[Out] 3*ArcTan[x] - (3*ArcTan[x/Sqrt[2]])/Sqrt[2] + 2*Log[1 + x^2] - 2*Log[2 + x^
2]
```

Rule 1010

```
Int[((g_) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q
_), x_Symbol] :> Dist[g, Int[(a + c*x^2)^p*(d + f*x^2)^q, x], x] + Dist[h,
Int[x*(a + c*x^2)^p*(d + f*x^2)^q, x], x] /; FreeQ[{a, c, d, f, g, h, p, q}
, x]
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{3+4x}{\left(1+x^2\right)\left(2+x^2\right)}\,dx &= 3\int \frac{1}{\left(1+x^2\right)\left(2+x^2\right)}\,dx+4\int \frac{x}{\left(1+x^2\right)\left(2+x^2\right)}\,dx\\
&= 2\,\text{Subst}\left(\int \frac{1}{(1+x)(2+x)}\,dx,x,x^2\right)+3\int \frac{1}{1+x^2}\,dx-3\int \frac{1}{2+x^2}\,dx\\
&= 3\tan^{-1}(x)-\frac{3\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}+2\,\text{Subst}\left(\int \frac{1}{1+x}\,dx,x,x^2\right)-2\,\text{Subst}\left(\int \frac{1}{2+x}\,dx,x,x^2\right)\\
&= 3\tan^{-1}(x)-\frac{3\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}+2\log\left(1+x^2\right)-2\log\left(2+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.0148312, size = 36, normalized size = 1.

$$2\log\left(x^2+1\right)-2\log\left(x^2+2\right)+3\tan^{-1}(x)-\frac{3\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 4*x)/((1 + x^2)*(2 + x^2)),x]
```

```
[Out] 3*ArcTan[x] - (3*ArcTan[x/Sqrt[2]])/Sqrt[2] + 2*Log[1 + x^2] - 2*Log[2 + x^
2]
```

Maple [A] time = 0.003, size = 34, normalized size = 0.9

$$3\arctan(x)+2\ln\left(x^2+1\right)-2\ln\left(x^2+2\right)-\frac{3\sqrt{2}}{2}\arctan\left(\frac{x\sqrt{2}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3+4*x)/(x^2+1)/(x^2+2),x)
```

```
[Out] 3*arctan(x)+2*ln(x^2+1)-2*ln(x^2+2)-3/2*arctan(1/2*x*2^(1/2))*2^(1/2)
```

Maxima [A] time = 1.45174, size = 45, normalized size = 1.25

$$-\frac{3}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+3\arctan(x)-2\log\left(x^2+2\right)+2\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+4*x)/(x^2+1)/(x^2+2),x, algorithm="maxima")
```

```
[Out] -3/2*sqrt(2)*arctan(1/2*sqrt(2)*x) + 3*arctan(x) - 2*log(x^2 + 2) + 2*log(x
^2 + 1)
```

Fricas [A] time = 1.51031, size = 113, normalized size = 3.14

$$-\frac{3}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+3\arctan(x)-2\log\left(x^2+2\right)+2\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+4*x)/(x^2+1)/(x^2+2),x, algorithm="fricas")
```

```
[Out] -3/2*sqrt(2)*arctan(1/2*sqrt(2)*x) + 3*arctan(x) - 2*log(x^2 + 2) + 2*log(x
^2 + 1)
```

Sympy [A] time = 0.16702, size = 39, normalized size = 1.08

$$2\log\left(x^2+1\right)-2\log\left(x^2+2\right)+3\operatorname{atan}(x)-\frac{3\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+4*x)/(x**2+1)/(x**2+2),x)
```

```
[Out] 2*log(x**2 + 1) - 2*log(x**2 + 2) + 3*atan(x) - 3*sqrt(2)*atan(sqrt(2)*x/2)
/2
```

Giac [A] time = 1.18855, size = 45, normalized size = 1.25

$$-\frac{3}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+3\arctan(x)-2\log\left(x^2+2\right)+2\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+4*x)/(x^2+1)/(x^2+2),x, algorithm="giac")
```

```
[Out] -3/2*sqrt(2)*arctan(1/2*sqrt(2)*x) + 3*arctan(x) - 2*log(x^2 + 2) + 2*log(x
^2 + 1)
```

3.289 $\int \frac{2+x}{(1+x^2)(4+x^2)} \, dx$

Optimal. Leaf size=37

$$\frac{1}{6}\log\left(x^2+1\right)-\frac{1}{6}\log\left(x^2+4\right)-\frac{1}{3}\tan^{-1}\left(\frac{x}{2}\right)+\frac{2}{3}\tan^{-1}(x)$$

```
[Out] -ArcTan[x/2]/3 + (2*ArcTan[x])/3 + Log[1 + x^2]/6 - Log[4 + x^2]/6
```

Rubi [A] time = 0.023859, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1010, 391, 203, 444, 36, 31}

$$\frac{1}{6}\log\left(x^2+1\right)-\frac{1}{6}\log\left(x^2+4\right)-\frac{1}{3}\tan^{-1}\left(\frac{x}{2}\right)+\frac{2}{3}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x)/((1 + x^2)*(4 + x^2)),x]
```

```
[Out] -ArcTan[x/2]/3 + (2*ArcTan[x])/3 + Log[1 + x^2]/6 - Log[4 + x^2]/6
```

Rule 1010

```
Int[((g_) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q
_), x_Symbol] :> Dist[g, Int[(a + c*x^2)^p*(d + f*x^2)^q, x], x] + Dist[h,
Int[x*(a + c*x^2)^p*(d + f*x^2)^q, x], x] /; FreeQ[{a, c, d, f, g, h, p, q}
, x]
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{2+x}{(1+x^2)(4+x^2)}\,dx &= 2\int \frac{1}{(1+x^2)(4+x^2)}\,dx+\int \frac{x}{(1+x^2)(4+x^2)}\,dx\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{(1+x)(4+x)}\,dx,x,x^2\right)+\frac{2}{3}\int \frac{1}{1+x^2}\,dx-\frac{2}{3}\int \frac{1}{4+x^2}\,dx\\
&= -\frac{1}{3}\tan^{-1}\left(\frac{x}{2}\right)+\frac{2}{3}\tan^{-1}(x)+\frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{1+x}\,dx,x,x^2\right)-\frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{4+x}\,dx,x,x^2\right)\\
&= -\frac{1}{3}\tan^{-1}\left(\frac{x}{2}\right)+\frac{2}{3}\tan^{-1}(x)+\frac{1}{6}\log\left(1+x^2\right)-\frac{1}{6}\log\left(4+x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.0075078, size = 37, normalized size = 1.

$$\frac{1}{6}\log\left(x^2+1\right)-\frac{1}{6}\log\left(x^2+4\right)-\frac{1}{3}\tan^{-1}\left(\frac{x}{2}\right)+\frac{2}{3}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x)/((1 + x^2)*(4 + x^2)),x]
```

```
[Out] -ArcTan[x/2]/3 + (2*ArcTan[x])/3 + Log[1 + x^2]/6 - Log[4 + x^2]/6
```

Maple [A] time = 0.004, size = 28, normalized size = 0.8

$$-\frac{1}{3}\arctan\left(\frac{x}{2}\right)+\frac{2\arctan(x)}{3}+\frac{\ln\left(x^2+1\right)}{6}-\frac{\ln\left(x^2+4\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+x)/(x^2+1)/(x^2+4),x)
```

```
[Out] -1/3*arctan(1/2*x)+2/3*arctan(x)+1/6*ln(x^2+1)-1/6*ln(x^2+4)
```

Maxima [A] time = 1.46583, size = 36, normalized size = 0.97

$$-\frac{1}{3}\arctan\left(\frac{1}{2}x\right)+\frac{2}{3}\arctan(x)-\frac{1}{6}\log\left(x^2+4\right)+\frac{1}{6}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x^2+1)/(x^2+4),x, algorithm="maxima")
```

```
[Out] -1/3*arctan(1/2*x) + 2/3*arctan(x) - 1/6*log(x^2 + 4) + 1/6*log(x^2 + 1)
```

Fricas [A] time = 1.55561, size = 100, normalized size = 2.7

$$-\frac{1}{3}\arctan\left(\frac{1}{2}x\right)+\frac{2}{3}\arctan(x)-\frac{1}{6}\log\left(x^2+4\right)+\frac{1}{6}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x^2+1)/(x^2+4),x, algorithm="fricas")
```

```
[Out] -1/3*arctan(1/2*x) + 2/3*arctan(x) - 1/6*log(x^2 + 4) + 1/6*log(x^2 + 1)
```

Sympy [A] time = 0.152719, size = 29, normalized size = 0.78

$$\frac{\log\left(x^2+1\right)}{6}-\frac{\log\left(x^2+4\right)}{6}-\frac{\operatorname{atan}\left(\frac{x}{2}\right)}{3}+\frac{2\operatorname{atan}(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x**2+1)/(x**2+4),x)
```

```
[Out] log(x**2 + 1)/6 - log(x**2 + 4)/6 - atan(x/2)/3 + 2*atan(x)/3
```

Giac [A] time = 1.23479, size = 36, normalized size = 0.97

$$-\frac{1}{3}\arctan\left(\frac{1}{2}x\right)+\frac{2}{3}\arctan\left(x\right)-\frac{1}{6}\log\left(x^2+4\right)+\frac{1}{6}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x^2+1)/(x^2+4),x, algorithm="giac")
```

```
[Out] -1/3*arctan(1/2*x) + 2/3*arctan(x) - 1/6*log(x^2 + 4) + 1/6*log(x^2 + 1)
```

3.290 $\int \frac{2-x+x^3}{-7-6x+x^2}\,dx$

Optimal. Leaf size=29

$$\frac{x^2}{2}+6x+\frac{169}{4}\log(7-x)-\frac{1}{4}\log(x+1)$$

```
[Out] 6*x + x^2/2 + (169*Log[7 - x])/4 - Log[1 + x]/4
```

Rubi [A] time = 0.0165118, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1657, 632, 31}

$$\frac{x^2}{2}+6x+\frac{169}{4}\log(7-x)-\frac{1}{4}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(2 - x + x^3)/(-7 - 6*x + x^2),x]
```

```
[Out] 6*x + x^2/2 + (169*Log[7 - x])/4 - Log[1 + x]/4
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{2-x+x^3}{-7-6x+x^2}\,dx &= \int \left(6+x+\frac{2(22+21x)}{-7-6x+x^2}\right)dx\\
&= 6x+\frac{x^2}{2}+2\int \frac{22+21x}{-7-6x+x^2}\,dx\\
&= 6x+\frac{x^2}{2}-\frac{1}{4}\int \frac{1}{1+x}\,dx+\frac{169}{4}\int \frac{1}{-7+x}\,dx\\
&= 6x+\frac{x^2}{2}+\frac{169}{4}\log(7-x)-\frac{1}{4}\log(1+x)
\end{aligned}
$$

Mathematica [A] time = 0.0058195, size = 29, normalized size = 1.

$$\frac{x^2}{2}+6x+\frac{169}{4}\log(7-x)-\frac{1}{4}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - x + x^3)/(-7 - 6*x + x^2),x]
```

```
[Out] 6*x + x^2/2 + (169*Log[7 - x])/4 - Log[1 + x]/4
```

Maple [A] time = 0.006, size = 22, normalized size = 0.8

$$\frac{x^2}{2}+6\,x+\frac{169\,\ln(x-7)}{4}-\frac{\ln(1+x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-x+2)/(x^2-6*x-7),x)
```

```
[Out] 1/2*x^2+6*x+169/4*ln(x-7)-1/4*ln(1+x)
```

Maxima [A] time = 1.07431, size = 28, normalized size = 0.97

$$\frac{1}{2}\,x^2+6\,x-\frac{1}{4}\,\log(x+1)+\frac{169}{4}\,\log(x-7)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x+2)/(x^2-6*x-7),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 + 6*x - 1/4*log(x + 1) + 169/4*log(x - 7)
```

Fricas [A] time = 1.49636, size = 69, normalized size = 2.38

$$\frac{1}{2}x^2 + 6x - \frac{1}{4}\log(x+1) + \frac{169}{4}\log(x-7)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x+2)/(x^2-6*x-7),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 + 6*x - 1/4*log(x + 1) + 169/4*log(x - 7)
```

Sympy [A] time = 0.101816, size = 22, normalized size = 0.76

$$\frac{x^2}{2} + 6x + \frac{169\log(x-7)}{4} - \frac{\log(x+1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-x+2)/(x**2-6*x-7),x)
```

```
[Out] x**2/2 + 6*x + 169*log(x - 7)/4 - log(x + 1)/4
```

Giac [A] time = 1.13385, size = 31, normalized size = 1.07

$$\frac{1}{2}x^2 + 6x - \frac{1}{4}\log(|x+1|) + \frac{169}{4}\log(|x-7|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x+2)/(x^2-6*x-7),x, algorithm="giac")
```

```
[Out] 1/2*x^2 + 6*x - 1/4*log(abs(x + 1)) + 169/4*log(abs(x - 7))
```

3.291 $\int \frac{-1+x^5}{-1+x^2} \, dx$

Optimal. Leaf size=19

$$\frac{x^4}{4}+\frac{x^2}{2}+\log(x+1)$$

```
[Out] x^2/2 + x^4/4 + Log[1 + x]
```

Rubi [A] time = 0.0132435, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1810, 627, 31}

$$\frac{x^4}{4}+\frac{x^2}{2}+\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^5)/(-1 + x^2),x]
```

```
[Out] x^2/2 + x^4/4 + Log[1 + x]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{-1+x^5}{-1+x^2}\,dx &= \int \left(x+x^3-\frac{1-x}{-1+x^2}\right)dx\\
&= \frac{x^2}{2}+\frac{x^4}{4}-\int \frac{1-x}{-1+x^2}\,dx\\
&= \frac{x^2}{2}+\frac{x^4}{4}-\int \frac{1}{-1-x}\,dx\\
&= \frac{x^2}{2}+\frac{x^4}{4}+\log(1+x)
\end{aligned}
$$

Mathematica [A] time = 0.0041398, size = 19, normalized size = 1.

$$\frac{x^4}{4}+\frac{x^2}{2}+\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^5)/(-1 + x^2),x]
```

```
[Out] x^2/2 + x^4/4 + Log[1 + x]
```

Maple [A] time = 0.001, size = 16, normalized size = 0.8

$$\frac{x^2}{2}+\frac{x^4}{4}+\ln(1+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5-1)/(x^2-1),x)
```

```
[Out] 1/2*x^2+1/4*x^4+ln(1+x)
```

Maxima [A] time = 0.962015, size = 20, normalized size = 1.05

$$\frac{1}{4}x^4+\frac{1}{2}x^2+\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-1)/(x^2-1),x, algorithm="maxima")
```

```
[Out] 1/4*x^4 + 1/2*x^2 + log(x + 1)
```

Fricas [A] time = 1.4384, size = 43, normalized size = 2.26

$$\frac{1}{4}x^4 + \frac{1}{2}x^2 + \log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-1)/(x^2-1),x, algorithm="fricas")
```

```
[Out] 1/4*x^4 + 1/2*x^2 + log(x + 1)
```

Sympy [A] time = 0.075318, size = 14, normalized size = 0.74

$$\frac{x^4}{4} + \frac{x^2}{2} + \log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**5-1)/(x**2-1),x)
```

```
[Out] x**4/4 + x**2/2 + log(x + 1)
```

Giac [A] time = 1.17694, size = 22, normalized size = 1.16

$$\frac{1}{4}x^4 + \frac{1}{2}x^2 + \log(|x+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-1)/(x^2-1),x, algorithm="giac")
```

```
[Out] 1/4*x^4 + 1/2*x^2 + log(abs(x + 1))
```

3.292 $\int \frac{5+2x-x^2+x^3}{1+x+x^2} dx$

Optimal. Leaf size=41

$$\frac{x^2}{2}+\frac{3}{2}\log \left(x^2+x+1\right)-2x+\frac{11\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] -2*x + x^2/2 + (11*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] + (3*Log[1 + x + x^2]
)/2
```

Rubi [A] time = 0.0278992, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1657, 634, 618, 204, 628}

$$\frac{x^2}{2}+\frac{3}{2}\log \left(x^2+x+1\right)-2x+\frac{11\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + 2*x - x^2 + x^3)/(1 + x + x^2),x]
```

```
[Out] -2*x + x^2/2 + (11*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] + (3*Log[1 + x + x^2]
)/2
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{5+2x-x^2+x^3}{1+x+x^2}\,dx &= \int \left(-2+x+\frac{7+3x}{1+x+x^2}\right)dx \\
&= -2x+\frac{x^2}{2}+\int \frac{7+3x}{1+x+x^2}\,dx \\
&= -2x+\frac{x^2}{2}+\frac{3}{2}\int \frac{1+2x}{1+x+x^2}\,dx+\frac{11}{2}\int \frac{1}{1+x+x^2}\,dx \\
&= -2x+\frac{x^2}{2}+\frac{3}{2}\log\left(1+x+x^2\right)-11\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2x\right) \\
&= -2x+\frac{x^2}{2}+\frac{11\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{3}{2}\log\left(1+x+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.0158124, size = 41, normalized size = 1.

$$\frac{x^2}{2}+\frac{3}{2}\log\left(x^2+x+1\right)-2x+\frac{11\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + 2*x - x^2 + x^3)/(1 + x + x^2),x]
```

```
[Out] -2*x + x^2/2 + (11*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] + (3*Log[1 + x + x^2]
)/2
```

Maple [A] time = 0.003, size = 35, normalized size = 0.9

$$-2\,x+\frac{x^{2}}{2}+\frac{3\,\ln\left(x^{2}+x+1\right)}{2}+\frac{11\,\sqrt{3}}{3}\arctan\left(\frac{(1+2\,x)\,\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-x^2+2*x+5)/(x^2+x+1),x)
```

```
[Out] -2*x+1/2*x^2+3/2*ln(x^2+x+1)+11/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Maxima [A] time = 1.48315, size = 46, normalized size = 1.12

$$\frac{1}{2}\,x^{2}+\frac{11}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)-2\,x+\frac{3}{2}\,\log\left(x^{2}+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x^2+2*x+5)/(x^2+x+1),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 + 11/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 2*x + 3/2*log(x^2 +
x + 1)
```

Fricas [A] time = 1.46313, size = 112, normalized size = 2.73

$$\frac{1}{2}\,x^{2}+\frac{11}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)-2\,x+\frac{3}{2}\,\log\left(x^{2}+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x^2+2*x+5)/(x^2+x+1),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 + 11/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 2*x + 3/2*log(x^2 +
x + 1)
```

Sympy [A] time = 0.104471, size = 46, normalized size = 1.12

$$\frac{x^2}{2} - 2x + \frac{3\log\left(x^2 + x + 1\right)}{2} + \frac{11\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3} + \frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-x**2+2*x+5)/(x**2+x+1),x)
```

```
[Out] x**2/2 - 2*x + 3*log(x**2 + x + 1)/2 + 11*sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt
(3)/3)/3
```

Giac [A] time = 1.23359, size = 46, normalized size = 1.12

$$\frac{1}{2}\,x^2 + \frac{11}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x + 1)\right) - 2\,x + \frac{3}{2}\,\log\left(x^2 + x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x^2+2*x+5)/(x^2+x+1),x, algorithm="giac")
```

```
[Out] 1/2*x^2 + 11/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 2*x + 3/2*log(x^2 +
x + 1)
```

3.293 $\int \frac{-3+x-2x^3+x^4}{10-8x+2x^2}\,dx$

Optimal. Leaf size=41

$$\frac{x^3}{6}+\frac{x^2}{2}+\frac{3}{4}\log\left(x^2-4x+5\right)+\frac{3x}{2}+6\tan^{-1}(2-x)$$

```
[Out] (3*x)/2 + x^2/2 + x^3/6 + 6*ArcTan[2 - x] + (3*Log[5 - 4*x + x^2])/4
```

Rubi [A] time = 0.0276182, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1657, 634, 618, 204, 628}

$$\frac{x^3}{6}+\frac{x^2}{2}+\frac{3}{4}\log\left(x^2-4x+5\right)+\frac{3x}{2}+6\tan^{-1}(2-x)$$

Antiderivative was successfully verified.

```
[In] Int[(-3 + x - 2*x^3 + x^4)/(10 - 8*x + 2*x^2),x]
```

```
[Out] (3*x)/2 + x^2/2 + x^3/6 + 6*ArcTan[2 - x] + (3*Log[5 - 4*x + x^2])/4
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{-3+x-2x^3+x^4}{10-8x+2x^2}\,dx &= \int \left(\frac{3}{2}+x+\frac{x^2}{2}-\frac{3(6-x)}{10-8x+2x^2}\right)dx\\ &= \frac{3x}{2}+\frac{x^2}{2}+\frac{x^3}{6}-3\int \frac{6-x}{10-8x+2x^2}\,dx\\ &= \frac{3x}{2}+\frac{x^2}{2}+\frac{x^3}{6}+\frac{3}{4}\int \frac{-8+4x}{10-8x+2x^2}\,dx-12\int \frac{1}{10-8x+2x^2}\,dx\\ &= \frac{3x}{2}+\frac{x^2}{2}+\frac{x^3}{6}+\frac{3}{4}\log\left(5-4x+x^2\right)+24\,\text{Subst}\left(\int \frac{1}{-16-x^2}\,dx,x,-8+4x\right)\\ &= \frac{3x}{2}+\frac{x^2}{2}+\frac{x^3}{6}+6\tan^{-1}(2-x)+\frac{3}{4}\log\left(5-4x+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.009641, size = 39, normalized size = 0.95

$$\frac{1}{2}\left(\frac{x^3}{3}+x^2+\frac{3}{2}\log\left(x^2-4x+5\right)+3x+12\tan^{-1}(2-x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 + x - 2*x^3 + x^4)/(10 - 8*x + 2*x^2),x]
```

```
[Out] (3*x + x^2 + x^3/3 + 12*ArcTan[2 - x] + (3*Log[5 - 4*x + x^2])/2)/2
```

Maple [A] time = 0.004, size = 32, normalized size = 0.8

$$\frac{3\,x}{2}+\frac{x^2}{2}+\frac{x^3}{6}-6\,\arctan\left(-2+x\right)+\frac{3\,\ln\left(x^2-4\,x+5\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-2*x^3+x-3)/(2*x^2-8*x+10),x)
```

```
[Out] 3/2*x+1/2*x^2+1/6*x^3-6*arctan(-2+x)+3/4*ln(x^2-4*x+5)
```

Maxima [A] time = 1.45813, size = 42, normalized size = 1.02

$$\frac{1}{6}\,x^3+\frac{1}{2}\,x^2+\frac{3}{2}\,x-6\,\arctan\left(x-2\right)+\frac{3}{4}\,\log\left(x^2-4\,x+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-2*x^3+x-3)/(2*x^2-8*x+10),x, algorithm="maxima")
```

```
[Out] 1/6*x^3 + 1/2*x^2 + 3/2*x - 6*arctan(x - 2) + 3/4*log(x^2 - 4*x + 5)
```

Fricas [A] time = 1.56159, size = 95, normalized size = 2.32

$$\frac{1}{6}\,x^3+\frac{1}{2}\,x^2+\frac{3}{2}\,x-6\,\arctan\left(x-2\right)+\frac{3}{4}\,\log\left(x^2-4\,x+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-2*x^3+x-3)/(2*x^2-8*x+10),x, algorithm="fricas")
```

```
[Out] 1/6*x^3 + 1/2*x^2 + 3/2*x - 6*arctan(x - 2) + 3/4*log(x^2 - 4*x + 5)
```

Sympy [A] time = 0.110648, size = 34, normalized size = 0.83

$$\frac{x^3}{6}+\frac{x^2}{2}+\frac{3x}{2}+\frac{3\log\left(x^2-4x+5\right)}{4}-6\,\mathrm{atan}\left(x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-2*x**3+x-3)/(2*x**2-8*x+10),x)
```

```
[Out] x**3/6 + x**2/2 + 3*x/2 + 3*log(x**2 - 4*x + 5)/4 - 6*atan(x - 2)
```

Giac [A] time = 1.12129, size = 42, normalized size = 1.02

$$\frac{1}{6}x^3 + \frac{1}{2}x^2 + \frac{3}{2}x - 6\arctan(x-2) + \frac{3}{4}\log\left(x^2 - 4x + 5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-2*x^3+x-3)/(2*x^2-8*x+10),x, algorithm="giac")
```

```
[Out] 1/6*x^3 + 1/2*x^2 + 3/2*x - 6*arctan(x - 2) + 3/4*log(x^2 - 4*x + 5)
```

3.294 $\int \frac{1+2x+3x^2+x^3}{(-3+x)(-2+x)(-1+x)}\,dx$

Optimal. Leaf size=30

$$x+\frac{7}{2}\log(1-x)-25\log(2-x)+\frac{61}{2}\log(3-x)$$

```
[Out] x + (7*Log[1 - x])/2 - 25*Log[2 - x] + (61*Log[3 - x])/2
```

Rubi [A] time = 0.0576631, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {1612}

$$x+\frac{7}{2}\log(1-x)-25\log(2-x)+\frac{61}{2}\log(3-x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x + 3*x^2 + x^3)/((-3 + x)*(-2 + x)*(-1 + x)),x]
```

```
[Out] x + (7*Log[1 - x])/2 - 25*Log[2 - x] + (61*Log[3 - x])/2
```

Rule 1612

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^
n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px,
 x] && IntegersQ[m, n]
```

Rubi steps

$$\begin{aligned}\int \frac{1+2x+3x^2+x^3}{(-3+x)(-2+x)(-1+x)}\,dx &= \int \left(1+\frac{61}{2(-3+x)}-\frac{25}{-2+x}+\frac{7}{2(-1+x)}\right)dx\\ &= x+\frac{7}{2}\log(1-x)-25\log(2-x)+\frac{61}{2}\log(3-x)\end{aligned}$$

Mathematica [A] time = 0.011634, size = 24, normalized size = 0.8

$$x+\frac{61}{2}\log(x-3)-25\log(x-2)+\frac{7}{2}\log(x-1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x + 3*x^2 + x^3)/((-3 + x)*(-2 + x)*(-1 + x)),x]
```

```
[Out] x + (61*Log[-3 + x])/2 - 25*Log[-2 + x] + (7*Log[-1 + x])/2
```

Maple [A] time = 0.007, size = 21, normalized size = 0.7

$$x+\frac{7\ln(x-1)}{2}+\frac{61\ln(-3+x)}{2}-25\ln(-2+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+3*x^2+2*x+1)/(-3+x)/(-2+x)/(x-1),x)
```

```
[Out] x+7/2*ln(x-1)+61/2*ln(-3+x)-25*ln(-2+x)
```

Maxima [A] time = 0.967553, size = 27, normalized size = 0.9

$$x+\frac{7}{2}\log(x-1)-25\log(x-2)+\frac{61}{2}\log(x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+3*x^2+2*x+1)/(-3+x)/(-2+x)/(-1+x),x, algorithm="maxima")
```

```
[Out] x + 7/2*log(x - 1) - 25*log(x - 2) + 61/2*log(x - 3)
```

Fricas [A] time = 1.5394, size = 73, normalized size = 2.43

$$x+\frac{7}{2}\log(x-1)-25\log(x-2)+\frac{61}{2}\log(x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+3*x^2+2*x+1)/(-3+x)/(-2+x)/(-1+x),x, algorithm="fricas")
```

```
[Out] x + 7/2*log(x - 1) - 25*log(x - 2) + 61/2*log(x - 3)
```

Sympy [A] time = 0.13527, size = 24, normalized size = 0.8

$$x + \frac{61 \log(x-3)}{2} - 25 \log(x-2) + \frac{7 \log(x-1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+3*x**2+2*x+1)/(-3+x)/(-2+x)/(-1+x),x)
```

```
[Out] x + 61*log(x - 3)/2 - 25*log(x - 2) + 7*log(x - 1)/2
```

Giac [A] time = 1.12603, size = 31, normalized size = 1.03

$$x + \frac{7}{2} \log(|x-1|) - 25 \log(|x-2|) + \frac{61}{2} \log(|x-3|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+3*x^2+2*x+1)/(-3+x)/(-2+x)/(-1+x),x, algorithm="giac")
```

```
[Out] x + 7/2*log(abs(x - 1)) - 25*log(abs(x - 2)) + 61/2*log(abs(x - 3))
```

3.295 $\int \frac{2-7x+x^2-x^3+x^4}{-24-14x+x^2+x^3} \, dx$

Optimal. Leaf size=35

$$\frac{x^2}{2}-2x+\frac{13}{3}\log(4-x)-\frac{22}{3}\log(x+2)+20\log(x+3)$$

```
[Out] -2*x + x^2/2 + (13*Log[4 - x])/3 - (22*Log[2 + x])/3 + 20*Log[3 + x]
```

Rubi [A] time = 0.0397596, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {2074}

$$\frac{x^2}{2}-2x+\frac{13}{3}\log(4-x)-\frac{22}{3}\log(x+2)+20\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Int[(2 - 7*x + x^2 - x^3 + x^4)/(-24 - 14*x + x^2 + x^3),x]
```

```
[Out] -2*x + x^2/2 + (13*Log[4 - x])/3 - (22*Log[2 + x])/3 + 20*Log[3 + x]
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\begin{aligned}\int \frac{2-7x+x^2-x^3+x^4}{-24-14x+x^2+x^3} \, dx &= \int \left(-2+\frac{13}{3(-4+x)}+x-\frac{22}{3(2+x)}+\frac{20}{3+x}\right) dx\\ &= -2x+\frac{x^2}{2}+\frac{13}{3}\log(4-x)-\frac{22}{3}\log(2+x)+20\log(3+x)\end{aligned}$$

Mathematica [A] time = 0.0097404, size = 35, normalized size = 1.

$$\frac{x^2}{2}-2x+\frac{13}{3}\log(4-x)-\frac{22}{3}\log(x+2)+20\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - 7*x + x^2 - x^3 + x^4)/(-24 - 14*x + x^2 + x^3),x]
```

```
[Out] -2*x + x^2/2 + (13*Log[4 - x])/3 - (22*Log[2 + x])/3 + 20*Log[3 + x]
```

Maple [A] time = 0.008, size = 28, normalized size = 0.8

$$-2x+\frac{x^2}{2}-\frac{22\ln(2+x)}{3}+\frac{13\ln(x-4)}{3}+20\ln(3+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-x^3+x^2-7*x+2)/(x^3+x^2-14*x-24),x)
```

```
[Out] -2*x+1/2*x^2-22/3*ln(2+x)+13/3*ln(x-4)+20*ln(3+x)
```

Maxima [A] time = 0.962659, size = 36, normalized size = 1.03

$$\frac{1}{2}x^2-2x+20\log(x+3)-\frac{22}{3}\log(x+2)+\frac{13}{3}\log(x-4)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^3+x^2-7*x+2)/(x^3+x^2-14*x-24),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - 2*x + 20*log(x + 3) - 22/3*log(x + 2) + 13/3*log(x - 4)
```

Fricas [A] time = 1.53091, size = 90, normalized size = 2.57

$$\frac{1}{2}x^2-2x+20\log(x+3)-\frac{22}{3}\log(x+2)+\frac{13}{3}\log(x-4)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^3+x^2-7*x+2)/(x^3+x^2-14*x-24),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 - 2*x + 20*log(x + 3) - 22/3*log(x + 2) + 13/3*log(x - 4)
```

Sympy [A] time = 0.132018, size = 31, normalized size = 0.89

$$\frac{x^2}{2} - 2x + \frac{13\log(x-4)}{3} - \frac{22\log(x+2)}{3} + 20\log(x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-x**3+x**2-7*x+2)/(x**3+x**2-14*x-24),x)
```

```
[Out] x**2/2 - 2*x + 13*log(x - 4)/3 - 22*log(x + 2)/3 + 20*log(x + 3)
```

Giac [A] time = 1.12382, size = 41, normalized size = 1.17

$$\frac{1}{2}x^2 - 2x + 20\log(|x+3|) - \frac{22}{3}\log(|x+2|) + \frac{13}{3}\log(|x-4|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^3+x^2-7*x+2)/(x^3+x^2-14*x-24),x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 2*x + 20*log(abs(x + 3)) - 22/3*log(abs(x + 2)) + 13/3*log(abs(x
- 4))
```

3.296 $\int \frac{2+x^2}{(-1+x)^2 x(1+x)} dx$

Optimal. Leaf size=34

$$\frac{3}{2(1-x)} - \frac{5}{4}\log(1-x) + 2\log(x) - \frac{3}{4}\log(x+1)$$

```
[Out] 3/(2*(1 - x)) - (5*Log[1 - x])/4 + 2*Log[x] - (3*Log[1 + x])/4
```

Rubi [A] time = 0.0574547, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1612}

$$\frac{3}{2(1-x)} - \frac{5}{4}\log(1-x) + 2\log(x) - \frac{3}{4}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x^2)/((-1 + x)^2*x*(1 + x)),x]
```

```
[Out] 3/(2*(1 - x)) - (5*Log[1 - x])/4 + 2*Log[x] - (3*Log[1 + x])/4
```

Rule 1612

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^
n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px,
 x] && IntegersQ[m, n]
```

Rubi steps

$$\begin{aligned}\int \frac{2+x^2}{(-1+x)^2 x(1+x)} dx &= \int \left(\frac{3}{2(-1+x)^2} - \frac{5}{4(-1+x)} + \frac{2}{x} - \frac{3}{4(1+x)}\right) dx \\ &= \frac{3}{2(1-x)} - \frac{5}{4}\log(1-x) + 2\log(x) - \frac{3}{4}\log(1+x)\end{aligned}$$

Mathematica [A] time = 0.0166073, size = 32, normalized size = 0.94

$$-\frac{3}{2(x-1)} - \frac{5}{4}\log(1-x) + 2\log(x) - \frac{3}{4}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x^2)/((-1 + x)^2*x*(1 + x)),x]
```

```
[Out] -3/(2*(-1 + x)) - (5*Log[1 - x])/4 + 2*Log[x] - (3*Log[1 + x])/4
```

Maple [A] time = 0.007, size = 25, normalized size = 0.7

$$-\frac{3}{2x-2}-\frac{5\ln(x-1)}{4}+2\ln(x)-\frac{3\ln(1+x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+2)/(x-1)^2/x/(1+x),x)
```

```
[Out] -3/2/(x-1)-5/4*ln(x-1)+2*ln(x)-3/4*ln(1+x)
```

Maxima [A] time = 0.984737, size = 32, normalized size = 0.94

$$-\frac{3}{2(x-1)}-\frac{3}{4}\log(x+1)-\frac{5}{4}\log(x-1)+2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2)/(-1+x)^2/x/(1+x),x, algorithm="maxima")
```

```
[Out] -3/2/(x - 1) - 3/4*log(x + 1) - 5/4*log(x - 1) + 2*log(x)
```

Fricas [A] time = 1.53738, size = 112, normalized size = 3.29

$$-\frac{3(x-1)\log(x+1)+5(x-1)\log(x-1)-8(x-1)\log(x)+6}{4(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2)/(-1+x)^2/x/(1+x),x, algorithm="fricas")
```

```
[Out] -1/4*(3*(x - 1)*log(x + 1) + 5*(x - 1)*log(x - 1) - 8*(x - 1)*log(x) + 6)/(
x - 1)
```

Sympy [A] time = 0.133397, size = 27, normalized size = 0.79

$$2\log(x) - \frac{5\log(x-1)}{4} - \frac{3\log(x+1)}{4} - \frac{3}{2x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+2)/(-1+x)**2/x/(1+x),x)
```

```
[Out] 2*log(x) - 5*log(x - 1)/4 - 3*log(x + 1)/4 - 3/(2*x - 2)
```

Giac [A] time = 1.15256, size = 46, normalized size = 1.35

$$-\frac{3}{2(x-1)} + 2\log\left(\left|-\frac{1}{x-1} - 1\right|\right) - \frac{3}{4}\log\left(\left|-\frac{2}{x-1} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2)/(-1+x)^2/x/(1+x),x, algorithm="giac")
```

```
[Out] -3/2/(x - 1) + 2*log(abs(-1/(x - 1) - 1)) - 3/4*log(abs(-2/(x - 1) - 1))
```

3.297 $\int \frac{3+x^2+x^3}{(2+x^2)^2} dx$

Optimal. Leaf size=42

$$\frac{x+4}{4(x^2+2)}+\frac{1}{2}\log\left(x^2+2\right)+\frac{5\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{4\sqrt{2}}$$

```
[Out] (4 + x)/(4*(2 + x^2)) + (5*ArcTan[x/Sqrt[2]])/(4*Sqrt[2]) + Log[2 + x^2]/2
```

Rubi [A] time = 0.0209896, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1814, 635, 203, 260}

$$\frac{x+4}{4(x^2+2)}+\frac{1}{2}\log\left(x^2+2\right)+\frac{5\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + x^2 + x^3)/(2 + x^2)^2,x]
```

```
[Out] (4 + x)/(4*(2 + x^2)) + (5*ArcTan[x/Sqrt[2]])/(4*Sqrt[2]) + Log[2 + x^2]/2
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{3+x^2+x^3}{\left(2+x^2\right)^2}\,dx &= \frac{4+x}{4\left(2+x^2\right)} - \frac{1}{4}\int \frac{-5-4x}{2+x^2}\,dx \\
&= \frac{4+x}{4\left(2+x^2\right)} + \frac{5}{4}\int \frac{1}{2+x^2}\,dx + \int \frac{x}{2+x^2}\,dx \\
&= \frac{4+x}{4\left(2+x^2\right)} + \frac{5\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{4\sqrt{2}} + \frac{1}{2}\log\left(2+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.0360976, size = 42, normalized size = 1.

$$\frac{x+4}{4\left(x^2+2\right)} + \frac{1}{2}\log\left(x^2+2\right) + \frac{5\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + x^2 + x^3)/(2 + x^2)^2,x]
```

```
[Out] (4 + x)/(4*(2 + x^2)) + (5*ArcTan[x/Sqrt[2]])/(4*Sqrt[2]) + Log[2 + x^2]/2
```

Maple [A] time = 0.007, size = 35, normalized size = 0.8

$$\frac{1}{x^2+2}\left(\frac{x}{4}+1\right) + \frac{\ln\left(x^2+2\right)}{2} + \frac{5\sqrt{2}}{8}\arctan\left(\frac{x\sqrt{2}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x^2+3)/(x^2+2)^2,x)
```

```
[Out] (1/4*x+1)/(x^2+2)+1/2*ln(x^2+2)+5/8*arctan(1/2*x*2^(1/2))*2^(1/2)
```

Maxima [A] time = 1.46403, size = 45, normalized size = 1.07

$$\frac{5}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+\frac{x+4}{4\left(x^2+2\right)}+\frac{1}{2}\log\left(x^2+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+3)/(x^2+2)^2,x, algorithm="maxima")
```

```
[Out] 5/8*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/4*(x + 4)/(x^2 + 2) + 1/2*log(x^2 + 2
)
```

Fricas [A] time = 1.47632, size = 130, normalized size = 3.1

$$\frac{5\sqrt{2}\left(x^2+2\right)\arctan\left(\frac{1}{2}\sqrt{2}x\right)+4\left(x^2+2\right)\log\left(x^2+2\right)+2x+8}{8\left(x^2+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+3)/(x^2+2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(5*sqrt(2)*(x^2 + 2)*arctan(1/2*sqrt(2)*x) + 4*(x^2 + 2)*log(x^2 + 2) +
 2*x + 8)/(x^2 + 2)
```

Sympy [A] time = 0.122285, size = 36, normalized size = 0.86

$$\frac{x+4}{4x^2+8}+\frac{\log\left(x^2+2\right)}{2}+\frac{5\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x**2+3)/(x**2+2)**2,x)
```

```
[Out] (x + 4)/(4*x**2 + 8) + log(x**2 + 2)/2 + 5*sqrt(2)*atan(sqrt(2)*x/2)/8
```

Giac [A] time = 1.1487, size = 45, normalized size = 1.07

$$\frac{5}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+\frac{x+4}{4\left(x^2+2\right)}+\frac{1}{2}\log\left(x^2+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+3)/(x^2+2)^2,x, algorithm="giac")
```

```
[Out] 5/8*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/4*(x + 4)/(x^2 + 2) + 1/2*log(x^2 + 2
)
```

3.298 $\int \frac{-35+70x-4x^2+2x^3}{(26-10x+x^2)(17-2x+x^2)}\,dx$

Optimal. Leaf size=49

$$\frac{1003\log\left(x^2-10x+26\right)}{1025}+\frac{22\log\left(x^2-2x+17\right)}{1025}-\frac{15033\tan^{-1}(5-x)}{1025}-\frac{4607\tan^{-1}\left(\frac{x-1}{4}\right)}{4100}$$

```
[Out] (-15033*ArcTan[5 - x])/1025 - (4607*ArcTan[(-1 + x)/4])/4100 + (1003*Log[26
 - 10*x + x^2])/1025 + (22*Log[17 - 2*x + x^2])/1025
```

Rubi [A] time = 0.156707, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {6728, 634, 618, 204, 628}

$$\frac{1003\log\left(x^2-10x+26\right)}{1025}+\frac{22\log\left(x^2-2x+17\right)}{1025}-\frac{15033\tan^{-1}(5-x)}{1025}-\frac{4607\tan^{-1}\left(\frac{x-1}{4}\right)}{4100}$$

Antiderivative was successfully verified.

```
[In] Int[(-35 + 70*x - 4*x^2 + 2*x^3)/((26 - 10*x + x^2)*(17 - 2*x + x^2)),x]
```

```
[Out] (-15033*ArcTan[5 - x])/1025 - (4607*ArcTan[(-1 + x)/4])/4100 + (1003*Log[26
 - 10*x + x^2])/1025 + (22*Log[17 - 2*x + x^2])/1025
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{-35+70x-4x^2+2x^3}{\left(26-10x+x^2\right)\left(17-2x+x^2\right)}\,dx = \int \left(\frac{5003+2006x}{1025\left(26-10x+x^2\right)}+\frac{-4651+44x}{1025\left(17-2x+x^2\right)}\right)dx$$
$$= \frac{\int \frac{5003+2006x}{26-10x+x^2}\,dx}{1025}+\frac{\int \frac{-4651+44x}{17-2x+x^2}\,dx}{1025}$$
$$= \frac{22\int \frac{-2+2x}{17-2x+x^2}\,dx}{1025}+\frac{1003\int \frac{-10+2x}{26-10x+x^2}\,dx}{1025}-\frac{4607\int \frac{1}{17-2x+x^2}\,dx}{1025}+\frac{15033\int \frac{1}{26-10x+x}}{1025}$$
$$= \frac{1003\log\left(26-10x+x^2\right)}{1025}+\frac{22\log\left(17-2x+x^2\right)}{1025}+\frac{9214\,\text{Subst}\left(\int \frac{1}{-64-x^2}\,dx,x,\right.}{1025}$$
$$= -\frac{15033\tan^{-1}(5-x)}{1025}-\frac{4607\tan^{-1}\left(\frac{1}{4}(-1+x)\right)}{4100}+\frac{1003\log\left(26-10x+x^2\right)}{1025}+\frac{22}{}$$

Mathematica [A] time = 0.0135432, size = 49, normalized size = 1.

$$\frac{1003\log\left(x^2-10x+26\right)}{1025}+\frac{22\log\left(x^2-2x+17\right)}{1025}-\frac{15033\tan^{-1}(5-x)}{1025}-\frac{4607\tan^{-1}\left(\frac{x-1}{4}\right)}{4100}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-35 + 70*x - 4*x^2 + 2*x^3)/((26 - 10*x + x^2)*(17 - 2*x + x^2))
,x]
```

```
[Out] (-15033*ArcTan[5 - x])/1025 - (4607*ArcTan[(-1 + x)/4])/4100 + (1003*Log[26
 - 10*x + x^2])/1025 + (22*Log[17 - 2*x + x^2])/1025
```

Maple [A] time = 0.007, size = 38, normalized size = 0.8

$$\frac{15033\arctan\left(-5+x\right)}{1025}-\frac{4607}{4100}\arctan\left(\frac{x}{4}-\frac{1}{4}\right)+\frac{1003\ln\left(x^{2}-10x+26\right)}{1025}+\frac{22\ln\left(x^{2}-2x+17\right)}{1025}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3-4*x^2+70*x-35)/(x^2-10*x+26)/(x^2-2*x+17),x)
```

```
[Out] 15033/1025*arctan(-5+x)-4607/4100*arctan(1/4*x-1/4)+1003/1025*ln(x^2-10*x+2
6)+22/1025*ln(x^2-2*x+17)
```

Maxima [A] time = 1.4725, size = 50, normalized size = 1.02

$$\frac{15033}{1025}\arctan\left(x-5\right)-\frac{4607}{4100}\arctan\left(\frac{1}{4}x-\frac{1}{4}\right)+\frac{22}{1025}\log\left(x^{2}-2x+17\right)+\frac{1003}{1025}\log\left(x^{2}-10x+26\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3-4*x^2+70*x-35)/(x^2-10*x+26)/(x^2-2*x+17),x, algorithm="ma
xima")
```

```
[Out] 15033/1025*arctan(x - 5) - 4607/4100*arctan(1/4*x - 1/4) + 22/1025*log(x^2
- 2*x + 17) + 1003/1025*log(x^2 - 10*x + 26)
```

Fricas [A] time = 1.67226, size = 163, normalized size = 3.33

$$\frac{15033}{1025}\arctan\left(x-5\right)-\frac{4607}{4100}\arctan\left(\frac{1}{4}x-\frac{1}{4}\right)+\frac{22}{1025}\log\left(x^{2}-2x+17\right)+\frac{1003}{1025}\log\left(x^{2}-10x+26\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3-4*x^2+70*x-35)/(x^2-10*x+26)/(x^2-2*x+17),x, algorithm="fr
icas")
```

```
[Out] 15033/1025*arctan(x - 5) - 4607/4100*arctan(1/4*x - 1/4) + 22/1025*log(x^2
- 2*x + 17) + 1003/1025*log(x^2 - 10*x + 26)
```

Sympy [A] time = 0.195851, size = 46, normalized size = 0.94

$$\frac{1003\log\left(x^2-10x+26\right)}{1025}+\frac{22\log\left(x^2-2x+17\right)}{1025}-\frac{4607\,\mathrm{atan}\left(\frac{x}{4}-\frac{1}{4}\right)}{4100}+\frac{15033\,\mathrm{atan}\left(x-5\right)}{1025}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**3-4*x**2+70*x-35)/(x**2-10*x+26)/(x**2-2*x+17),x)
```

```
[Out] 1003*log(x**2 - 10*x + 26)/1025 + 22*log(x**2 - 2*x + 17)/1025 - 4607*atan(
x/4 - 1/4)/4100 + 15033*atan(x - 5)/1025
```

Giac [A] time = 1.12058, size = 50, normalized size = 1.02

$$\frac{15033}{1025}\arctan\left(x-5\right)-\frac{4607}{4100}\arctan\left(\frac{1}{4}x-\frac{1}{4}\right)+\frac{22}{1025}\log\left(x^2-2x+17\right)+\frac{1003}{1025}\log\left(x^2-10x+26\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3-4*x^2+70*x-35)/(x^2-10*x+26)/(x^2-2*x+17),x, algorithm="gi
ac")
```

```
[Out] 15033/1025*arctan(x - 5) - 4607/4100*arctan(1/4*x - 1/4) + 22/1025*log(x^2
- 2*x + 17) + 1003/1025*log(x^2 - 10*x + 26)
```

3.299 $\int \frac{2+x^2}{(-5+x)(-3+x)(4+x)}\,dx$

Optimal. Leaf size=29

$$-\frac{11}{14}\log(3-x)+\frac{3}{2}\log(5-x)+\frac{2}{7}\log(x+4)$$

```
[Out] (-11*Log[3 - x])/14 + (3*Log[5 - x])/2 + (2*Log[4 + x])/7
```

Rubi [A] time = 0.0528289, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {1612}

$$-\frac{11}{14}\log(3-x)+\frac{3}{2}\log(5-x)+\frac{2}{7}\log(x+4)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x^2)/((-5 + x)*(-3 + x)*(4 + x)),x]
```

```
[Out] (-11*Log[3 - x])/14 + (3*Log[5 - x])/2 + (2*Log[4 + x])/7
```

Rule 1612

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^
n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px,
 x] && IntegersQ[m, n]
```

Rubi steps

$$\begin{aligned}\int \frac{2+x^2}{(-5+x)(-3+x)(4+x)}\,dx &= \int \left(\frac{3}{2(-5+x)}-\frac{11}{14(-3+x)}+\frac{2}{7(4+x)}\right)dx\\ &= -\frac{11}{14}\log(3-x)+\frac{3}{2}\log(5-x)+\frac{2}{7}\log(4+x)\end{aligned}$$

Mathematica [A] time = 0.0074461, size = 29, normalized size = 1.

$$-\frac{11}{14}\log(3-x)+\frac{3}{2}\log(5-x)+\frac{2}{7}\log(x+4)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x^2)/((-5 + x)*(-3 + x)*(4 + x)),x]
```

```
[Out] (-11*Log[3 - x])/14 + (3*Log[5 - x])/2 + (2*Log[4 + x])/7
```

Maple [A] time = 0.009, size = 20, normalized size = 0.7

$$-\frac{11\ln(-3+x)}{14}+\frac{2\ln(4+x)}{7}+\frac{3\ln(-5+x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+2)/(-5+x)/(-3+x)/(4+x),x)
```

```
[Out] -11/14*ln(-3+x)+2/7*ln(4+x)+3/2*ln(-5+x)
```

Maxima [A] time = 0.971552, size = 26, normalized size = 0.9

$$\frac{2}{7}\log(x+4)-\frac{11}{14}\log(x-3)+\frac{3}{2}\log(x-5)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2)/(-5+x)/(-3+x)/(4+x),x, algorithm="maxima")
```

```
[Out] 2/7*log(x + 4) - 11/14*log(x - 3) + 3/2*log(x - 5)
```

Fricas [A] time = 1.5611, size = 70, normalized size = 2.41

$$\frac{2}{7}\log(x+4)-\frac{11}{14}\log(x-3)+\frac{3}{2}\log(x-5)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2)/(-5+x)/(-3+x)/(4+x),x, algorithm="fricas")
```

```
[Out] 2/7*log(x + 4) - 11/14*log(x - 3) + 3/2*log(x - 5)
```

Sympy [A] time = 0.130081, size = 24, normalized size = 0.83

$$\frac{3\log(x-5)}{2} - \frac{11\log(x-3)}{14} + \frac{2\log(x+4)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+2)/(-5+x)/(-3+x)/(4+x),x)
```

```
[Out] 3*log(x - 5)/2 - 11*log(x - 3)/14 + 2*log(x + 4)/7
```

Giac [A] time = 1.15394, size = 30, normalized size = 1.03

$$\frac{2}{7}\log(|x+4|) - \frac{11}{14}\log(|x-3|) + \frac{3}{2}\log(|x-5|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2)/(-5+x)/(-3+x)/(4+x),x, algorithm="giac")
```

```
[Out] 2/7*log(abs(x + 4)) - 11/14*log(abs(x - 3)) + 3/2*log(abs(x - 5))
```

3.300 $\int \frac{x^4}{(-1+x)(2+x^2)} dx$

Optimal. Leaf size=46

$$\frac{x^2}{2} - \frac{2}{3}\log\left(x^2+2\right) + x + \frac{1}{3}\log(1-x) - \frac{2}{3}\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

```
[Out] x + x^2/2 - (2*Sqrt[2]*ArcTan[x/Sqrt[2]])/3 + Log[1 - x]/3 - (2*Log[2 + x^2
])/3
```

Rubi [A] time = 0.0444875, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1629, 635, 203, 260}

$$\frac{x^2}{2} - \frac{2}{3}\log\left(x^2+2\right) + x + \frac{1}{3}\log(1-x) - \frac{2}{3}\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((-1 + x)*(2 + x^2)),x]
```

```
[Out] x + x^2/2 - (2*Sqrt[2]*ArcTan[x/Sqrt[2]])/3 + Log[1 - x]/3 - (2*Log[2 + x^2
])/3
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{(-1+x)\left(2+x^2\right)}\,dx &= \int \left(1+\frac{1}{3(-1+x)}+x-\frac{4(1+x)}{3\left(2+x^2\right)}\right)dx\\ &= x+\frac{x^2}{2}+\frac{1}{3}\log(1-x)-\frac{4}{3}\int \frac{1+x}{2+x^2}\,dx\\ &= x+\frac{x^2}{2}+\frac{1}{3}\log(1-x)-\frac{4}{3}\int \frac{1}{2+x^2}\,dx-\frac{4}{3}\int \frac{x}{2+x^2}\,dx\\ &= x+\frac{x^2}{2}-\frac{2}{3}\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)+\frac{1}{3}\log(1-x)-\frac{2}{3}\log\left(2+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.0164217, size = 43, normalized size = 0.93

$$\frac{1}{6}\left(3x^2-4\log\left(x^2+2\right)+6x+2\log(x-1)-4\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)-9\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((-1 + x)*(2 + x^2)),x]
```

```
[Out] (-9 + 6*x + 3*x^2 - 4*Sqrt[2]*ArcTan[x/Sqrt[2]] + 2*Log[-1 + x] - 4*Log[2 +
 x^2])/6
```

Maple [A] time = 0.004, size = 34, normalized size = 0.7

$$\frac{x^2}{2}+x+\frac{\ln(x-1)}{3}-\frac{2\ln\left(x^2+2\right)}{3}-\frac{2\sqrt{2}}{3}\arctan\left(\frac{x\sqrt{2}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x-1)/(x^2+2),x)
```

```
[Out] 1/2*x^2+x+1/3*ln(x-1)-2/3*ln(x^2+2)-2/3*arctan(1/2*x*2^(1/2))*2^(1/2)
```

Maxima [A] time = 1.47715, size = 45, normalized size = 0.98

$$\frac{1}{2}x^2 - \frac{2}{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) + x - \frac{2}{3}\log\left(x^2+2\right) + \frac{1}{3}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-1+x)/(x^2+2),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - 2/3*sqrt(2)*arctan(1/2*sqrt(2)*x) + x - 2/3*log(x^2 + 2) + 1/3*lo
g(x - 1)
```

Fricas [A] time = 1.52902, size = 115, normalized size = 2.5

$$\frac{1}{2}x^2 - \frac{2}{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) + x - \frac{2}{3}\log\left(x^2+2\right) + \frac{1}{3}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-1+x)/(x^2+2),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 - 2/3*sqrt(2)*arctan(1/2*sqrt(2)*x) + x - 2/3*log(x^2 + 2) + 1/3*lo
g(x - 1)
```

Sympy [A] time = 0.127096, size = 41, normalized size = 0.89

$$\frac{x^2}{2} + x + \frac{\log(x-1)}{3} - \frac{2\log\left(x^2+2\right)}{3} - \frac{2\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-1+x)/(x**2+2),x)
```

```
[Out] x**2/2 + x + log(x - 1)/3 - 2*log(x**2 + 2)/3 - 2*sqrt(2)*atan(sqrt(2)*x/2)
/3
```

Giac [A] time = 1.2218, size = 46, normalized size = 1.

$$\frac{1}{2}x^2 - \frac{2}{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) + x - \frac{2}{3}\log\left(x^2+2\right) + \frac{1}{3}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-1+x)/(x^2+2),x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 2/3*sqrt(2)*arctan(1/2*sqrt(2)*x) + x - 2/3*log(x^2 + 2) + 1/3*lo
g(abs(x - 1))
```

3.301 $\int \frac{-1+7x+2x^2}{-1-x+x^2+x^3} \, dx$

Optimal. Leaf size=16

$$2\log(1-x) - \frac{3}{x+1}$$

```
[Out] -3/(1 + x) + 2*Log[1 - x]
```

Rubi [A] time = 0.0249667, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {2074}

$$2\log(1-x) - \frac{3}{x+1}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 7*x + 2*x^2)/(-1 - x + x^2 + x^3),x]
```

```
[Out] -3/(1 + x) + 2*Log[1 - x]
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+7x+2x^2}{-1-x+x^2+x^3} \, dx &= \int \left(\frac{2}{-1+x} + \frac{3}{(1+x)^2}\right) dx\\ &= -\frac{3}{1+x} + 2\log(1-x)\end{aligned}$$

Mathematica [A] time = 0.0081222, size = 14, normalized size = 0.88

$$2\log(x-1) - \frac{3}{x+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + 7*x + 2*x^2)/(-1 - x + x^2 + x^3),x]
```

```
[Out] -3/(1 + x) + 2*Log[-1 + x]
```

Maple [A] time = 0.007, size = 15, normalized size = 0.9

$$2\ln(x-1)-3(1+x)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+7*x-1)/(x^3+x^2-x-1),x)
```

```
[Out] 2*ln(x-1)-3/(1+x)
```

Maxima [A] time = 0.969146, size = 19, normalized size = 1.19

$$-\frac{3}{x+1}+2\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+7*x-1)/(x^3+x^2-x-1),x, algorithm="maxima")
```

```
[Out] -3/(x + 1) + 2*log(x - 1)
```

Fricas [A] time = 1.54451, size = 49, normalized size = 3.06

$$\frac{2(x+1)\log(x-1)-3}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+7*x-1)/(x^3+x^2-x-1),x, algorithm="fricas")
```

```
[Out] (2*(x + 1)*log(x - 1) - 3)/(x + 1)
```

Sympy [A] time = 0.08878, size = 10, normalized size = 0.62

$$2\log(x-1)-\frac{3}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+7*x-1)/(x**3+x**2-x-1),x)
```

```
[Out] 2*log(x - 1) - 3/(x + 1)
```

Giac [A] time = 1.12265, size = 20, normalized size = 1.25

$$-\frac{3}{x+1}+2\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+7*x-1)/(x^3+x^2-x-1),x, algorithm="giac")
```

```
[Out] -3/(x + 1) + 2*log(abs(x - 1))
```

3.302 $\int \frac{1+2x}{-1+3x-3x^2+x^3}\,dx$

Optimal. Leaf size=21

$$\frac{2}{1-x}-\frac{3}{2(1-x)^2}$$

```
[Out] -3/(2*(1 - x)^2) + 2/(1 - x)
```

Rubi [A] time = 0.0192289, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2074}

$$\frac{2}{1-x}-\frac{3}{2(1-x)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)/(-1 + 3*x - 3*x^2 + x^3),x]
```

```
[Out] -3/(2*(1 - x)^2) + 2/(1 - x)
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1+2x}{-1+3x-3x^2+x^3}\,dx &= \int \left(\frac{3}{(-1+x)^3}+\frac{2}{(-1+x)^2}\right)dx\\ &= -\frac{3}{2(1-x)^2}+\frac{2}{1-x}\end{aligned}$$

Mathematica [A] time = 0.0033034, size = 14, normalized size = 0.67

$$\frac{1-4x}{2(x-1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)/(-1 + 3*x - 3*x^2 + x^3),x]
```

```
[Out] (1 - 4*x)/(2*(-1 + x)^2)
```

Maple [A] time = 0.004, size = 16, normalized size = 0.8

$$-2\,(x-1)^{-1}-\frac{3}{2\,(x-1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)/(x^3-3*x^2+3*x-1),x)
```

```
[Out] -2/(x-1)-3/2/(x-1)^2
```

Maxima [A] time = 0.998447, size = 23, normalized size = 1.1

$$-\frac{4\,x-1}{2\left(x^2-2\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x^3-3*x^2+3*x-1),x, algorithm="maxima")
```

```
[Out] -1/2*(4*x - 1)/(x^2 - 2*x + 1)
```

Fricas [A] time = 1.45897, size = 43, normalized size = 2.05

$$-\frac{4\,x-1}{2\left(x^2-2\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x^3-3*x^2+3*x-1),x, algorithm="fricas")
```

```
[Out] -1/2*(4*x - 1)/(x^2 - 2*x + 1)
```

Sympy [A] time = 0.084257, size = 15, normalized size = 0.71

$$-\frac{4x-1}{2x^2-4x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x**3-3*x**2+3*x-1),x)
```

```
[Out] -(4*x - 1)/(2*x**2 - 4*x + 2)
```

Giac [A] time = 1.13127, size = 16, normalized size = 0.76

$$-\frac{4\,x-1}{2\,(x-1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x^3-3*x^2+3*x-1),x, algorithm="giac")
```

```
[Out] -1/2*(4*x - 1)/(x - 1)^2
```

3.303 $\int \frac{5-5x+7x^2+x^3}{(-1+x)^2(1+x)^3} \, dx$

Optimal. Leaf size=15

$$\frac{1}{1-x}-\frac{2}{(x+1)^2}$$

```
[Out] (1 - x)^(-1) - 2/(1 + x)^2
```

Rubi [A] time = 0.0270943, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {1620}

$$\frac{1}{1-x}-\frac{2}{(x+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(5 - 5*x + 7*x^2 + x^3)/((-1 + x)^2*(1 + x)^3),x]
```

```
[Out] (1 - x)^(-1) - 2/(1 + x)^2
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\begin{aligned}\int \frac{5-5x+7x^2+x^3}{(-1+x)^2(1+x)^3} \, dx &= \int \left(\frac{1}{(-1+x)^2}+\frac{4}{(1+x)^3}\right) \, dx\\ &= \frac{1}{1-x}-\frac{2}{(1+x)^2}\end{aligned}$$

Mathematica [A] time = 0.011732, size = 15, normalized size = 1.

$$-\frac{2}{(x+1)^2}-\frac{1}{x-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 - 5*x + 7*x^2 + x^3)/((-1 + x)^2*(1 + x)^3),x]
```

```
[Out] -(-1 + x)^(-1) - 2/(1 + x)^2
```

Maple [A] time = 0.004, size = 16, normalized size = 1.1

$$-(x-1)^{-1} - 2(1+x)^{-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+7*x^2-5*x+5)/(x-1)^2/(1+x)^3,x)
```

```
[Out] -1/(x-1)-2/(1+x)^2
```

Maxima [A] time = 0.970745, size = 31, normalized size = 2.07

$$-\frac{x^2+4x-1}{x^3+x^2-x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+7*x^2-5*x+5)/(-1+x)^2/(1+x)^3,x, algorithm="maxima")
```

```
[Out] -(x^2 + 4*x - 1)/(x^3 + x^2 - x - 1)
```

Fricas [A] time = 1.47772, size = 51, normalized size = 3.4

$$-\frac{x^2+4x-1}{x^3+x^2-x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+7*x^2-5*x+5)/(-1+x)^2/(1+x)^3,x, algorithm="fricas")
```

```
[Out] -(x^2 + 4*x - 1)/(x^3 + x^2 - x - 1)
```

Sympy [A] time = 0.110862, size = 19, normalized size = 1.27

$$-\frac{x^2+4x-1}{x^3+x^2-x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+7*x**2-5*x+5)/(-1+x)**2/(1+x)**3,x)
```

```
[Out] -(x**2 + 4*x - 1)/(x**3 + x**2 - x - 1)
```

Giac [B] time = 1.11392, size = 41, normalized size = 2.73

$$-\frac{1}{x-1}+\frac{\frac{4}{x-1}+1}{2\left(\frac{2}{x-1}+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+7*x^2-5*x+5)/(-1+x)^2/(1+x)^3,x, algorithm="giac")
```

```
[Out] -1/(x - 1) + 1/2*(4/(x - 1) + 1)/(2/(x - 1) + 1)^2
```

3.304 $\int \frac{1+3x+3x^2}{1+2x+2x^2+x^3} dx$

Optimal. Leaf size=31

$$\log\left(x^2+x+1\right)+\log(x+1)-\frac{2\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] (-2*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] + Log[1 + x] + Log[1 + x + x^2]
```

Rubi [A] time = 0.0421315, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2074, 634, 618, 204, 628}

$$\log\left(x^2+x+1\right)+\log(x+1)-\frac{2\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 3*x + 3*x^2)/(1 + 2*x + 2*x^2 + x^3),x]
```

```
[Out] (-2*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] + Log[1 + x] + Log[1 + x + x^2]
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+3x+3x^2}{1+2x+2x^2+x^3}\,dx &= \int \left(\frac{1}{1+x}+\frac{2x}{1+x+x^2}\right)dx\\
&= \log(1+x)+2\int \frac{x}{1+x+x^2}\,dx\\
&= \log(1+x)-\int \frac{1}{1+x+x^2}\,dx+\int \frac{1+2x}{1+x+x^2}\,dx\\
&= \log(1+x)+\log\left(1+x+x^2\right)+2\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2x\right)\\
&= -\frac{2\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\log(1+x)+\log\left(1+x+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.0120442, size = 31, normalized size = 1.

$$\log\left(x^2+x+1\right)+\log(x+1)-\frac{2\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 3*x + 3*x^2)/(1 + 2*x + 2*x^2 + x^3),x]

[Out] (-2*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] + Log[1 + x] + Log[1 + x + x^2]
```

Maple [A] time = 0.005, size = 29, normalized size = 0.9

$$\ln(1+x)+\ln\left(x^2+x+1\right)-\frac{2\sqrt{3}}{3}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+3*x+1)/(x^3+2*x^2+2*x+1),x)
```

```
[Out] ln(1+x)+ln(x^2+x+1)-2/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Maxima [A] time = 1.46966, size = 38, normalized size = 1.23

$$-\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\log\left(x^2+x+1\right)+\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+3*x+1)/(x^3+2*x^2+2*x+1),x, algorithm="maxima")
```

```
[Out] -2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + log(x^2 + x + 1) + log(x + 1)
```

Fricas [A] time = 1.53569, size = 103, normalized size = 3.32

$$-\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\log\left(x^2+x+1\right)+\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+3*x+1)/(x^3+2*x^2+2*x+1),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + log(x^2 + x + 1) + log(x + 1)
```

Sympy [A] time = 0.115544, size = 3, normalized size = 0.1

$$\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+3*x+1)/(x**3+2*x**2+2*x+1),x)
```

```
[Out] log(x + 1)
```

Giac [A] time = 1.14224, size = 39, normalized size = 1.26

$$-\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\log\left(x^2+x+1\right)+\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+3*x+1)/(x^3+2*x^2+2*x+1),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + log(x^2 + x + 1) + log(abs(x +
 1))
```

3.305 $\int \frac{-1+2x+x^2}{-2x+3x^2+2x^3} \, dx$

Optimal. Leaf size=25

$$\frac{1}{10}\log(1-2x)+\frac{\log(x)}{2}-\frac{1}{10}\log(x+2)$$

```
[Out] Log[1 - 2*x]/10 + Log[x]/2 - Log[2 + x]/10
```

Rubi [A] time = 0.0417384, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {1594, 1628}

$$\frac{1}{10}\log(1-2x)+\frac{\log(x)}{2}-\frac{1}{10}\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 2*x + x^2)/(-2*x + 3*x^2 + 2*x^3),x]
```

```
[Out] Log[1 - 2*x]/10 + Log[x]/2 - Log[2 + x]/10
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{-1+2x+x^2}{-2x+3x^2+2x^3}\,dx = \int \frac{-1+2x+x^2}{x\left(-2+3x+2x^2\right)}\,dx$$
$$= \int \left(\frac{1}{2x} - \frac{1}{10(2+x)} + \frac{1}{5(-1+2x)}\right) dx$$
$$= \frac{1}{10}\log(1-2x) + \frac{\log(x)}{2} - \frac{1}{10}\log(2+x)$$

Mathematica [A] time = 0.0065663, size = 25, normalized size = 1.

$$\frac{1}{10}\log(1-2x) + \frac{\log(x)}{2} - \frac{1}{10}\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + 2*x + x^2)/(-2*x + 3*x^2 + 2*x^3),x]
```

```
[Out] Log[1 - 2*x]/10 + Log[x]/2 - Log[2 + x]/10
```

Maple [A] time = 0.007, size = 20, normalized size = 0.8

$$\frac{\ln(x)}{2} + \frac{\ln(2x-1)}{10} - \frac{\ln(2+x)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+2*x-1)/(2*x^3+3*x^2-2*x),x)
```

```
[Out] 1/2*ln(x)+1/10*ln(2*x-1)-1/10*ln(2+x)
```

Maxima [A] time = 0.984794, size = 26, normalized size = 1.04

$$\frac{1}{10}\log(2x-1) - \frac{1}{10}\log(x+2) + \frac{1}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x-1)/(2*x^3+3*x^2-2*x),x, algorithm="maxima")
```

```
[Out] 1/10*log(2*x - 1) - 1/10*log(x + 2) + 1/2*log(x)
```

Fricas [A] time = 1.62567, size = 68, normalized size = 2.72

$$\frac{1}{10}\log(2x-1) - \frac{1}{10}\log(x+2) + \frac{1}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x-1)/(2*x^3+3*x^2-2*x),x, algorithm="fricas")
```

```
[Out] 1/10*log(2*x - 1) - 1/10*log(x + 2) + 1/2*log(x)
```

Sympy [A] time = 0.1236, size = 19, normalized size = 0.76

$$\frac{\log(x)}{2} + \frac{\log\left(x-\frac{1}{2}\right)}{10} - \frac{\log(x+2)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+2*x-1)/(2*x**3+3*x**2-2*x),x)
```

```
[Out] log(x)/2 + log(x - 1/2)/10 - log(x + 2)/10
```

Giac [A] time = 1.16249, size = 30, normalized size = 1.2

$$\frac{1}{10}\log(|2x-1|) - \frac{1}{10}\log(|x+2|) + \frac{1}{2}\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x-1)/(2*x^3+3*x^2-2*x),x, algorithm="giac")
```

```
[Out] 1/10*log(abs(2*x - 1)) - 1/10*log(abs(x + 2)) + 1/2*log(abs(x))
```

3.306 $\int \frac{1+4x-2x^2+x^4}{1-x-x^2+x^3} \, dx$

Optimal. Leaf size=30

$$\frac{x^2}{2}+x+\frac{2}{1-x}+\log(1-x)-\log(x+1)$$

```
[Out] 2/(1 - x) + x + x^2/2 + Log[1 - x] - Log[1 + x]
```

Rubi [A] time = 0.0298323, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {2074}

$$\frac{x^2}{2}+x+\frac{2}{1-x}+\log(1-x)-\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 4*x - 2*x^2 + x^4)/(1 - x - x^2 + x^3),x]

[Out] 2/(1 - x) + x + x^2/2 + Log[1 - x] - Log[1 + x]
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1+4x-2x^2+x^4}{1-x-x^2+x^3} \, dx &= \int \left(1+\frac{1}{-1-x}+\frac{2}{(-1+x)^2}+\frac{1}{-1+x}+x\right) \, dx\\ &= \frac{2}{1-x}+x+\frac{x^2}{2}+\log(1-x)-\log(1+x)\end{aligned}$$

Mathematica [A] time = 0.0156203, size = 29, normalized size = 0.97

$$\frac{1}{2}(x+1)^2-\frac{2}{x-1}+\log(1-x)-\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 4*x - 2*x^2 + x^4)/(1 - x - x^2 + x^3),x]
```

```
[Out] -2/(-1 + x) + (1 + x)^2/2 + Log[1 - x] - Log[1 + x]
```

Maple [A] time = 0.007, size = 25, normalized size = 0.8

$$\frac{x^2}{2} + x + \ln(x-1) - 2\,(x-1)^{-1} - \ln(1+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-2*x^2+4*x+1)/(x^3-x^2-x+1),x)
```

```
[Out] 1/2*x^2+x+ln(x-1)-2/(x-1)-ln(1+x)
```

Maxima [A] time = 1.00426, size = 32, normalized size = 1.07

$$\frac{1}{2}\,x^2 + x - \frac{2}{x-1} - \log(x+1) + \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-2*x^2+4*x+1)/(x^3-x^2-x+1),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 + x - 2/(x - 1) - log(x + 1) + log(x - 1)
```

Fricas [A] time = 1.89655, size = 109, normalized size = 3.63

$$\frac{x^3 + x^2 - 2\,(x-1)\log(x+1) + 2\,(x-1)\log(x-1) - 2\,x - 4}{2\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-2*x^2+4*x+1)/(x^3-x^2-x+1),x, algorithm="fricas")
```

```
[Out] 1/2*(x^3 + x^2 - 2*(x - 1)*log(x + 1) + 2*(x - 1)*log(x - 1) - 2*x - 4)/(x
- 1)
```

Sympy [A] time = 0.091519, size = 20, normalized size = 0.67

$$\frac{x^2}{2}+x+\log(x-1)-\log(x+1)-\frac{2}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-2*x**2+4*x+1)/(x**3-x**2-x+1),x)
```

```
[Out] x**2/2 + x + log(x - 1) - log(x + 1) - 2/(x - 1)
```

Giac [A] time = 1.15927, size = 35, normalized size = 1.17

$$\frac{1}{2}x^2+x-\frac{2}{x-1}-\log(|x+1|)+\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-2*x^2+4*x+1)/(x^3-x^2-x+1),x, algorithm="giac")
```

```
[Out] 1/2*x^2 + x - 2/(x - 1) - log(abs(x + 1)) + log(abs(x - 1))
```

3.307 $\int \frac{4-x+2x^2}{4x+x^3} \, dx$

Optimal. Leaf size=23

$$\frac{1}{2}\log\left(x^2+4\right)+\log(x)-\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)$$

```
[Out] -ArcTan[x/2]/2 + Log[x] + Log[4 + x^2]/2
```

Rubi [A] time = 0.0378514, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1593, 1802, 635, 203, 260}

$$\frac{1}{2}\log\left(x^2+4\right)+\log(x)-\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(4 - x + 2*x^2)/(4*x + x^3),x]
```

```
[Out] -ArcTan[x/2]/2 + Log[x] + Log[4 + x^2]/2
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{4-x+2x^2}{4x+x^3}\,dx &= \int \frac{4-x+2x^2}{x\left(4+x^2\right)}\,dx\\
&= \int \left(\frac{1}{x}+\frac{-1+x}{4+x^2}\right)dx\\
&= \log(x)+\int \frac{-1+x}{4+x^2}\,dx\\
&= \log(x)-\int \frac{1}{4+x^2}\,dx+\int \frac{x}{4+x^2}\,dx\\
&= -\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)+\log(x)+\frac{1}{2}\log\left(4+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.0047499, size = 23, normalized size = 1.

$$\frac{1}{2}\log\left(x^2+4\right)+\log(x)-\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 - x + 2*x^2)/(4*x + x^3),x]
```

```
[Out] -ArcTan[x/2]/2 + Log[x] + Log[4 + x^2]/2
```

Maple [A] time = 0.005, size = 18, normalized size = 0.8

$$-\frac{1}{2}\arctan\left(\frac{x}{2}\right)+\ln(x)+\frac{\ln\left(x^2+4\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+4)/(x^3+4*x),x)
```

```
[Out] -1/2*arctan(1/2*x)+ln(x)+1/2*ln(x^2+4)
```

Maxima [A] time = 1.49329, size = 23, normalized size = 1.

$$-\frac{1}{2}\arctan\left(\frac{1}{2}x\right)+\frac{1}{2}\log\left(x^2+4\right)+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+4)/(x^3+4*x),x, algorithm="maxima")
```

```
[Out] -1/2*arctan(1/2*x) + 1/2*log(x^2 + 4) + log(x)
```

Fricas [A] time = 1.83692, size = 65, normalized size = 2.83

$$-\frac{1}{2}\arctan\left(\frac{1}{2}x\right)+\frac{1}{2}\log\left(x^2+4\right)+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+4)/(x^3+4*x),x, algorithm="fricas")
```

```
[Out] -1/2*arctan(1/2*x) + 1/2*log(x^2 + 4) + log(x)
```

Sympy [A] time = 0.121869, size = 17, normalized size = 0.74

$$\log\left(x\right)+\frac{\log\left(x^2+4\right)}{2}-\frac{\operatorname{atan}\left(\frac{x}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+4)/(x**3+4*x),x)
```

```
[Out] log(x) + log(x**2 + 4)/2 - atan(x/2)/2
```

Giac [A] time = 1.15423, size = 24, normalized size = 1.04

$$-\frac{1}{2}\arctan\left(\frac{1}{2}x\right)+\frac{1}{2}\log\left(x^2+4\right)+\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+4)/(x^3+4*x),x, algorithm="giac")
```

```
[Out] -1/2*arctan(1/2*x) + 1/2*log(x^2 + 4) + log(abs(x))
```

3.308 $\int \frac{1+x^2+x^3}{(-1+x)x\left(1+x^2\right)^3\left(1+x+x^2\right)}\,dx$

Optimal. Leaf size=103

$$-\frac{3(1-x)}{8\left(x^2+1\right)}+\frac{3x}{16\left(x^2+1\right)}+\frac{x+1}{8\left(x^2+1\right)^2}+\frac{15}{16}\log\left(x^2+1\right)-\frac{1}{2}\log\left(x^2+x+1\right)+\frac{1}{8}\log(1-x)-\log(x)+\frac{7}{16}\tan^{-1}(x)-$$

```
[Out] (1 + x)/(8*(1 + x^2)^2) - (3*(1 - x))/(8*(1 + x^2)) + (3*x)/(16*(1 + x^2))
+ (7*ArcTan[x])/16 - ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 - x]/8 - Log
[x] + (15*Log[1 + x^2])/16 - Log[1 + x + x^2]/2
```

Rubi [A] time = 0.475819, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {6728, 639, 199, 203, 635, 260, 634, 618, 204, 628}

$$-\frac{3(1-x)}{8\left(x^2+1\right)}+\frac{3x}{16\left(x^2+1\right)}+\frac{x+1}{8\left(x^2+1\right)^2}+\frac{15}{16}\log\left(x^2+1\right)-\frac{1}{2}\log\left(x^2+x+1\right)+\frac{1}{8}\log(1-x)-\log(x)+\frac{7}{16}\tan^{-1}(x)-$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2 + x^3)/((-1 + x)*x*(1 + x^2)^3*(1 + x + x^2)),x]
```

```
[Out] (1 + x)/(8*(1 + x^2)^2) - (3*(1 - x))/(8*(1 + x^2)) + (3*x)/(16*(1 + x^2))
+ (7*ArcTan[x])/16 - ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 - x]/8 - Log
[x] + (15*Log[1 + x^2])/16 - Log[1 + x + x^2]/2
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
```

```
Q[p, -1] && NeQ[p, -3/2]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1+x^2+x^3}{(-1+x)x\left(1+x^2\right)^3\left(1+x+x^2\right)}\,dx = \int \left(\frac{1}{8(-1+x)} - \frac{1}{x} + \frac{1-x}{2\left(1+x^2\right)^3} + \frac{3(1+x)}{4\left(1+x^2\right)^2} + \frac{-1+15x}{8\left(1+x^2\right)} + \frac{-1-x}{1+x+x^2}\right) dx$$

$$= \frac{1}{8}\log(1-x) - \log(x) + \frac{1}{8}\int \frac{-1+15x}{1+x^2}\,dx + \frac{1}{2}\int \frac{1-x}{\left(1+x^2\right)^3}\,dx + \frac{3}{4}\int \frac{1+x}{\left(1+x^2\right)}$$

$$= \frac{1+x}{8\left(1+x^2\right)^2} - \frac{3(1-x)}{8\left(1+x^2\right)} + \frac{1}{8}\log(1-x) - \log(x) - \frac{1}{8}\int \frac{1}{1+x^2}\,dx + \frac{3}{8}\int \frac{1}{(1+}$$

$$= \frac{1+x}{8\left(1+x^2\right)^2} - \frac{3(1-x)}{8\left(1+x^2\right)} + \frac{3x}{16\left(1+x^2\right)} + \frac{1}{4}\tan^{-1}(x) + \frac{1}{8}\log(1-x) - \log(x) +$$

$$= \frac{1+x}{8\left(1+x^2\right)^2} - \frac{3(1-x)}{8\left(1+x^2\right)} + \frac{3x}{16\left(1+x^2\right)} + \frac{7}{16}\tan^{-1}(x) - \frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{8}\log($$

Mathematica [A] time = 0.0439017, size = 93, normalized size = 0.9

$$\frac{1}{48}\left(\frac{6(x+1)}{\left(x^2+1\right)^2} + \frac{9(3x-2)}{x^2+1} + 45\log\left(x^2+1\right) - 10\log\left(x^2+x+1\right) - 14\log\left(1-x^3\right) + 20\log(1-x) - 48\log(x) + 21\tan$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2 + x^3)/((-1 + x)*x*(1 + x^2)^3*(1 + x + x^2)),x]
```

```
[Out] ((6*(1 + x))/(1 + x^2)^2 + (9*(-2 + 3*x))/(1 + x^2) + 21*ArcTan[x] - 16*Sqr
t[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + 20*Log[1 - x] - 48*Log[x] + 45*Log[1 + x^2
] - 10*Log[1 + x + x^2] - 14*Log[1 - x^3])/48
```

Maple [A] time = 0.012, size = 73, normalized size = 0.7

$$\frac{1}{8\left(x^2+1\right)^2}\left(\frac{9x^3}{2}-3x^2+\frac{11x}{2}-2\right)+\frac{15\ln\left(x^2+1\right)}{16}+\frac{7\arctan(x)}{16}+\frac{\ln(x-1)}{8}-\ln(x)-\frac{\ln\left(x^2+x+1\right)}{2}-\frac{\sqrt{3}}{3}\operatorname{arcta}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x^2+1)/(x-1)/x/(x^2+1)^3/(x^2+x+1),x)
```

```
[Out] 1/8*(9/2*x^3-3*x^2+11/2*x-2)/(x^2+1)^2+15/16*ln(x^2+1)+7/16*arctan(x)+1/8*l
n(x-1)-ln(x)-1/2*ln(x^2+x+1)-1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Maxima [A] time = 1.72619, size = 104, normalized size = 1.01

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{9x^3-6x^2+11x-4}{16\left(x^4+2x^2+1\right)}+\frac{7}{16}\arctan(x)-\frac{1}{2}\log\left(x^2+x+1\right)+\frac{15}{16}\log\left(x^2+1\right)+\frac{1}{8}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+1)/(-1+x)/x/(x^2+1)^3/(x^2+x+1),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/16*(9*x^3 - 6*x^2 + 11*x - 4
)/(x^4 + 2*x^2 + 1) + 7/16*arctan(x) - 1/2*log(x^2 + x + 1) + 15/16*log(x^2
 + 1) + 1/8*log(x - 1) - log(x)
```

Fricas [A] time = 1.74576, size = 387, normalized size = 3.76

$$\frac{27x^3-16\sqrt{3}\left(x^4+2x^2+1\right)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-18x^2+21\left(x^4+2x^2+1\right)\arctan(x)-24\left(x^4+2x^2+1\right)\log\left(x^2+\right.}{48\left(x^4+2x^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+1)/(-1+x)/x/(x^2+1)^3/(x^2+x+1),x, algorithm="fricas")
```

```
[Out] 1/48*(27*x^3 - 16*sqrt(3)*(x^4 + 2*x^2 + 1)*arctan(1/3*sqrt(3)*(2*x + 1)) -
 18*x^2 + 21*(x^4 + 2*x^2 + 1)*arctan(x) - 24*(x^4 + 2*x^2 + 1)*log(x^2 + x
 + 1) + 45*(x^4 + 2*x^2 + 1)*log(x^2 + 1) + 6*(x^4 + 2*x^2 + 1)*log(x - 1)
```

```
- 48*(x^4 + 2*x^2 + 1)*log(x) + 33*x - 12)/(x^4 + 2*x^2 + 1)
```

Sympy [A] time = 0.46186, size = 88, normalized size = 0.85

$$-\log(x)+\frac{\log(x-1)}{8}+\frac{15\log\left(x^2+1\right)}{16}-\frac{\log\left(x^2+x+1\right)}{2}+\frac{7\,\mathrm{atan}(x)}{16}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{3}+\frac{9x^3-6x^2+11x-4}{16x^4+32x^2+16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x**2+1)/(-1+x)/x/(x**2+1)**3/(x**2+x+1),x)
```

```
[Out] -log(x) + log(x - 1)/8 + 15*log(x**2 + 1)/16 - log(x**2 + x + 1)/2 + 7*atan
(x)/16 - sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/3 + (9*x**3 - 6*x**2 + 11*
x - 4)/(16*x**4 + 32*x**2 + 16)
```

Giac [A] time = 1.14488, size = 100, normalized size = 0.97

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{9x^3-6x^2+11x-4}{16\left(x^2+1\right)^2}+\frac{7}{16}\arctan(x)-\frac{1}{2}\log\left(x^2+x+1\right)+\frac{15}{16}\log\left(x^2+1\right)+\frac{1}{8}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+1)/(-1+x)/x/(x^2+1)^3/(x^2+x+1),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/16*(9*x^3 - 6*x^2 + 11*x - 4
)/(x^2 + 1)^2 + 7/16*arctan(x) - 1/2*log(x^2 + x + 1) + 15/16*log(x^2 + 1)
+ 1/8*log(abs(x - 1)) - log(abs(x))
```

3.309 $\int \frac{1-3x+2x^2-x^3}{(1+x^2)^2}\,dx$

Optimal. Leaf size=33

$$\frac{2-x}{2(x^2+1)}-\frac{1}{2}\log\left(x^2+1\right)+\frac{3}{2}\tan^{-1}(x)$$

```
[Out] (2 - x)/(2*(1 + x^2)) + (3*ArcTan[x])/2 - Log[1 + x^2]/2
```

Rubi [A] time = 0.0167461, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {1814, 635, 203, 260}

$$\frac{2-x}{2(x^2+1)}-\frac{1}{2}\log\left(x^2+1\right)+\frac{3}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 3*x + 2*x^2 - x^3)/(1 + x^2)^2,x]
```

```
[Out] (2 - x)/(2*(1 + x^2)) + (3*ArcTan[x])/2 - Log[1 + x^2]/2
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{1-3x+2x^2-x^3}{\left(1+x^2\right)^2}\,dx &= \frac{2-x}{2\left(1+x^2\right)}-\frac{1}{2}\int \frac{-3+2x}{1+x^2}\,dx\\ &= \frac{2-x}{2\left(1+x^2\right)}+\frac{3}{2}\int \frac{1}{1+x^2}\,dx-\int \frac{x}{1+x^2}\,dx\\ &= \frac{2-x}{2\left(1+x^2\right)}+\frac{3}{2}\tan^{-1}(x)-\frac{1}{2}\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.0111377, size = 30, normalized size = 0.91

$$\frac{1}{2}\left(\frac{2-x}{x^2+1}-\log\left(x^2+1\right)+3\tan^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 3*x + 2*x^2 - x^3)/(1 + x^2)^2,x]
```

```
[Out] ((2 - x)/(1 + x^2) + 3*ArcTan[x] - Log[1 + x^2])/2
```

Maple [A] time = 0.006, size = 28, normalized size = 0.9

$$-\frac{1}{x^2+1}\left(\frac{x}{2}-1\right)-\frac{\ln\left(x^2+1\right)}{2}+\frac{3\arctan(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+2*x^2-3*x+1)/(x^2+1)^2,x)
```

```
[Out] -(1/2*x-1)/(x^2+1)-1/2*ln(x^2+1)+3/2*arctan(x)
```

Maxima [A] time = 1.62154, size = 34, normalized size = 1.03

$$-\frac{x-2}{2\left(x^2+1\right)}+\frac{3}{2}\arctan\left(x\right)-\frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+2*x^2-3*x+1)/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(x - 2)/(x^2 + 1) + 3/2*arctan(x) - 1/2*log(x^2 + 1)
```

Fricas [A] time = 1.56371, size = 97, normalized size = 2.94

$$\frac{3\left(x^2+1\right)\arctan\left(x\right)-\left(x^2+1\right)\log\left(x^2+1\right)-x+2}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+2*x^2-3*x+1)/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(3*(x^2 + 1)*arctan(x) - (x^2 + 1)*log(x^2 + 1) - x + 2)/(x^2 + 1)
```

Sympy [A] time = 0.116215, size = 24, normalized size = 0.73

$$-\frac{x-2}{2x^2+2}-\frac{\log\left(x^2+1\right)}{2}+\frac{3\operatorname{atan}\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+2*x**2-3*x+1)/(x**2+1)**2,x)
```

```
[Out] -(x - 2)/(2*x**2 + 2) - log(x**2 + 1)/2 + 3*atan(x)/2
```

Giac [A] time = 1.38542, size = 34, normalized size = 1.03

$$-\frac{x-2}{2\left(x^2+1\right)}+\frac{3}{2}\arctan\left(x\right)-\frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+2*x^2-3*x+1)/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] -1/2*(x - 2)/(x^2 + 1) + 3/2*arctan(x) - 1/2*log(x^2 + 1)
```

3.310 $\int \frac{1-3x+2x^2-x^3}{x(1+x^2)^2}\,dx$

Optimal. Leaf size=33

$$-\frac{2x+1}{2\left(x^2+1\right)}-\frac{1}{2}\log\left(x^2+1\right)+\log(x)-2\tan^{-1}(x)$$

```
[Out] -(1 + 2*x)/(2*(1 + x^2)) - 2*ArcTan[x] + Log[x] - Log[1 + x^2]/2
```

Rubi [A] time = 0.0409266, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1805, 801, 635, 203, 260}

$$-\frac{2x+1}{2\left(x^2+1\right)}-\frac{1}{2}\log\left(x^2+1\right)+\log(x)-2\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 3*x + 2*x^2 - x^3)/(x*(1 + x^2)^2),x]
```

```
[Out] -(1 + 2*x)/(2*(1 + x^2)) - 2*ArcTan[x] + Log[x] - Log[1 + x^2]/2
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1-3x+2x^2-x^3}{x\left(1+x^2\right)^2}\,dx &= -\frac{1+2x}{2\left(1+x^2\right)}-\frac{1}{2}\int \frac{-2+4x}{x\left(1+x^2\right)}\,dx\\
&= -\frac{1+2x}{2\left(1+x^2\right)}-\frac{1}{2}\int \left(-\frac{2}{x}+\frac{2(2+x)}{1+x^2}\right)dx\\
&= -\frac{1+2x}{2\left(1+x^2\right)}+\log(x)-\int \frac{2+x}{1+x^2}\,dx\\
&= -\frac{1+2x}{2\left(1+x^2\right)}+\log(x)-2\int \frac{1}{1+x^2}\,dx-\int \frac{x}{1+x^2}\,dx\\
&= -\frac{1+2x}{2\left(1+x^2\right)}-2\tan^{-1}(x)+\log(x)-\frac{1}{2}\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.0188928, size = 33, normalized size = 1.

$$\frac{-2x-1}{2\left(x^2+1\right)}-\frac{1}{2}\log\left(x^2+1\right)+\log(x)-2\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 3*x + 2*x^2 - x^3)/(x*(1 + x^2)^2),x]
```

```
[Out] (-1 - 2*x)/(2*(1 + x^2)) - 2*ArcTan[x] + Log[x] - Log[1 + x^2]/2
```

Maple [A] time = 0.007, size = 28, normalized size = 0.9

$$-\frac{1}{x^2+1}\left(x+\frac{1}{2}\right)-\frac{\ln\left(x^2+1\right)}{2}-2\arctan(x)+\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+2*x^2-3*x+1)/x/(x^2+1)^2,x)
```

```
[Out] -(x+1/2)/(x^2+1)-1/2*ln(x^2+1)-2*arctan(x)+ln(x)
```

Maxima [A] time = 1.62908, size = 39, normalized size = 1.18

$$-\frac{2x+1}{2\left(x^2+1\right)}-2\arctan(x)-\frac{1}{2}\log\left(x^2+1\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+2*x^2-3*x+1)/x/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*x + 1)/(x^2 + 1) - 2*arctan(x) - 1/2*log(x^2 + 1) + log(x)
```

Fricas [A] time = 1.56285, size = 130, normalized size = 3.94

$$-\frac{4\left(x^2+1\right)\arctan(x)+\left(x^2+1\right)\log\left(x^2+1\right)-2\left(x^2+1\right)\log(x)+2x+1}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+2*x^2-3*x+1)/x/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(4*(x^2 + 1)*arctan(x) + (x^2 + 1)*log(x^2 + 1) - 2*(x^2 + 1)*log(x) +
 2*x + 1)/(x^2 + 1)
```

Sympy [A] time = 0.132646, size = 27, normalized size = 0.82

$$-\frac{2x+1}{2x^2+2}+\log(x)-\frac{\log\left(x^2+1\right)}{2}-2\,\mathrm{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+2*x**2-3*x+1)/x/(x**2+1)**2,x)
```

```
[Out] -(2*x + 1)/(2*x**2 + 2) + log(x) - log(x**2 + 1)/2 - 2*atan(x)
```

Giac [A] time = 1.18865, size = 41, normalized size = 1.24

$$-\frac{2\,x+1}{2\left(x^2+1\right)}-2\,\arctan(x)-\frac{1}{2}\,\log\left(x^2+1\right)+\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+2*x^2-3*x+1)/x/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*x + 1)/(x^2 + 1) - 2*arctan(x) - 1/2*log(x^2 + 1) + log(abs(x))
```

3.311 $\int \frac{1-x-x^2+x^3+x^4}{-x+x^3} dx$

Optimal. Leaf size=25

$$\frac{x^2}{2}+\frac{1}{2}\log\left(1-x^2\right)+x-\log(x)$$

```
[Out] x + x^2/2 - Log[x] + Log[1 - x^2]/2
```

Rubi [A] time = 0.040968, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1593, 1802, 260}

$$\frac{x^2}{2}+\frac{1}{2}\log\left(1-x^2\right)+x-\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x - x^2 + x^3 + x^4)/(-x + x^3),x]
```

```
[Out] x + x^2/2 - Log[x] + Log[1 - x^2]/2
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{1-x-x^2+x^3+x^4}{-x+x^3}\,dx = \int \frac{1-x-x^2+x^3+x^4}{x\left(-1+x^2\right)}\,dx$$
$$= \int \left(1-\frac{1}{x}+x+\frac{x}{-1+x^2}\right)dx$$
$$= x+\frac{x^2}{2}-\log(x)+\int \frac{x}{-1+x^2}\,dx$$
$$= x+\frac{x^2}{2}-\log(x)+\frac{1}{2}\log\left(1-x^2\right)$$

Mathematica [A] time = 0.0052358, size = 25, normalized size = 1.

$$\frac{x^2}{2}+\frac{1}{2}\log\left(1-x^2\right)+x-\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x - x^2 + x^3 + x^4)/(-x + x^3),x]
```

```
[Out] x + x^2/2 - Log[x] + Log[1 - x^2]/2
```

Maple [A] time = 0.006, size = 24, normalized size = 1.

$$\frac{x^2}{2}+x+\frac{\ln(x-1)}{2}-\ln(x)+\frac{\ln(1+x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+x^3-x^2-x+1)/(x^3-x),x)
```

```
[Out] 1/2*x^2+x+1/2*ln(x-1)-ln(x)+1/2*ln(1+x)
```

Maxima [A] time = 1.05289, size = 31, normalized size = 1.24

$$\frac{1}{2}x^2+x+\frac{1}{2}\log(x+1)+\frac{1}{2}\log(x-1)-\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^3-x^2-x+1)/(x^3-x),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 + x + 1/2*log(x + 1) + 1/2*log(x - 1) - log(x)
```

Fricas [A] time = 1.58798, size = 55, normalized size = 2.2

$$\frac{1}{2}x^2 + x + \frac{1}{2}\log\left(x^2 - 1\right) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^3-x^2-x+1)/(x^3-x),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 + x + 1/2*log(x^2 - 1) - log(x)
```

Sympy [A] time = 0.08867, size = 17, normalized size = 0.68

$$\frac{x^2}{2} + x - \log(x) + \frac{\log\left(x^2 - 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+x**3-x**2-x+1)/(x**3-x),x)
```

```
[Out] x**2/2 + x - log(x) + log(x**2 - 1)/2
```

Giac [A] time = 1.14706, size = 35, normalized size = 1.4

$$\frac{1}{2}x^2 + x + \frac{1}{2}\log(|x + 1|) + \frac{1}{2}\log(|x - 1|) - \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^3-x^2-x+1)/(x^3-x),x, algorithm="giac")
```

```
[Out] 1/2*x^2 + x + 1/2*log(abs(x + 1)) + 1/2*log(abs(x - 1)) - log(abs(x))
```

3.312 $\int \frac{2-4x^2+x^3}{(1+x^2)(2+x^2)}\,dx$

Optimal. Leaf size=36

$$-\frac{1}{2}\log\left(x^2+1\right)+\log\left(x^2+2\right)+6\tan^{-1}(x)-5\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

```
[Out] 6*ArcTan[x] - 5*Sqrt[2]*ArcTan[x/Sqrt[2]] - Log[1 + x^2]/2 + Log[2 + x^2]
```

Rubi [A] time = 0.115305, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {6725, 635, 203, 260}

$$-\frac{1}{2}\log\left(x^2+1\right)+\log\left(x^2+2\right)+6\tan^{-1}(x)-5\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2 - 4*x^2 + x^3)/((1 + x^2)*(2 + x^2)),x]
```

```
[Out] 6*ArcTan[x] - 5*Sqrt[2]*ArcTan[x/Sqrt[2]] - Log[1 + x^2]/2 + Log[2 + x^2]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{2-4x^2+x^3}{(1+x^2)(2+x^2)}\,dx &= \int \left(\frac{6-x}{1+x^2}+\frac{2(-5+x)}{2+x^2}\right)dx \\
&= 2\int \frac{-5+x}{2+x^2}\,dx+\int \frac{6-x}{1+x^2}\,dx \\
&= 2\int \frac{x}{2+x^2}\,dx+6\int \frac{1}{1+x^2}\,dx-10\int \frac{1}{2+x^2}\,dx-\int \frac{x}{1+x^2}\,dx \\
&= 6\tan^{-1}(x)-5\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)-\frac{1}{2}\log\left(1+x^2\right)+\log\left(2+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.0151129, size = 36, normalized size = 1.

$$-\frac{1}{2}\log\left(x^2+1\right)+\log\left(x^2+2\right)+6\tan^{-1}(x)-5\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - 4*x^2 + x^3)/((1 + x^2)*(2 + x^2)),x]
```

```
[Out] 6*ArcTan[x] - 5*Sqrt[2]*ArcTan[x/Sqrt[2]] - Log[1 + x^2]/2 + Log[2 + x^2]
```

Maple [A] time = 0.004, size = 32, normalized size = 0.9

$$6\arctan(x)-\frac{\ln\left(x^2+1\right)}{2}+\ln\left(x^2+2\right)-5\arctan\left(1/2\,x\sqrt{2}\right)\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-4*x^2+2)/(x^2+1)/(x^2+2),x)
```

```
[Out] 6*arctan(x)-1/2*ln(x^2+1)+ln(x^2+2)-5*arctan(1/2*x*2^(1/2))*2^(1/2)
```

Maxima [A] time = 1.68476, size = 42, normalized size = 1.17

$$-5\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+6\arctan(x)+\log\left(x^2+2\right)-\frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-4*x^2+2)/(x^2+1)/(x^2+2),x, algorithm="maxima")
```

```
[Out] -5*sqrt(2)*arctan(1/2*sqrt(2)*x) + 6*arctan(x) + log(x^2 + 2) - 1/2*log(x^2
 + 1)
```

Fricas [A] time = 1.53561, size = 111, normalized size = 3.08

$$-5\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+6\arctan(x)+\log\left(x^2+2\right)-\frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-4*x^2+2)/(x^2+1)/(x^2+2),x, algorithm="fricas")
```

```
[Out] -5*sqrt(2)*arctan(1/2*sqrt(2)*x) + 6*arctan(x) + log(x^2 + 2) - 1/2*log(x^2
 + 1)
```

Sympy [A] time = 0.170663, size = 36, normalized size = 1.

$$-\frac{\log\left(x^2+1\right)}{2}+\log\left(x^2+2\right)+6\operatorname{atan}(x)-5\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-4*x**2+2)/(x**2+1)/(x**2+2),x)
```

```
[Out] -log(x**2 + 1)/2 + log(x**2 + 2) + 6*atan(x) - 5*sqrt(2)*atan(sqrt(2)*x/2)
```

Giac [A] time = 1.16949, size = 42, normalized size = 1.17

$$-5\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+6\arctan\left(x\right)+\log\left(x^{2}+2\right)-\frac{1}{2}\log\left(x^{2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-4*x^2+2)/(x^2+1)/(x^2+2),x, algorithm="giac")
```

```
[Out] -5*sqrt(2)*arctan(1/2*sqrt(2)*x) + 6*arctan(x) + log(x^2 + 2) - 1/2*log(x^2
 + 1)
```

3.313 $\int \frac{1+x^2+x^4}{(1+x^2)(4+x^2)^2} dx$

Optimal. Leaf size=29

$$-\frac{13x}{24(x^2+4)}+\frac{25}{144}\tan^{-1}\left(\frac{x}{2}\right)+\frac{1}{9}\tan^{-1}(x)$$

```
[Out] (-13*x)/(24*(4 + x^2)) + (25*ArcTan[x/2])/144 + ArcTan[x]/9
```

Rubi [A] time = 0.10685, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$, Rules used = {6725, 203, 199}

$$-\frac{13x}{24(x^2+4)}+\frac{25}{144}\tan^{-1}\left(\frac{x}{2}\right)+\frac{1}{9}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2 + x^4)/((1 + x^2)*(4 + x^2)^2),x]
```

```
[Out] (-13*x)/(24*(4 + x^2)) + (25*ArcTan[x/2])/144 + ArcTan[x]/9
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
```

```
ator[p + 1/n] < Denominator[p])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1+x^2+x^4}{\left(1+x^2\right)\left(4+x^2\right)^2}\,dx &= \int \left(\frac{1}{9\left(1+x^2\right)}-\frac{13}{3\left(4+x^2\right)^2}+\frac{8}{9\left(4+x^2\right)}\right)dx\\
&= \frac{1}{9}\int \frac{1}{1+x^2}\,dx+\frac{8}{9}\int \frac{1}{4+x^2}\,dx-\frac{13}{3}\int \frac{1}{\left(4+x^2\right)^2}\,dx\\
&= -\frac{13x}{24\left(4+x^2\right)}+\frac{4}{9}\tan^{-1}\left(\frac{x}{2}\right)+\frac{1}{9}\tan^{-1}(x)-\frac{13}{24}\int \frac{1}{4+x^2}\,dx\\
&= -\frac{13x}{24\left(4+x^2\right)}+\frac{25}{144}\tan^{-1}\left(\frac{x}{2}\right)+\frac{1}{9}\tan^{-1}(x)
\end{aligned}
$$

Mathematica [A] time = 0.0181231, size = 29, normalized size = 1.

$$-\frac{13x}{24\left(x^2+4\right)}+\frac{25}{144}\tan^{-1}\left(\frac{x}{2}\right)+\frac{1}{9}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2 + x^4)/((1 + x^2)*(4 + x^2)^2),x]
```

```
[Out] (-13*x)/(24*(4 + x^2)) + (25*ArcTan[x/2])/144 + ArcTan[x]/9
```

Maple [A] time = 0.007, size = 22, normalized size = 0.8

$$-\frac{13\,x}{24\,x^2+96}+\frac{25}{144}\arctan\left(\frac{x}{2}\right)+\frac{\arctan(x)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+x^2+1)/(x^2+1)/(x^2+4)^2,x)
```

```
[Out] -13/24*x/(x^2+4)+25/144*arctan(1/2*x)+1/9*arctan(x)
```

Maxima [A] time = 1.61975, size = 28, normalized size = 0.97

$$-\frac{13\,x}{24\left(x^2+4\right)}+\frac{25}{144}\,\arctan\left(\frac{1}{2}\,x\right)+\frac{1}{9}\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^2+1)/(x^2+1)/(x^2+4)^2,x, algorithm="maxima")
```

```
[Out] -13/24*x/(x^2 + 4) + 25/144*arctan(1/2*x) + 1/9*arctan(x)
```

Fricas [A] time = 1.5289, size = 105, normalized size = 3.62

$$\frac{25\left(x^2+4\right)\arctan\left(\frac{1}{2}\,x\right)+16\left(x^2+4\right)\arctan\left(x\right)-78\,x}{144\left(x^2+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^2+1)/(x^2+1)/(x^2+4)^2,x, algorithm="fricas")
```

```
[Out] 1/144*(25*(x^2 + 4)*arctan(1/2*x) + 16*(x^2 + 4)*arctan(x) - 78*x)/(x^2 + 4
)
```

Sympy [A] time = 0.159951, size = 22, normalized size = 0.76

$$-\frac{13x}{24x^2+96}+\frac{25\,\mathrm{atan}\left(\frac{x}{2}\right)}{144}+\frac{\mathrm{atan}\left(x\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+x**2+1)/(x**2+1)/(x**2+4)**2,x)
```

```
[Out] -13*x/(24*x**2 + 96) + 25*atan(x/2)/144 + atan(x)/9
```

Giac [A] time = 1.20342, size = 28, normalized size = 0.97

$$-\frac{13\,x}{24\,\left(x^{2}+4\right)}+\frac{25}{144}\,\arctan\left(\frac{1}{2}\,x\right)+\frac{1}{9}\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^2+1)/(x^2+1)/(x^2+4)^2,x, algorithm="giac")
```

```
[Out] -13/24*x/(x^2 + 4) + 25/144*arctan(1/2*x) + 1/9*arctan(x)
```

3.314 $\int \frac{1+x^2+x^3}{2x^2+x^3+x^4} dx$

Optimal. Leaf size=46

$$\frac{5}{8}\log\left(x^2+x+2\right)-\frac{1}{2x}-\frac{\log(x)}{4}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{7}}\right)}{4\sqrt{7}}$$

```
[Out] -1/(2*x) + ArcTan[(1 + 2*x)/Sqrt[7]]/(4*Sqrt[7]) - Log[x]/4 + (5*Log[2 + x
+ x^2])/8
```

Rubi [A] time = 0.0578528, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {1594, 1628, 634, 618, 204, 628}

$$\frac{5}{8}\log\left(x^2+x+2\right)-\frac{1}{2x}-\frac{\log(x)}{4}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{7}}\right)}{4\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2 + x^3)/(2*x^2 + x^3 + x^4),x]
```

```
[Out] -1/(2*x) + ArcTan[(1 + 2*x)/Sqrt[7]]/(4*Sqrt[7]) - Log[x]/4 + (5*Log[2 + x
+ x^2])/8
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1+x^2+x^3}{2x^2+x^3+x^4}\,dx &= \int \frac{1+x^2+x^3}{x^2\left(2+x+x^2\right)}\,dx \\
&= \int \left(\frac{1}{2x^2}-\frac{1}{4x}+\frac{3+5x}{4\left(2+x+x^2\right)}\right)dx \\
&= -\frac{1}{2x}-\frac{\log(x)}{4}+\frac{1}{4}\int \frac{3+5x}{2+x+x^2}\,dx \\
&= -\frac{1}{2x}-\frac{\log(x)}{4}+\frac{1}{8}\int \frac{1}{2+x+x^2}\,dx+\frac{5}{8}\int \frac{1+2x}{2+x+x^2}\,dx \\
&= -\frac{1}{2x}-\frac{\log(x)}{4}+\frac{5}{8}\log\left(2+x+x^2\right)-\frac{1}{4}\,\text{Subst}\left(\int \frac{1}{-7-x^2}\,dx,x,1+2x\right) \\
&= -\frac{1}{2x}+\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{7}}\right)}{4\sqrt{7}}-\frac{\log(x)}{4}+\frac{5}{8}\log\left(2+x+x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.0262491, size = 46, normalized size = 1.

$$\frac{5}{8}\log\left(x^2+x+2\right)-\frac{1}{2x}-\frac{\log(x)}{4}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{7}}\right)}{4\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2 + x^3)/(2*x^2 + x^3 + x^4),x]
```

```
[Out] -1/(2*x) + ArcTan[(1 + 2*x)/Sqrt[7]]/(4*Sqrt[7]) - Log[x]/4 + (5*Log[2 + x
+ x^2])/8
```

Maple [A] time = 0.007, size = 36, normalized size = 0.8

$$-\frac{1}{2x}-\frac{\ln(x)}{4}+\frac{5\ln\left(x^2+x+2\right)}{8}+\frac{\sqrt{7}}{28}\arctan\left(\frac{(1+2x)\sqrt{7}}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x^2+1)/(x^4+x^3+2*x^2),x)
```

```
[Out] -1/2/x-1/4*ln(x)+5/8*ln(x^2+x+2)+1/28*arctan(1/7*(1+2*x)*7^(1/2))*7^(1/2)
```

Maxima [A] time = 1.57762, size = 47, normalized size = 1.02

$$\frac{1}{28}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(2x+1)\right)-\frac{1}{2x}+\frac{5}{8}\log\left(x^2+x+2\right)-\frac{1}{4}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+1)/(x^4+x^3+2*x^2),x, algorithm="maxima")
```

```
[Out] 1/28*sqrt(7)*arctan(1/7*sqrt(7)*(2*x + 1)) - 1/2/x + 5/8*log(x^2 + x + 2) -
 1/4*log(x)
```

Fricas [A] time = 1.49417, size = 128, normalized size = 2.78

$$\frac{2\sqrt{7}x\arctan\left(\frac{1}{7}\sqrt{7}(2x+1)\right)+35x\log\left(x^2+x+2\right)-14x\log(x)-28}{56x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+1)/(x^4+x^3+2*x^2),x, algorithm="fricas")
```

```
[Out] 1/56*(2*sqrt(7)*x*arctan(1/7*sqrt(7)*(2*x + 1)) + 35*x*log(x^2 + x + 2) - 1
4*x*log(x) - 28)/x
```

Sympy [A] time = 0.145931, size = 46, normalized size = 1.

$$-\frac{\log(x)}{4}+\frac{5\log\left(x^2+x+2\right)}{8}+\frac{\sqrt{7}\operatorname{atan}\left(\frac{2\sqrt{7}x}{7}+\frac{\sqrt{7}}{7}\right)}{28}-\frac{1}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x**2+1)/(x**4+x**3+2*x**2),x)
```

```
[Out] -log(x)/4 + 5*log(x**2 + x + 2)/8 + sqrt(7)*atan(2*sqrt(7)*x/7 + sqrt(7)/7)
/28 - 1/(2*x)
```

Giac [A] time = 1.22634, size = 49, normalized size = 1.07

$$\frac{1}{28}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(2x+1)\right)-\frac{1}{2x}+\frac{5}{8}\log\left(x^2+x+2\right)-\frac{1}{4}\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+1)/(x^4+x^3+2*x^2),x, algorithm="giac")
```

```
[Out] 1/28*sqrt(7)*arctan(1/7*sqrt(7)*(2*x + 1)) - 1/2/x + 5/8*log(x^2 + x + 2) -
 1/4*log(abs(x))
```

3.315 $\int \frac{1-12x+x^2+x^3}{-12+x+x^2}\,dx$

Optimal. Leaf size=22

$$\frac{x^2}{2}-\frac{2}{7}\tanh^{-1}\left(\frac{1}{7}(2x+1)\right)$$

```
[Out] x^2/2 - (2*ArcTanh[(1 + 2*x)/7])/7
```

Rubi [A] time = 0.0150512, antiderivative size = 26, normalized size of antiderivative = 1.18, number of steps used = 5, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {1657, 616, 31}

$$\frac{x^2}{2}+\frac{1}{7}\log(3-x)-\frac{1}{7}\log(x+4)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 12*x + x^2 + x^3)/(-12 + x + x^2),x]
```

```
[Out] x^2/2 + Log[3 - x]/7 - Log[4 + x]/7
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1-12x+x^2+x^3}{-12+x+x^2}\,dx &= \int \left(x+\frac{1}{-12+x+x^2}\right)dx\\ &= \frac{x^2}{2}+\int \frac{1}{-12+x+x^2}\,dx\\ &= \frac{x^2}{2}+\frac{1}{7}\int \frac{1}{-3+x}\,dx-\frac{1}{7}\int \frac{1}{4+x}\,dx\\ &= \frac{x^2}{2}+\frac{1}{7}\log(3-x)-\frac{1}{7}\log(4+x)\end{aligned}$$

Mathematica [A] time = 0.0056404, size = 26, normalized size = 1.18

$$\frac{x^2}{2}+\frac{1}{7}\log(3-x)-\frac{1}{7}\log(x+4)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 12*x + x^2 + x^3)/(-12 + x + x^2),x]
```

```
[Out] x^2/2 + Log[3 - x]/7 - Log[4 + x]/7
```

Maple [A] time = 0.007, size = 19, normalized size = 0.9

$$\frac{x^2}{2}+\frac{\ln(-3+x)}{7}-\frac{\ln(4+x)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x^2-12*x+1)/(x^2+x-12),x)
```

```
[Out] 1/2*x^2+1/7*ln(-3+x)-1/7*ln(4+x)
```

Maxima [A] time = 1.02487, size = 24, normalized size = 1.09

$$\frac{1}{2}x^2-\frac{1}{7}\log(x+4)+\frac{1}{7}\log(x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2-12*x+1)/(x^2+x-12),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - 1/7*log(x + 4) + 1/7*log(x - 3)
```

Fricas [A] time = 1.46019, size = 58, normalized size = 2.64

$$\frac{1}{2}x^2 - \frac{1}{7}\log(x+4) + \frac{1}{7}\log(x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2-12*x+1)/(x^2+x-12),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 - 1/7*log(x + 4) + 1/7*log(x - 3)
```

Sympy [A] time = 0.089506, size = 17, normalized size = 0.77

$$\frac{x^2}{2} + \frac{\log(x-3)}{7} - \frac{\log(x+4)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x**2-12*x+1)/(x**2+x-12),x)
```

```
[Out] x**2/2 + log(x - 3)/7 - log(x + 4)/7
```

Giac [A] time = 1.12496, size = 27, normalized size = 1.23

$$\frac{1}{2}x^2 - \frac{1}{7}\log(|x+4|) + \frac{1}{7}\log(|x-3|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2-12*x+1)/(x^2+x-12),x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 1/7*log(abs(x + 4)) + 1/7*log(abs(x - 3))
```

3.316 $\int \frac{-3+5x+6x^2}{-3x+2x^2+x^3} \, dx$

Optimal. Leaf size=17

$$2\log(1-x)+\log(x)+3\log(x+3)$$

```
[Out] 2*Log[1 - x] + Log[x] + 3*Log[3 + x]
```

Rubi [A] time = 0.0391746, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {1594, 1628}

$$2\log(1-x)+\log(x)+3\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Int[(-3 + 5*x + 6*x^2)/(-3*x + 2*x^2 + x^3),x]
```

```
[Out] 2*Log[1 - x] + Log[x] + 3*Log[3 + x]
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{-3+5x+6x^2}{-3x+2x^2+x^3} \, dx &= \int \frac{-3+5x+6x^2}{x\left(-3+2x+x^2\right)} \, dx \\
&= \int \left(\frac{2}{-1+x}+\frac{1}{x}+\frac{3}{3+x}\right) dx \\
&= 2\log(1-x)+\log(x)+3\log(3+x)
\end{aligned}$$

Mathematica [A] time = 0.0058524, size = 17, normalized size = 1.

$$2\log(1-x)+\log(x)+3\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 + 5*x + 6*x^2)/(-3*x + 2*x^2 + x^3),x]
```

```
[Out] 2*Log[1 - x] + Log[x] + 3*Log[3 + x]
```

Maple [A] time = 0.007, size = 16, normalized size = 0.9

$$2\ln(x-1)+\ln(x)+3\ln(3+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((6*x^2+5*x-3)/(x^3+2*x^2-3*x),x)
```

```
[Out] 2*ln(x-1)+ln(x)+3*ln(3+x)
```

Maxima [A] time = 1.08153, size = 20, normalized size = 1.18

$$3\log(x+3)+2\log(x-1)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((6*x^2+5*x-3)/(x^3+2*x^2-3*x),x, algorithm="maxima")
```

```
[Out] 3*log(x + 3) + 2*log(x - 1) + log(x)
```

Fricas [A] time = 1.48899, size = 51, normalized size = 3.

$$3\log(x+3)+2\log(x-1)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((6*x^2+5*x-3)/(x^3+2*x^2-3*x),x, algorithm="fricas")
```

```
[Out] 3*log(x + 3) + 2*log(x - 1) + log(x)
```

Sympy [A] time = 0.11806, size = 15, normalized size = 0.88

$$\log(x) + 2\log(x - 1) + 3\log(x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((6*x**2+5*x-3)/(x**3+2*x**2-3*x),x)
```

```
[Out] log(x) + 2*log(x - 1) + 3*log(x + 3)
```

Giac [A] time = 1.44645, size = 24, normalized size = 1.41

$$3\,\log(|x + 3|) + 2\,\log(|x - 1|) + \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((6*x^2+5*x-3)/(x^3+2*x^2-3*x),x, algorithm="giac")
```

```
[Out] 3*log(abs(x + 3)) + 2*log(abs(x - 1)) + log(abs(x))
```

3.317 $\int \frac{-2+3x+5x^2}{2x^2+x^3} dx$

Optimal. Leaf size=14

$$\frac{1}{x} + 2\log(x) + 3\log(x+2)$$

```
[Out] x^(-1) + 2*Log[x] + 3*Log[2 + x]
```

Rubi [A] time = 0.0245166, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1593, 893}

$$\frac{1}{x} + 2\log(x) + 3\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[(-2 + 3*x + 5*x^2)/(2*x^2 + x^3),x]
```

```
[Out] x^(-1) + 2*Log[x] + 3*Log[2 + x]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rubi steps

$$\int \frac{-2+3x+5x^2}{2x^2+x^3}\,dx = \int \frac{-2+3x+5x^2}{x^2(2+x)}\,dx$$
$$= \int \left(-\frac{1}{x^2}+\frac{2}{x}+\frac{3}{2+x}\right)dx$$
$$= \frac{1}{x}+2\log(x)+3\log(2+x)$$

Mathematica [A] time = 0.0037496, size = 14, normalized size = 1.

$$\frac{1}{x}+2\log(x)+3\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 + 3*x + 5*x^2)/(2*x^2 + x^3),x]
```

```
[Out] x^(-1) + 2*Log[x] + 3*Log[2 + x]
```

Maple [A] time = 0.006, size = 15, normalized size = 1.1

$$x^{-1}+2\ln(x)+3\ln(2+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x-2)/(x^3+2*x^2),x)
```

```
[Out] 1/x+2*ln(x)+3*ln(2+x)
```

Maxima [A] time = 1.12979, size = 19, normalized size = 1.36

$$\frac{1}{x}+3\log(x+2)+2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x-2)/(x^3+2*x^2),x, algorithm="maxima")
```

```
[Out] 1/x + 3*log(x + 2) + 2*log(x)
```

Fricas [A] time = 1.46122, size = 50, normalized size = 3.57

$$\frac{3\,x\log\left(x+2\right)+2\,x\log\left(x\right)+1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x-2)/(x^3+2*x^2),x, algorithm="fricas")
```

```
[Out] (3*x*log(x + 2) + 2*x*log(x) + 1)/x
```

Sympy [A] time = 0.10341, size = 14, normalized size = 1.

$$2\log\left(x\right)+3\log\left(x+2\right)+\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x-2)/(x**3+2*x**2),x)
```

```
[Out] 2*log(x) + 3*log(x + 2) + 1/x
```

Giac [A] time = 1.21113, size = 22, normalized size = 1.57

$$\frac{1}{x}+3\,\log\left(|x+2|\right)+2\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x-2)/(x^3+2*x^2),x, algorithm="giac")
```

```
[Out] 1/x + 3*log(abs(x + 2)) + 2*log(abs(x))
```

3.318 $\int \frac{18-2x-4x^2}{-6+x+4x^2+x^3}\,dx$

Optimal. Leaf size=19

$$\log(1-x)-2\log(x+2)-3\log(x+3)$$

```
[Out] Log[1 - x] - 2*Log[2 + x] - 3*Log[3 + x]
```

Rubi [A] time = 0.0273159, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {2074}

$$\log(1-x)-2\log(x+2)-3\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Int[(18 - 2*x - 4*x^2)/(-6 + x + 4*x^2 + x^3),x]

[Out] Log[1 - x] - 2*Log[2 + x] - 3*Log[3 + x]
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\begin{aligned}\int \frac{18-2x-4x^2}{-6+x+4x^2+x^3}\,dx &= \int \left(\frac{1}{-1+x}-\frac{2}{2+x}-\frac{3}{3+x}\right)dx\\ &= \log(1-x)-2\log(2+x)-3\log(3+x)\end{aligned}$$

Mathematica [A] time = 0.0074008, size = 25, normalized size = 1.32

$$-2\left(-\frac{1}{2}\log(1-x)+\log(x+2)+\frac{3}{2}\log(x+3)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(18 - 2*x - 4*x^2)/(-6 + x + 4*x^2 + x^3),x]
```

```
[Out] -2*(-Log[1 - x]/2 + Log[2 + x] + (3*Log[3 + x])/2)
```

Maple [A] time = 0.007, size = 18, normalized size = 1.

$$\ln(x-1) - 2\ln(2+x) - 3\ln(3+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4*x^2-2*x+18)/(x^3+4*x^2+x-6),x)
```

```
[Out] ln(x-1)-2*ln(2+x)-3*ln(3+x)
```

Maxima [A] time = 1.10956, size = 23, normalized size = 1.21

$$-3\log(x+3) - 2\log(x+2) + \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2-2*x+18)/(x^3+4*x^2+x-6),x, algorithm="maxima")
```

```
[Out] -3*log(x + 3) - 2*log(x + 2) + log(x - 1)
```

Fricas [A] time = 1.47699, size = 58, normalized size = 3.05

$$-3\log(x+3) - 2\log(x+2) + \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2-2*x+18)/(x^3+4*x^2+x-6),x, algorithm="fricas")
```

```
[Out] -3*log(x + 3) - 2*log(x + 2) + log(x - 1)
```

Sympy [A] time = 0.116739, size = 17, normalized size = 0.89

$$\log(x-1) - 2\log(x+2) - 3\log(x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x**2-2*x+18)/(x**3+4*x**2+x-6),x)
```

```
[Out] log(x - 1) - 2*log(x + 2) - 3*log(x + 3)
```

Giac [A] time = 1.14033, size = 27, normalized size = 1.42

$$-3\log(|x+3|) - 2\log(|x+2|) + \log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2-2*x+18)/(x^3+4*x^2+x-6),x, algorithm="giac")
```

```
[Out] -3*log(abs(x + 3)) - 2*log(abs(x + 2)) + log(abs(x - 1))
```

3.319 $\int \frac{1+x-2x^2+x^3}{4+5x^2+x^4} \, dx$

Optimal. Leaf size=23

$$\frac{1}{2}\log\left(x^2+4\right)-\frac{3}{2}\tan^{-1}\left(\frac{x}{2}\right)+\tan^{-1}(x)$$

```
[Out] (-3*ArcTan[x/2])/2 + ArcTan[x] + Log[4 + x^2]/2
```

Rubi [A] time = 0.0289438, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1673, 1166, 203, 1247, 626, 31}

$$\frac{1}{2}\log\left(x^2+4\right)-\frac{3}{2}\tan^{-1}\left(\frac{x}{2}\right)+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x - 2*x^2 + x^3)/(4 + 5*x^2 + x^4),x]
```

```
[Out] (-3*ArcTan[x/2])/2 + ArcTan[x] + Log[4 + x^2]/2
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x-2x^2+x^3}{4+5x^2+x^4}\,dx &= \int \frac{1-2x^2}{4+5x^2+x^4}\,dx+\int \frac{x\left(1+x^2\right)}{4+5x^2+x^4}\,dx\\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{1+x}{4+5x+x^2}\,dx,x,x^2\right)-3\int \frac{1}{4+x^2}\,dx+\int \frac{1}{1+x^2}\,dx\\
&= -\frac{3}{2}\tan^{-1}\left(\frac{x}{2}\right)+\tan^{-1}(x)+\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{4+x}\,dx,x,x^2\right)\\
&= -\frac{3}{2}\tan^{-1}\left(\frac{x}{2}\right)+\tan^{-1}(x)+\frac{1}{2}\log\left(4+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.0092757, size = 23, normalized size = 1.

$$\frac{1}{2}\log\left(x^2+4\right)-\frac{3}{2}\tan^{-1}\left(\frac{x}{2}\right)+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x - 2*x^2 + x^3)/(4 + 5*x^2 + x^4),x]
```

```
[Out] (-3*ArcTan[x/2])/2 + ArcTan[x] + Log[4 + x^2]/2
```

Maple [A] time = 0.004, size = 18, normalized size = 0.8

$$-\frac{3}{2}\arctan\left(\frac{x}{2}\right)+\arctan\left(x\right)+\frac{\ln\left(x^{2}+4\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-2*x^2+x+1)/(x^4+5*x^2+4),x)
```

```
[Out] -3/2*arctan(1/2*x)+arctan(x)+1/2*ln(x^2+4)
```

Maxima [A] time = 1.56833, size = 23, normalized size = 1.

$$-\frac{3}{2}\arctan\left(\frac{1}{2}x\right)+\arctan\left(x\right)+\frac{1}{2}\log\left(x^{2}+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2*x^2+x+1)/(x^4+5*x^2+4),x, algorithm="maxima")
```

```
[Out] -3/2*arctan(1/2*x) + arctan(x) + 1/2*log(x^2 + 4)
```

Fricas [A] time = 1.46552, size = 69, normalized size = 3.

$$-\frac{3}{2}\arctan\left(\frac{1}{2}x\right)+\arctan\left(x\right)+\frac{1}{2}\log\left(x^{2}+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2*x^2+x+1)/(x^4+5*x^2+4),x, algorithm="fricas")
```

```
[Out] -3/2*arctan(1/2*x) + arctan(x) + 1/2*log(x^2 + 4)
```

Sympy [A] time = 0.15904, size = 19, normalized size = 0.83

$$\frac{\log\left(x^{2}+4\right)}{2}-\frac{3\operatorname{atan}\left(\frac{x}{2}\right)}{2}+\operatorname{atan}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-2*x**2+x+1)/(x**4+5*x**2+4),x)
```

```
[Out] log(x**2 + 4)/2 - 3*atan(x/2)/2 + atan(x)
```

Giac [A] time = 1.1427, size = 23, normalized size = 1.

$$-\frac{3}{2}\,\arctan\left(\frac{1}{2}\,x\right) + \arctan\left(x\right) + \frac{1}{2}\,\log\left(x^2 + 4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2*x^2+x+1)/(x^4+5*x^2+4),x, algorithm="giac")
```

```
[Out] -3/2*arctan(1/2*x) + arctan(x) + 1/2*log(x^2 + 4)
```

3.320 $\int \frac{-32+5x-27x^2+4x^3}{-70-299x-286x^2+50x^3-13x^4+30x^5} \, dx$

Optimal. Leaf size=63

$$\frac{11049 \log \left(x^2+x+5\right)}{260015}-\frac{3146 \log (7-3 x)}{80155}-\frac{334}{323} \log (2 x+1)+\frac{4822 \log (5 x+2)}{4879}+\frac{3988 \tan ^{-1}\left(\frac{2 x+1}{\sqrt{19}}\right)}{13685 \sqrt{19}}$$

```
[Out] (3988*ArcTan[(1 + 2*x)/Sqrt[19]])/(13685*Sqrt[19]) - (3146*Log[7 - 3*x])/80
155 - (334*Log[1 + 2*x])/323 + (4822*Log[2 + 5*x])/4879 + (11049*Log[5 + x
+ x^2])/260015
```

Rubi [A] time = 0.0877549, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.116, Rules used = {2074, 634, 618, 204, 628}

$$\frac{11049 \log \left(x^2+x+5\right)}{260015}-\frac{3146 \log (7-3 x)}{80155}-\frac{334}{323} \log (2 x+1)+\frac{4822 \log (5 x+2)}{4879}+\frac{3988 \tan ^{-1}\left(\frac{2 x+1}{\sqrt{19}}\right)}{13685 \sqrt{19}}$$

Antiderivative was successfully verified.

```
[In] Int[(-32 + 5*x - 27*x^2 + 4*x^3)/(-70 - 299*x - 286*x^2 + 50*x^3 - 13*x^4 +
 30*x^5),x]
```

```
[Out] (3988*ArcTan[(1 + 2*x)/Sqrt[19]])/(13685*Sqrt[19]) - (3146*Log[7 - 3*x])/80
155 - (334*Log[1 + 2*x])/323 + (4822*Log[2 + 5*x])/4879 + (11049*Log[5 + x
+ x^2])/260015
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{-32+5x-27x^2+4x^3}{-70-299x-286x^2+50x^3-13x^4+30x^5}\,dx = \int\left(-\frac{668}{323(1+2x)}-\frac{9438}{80155(-7+3x)}+\frac{24110}{4879(2+5x)}+\frac{48935+22}{260015\left(5+\right.}\right.$$

$$= -\frac{3146\log(7-3x)}{80155}-\frac{334}{323}\log(1+2x)+\frac{4822\log(2+5x)}{4879}+\frac{\int\frac{48935+}{5+x}}{260}$$

$$= -\frac{3146\log(7-3x)}{80155}-\frac{334}{323}\log(1+2x)+\frac{4822\log(2+5x)}{4879}+\frac{11049\int}{260}$$

$$= -\frac{3146\log(7-3x)}{80155}-\frac{334}{323}\log(1+2x)+\frac{4822\log(2+5x)}{4879}+\frac{11049\,\mathrm{lo}}{2}$$

$$= \frac{3988\tan^{-1}\left(\frac{1+2x}{\sqrt{19}}\right)}{13685\sqrt{19}}-\frac{3146\log(7-3x)}{80155}-\frac{334}{323}\log(1+2x)+\frac{4822\log}{48}$$

Mathematica [A] time = 0.0266384, size = 57, normalized size = 0.9

$$\frac{453009\log\left(x^2+x+5\right)-418418\log(7-3x)-11023670\log(2x+1)+10536070\log(5x+2)+163508\sqrt{19}\tan^{-1}\left(\frac{2x+1}{\sqrt{19}}\right.}{10660615}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-32 + 5*x - 27*x^2 + 4*x^3)/(-70 - 299*x - 286*x^2 + 50*x^3 - 13
*x^4 + 30*x^5),x]
```

```
[Out] (163508*Sqrt[19]*ArcTan[(1 + 2*x)/Sqrt[19]] - 418418*Log[7 - 3*x] - 1102367
0*Log[1 + 2*x] + 10536070*Log[2 + 5*x] + 453009*Log[5 + x + x^2])/10660615
```

Maple [A] time = 0.009, size = 51, normalized size = 0.8

$$\frac{4822\,\ln(2+5\,x)}{4879}+\frac{11049\,\ln\left(x^2+x+5\right)}{260015}+\frac{3988\,\sqrt{19}}{260015}\arctan\left(\frac{(1+2\,x)\sqrt{19}}{19}\right)-\frac{334\,\ln(1+2\,x)}{323}-\frac{3146\,\ln(3\,x-7)}{80155}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^3-27*x^2+5*x-32)/(30*x^5-13*x^4+50*x^3-286*x^2-299*x-70),x)
```

```
[Out] 4822/4879*ln(2+5*x)+11049/260015*ln(x^2+x+5)+3988/260015*arctan(1/19*(1+2*x
)*19^(1/2))*19^(1/2)-334/323*ln(1+2*x)-3146/80155*ln(3*x-7)
```

Maxima [A] time = 1.55734, size = 68, normalized size = 1.08

$$\frac{3988}{260015}\sqrt{19}\arctan\left(\frac{1}{19}\sqrt{19}(2\,x+1)\right)+\frac{11049}{260015}\log\left(x^2+x+5\right)+\frac{4822}{4879}\log(5\,x+2)-\frac{3146}{80155}\log(3\,x-7)-\frac{334}{323}\,\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^3-27*x^2+5*x-32)/(30*x^5-13*x^4+50*x^3-286*x^2-299*x-70),x,
algorithm="maxima")
```

```
[Out] 3988/260015*sqrt(19)*arctan(1/19*sqrt(19)*(2*x + 1)) + 11049/260015*log(x^2
 + x + 5) + 4822/4879*log(5*x + 2) - 3146/80155*log(3*x - 7) - 334/323*log(
2*x + 1)
```

Fricas [A] time = 1.56848, size = 216, normalized size = 3.43

$$\frac{3988}{260015}\sqrt{19}\arctan\left(\frac{1}{19}\sqrt{19}(2\,x+1)\right)+\frac{11049}{260015}\log\left(x^2+x+5\right)+\frac{4822}{4879}\log(5\,x+2)-\frac{3146}{80155}\log(3\,x-7)-\frac{334}{323}\,\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^3-27*x^2+5*x-32)/(30*x^5-13*x^4+50*x^3-286*x^2-299*x-70),x,
algorithm="fricas")
```

```
[Out] 3988/260015*sqrt(19)*arctan(1/19*sqrt(19)*(2*x + 1)) + 11049/260015*log(x^2
 + x + 5) + 4822/4879*log(5*x + 2) - 3146/80155*log(3*x - 7) - 334/323*log(
2*x + 1)
```

Sympy [A] time = 0.313844, size = 68, normalized size = 1.08

$$-\frac{3146\log\left(x-\frac{7}{3}\right)}{80155}+\frac{4822\log\left(x+\frac{2}{5}\right)}{4879}-\frac{334\log\left(x+\frac{1}{2}\right)}{323}+\frac{11049\log\left(x^2+x+5\right)}{260015}+\frac{3988\sqrt{19}\,\operatorname{atan}\left(\frac{2\sqrt{19}x}{19}+\frac{\sqrt{19}}{19}\right)}{260015}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**3-27*x**2+5*x-32)/(30*x**5-13*x**4+50*x**3-286*x**2-299*x-7
0),x)
```

```
[Out] -3146*log(x - 7/3)/80155 + 4822*log(x + 2/5)/4879 - 334*log(x + 1/2)/323 +
11049*log(x**2 + x + 5)/260015 + 3988*sqrt(19)*atan(2*sqrt(19)*x/19 + sqrt(
19)/19)/260015
```

Giac [A] time = 1.31912, size = 72, normalized size = 1.14

$$\frac{3988}{260015}\sqrt{19}\arctan\left(\frac{1}{19}\sqrt{19}(2x+1)\right)+\frac{11049}{260015}\log\left(x^2+x+5\right)+\frac{4822}{4879}\log\left(|5x+2|\right)-\frac{3146}{80155}\log\left(|3x-7|\right)-\frac{334}{323}\,\mathrm{l}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^3-27*x^2+5*x-32)/(30*x^5-13*x^4+50*x^3-286*x^2-299*x-70),x,
algorithm="giac")
```

```
[Out] 3988/260015*sqrt(19)*arctan(1/19*sqrt(19)*(2*x + 1)) + 11049/260015*log(x^2
 + x + 5) + 4822/4879*log(abs(5*x + 2)) - 3146/80155*log(abs(3*x - 7)) - 33
4/323*log(abs(2*x + 1))
```

3.321 $\int \frac{8-13x^2-7x^3+12x^5}{4-20x+41x^2-80x^3+116x^4-80x^5+100x^6} \, dx$

Optimal. Leaf size=69

$$-\frac{502x+313}{1452(2x^2+1)}+\frac{2843\log(2x^2+1)}{7986}+\frac{5828}{9075(2-5x)}-\frac{59096\log(2-5x)}{99825}+\frac{503\tan^{-1}(\sqrt{2}x)}{7986\sqrt{2}}$$

```
[Out] 5828/(9075*(2 - 5*x)) - (313 + 502*x)/(1452*(1 + 2*x^2)) + (503*ArcTan[Sqrt
[2]*x])/(7986*Sqrt[2]) - (59096*Log[2 - 5*x])/99825 + (2843*Log[1 + 2*x^2])
/7986
```

Rubi [A] time = 0.0989439, antiderivative size = 86, normalized size of antiderivative = 1.25, number of steps used = 7, number of rules used = 5, integrand size = 50, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {2074, 639, 203, 635, 260}

$$-\frac{502x+313}{1452(2x^2+1)}+\frac{2843\log(2x^2+1)}{7986}+\frac{5828}{9075(2-5x)}-\frac{59096\log(2-5x)}{99825}+\frac{272\sqrt{2}\tan^{-1}(\sqrt{2}x)}{1331}-\frac{251\tan^{-1}(\sqrt{2}x)}{726\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(8 - 13*x^2 - 7*x^3 + 12*x^5)/(4 - 20*x + 41*x^2 - 80*x^3 + 116*x^4 - 8
0*x^5 + 100*x^6),x]
```

```
[Out] 5828/(9075*(2 - 5*x)) - (313 + 502*x)/(1452*(1 + 2*x^2)) - (251*ArcTan[Sqrt
[2]*x])/(726*Sqrt[2]) + (272*Sqrt[2]*ArcTan[Sqrt[2]*x])/1331 - (59096*Log[2
 - 5*x])/99825 + (2843*Log[1 + 2*x^2])/7986
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{8-13x^2-7x^3+12x^5}{4-20x+41x^2-80x^3+116x^4-80x^5+100x^6}\,dx = \int \left(\frac{5828}{1815(-2+5x)^2} - \frac{59096}{19965(-2+5x)} + \frac{-251+313x}{363\left(1+2x^2\right)^2} + \frac{2(81\cdots}{399\cdots}\right.$$

$$= \frac{5828}{9075(2-5x)} - \frac{59096\log(2-5x)}{99825} + \frac{2\int \frac{816+2843x}{1+2x^2}\,dx}{3993} + \frac{1}{363}\int \cdots$$

$$= \frac{5828}{9075(2-5x)} - \frac{313+502x}{1452\left(1+2x^2\right)} - \frac{59096\log(2-5x)}{99825} - \frac{251}{726}\int \frac{\cdots}{1\cdots}$$

$$= \frac{5828}{9075(2-5x)} - \frac{313+502x}{1452\left(1+2x^2\right)} - \frac{251\tan^{-1}\left(\sqrt{2}x\right)}{726\sqrt{2}} + \frac{272\sqrt{2}\tan\cdots}{13\cdots}$$

Mathematica [A] time = 0.0460269, size = 67, normalized size = 0.97

$$\frac{-\frac{33\left(36458x^2+4675x+2554\right)}{10x^3-4x^2+5x-2}+142150\log\left(2x^2+1\right)-236384\log(2-5x)+12575\sqrt{2}\tan^{-1}\left(\sqrt{2}x\right)}{399300}$$

Antiderivative was successfully verified.

```
[In] Integrate[(8 - 13*x^2 - 7*x^3 + 12*x^5)/(4 - 20*x + 41*x^2 - 80*x^3 + 116*x
^4 - 80*x^5 + 100*x^6),x]
```

```
[Out] ((-33*(2554 + 4675*x + 36458*x^2))/(-2 + 5*x - 4*x^2 + 10*x^3) + 12575*Sqrt
[2]*ArcTan[Sqrt[2]*x] - 236384*Log[2 - 5*x] + 142150*Log[1 + 2*x^2])/399300
```

Maple [A] time = 0.013, size = 54, normalized size = 0.8

$$-\frac{5828}{45375\,x-18150}-\frac{59096\,\ln\left(5\,x-2\right)}{99825}+\frac{1}{3993}\left(-\frac{2761\,x}{4}-\frac{3443}{8}\right)\left(x^2+\frac{1}{2}\right)^{-1}+\frac{2843\,\ln\left(2\,x^2+1\right)}{7986}+\frac{503\,\arctan\left(x\sqrt{}\right.}{15972}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((12*x^5-7*x^3-13*x^2+8)/(100*x^6-80*x^5+116*x^4-80*x^3+41*x^2-20*x+4),x
)
```

```
[Out] -5828/9075/(5*x-2)-59096/99825*ln(5*x-2)+1/3993*(-2761/4*x-3443/8)/(x^2+1/2
)+2843/7986*ln(2*x^2+1)+503/15972*arctan(x*2^(1/2))*2^(1/2)
```

Maxima [A] time = 1.54023, size = 80, normalized size = 1.16

$$\frac{503}{15972}\,\sqrt{2}\arctan\left(\sqrt{2}x\right)-\frac{36458\,x^2+4675\,x+2554}{12100\left(10\,x^3-4\,x^2+5\,x-2\right)}+\frac{2843}{7986}\,\log\left(2\,x^2+1\right)-\frac{59096}{99825}\,\log\left(5\,x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((12*x^5-7*x^3-13*x^2+8)/(100*x^6-80*x^5+116*x^4-80*x^3+41*x^2-20*
x+4),x, algorithm="maxima")
```

```
[Out] 503/15972*sqrt(2)*arctan(sqrt(2)*x) - 1/12100*(36458*x^2 + 4675*x + 2554)/(
10*x^3 - 4*x^2 + 5*x - 2) + 2843/7986*log(2*x^2 + 1) - 59096/99825*log(5*x
- 2)
```

Fricas [A] time = 1.49578, size = 312, normalized size = 4.52

$$\frac{12575\,\sqrt{2}\left(10\,x^3-4\,x^2+5\,x-2\right)\arctan\left(\sqrt{2}x\right)-1203114\,x^2+142150\left(10\,x^3-4\,x^2+5\,x-2\right)\log\left(2\,x^2+1\right)-236384}{399300\left(10\,x^3-4\,x^2+5\,x-2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((12*x^5-7*x^3-13*x^2+8)/(100*x^6-80*x^5+116*x^4-80*x^3+41*x^2-20*
x+4),x, algorithm="fricas")
```

```
[Out] 1/399300*(12575*sqrt(2)*(10*x^3 - 4*x^2 + 5*x - 2)*arctan(sqrt(2)*x) - 1203
114*x^2 + 142150*(10*x^3 - 4*x^2 + 5*x - 2)*log(2*x^2 + 1) - 236384*(10*x^3
 - 4*x^2 + 5*x - 2)*log(5*x - 2) - 154275*x - 84282)/(10*x^3 - 4*x^2 + 5*x
- 2)
```

Sympy [A] time = 0.196721, size = 63, normalized size = 0.91

$$-\frac{36458x^2 + 4675x + 2554}{121000x^3 - 48400x^2 + 60500x - 24200} - \frac{59096\log\left(x - \frac{2}{5}\right)}{99825} + \frac{2843\log\left(x^2 + \frac{1}{2}\right)}{7986} + \frac{503\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x\right)}{15972}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((12*x**5-7*x**3-13*x**2+8)/(100*x**6-80*x**5+116*x**4-80*x**3+41*
x**2-20*x+4),x)
```

```
[Out] -(36458*x**2 + 4675*x + 2554)/(121000*x**3 - 48400*x**2 + 60500*x - 24200)
- 59096*log(x - 2/5)/99825 + 2843*log(x**2 + 1/2)/7986 + 503*sqrt(2)*atan(s
qrt(2)*x)/15972
```

Giac [A] time = 1.40431, size = 80, normalized size = 1.16

$$\frac{503}{15972}\sqrt{2}\arctan\left(\sqrt{2}x\right) - \frac{36458\,x^2 + 4675\,x + 2554}{12100\left(2\,x^2 + 1\right)(5\,x - 2)} + \frac{2843}{7986}\log\left(2\,x^2 + 1\right) - \frac{59096}{99825}\log\left(|5\,x - 2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((12*x^5-7*x^3-13*x^2+8)/(100*x^6-80*x^5+116*x^4-80*x^3+41*x^2-20*
x+4),x, algorithm="giac")
```

```
[Out] 503/15972*sqrt(2)*arctan(sqrt(2)*x) - 1/12100*(36458*x^2 + 4675*x + 2554)/(
(2*x^2 + 1)*(5*x - 2)) + 2843/7986*log(2*x^2 + 1) - 59096/99825*log(abs(5*x
 - 2))
```

3.322 $\int \frac{9+x^4}{x^2(9+x^2)}\,dx$

Optimal. Leaf size=17

$$x-\frac{1}{x}-\frac{10}{3}\tan^{-1}\left(\frac{x}{3}\right)$$

```
[Out] -x^(-1) + x - (10*ArcTan[x/3])/3
```

Rubi [A] time = 0.0128914, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1262, 203}

$$x-\frac{1}{x}-\frac{10}{3}\tan^{-1}\left(\frac{x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(9 + x^4)/(x^2*(9 + x^2)),x]
```

```
[Out] -x^(-1) + x - (10*ArcTan[x/3])/3
```

Rule 1262

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p,
x], x] /; FreeQ[{a, c, d, e, f, m, q}, x] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{9+x^4}{x^2\left(9+x^2\right)}\,dx = \int \left(1+\frac{1}{x^2}-\frac{10}{9+x^2}\right)dx$$
$$= -\frac{1}{x}+x-10\int \frac{1}{9+x^2}\,dx$$
$$= -\frac{1}{x}+x-\frac{10}{3}\tan^{-1}\left(\frac{x}{3}\right)$$

Mathematica [A] time = 0.0059834, size = 17, normalized size = 1.

$$x-\frac{1}{x}-\frac{10}{3}\tan^{-1}\left(\frac{x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(9 + x^4)/(x^2*(9 + x^2)),x]
```

```
[Out] -x^(-1) + x - (10*ArcTan[x/3])/3
```

Maple [A] time = 0.004, size = 14, normalized size = 0.8

$$-x^{-1}+x-\frac{10}{3}\arctan\left(\frac{x}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+9)/x^2/(x^2+9),x)
```

```
[Out] -1/x+x-10/3*arctan(1/3*x)
```

Maxima [A] time = 1.56195, size = 18, normalized size = 1.06

$$x-\frac{1}{x}-\frac{10}{3}\arctan\left(\frac{1}{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+9)/x^2/(x^2+9),x, algorithm="maxima")
```

```
[Out] x - 1/x - 10/3*arctan(1/3*x)
```

Fricas [A] time = 1.52136, size = 54, normalized size = 3.18

$$\frac{3x^2 - 10x\arctan\left(\frac{1}{3}x\right) - 3}{3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+9)/x^2/(x^2+9),x, algorithm="fricas")
```

```
[Out] 1/3*(3*x^2 - 10*x*arctan(1/3*x) - 3)/x
```

Sympy [A] time = 0.098892, size = 12, normalized size = 0.71

$$x - \frac{10\operatorname{atan}\left(\frac{x}{3}\right)}{3} - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+9)/x**2/(x**2+9),x)
```

```
[Out] x - 10*atan(x/3)/3 - 1/x
```

Giac [A] time = 1.22681, size = 18, normalized size = 1.06

$$x - \frac{1}{x} - \frac{10}{3}\arctan\left(\frac{1}{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+9)/x^2/(x^2+9),x, algorithm="giac")
```

```
[Out] x - 1/x - 10/3*arctan(1/3*x)
```

3.323 $\int \frac{2x+x^4}{1+x^2} \, dx$

Optimal. Leaf size=19

$$\frac{x^3}{3} + \log\left(x^2+1\right) - x + \tan^{-1}(x)$$

```
[Out] -x + x^3/3 + ArcTan[x] + Log[1 + x^2]
```

Rubi [A] time = 0.0267939, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1593, 1802, 635, 203, 260}

$$\frac{x^3}{3} + \log\left(x^2+1\right) - x + \tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(2*x + x^4)/(1 + x^2),x]
```

```
[Out] -x + x^3/3 + ArcTan[x] + Log[1 + x^2]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{2x+x^4}{1+x^2}\,dx &= \int \frac{x\left(2+x^3\right)}{1+x^2}\,dx\\ &= \int \left(-1+x^2+\frac{1+2x}{1+x^2}\right)dx\\ &= -x+\frac{x^3}{3}+\int \frac{1+2x}{1+x^2}\,dx\\ &= -x+\frac{x^3}{3}+2\int \frac{x}{1+x^2}\,dx+\int \frac{1}{1+x^2}\,dx\\ &= -x+\frac{x^3}{3}+\tan^{-1}(x)+\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.0049293, size = 19, normalized size = 1.

$$\frac{x^3}{3}+\log\left(x^2+1\right)-x+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*x + x^4)/(1 + x^2),x]
```

```
[Out] -x + x^3/3 + ArcTan[x] + Log[1 + x^2]
```

Maple [A] time = 0.001, size = 18, normalized size = 1.

$$-x+\frac{x^3}{3}+\arctan(x)+\ln\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+2*x)/(x^2+1),x)
```

```
[Out] -x+1/3*x^3+arctan(x)+ln(x^2+1)
```

Maxima [A] time = 1.46024, size = 23, normalized size = 1.21

$$\frac{1}{3}x^3 - x + \arctan(x) + \log\left(x^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+2*x)/(x^2+1),x, algorithm="maxima")
```

```
[Out] 1/3*x^3 - x + arctan(x) + log(x^2 + 1)
```

Fricas [A] time = 1.46225, size = 54, normalized size = 2.84

$$\frac{1}{3}x^3 - x + \arctan(x) + \log\left(x^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+2*x)/(x^2+1),x, algorithm="fricas")
```

```
[Out] 1/3*x^3 - x + arctan(x) + log(x^2 + 1)
```

Sympy [A] time = 0.090904, size = 15, normalized size = 0.79

$$\frac{x^3}{3} - x + \log\left(x^2 + 1\right) + \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+2*x)/(x**2+1),x)
```

```
[Out] x**3/3 - x + log(x**2 + 1) + atan(x)
```

Giac [A] time = 1.2895, size = 23, normalized size = 1.21

$$\frac{1}{3}x^3 - x + \arctan(x) + \log\left(x^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+2*x)/(x^2+1),x, algorithm="giac")
```

```
[Out] 1/3*x^3 - x + arctan(x) + log(x^2 + 1)
```

3.324 $\int \frac{-x+x^3}{(-1+x)^2(1+x^2)}\,dx$

Optimal. Leaf size=9

$$\log(1-x)+\tan^{-1}(x)$$

```
[Out] ArcTan[x] + Log[1 - x]
```

Rubi [A] time = 0.0752009, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1586, 1593, 1629, 203}

$$\log(1-x)+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-x + x^3)/((-1 + x)^2*(1 + x^2)),x]
```

```
[Out] ArcTan[x] + Log[1 - x]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{-x+x^3}{(-1+x)^2\left(1+x^2\right)}\,dx &= \int \frac{x+x^2}{(-1+x)\left(1+x^2\right)}\,dx\\
&= \int \frac{x(1+x)}{(-1+x)\left(1+x^2\right)}\,dx\\
&= \int \left(\frac{1}{-1+x}+\frac{1}{1+x^2}\right)dx\\
&= \log(1-x)+\int \frac{1}{1+x^2}\,dx\\
&= \tan^{-1}(x)+\log(1-x)
\end{aligned}
$$

Mathematica [A] time = 0.0058596, size = 9, normalized size = 1.

$$\log(1-x)+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x + x^3)/((-1 + x)^2*(1 + x^2)),x]
```

```
[Out] ArcTan[x] + Log[1 - x]
```

Maple [A] time = 0.004, size = 8, normalized size = 0.9

$$\arctan(x)+\ln(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-x)/(x-1)^2/(x^2+1),x)
```

```
[Out] arctan(x)+ln(x-1)
```

Maxima [A] time = 1.58857, size = 9, normalized size = 1.

$$\arctan(x) + \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x)/(-1+x)^2/(x^2+1),x, algorithm="maxima")

[Out] arctan(x) + log(x - 1)
```

Fricas [A] time = 1.60499, size = 32, normalized size = 3.56

$$\arctan(x) + \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x)/(-1+x)^2/(x^2+1),x, algorithm="fricas")

[Out] arctan(x) + log(x - 1)
```

Sympy [A] time = 0.121915, size = 7, normalized size = 0.78

$$\log(x - 1) + \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-x)/(-1+x)**2/(x**2+1),x)

[Out] log(x - 1) + atan(x)
```

Giac [B] time = 1.22508, size = 38, normalized size = 4.22

$$\frac{1}{4}\pi - \pi\left\lfloor \frac{\pi + 4\arctan(x)}{4\pi} + \frac{1}{2} \right\rfloor + \arctan(x) + \log(|x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x)/(-1+x)^2/(x^2+1),x, algorithm="giac")
```

```
[Out] 1/4*pi - pi*floor(1/4*(pi + 4*arctan(x))/pi + 1/2) + arctan(x) + log(abs(x
- 1))
```

3.325 $\int \frac{2+5x+3x^2+2x^3}{1+x+x^2} dx$

Optimal. Leaf size=12

$$x^2 + \log\left(x^2 + x + 1\right) + x$$

```
[Out] x + x^2 + Log[1 + x + x^2]
```

Rubi [A] time = 0.0162013, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1657, 628}

$$x^2 + \log\left(x^2 + x + 1\right) + x$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 5*x + 3*x^2 + 2*x^3)/(1 + x + x^2),x]
```

```
[Out] x + x^2 + Log[1 + x + x^2]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{2+5x+3x^2+2x^3}{1+x+x^2} \, dx &= \int \left(1+2x+\frac{1+2x}{1+x+x^2}\right) dx \\
&= x+x^2+\int \frac{1+2x}{1+x+x^2} \, dx \\
&= x+x^2+\log\left(1+x+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.0053581, size = 12, normalized size = 1.

$$x^2 + \log\left(x^2 + x + 1\right) + x$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 5*x + 3*x^2 + 2*x^3)/(1 + x + x^2),x]
```

```
[Out] x + x^2 + Log[1 + x + x^2]
```

Maple [A] time = 0.001, size = 13, normalized size = 1.1

$$x + x^2 + \ln\left(x^2 + x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3+3*x^2+5*x+2)/(x^2+x+1),x)
```

```
[Out] x+x^2+ln(x^2+x+1)
```

Maxima [A] time = 1.04541, size = 16, normalized size = 1.33

$$x^2 + x + \log\left(x^2 + x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+5*x+2)/(x^2+x+1),x, algorithm="maxima")
```

```
[Out] x^2 + x + log(x^2 + x + 1)
```

Fricas [A] time = 1.52452, size = 38, normalized size = 3.17

$$x^2 + x + \log\left(x^2 + x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+5*x+2)/(x^2+x+1),x, algorithm="fricas")
```

```
[Out] x^2 + x + log(x^2 + x + 1)
```

Sympy [A] time = 0.079665, size = 12, normalized size = 1.

$$x^2 + x + \log\left(x^2 + x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**3+3*x**2+5*x+2)/(x**2+x+1),x)
```

```
[Out] x**2 + x + log(x**2 + x + 1)
```

Giac [A] time = 1.12055, size = 16, normalized size = 1.33

$$x^2 + x + \log\left(x^2 + x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+5*x+2)/(x^2+x+1),x, algorithm="giac")
```

```
[Out] x^2 + x + log(x^2 + x + 1)
```

3.326 $\int \frac{3-4x-5x^2+3x^3}{x^3\left(-1+x+x^2\right)}\,dx$

Optimal. Leaf size=65

$$\frac{3}{2x^2}-\frac{1}{x}+3\log(x)-\frac{1}{10}\left(15-\sqrt{5}\right)\log\left(2x-\sqrt{5}+1\right)-\frac{1}{10}\left(15+\sqrt{5}\right)\log\left(2x+\sqrt{5}+1\right)$$

```
[Out] 3/(2*x^2) - x^(-1) + 3*Log[x] - ((15 - Sqrt[5])*Log[1 - Sqrt[5] + 2*x])/10
- ((15 + Sqrt[5])*Log[1 + Sqrt[5] + 2*x])/10
```

Rubi [A] time = 0.0615783, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1628, 632, 31}

$$\frac{3}{2x^2}-\frac{1}{x}+3\log(x)-\frac{1}{10}\left(15-\sqrt{5}\right)\log\left(2x-\sqrt{5}+1\right)-\frac{1}{10}\left(15+\sqrt{5}\right)\log\left(2x+\sqrt{5}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 4*x - 5*x^2 + 3*x^3)/(x^3*(-1 + x + x^2)),x]
```

```
[Out] 3/(2*x^2) - x^(-1) + 3*Log[x] - ((15 - Sqrt[5])*Log[1 - Sqrt[5] + 2*x])/10
- ((15 + Sqrt[5])*Log[1 + Sqrt[5] + 2*x])/10
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{3-4x-5x^2+3x^3}{x^3\left(-1+x+x^2\right)}\,dx &= \int \left(-\frac{3}{x^3}+\frac{1}{x^2}+\frac{3}{x}+\frac{-1-3x}{-1+x+x^2}\right)dx \\
&= \frac{3}{2x^2}-\frac{1}{x}+3\log(x)+\int \frac{-1-3x}{-1+x+x^2}\,dx \\
&= \frac{3}{2x^2}-\frac{1}{x}+3\log(x)+\frac{1}{10}\left(-15+\sqrt{5}\right)\int \frac{1}{\frac{1}{2}-\frac{\sqrt{5}}{2}+x}\,dx-\frac{1}{10}\left(15+\sqrt{5}\right)\int \frac{1}{\frac{1}{2}+\frac{\sqrt{5}}{2}+x}\,dx \\
&= \frac{3}{2x^2}-\frac{1}{x}+3\log(x)-\frac{1}{10}\left(15-\sqrt{5}\right)\log\left(1-\sqrt{5}+2x\right)-\frac{1}{10}\left(15+\sqrt{5}\right)\log\left(1+\sqrt{5}+2x\right)
\end{aligned}
$$

Mathematica [A] time = 0.0368658, size = 58, normalized size = 0.89

$$\frac{1}{10}\left(\frac{15}{x^2}-\frac{10}{x}+\left(\sqrt{5}-15\right)\log\left(-2x+\sqrt{5}-1\right)+30\log(x)-\left(15+\sqrt{5}\right)\log\left(2x+\sqrt{5}+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 4*x - 5*x^2 + 3*x^3)/(x^3*(-1 + x + x^2)),x]
```

```
[Out] (15/x^2 - 10/x + (-15 + Sqrt[5])*Log[-1 + Sqrt[5] - 2*x] + 30*Log[x] - (15
+ Sqrt[5])*Log[1 + Sqrt[5] + 2*x])/10
```

Maple [A] time = 0.007, size = 41, normalized size = 0.6

$$-x^{-1}+\frac{3}{2\,x^2}+3\,\ln(x)-\frac{3\,\ln\left(x^2+x-1\right)}{2}-\frac{\sqrt{5}}{5}\mathrm{Artanh}\left(\frac{(1+2\,x)\sqrt{5}}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^3-5*x^2-4*x+3)/x^3/(x^2+x-1),x)
```

```
[Out] -1/x+3/2/x^2+3*ln(x)-3/2*ln(x^2+x-1)-1/5*5^(1/2)*arctanh(1/5*(1+2*x)*5^(1/2
))
```

Maxima [A] time = 1.67304, size = 69, normalized size = 1.06

$$\frac{1}{10}\sqrt{5}\log\left(\frac{2x-\sqrt{5}+1}{2x+\sqrt{5}+1}\right)-\frac{2x-3}{2x^2}-\frac{3}{2}\log\left(x^2+x-1\right)+3\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^3-5*x^2-4*x+3)/x^3/(x^2+x-1),x, algorithm="maxima")
```

```
[Out] 1/10*sqrt(5)*log((2*x - sqrt(5) + 1)/(2*x + sqrt(5) + 1)) - 1/2*(2*x - 3)/x
^2 - 3/2*log(x^2 + x - 1) + 3*log(x)
```

Fricas [A] time = 1.49938, size = 182, normalized size = 2.8

$$\frac{\sqrt{5}x^2\log\left(\frac{2x^2-\sqrt{5}(2x+1)+2x+3}{x^2+x-1}\right)-15x^2\log\left(x^2+x-1\right)+30x^2\log\left(x\right)-10x+15}{10x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^3-5*x^2-4*x+3)/x^3/(x^2+x-1),x, algorithm="fricas")
```

```
[Out] 1/10*(sqrt(5)*x^2*log((2*x^2 - sqrt(5)*(2*x + 1) + 2*x + 3)/(x^2 + x - 1))
- 15*x^2*log(x^2 + x - 1) + 30*x^2*log(x) - 10*x + 15)/x^2
```

Sympy [A] time = 0.496657, size = 99, normalized size = 1.52

$$3\log\left(x\right)+\left(-\frac{3}{2}+\frac{\sqrt{5}}{10}\right)\log\left(x-\frac{405}{202}-\frac{35\sqrt{5}}{202}+\frac{110\left(-\frac{3}{2}+\frac{\sqrt{5}}{10}\right)^2}{101}\right)+\left(-\frac{3}{2}-\frac{\sqrt{5}}{10}\right)\log\left(x-\frac{405}{202}+\frac{35\sqrt{5}}{202}+\frac{110\left(-\frac{3}{2}-\frac{\sqrt{5}}{10}\right)}{101}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**3-5*x**2-4*x+3)/x**3/(x**2+x-1),x)
```

```
[Out] 3*log(x) + (-3/2 + sqrt(5)/10)*log(x - 405/202 - 35*sqrt(5)/202 + 110*(-3/2
 + sqrt(5)/10)**2/101) + (-3/2 - sqrt(5)/10)*log(x - 405/202 + 35*sqrt(5)/2
```

```
02 + 110*(-3/2 - sqrt(5)/10)**2/101) - (2*x - 3)/(2*x**2)
```

Giac [A] time = 1.20338, size = 74, normalized size = 1.14

$$\frac{1}{10}\sqrt{5}\log\left(\frac{\left|2x-\sqrt{5}+1\right|}{\left|2x+\sqrt{5}+1\right|}\right)-\frac{2x-3}{2x^2}-\frac{3}{2}\log\left(\left|x^2+x-1\right|\right)+3\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^3-5*x^2-4*x+3)/x^3/(x^2+x-1),x, algorithm="giac")
```

```
[Out] 1/10*sqrt(5)*log(abs(2*x - sqrt(5) + 1)/abs(2*x + sqrt(5) + 1)) - 1/2*(2*x
- 3)/x^2 - 3/2*log(abs(x^2 + x - 1)) + 3*log(abs(x))
```

3.327 $\int \frac{4+8x+5x^2+2x^3}{(2+2x+x^2)^2}\,dx$

Optimal. Leaf size=28

$$-\frac{1}{x^2+2x+2}+\log\left(x^2+2x+2\right)-\tan^{-1}(x+1)$$

```
[Out] -(2 + 2*x + x^2)^(-1) - ArcTan[1 + x] + Log[2 + 2*x + x^2]
```

Rubi [A] time = 0.0219627, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1660, 634, 617, 204, 628}

$$-\frac{1}{x^2+2x+2}+\log\left(x^2+2x+2\right)-\tan^{-1}(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(4 + 8*x + 5*x^2 + 2*x^3)/(2 + 2*x + x^2)^2,x]
```

```
[Out] -(2 + 2*x + x^2)^(-1) - ArcTan[1 + x] + Log[2 + 2*x + x^2]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{4+8x+5x^2+2x^3}{\left(2+2x+x^2\right)^2}\,dx &= -\frac{1}{2+2x+x^2}+\frac{1}{4}\int \frac{4+8x}{2+2x+x^2}\,dx \\
&= -\frac{1}{2+2x+x^2}-\int \frac{1}{2+2x+x^2}\,dx+\int \frac{2+2x}{2+2x+x^2}\,dx \\
&= -\frac{1}{2+2x+x^2}+\log\left(2+2x+x^2\right)+\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1+x\right) \\
&= -\frac{1}{2+2x+x^2}-\tan^{-1}(1+x)+\log\left(2+2x+x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.0096268, size = 28, normalized size = 1.

$$
-\frac{1}{x^2+2x+2}+\log\left(x^2+2x+2\right)-\tan^{-1}(x+1)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + 8*x + 5*x^2 + 2*x^3)/(2 + 2*x + x^2)^2,x]
```

```
[Out] -(2 + 2*x + x^2)^(-1) - ArcTan[1 + x] + Log[2 + 2*x + x^2]
```

Maple [A] time = 0.004, size = 29, normalized size = 1.

$$-\left(x^2+2x+2\right)^{-1}-\arctan(1+x)+\ln\left(x^2+2x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3+5*x^2+8*x+4)/(x^2+2*x+2)^2,x)
```

```
[Out] -1/(x^2+2*x+2)-arctan(1+x)+ln(x^2+2*x+2)
```

Maxima [A] time = 1.68009, size = 38, normalized size = 1.36

$$-\frac{1}{x^2+2x+2}-\arctan(x+1)+\log\left(x^2+2x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+5*x^2+8*x+4)/(x^2+2*x+2)^2,x, algorithm="maxima")
```

```
[Out] -1/(x^2 + 2*x + 2) - arctan(x + 1) + log(x^2 + 2*x + 2)
```

Fricas [A] time = 1.54265, size = 123, normalized size = 4.39

$$-\frac{\left(x^2+2x+2\right)\arctan(x+1)-\left(x^2+2x+2\right)\log\left(x^2+2x+2\right)+1}{x^2+2x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+5*x^2+8*x+4)/(x^2+2*x+2)^2,x, algorithm="fricas")
```

```
[Out] -((x^2 + 2*x + 2)*arctan(x + 1) - (x^2 + 2*x + 2)*log(x^2 + 2*x + 2) + 1)/(
x^2 + 2*x + 2)
```

Sympy [A] time = 0.121162, size = 24, normalized size = 0.86

$$\log\left(x^2+2x+2\right)-\operatorname{atan}(x+1)-\frac{1}{x^2+2x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**3+5*x**2+8*x+4)/(x**2+2*x+2)**2,x)
```

```
[Out] log(x**2 + 2*x + 2) - atan(x + 1) - 1/(x**2 + 2*x + 2)
```

Giac [A] time = 1.22375, size = 38, normalized size = 1.36

$$-\frac{1}{x^2+2\,x+2}-\arctan\left(x+1\right)+\log\left(x^2+2\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+5*x^2+8*x+4)/(x^2+2*x+2)^2,x, algorithm="giac")
```

```
[Out] -1/(x^2 + 2*x + 2) - arctan(x + 1) + log(x^2 + 2*x + 2)
```

3.328 $\int \frac{(-1+x)^4 x^4}{1+x^2} \, dx$

Optimal. Leaf size=32

$$\frac{x^7}{7} - \frac{2x^6}{3} + x^5 - \frac{4x^3}{3} + 4x - 4\tan^{-1}(x)$$

```
[Out] 4*x - (4*x^3)/3 + x^5 - (2*x^6)/3 + x^7/7 - 4*ArcTan[x]
```

Rubi [A] time = 0.0400596, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1629, 203}

$$\frac{x^7}{7} - \frac{2x^6}{3} + x^5 - \frac{4x^3}{3} + 4x - 4\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x)^4*x^4)/(1 + x^2),x]
```

```
[Out] 4*x - (4*x^3)/3 + x^5 - (2*x^6)/3 + x^7/7 - 4*ArcTan[x]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{(-1+x)^4 x^4}{1+x^2}\,dx = \int \left(4-4x^2+5x^4-4x^5+x^6-\frac{4}{1+x^2}\right)dx$$
$$= 4x-\frac{4x^3}{3}+x^5-\frac{2x^6}{3}+\frac{x^7}{7}-4\int \frac{1}{1+x^2}\,dx$$
$$= 4x-\frac{4x^3}{3}+x^5-\frac{2x^6}{3}+\frac{x^7}{7}-4\tan^{-1}(x)$$

Mathematica [A] time = 0.0240289, size = 32, normalized size = 1.

$$\frac{x^7}{7}-\frac{2x^6}{3}+x^5-\frac{4x^3}{3}+4x-4\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x)^4*x^4)/(1 + x^2),x]
```

```
[Out] 4*x - (4*x^3)/3 + x^5 - (2*x^6)/3 + x^7/7 - 4*ArcTan[x]
```

Maple [A] time = 0.003, size = 27, normalized size = 0.8

$$4\,x-\frac{4\,x^3}{3}+x^5-\frac{2\,x^6}{3}+\frac{x^7}{7}-4\,\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-1)^4*x^4/(x^2+1),x)
```

```
[Out] 4*x-4/3*x^3+x^5-2/3*x^6+1/7*x^7-4*arctan(x)
```

Maxima [A] time = 1.71845, size = 35, normalized size = 1.09

$$\frac{1}{7}\,x^7-\frac{2}{3}\,x^6+x^5-\frac{4}{3}\,x^3+4\,x-4\,\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^4*x^4/(x^2+1),x, algorithm="maxima")
```

```
[Out] 1/7*x^7 - 2/3*x^6 + x^5 - 4/3*x^3 + 4*x - 4*arctan(x)
```

Fricas [A] time = 1.44205, size = 74, normalized size = 2.31

$$\frac{1}{7}x^7 - \frac{2}{3}x^6 + x^5 - \frac{4}{3}x^3 + 4x - 4\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^4*x^4/(x^2+1),x, algorithm="fricas")
```

```
[Out] 1/7*x^7 - 2/3*x^6 + x^5 - 4/3*x^3 + 4*x - 4*arctan(x)
```

Sympy [A] time = 0.090415, size = 29, normalized size = 0.91

$$\frac{x^7}{7} - \frac{2x^6}{3} + x^5 - \frac{4x^3}{3} + 4x - 4\operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)**4*x**4/(x**2+1),x)
```

```
[Out] x**7/7 - 2*x**6/3 + x**5 - 4*x**3/3 + 4*x - 4*atan(x)
```

Giac [A] time = 1.43134, size = 35, normalized size = 1.09

$$\frac{1}{7}x^7 - \frac{2}{3}x^6 + x^5 - \frac{4}{3}x^3 + 4x - 4\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^4*x^4/(x^2+1),x, algorithm="giac")
```

```
[Out] 1/7*x^7 - 2/3*x^6 + x^5 - 4/3*x^3 + 4*x - 4*arctan(x)
```

3.329 $\int \frac{-20x+4x^2}{9-10x^2+x^4}\,dx$

Optimal. Leaf size=31

$$\log(1-x)-\frac{1}{2}\log(3-x)+\frac{3}{2}\log(x+1)-2\log(x+3)$$

```
[Out] Log[1 - x] - Log[3 - x]/2 + (3*Log[1 + x])/2 - 2*Log[3 + x]
```

Rubi [A] time = 0.0487104, antiderivative size = 41, normalized size of antiderivative = 1.32, number of steps used = 11, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {1593, 1662, 12, 1107, 616, 31, 1130, 207}

$$\frac{5}{4}\log\left(1-x^2\right)-\frac{5}{4}\log\left(9-x^2\right)-\frac{3}{2}\tanh^{-1}\left(\frac{x}{3}\right)+\frac{1}{2}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-20*x + 4*x^2)/(9 - 10*x^2 + x^4),x]
```

```
[Out] (-3*ArcTanh[x/3])/2 + ArcTanh[x]/2 + (5*Log[1 - x^2])/4 - (5*Log[9 - x^2])/
4
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1662

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
 + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !Po
lyQ[Pq, x^2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 1130

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{-20x+4x^2}{9-10x^2+x^4}\,dx &= \int \frac{x(-20+4x)}{9-10x^2+x^4}\,dx\\ &= \int -\frac{20x}{9-10x^2+x^4}\,dx + \int \frac{4x^2}{9-10x^2+x^4}\,dx\\ &= 4\int \frac{x^2}{9-10x^2+x^4}\,dx - 20\int \frac{x}{9-10x^2+x^4}\,dx\\ &= -\left(\frac{1}{2}\int \frac{1}{-1+x^2}\,dx\right) + \frac{9}{2}\int \frac{1}{-9+x^2}\,dx - 10\,\mathrm{Subst}\left(\int \frac{1}{9-10x+x^2}\,dx, x, x^2\right)\\ &= -\frac{3}{2}\tanh^{-1}\left(\frac{x}{3}\right) + \frac{1}{2}\tanh^{-1}(x) - \frac{5}{4}\,\mathrm{Subst}\left(\int \frac{1}{-9+x}\,dx, x, x^2\right) + \frac{5}{4}\,\mathrm{Subst}\left(\int \frac{1}{-1+x}\,dx, x, x^2\right)\\ &= -\frac{3}{2}\tanh^{-1}\left(\frac{x}{3}\right) + \frac{1}{2}\tanh^{-1}(x) + \frac{5}{4}\log\left(1-x^2\right) - \frac{5}{4}\log\left(9-x^2\right)\end{aligned}$$

Mathematica [A] time = 0.0069375, size = 39, normalized size = 1.26

$$4\left(\frac{1}{4}\log(1-x) - \frac{1}{8}\log(3-x) + \frac{3}{8}\log(x+1) - \frac{1}{2}\log(x+3)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-20*x + 4*x^2)/(9 - 10*x^2 + x^4),x]
```

```
[Out] 4*(Log[1 - x]/4 - Log[3 - x]/8 + (3*Log[1 + x])/8 - Log[3 + x]/2)
```

Maple [A] time = 0.007, size = 24, normalized size = 0.8

$$\ln(x-1) - \frac{\ln(-3+x)}{2} + \frac{3\ln(1+x)}{2} - 2\ln(3+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2-20*x)/(x^4-10*x^2+9),x)
```

```
[Out] ln(x-1)-1/2*ln(-3+x)+3/2*ln(1+x)-2*ln(3+x)
```

Maxima [A] time = 1.00401, size = 31, normalized size = 1.

$$-2 \log(x+3) + \frac{3}{2} \log(x+1) + \log(x-1) - \frac{1}{2} \log(x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-20*x)/(x^4-10*x^2+9),x, algorithm="maxima")
```

```
[Out] -2*log(x + 3) + 3/2*log(x + 1) + log(x - 1) - 1/2*log(x - 3)
```

Fricas [A] time = 1.46842, size = 84, normalized size = 2.71

$$-2 \log(x+3) + \frac{3}{2} \log(x+1) + \log(x-1) - \frac{1}{2} \log(x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-20*x)/(x^4-10*x^2+9),x, algorithm="fricas")
```

```
[Out] -2*log(x + 3) + 3/2*log(x + 1) + log(x - 1) - 1/2*log(x - 3)
```

Sympy [A] time = 0.170447, size = 26, normalized size = 0.84

$$-\frac{\log(x-3)}{2} + \log(x-1) + \frac{3\log(x+1)}{2} - 2\log(x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2-20*x)/(x**4-10*x**2+9),x)
```

```
[Out] -log(x - 3)/2 + log(x - 1) + 3*log(x + 1)/2 - 2*log(x + 3)
```

Giac [A] time = 1.12549, size = 36, normalized size = 1.16

$$-2 \log(|x+3|) + \frac{3}{2} \log(|x+1|) + \log(|x-1|) - \frac{1}{2} \log(|x-3|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-20*x)/(x^4-10*x^2+9),x, algorithm="giac")
```

```
[Out] -2*log(abs(x + 3)) + 3/2*log(abs(x + 1)) + log(abs(x - 1)) - 1/2*log(abs(x
- 3))
```

3.330 $\int \frac{-1+x+4x^3}{(-1+x)x^2(1+x^2)}\,dx$

Optimal. Leaf size=24

$$-\log\left(x^2+1\right)-\frac{1}{x}+2\log(1-x)+\tan^{-1}(x)$$

```
[Out] -x^(-1) + ArcTan[x] + 2*Log[1 - x] - Log[1 + x^2]
```

Rubi [A] time = 0.1668, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6725, 635, 203, 260}

$$-\log\left(x^2+1\right)-\frac{1}{x}+2\log(1-x)+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x + 4*x^3)/((-1 + x)*x^2*(1 + x^2)),x]
```

```
[Out] -x^(-1) + ArcTan[x] + 2*Log[1 - x] - Log[1 + x^2]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+x+4x^3}{(-1+x)x^2(1+x^2)}\,dx &= \int \left(\frac{2}{-1+x}+\frac{1}{x^2}+\frac{1-2x}{1+x^2}\right)dx\\ &= -\frac{1}{x}+2\log(1-x)+\int \frac{1-2x}{1+x^2}\,dx\\ &= -\frac{1}{x}+2\log(1-x)-2\int \frac{x}{1+x^2}\,dx+\int \frac{1}{1+x^2}\,dx\\ &= -\frac{1}{x}+\tan^{-1}(x)+2\log(1-x)-\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.0085351, size = 24, normalized size = 1.

$$-\log\left(x^2+1\right)-\frac{1}{x}+2\log(1-x)+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x + 4*x^3)/((-1 + x)*x^2*(1 + x^2)),x]
```

```
[Out] -x^(-1) + ArcTan[x] + 2*Log[1 - x] - Log[1 + x^2]
```

Maple [A] time = 0.007, size = 23, normalized size = 1.

$$-\ln\left(x^2+1\right)+\arctan(x)+2\,\ln(x-1)-x^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^3+x-1)/(x-1)/x^2/(x^2+1),x)
```

```
[Out] -ln(x^2+1)+arctan(x)+2*ln(x-1)-1/x
```

Maxima [A] time = 1.48614, size = 30, normalized size = 1.25

$$-\frac{1}{x}+\arctan(x)-\log\left(x^2+1\right)+2\,\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^3+x-1)/(-1+x)/x^2/(x^2+1),x, algorithm="maxima")
```

```
[Out] -1/x + arctan(x) - log(x^2 + 1) + 2*log(x - 1)
```

Fricas [A] time = 1.46124, size = 74, normalized size = 3.08

$$\frac{x\arctan\left(x\right) - x\log\left(x^2+1\right) + 2\,x\log\left(x-1\right) - 1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^3+x-1)/(-1+x)/x^2/(x^2+1),x, algorithm="fricas")
```

```
[Out] (x*arctan(x) - x*log(x^2 + 1) + 2*x*log(x - 1) - 1)/x
```

Sympy [A] time = 0.142021, size = 19, normalized size = 0.79

$$2\log\left(x-1\right) - \log\left(x^2+1\right) + \operatorname{atan}\left(x\right) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**3+x-1)/(-1+x)/x**2/(x**2+1),x)
```

```
[Out] 2*log(x - 1) - log(x**2 + 1) + atan(x) - 1/x
```

Giac [A] time = 1.13051, size = 31, normalized size = 1.29

$$-\frac{1}{x} + \arctan\left(x\right) - \log\left(x^2+1\right) + 2\,\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^3+x-1)/(-1+x)/x^2/(x^2+1),x, algorithm="giac")
```

```
[Out] -1/x + arctan(x) - log(x^2 + 1) + 2*log(abs(x - 1))
```

3.331 $\int \frac{1-3x+2x^2-4x^3+x^4}{\left(1+x^2\right)^3} \, dx$

Optimal. Leaf size=23

$$\frac{2}{x^2+1}-\frac{1}{4\left(x^2+1\right)^2}+\tan^{-1}(x)$$

```
[Out] -1/(4*(1 + x^2)^2) + 2/(1 + x^2) + ArcTan[x]
```

Rubi [A] time = 0.0246163, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1814, 12, 203}

$$\frac{2}{x^2+1}-\frac{1}{4\left(x^2+1\right)^2}+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 3*x + 2*x^2 - 4*x^3 + x^4)/(1 + x^2)^3,x]
```

```
[Out] -1/(4*(1 + x^2)^2) + 2/(1 + x^2) + ArcTan[x]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1-3x+2x^2-4x^3+x^4}{\left(1+x^2\right)^3}\,dx &= -\frac{1}{4\left(1+x^2\right)^2}-\frac{1}{4}\int \frac{-4+16x-4x^2}{\left(1+x^2\right)^2}\,dx\\
&= -\frac{1}{4\left(1+x^2\right)^2}+\frac{2}{1+x^2}+\frac{1}{8}\int \frac{8}{1+x^2}\,dx\\
&= -\frac{1}{4\left(1+x^2\right)^2}+\frac{2}{1+x^2}+\int \frac{1}{1+x^2}\,dx\\
&= -\frac{1}{4\left(1+x^2\right)^2}+\frac{2}{1+x^2}+\tan^{-1}(x)
\end{aligned}
$$

Mathematica [A] time = 0.0112247, size = 23, normalized size = 1.

$$\frac{2}{x^2+1}-\frac{1}{4\left(x^2+1\right)^2}+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 3*x + 2*x^2 - 4*x^3 + x^4)/(1 + x^2)^3,x]
```

```
[Out] -1/(4*(1 + x^2)^2) + 2/(1 + x^2) + ArcTan[x]
```

Maple [A] time = 0.004, size = 19, normalized size = 0.8

$$\frac{1}{\left(x^2+1\right)^2}\left(2\,x^2+\frac{7}{4}\right)+\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-4*x^3+2*x^2-3*x+1)/(x^2+1)^3,x)
```

```
[Out] (2*x^2+7/4)/(x^2+1)^2+arctan(x)
```

Maxima [A] time = 1.66992, size = 32, normalized size = 1.39

$$\frac{8\,x^2+7}{4\left(x^4+2\,x^2+1\right)}+\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-4*x^3+2*x^2-3*x+1)/(x^2+1)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(8*x^2 + 7)/(x^4 + 2*x^2 + 1) + arctan(x)
```

Fricas [A] time = 1.46814, size = 90, normalized size = 3.91

$$\frac{8\,x^2+4\left(x^4+2\,x^2+1\right)\arctan\left(x\right)+7}{4\left(x^4+2\,x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-4*x^3+2*x^2-3*x+1)/(x^2+1)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(8*x^2 + 4*(x^4 + 2*x^2 + 1)*arctan(x) + 7)/(x^4 + 2*x^2 + 1)
```

Sympy [A] time = 0.122589, size = 20, normalized size = 0.87

$$\frac{8x^2+7}{4x^4+8x^2+4}+\operatorname{atan}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-4*x**3+2*x**2-3*x+1)/(x**2+1)**3,x)
```

```
[Out] (8*x**2 + 7)/(4*x**4 + 8*x**2 + 4) + atan(x)
```

Giac [A] time = 1.18253, size = 26, normalized size = 1.13

$$\frac{8\,x^2+7}{4\left(x^2+1\right)^2}+\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-4*x^3+2*x^2-3*x+1)/(x^2+1)^3,x, algorithm="giac")
```

```
[Out] 1/4*(8*x^2 + 7)/(x^2 + 1)^2 + arctan(x)
```

3.332 $\int \frac{1-3x+2x^2-4x^3+x^4}{1+3x^2+3x^4+x^6}\,dx$

Optimal. Leaf size=23

$$\frac{2}{x^2+1}-\frac{1}{4\left(x^2+1\right)^2}+\tan^{-1}(x)$$

```
[Out] -1/(4*(1 + x^2)^2) + 2/(1 + x^2) + ArcTan[x]
```

Rubi [A] time = 0.0387941, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2073, 261, 203}

$$\frac{2}{x^2+1}-\frac{1}{4\left(x^2+1\right)^2}+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 3*x + 2*x^2 - 4*x^3 + x^4)/(1 + 3*x^2 + 3*x^4 + x^6),x]
```

```
[Out] -1/(4*(1 + x^2)^2) + 2/(1 + x^2) + ArcTan[x]
```

Rule 2073

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P /. x -> Sqrt[x]]
}, Int[ExpandIntegrand[(PP /. x -> x^2)^p*Q^q, x], x] /;  !SumQ[NonfreeFact
ors[PP, x]]] /; FreeQ[q, x] && PolyQ[P, x^2] && PolyQ[Q, x] && ILtQ[p, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1-3x+2x^2-4x^3+x^4}{1+3x^2+3x^4+x^6}\,dx &= \int \left(\frac{x}{(1+x^2)^3}-\frac{4x}{(1+x^2)^2}+\frac{1}{1+x^2}\right)dx\\ &= -\left(4\int \frac{x}{(1+x^2)^2}\,dx\right)+\int \frac{x}{(1+x^2)^3}\,dx+\int \frac{1}{1+x^2}\,dx\\ &= -\frac{1}{4(1+x^2)^2}+\frac{2}{1+x^2}+\tan^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.0054313, size = 23, normalized size = 1.

$$\frac{2}{x^2+1}-\frac{1}{4(x^2+1)^2}+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 3*x + 2*x^2 - 4*x^3 + x^4)/(1 + 3*x^2 + 3*x^4 + x^6),x]
```

```
[Out] -1/(4*(1 + x^2)^2) + 2/(1 + x^2) + ArcTan[x]
```

Maple [A] time = 0.005, size = 19, normalized size = 0.8

$$\frac{1}{(x^2+1)^2}\left(2x^2+\frac{7}{4}\right)+\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-4*x^3+2*x^2-3*x+1)/(x^6+3*x^4+3*x^2+1),x)
```

```
[Out] (2*x^2+7/4)/(x^2+1)^2+arctan(x)
```

Maxima [A] time = 1.49552, size = 32, normalized size = 1.39

$$\frac{8x^2+7}{4(x^4+2x^2+1)}+\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-4*x^3+2*x^2-3*x+1)/(x^6+3*x^4+3*x^2+1),x, algorithm="maxima"
)
```

```
[Out] 1/4*(8*x^2 + 7)/(x^4 + 2*x^2 + 1) + arctan(x)
```

Fricas [A] time = 1.45886, size = 90, normalized size = 3.91

$$\frac{8\,x^2+4\left(x^4+2\,x^2+1\right)\arctan\left(x\right)+7}{4\left(x^4+2\,x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-4*x^3+2*x^2-3*x+1)/(x^6+3*x^4+3*x^2+1),x, algorithm="fricas"
)
```

```
[Out] 1/4*(8*x^2 + 4*(x^4 + 2*x^2 + 1)*arctan(x) + 7)/(x^4 + 2*x^2 + 1)
```

Sympy [A] time = 0.122828, size = 20, normalized size = 0.87

$$\frac{8x^2+7}{4x^4+8x^2+4}+\operatorname{atan}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-4*x**3+2*x**2-3*x+1)/(x**6+3*x**4+3*x**2+1),x)
```

```
[Out] (8*x**2 + 7)/(4*x**4 + 8*x**2 + 4) + atan(x)
```

Giac [A] time = 1.15276, size = 26, normalized size = 1.13

$$\frac{8\,x^2+7}{4\left(x^2+1\right)^2}+\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-4*x^3+2*x^2-3*x+1)/(x^6+3*x^4+3*x^2+1),x, algorithm="giac")

[Out] 1/4*(8*x^2 + 7)/(x^2 + 1)^2 + arctan(x)
```

3.333 $\int \frac{1+x+2x^2+2x^3}{x^2+x^3+x^4} \, dx$

Optimal. Leaf size=13

$$\log \left(x^2+x+1\right)-\frac{1}{x}$$

```
[Out] -x^(-1) + Log[1 + x + x^2]
```

Rubi [A] time = 0.0490023, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1594, 1628, 628}

$$\log \left(x^2+x+1\right)-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + 2*x^2 + 2*x^3)/(x^2 + x^3 + x^4),x]
```

```
[Out] -x^(-1) + Log[1 + x + x^2]
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1+x+2x^2+2x^3}{x^2+x^3+x^4}\,dx = \int \frac{1+x+2x^2+2x^3}{x^2\left(1+x+x^2\right)}\,dx$$
$$= \int \left(\frac{1}{x^2}+\frac{1+2x}{1+x+x^2}\right)dx$$
$$= -\frac{1}{x}+\int \frac{1+2x}{1+x+x^2}\,dx$$
$$= -\frac{1}{x}+\log\left(1+x+x^2\right)$$

Mathematica [A] time = 0.005564, size = 13, normalized size = 1.

$$\log\left(x^2+x+1\right)-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + 2*x^2 + 2*x^3)/(x^2 + x^3 + x^4),x]
```

```
[Out] -x^(-1) + Log[1 + x + x^2]
```

Maple [A] time = 0.003, size = 14, normalized size = 1.1

$$-x^{-1}+\ln\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3+2*x^2+x+1)/(x^4+x^3+x^2),x)
```

```
[Out] -1/x+ln(x^2+x+1)
```

Maxima [A] time = 1.03172, size = 18, normalized size = 1.38

$$-\frac{1}{x}+\log\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+2*x^2+x+1)/(x^4+x^3+x^2),x, algorithm="maxima")
```

```
[Out] -1/x + log(x^2 + x + 1)
```

Fricas [A] time = 1.43135, size = 38, normalized size = 2.92

$$\frac{x\log\left(x^2+x+1\right)-1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+2*x^2+x+1)/(x^4+x^3+x^2),x, algorithm="fricas")
```

```
[Out] (x*log(x^2 + x + 1) - 1)/x
```

Sympy [A] time = 0.092617, size = 10, normalized size = 0.77

$$\log\left(x^2+x+1\right)-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**3+2*x**2+x+1)/(x**4+x**3+x**2),x)
```

```
[Out] log(x**2 + x + 1) - 1/x
```

Giac [A] time = 1.13992, size = 18, normalized size = 1.38

$$-\frac{1}{x}+\log\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+2*x^2+x+1)/(x^4+x^3+x^2),x, algorithm="giac")
```

```
[Out] -1/x + log(x^2 + x + 1)
```

3.334 $\int \frac{x^2(c+dx)^2}{a+bx^3}\,dx$

Optimal. Leaf size=206

$$\frac{\sqrt[3]{a}d\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{5/3}}-\frac{\sqrt[3]{a}d\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{5/3}}+\frac{\sqrt[3]{a}d\left(\sqrt[3]{a}d+2\sqrt[3]{b}c\right)\tan^{-1}\left(\ldots\right.}{\sqrt{3}b^{5/3}}$$

```
[Out] (2*c*d*x)/b + (d^2*x^2)/(2*b) + (a^(1/3)*d*(2*b^(1/3)*c + a^(1/3)*d)*ArcTan
[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(5/3)) - (a^(1/3)*d
*(2*b^(1/3)*c - a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(5/3)) + (a^(1/3)
*d*(2*b^(1/3)*c - a^(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]
)/(6*b^(5/3)) + (c^2*Log[a + b*x^3])/(3*b)
```

Rubi [A] time = 0.273597, antiderivative size = 206, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.45, Rules used = {1887, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{\sqrt[3]{a}d\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{5/3}}-\frac{\sqrt[3]{a}d\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{5/3}}+\frac{\sqrt[3]{a}d\left(\sqrt[3]{a}d+2\sqrt[3]{b}c\right)\tan^{-1}\left(\ldots\right.}{\sqrt{3}b^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x)^2)/(a + b*x^3),x]
```

```
[Out] (2*c*d*x)/b + (d^2*x^2)/(2*b) + (a^(1/3)*d*(2*b^(1/3)*c + a^(1/3)*d)*ArcTan
[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(5/3)) - (a^(1/3)*d
*(2*b^(1/3)*c - a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(5/3)) + (a^(1/3)
*d*(2*b^(1/3)*c - a^(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]
)/(6*b^(5/3)) + (c^2*Log[a + b*x^3])/(3*b)
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
```

```
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{x^2(c+dx)^2}{a+bx^3}\,dx = \int \left(\frac{2cd}{b}+\frac{d^2x}{b}-\frac{2acd+ad^2x-bc^2x^2}{b\left(a+bx^3\right)}\right)dx$$

$$= \frac{2cdx}{b}+\frac{d^2x^2}{2b}-\frac{\int \frac{2acd+ad^2x-bc^2x^2}{a+bx^3}\,dx}{b}$$

$$= \frac{2cdx}{b}+\frac{d^2x^2}{2b}-\frac{\int \frac{2acd+ad^2x}{a+bx^3}\,dx}{b}+c^2\int \frac{x^2}{a+bx^3}\,dx$$

$$= \frac{2cdx}{b}+\frac{d^2x^2}{2b}+\frac{c^2\log\left(a+bx^3\right)}{3b}-\frac{\int \frac{\sqrt[3]{a}\left(4a\sqrt[3]{b}cd+a^{4/3}d^2\right)+\sqrt[3]{b}\left(-2a\sqrt[3]{b}cd+a^{4/3}d^2\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{2/3}b^{4/3}}-\frac{\left(\sqrt[3]{a}d\left(2c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int$$

$$= \frac{2cdx}{b}+\frac{d^2x^2}{2b}-\frac{\sqrt[3]{a}d\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{5/3}}+\frac{c^2\log\left(a+bx^3\right)}{3b}+\frac{\left(\sqrt[3]{a}d\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\right)\int$$

$$= \frac{2cdx}{b}+\frac{d^2x^2}{2b}-\frac{\sqrt[3]{a}d\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{5/3}}+\frac{\sqrt[3]{a}d\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}\right.}{6b^{5/3}}$$

$$= \frac{2cdx}{b}+\frac{d^2x^2}{2b}+\frac{\sqrt[3]{a}d\left(2\sqrt[3]{b}c+\sqrt[3]{a}d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{5/3}}-\frac{\sqrt[3]{a}d\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{5/3}}+$$

Mathematica [A] time = 0.116305, size = 193, normalized size = 0.94

$$\frac{-\sqrt[3]{a}d\left(\sqrt[3]{a}d-2\sqrt[3]{b}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+2b^{2/3}c^2\log\left(a+bx^3\right)+2\sqrt[3]{a}d\left(\sqrt[3]{a}d-2\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+2\sqrt{3}}{6b^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x)^2)/(a + b*x^3),x]
```

```
[Out] (12*b^(2/3)*c*d*x + 3*b^(2/3)*d^2*x^2 + 2*Sqrt[3]*a^(1/3)*d*(2*b^(1/3)*c +
a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*a^(1/3)*d*(-2*b^
(1/3)*c + a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x] - a^(1/3)*d*(-2*b^(1/3)*c + a
^(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + 2*b^(2/3)*c^2*Lo
```

```
g[a + b*x^3])/(6*b^(5/3))
```

Maple [A] time = 0.006, size = 236, normalized size = 1.2

$$\frac{d^2x^2}{2b}+2\frac{cdx}{b}-\frac{2\,acd}{3\,b^2}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{acd}{3\,b^2}\ln\left(x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{2\,acd\sqrt{3}}{3\,b^2}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\left(\frac{a}{b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d*x+c)^2/(b*x^3+a),x)
```

```
[Out] 1/2*d^2*x^2/b+2*c*d*x/b-2/3/b^2*a*c*d/(a/b)^(2/3)*ln(x+(a/b)^(1/3))+1/3/b^2
*a*c*d/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-2/3/b^2*a*c*d/(a/b)^(2
/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+1/3/b^2*a*d^2/(a/b)^(1/
3)*ln(x+(a/b)^(1/3))-1/6/b^2*a*d^2/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(
2/3))-1/3/b^2*a*d^2*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x
-1))+1/3*c^2*ln(b*x^3+a)/b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x+c)^2/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 9.5983, size = 9873, normalized size = 47.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x+c)^2/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/12*(6*d^2*x^2 + 24*c*d*x - 2*(2*(1/2)^(2/3)*(c^4/b^2 - (b*c^4 + 2*a*c*d^3
)/b^3)*(-I*sqrt(3) + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4
 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3) + (1
/2)^(1/3)*(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*
c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3)*(I*sqrt(3) + 1) -
2*c^2/b)*b*log(1/4*(2*(1/2)^(2/3)*(c^4/b^2 - (b*c^4 + 2*a*c*d^3)/b^3)*(-I*s
qrt(3) + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3
)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3) + (1/2)^(1/3)*(2
*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b
^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3)*(I*sqrt(3) + 1) - 2*c^2/b)^2*b
^3 + 3*(2*(1/2)^(2/3)*(c^4/b^2 - (b*c^4 + 2*a*c*d^3)/b^3)*(-I*sqrt(3) + 1)/
(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 +
(b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3) + (1/2)^(1/3)*(2*c^6/b^3 + (
8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a
*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3)*(I*sqrt(3) + 1) - 2*c^2/b)*b^2*c^2 + 5*b*c
^4 + 4*a*c*d^3 + (8*b*c^3*d + a*d^4)*x) + ((2*(1/2)^(2/3)*(c^4/b^2 - (b*c^4
 + 2*a*c*d^3)/b^3)*(-I*sqrt(3) + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^
5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5
)^(1/3) + (1/2)^(1/3)*(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 +
 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3)*(I*sqr
t(3) + 1) - 2*c^2/b)*b + 6*c^2 + 3*sqrt(1/3)*b*sqrt(-((2*(1/2)^(2/3)*(c^4/b
^2 - (b*c^4 + 2*a*c*d^3)/b^3)*(-I*sqrt(3) + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^
3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a
^2*d^6)/b^5)^(1/3) + (1/2)^(1/3)*(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 -
 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(
1/3)*(I*sqrt(3) + 1) - 2*c^2/b)^2*b^3 + 4*(2*(1/2)^(2/3)*(c^4/b^2 - (b*c^4
+ 2*a*c*d^3)/b^3)*(-I*sqrt(3) + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5
 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)
^(1/3) + (1/2)^(1/3)*(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 +
2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3)*(I*sqrt
(3) + 1) - 2*c^2/b)*b^2*c^2 + 4*b*c^4 + 32*a*c*d^3)/b^3))*log(-1/4*(2*(1/2)
^(2/3)*(c^4/b^2 - (b*c^4 + 2*a*c*d^3)/b^3)*(-I*sqrt(3) + 1)/(2*c^6/b^3 + (8
*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*
b*c^3*d^3 + a^2*d^6)/b^5)^(1/3) + (1/2)^(1/3)*(2*c^6/b^3 + (8*b*c^3 + a*d^3
)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^
2*d^6)/b^5)^(1/3)*(I*sqrt(3) + 1) - 2*c^2/b)^2*b^3 - 3*(2*(1/2)^(2/3)*(c^4/
b^2 - (b*c^4 + 2*a*c*d^3)/b^3)*(-I*sqrt(3) + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d
^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 +
a^2*d^6)/b^5)^(1/3) + (1/2)^(1/3)*(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5
- 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^
(1/3)*(I*sqrt(3) + 1) - 2*c^2/b)*b^2*c^2 - 5*b*c^4 - 4*a*c*d^3 + 2*(8*b*c^3
*d + a*d^4)*x + 3/4*sqrt(1/3)*((2*(1/2)^(2/3)*(c^4/b^2 - (b*c^4 + 2*a*c*d^3
)/b^3)*(-I*sqrt(3) + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4
 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3) + (1
/2)^(1/3)*(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*
```

```
c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3)*(I*sqrt(3) + 1) -
2*c^2/b)*b^3 - 6*b^2*c^2)*sqrt(-((2*(1/2)^(2/3)*(c^4/b^2 - (b*c^4 + 2*a*c*d
^3)/b^3)*(-I*sqrt(3) + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c
^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3) +
(1/2)^(1/3)*(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3
)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3)*(I*sqrt(3) + 1)
 - 2*c^2/b)^2*b^3 + 4*(2*(1/2)^(2/3)*(c^4/b^2 - (b*c^4 + 2*a*c*d^3)/b^3)*(-I
*sqrt(3) + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d
^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3) + (1/2)^(1/3)*
(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 +
(b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3)*(I*sqrt(3) + 1) - 2*c^2/b)*b
^2*c^2 + 4*b*c^4 + 32*a*c*d^3)/b^3)) + ((2*(1/2)^(2/3)*(c^4/b^2 - (b*c^4 +
2*a*c*d^3)/b^3)*(-I*sqrt(3) + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 -
 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(
1/3) + (1/2)^(1/3)*(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*
a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3)*(I*sqrt(3
) + 1) - 2*c^2/b)*b + 6*c^2 - 3*sqrt(1/3)*b*sqrt(-((2*(1/2)^(2/3)*(c^4/b^2
 - (b*c^4 + 2*a*c*d^3)/b^3)*(-I*sqrt(3) + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^3)*
a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*
d^6)/b^5)^(1/3) + (1/2)^(1/3)*(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*
(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3
)*(I*sqrt(3) + 1) - 2*c^2/b)^2*b^3 + 4*(2*(1/2)^(2/3)*(c^4/b^2 - (b*c^4 + 2
*a*c*d^3)/b^3)*(-I*sqrt(3) + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 -
3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1
/3) + (1/2)^(1/3)*(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a
*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3)*(I*sqrt(3)
 + 1) - 2*c^2/b)*b^2*c^2 + 4*b*c^4 + 32*a*c*d^3)/b^3))*log(-1/4*(2*(1/2)^(2
/3)*(c^4/b^2 - (b*c^4 + 2*a*c*d^3)/b^3)*(-I*sqrt(3) + 1)/(2*c^6/b^3 + (8*b*
c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c
^3*d^3 + a^2*d^6)/b^5)^(1/3) + (1/2)^(1/3)*(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a
*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d
^6)/b^5)^(1/3)*(I*sqrt(3) + 1) - 2*c^2/b)^2*b^3 - 3*(2*(1/2)^(2/3)*(c^4/b^2
 - (b*c^4 + 2*a*c*d^3)/b^3)*(-I*sqrt(3) + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^3)
*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2
*d^6)/b^5)^(1/3) + (1/2)^(1/3)*(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3
*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/
3)*(I*sqrt(3) + 1) - 2*c^2/b)*b^2*c^2 - 5*b*c^4 - 4*a*c*d^3 + 2*(8*b*c^3*d
+ a*d^4)*x - 3/4*sqrt(1/3)*((2*(1/2)^(2/3)*(c^4/b^2 - (b*c^4 + 2*a*c*d^3)/b
^3)*(-I*sqrt(3) + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 +
2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3) + (1/2)
^(1/3)*(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2
/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3)*(I*sqrt(3) + 1) - 2*c
^2/b)*b^3 - 6*b^2*c^2)*sqrt(-((2*(1/2)^(2/3)*(c^4/b^2 - (b*c^4 + 2*a*c*d^3)
/b^3)*(-I*sqrt(3) + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4
+ 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3) + (1/
```

```
2)^(1/3)*(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c
^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3)*(I*sqrt(3) + 1) - 2
*c^2/b)^2*b^3 + 4*(2*(1/2)^(2/3)*(c^4/b^2 - (b*c^4 + 2*a*c*d^3)/b^3)*(-I*sq
rt(3) + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)
*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3) + (1/2)^(1/3)*(2*
c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^
2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3)*(I*sqrt(3) + 1) - 2*c^2/b)*b^2*
c^2 + 4*b*c^4 + 32*a*c*d^3)/b^3)))/b
```

Sympy [A] time = 1.30313, size = 138, normalized size = 0.67

$$\operatorname{RootSum}\left(27t^3b^5 - 27t^2b^4c^2 + t\left(18ab^2cd^3 + 9b^3c^4\right) - a^2d^6 + 2abc^3d^3 - b^2c^6, \left(t \mapsto t\log\left(x + \frac{9t^2b^3 - 18tb^2c^2 + 4acd^3}{ad^4 + 8bc^3d}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x+c)**2/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*b**5 - 27*_t**2*b**4*c**2 + _t*(18*a*b**2*c*d**3 + 9*b**3*
c**4) - a**2*d**6 + 2*a*b*c**3*d**3 - b**2*c**6, Lambda(_t, _t*log(x + (9*_
t**2*b**3 - 18*_t*b**2*c**2 + 4*a*c*d**3 + 5*b*c**4)/(a*d**4 + 8*b*c**3*d))
)) + 2*c*d*x/b + d**2*x**2/(2*b)
```

Giac [A] time = 1.19243, size = 302, normalized size = 1.47

$$\frac{c^2\log\left(\left|bx^3+a\right|\right)}{3\,b} + \frac{bd^2x^2 + 4\,bcdx}{2\,b^2} - \frac{\sqrt{3}\left(2\,\left(-ab^2\right)^{\frac{1}{3}}ab^2cd - \left(-ab^2\right)^{\frac{2}{3}}abd^2\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab^4} - \frac{\left(2\,\left(-ab^2\right)^{\frac{1}{3}}ab^2cd\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x+c)^2/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*c^2*log(abs(b*x^3 + a))/b + 1/2*(b*d^2*x^2 + 4*b*c*d*x)/b^2 - 1/3*sqrt(
3)*(2*(-a*b^2)^(1/3)*a*b^2*c*d - (-a*b^2)^(2/3)*a*b*d^2)*arctan(1/3*sqrt(3)
*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b^4) - 1/6*(2*(-a*b^2)^(1/3)*a*b^2*c
*d + (-a*b^2)^(2/3)*a*b*d^2)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^
4) + 1/3*(a*b^4*d^2*(-a/b)^(1/3) + 2*a*b^4*c*d)*(-a/b)^(1/3)*log(abs(x - (-
a/b)^(1/3)))/(a*b^5)
```

3.335 $\int \frac{-x+2x^3+4x^5}{(3+2x^2+x^4)^2}\,dx$

Optimal. Leaf size=45

$$\frac{5-7x^2}{8(x^4+2x^2+3)}+\frac{9\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{8\sqrt{2}}$$

```
[Out] (5 - 7*x^2)/(8*(3 + 2*x^2 + x^4)) + (9*ArcTan[(1 + x^2)/Sqrt[2]])/(8*Sqrt[2
])
```

Rubi [A] time = 0.0669286, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1594, 1663, 1660, 12, 618, 204}

$$\frac{5-7x^2}{8(x^4+2x^2+3)}+\frac{9\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-x + 2*x^3 + 4*x^5)/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] (5 - 7*x^2)/(8*(3 + 2*x^2 + x^4)) + (9*ArcTan[(1 + x^2)/Sqrt[2]])/(8*Sqrt[2
])
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{-x+2x^3+4x^5}{\left(3+2x^2+x^4\right)^2}\,dx &= \int \frac{x\left(-1+2x^2+4x^4\right)}{\left(3+2x^2+x^4\right)^2}\,dx \\
&= \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{-1+2x+4x^2}{\left(3+2x+x^2\right)^2}\,dx,x,x^2\right) \\
&= \frac{5-7x^2}{8\left(3+2x^2+x^4\right)}+\frac{1}{16}\,\mathrm{Subst}\left(\int \frac{18}{3+2x+x^2}\,dx,x,x^2\right) \\
&= \frac{5-7x^2}{8\left(3+2x^2+x^4\right)}+\frac{9}{8}\,\mathrm{Subst}\left(\int \frac{1}{3+2x+x^2}\,dx,x,x^2\right) \\
&= \frac{5-7x^2}{8\left(3+2x^2+x^4\right)}-\frac{9}{4}\,\mathrm{Subst}\left(\int \frac{1}{-8-x^2}\,dx,x,2\left(1+x^2\right)\right) \\
&= \frac{5-7x^2}{8\left(3+2x^2+x^4\right)}+\frac{9\tan^{-1}\left(\frac{1+x^2}{\sqrt{2}}\right)}{8\sqrt{2}}
\end{aligned}
$$

Mathematica [A] time = 0.0254557, size = 45, normalized size = 1.

$$
\frac{5-7x^2}{8\left(x^4+2x^2+3\right)}+\frac{9\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{8\sqrt{2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x + 2*x^3 + 4*x^5)/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] (5 - 7*x^2)/(8*(3 + 2*x^2 + x^4)) + (9*ArcTan[(1 + x^2)/Sqrt[2]])/(8*Sqrt[2
])
```

Maple [A] time = 0.008, size = 41, normalized size = 0.9

$$
\frac{1}{2\,x^4+4\,x^2+6}\left(-\frac{7\,x^2}{4}+\frac{5}{4}\right)+\frac{9\,\sqrt{2}}{16}\arctan\left(\frac{\left(2\,x^2+2\right)\sqrt{2}}{4}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^5+2*x^3-x)/(x^4+2*x^2+3)^2,x)
```

```
[Out] 1/2*(-7/4*x^2+5/4)/(x^4+2*x^2+3)+9/16*2^(1/2)*arctan(1/4*(2*x^2+2)*2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{7\,x^2-5}{8\,(x^4+2\,x^2+3)}+\frac{9}{4}\int\frac{x}{x^4+2\,x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^5+2*x^3-x)/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

```
[Out] -1/8*(7*x^2 - 5)/(x^4 + 2*x^2 + 3) + 9/4*integrate(x/(x^4 + 2*x^2 + 3), x)
```

Fricas [A] time = 1.48295, size = 132, normalized size = 2.93

$$\frac{9\,\sqrt{2}(x^4+2\,x^2+3)\arctan\left(\frac{1}{2}\,\sqrt{2}(x^2+1)\right)-14\,x^2+10}{16\,(x^4+2\,x^2+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^5+2*x^3-x)/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(9*sqrt(2)*(x^4 + 2*x^2 + 3)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 14*x^2 +
10)/(x^4 + 2*x^2 + 3)
```

Sympy [A] time = 0.147528, size = 44, normalized size = 0.98

$$-\frac{7x^2-5}{8x^4+16x^2+24}+\frac{9\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x^2}{2}+\frac{\sqrt{2}}{2}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**5+2*x**3-x)/(x**4+2*x**2+3)**2,x)
```

```
[Out] -(7*x**2 - 5)/(8*x**4 + 16*x**2 + 24) + 9*sqrt(2)*atan(sqrt(2)*x**2/2 + sqr
t(2)/2)/16
```

Giac [A] time = 1.11674, size = 51, normalized size = 1.13

$$\frac{9}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(x^2+1\right)\right)-\frac{7x^2-5}{8\left(x^4+2x^2+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^5+2*x^3-x)/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

```
[Out] 9/16*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 1/8*(7*x^2 - 5)/(x^4 + 2*x^2 +
 3)
```

3.336 $\int \frac{x+x^5}{\left(1+2x^2+2x^4\right)^3}\,dx$

Optimal. Leaf size=59

$$\frac{2x^2+1}{2\left(2x^4+2x^2+1\right)}+\frac{4x^2+3}{16\left(2x^4+2x^2+1\right)^2}+\tan^{-1}\left(2x^2+1\right)$$

```
[Out] (3 + 4*x^2)/(16*(1 + 2*x^2 + 2*x^4)^2) + (1 + 2*x^2)/(2*(1 + 2*x^2 + 2*x^4)
) + ArcTan[1 + 2*x^2]
```

Rubi [A] time = 0.0642882, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {1593, 1663, 1660, 12, 614, 617, 204}

$$\frac{2x^2+1}{2\left(2x^4+2x^2+1\right)}+\frac{4x^2+3}{16\left(2x^4+2x^2+1\right)^2}+\tan^{-1}\left(2x^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x + x^5)/(1 + 2*x^2 + 2*x^4)^3,x]
```

```
[Out] (3 + 4*x^2)/(16*(1 + 2*x^2 + 2*x^4)^2) + (1 + 2*x^2)/(2*(1 + 2*x^2 + 2*x^4)
) + ArcTan[1 + 2*x^2]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x+x^5}{\left(1+2x^2+2x^4\right)^3}\,dx &= \int \frac{x\left(1+x^4\right)}{\left(1+2x^2+2x^4\right)^3}\,dx \\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1+x^2}{\left(1+2x+2x^2\right)^3}\,dx, x, x^2\right) \\
&= \frac{3+4x^2}{16\left(1+2x^2+2x^4\right)^2} + \frac{1}{16}\operatorname{Subst}\left(\int \frac{16}{\left(1+2x+2x^2\right)^2}\,dx, x, x^2\right) \\
&= \frac{3+4x^2}{16\left(1+2x^2+2x^4\right)^2} + \operatorname{Subst}\left(\int \frac{1}{\left(1+2x+2x^2\right)^2}\,dx, x, x^2\right) \\
&= \frac{3+4x^2}{16\left(1+2x^2+2x^4\right)^2} + \frac{1+2x^2}{2\left(1+2x^2+2x^4\right)} + \operatorname{Subst}\left(\int \frac{1}{1+2x+2x^2}\,dx, x, x^2\right) \\
&= \frac{3+4x^2}{16\left(1+2x^2+2x^4\right)^2} + \frac{1+2x^2}{2\left(1+2x^2+2x^4\right)} - \operatorname{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+2x^2\right) \\
&= \frac{3+4x^2}{16\left(1+2x^2+2x^4\right)^2} + \frac{1+2x^2}{2\left(1+2x^2+2x^4\right)} + \tan^{-1}\left(1+2x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.02139, size = 44, normalized size = 0.75

$$
\frac{32x^6+48x^4+36x^2+11}{16\left(2x^4+2x^2+1\right)^2} + \tan^{-1}\left(2x^2+1\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + x^5)/(1 + 2*x^2 + 2*x^4)^3,x]
```

```
[Out] (11 + 36*x^2 + 48*x^4 + 32*x^6)/(16*(1 + 2*x^2 + 2*x^4)^2) + ArcTan[1 + 2*x
^2]
```

Maple [A] time = 0.009, size = 41, normalized size = 0.7

$$
2\,\frac{1}{\left(2\,x^4+2\,x^2+1\right)^2}\left(x^6+3/2\,x^4+\frac{9\,x^2}{8}+\frac{11}{32}\right)+\arctan\left(2\,x^2+1\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5+x)/(2*x^4+2*x^2+1)^3,x)
```

```
[Out] 2*(x^6+3/2*x^4+9/8*x^2+11/32)/(2*x^4+2*x^2+1)^2+arctan(2*x^2+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{32\,x^6+48\,x^4+36\,x^2+11}{16\left(4\,x^8+8\,x^6+8\,x^4+4\,x^2+1\right)}+2\int\frac{x}{2\,x^4+2\,x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+x)/(2*x^4+2*x^2+1)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(32*x^6 + 48*x^4 + 36*x^2 + 11)/(4*x^8 + 8*x^6 + 8*x^4 + 4*x^2 + 1) +
2*integrate(x/(2*x^4 + 2*x^2 + 1), x)
```

Fricas [A] time = 1.54041, size = 180, normalized size = 3.05

$$\frac{32\,x^6+48\,x^4+36\,x^2+16\left(4\,x^8+8\,x^6+8\,x^4+4\,x^2+1\right)\arctan\left(2\,x^2+1\right)+11}{16\left(4\,x^8+8\,x^6+8\,x^4+4\,x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+x)/(2*x^4+2*x^2+1)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(32*x^6 + 48*x^4 + 36*x^2 + 16*(4*x^8 + 8*x^6 + 8*x^4 + 4*x^2 + 1)*arc
tan(2*x^2 + 1) + 11)/(4*x^8 + 8*x^6 + 8*x^4 + 4*x^2 + 1)
```

Sympy [A] time = 0.182194, size = 46, normalized size = 0.78

$$\frac{32x^6+48x^4+36x^2+11}{64x^8+128x^6+128x^4+64x^2+16}+\operatorname{atan}\left(2x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**5+x)/(2*x**4+2*x**2+1)**3,x)
```

```
[Out] (32*x**6 + 48*x**4 + 36*x**2 + 11)/(64*x**8 + 128*x**6 + 128*x**4 + 64*x**2
 + 16) + atan(2*x**2 + 1)
```

Giac [A] time = 1.10982, size = 57, normalized size = 0.97

$$\frac{32\,x^{6}+48\,x^{4}+36\,x^{2}+11}{16\left(2\,x^{4}+2\,x^{2}+1\right)^{2}}+\arctan\left(2\,x^{2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+x)/(2*x^4+2*x^2+1)^3,x, algorithm="giac")
```

```
[Out] 1/16*(32*x^6 + 48*x^4 + 36*x^2 + 11)/(2*x^4 + 2*x^2 + 1)^2 + arctan(2*x^2 +
 1)
```

3.337 $\int \frac{a+bx+cx^2}{d+ex^2+fx^4}\,dx$

Optimal. Leaf size=209

$$\frac{\left(c-\frac{ce-2af}{\sqrt{e^2-4df}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{e-\sqrt{e^2-4df}}}\right)}{\sqrt{2}\sqrt{f}\sqrt{e-\sqrt{e^2-4df}}}+\frac{\left(\frac{ce-2af}{\sqrt{e^2-4df}}+c\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{\sqrt{e^2-4df}+e}}\right)}{\sqrt{2}\sqrt{f}\sqrt{\sqrt{e^2-4df}+e}}-\frac{b\tanh^{-1}\left(\frac{e+2fx^2}{\sqrt{e^2-4df}}\right)}{\sqrt{e^2-4df}}$$

```
[Out] ((c - (c*e - 2*a*f)/Sqrt[e^2 - 4*d*f])*ArcTan[(Sqrt[2]*Sqrt[f]*x)/Sqrt[e -
Sqrt[e^2 - 4*d*f]]])/(Sqrt[2]*Sqrt[f]*Sqrt[e - Sqrt[e^2 - 4*d*f]]) + ((c +
(c*e - 2*a*f)/Sqrt[e^2 - 4*d*f])*ArcTan[(Sqrt[2]*Sqrt[f]*x)/Sqrt[e + Sqrt[e
^2 - 4*d*f]]])/(Sqrt[2]*Sqrt[f]*Sqrt[e + Sqrt[e^2 - 4*d*f]]) - (b*ArcTanh[(
e + 2*f*x^2)/Sqrt[e^2 - 4*d*f]])/Sqrt[e^2 - 4*d*f]
```

Rubi [A] time = 0.370806, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {1673, 1166, 205, 12, 1107, 618, 206}

$$\frac{\left(c-\frac{ce-2af}{\sqrt{e^2-4df}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{e-\sqrt{e^2-4df}}}\right)}{\sqrt{2}\sqrt{f}\sqrt{e-\sqrt{e^2-4df}}}+\frac{\left(\frac{ce-2af}{\sqrt{e^2-4df}}+c\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{\sqrt{e^2-4df}+e}}\right)}{\sqrt{2}\sqrt{f}\sqrt{\sqrt{e^2-4df}+e}}-\frac{b\tanh^{-1}\left(\frac{e+2fx^2}{\sqrt{e^2-4df}}\right)}{\sqrt{e^2-4df}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(d + e*x^2 + f*x^4),x]
```

```
[Out] ((c - (c*e - 2*a*f)/Sqrt[e^2 - 4*d*f])*ArcTan[(Sqrt[2]*Sqrt[f]*x)/Sqrt[e -
Sqrt[e^2 - 4*d*f]]])/(Sqrt[2]*Sqrt[f]*Sqrt[e - Sqrt[e^2 - 4*d*f]]) + ((c +
(c*e - 2*a*f)/Sqrt[e^2 - 4*d*f])*ArcTan[(Sqrt[2]*Sqrt[f]*x)/Sqrt[e + Sqrt[e
^2 - 4*d*f]]])/(Sqrt[2]*Sqrt[f]*Sqrt[e + Sqrt[e^2 - 4*d*f]]) - (b*ArcTanh[(
e + 2*f*x^2)/Sqrt[e^2 - 4*d*f]])/Sqrt[e^2 - 4*d*f]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
```

```
&& !PolyQ[Pq, x^2]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{a+bx+cx^2}{d+ex^2+fx^4}\,dx = \int \frac{bx}{d+ex^2+fx^4}\,dx + \int \frac{a+cx^2}{d+ex^2+fx^4}\,dx$$
$$= b\int \frac{x}{d+ex^2+fx^4}\,dx + \frac{1}{2}\left(c-\frac{ce-2af}{\sqrt{e^2-4df}}\right)\int \frac{1}{\frac{e}{2}-\frac{1}{2}\sqrt{e^2-4df}+fx^2}\,dx + \frac{1}{2}\left(c+\frac{ce-2af}{\sqrt{e^2-4df}}\right)\int$$
$$= \frac{\left(c-\frac{ce-2af}{\sqrt{e^2-4df}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{e-\sqrt{e^2-4df}}}\right)}{\sqrt{2}\sqrt{f}\sqrt{e-\sqrt{e^2-4df}}} + \frac{\left(c+\frac{ce-2af}{\sqrt{e^2-4df}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{e+\sqrt{e^2-4df}}}\right)}{\sqrt{2}\sqrt{f}\sqrt{e+\sqrt{e^2-4df}}} + \frac{1}{2}b\,\text{Subst}\left(\int \frac{1}{d+ex+f}\right.$$
$$= \frac{\left(c-\frac{ce-2af}{\sqrt{e^2-4df}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{e-\sqrt{e^2-4df}}}\right)}{\sqrt{2}\sqrt{f}\sqrt{e-\sqrt{e^2-4df}}} + \frac{\left(c+\frac{ce-2af}{\sqrt{e^2-4df}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{e+\sqrt{e^2-4df}}}\right)}{\sqrt{2}\sqrt{f}\sqrt{e+\sqrt{e^2-4df}}} - b\,\text{Subst}\left(\int \frac{1}{e^2-4df-x^2}\right.$$
$$= \frac{\left(c-\frac{ce-2af}{\sqrt{e^2-4df}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{e-\sqrt{e^2-4df}}}\right)}{\sqrt{2}\sqrt{f}\sqrt{e-\sqrt{e^2-4df}}} + \frac{\left(c+\frac{ce-2af}{\sqrt{e^2-4df}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{e+\sqrt{e^2-4df}}}\right)}{\sqrt{2}\sqrt{f}\sqrt{e+\sqrt{e^2-4df}}} - \frac{b\tanh^{-1}\left(\frac{e+2fx^2}{\sqrt{e^2-4df}}\right)}{\sqrt{e^2-4df}}$$

Mathematica [A] time = 0.254728, size = 234, normalized size = 1.12

$$\frac{\frac{\sqrt{2}\left(2af+c\left(\sqrt{e^2-4df}-e\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{e-\sqrt{e^2-4df}}}\right)}{\sqrt{f}\sqrt{e-\sqrt{e^2-4df}}} + \frac{\sqrt{2}\left(c\left(\sqrt{e^2-4df}+e\right)-2af\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{\sqrt{e^2-4df}+e}}\right)}{\sqrt{f}\sqrt{\sqrt{e^2-4df}+e}} + b\log\left(\sqrt{e^2-4df}-e-2fx^2\right) - b\log\left(\sqrt{e^2-\right.}{2\sqrt{e^2-4df}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/(d + e*x^2 + f*x^4),x]
```

```
[Out] ((Sqrt[2]*(2*a*f + c*(-e + Sqrt[e^2 - 4*d*f]))*ArcTan[(Sqrt[2]*Sqrt[f]*x)/S
qrt[e - Sqrt[e^2 - 4*d*f]]])/(Sqrt[f]*Sqrt[e - Sqrt[e^2 - 4*d*f]]) + (Sqrt[
2]*(-2*a*f + c*(e + Sqrt[e^2 - 4*d*f]))*ArcTan[(Sqrt[2]*Sqrt[f]*x)/Sqrt[e +
 Sqrt[e^2 - 4*d*f]]])/(Sqrt[f]*Sqrt[e + Sqrt[e^2 - 4*d*f]]) + b*Log[-e + Sq
rt[e^2 - 4*d*f] - 2*f*x^2] - b*Log[e + Sqrt[e^2 - 4*d*f] + 2*f*x^2])/(2*Sqr
t[e^2 - 4*d*f])
```

Maple [B] time = 0.044, size = 616, normalized size = 3.

$$\frac{b}{8df-2e^2}\sqrt{-4df+e^2}\ln\left(2fx^2+\sqrt{-4df+e^2}+e\right)+2\frac{f\sqrt{2}cd}{\left(4df-e^2\right)\sqrt{\left(e+\sqrt{-4df+e^2}\right)f}}\arctan\left(\frac{fx\sqrt{2}}{\sqrt{\left(e+\sqrt{-4df+e^2}\right)f}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(f*x^4+e*x^2+d),x)
```

```
[Out] 1/2*(-4*d*f+e^2)^(1/2)/(4*d*f-e^2)*b*ln(2*f*x^2+(-4*d*f+e^2)^(1/2)+e)+2*f/(
4*d*f-e^2)*2^(1/2)/((e+(-4*d*f+e^2)^(1/2))*f)^(1/2)*arctan(f*x*2^(1/2)/((e+
(-4*d*f+e^2)^(1/2))*f)^(1/2))*c*d-1/2/(4*d*f-e^2)*2^(1/2)/((e+(-4*d*f+e^2)^
(1/2))*f)^(1/2)*arctan(f*x*2^(1/2)/((e+(-4*d*f+e^2)^(1/2))*f)^(1/2))*c*e^2+
f*(-4*d*f+e^2)^(1/2)/(4*d*f-e^2)*2^(1/2)/((e+(-4*d*f+e^2)^(1/2))*f)^(1/2)*a
rctan(f*x*2^(1/2)/((e+(-4*d*f+e^2)^(1/2))*f)^(1/2))*a-1/2*(-4*d*f+e^2)^(1/2
)/(4*d*f-e^2)*2^(1/2)/((e+(-4*d*f+e^2)^(1/2))*f)^(1/2)*arctan(f*x*2^(1/2)/(
(e+(-4*d*f+e^2)^(1/2))*f)^(1/2))*c*e-1/2*(-4*d*f+e^2)^(1/2)/(4*d*f-e^2)*b*l
n(-2*f*x^2+(-4*d*f+e^2)^(1/2)-e)-2*f/(4*d*f-e^2)*2^(1/2)/(((-4*d*f+e^2)^(1/
2)-e)*f)^(1/2)*arctanh(f*x*2^(1/2)/(((-4*d*f+e^2)^(1/2)-e)*f)^(1/2))*c*d+1/
2/(4*d*f-e^2)*2^(1/2)/(((-4*d*f+e^2)^(1/2)-e)*f)^(1/2)*arctanh(f*x*2^(1/2)/
(((-4*d*f+e^2)^(1/2)-e)*f)^(1/2))*c*e^2+f*(-4*d*f+e^2)^(1/2)/(4*d*f-e^2)*2^
(1/2)/(((-4*d*f+e^2)^(1/2)-e)*f)^(1/2)*arctanh(f*x*2^(1/2)/(((-4*d*f+e^2)^(
1/2)-e)*f)^(1/2))*a-1/2*(-4*d*f+e^2)^(1/2)/(4*d*f-e^2)*2^(1/2)/(((-4*d*f+e^
2)^(1/2)-e)*f)^(1/2)*arctanh(f*x*2^(1/2)/(((-4*d*f+e^2)^(1/2)-e)*f)^(1/2))*
c*e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{cx^2+bx+a}{fx^4+ex^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(f*x^4+e*x^2+d),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)/(f*x^4 + e*x^2 + d), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(f*x^4+e*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(f*x**4+e*x**2+d),x)
```

```
[Out] Timed out
```

Giac [C] time = 3.09984, size = 9341, normalized size = 44.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(f*x^4+e*x^2+d),x, algorithm="giac")
```

```
[Out] 1/2*(3*(4*(d*f^3)^(3/4)*d*f - (d*f^3)^(3/4)*sqrt(-4*d*f + e^2)*e - (d*f^3)^
(3/4)*e^2)*c*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f))))
)^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))^3*sin(5/4*pi +
1/2*real_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f))))) - (4*(d*f^3)^(3/4)*d*f -
 (d*f^3)^(3/4)*sqrt(-4*d*f + e^2)*e - (d*f^3)^(3/4)*e^2)*c*cosh(1/2*imag_pa
rt(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))^3*sin(5/4*pi + 1/2*real_part(arcsin
(1/2*sqrt(d*f)*e/(d*abs(f)))))^3 - 9*(4*(d*f^3)^(3/4)*d*f - (d*f^3)^(3/4)*s
qrt(-4*d*f + e^2)*e - (d*f^3)^(3/4)*e^2)*c*cos(5/4*pi + 1/2*real_part(arcsi
n(1/2*sqrt(d*f)*e/(d*abs(f)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(d*f)*e
/(d*abs(f)))))^2*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f
)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f))))) + 3*(4*(d*f^3
)^(3/4)*d*f - (d*f^3)^(3/4)*sqrt(-4*d*f + e^2)*e - (d*f^3)^(3/4)*e^2)*c*cos
h(1/2*imag_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))^2*sin(5/4*pi + 1/2*rea
l_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))^3*sinh(1/2*imag_part(arcsin(1/2
*sqrt(d*f)*e/(d*abs(f))))) + 9*(4*(d*f^3)^(3/4)*d*f - (d*f^3)^(3/4)*sqrt(-4
```

```
*d*f + e^2)*e - (d*f^3)^(3/4)*e^2)*c*cos(5/4*pi + 1/2*real_part(arcsin(1/2*
sqrt(d*f)*e/(d*abs(f)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(d*f)*e/(d*ab
s(f)))))*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))*si
nh(1/2*imag_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))^2 - 3*(4*(d*f^3)^(3/4
)*d*f - (d*f^3)^(3/4)*sqrt(-4*d*f + e^2)*e - (d*f^3)^(3/4)*e^2)*c*cosh(1/2*
imag_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))*sin(5/4*pi + 1/2*real_part(a
rcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(d*
f)*e/(d*abs(f)))))^2 - 3*(4*(d*f^3)^(3/4)*d*f - (d*f^3)^(3/4)*sqrt(-4*d*f +
 e^2)*e - (d*f^3)^(3/4)*e^2)*c*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(d
*f)*e/(d*abs(f)))))^2*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(d*f)*e/(d*
abs(f)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))^3 + (4*(
d*f^3)^(3/4)*d*f - (d*f^3)^(3/4)*sqrt(-4*d*f + e^2)*e - (d*f^3)^(3/4)*e^2)*
c*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))^3*sinh(1/
2*imag_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))^3 - 2*(4*sqrt(d*f)*d*f^3 +
 sqrt(d*f)*sqrt(-4*d*f + e^2)*f^2*e + sqrt(d*f)*f^2*e^2)*b*cos(5/4*pi + 1/2
*real_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))*cosh(1/2*imag_part(arcsin(1
/2*sqrt(d*f)*e/(d*abs(f)))))^2*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(d
*f)*e/(d*abs(f))))) + 4*(4*sqrt(d*f)*d*f^3 - sqrt(d*f)*sqrt(-4*d*f + e^2)*f
^2*e + sqrt(d*f)*f^2*e^2)*b*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(d*f)
*e/(d*abs(f)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))*si
n(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))*sinh(1/2*imag
_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f))))) - 2*(4*sqrt(d*f)*d*f^3 - sqrt(d*
f)*sqrt(-4*d*f + e^2)*f^2*e - sqrt(d*f)*f^2*e^2)*b*cos(5/4*pi + 1/2*real_pa
rt(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))*sin(5/4*pi + 1/2*real_part(arcsin(1
/2*sqrt(d*f)*e/(d*abs(f)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(d*f)*e/(d*a
bs(f)))))^2 + (4*(d*f^3)^(1/4)*d*f^3 - (d*f^3)^(1/4)*sqrt(-4*d*f + e^2)*f^2
*e - (d*f^3)^(1/4)*f^2*e^2)*a*cosh(1/2*imag_part(arcsin(1/2*sqrt(d*f)*e/(d*
abs(f)))))*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))
- (4*(d*f^3)^(1/4)*d*f^3 - (d*f^3)^(1/4)*sqrt(-4*d*f + e^2)*f^2*e - (d*f^3)
^(1/4)*f^2*e^2)*a*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(
f)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f))))))*arctan(-((d
/f)^(1/4)*cos(5/4*pi + 1/2*arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))) - x)/((d/f)^
(1/4)*sin(5/4*pi + 1/2*arcsin(1/2*sqrt(d*f)*e/(d*abs(f))))))/(4*d^2*f^4 - d
*f^3*e^2) + 1/2*(3*(4*(d*f^3)^(3/4)*d*f - (d*f^3)^(3/4)*sqrt(-4*d*f + e^2)*
e - (d*f^3)^(3/4)*e^2)*c*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(d*f)*e/
(d*abs(f)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))^3*s
in(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f))))) - (4*(d*f^3)
^(3/4)*d*f - (d*f^3)^(3/4)*sqrt(-4*d*f + e^2)*e - (d*f^3)^(3/4)*e^2)*c*cosh
(1/2*imag_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))^3*sin(1/4*pi + 1/2*real
_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))^3 - 9*(4*(d*f^3)^(3/4)*d*f - (d*
f^3)^(3/4)*sqrt(-4*d*f + e^2)*e - (d*f^3)^(3/4)*e^2)*c*cos(1/4*pi + 1/2*rea
l_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))^2*cosh(1/2*imag_part(arcsin(1/2
*sqrt(d*f)*e/(d*abs(f)))))^2*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(d*f
)*e/(d*abs(f)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f))))) +
 3*(4*(d*f^3)^(3/4)*d*f - (d*f^3)^(3/4)*sqrt(-4*d*f + e^2)*e - (d*f^3)^(3/4
```

)*e^2)*c*cosh(1/2*imag_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))^2*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f))))) + 9*(4*(d*f^3)^(3/4)*d*f - (d*f^3)^(3/4)*sqrt(-4*d*f + e^2)*e - (d*f^3)^(3/4)*e^2)*c*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))^2 - 3*(4*(d*f^3)^(3/4)*d*f - (d*f^3)^(3/4)*sqrt(-4*d*f + e^2)*e - (d*f^3)^(3/4)*e^2)*c*cosh(1/2*imag_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))^2 - 3*(4*(d*f^3)^(3/4)*d*f - (d*f^3)^(3/4)*sqrt(-4*d*f + e^2)*e - (d*f^3)^(3/4)*e^2)*c*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))^2*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))^3 + (4*(d*f^3)^(3/4)*d*f - (d*f^3)^(3/4)*sqrt(-4*d*f + e^2)*e - (d*f^3)^(3/4)*e^2)*c*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))^3 + 2*(4*sqrt(d*f)*d*f^3 + sqrt(d*f)*sqrt(-4*d*f + e^2)*f^2*e - sqrt(d*f)*f^2*e^2)*b*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))^2*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f))))) + 4*(4*sqrt(d*f)*d*f^3 + sqrt(d*f)*sqrt(-4*d*f + e^2)*f^2*e - sqrt(d*f)*f^2*e^2)*b*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f))))) - 2*(4*sqrt(d*f)*d*f^3 + sqrt(d*f)*sqrt(-4*d*f + e^2)*f^2*e + sqrt(d*f)*f^2*e^2)*b*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))^2 + (4*(d*f^3)^(1/4)*d*f^3 - (d*f^3)^(1/4)*sqrt(-4*d*f + e^2)*f^2*e - (d*f^3)^(1/4)*f^2*e^2)*a*cosh(1/2*imag_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f))))) - (4*(d*f^3)^(1/4)*d*f^3 - (d*f^3)^(1/4)*sqrt(-4*d*f + e^2)*f^2*e - (d*f^3)^(1/4)*f^2*e^2)*a*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f))))))*arctan(-((d/f)^(1/4)*cos(1/4*pi + 1/2*arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))) - x)/((d/f)^(1/4)*sin(1/4*pi + 1/2*arcsin(1/2*sqrt(d*f)*e/(d*abs(f))))))/(4*d^2*f^4 - d*f^3*e^2) - 1/4*((4*(d*f^3)^(3/4)*d*f - (d*f^3)^(3/4)*sqrt(-4*d*f + e^2)*e - (d*f^3)^(3/4)*e^2)*c*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))^3*cosh(1/2*imag_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))^3 - 3*(4*(d*f^3)^(3/4)*d*f - (d*f^3)^(3/4)*sqrt(-4*d*f + e^2)*e - (d*f^3)^(3/4)*e^2)*c*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))^3*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))^2 - 3*(4*(d*f^3)^(3/4)*d*f - (d*f^3)^(3/4)*sqrt(-4*d*f + e^2)*e - (d*f^3)^(3/4)*e^2)*c*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))^3*cosh(1/2*imag_par

t(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt
(d*f)*e/(d*abs(f))))) + 9*(4*(d*f^3)^(3/4)*d*f - (d*f^3)^(3/4)*sqrt(-4*d*f
+ e^2)*e - (d*f^3)^(3/4)*e^2)*c*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(
d*f)*e/(d*abs(f)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f))))
)^2*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))^2*sinh(
1/2*imag_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f))))) + 3*(4*(d*f^3)^(3/4)*d*f
 - (d*f^3)^(3/4)*sqrt(-4*d*f + e^2)*e - (d*f^3)^(3/4)*e^2)*c*cos(5/4*pi + 1
/2*real_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))^3*cosh(1/2*imag_part(arcs
in(1/2*sqrt(d*f)*e/(d*abs(f)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(d*f)*e/
(d*abs(f)))))^2 - 9*(4*(d*f^3)^(3/4)*d*f - (d*f^3)^(3/4)*sqrt(-4*d*f + e^2)
*e - (d*f^3)^(3/4)*e^2)*c*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(d*f)*e
/(d*abs(f)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))*sin(
5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))^2*sinh(1/2*imag
_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))^2 - (4*(d*f^3)^(3/4)*d*f - (d*f^
3)^(3/4)*sqrt(-4*d*f + e^2)*e - (d*f^3)^(3/4)*e^2)*c*cos(5/4*pi + 1/2*real_
part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))^3*sinh(1/2*imag_part(arcsin(1/2*s
qrt(d*f)*e/(d*abs(f)))))^3 + 3*(4*(d*f^3)^(3/4)*d*f - (d*f^3)^(3/4)*sqrt(-4
*d*f + e^2)*e - (d*f^3)^(3/4)*e^2)*c*cos(5/4*pi + 1/2*real_part(arcsin(1/2*
sqrt(d*f)*e/(d*abs(f)))))*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(d*f)*e
/(d*abs(f)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))^3
+ (4*sqrt(d*f)*d*f^3 + sqrt(d*f)*sqrt(-4*d*f + e^2)*f^2*e + sqrt(d*f)*f^2*e
^2)*b*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))^2*cos
h(1/2*imag_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))^2 - (4*sqrt(d*f)*d*f^3
 + sqrt(d*f)*sqrt(-4*d*f + e^2)*f^2*e - sqrt(d*f)*f^2*e^2)*b*cosh(1/2*imag_
part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))^2*sin(5/4*pi + 1/2*real_part(arcs
in(1/2*sqrt(d*f)*e/(d*abs(f)))))^2 - 2*(4*sqrt(d*f)*d*f^3 + sqrt(d*f)*sqrt(
-4*d*f + e^2)*f^2*e - sqrt(d*f)*f^2*e^2)*b*cos(5/4*pi + 1/2*real_part(arcsi
n(1/2*sqrt(d*f)*e/(d*abs(f)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(d*f)*e
/(d*abs(f)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f))))) + 2*
(4*sqrt(d*f)*d*f^3 + sqrt(d*f)*sqrt(-4*d*f + e^2)*f^2*e - sqrt(d*f)*f^2*e^2
)*b*cosh(1/2*imag_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))*sin(5/4*pi + 1/
2*real_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))^2*sinh(1/2*imag_part(arcsi
n(1/2*sqrt(d*f)*e/(d*abs(f))))) + (4*sqrt(d*f)*d*f^3 - sqrt(d*f)*sqrt(-4*d*
f + e^2)*f^2*e - sqrt(d*f)*f^2*e^2)*b*cos(5/4*pi + 1/2*real_part(arcsin(1/2
*sqrt(d*f)*e/(d*abs(f)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(d*f)*e/(d*a
bs(f)))))^2 - (4*sqrt(d*f)*d*f^3 - sqrt(d*f)*sqrt(-4*d*f + e^2)*f^2*e + sqr
t(d*f)*f^2*e^2)*b*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(
f)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))^2 + (4*(d*
f^3)^(1/4)*d*f^3 - (d*f^3)^(1/4)*sqrt(-4*d*f + e^2)*f^2*e - (d*f^3)^(1/4)*f
^2*e^2)*a*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))*c
osh(1/2*imag_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f))))) - (4*(d*f^3)^(1/4)*d
*f^3 - (d*f^3)^(1/4)*sqrt(-4*d*f + e^2)*f^2*e - (d*f^3)^(1/4)*f^2*e^2)*a*co
s(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))*sinh(1/2*imag
_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f))))))*log(-2*x*(d/f)^(1/4)*cos(5/4*pi
 + 1/2*arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))) + x^2 + sqrt(d/f))/(4*d^2*f^4 -

d*f^3*e^2) - 1/4*((4*(d*f^3)^(3/4)*d*f - (d*f^3)^(3/4)*sqrt(-4*d*f + e^2)*e - (d*f^3)^(3/4)*e^2)*c*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))^3*cosh(1/2*imag_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))^3 - 3*(4*(d*f^3)^(3/4)*d*f - (d*f^3)^(3/4)*sqrt(-4*d*f + e^2)*e - (d*f^3)^(3/4)*e^2)*c*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))^3*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))^2 - 3*(4*(d*f^3)^(3/4)*d*f - (d*f^3)^(3/4)*sqrt(-4*d*f + e^2)*e - (d*f^3)^(3/4)*e^2)*c*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))^3*cosh(1/2*imag_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f))))) + 9*(4*(d*f^3)^(3/4)*d*f - (d*f^3)^(3/4)*sqrt(-4*d*f + e^2)*e - (d*f^3)^(3/4)*e^2)*c*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))^2*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f))))) + 3*(4*(d*f^3)^(3/4)*d*f - (d*f^3)^(3/4)*sqrt(-4*d*f + e^2)*e - (d*f^3)^(3/4)*e^2)*c*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))^3*cosh(1/2*imag_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))^2 - 9*(4*(d*f^3)^(3/4)*d*f - (d*f^3)^(3/4)*sqrt(-4*d*f + e^2)*e - (d*f^3)^(3/4)*e^2)*c*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))^2 - (4*(d*f^3)^(3/4)*d*f - (d*f^3)^(3/4)*sqrt(-4*d*f + e^2)*e - (d*f^3)^(3/4)*e^2)*c*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))^3 + 3*(4*(d*f^3)^(3/4)*d*f - (d*f^3)^(3/4)*sqrt(-4*d*f + e^2)*e - (d*f^3)^(3/4)*e^2)*c*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))^3 - (4*sqrt(d*f)*d*f^3 + sqrt(d*f)*sqrt(-4*d*f + e^2)*f^2*e - sqrt(d*f)*f^2*e^2)*b*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))^2 - (4*sqrt(d*f)*d*f^3 + sqrt(d*f)*sqrt(-4*d*f + e^2)*f^2*e - sqrt(d*f)*f^2*e^2)*b*cosh(1/2*imag_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))^2*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))^2 - 2*(4*sqrt(d*f)*d*f^3 + sqrt(d*f)*sqrt(-4*d*f + e^2)*f^2*e - sqrt(d*f)*f^2*e^2)*b*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f))))) + 2*(4*sqrt(d*f)*d*f^3 - sqrt(d*f)*sqrt(-4*d*f + e^2)*f^2*e + sqrt(d*f)*f^2*e^2)*b*cosh(1/2*imag_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f))))) - (4*sqrt(d*f)*d*f^3 - sqrt(d*f)*sqrt(-4*d*f + e^2)*f^2*e - sqrt(d*f)*f^2*e^2)*b*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))^2 + (4*sqrt(d*f)*d*f^3 + sqrt(d*f)*sqrt(-4*d*f + e^2)*f^2*e + sqrt(d*f)*f^2*

```
e^2)*b*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))^2*si
nh(1/2*imag_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))^2 + (4*(d*f^3)^(1/4)*
d*f^3 - (d*f^3)^(1/4)*sqrt(-4*d*f + e^2)*f^2*e - (d*f^3)^(1/4)*f^2*e^2)*a*c
os(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))*cosh(1/2*ima
g_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f))))) - (4*(d*f^3)^(1/4)*d*f^3 - (d*f
^3)^(1/4)*sqrt(-4*d*f + e^2)*f^2*e - (d*f^3)^(1/4)*f^2*e^2)*a*cos(1/4*pi +
1/2*real_part(arcsin(1/2*sqrt(d*f)*e/(d*abs(f)))))*sinh(1/2*imag_part(arcsi
n(1/2*sqrt(d*f)*e/(d*abs(f))))))*log(-2*x*(d/f)^(1/4)*cos(1/4*pi + 1/2*arcs
in(1/2*sqrt(d*f)*e/(d*abs(f)))) + x^2 + sqrt(d/f))/(4*d^2*f^4 - d*f^3*e^2)
```

3.338 $\int \frac{(d+ex)^2}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=224

$$\frac{\left(\frac{2cd^2-be^2}{\sqrt{b^2-4ac}}+e^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(e^2-\frac{2cd^2-be^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{2de\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

```
[Out] ((e^2 + (2*c*d^2 - b*e^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqr
t[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) +
((e^2 - (2*c*d^2 - b*e^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqr
t[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) -
(2*d*e*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4*a*c]
```

Rubi [A] time = 0.389424, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {1673, 1166, 205, 12, 1107, 618, 206}

$$\frac{\left(\frac{2cd^2-be^2}{\sqrt{b^2-4ac}}+e^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(e^2-\frac{2cd^2-be^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{2de\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(a + b*x^2 + c*x^4),x]
```


```
[Out] ((e^2 + (2*c*d^2 - b*e^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqr
t[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) +
((e^2 - (2*c*d^2 - b*e^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqr
t[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) -
(2*d*e*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4*a*c]
```


Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
```

```
&& !PolyQ[Pq, x^2]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(d+ex)^2}{a+bx^2+cx^4}\,dx = \int \frac{2dex}{a+bx^2+cx^4}\,dx + \int \frac{d^2+e^2x^2}{a+bx^2+cx^4}\,dx$$

$$= (2de)\int \frac{x}{a+bx^2+cx^4}\,dx + \frac{1}{2}\left(e^2-\frac{2cd^2-be^2}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx + \frac{1}{2}\left(e^2+\frac{2cd^2-be^2}{\sqrt{b^2-4ac}}\right.$$

$$= \frac{\left(e^2+\frac{2cd^2-be^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(e^2-\frac{2cd^2-be^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}} + (de)\,\text{Subst}\left(\int \frac{1}{a+bx+\right.$$

$$= \frac{\left(e^2+\frac{2cd^2-be^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(e^2-\frac{2cd^2-be^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}} - (2de)\,\text{Subst}\left(\int \frac{1}{b^2-4a\right.$$

$$= \frac{\left(e^2+\frac{2cd^2-be^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(e^2-\frac{2cd^2-be^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}} - \frac{2de\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

Mathematica [A] time = 0.268519, size = 245, normalized size = 1.09

$$\frac{\frac{\sqrt{2}\left(e^2\left(\sqrt{b^2-4ac}-b\right)+2cd^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\left(e^2\left(\sqrt{b^2-4ac}+b\right)-2cd^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}} + 2de\log\left(\sqrt{b^2-4ac}-b-2cx^2\right) - 2de\log\left(\right.}{2\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((Sqrt[2]*(2*c*d^2 + (-b + Sqrt[b^2 - 4*a*c])*e^2)*ArcTan[(Sqrt[2]*Sqrt[c]*
x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (S
qrt[2]*(-2*c*d^2 + (b + Sqrt[b^2 - 4*a*c])*e^2)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/
Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + 2*d*e
*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2] - 2*d*e*Log[b + Sqrt[b^2 - 4*a*c] +
2*c*x^2])/(2*Sqrt[b^2 - 4*a*c])
```

Maple [B] time = 0.027, size = 633, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(c*x^4+b*x^2+a),x)
```

```
[Out] (-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*d*e*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)+2*c/(4*
a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-
4*a*c+b^2)^(1/2))*c)^(1/2))*e^2*a-1/2/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^
(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*e^2*b^
2-1/2*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/
2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*e^2+c*(-4*a*c+b^2
)^(1/2)/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(
1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*d^2-(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*d
*e*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)-2*c/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b
^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*
e^2*a+1/2/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x
*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*e^2*b^2-1/2*(-4*a*c+b^2)^(1/2)/
(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/(
(-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*e^2+c*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*2^
(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^
2)^(1/2))*c)^(1/2))*d^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^2}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^2/(c*x^4 + b*x^2 + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [C] time = 3.06879, size = 9196, normalized size = 41.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*(3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 -
4*a*c)*b)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2
*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*e^2*sin(5/4*pi +
 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - ((a*c^3)^(3/4)*b^2 -
4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cosh(1/2*imag_part
(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*e^2*sin(5/4*pi + 1/2*real_part(arcs
in(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 - 9*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)
*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cos(5/4*pi + 1/2*real_part(arcsin
(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/
(a*abs(c)))))^2*e^2*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*ab
s(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 3*((a*c^
3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cos
h(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*e^2*sin(5/4*pi + 1/2
*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin
(1/2*sqrt(a*c)*b/(a*abs(c))))) + 9*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c
 + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cos(5/4*pi + 1/2*real_part(arcsin(1/2
*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*a
bs(c)))))*e^2*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))
))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 3*((a*c^3)^(
```

```
3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cosh(1/
2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*e^2*sin(5/4*pi + 1/2*real_
part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*s
qrt(a*c)*b/(a*abs(c)))))^2 - 3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (
a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqr
t(a*c)*b/(a*abs(c)))))^2*e^2*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c
)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3
 + ((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*
c)*b)*e^2*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3
*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 + 4*(sqrt(a*c)*b
^2*c^2 + 4*sqrt(a*c)*a*c^3 + sqrt(b^2 - 4*a*c)*sqrt(a*c)*b*c^2)*d*cos(5/4*p
i + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(a
rcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*e*sin(5/4*pi + 1/2*real_part(arcsin(1
/2*sqrt(a*c)*b/(a*abs(c))))) - 8*(sqrt(a*c)*b^2*c^2 + 4*sqrt(a*c)*a*c^3 - s
qrt(b^2 - 4*a*c)*sqrt(a*c)*b*c^2)*d*cos(5/4*pi + 1/2*real_part(arcsin(1/2*s
qrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c
)))))*e*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin
h(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - 4*(sqrt(a*c)*b^2*c^2
 - 4*sqrt(a*c)*a*c^3 + sqrt(b^2 - 4*a*c)*sqrt(a*c)*b*c^2)*d*cos(5/4*pi + 1/
2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*e*sin(5/4*pi + 1/2*real_pa
rt(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(
a*c)*b/(a*abs(c)))))^2 + ((a*c^3)^(1/4)*b^2*c^2 - 4*(a*c^3)^(1/4)*a*c^3 + (
a*c^3)^(1/4)*sqrt(b^2 - 4*a*c)*b*c^2)*d^2*cosh(1/2*imag_part(arcsin(1/2*sqr
t(a*c)*b/(a*abs(c)))))*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a
*abs(c))))) - ((a*c^3)^(1/4)*b^2*c^2 - 4*(a*c^3)^(1/4)*a*c^3 + (a*c^3)^(1/4
)*sqrt(b^2 - 4*a*c)*b*c^2)*d^2*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a
*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))
)*arctan(-((a/c)^(1/4)*cos(5/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))
 - x)/((a/c)^(1/4)*sin(5/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))/(
a*b^2*c^3 - 4*a^2*c^4) + 1/2*(3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c +
(a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sq
rt(a*c)*b/(a*abs(c)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(
c)))))^3*e^2*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))
) - ((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a
*c)*b)*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*e^2*sin(1/
4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 - 9*((a*c^3)^(3
/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cos(1/4*
pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(1/2*imag_par
t(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*e^2*sin(1/4*pi + 1/2*real_part(arc
sin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b
/(a*abs(c))))) + 3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)
*sqrt(b^2 - 4*a*c)*b)*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))
))^2*e^2*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*
sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 9*((a*c^3)^(3/4)*
b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cos(1/4*pi +
```

```
 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(1/2*imag_part(ar
csin(1/2*sqrt(a*c)*b/(a*abs(c)))))*e^2*sin(1/4*pi + 1/2*real_part(arcsin(1/
2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*ab
s(c)))))^2 - 3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqr
t(b^2 - 4*a*c)*b)*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*e
^2*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1
/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 3*((a*c^3)^(3/4)*b^2
 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cos(1/4*pi + 1/2
*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*e^2*sin(1/4*pi + 1/2*real
_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sq
rt(a*c)*b/(a*abs(c)))))^3 + ((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c
^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*e^2*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sq
rt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(
c)))))^3 + 4*(sqrt(a*c)*b^2*c^2 - 4*sqrt(a*c)*a*c^3 - sqrt(b^2 - 4*a*c)*sqr
t(a*c)*b*c^2)*d*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)
))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*e*sin(1/4*pi
 + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 8*(sqrt(a*c)*b^2*c^
2 - 4*sqrt(a*c)*a*c^3 - sqrt(b^2 - 4*a*c)*sqrt(a*c)*b*c^2)*d*cos(1/4*pi + 1
/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin
(1/2*sqrt(a*c)*b/(a*abs(c)))))*e*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt
(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))
)) + 4*(sqrt(a*c)*b^2*c^2 + 4*sqrt(a*c)*a*c^3 + sqrt(b^2 - 4*a*c)*sqrt(a*c)
*b*c^2)*d*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*e
*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*i
mag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 + ((a*c^3)^(1/4)*b^2*c^2 -
4*(a*c^3)^(1/4)*a*c^3 + (a*c^3)^(1/4)*sqrt(b^2 - 4*a*c)*b*c^2)*d^2*cosh(1/2
*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(1/4*pi + 1/2*real_part(
arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - ((a*c^3)^(1/4)*b^2*c^2 - 4*(a*c^3)^(
1/4)*a*c^3 + (a*c^3)^(1/4)*sqrt(b^2 - 4*a*c)*b*c^2)*d^2*sin(1/4*pi + 1/2*re
al_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*
sqrt(a*c)*b/(a*abs(c))))))*arctan(-((a/c)^(1/4)*cos(1/4*pi + 1/2*arcsin(1/2
*sqrt(a*c)*b/(a*abs(c)))) - x)/((a/c)^(1/4)*sin(1/4*pi + 1/2*arcsin(1/2*sqr
t(a*c)*b/(a*abs(c))))))/(a*b^2*c^3 - 4*a^2*c^4) - 1/4*(((a*c^3)^(3/4)*b^2 -
 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cos(5/4*pi + 1/2*
real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*cosh(1/2*imag_part(arcsin(
1/2*sqrt(a*c)*b/(a*abs(c)))))^3*e^2 - 3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4
)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cos(5/4*pi + 1/2*real_part(arcsi
n(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(
a*abs(c)))))^3*e^2*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs
(c)))))^2 - 3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt
(b^2 - 4*a*c)*b)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c
)))))^3*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*e^2*sinh(
1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 9*((a*c^3)^(3/4)*b^2 -
 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cos(5/4*pi + 1/2*
real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/
```

```
2*sqrt(a*c)*b/(a*abs(c)))))^2*e^2*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqr
t(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c
))))) + 3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2
 - 4*a*c)*b)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))
)^3*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*e^2*sinh(1/2*im
ag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 9*((a*c^3)^(3/4)*b^2 - 4*(
a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cos(5/4*pi + 1/2*real
_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sq
rt(a*c)*b/(a*abs(c)))))*e^2*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)
*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^
2 - ((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a
*c)*b)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*e^
2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 + 3*((a*c^3)^(3
/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cos(5/4*
pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*e^2*sin(5/4*pi + 1/
2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsi
n(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 - 2*(sqrt(a*c)*b^2*c^2 + 4*sqrt(a*c)*a*c^
3 + sqrt(b^2 - 4*a*c)*sqrt(a*c)*b*c^2)*d*cos(5/4*pi + 1/2*real_part(arcsin(
1/2*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(
a*abs(c)))))^2*e - 2*(sqrt(a*c)*b^2*c^2 - 4*sqrt(a*c)*a*c^3 - sqrt(b^2 - 4*
a*c)*sqrt(a*c)*b*c^2)*d*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)
))))^2*e*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2
- 4*(sqrt(a*c)*b^2*c^2 - 4*sqrt(a*c)*a*c^3 - sqrt(b^2 - 4*a*c)*sqrt(a*c)*b*
c^2)*d*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*co
sh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*e*sinh(1/2*imag_part(
arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 4*(sqrt(a*c)*b^2*c^2 - 4*sqrt(a*c)*a
*c^3 - sqrt(b^2 - 4*a*c)*sqrt(a*c)*b*c^2)*d*cosh(1/2*imag_part(arcsin(1/2*s
qrt(a*c)*b/(a*abs(c)))))*e*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*
b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) +
 2*(sqrt(a*c)*b^2*c^2 - 4*sqrt(a*c)*a*c^3 + sqrt(b^2 - 4*a*c)*sqrt(a*c)*b*c
^2)*d*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*e*s
inh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 + 2*(sqrt(a*c)*b^2
*c^2 + 4*sqrt(a*c)*a*c^3 - sqrt(b^2 - 4*a*c)*sqrt(a*c)*b*c^2)*d*e*sin(5/4*p
i + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part
(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 + ((a*c^3)^(1/4)*b^2*c^2 - 4*(a*c^3
)^(1/4)*a*c^3 + (a*c^3)^(1/4)*sqrt(b^2 - 4*a*c)*b*c^2)*d^2*cos(5/4*pi + 1/2
*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1
/2*sqrt(a*c)*b/(a*abs(c))))) - ((a*c^3)^(1/4)*b^2*c^2 - 4*(a*c^3)^(1/4)*a*c
^3 + (a*c^3)^(1/4)*sqrt(b^2 - 4*a*c)*b*c^2)*d^2*cos(5/4*pi + 1/2*real_part(
arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c
)*b/(a*abs(c))))))*log(-2*x*(a/c)^(1/4)*cos(5/4*pi + 1/2*arcsin(1/2*sqrt(a*
c)*b/(a*abs(c)))) + x^2 + sqrt(a/c))/(a*b^2*c^3 - 4*a^2*c^4) - 1/4*(((a*c^3
)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cos(
1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*cosh(1/2*imag
_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*e^2 - 3*((a*c^3)^(3/4)*b^2 - 4
```

*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cos(1/4*pi + 1/2*re
al_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*
sqrt(a*c)*b/(a*abs(c)))))^3*e^2*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(
a*c)*b/(a*abs(c)))))^2 - 3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^
3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*
c)*b/(a*abs(c)))))^3*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))
)^2*e^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 9*((a*c^3
)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cos(
1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_p
art(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*e^2*sin(1/4*pi + 1/2*real_part(a
rcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*
c)*b/(a*abs(c))))) + 3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(
3/4)*sqrt(b^2 - 4*a*c)*b)*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b
/(a*abs(c)))))^3*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*e^
2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 9*((a*c^3)^(3
/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cos(1/4*
pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(
arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*e^2*sin(1/4*pi + 1/2*real_part(arcsin(
1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(
a*abs(c)))))^2 - ((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*s
qrt(b^2 - 4*a*c)*b)*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*ab
s(c)))))^3*e^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 +
3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c
)*b)*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*e^2*si
n(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*im
ag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 - 2*(sqrt(a*c)*b^2*c^2 - 4*s
qrt(a*c)*a*c^3 - sqrt(b^2 - 4*a*c)*sqrt(a*c)*b*c^2)*d*cos(1/4*pi + 1/2*real
_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(1/2*imag_part(arcsin(1/2*
sqrt(a*c)*b/(a*abs(c)))))^2*e - 2*(sqrt(a*c)*b^2*c^2 - 4*sqrt(a*c)*a*c^3 -
sqrt(b^2 - 4*a*c)*sqrt(a*c)*b*c^2)*d*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c
)*b/(a*abs(c)))))^2*e*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*
abs(c)))))^2 - 4*(sqrt(a*c)*b^2*c^2 - 4*sqrt(a*c)*a*c^3 - sqrt(b^2 - 4*a*c)
*sqrt(a*c)*b*c^2)*d*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*ab
s(c)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*e*sinh(1
/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - 4*(sqrt(a*c)*b^2*c^2 +
4*sqrt(a*c)*a*c^3 - sqrt(b^2 - 4*a*c)*sqrt(a*c)*b*c^2)*d*cosh(1/2*imag_part
(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*e*sin(1/4*pi + 1/2*real_part(arcsin(1
/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a
abs(c))))) - 2(sqrt(a*c)*b^2*c^2 - 4*sqrt(a*c)*a*c^3 + sqrt(b^2 - 4*a*c)*
sqrt(a*c)*b*c^2)*d*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs
(c)))))^2*e*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 2*(
sqrt(a*c)*b^2*c^2 + 4*sqrt(a*c)*a*c^3 + sqrt(b^2 - 4*a*c)*sqrt(a*c)*b*c^2)*
d*e*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(
1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 + ((a*c^3)^(1/4)*b^2*c
^2 - 4*(a*c^3)^(1/4)*a*c^3 + (a*c^3)^(1/4)*sqrt(b^2 - 4*a*c)*b*c^2)*d^2*cos

```
(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_
part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - ((a*c^3)^(1/4)*b^2*c^2 - 4*(a*c
^3)^(1/4)*a*c^3 + (a*c^3)^(1/4)*sqrt(b^2 - 4*a*c)*b*c^2)*d^2*cos(1/4*pi + 1
/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin
(1/2*sqrt(a*c)*b/(a*abs(c))))))*log(-2*x*(a/c)^(1/4)*cos(1/4*pi + 1/2*arcsi
n(1/2*sqrt(a*c)*b/(a*abs(c)))) + x^2 + sqrt(a/c))/(a*b^2*c^3 - 4*a^2*c^4)
```

3.339 $\int \frac{x^2}{(a+bx)(c+dx)}\,dx$

Optimal. Leaf size=56

$$\frac{a^2\log(a+bx)}{b^2(bc-ad)}-\frac{c^2\log(c+dx)}{d^2(bc-ad)}+\frac{x}{bd}$$

```
[Out] x/(b*d) + (a^2*Log[a + b*x])/(b^2*(b*c - a*d)) - (c^2*Log[c + d*x])/(d^2*(b
*c - a*d))
```

Rubi [A] time = 0.0466322, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {72}

$$\frac{a^2\log(a+bx)}{b^2(bc-ad)}-\frac{c^2\log(c+dx)}{d^2(bc-ad)}+\frac{x}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x)*(c + d*x)),x]
```

```
[Out] x/(b*d) + (a^2*Log[a + b*x])/(b^2*(b*c - a*d)) - (c^2*Log[c + d*x])/(d^2*(b
*c - a*d))
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(a+bx)(c+dx)}\,dx &= \int \left(\frac{1}{bd}+\frac{a^2}{b(bc-ad)(a+bx)}+\frac{c^2}{d(-bc+ad)(c+dx)}\right)dx\\ &= \frac{x}{bd}+\frac{a^2\log(a+bx)}{b^2(bc-ad)}-\frac{c^2\log(c+dx)}{d^2(bc-ad)}\end{aligned}$$

Mathematica [A] time = 0.0352156, size = 56, normalized size = 1.

$$\frac{a^2 \log(a+bx)}{b^2(bc-ad)} - \frac{c^2 \log(c+dx)}{d^2(bc-ad)} + \frac{x}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x)*(c + d*x)),x]
```

```
[Out] x/(b*d) + (a^2*Log[a + b*x])/(b^2*(b*c - a*d)) - (c^2*Log[c + d*x])/(d^2*(b
*c - a*d))
```

Maple [A] time = 0.005, size = 57, normalized size = 1.

$$\frac{x}{bd} - \frac{a^2 \ln(bx+a)}{b^2(ad-bc)} + \frac{c^2 \ln(dx+c)}{d^2(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x+a)/(d*x+c),x)
```

```
[Out] x/b/d-1/b^2*a^2/(a*d-b*c)*ln(b*x+a)+1/d^2*c^2/(a*d-b*c)*ln(d*x+c)
```

Maxima [A] time = 1.23451, size = 81, normalized size = 1.45

$$\frac{a^2 \log(bx+a)}{b^3c - ab^2d} - \frac{c^2 \log(dx+c)}{bcd^2 - ad^3} + \frac{x}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] a^2*log(b*x + a)/(b^3*c - a*b^2*d) - c^2*log(d*x + c)/(b*c*d^2 - a*d^3) + x
/(b*d)
```

Fricas [A] time = 1.45653, size = 128, normalized size = 2.29

$$\frac{a^2d^2 \log(bx+a) - b^2c^2 \log(dx+c) + \left(b^2cd - abd^2\right)x}{b^3cd^2 - ab^2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] (a^2*d^2*log(b*x + a) - b^2*c^2*log(d*x + c) + (b^2*c*d - a*b*d^2)*x)/(b^3*
c*d^2 - a*b^2*d^3)
```

Sympy [B] time = 1.04137, size = 190, normalized size = 3.39

$$-\frac{a^2\log\left(x+\frac{\frac{a^4d^3}{b(ad-bc)}-\frac{2a^3cd^2}{ad-bc}+\frac{a^2bc^2d}{ad-bc}+a^2cd+abc^2}{a^2d^2+b^2c^2}\right)}{b^2\,(ad-bc)}+\frac{c^2\log\left(x+\frac{-\frac{a^2bc^2d}{ad-bc}+a^2cd+\frac{2ab^2c^3}{ad-bc}+abc^2-\frac{b^3c^4}{d(ad-bc)}}{a^2d^2+b^2c^2}\right)}{d^2\,(ad-bc)}+\frac{x}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x+a)/(d*x+c),x)
```

```
[Out] -a**2*log(x + (a**4*d**3/(b*(a*d - b*c)) - 2*a**3*c*d**2/(a*d - b*c) + a**2
*b*c**2*d/(a*d - b*c) + a**2*c*d + a*b*c**2)/(a**2*d**2 + b**2*c**2))/(b**2
*(a*d - b*c)) + c**2*log(x + (-a**2*b*c**2*d/(a*d - b*c) + a**2*c*d + 2*a*b
**2*c**3/(a*d - b*c) + a*b*c**2 - b**3*c**4/(d*(a*d - b*c)))/(a**2*d**2 + b
**2*c**2))/(d**2*(a*d - b*c)) + x/(b*d)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.340 $\int \frac{x^2}{(c+dx)(a+bx^2)}\,dx$

Optimal. Leaf size=96

$$\frac{ad\log\left(a+bx^2\right)}{2b\left(ad^2+bc^2\right)}+\frac{c^2\log(c+dx)}{d\left(ad^2+bc^2\right)}-\frac{\sqrt{a}c\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}\left(ad^2+bc^2\right)}$$

```
[Out] -((Sqrt[a]*c*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[b]*(b*c^2 + a*d^2))) + (c^2
*Log[c + d*x])/(d*(b*c^2 + a*d^2)) + (a*d*Log[a + b*x^2])/(2*b*(b*c^2 + a*d
^2))
```

Rubi [A] time = 0.108043, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1629, 635, 205, 260}

$$\frac{ad\log\left(a+bx^2\right)}{2b\left(ad^2+bc^2\right)}+\frac{c^2\log(c+dx)}{d\left(ad^2+bc^2\right)}-\frac{\sqrt{a}c\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}\left(ad^2+bc^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((c + d*x)*(a + b*x^2)),x]
```

```
[Out] -((Sqrt[a]*c*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[b]*(b*c^2 + a*d^2))) + (c^2
*Log[c + d*x])/(d*(b*c^2 + a*d^2)) + (a*d*Log[a + b*x^2])/(2*b*(b*c^2 + a*d
^2))
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2}{(c+dx)\left(a+bx^2\right)}\,dx &= \int \left(\frac{c^2}{\left(bc^2+ad^2\right)(c+dx)}-\frac{a(c-dx)}{\left(bc^2+ad^2\right)\left(a+bx^2\right)}\right)dx\\
&= \frac{c^2\log(c+dx)}{d\left(bc^2+ad^2\right)}-\frac{a\int \frac{c-dx}{a+bx^2}\,dx}{bc^2+ad^2}\\
&= \frac{c^2\log(c+dx)}{d\left(bc^2+ad^2\right)}-\frac{(ac)\int \frac{1}{a+bx^2}\,dx}{bc^2+ad^2}+\frac{(ad)\int \frac{x}{a+bx^2}\,dx}{bc^2+ad^2}\\
&= -\frac{\sqrt{a}c\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}\left(bc^2+ad^2\right)}+\frac{c^2\log(c+dx)}{d\left(bc^2+ad^2\right)}+\frac{ad\log\left(a+bx^2\right)}{2b\left(bc^2+ad^2\right)}
\end{aligned}
$$

Mathematica [A] time = 0.0418023, size = 73, normalized size = 0.76

$$
\frac{-2\sqrt{a}\sqrt{b}cd\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)+ad^2\log\left(a+bx^2\right)+2bc^2\log(c+dx)}{2abd^3+2b^2c^2d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((c + d*x)*(a + b*x^2)),x]
```

```
[Out] (-2*Sqrt[a]*Sqrt[b]*c*d*ArcTan[(Sqrt[b]*x)/Sqrt[a]] + 2*b*c^2*Log[c + d*x]
+ a*d^2*Log[a + b*x^2])/(2*b^2*c^2*d + 2*a*b*d^3)
```

Maple [A] time = 0.007, size = 87, normalized size = 0.9

$$
\frac{ad\ln\left(bx^2+a\right)}{2\,b\left(ad^2+c^2b\right)}-\frac{ac}{ad^2+c^2b}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{c^2\ln(dx+c)}{d\left(ad^2+c^2b\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(d*x+c)/(b*x^2+a),x)
```

```
[Out] 1/2*a*d*ln(b*x^2+a)/b/(a*d^2+b*c^2)-a/(a*d^2+b*c^2)*c/(a*b)^(1/2)*arctan(b*
x/(a*b)^(1/2))+c^2*ln(d*x+c)/d/(a*d^2+b*c^2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(d*x+c)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.77094, size = 354, normalized size = 3.69

$$\left[\frac{bcd\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+ad^2\log\left(bx^2+a\right)+2\,bc^2\log\left(dx+c\right)}{2\left(b^2c^2d+abd^3\right)},-\frac{2\,bcd\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)-ad^2\log\left(bx^2+a\right)-2}{2\left(b^2c^2d+abd^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(d*x+c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/2*(b*c*d*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + a*
d^2*log(b*x^2 + a) + 2*b*c^2*log(d*x + c))/(b^2*c^2*d + a*b*d^3), -1/2*(2*b
*c*d*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - a*d^2*log(b*x^2 + a) - 2*b*c^2*log
(d*x + c))/(b^2*c^2*d + a*b*d^3)]
```

Sympy [B] time = 7.01037, size = 1355, normalized size = 14.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(d*x+c)/(b*x**2+a),x)
```

```
[Out] c**2*log(x + (-4*a**3*b*c**4*d**5/(a*d**2 + b*c**2)**2 + 2*a**3*c**2*d**5/(
a*d**2 + b*c**2) + 4*a**2*b**2*c**6*d**3/(a*d**2 + b*c**2)**2 - 4*a**2*b*c*
*4*d**3/(a*d**2 + b*c**2) - a**2*c**2*d**3 + 20*a*b**3*c**8*d/(a*d**2 + b*c
**2)**2 - 14*a*b**2*c**6*d/(a*d**2 + b*c**2) + 7*a*b*c**4*d + 12*b**4*c**10
/(d*(a*d**2 + b*c**2)**2) - 8*b**3*c**8/(d*(a*d**2 + b*c**2)))/(a**2*c*d**4
 - 3*a*b*c**3*d**2 + 4*b**2*c**5))/(d*(a*d**2 + b*c**2)) + (a*d/(2*b*(a*d**
2 + b*c**2)) - c*sqrt(-a*b**3)/(2*b**2*(a*d**2 + b*c**2)))*log(x + (-4*a**3
*b*d**7*(a*d/(2*b*(a*d**2 + b*c**2)) - c*sqrt(-a*b**3)/(2*b**2*(a*d**2 + b*
c**2)))**2 + 2*a**3*d**6*(a*d/(2*b*(a*d**2 + b*c**2)) - c*sqrt(-a*b**3)/(2*
b**2*(a*d**2 + b*c**2))) + 4*a**2*b**2*c**2*d**5*(a*d/(2*b*(a*d**2 + b*c**2
)) - c*sqrt(-a*b**3)/(2*b**2*(a*d**2 + b*c**2)))**2 - 4*a**2*b*c**2*d**4*(a
*d/(2*b*(a*d**2 + b*c**2)) - c*sqrt(-a*b**3)/(2*b**2*(a*d**2 + b*c**2))) -
a**2*c**2*d**3 + 20*a*b**3*c**4*d**3*(a*d/(2*b*(a*d**2 + b*c**2)) - c*sqrt(
-a*b**3)/(2*b**2*(a*d**2 + b*c**2)))**2 - 14*a*b**2*c**4*d**2*(a*d/(2*b*(a*
d**2 + b*c**2)) - c*sqrt(-a*b**3)/(2*b**2*(a*d**2 + b*c**2))) + 7*a*b*c**4*
d + 12*b**4*c**6*d*(a*d/(2*b*(a*d**2 + b*c**2)) - c*sqrt(-a*b**3)/(2*b**2*(
a*d**2 + b*c**2)))**2 - 8*b**3*c**6*(a*d/(2*b*(a*d**2 + b*c**2)) - c*sqrt(-
a*b**3)/(2*b**2*(a*d**2 + b*c**2))))/(a**2*c*d**4 - 3*a*b*c**3*d**2 + 4*b**
2*c**5)) + (a*d/(2*b*(a*d**2 + b*c**2)) + c*sqrt(-a*b**3)/(2*b**2*(a*d**2 +
 b*c**2)))*log(x + (-4*a**3*b*d**7*(a*d/(2*b*(a*d**2 + b*c**2)) + c*sqrt(-a
*b**3)/(2*b**2*(a*d**2 + b*c**2)))**2 + 2*a**3*d**6*(a*d/(2*b*(a*d**2 + b*c
**2)) + c*sqrt(-a*b**3)/(2*b**2*(a*d**2 + b*c**2))) + 4*a**2*b**2*c**2*d**5
*(a*d/(2*b*(a*d**2 + b*c**2)) + c*sqrt(-a*b**3)/(2*b**2*(a*d**2 + b*c**2)))
**2 - 4*a**2*b*c**2*d**4*(a*d/(2*b*(a*d**2 + b*c**2)) + c*sqrt(-a*b**3)/(2*
b**2*(a*d**2 + b*c**2))) - a**2*c**2*d**3 + 20*a*b**3*c**4*d**3*(a*d/(2*b*(
a*d**2 + b*c**2)) + c*sqrt(-a*b**3)/(2*b**2*(a*d**2 + b*c**2)))**2 - 14*a*b
**2*c**4*d**2*(a*d/(2*b*(a*d**2 + b*c**2)) + c*sqrt(-a*b**3)/(2*b**2*(a*d**
2 + b*c**2))) + 7*a*b*c**4*d + 12*b**4*c**6*d*(a*d/(2*b*(a*d**2 + b*c**2))
+ c*sqrt(-a*b**3)/(2*b**2*(a*d**2 + b*c**2)))**2 - 8*b**3*c**6*(a*d/(2*b*(a
*d**2 + b*c**2)) + c*sqrt(-a*b**3)/(2*b**2*(a*d**2 + b*c**2))))/(a**2*c*d**
4 - 3*a*b*c**3*d**2 + 4*b**2*c**5))
```

Giac [A] time = 1.18663, size = 115, normalized size = 1.2

$$\frac{ad\log\left(bx^2+a\right)}{2\left(b^2c^2+abd^2\right)}+\frac{c^2\log\left(|dx+c|\right)}{bc^2d+ad^3}-\frac{ac\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\left(bc^2+ad^2\right)\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(d*x+c)/(b*x^2+a),x, algorithm="giac")

[Out] 1/2*a*d*log(b*x^2 + a)/(b^2*c^2 + a*b*d^2) + c^2*log(abs(d*x + c))/(b*c^2*d
 + a*d^3) - a*c*arctan(b*x/sqrt(a*b))/((b*c^2 + a*d^2)*sqrt(a*b))
```

3.341 $\int \frac{x^2}{(c+dx)\left(a+bx^3\right)}\,dx$

Optimal. Leaf size=264

$$-\frac{\sqrt[3]{a}d\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{2/3}\left(bc^3-ad^3\right)}-\frac{\sqrt[3]{a}d\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{2/3}\left(a^{2/3}d^2+\sqrt[3]{a}\sqrt[3]{b}cd+b^{2/3}c^2\right)}+\frac{\sqrt[3]{a}d\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{\cdots}\right.}{3b^{2/3}\left(bc^3-ad^3\right)}$$

```
[Out] -((a^(1/3)*d*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^
(2/3)*(b^(2/3)*c^2 + a^(1/3)*b^(1/3)*c*d + a^(2/3)*d^2))) + (a^(1/3)*d*(b^(
1/3)*c + a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(2/3)*(b*c^3 - a*d^3)) -
 (c^2*Log[c + d*x])/(b*c^3 - a*d^3) - (a^(1/3)*d*(b^(1/3)*c + a^(1/3)*d)*Lo
g[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(2/3)*(b*c^3 - a*d^3)) +
 (c^2*Log[a + b*x^3])/(3*(b*c^3 - a*d^3))
```

Rubi [A] time = 0.472221, antiderivative size = 264, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.45, Rules used = {6725, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{\sqrt[3]{a}d\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{2/3}\left(bc^3-ad^3\right)}-\frac{\sqrt[3]{a}d\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{2/3}\left(a^{2/3}d^2+\sqrt[3]{a}\sqrt[3]{b}cd+b^{2/3}c^2\right)}+\frac{\sqrt[3]{a}d\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{\cdots}\right.}{3b^{2/3}\left(bc^3-ad^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((c + d*x)*(a + b*x^3)),x]
```



```
[Out] -((a^(1/3)*d*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^
(2/3)*(b^(2/3)*c^2 + a^(1/3)*b^(1/3)*c*d + a^(2/3)*d^2))) + (a^(1/3)*d*(b^(
1/3)*c + a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(2/3)*(b*c^3 - a*d^3)) -
 (c^2*Log[c + d*x])/(b*c^3 - a*d^3) - (a^(1/3)*d*(b^(1/3)*c + a^(1/3)*d)*Lo
g[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(2/3)*(b*c^3 - a*d^3)) +
 (c^2*Log[a + b*x^3])/(3*(b*c^3 - a*d^3))
```


Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{x^2}{(c+dx)\left(a+bx^3\right)}\,dx = \int \left(-\frac{c^2 d}{\left(bc^3-ad^3\right)(c+dx)} + \frac{acd - ad^2x + bc^2x^2}{\left(bc^3-ad^3\right)\left(a+bx^3\right)}\right) dx$$

$$= -\frac{c^2\log(c+dx)}{bc^3-ad^3} + \frac{\int \frac{acd-ad^2x+bc^2x^2}{a+bx^3}\,dx}{bc^3-ad^3}$$

$$= -\frac{c^2\log(c+dx)}{bc^3-ad^3} + \frac{\int \frac{acd-ad^2x}{a+bx^3}\,dx}{bc^3-ad^3} + \frac{\left(bc^2\right)\int \frac{x^2}{a+bx^3}\,dx}{bc^3-ad^3}$$

$$= -\frac{c^2\log(c+dx)}{bc^3-ad^3} + \frac{c^2\log\left(a+bx^3\right)}{3\left(bc^3-ad^3\right)} + \frac{\int \frac{\sqrt[3]{a}\left(2a\sqrt[3]{b}cd-a^{4/3}d^2\right)+\sqrt[3]{b}\left(-a\sqrt[3]{b}cd-a^{4/3}d^2\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{2/3}\sqrt[3]{b}\left(bc^3-ad^3\right)} + \frac{\left(\sqrt[3]{a}d\left(c+\frac{\sqrt[3]{a}}{\sqrt[3]{b}}\right.\right.}{3\left(bc\right.}$$

$$= \frac{\sqrt[3]{a}d\left(\sqrt[3]{b}c+\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{2/3}\left(bc^3-ad^3\right)} - \frac{c^2\log(c+dx)}{bc^3-ad^3} + \frac{c^2\log\left(a+bx^3\right)}{3\left(bc^3-ad^3\right)} + \frac{\left(a^{2/3}d\right)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{}}}{2\sqrt[3]{b}\left(b^{2/3}c^2+\sqrt[3]{a}\sqrt[3]{b}\right.}$$

$$= \frac{\sqrt[3]{a}d\left(\sqrt[3]{b}c+\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{2/3}\left(bc^3-ad^3\right)} - \frac{c^2\log(c+dx)}{bc^3-ad^3} - \frac{\sqrt[3]{a}d\left(\sqrt[3]{b}c+\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+\right.}{6b^{2/3}\left(bc^3-ad^3\right)}$$

$$= -\frac{\sqrt[3]{a}d\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{2/3}\left(b^{2/3}c^2+\sqrt[3]{a}\sqrt[3]{b}cd+a^{2/3}d^2\right)} + \frac{\sqrt[3]{a}d\left(\sqrt[3]{b}c+\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{2/3}\left(bc^3-ad^3\right)} - \frac{c^2\log(c+dx)}{bc^3-ad^3} -$$

Mathematica [A] time = 0.0945356, size = 228, normalized size = 0.86

$$\frac{-\sqrt[3]{a}\sqrt[3]{b}cd\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-a^{2/3}d^2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+2b^{2/3}c^2\log\left(a+bx^3\right)+2\sqrt[3]{a}d\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)}{6b^{2/3}\left(bc^3-ad^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((c + d*x)*(a + b*x^3)),x]
```

```
[Out] (2*Sqrt[3]*a^(1/3)*d*(-(b^(1/3)*c) + a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/a
^(1/3))/Sqrt[3]] + 2*a^(1/3)*d*(b^(1/3)*c + a^(1/3)*d)*Log[a^(1/3) + b^(1/3
)*x] - 6*b^(2/3)*c^2*Log[c + d*x] - a^(1/3)*b^(1/3)*c*d*Log[a^(2/3) - a^(1/
3)*b^(1/3)*x + b^(2/3)*x^2] - a^(2/3)*d^2*Log[a^(2/3) - a^(1/3)*b^(1/3)*x +
 b^(2/3)*x^2] + 2*b^(2/3)*c^2*Log[a + b*x^3])/(6*b^(2/3)*(b*c^3 - a*d^3))
```

Maple [A] time = 0.006, size = 336, normalized size = 1.3

$$-\frac{acd}{\left(3\,ad^{3}-3\,bc^{3}\right)b}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{acd}{\left(6\,ad^{3}-6\,bc^{3}\right)b}\ln\left(x^{2}-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{acd\sqrt{3}}{\left(3\,ad^{3}-3\,bc^{3}\right)b}\arctan\left(\frac{\sqrt{3}}{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(d*x+c)/(b*x^3+a),x)
```

```
[Out] -1/3/(a*d^3-b*c^3)*a*c*d/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))+1/6/(a*d^3-b*c^3)*
a*c*d/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-1/3/(a*d^3-b*c^3)*a*c
*d/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-1/3/(a*d^3
-b*c^3)*a*d^2/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/(a*d^3-b*c^3)*a*d^2/b/(a/
b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/(a*d^3-b*c^3)*a*d^2*3^(1/2)/
b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-1/3/(a*d^3-b*c^3)*c^2
*ln(b*x^3+a)+c^2/(a*d^3-b*c^3)*ln(d*x+c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(d*x+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 10.9875, size = 11777, normalized size = 44.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(d*x+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/12*(2*(b*c^3 - a*d^3)*(2*(1/2)^(2/3)*(c^4/(b*c^3 - a*d^3)^2 - c/(b^2*c^3
 - a*b*d^3))*(-I*sqrt(3) + 1)/(2*c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 -
a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*
b^2*d^3))^(1/3) + (1/2)^(1/3)*(2*c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 -
a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*
b^2*d^3))^(1/3)*(I*sqrt(3) + 1) - 2*c^2/(b*c^3 - a*d^3))*log(-3/2*(2*(1/2)^
(2/3)*(c^4/(b*c^3 - a*d^3)^2 - c/(b^2*c^3 - a*b*d^3))*(-I*sqrt(3) + 1)/(2*c
^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/
((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3) + (1/2)^(1/3)*(2*c
^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/
((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3)*(I*sqrt(3) + 1) -
2*c^2/(b*c^3 - a*d^3))*b*c^2 - 1/4*(b^2*c^3 - a*b*d^3)*(2*(1/2)^(2/3)*(c^4/
(b*c^3 - a*d^3)^2 - c/(b^2*c^3 - a*b*d^3))*(-I*sqrt(3) + 1)/(2*c^6/(b*c^3 -
 a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a
*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3) + (1/2)^(1/3)*(2*c^6/(b*c^3 -
 a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a
*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3)*(I*sqrt(3) + 1) - 2*c^2/(b*c^
3 - a*d^3))^2 + d*x - 2*c) + 12*c^2*log(d*x + c) - ((b*c^3 - a*d^3)*(2*(1/2
)^(2/3)*(c^4/(b*c^3 - a*d^3)^2 - c/(b^2*c^3 - a*b*d^3))*(-I*sqrt(3) + 1)/(2
*c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^
3/((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3) + (1/2)^(1/3)*(2
*c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^
3/((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3)*(I*sqrt(3) + 1)
 - 2*c^2/(b*c^3 - a*d^3)) + 6*c^2 - 3*sqrt(1/3)*(b*c^3 - a*d^3)*sqrt(-(4*b*c
^4 - 16*a*c*d^3 + (b^3*c^6 - 2*a*b^2*c^3*d^3 + a^2*b*d^6)*(2*(1/2)^(2/3)*(c
^4/(b*c^3 - a*d^3)^2 - c/(b^2*c^3 - a*b*d^3))*(-I*sqrt(3) + 1)/(2*c^6/(b*c^
3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3
 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3) + (1/2)^(1/3)*(2*c^6/(b*c^
3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3
 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3)*(I*sqrt(3) + 1) - 2*c^2/(b
*c^3 - a*d^3))^2 + 4*(b^2*c^5 - a*b*c^2*d^3)*(2*(1/2)^(2/3)*(c^4/(b*c^3 - a
*d^3)^2 - c/(b^2*c^3 - a*b*d^3))*(-I*sqrt(3) + 1)/(2*c^6/(b*c^3 - a*d^3)^3
 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a*d^3)^2*b^
2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3) + (1/2)^(1/3)*(2*c^6/(b*c^3 - a*d^3)^3
 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a*d^3)^2*b^
2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3)*(I*sqrt(3) + 1) - 2*c^2/(b*c^3 - a*d^3)
```

```
))/(b^3*c^6 - 2*a*b^2*c^3*d^3 + a^2*b*d^6)))*log(3/2*(2*(1/2)^(2/3)*(c^4/(b
*c^3 - a*d^3)^2 - c/(b^2*c^3 - a*b*d^3))*(-I*sqrt(3) + 1)/(2*c^6/(b*c^3 - a
*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a*d
^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3) + (1/2)^(1/3)*(2*c^6/(b*c^3 - a
*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a*d
^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3)*(I*sqrt(3) + 1) - 2*c^2/(b*c^3
- a*d^3))*b*c^2 + 1/4*(b^2*c^3 - a*b*d^3)*(2*(1/2)^(2/3)*(c^4/(b*c^3 - a*d^
3)^2 - c/(b^2*c^3 - a*b*d^3))*(-I*sqrt(3) + 1)/(2*c^6/(b*c^3 - a*d^3)^3 - 3
*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a*d^3)^2*b^2)
+ 1/(b^3*c^3 - a*b^2*d^3))^(1/3) + (1/2)^(1/3)*(2*c^6/(b*c^3 - a*d^3)^3 - 3
*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a*d^3)^2*b^2)
+ 1/(b^3*c^3 - a*b^2*d^3))^(1/3)*(I*sqrt(3) + 1) - 2*c^2/(b*c^3 - a*d^3))^2
 + 3/4*sqrt(1/3)*(b^2*c^3 - a*b*d^3)*(2*(1/2)^(2/3)*(c^4/(b*c^3 - a*d^3)^2
- c/(b^2*c^3 - a*b*d^3))*(-I*sqrt(3) + 1)/(2*c^6/(b*c^3 - a*d^3)^3 - 3*c^3/
((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a*d^3)^2*b^2) + 1/(
b^3*c^3 - a*b^2*d^3))^(1/3) + (1/2)^(1/3)*(2*c^6/(b*c^3 - a*d^3)^3 - 3*c^3/
((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a*d^3)^2*b^2) + 1/(
b^3*c^3 - a*b^2*d^3))^(1/3)*(I*sqrt(3) + 1) - 2*c^2/(b*c^3 - a*d^3))*sqrt(-
(4*b*c^4 - 16*a*c*d^3 + (b^3*c^6 - 2*a*b^2*c^3*d^3 + a^2*b*d^6)*(2*(1/2)^(2
/3)*(c^4/(b*c^3 - a*d^3)^2 - c/(b^2*c^3 - a*b*d^3))*(-I*sqrt(3) + 1)/(2*c^6
/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((
b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3) + (1/2)^(1/3)*(2*c^6
/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((
b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3)*(I*sqrt(3) + 1) - 2*
c^2/(b*c^3 - a*d^3))^2 + 4*(b^2*c^5 - a*b*c^2*d^3)*(2*(1/2)^(2/3)*(c^4/(b*c
^3 - a*d^3)^2 - c/(b^2*c^3 - a*b*d^3))*(-I*sqrt(3) + 1)/(2*c^6/(b*c^3 - a*d
^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a*d^3
)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3) + (1/2)^(1/3)*(2*c^6/(b*c^3 - a*d
^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a*d^3
)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3)*(I*sqrt(3) + 1) - 2*c^2/(b*c^3 -
a*d^3)))/(b^3*c^6 - 2*a*b^2*c^3*d^3 + a^2*b*d^6)) + 2*d*x + 2*c) - ((b*c^3
- a*d^3)*(2*(1/2)^(2/3)*(c^4/(b*c^3 - a*d^3)^2 - c/(b^2*c^3 - a*b*d^3))*(-I
*sqrt(3) + 1)/(2*c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3
- a*d^3)) + a*d^3/((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3)
+ (1/2)^(1/3)*(2*c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3
- a*d^3)) + a*d^3/((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3)*
(I*sqrt(3) + 1) - 2*c^2/(b*c^3 - a*d^3)) + 6*c^2 + 3*sqrt(1/3)*(b*c^3 - a*d
^3)*sqrt(-(4*b*c^4 - 16*a*c*d^3 + (b^3*c^6 - 2*a*b^2*c^3*d^3 + a^2*b*d^6)*(
2*(1/2)^(2/3)*(c^4/(b*c^3 - a*d^3)^2 - c/(b^2*c^3 - a*b*d^3))*(-I*sqrt(3) +
 1)/(2*c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3))
+ a*d^3/((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3) + (1/2)^(1
/3)*(2*c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3))
+ a*d^3/((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3)*(I*sqrt(3)
 + 1) - 2*c^2/(b*c^3 - a*d^3))^2 + 4*(b^2*c^5 - a*b*c^2*d^3)*(2*(1/2)^(2/3)
*(c^4/(b*c^3 - a*d^3)^2 - c/(b^2*c^3 - a*b*d^3))*(-I*sqrt(3) + 1)/(2*c^6/(b
```

```
*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c
^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3) + (1/2)^(1/3)*(2*c^6/(b
*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c
^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3)*(I*sqrt(3) + 1) - 2*c^2
/(b*c^3 - a*d^3)))/(b^3*c^6 - 2*a*b^2*c^3*d^3 + a^2*b*d^6)))*log(3/2*(2*(1/
2)^(2/3)*(c^4/(b*c^3 - a*d^3)^2 - c/(b^2*c^3 - a*b*d^3))*(-I*sqrt(3) + 1)/(
2*c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d
^3/((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3) + (1/2)^(1/3)*(
2*c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d
^3/((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3)*(I*sqrt(3) + 1)
 - 2*c^2/(b*c^3 - a*d^3))*b*c^2 + 1/4*(b^2*c^3 - a*b*d^3)*(2*(1/2)^(2/3)*(c
^4/(b*c^3 - a*d^3)^2 - c/(b^2*c^3 - a*b*d^3))*(-I*sqrt(3) + 1)/(2*c^6/(b*c^
3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3
- a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3) + (1/2)^(1/3)*(2*c^6/(b*c^
3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3
- a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3)*(I*sqrt(3) + 1) - 2*c^2/(b
*c^3 - a*d^3))^2 - 3/4*sqrt(1/3)*(b^2*c^3 - a*b*d^3)*(2*(1/2)^(2/3)*(c^4/(b
*c^3 - a*d^3)^2 - c/(b^2*c^3 - a*b*d^3))*(-I*sqrt(3) + 1)/(2*c^6/(b*c^3 - a
*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a*d
^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3) + (1/2)^(1/3)*(2*c^6/(b*c^3 - a
*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a*d
^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3)*(I*sqrt(3) + 1) - 2*c^2/(b*c^3
- a*d^3))*sqrt(-(4*b*c^4 - 16*a*c*d^3 + (b^3*c^6 - 2*a*b^2*c^3*d^3 + a^2*b*
d^6)*(2*(1/2)^(2/3)*(c^4/(b*c^3 - a*d^3)^2 - c/(b^2*c^3 - a*b*d^3))*(-I*sqr
t(3) + 1)/(2*c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*
d^3)) + a*d^3/((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3) + (1
/2)^(1/3)*(2*c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*
d^3)) + a*d^3/((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3)*(I*s
qrt(3) + 1) - 2*c^2/(b*c^3 - a*d^3))^2 + 4*(b^2*c^5 - a*b*c^2*d^3)*(2*(1/2)
^(2/3)*(c^4/(b*c^3 - a*d^3)^2 - c/(b^2*c^3 - a*b*d^3))*(-I*sqrt(3) + 1)/(2*
c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3
/((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3) + (1/2)^(1/3)*(2*
c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3
/((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3)*(I*sqrt(3) + 1) -
 2*c^2/(b*c^3 - a*d^3)))/(b^3*c^6 - 2*a*b^2*c^3*d^3 + a^2*b*d^6)) + 2*d*x +
 2*c))/(b*c^3 - a*d^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(d*x+c)/(b*x**3+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20513, size = 432, normalized size = 1.64

$$-\frac{c^2 d \log\left(|dx+c|\right)}{bc^3 d - ad^4} + \frac{c^2 \log\left(\left|bx^3+a\right|\right)}{3\left(bc^3-ad^3\right)} + \frac{\left(-ab^2\right)^{\frac{1}{3}} d \arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}b^2c^2 - \sqrt{3}\left(-ab^2\right)^{\frac{1}{3}} bcd + \sqrt{3}\left(-ab^2\right)^{\frac{2}{3}} d^2} + \frac{\left(ab^2c^3d^2\left(-\frac{a}{b}\right)^{\frac{1}{3}} - a^2bd^5\left(-\frac{a}{b}\right)^{\frac{1}{3}} - \right.}{3\left(ab^3c^6 - \right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(d*x+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -c^2*d*log(abs(d*x + c))/(b*c^3*d - a*d^4) + 1/3*c^2*log(abs(b*x^3 + a))/(b
*c^3 - a*d^3) + (-a*b^2)^(1/3)*d*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-
a/b)^(1/3))/(sqrt(3)*b^2*c^2 - sqrt(3)*(-a*b^2)^(1/3)*b*c*d + sqrt(3)*(-a*b
^2)^(2/3)*d^2) + 1/3*(a*b^2*c^3*d^2*(-a/b)^(1/3) - a^2*b*d^5*(-a/b)^(1/3) -
 a*b^2*c^4*d + a^2*b*c*d^4)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^3*
c^6 - 2*a^2*b^2*c^3*d^3 + a^3*b*d^6) + 1/6*((-a*b^2)^(1/3)*b*c*d - (-a*b^2)
^(2/3)*d^2)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(b^3*c^3 - a*b^2*d^3)
```

3.342 $\int \frac{x^2}{(c+dx)\left(a+bx^4\right)}\,dx$

Optimal. Leaf size=417

$$-\frac{c^2 d\log\left(a+bx^4\right)}{4\left(ad^4+bc^4\right)}+\frac{c\left(\sqrt{a}d^2+\sqrt{b}c^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\left(ad^4+bc^4\right)}-\frac{c\left(\sqrt{a}d^2+\sqrt{b}c^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\left(ad^4+bc^4\right)}$$

```
[Out] (Sqrt[a]*d^3*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[b]*(b*c^4 + a*d^4)) - (
c*(Sqrt[b]*c^2 - Sqrt[a]*d^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*S
qrt[2]*a^(1/4)*b^(1/4)*(b*c^4 + a*d^4)) + (c*(Sqrt[b]*c^2 - Sqrt[a]*d^2)*Ar
cTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(1/4)*b^(1/4)*(b*c^4 +
a*d^4)) + (c^2*d*Log[c + d*x])/(b*c^4 + a*d^4) + (c*(Sqrt[b]*c^2 + Sqrt[a]*
d^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(
1/4)*b^(1/4)*(b*c^4 + a*d^4)) - (c*(Sqrt[b]*c^2 + Sqrt[a]*d^2)*Log[Sqrt[a]
+ Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(1/4)*b^(1/4)*(b*c
^4 + a*d^4)) - (c^2*d*Log[a + b*x^4])/(4*(b*c^4 + a*d^4))
```

Rubi [A] time = 0.546728, antiderivative size = 417, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 12, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {6725, 1461, 1168, 1162, 617, 204, 1165, 628, 1248, 635, 205, 260}

$$-\frac{c^2 d\log\left(a+bx^4\right)}{4\left(ad^4+bc^4\right)}+\frac{c\left(\sqrt{a}d^2+\sqrt{b}c^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\left(ad^4+bc^4\right)}-\frac{c\left(\sqrt{a}d^2+\sqrt{b}c^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\left(ad^4+bc^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((c + d*x)*(a + b*x^4)),x]
```

```
[Out] (Sqrt[a]*d^3*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[b]*(b*c^4 + a*d^4)) - (
c*(Sqrt[b]*c^2 - Sqrt[a]*d^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*S
qrt[2]*a^(1/4)*b^(1/4)*(b*c^4 + a*d^4)) + (c*(Sqrt[b]*c^2 - Sqrt[a]*d^2)*Ar
cTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(1/4)*b^(1/4)*(b*c^4 +
a*d^4)) + (c^2*d*Log[c + d*x])/(b*c^4 + a*d^4) + (c*(Sqrt[b]*c^2 + Sqrt[a]*
d^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(
1/4)*b^(1/4)*(b*c^4 + a*d^4)) - (c*(Sqrt[b]*c^2 + Sqrt[a]*d^2)*Log[Sqrt[a]
+ Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(1/4)*b^(1/4)*(b*c
^4 + a*d^4)) - (c^2*d*Log[a + b*x^4])/(4*(b*c^4 + a*d^4))
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 1461

```
Int[((A_) + (B_.)*(x_)^(m_.))*((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (c_.)*
(x_)^(n2_))^(p_.), x_Symbol] :> Dist[A, Int[(d + e*x^n)^q*(a + c*x^(2*n))^p
, x], x] + Dist[B, Int[x^m*(d + e*x^n)^q*(a + c*x^(2*n))^p, x], x] /; FreeQ
[{a, c, d, e, A, B, m, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[m - n + 1, 0]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{x^2}{(c+dx)\left(a+bx^4\right)}\,dx = \int \left(\frac{c^2d^2}{\left(bc^4+ad^4\right)(c+dx)} + \frac{(c-dx)\left(-ad^2+bc^2x^2\right)}{\left(bc^4+ad^4\right)\left(a+bx^4\right)}\right) dx$$

$$= \frac{c^2d\log(c+dx)}{bc^4+ad^4} + \frac{\int \frac{(c-dx)\left(-ad^2+bc^2x^2\right)}{a+bx^4}\,dx}{bc^4+ad^4}$$

$$= \frac{c^2d\log(c+dx)}{bc^4+ad^4} + \frac{c\int \frac{-ad^2+bc^2x^2}{a+bx^4}\,dx}{bc^4+ad^4} - \frac{d\int \frac{x\left(-ad^2+bc^2x^2\right)}{a+bx^4}\,dx}{bc^4+ad^4}$$

$$= \frac{c^2d\log(c+dx)}{bc^4+ad^4} - \frac{d\,\text{Subst}\left(\int \frac{-ad^2+bc^2x}{a+bx^2}\,dx, x, x^2\right)}{2\left(bc^4+ad^4\right)} + \frac{\left(c\left(c^2-\frac{\sqrt{a}d^2}{\sqrt{b}}\right)\right)\int \frac{\sqrt{a}\sqrt{b}+bx^2}{a+bx^4}\,dx}{2\left(bc^4+ad^4\right)} - \frac{\left(c\left(c^2+\right.\right.}{}$$

$$= \frac{c^2d\log(c+dx)}{bc^4+ad^4} - \frac{\left(bc^2d\right)\text{Subst}\left(\int \frac{x}{a+bx^2}\,dx, x, x^2\right)}{2\left(bc^4+ad^4\right)} + \frac{\left(ad^3\right)\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, x^2\right)}{2\left(bc^4+ad^4\right)} + \frac{\left(c\left(c^2\right.\right.}{}$$

$$= \frac{\sqrt{a}d^3\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}\left(bc^4+ad^4\right)} + \frac{c^2d\log(c+dx)}{bc^4+ad^4} + \frac{\sqrt[4]{b}c\left(c^2+\frac{\sqrt{a}d^2}{\sqrt{b}}\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{a}\left(bc^4+ad^4\right)} - \frac{\sqrt[4]{}}{}$$

$$= \frac{\sqrt{a}d^3\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}\left(bc^4+ad^4\right)} - \frac{\sqrt[4]{b}c\left(c^2-\frac{\sqrt{a}d^2}{\sqrt{b}}\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}\left(bc^4+ad^4\right)} + \frac{\sqrt[4]{b}c\left(c^2-\frac{\sqrt{a}d^2}{\sqrt{b}}\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{}}{\sqrt[4]{a}}\right.}{2\sqrt{2}\sqrt[4]{a}\left(bc^4+ad^4\right)}$$

Mathematica [A] time = 0.250189, size = 370, normalized size = 0.89

$$\frac{-2\left(2a^{3/4}d^3-\sqrt{2}\sqrt{a}\sqrt[4]{b}cd^2+\sqrt{2}b^{3/4}c^3\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+2\left(-2a^{3/4}d^3-\sqrt{2}\sqrt{a}\sqrt[4]{b}cd^2+\sqrt{2}b^{3/4}c^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((c + d*x)*(a + b*x^4)),x]
```

```
[Out] (-2*(Sqrt[2]*b^(3/4)*c^3 - Sqrt[2]*Sqrt[a]*b^(1/4)*c*d^2 + 2*a^(3/4)*d^3)*A
rcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*(Sqrt[2]*b^(3/4)*c^3 - Sqrt[2]*S
qrt[a]*b^(1/4)*c*d^2 - 2*a^(3/4)*d^3)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4
)] + b^(1/4)*c*(8*a^(1/4)*b^(1/4)*c*d*Log[c + d*x] + Sqrt[2]*(Sqrt[b]*c^2 +
 Sqrt[a]*d^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] - Sqrt
[2]*Sqrt[b]*c^2*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] - Sq
```

```
rt[2]*Sqrt[a]*d^2*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] -
2*a^(1/4)*b^(1/4)*c*d*Log[a + b*x^4]))/(8*a^(1/4)*Sqrt[b]*(b*c^4 + a*d^4))
```

Maple [A] time = 0.013, size = 422, normalized size = 1.

$$-\frac{cd^2\sqrt{2}}{4\,ad^4+4\,bc^4}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)-\frac{cd^2\sqrt{2}}{8\,ad^4+8\,bc^4}\sqrt[4]{\frac{a}{b}}\ln\left(\left(x^2+\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x^2-\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)-\frac{c}{4\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(d*x+c)/(b*x^4+a),x)
```

```
[Out] -1/4/(a*d^4+b*c^4)*c*d^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1
)-1/8/(a*d^4+b*c^4)*c*d^2*(a/b)^(1/4)*2^(1/2)*ln((x^2+(a/b)^(1/4)*x*2^(1/2)
+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))-1/4/(a*d^4+b*c^4)*c*
d^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+1/2/(a*d^4+b*c^4)*a
*d^3/(a*b)^(1/2)*arctan(x^2*(b/a)^(1/2))+1/8/(a*d^4+b*c^4)*c^3/(a/b)^(1/4)*
2^(1/2)*ln((x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*x*2^(1/
2)+(a/b)^(1/2)))+1/4/(a*d^4+b*c^4)*c^3/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(
a/b)^(1/4)*x+1)+1/4/(a*d^4+b*c^4)*c^3/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a
/b)^(1/4)*x-1)-1/4*c^2*d*ln(b*x^4+a)/(a*d^4+b*c^4)+c^2*d*ln(d*x+c)/(a*d^4+b
*c^4)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(d*x+c)/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(d*x+c)/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(d*x+c)/(b*x**4+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20294, size = 575, normalized size = 1.38

$$\frac{c^2d^2\log\left(|dx+c|\right)}{bc^4d+ad^5}-\frac{c^2d\log\left(\left|bx^4+a\right|\right)}{4\left(bc^4+ad^4\right)}-\frac{\left(\sqrt{2}a^2b^3d-\left(ab^3\right)^{\frac{3}{4}}abc\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}a^2b^4c^2+\sqrt{2}\sqrt{ab}a^2b^3d^2-2\left(ab^3\right)^{\frac{1}{4}}a^2b^3cd\right)}+\frac{\left(\sqrt{2}a^2b^3d+\left(ab^3\right)^{\frac{3}{4}}abc\right)}{2\left(\sqrt{2}a^2b^4c^2+\sqrt{2}\sqrt{ab}a^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(d*x+c)/(b*x^4+a),x, algorithm="giac")
```

```
[Out] c^2*d^2*log(abs(d*x + c))/(b*c^4*d + a*d^5) - 1/4*c^2*d*log(abs(b*x^4 + a))
/(b*c^4 + a*d^4) - 1/2*(sqrt(2)*a^2*b^3*d - (a*b^3)^(3/4)*a*b*c)*arctan(1/2
*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*a^2*b^4*c^2 + sq
rt(2)*sqrt(a*b)*a^2*b^3*d^2 - 2*(a*b^3)^(1/4)*a^2*b^3*c*d) + 1/2*(sqrt(2)*a
^2*b^3*d + (a*b^3)^(3/4)*a*b*c)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/
4))/(a/b)^(1/4))/(sqrt(2)*a^2*b^4*c^2 + sqrt(2)*sqrt(a*b)*a^2*b^3*d^2 + 2*(
a*b^3)^(1/4)*a^2*b^3*c*d) - 1/4*((a*b^3)^(1/4)*a*b*c*d^2 + (a*b^3)^(3/4)*c^
3)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(sqrt(2)*a*b^3*c^4 + sqrt(2
)*a^2*b^2*d^4) + 1/4*((a*b^3)^(1/4)*a*b*c*d^2 + (a*b^3)^(3/4)*c^3)*log(x^2
- sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(sqrt(2)*a*b^3*c^4 + sqrt(2)*a^2*b^2*d
^4)
```

3.343 $\int \frac{x}{(1-x)(1+x)^2} \, dx$

Optimal. Leaf size=16

$$\frac{1}{2(x+1)}+\frac{1}{2}\tanh^{-1}(x)$$

```
[Out] 1/(2*(1 + x)) + ArcTanh[x]/2
```

Rubi [A] time = 0.0082498, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {77, 207}

$$\frac{1}{2(x+1)}+\frac{1}{2}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 - x)*(1 + x)^2),x]
```

```
[Out] 1/(2*(1 + x)) + ArcTanh[x]/2
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{x}{(1-x)(1+x)^2}\,dx = \int \left(-\frac{1}{2(1+x)^2} - \frac{1}{2\left(-1+x^2\right)}\right) dx$$
$$= \frac{1}{2(1+x)} - \frac{1}{2}\int \frac{1}{-1+x^2}\,dx$$
$$= \frac{1}{2(1+x)} + \frac{1}{2}\tanh^{-1}(x)$$

Mathematica [A] time = 0.0087884, size = 24, normalized size = 1.5

$$\frac{1}{4}\left(\frac{2}{x+1} - \log(1-x) + \log(x+1)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 - x)*(1 + x)^2),x]
```

```
[Out] (2/(1 + x) - Log[1 - x] + Log[1 + x])/4
```

Maple [A] time = 0.006, size = 21, normalized size = 1.3

$$-\frac{\ln(x-1)}{4} + \frac{1}{2+2x} + \frac{\ln(1+x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1-x)/(1+x)^2,x)
```

```
[Out] -1/4*ln(x-1)+1/2/(1+x)+1/4*ln(1+x)
```

Maxima [A] time = 1.5397, size = 27, normalized size = 1.69

$$\frac{1}{2(x+1)} + \frac{1}{4}\log(x+1) - \frac{1}{4}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1-x)/(1+x)^2,x, algorithm="maxima")
```

```
[Out] 1/2/(x + 1) + 1/4*log(x + 1) - 1/4*log(x - 1)
```

Fricas [B] time = 1.40792, size = 80, normalized size = 5.

$$\frac{(x+1)\log(x+1)-(x+1)\log(x-1)+2}{4(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1-x)/(1+x)^2,x, algorithm="fricas")
```

```
[Out] 1/4*((x + 1)*log(x + 1) - (x + 1)*log(x - 1) + 2)/(x + 1)
```

Sympy [A] time = 0.10024, size = 19, normalized size = 1.19

$$-\frac{\log(x-1)}{4}+\frac{\log(x+1)}{4}+\frac{1}{2x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1-x)/(1+x)**2,x)
```

```
[Out] -log(x - 1)/4 + log(x + 1)/4 + 1/(2*x + 2)
```

Giac [A] time = 1.15128, size = 28, normalized size = 1.75

$$\frac{1}{2(x+1)}-\frac{1}{4}\log\left(\left|-\frac{2}{x+1}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1-x)/(1+x)^2,x, algorithm="giac")
```

```
[Out] 1/2/(x + 1) - 1/4*log(abs(-2/(x + 1) + 1))
```

3.344 $\int \frac{x^2}{(1-x^2)(1+x^2)^2}\,dx$

Optimal. Leaf size=19

$$\frac{1}{4}\tanh^{-1}(x) - \frac{x}{4(x^2+1)}$$

```
[Out] -x/(4*(1 + x^2)) + ArcTanh[x]/4
```

Rubi [A] time = 0.0125273, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {471, 206}

$$\frac{1}{4}\tanh^{-1}(x) - \frac{x}{4(x^2+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((1 - x^2)*(1 + x^2)^2),x]
```

```
[Out] -x/(4*(1 + x^2)) + ArcTanh[x]/4
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^2}{\left(1-x^2\right)\left(1+x^2\right)^2}\,dx = -\frac{x}{4\left(1+x^2\right)} + \frac{1}{4}\int \frac{1}{1-x^2}\,dx$$
$$= -\frac{x}{4\left(1+x^2\right)} + \frac{1}{4}\tanh^{-1}(x)$$

Mathematica [A] time = 0.0112309, size = 27, normalized size = 1.42

$$\frac{1}{8}\left(-\frac{2x}{x^2+1} - \log(1-x) + \log(x+1)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((1 - x^2)*(1 + x^2)^2),x]
```

```
[Out] ((-2*x)/(1 + x^2) - Log[1 - x] + Log[1 + x])/8
```

Maple [A] time = 0.008, size = 24, normalized size = 1.3

$$-\frac{x}{4x^2+4} - \frac{\ln(x-1)}{8} + \frac{\ln(1+x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-x^2+1)/(x^2+1)^2,x)
```

```
[Out] -1/4*x/(x^2+1)-1/8*ln(x-1)+1/8*ln(1+x)
```

Maxima [A] time = 1.17111, size = 31, normalized size = 1.63

$$-\frac{x}{4\left(x^2+1\right)} + \frac{1}{8}\log(x+1) - \frac{1}{8}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+1)/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] -1/4*x/(x^2 + 1) + 1/8*log(x + 1) - 1/8*log(x - 1)
```

Fricas [B] time = 1.43784, size = 90, normalized size = 4.74

$$\frac{\left(x^2+1\right)\log\left(x+1\right)-\left(x^2+1\right)\log\left(x-1\right)-2\,x}{8\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+1)/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] 1/8*((x^2 + 1)*log(x + 1) - (x^2 + 1)*log(x - 1) - 2*x)/(x^2 + 1)
```

Sympy [A] time = 0.109978, size = 20, normalized size = 1.05

$$-\frac{x}{4x^2+4}-\frac{\log\left(x-1\right)}{8}+\frac{\log\left(x+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-x**2+1)/(x**2+1)**2,x)
```

```
[Out] -x/(4*x**2 + 4) - log(x - 1)/8 + log(x + 1)/8
```

Giac [B] time = 1.12892, size = 41, normalized size = 2.16

$$-\frac{1}{4\left(x+\frac{1}{x}\right)}+\frac{1}{16}\,\log\left(\left|x+\frac{1}{x}+2\right|\right)-\frac{1}{16}\,\log\left(\left|x+\frac{1}{x}-2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+1)/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] -1/4/(x + 1/x) + 1/16*log(abs(x + 1/x + 2)) - 1/16*log(abs(x + 1/x - 2))
```

3.345 $\int \frac{x^3}{(1-x^3)(1+x^3)^2} dx$

Optimal. Leaf size=97

$$-\frac{x}{6(x^3+1)}+\frac{1}{72}\log\left(x^2-x+1\right)+\frac{1}{24}\log\left(x^2+x+1\right)-\frac{1}{12}\log(1-x)-\frac{1}{36}\log(x+1)+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{12\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{4\sqrt{3}}$$

```
[Out] -x/(6*(1 + x^3)) + ArcTan[(1 - 2*x)/Sqrt[3]]/(12*Sqrt[3]) + ArcTan[(1 + 2*x
)/Sqrt[3]]/(4*Sqrt[3]) - Log[1 - x]/12 - Log[1 + x]/36 + Log[1 - x + x^2]/7
2 + Log[1 + x + x^2]/24
```

Rubi [A] time = 0.0707737, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {471, 522, 200, 31, 634, 618, 204, 628}

$$-\frac{x}{6(x^3+1)}+\frac{1}{72}\log\left(x^2-x+1\right)+\frac{1}{24}\log\left(x^2+x+1\right)-\frac{1}{12}\log(1-x)-\frac{1}{36}\log(x+1)+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{12\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((1 - x^3)*(1 + x^3)^2),x]
```


```
[Out] -x/(6*(1 + x^3)) + ArcTan[(1 - 2*x)/Sqrt[3]]/(12*Sqrt[3]) + ArcTan[(1 + 2*x
)/Sqrt[3]]/(4*Sqrt[3]) - Log[1 - x]/12 - Log[1 + x]/36 + Log[1 - x + x^2]/7
2 + Log[1 + x + x^2]/24
```


Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^3}{(1-x^3)(1+x^3)^2}\,dx = -\frac{x}{6(1+x^3)} + \frac{1}{6}\int \frac{1+2x^3}{(1-x^3)(1+x^3)}\,dx$$
$$= -\frac{x}{6(1+x^3)} - \frac{1}{12}\int \frac{1}{1+x^3}\,dx + \frac{1}{4}\int \frac{1}{1-x^3}\,dx$$
$$= -\frac{x}{6(1+x^3)} - \frac{1}{36}\int \frac{1}{1+x}\,dx - \frac{1}{36}\int \frac{2-x}{1-x+x^2}\,dx + \frac{1}{12}\int \frac{1}{1-x}\,dx + \frac{1}{12}\int \frac{2+x}{1+x+x^2}\,dx$$
$$= -\frac{x}{6(1+x^3)} - \frac{1}{12}\log(1-x) - \frac{1}{36}\log(1+x) + \frac{1}{72}\int \frac{-1+2x}{1-x+x^2}\,dx - \frac{1}{24}\int \frac{1}{1-x+x^2}\,dx + \frac{1}{24}$$
$$= -\frac{x}{6(1+x^3)} - \frac{1}{12}\log(1-x) - \frac{1}{36}\log(1+x) + \frac{1}{72}\log\left(1-x+x^2\right) + \frac{1}{24}\log\left(1+x+x^2\right) + \frac{1}{12}$$
$$= -\frac{x}{6(1+x^3)} + \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{12\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{12}\log(1-x) - \frac{1}{36}\log(1+x) + \frac{1}{72}\log\left(1-x+\right.$$

Mathematica [A] time = 0.0465136, size = 85, normalized size = 0.88

$$\frac{1}{72}\left(-\frac{12x}{x^3+1} + \log\left(x^2-x+1\right) + 3\log\left(x^2+x+1\right) - 6\log(1-x) - 2\log(x+1) - 2\sqrt{3}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right) + 6\sqrt{3}\tan^{-1}\left(\frac{2x}{\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((1 - x^3)*(1 + x^3)^2),x]
```

```
[Out] ((-12*x)/(1 + x^3) - 2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] + 6*Sqrt[3]*ArcTa
n[(1 + 2*x)/Sqrt[3]] - 6*Log[1 - x] - 2*Log[1 + x] + Log[1 - x + x^2] + 3*L
og[1 + x + x^2])/72
```

Maple [A] time = 0.012, size = 90, normalized size = 0.9

$$-\frac{\ln(x-1)}{12} + \frac{-2x-2}{36x^2-36x+36} + \frac{\ln\left(x^2-x+1\right)}{72} - \frac{\sqrt{3}}{36}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right) + \frac{1}{18+18x} - \frac{\ln(1+x)}{36} + \frac{\ln\left(x^2+x+1\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-x^3+1)/(x^3+1)^2,x)
```

```
[Out] -1/12*ln(x-1)+1/36*(-2*x-2)/(x^2-x+1)+1/72*ln(x^2-x+1)-1/36*3^(1/2)*arctan(
1/3*(2*x-1)*3^(1/2))+1/18/(1+x)-1/36*ln(1+x)+1/24*ln(x^2+x+1)+1/12*arctan(1
/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Maxima [A] time = 1.87009, size = 101, normalized size = 1.04

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{36}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{x}{6\left(x^3+1\right)}+\frac{1}{24}\log\left(x^2+x+1\right)+\frac{1}{72}\log\left(x^2-x+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^3+1)/(x^3+1)^2,x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/36*sqrt(3)*arctan(1/3*sqrt(3
)*(2*x - 1)) - 1/6*x/(x^3 + 1) + 1/24*log(x^2 + x + 1) + 1/72*log(x^2 - x +
 1) - 1/36*log(x + 1) - 1/12*log(x - 1)
```

Fricas [A] time = 1.51337, size = 320, normalized size = 3.3

$$\frac{6\sqrt{3}\left(x^3+1\right)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-2\sqrt{3}\left(x^3+1\right)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+3\left(x^3+1\right)\log\left(x^2+x+1\right)+\left(x^3+1\right)\log}{72\left(x^3+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^3+1)/(x^3+1)^2,x, algorithm="fricas")
```

```
[Out] 1/72*(6*sqrt(3)*(x^3 + 1)*arctan(1/3*sqrt(3)*(2*x + 1)) - 2*sqrt(3)*(x^3 +
1)*arctan(1/3*sqrt(3)*(2*x - 1)) + 3*(x^3 + 1)*log(x^2 + x + 1) + (x^3 + 1)
*log(x^2 - x + 1) - 2*(x^3 + 1)*log(x + 1) - 6*(x^3 + 1)*log(x - 1) - 12*x)
/(x^3 + 1)
```

Sympy [A] time = 0.329192, size = 92, normalized size = 0.95

$$-\frac{x}{6x^3+6}-\frac{\log(x-1)}{12}-\frac{\log(x+1)}{36}+\frac{\log\left(x^2-x+1\right)}{72}+\frac{\log\left(x^2+x+1\right)}{24}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{36}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}+\right.}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-x**3+1)/(x**3+1)**2,x)
```

```
[Out] -x/(6*x**3 + 6) - log(x - 1)/12 - log(x + 1)/36 + log(x**2 - x + 1)/72 + lo
g(x**2 + x + 1)/24 - sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/36 + sqrt(3)*a
tan(2*sqrt(3)*x/3 + sqrt(3)/3)/12
```

Giac [A] time = 1.1593, size = 104, normalized size = 1.07

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{36}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{x}{6\left(x^3+1\right)}+\frac{1}{24}\log\left(x^2+x+1\right)+\frac{1}{72}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^3+1)/(x^3+1)^2,x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/36*sqrt(3)*arctan(1/3*sqrt(3
)*(2*x - 1)) - 1/6*x/(x^3 + 1) + 1/24*log(x^2 + x + 1) + 1/72*log(x^2 - x +
 1) - 1/36*log(abs(x + 1)) - 1/12*log(abs(x - 1))
```

3.346 $\int \frac{9+x+3x^2+x^3}{(1+x^2)(3+x^2)} \, dx$

Optimal. Leaf size=15

$$\frac{1}{2}\log\left(x^2+3\right)+3\tan^{-1}(x)$$

```
[Out] 3*ArcTan[x] + Log[3 + x^2]/2
```

Rubi [A] time = 0.106478, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {6725, 203, 260}

$$\frac{1}{2}\log\left(x^2+3\right)+3\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(9 + x + 3*x^2 + x^3)/((1 + x^2)*(3 + x^2)),x]
```

```
[Out] 3*ArcTan[x] + Log[3 + x^2]/2
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{9+x+3x^2+x^3}{(1+x^2)(3+x^2)}\,dx &= \int \left(\frac{3}{1+x^2}+\frac{x}{3+x^2}\right)dx\\ &= 3\int \frac{1}{1+x^2}\,dx+\int \frac{x}{3+x^2}\,dx\\ &= 3\tan^{-1}(x)+\frac{1}{2}\log\left(3+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.0077366, size = 15, normalized size = 1.

$$\frac{1}{2}\log\left(x^2+3\right)+3\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(9 + x + 3*x^2 + x^3)/((1 + x^2)*(3 + x^2)),x]
```

```
[Out] 3*ArcTan[x] + Log[3 + x^2]/2
```

Maple [A] time = 0.004, size = 14, normalized size = 0.9

$$3\arctan(x)+\frac{\ln\left(x^2+3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+3*x^2+x+9)/(x^2+1)/(x^2+3),x)
```

```
[Out] 3*arctan(x)+1/2*ln(x^2+3)
```

Maxima [A] time = 1.9195, size = 18, normalized size = 1.2

$$3\arctan(x)+\frac{1}{2}\log\left(x^2+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+3*x^2+x+9)/(x^2+1)/(x^2+3),x, algorithm="maxima")
```

```
[Out] 3*arctan(x) + 1/2*log(x^2 + 3)
```

Fricas [A] time = 1.47228, size = 43, normalized size = 2.87

$$3\arctan(x)+\frac{1}{2}\log\left(x^2+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+3*x^2+x+9)/(x^2+1)/(x^2+3),x, algorithm="fricas")
```

```
[Out] 3*arctan(x) + 1/2*log(x^2 + 3)
```

Sympy [A] time = 0.109246, size = 12, normalized size = 0.8

$$\frac{\log\left(x^2+3\right)}{2}+3\operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+3*x**2+x+9)/(x**2+1)/(x**2+3),x)
```

```
[Out] log(x**2 + 3)/2 + 3*atan(x)
```

Giac [A] time = 1.14973, size = 18, normalized size = 1.2

$$3\arctan(x)+\frac{1}{2}\log\left(x^2+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+3*x^2+x+9)/(x^2+1)/(x^2+3),x, algorithm="giac")
```

```
[Out] 3*arctan(x) + 1/2*log(x^2 + 3)
```

3.347 $\int \frac{3+x+x^2+x^3}{(1+x^2)(3+x^2)}\,dx$

Optimal. Leaf size=13

$$\frac{1}{2}\log\left(x^2+3\right)+\tan^{-1}(x)$$

```
[Out] ArcTan[x] + Log[3 + x^2]/2
```

Rubi [A] time = 0.0940804, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6725, 203, 260}

$$\frac{1}{2}\log\left(x^2+3\right)+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(3 + x + x^2 + x^3)/((1 + x^2)*(3 + x^2)),x]

[Out] ArcTan[x] + Log[3 + x^2]/2
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{3+x+x^2+x^3}{(1+x^2)(3+x^2)}\,dx = \int \left(\frac{1}{1+x^2}+\frac{x}{3+x^2}\right)dx$$
$$= \int \frac{1}{1+x^2}\,dx + \int \frac{x}{3+x^2}\,dx$$
$$= \tan^{-1}(x) + \frac{1}{2}\log\left(3+x^2\right)$$

Mathematica [A] time = 0.0066322, size = 13, normalized size = 1.

$$\frac{1}{2}\log\left(x^2+3\right)+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + x + x^2 + x^3)/((1 + x^2)*(3 + x^2)),x]
```

```
[Out] ArcTan[x] + Log[3 + x^2]/2
```

Maple [A] time = 0.005, size = 12, normalized size = 0.9

$$\arctan(x)+\frac{\ln\left(x^2+3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x^2+x+3)/(x^2+1)/(x^2+3),x)
```

```
[Out] arctan(x)+1/2*ln(x^2+3)
```

Maxima [A] time = 2.28345, size = 15, normalized size = 1.15

$$\arctan(x)+\frac{1}{2}\log\left(x^2+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+3)/(x^2+1)/(x^2+3),x, algorithm="maxima")
```

```
[Out] arctan(x) + 1/2*log(x^2 + 3)
```

Fricas [A] time = 1.41621, size = 41, normalized size = 3.15

$$\arctan(x) + \frac{1}{2}\log\left(x^2 + 3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+3)/(x^2+1)/(x^2+3),x, algorithm="fricas")
```

```
[Out] arctan(x) + 1/2*log(x^2 + 3)
```

Sympy [A] time = 0.105318, size = 10, normalized size = 0.77

$$\frac{\log\left(x^2 + 3\right)}{2} + \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x**2+x+3)/(x**2+1)/(x**2+3),x)
```

```
[Out] log(x**2 + 3)/2 + atan(x)
```

Giac [A] time = 1.13548, size = 15, normalized size = 1.15

$$\arctan(x) + \frac{1}{2}\log\left(x^2 + 3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+3)/(x^2+1)/(x^2+3),x, algorithm="giac")
```

```
[Out] arctan(x) + 1/2*log(x^2 + 3)
```

3.348 $\int \frac{-4+6x-x^2+3x^3}{(1+x^2)(2+x^2)}\,dx$

Optimal. Leaf size=29

$$\frac{3}{2}\log\left(x^2+1\right)-3\tan^{-1}(x)+\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

```
[Out] -3*ArcTan[x] + Sqrt[2]*ArcTan[x/Sqrt[2]] + (3*Log[1 + x^2])/2
```

Rubi [A] time = 0.11757, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {6725, 635, 203, 260}

$$\frac{3}{2}\log\left(x^2+1\right)-3\tan^{-1}(x)+\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-4 + 6*x - x^2 + 3*x^3)/((1 + x^2)*(2 + x^2)),x]
```

```
[Out] -3*ArcTan[x] + Sqrt[2]*ArcTan[x/Sqrt[2]] + (3*Log[1 + x^2])/2
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{-4+6x-x^2+3x^3}{(1+x^2)(2+x^2)}\,dx &= \int \left(\frac{3(-1+x)}{1+x^2}+\frac{2}{2+x^2}\right)dx\\ &= 2\int \frac{1}{2+x^2}\,dx+3\int \frac{-1+x}{1+x^2}\,dx\\ &= \sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)-3\int \frac{1}{1+x^2}\,dx+3\int \frac{x}{1+x^2}\,dx\\ &= -3\tan^{-1}(x)+\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)+\frac{3}{2}\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.0136188, size = 29, normalized size = 1.

$$\frac{3}{2}\log\left(x^2+1\right)-3\tan^{-1}(x)+\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-4 + 6*x - x^2 + 3*x^3)/((1 + x^2)*(2 + x^2)),x]
```

```
[Out] -3*ArcTan[x] + Sqrt[2]*ArcTan[x/Sqrt[2]] + (3*Log[1 + x^2])/2
```

Maple [A] time = 0.004, size = 25, normalized size = 0.9

$$-3\arctan(x)+\frac{3\ln\left(x^2+1\right)}{2}+\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^3-x^2+6*x-4)/(x^2+1)/(x^2+2),x)
```

```
[Out] -3*arctan(x)+3/2*ln(x^2+1)+arctan(1/2*x*2^(1/2))*2^(1/2)
```

Maxima [A] time = 2.13953, size = 32, normalized size = 1.1

$$\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)-3\arctan\left(x\right)+\frac{3}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^3-x^2+6*x-4)/(x^2+1)/(x^2+2),x, algorithm="maxima")
```

```
[Out] sqrt(2)*arctan(1/2*sqrt(2)*x) - 3*arctan(x) + 3/2*log(x^2 + 1)
```

Fricas [A] time = 1.44878, size = 86, normalized size = 2.97

$$\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)-3\arctan\left(x\right)+\frac{3}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^3-x^2+6*x-4)/(x^2+1)/(x^2+2),x, algorithm="fricas")
```

```
[Out] sqrt(2)*arctan(1/2*sqrt(2)*x) - 3*arctan(x) + 3/2*log(x^2 + 1)
```

Sympy [A] time = 0.165354, size = 29, normalized size = 1.

$$\frac{3\log\left(x^2+1\right)}{2}-3\operatorname{atan}\left(x\right)+\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**3-x**2+6*x-4)/(x**2+1)/(x**2+2),x)
```

```
[Out] 3*log(x**2 + 1)/2 - 3*atan(x) + sqrt(2)*atan(sqrt(2)*x/2)
```

Giac [A] time = 1.32654, size = 32, normalized size = 1.1

$$\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)-3\arctan\left(x\right)+\frac{3}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^3-x^2+6*x-4)/(x^2+1)/(x^2+2),x, algorithm="giac")
```

```
[Out] sqrt(2)*arctan(1/2*sqrt(2)*x) - 3*arctan(x) + 3/2*log(x^2 + 1)
```

3.349 $\int \frac{1}{(4-4x+x^2)(5-4x+x^2)} dx$

Optimal. Leaf size=14

$$\frac{1}{2-x} + \tan^{-1}(2-x)$$

```
[Out] (2 - x)^(-1) + ArcTan[2 - x]
```

Rubi [A] time = 0.0106012, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {27, 693, 618, 204}

$$\frac{1}{2-x} + \tan^{-1}(2-x)$$

Antiderivative was successfully verified.

```
[In] Int[1/((4 - 4*x + x^2)*(5 - 4*x + x^2)),x]

[Out] (2 - x)^(-1) + ArcTan[2 - x]
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 693

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(-2*b*d*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(d^2*(m
 + 1)*(b^2 - 4*a*c)), x] + Dist[(b^2*(m + 2*p + 3))/(d^2*(m + 1)*(b^2 - 4*a
*c)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p +
 3, 0] && LtQ[m, -1] && (IntegerQ[2*p] || (IntegerQ[m] && RationalQ[p]) ||
IntegerQ[(m + 2*p + 3)/2])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(4-4x+x^2)(5-4x+x^2)}\,dx &= \int \frac{1}{(-2+x)^2(5-4x+x^2)}\,dx \\
&= \frac{1}{2-x} - \int \frac{1}{5-4x+x^2}\,dx \\
&= \frac{1}{2-x} + 2\,\mathrm{Subst}\left(\int \frac{1}{-4-x^2}\,dx, x, -4+2x\right) \\
&= \frac{1}{2-x} + \tan^{-1}(2-x)
\end{aligned}$$

Mathematica [A] time = 0.0085638, size = 14, normalized size = 1.

$$\tan^{-1}(2-x) - \frac{1}{x-2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((4 - 4*x + x^2)*(5 - 4*x + x^2)),x]
```

```
[Out] -(-2 + x)^(-1) + ArcTan[2 - x]
```

Maple [A] time = 0.005, size = 15, normalized size = 1.1

$$-\arctan(-2+x) - (-2+x)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2-4*x+4)/(x^2-4*x+5),x)
```

```
[Out] -arctan(-2+x)-1/(-2+x)
```

Maxima [A] time = 2.3651, size = 19, normalized size = 1.36

$$-\frac{1}{x-2}-\arctan\left(x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-4*x+4)/(x^2-4*x+5),x, algorithm="maxima")
```

```
[Out] -1/(x - 2) - arctan(x - 2)
```

Fricas [A] time = 1.47842, size = 51, normalized size = 3.64

$$-\frac{(x-2)\arctan\left(x-2\right)+1}{x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-4*x+4)/(x^2-4*x+5),x, algorithm="fricas")
```

```
[Out] -((x - 2)*arctan(x - 2) + 1)/(x - 2)
```

Sympy [A] time = 0.122933, size = 10, normalized size = 0.71

$$-\operatorname{atan}\left(x-2\right)-\frac{1}{x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2-4*x+4)/(x**2-4*x+5),x)
```

```
[Out] -atan(x - 2) - 1/(x - 2)
```

<u>Giac</u> [A] time = 1.16776, size = 19, normalized size = 1.36

$$-\frac{1}{x-2} - \arctan(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-4*x+4)/(x^2-4*x+5),x, algorithm="giac")
```

```
[Out] -1/(x - 2) - arctan(x - 2)
```

3.350 $\int \frac{-3+x+x^2}{(-3+x)x^2} dx$

Optimal. Leaf size=12

$$\log(3-x)-\frac{1}{x}$$

```
[Out] -x^(-1) + Log[3 - x]
```

Rubi [A] time = 0.0106617, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {893}

$$\log(3-x)-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(-3 + x + x^2)/((-3 + x)*x^2),x]
```

```
[Out] -x^(-1) + Log[3 - x]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rubi steps

$$\begin{aligned}\int \frac{-3+x+x^2}{(-3+x)x^2}\,dx &= \int \left(\frac{1}{-3+x}+\frac{1}{x^2}\right) dx\\ &= -\frac{1}{x}+\log(3-x)\end{aligned}$$

Mathematica [A] time = 0.0029219, size = 12, normalized size = 1.

$$\log(3-x)-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 + x + x^2)/((-3 + x)*x^2),x]
```

```
[Out] -x^(-1) + Log[3 - x]
```

Maple [A] time = 0.005, size = 11, normalized size = 0.9

$$\ln(-3+x)-x^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+x-3)/(-3+x)/x^2,x)
```

```
[Out] ln(-3+x)-1/x
```

Maxima [A] time = 0.98342, size = 14, normalized size = 1.17

$$-\frac{1}{x}+\log(x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x-3)/(-3+x)/x^2,x, algorithm="maxima")
```

```
[Out] -1/x + log(x - 3)
```

Fricas [A] time = 1.42738, size = 30, normalized size = 2.5

$$\frac{x\log(x-3)-1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x-3)/(-3+x)/x^2,x, algorithm="fricas")
```

```
[Out] (x*log(x - 3) - 1)/x
```

Sympy [A] time = 0.084146, size = 7, normalized size = 0.58

$$\log(x-3) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x-3)/(-3+x)/x**2,x)
```

```
[Out] log(x - 3) - 1/x
```

Giac [A] time = 1.14268, size = 15, normalized size = 1.25

$$-\frac{1}{x} + \log(|x-3|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x-3)/(-3+x)/x^2,x, algorithm="giac")
```

```
[Out] -1/x + log(abs(x - 3))
```

3.351 $\int \frac{1+x+4x^2}{x+4x^3} dx$

Optimal. Leaf size=11

$$\log(x) + \frac{1}{2}\tan^{-1}(2x)$$

```
[Out] ArcTan[2*x]/2 + Log[x]
```

Rubi [A] time = 0.0343319, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1593, 1802, 203}

$$\log(x) + \frac{1}{2}\tan^{-1}(2x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + 4*x^2)/(x + 4*x^3),x]
```

```
[Out] ArcTan[2*x]/2 + Log[x]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1+x+4x^2}{x+4x^3}\,dx &= \int \frac{1+x+4x^2}{x\left(1+4x^2\right)}\,dx\\ &= \int \left(\frac{1}{x}+\frac{1}{1+4x^2}\right)dx\\ &= \log(x)+\int \frac{1}{1+4x^2}\,dx\\ &= \frac{1}{2}\tan^{-1}(2x)+\log(x)\end{aligned}$$

Mathematica [A] time = 0.0047418, size = 11, normalized size = 1.

$$\log(x)+\frac{1}{2}\tan^{-1}(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + 4*x^2)/(x + 4*x^3),x]
```

```
[Out] ArcTan[2*x]/2 + Log[x]
```

Maple [A] time = 0.005, size = 10, normalized size = 0.9

$$\frac{\arctan(2x)}{2}+\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+x+1)/(4*x^3+x),x)
```

```
[Out] 1/2*arctan(2*x)+ln(x)
```

Maxima [A] time = 1.53159, size = 12, normalized size = 1.09

$$\frac{1}{2}\arctan(2x)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+x+1)/(4*x^3+x),x, algorithm="maxima")
```

```
[Out] 1/2*arctan(2*x) + log(x)
```

Fricas [A] time = 1.47463, size = 35, normalized size = 3.18

$$\frac{1}{2}\arctan\left(2x\right) + \log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+x+1)/(4*x^3+x),x, algorithm="fricas")
```

```
[Out] 1/2*arctan(2*x) + log(x)
```

Sympy [A] time = 0.114076, size = 8, normalized size = 0.73

$$\log\left(x\right) + \frac{\operatorname{atan}\left(2x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+x+1)/(4*x**3+x),x)
```

```
[Out] log(x) + atan(2*x)/2
```

Giac [A] time = 1.15745, size = 14, normalized size = 1.27

$$\frac{1}{2}\arctan\left(2x\right) + \log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+x+1)/(4*x^3+x),x, algorithm="giac")
```

```
[Out] 1/2*arctan(2*x) + log(abs(x))
```

3.352 $\int \frac{1-x+3x^2}{-x^2+x^3} \, dx$

Optimal. Leaf size=12

$$\frac{1}{x} + 3\log(1-x)$$

```
[Out] x^(-1) + 3*Log[1 - x]
```

Rubi [A] time = 0.0213736, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1593, 893}

$$\frac{1}{x} + 3\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x + 3*x^2)/(-x^2 + x^3),x]
```

```
[Out] x^(-1) + 3*Log[1 - x]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rubi steps

$$\int \frac{1-x+3x^2}{-x^2+x^3}\,dx = \int \frac{1-x+3x^2}{(-1+x)x^2}\,dx$$
$$= \int \left(\frac{3}{-1+x} - \frac{1}{x^2}\right)dx$$
$$= \frac{1}{x} + 3\log(1-x)$$

Mathematica [A] time = 0.0033556, size = 12, normalized size = 1.

$$\frac{1}{x} + 3\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x + 3*x^2)/(-x^2 + x^3),x]
```

```
[Out] x^(-1) + 3*Log[1 - x]
```

Maple [A] time = 0.006, size = 11, normalized size = 0.9

$$3\ln(x-1) + x^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2-x+1)/(x^3-x^2),x)
```

```
[Out] 3*ln(x-1)+1/x
```

Maxima [A] time = 1.10043, size = 14, normalized size = 1.17

$$\frac{1}{x} + 3\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+1)/(x^3-x^2),x, algorithm="maxima")
```

```
[Out] 1/x + 3*log(x - 1)
```

Fricas [A] time = 1.45993, size = 32, normalized size = 2.67

$$\frac{3\,x\log\left(x-1\right)+1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+1)/(x^3-x^2),x, algorithm="fricas")
```

```
[Out] (3*x*log(x - 1) + 1)/x
```

Sympy [A] time = 0.083881, size = 8, normalized size = 0.67

$$3\log\left(x-1\right)+\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2-x+1)/(x**3-x**2),x)
```

```
[Out] 3*log(x - 1) + 1/x
```

Giac [A] time = 1.1625, size = 15, normalized size = 1.25

$$\frac{1}{x}+3\,\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+1)/(x^3-x^2),x, algorithm="giac")
```

```
[Out] 1/x + 3*log(abs(x - 1))
```

3.353 $\int \frac{4+3x+x^2}{x+x^2} \, dx$

Optimal. Leaf size=12

$$x + 4\log(x) - 2\log(x + 1)$$

```
[Out] x + 4*Log[x] - 2*Log[1 + x]
```

Rubi [A] time = 0.0176414, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1593, 893}

$$x + 4\log(x) - 2\log(x + 1)$$

Antiderivative was successfully verified.

```
[In] Int[(4 + 3*x + x^2)/(x + x^2),x]
```

```
[Out] x + 4*Log[x] - 2*Log[1 + x]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rubi steps

$$\int \frac{4+3x+x^2}{x+x^2}\,dx = \int \frac{4+3x+x^2}{x(1+x)}\,dx$$
$$= \int \left(1+\frac{4}{x}-\frac{2}{1+x}\right)dx$$
$$= x+4\log(x)-2\log(1+x)$$

Mathematica [A] time = 0.0032719, size = 12, normalized size = 1.

$$x+4\log(x)-2\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + 3*x + x^2)/(x + x^2),x]
```

```
[Out] x + 4*Log[x] - 2*Log[1 + x]
```

Maple [A] time = 0.004, size = 13, normalized size = 1.1

$$x+4\ln(x)-2\ln(1+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+3*x+4)/(x^2+x),x)
```

```
[Out] x+4*ln(x)-2*ln(1+x)
```

Maxima [A] time = 1.00843, size = 16, normalized size = 1.33

$$x-2\log(x+1)+4\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3*x+4)/(x^2+x),x, algorithm="maxima")
```

```
[Out] x - 2*log(x + 1) + 4*log(x)
```

Fricas [A] time = 1.44783, size = 39, normalized size = 3.25

$$x - 2\,\log\left(x + 1\right) + 4\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3*x+4)/(x^2+x),x, algorithm="fricas")
```

```
[Out] x - 2*log(x + 1) + 4*log(x)
```

Sympy [A] time = 0.092957, size = 12, normalized size = 1.

$$x + 4\log\left(x\right) - 2\log\left(x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+3*x+4)/(x**2+x),x)
```

```
[Out] x + 4*log(x) - 2*log(x + 1)
```

Giac [A] time = 1.22549, size = 19, normalized size = 1.58

$$x - 2\,\log\left(|x + 1|\right) + 4\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3*x+4)/(x^2+x),x, algorithm="giac")
```

```
[Out] x - 2*log(abs(x + 1)) + 4*log(abs(x))
```

3.354 $\int \frac{4+x+3x^2}{x+x^3}\,dx$

Optimal. Leaf size=17

$$-\frac{1}{2}\log\left(x^2+1\right)+4\log(x)+\tan^{-1}(x)$$

```
[Out] ArcTan[x] + 4*Log[x] - Log[1 + x^2]/2
```

Rubi [A] time = 0.0370974, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1593, 1802, 635, 203, 260}

$$-\frac{1}{2}\log\left(x^2+1\right)+4\log(x)+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x + 3*x^2)/(x + x^3),x]
```

```
[Out] ArcTan[x] + 4*Log[x] - Log[1 + x^2]/2
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{4+x+3x^2}{x+x^3}\,dx &= \int \frac{4+x+3x^2}{x\left(1+x^2\right)}\,dx\\
&= \int \left(\frac{4}{x}+\frac{1-x}{1+x^2}\right)dx\\
&= 4\log(x)+\int \frac{1-x}{1+x^2}\,dx\\
&= 4\log(x)+\int \frac{1}{1+x^2}\,dx-\int \frac{x}{1+x^2}\,dx\\
&= \tan^{-1}(x)+4\log(x)-\frac{1}{2}\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.0046738, size = 17, normalized size = 1.

$$-\frac{1}{2}\log\left(x^2+1\right)+4\log(x)+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x + 3*x^2)/(x + x^3),x]
```

```
[Out] ArcTan[x] + 4*Log[x] - Log[1 + x^2]/2
```

Maple [A] time = 0.004, size = 16, normalized size = 0.9

$$\arctan(x)+4\ln(x)-\frac{\ln\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+x+4)/(x^3+x),x)
```

```
[Out] arctan(x)+4*ln(x)-1/2*ln(x^2+1)
```

Maxima [A] time = 1.4965, size = 20, normalized size = 1.18

$$\arctan(x) - \frac{1}{2}\log\left(x^2 + 1\right) + 4\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+x+4)/(x^3+x),x, algorithm="maxima")
```

```
[Out] arctan(x) - 1/2*log(x^2 + 1) + 4*log(x)
```

Fricas [A] time = 1.78526, size = 55, normalized size = 3.24

$$\arctan(x) - \frac{1}{2}\log\left(x^2 + 1\right) + 4\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+x+4)/(x^3+x),x, algorithm="fricas")
```

```
[Out] arctan(x) - 1/2*log(x^2 + 1) + 4*log(x)
```

Sympy [A] time = 0.115043, size = 15, normalized size = 0.88

$$4\log(x) - \frac{\log\left(x^2 + 1\right)}{2} + \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+x+4)/(x**3+x),x)
```

```
[Out] 4*log(x) - log(x**2 + 1)/2 + atan(x)
```

Giac [A] time = 1.12893, size = 22, normalized size = 1.29

$$\arctan(x) - \frac{1}{2}\log\left(x^2+1\right) + 4\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+x+4)/(x^3+x),x, algorithm="giac")
```

```
[Out] arctan(x) - 1/2*log(x^2 + 1) + 4*log(abs(x))
```

3.355 $\int \frac{7-4x+8x^2}{(1+4x)\left(1+x^2\right)} \, dx$

Optimal. Leaf size=13

$$2\log(4x+1)-\tan^{-1}(x)$$

```
[Out] -ArcTan[x] + 2*Log[1 + 4*x]
```

Rubi [A] time = 0.0347155, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {1629, 204}

$$2\log(4x+1)-\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(7 - 4*x + 8*x^2)/((1 + 4*x)*(1 + x^2)),x]
```

```
[Out] -ArcTan[x] + 2*Log[1 + 4*x]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{7-4x+8x^2}{(1+4x)\left(1+x^2\right)} \, dx &= \int \left(\frac{8}{1+4x}+\frac{1}{-1-x^2}\right) dx\\ &= 2\log(1+4x)+\int \frac{1}{-1-x^2} \, dx\\ &= -\tan^{-1}(x)+2\log(1+4x)\end{aligned}$$

Mathematica [A] time = 0.0086843, size = 13, normalized size = 1.

$$2\log(4x+1)-\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 - 4*x + 8*x^2)/((1 + 4*x)*(1 + x^2)),x]
```

```
[Out] -ArcTan[x] + 2*Log[1 + 4*x]
```

Maple [A] time = 0.006, size = 14, normalized size = 1.1

$$-\arctan(x)+2\ln(1+4x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*x^2-4*x+7)/(1+4*x)/(x^2+1),x)
```

```
[Out] -arctan(x)+2*ln(1+4*x)
```

Maxima [A] time = 1.67753, size = 18, normalized size = 1.38

$$-\arctan(x)+2\log(4x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^2-4*x+7)/(1+4*x)/(x^2+1),x, algorithm="maxima")
```

```
[Out] -arctan(x) + 2*log(4*x + 1)
```

Fricas [A] time = 1.78125, size = 39, normalized size = 3.

$$-\arctan(x)+2\log(4x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^2-4*x+7)/(1+4*x)/(x^2+1),x, algorithm="fricas")
```

```
[Out] -arctan(x) + 2*log(4*x + 1)
```

Sympy [A] time = 0.116184, size = 10, normalized size = 0.77

$$2\log\left(x+\frac{1}{4}\right)-\operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x**2-4*x+7)/(1+4*x)/(x**2+1),x)
```

```
[Out] 2*log(x + 1/4) - atan(x)
```

Giac [A] time = 1.11811, size = 19, normalized size = 1.46

$$-\arctan(x)+2\log(|4x+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^2-4*x+7)/(1+4*x)/(x^2+1),x, algorithm="giac")
```

```
[Out] -arctan(x) + 2*log(abs(4*x + 1))
```

3.356 $\int \frac{x^2}{(-1+x)\left(1+2x+x^2\right)}\,dx$

Optimal. Leaf size=28

$$\frac{1}{2(x+1)}+\frac{1}{4}\log(1-x)+\frac{3}{4}\log(x+1)$$

```
[Out] 1/(2*(1 + x)) + Log[1 - x]/4 + (3*Log[1 + x])/4
```

Rubi [A] time = 0.0111347, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {27, 88}

$$\frac{1}{2(x+1)}+\frac{1}{4}\log(1-x)+\frac{3}{4}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((-1 + x)*(1 + 2*x + x^2)),x]
```

```
[Out] 1/(2*(1 + x)) + Log[1 - x]/4 + (3*Log[1 + x])/4
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{x^2}{(-1+x)\left(1+2x+x^2\right)}\,dx = \int \frac{x^2}{(-1+x)(1+x)^2}\,dx$$
$$= \int \left(\frac{1}{4(-1+x)} - \frac{1}{2(1+x)^2} + \frac{3}{4(1+x)}\right) dx$$
$$= \frac{1}{2(1+x)} + \frac{1}{4}\log(1-x) + \frac{3}{4}\log(1+x)$$

Mathematica [A] time = 0.0141945, size = 22, normalized size = 0.79

$$\frac{1}{4}\left(\frac{2}{x+1} + \log(x-1) + 3\log(x+1)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((-1 + x)*(1 + 2*x + x^2)),x]
```

```
[Out] (2/(1 + x) + Log[-1 + x] + 3*Log[1 + x])/4
```

Maple [A] time = 0.008, size = 21, normalized size = 0.8

$$\frac{\ln(x-1)}{4} + \frac{1}{2+2x} + \frac{3\ln(1+x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x-1)/(x^2+2*x+1),x)
```

```
[Out] 1/4*ln(x-1)+1/2/(1+x)+3/4*ln(1+x)
```

Maxima [A] time = 1.0184, size = 27, normalized size = 0.96

$$\frac{1}{2(x+1)} + \frac{3}{4}\log(x+1) + \frac{1}{4}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-1+x)/(x^2+2*x+1),x, algorithm="maxima")
```

```
[Out] 1/2/(x + 1) + 3/4*log(x + 1) + 1/4*log(x - 1)
```

Fricas [A] time = 1.72075, size = 82, normalized size = 2.93

$$\frac{3(x+1)\log(x+1)+(x+1)\log(x-1)+2}{4(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-1+x)/(x^2+2*x+1),x, algorithm="fricas")
```

```
[Out] 1/4*(3*(x + 1)*log(x + 1) + (x + 1)*log(x - 1) + 2)/(x + 1)
```

Sympy [A] time = 0.104101, size = 20, normalized size = 0.71

$$\frac{\log(x-1)}{4}+\frac{3\log(x+1)}{4}+\frac{1}{2x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-1+x)/(x**2+2*x+1),x)
```

```
[Out] log(x - 1)/4 + 3*log(x + 1)/4 + 1/(2*x + 2)
```

Giac [A] time = 1.15438, size = 30, normalized size = 1.07

$$\frac{1}{2(x+1)}+\frac{3}{4}\log(|x+1|)+\frac{1}{4}\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-1+x)/(x^2+2*x+1),x, algorithm="giac")
```

```
[Out] 1/2/(x + 1) + 3/4*log(abs(x + 1)) + 1/4*log(abs(x - 1))
```

3.357 $\int \frac{-4+3x+x^2}{(-1+2x)^2(3+2x)}\,dx$

Optimal. Leaf size=32

$$-\frac{9}{32(1-2x)}+\frac{41}{128}\log(1-2x)-\frac{25}{128}\log(2x+3)$$

```
[Out] -9/(32*(1 - 2*x)) + (41*Log[1 - 2*x])/128 - (25*Log[3 + 2*x])/128
```

Rubi [A] time = 0.0261727, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {893}

$$-\frac{9}{32(1-2x)}+\frac{41}{128}\log(1-2x)-\frac{25}{128}\log(2x+3)$$

Antiderivative was successfully verified.

```
[In] Int[(-4 + 3*x + x^2)/((-1 + 2*x)^2*(3 + 2*x)),x]
```

```
[Out] -9/(32*(1 - 2*x)) + (41*Log[1 - 2*x])/128 - (25*Log[3 + 2*x])/128
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rubi steps

$$\begin{aligned}\int \frac{-4+3x+x^2}{(-1+2x)^2(3+2x)}\,dx &= \int \left(-\frac{9}{16(-1+2x)^2}+\frac{41}{64(-1+2x)}-\frac{25}{64(3+2x)}\right)dx\\ &= -\frac{9}{32(1-2x)}+\frac{41}{128}\log(1-2x)-\frac{25}{128}\log(3+2x)\end{aligned}$$

Mathematica [A] time = 0.0175288, size = 32, normalized size = 1.

$$\frac{9}{32(2x-1)}+\frac{41}{128}\log(1-2x)-\frac{25}{128}\log(2x+3)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-4 + 3*x + x^2)/((-1 + 2*x)^2*(3 + 2*x)),x]
```

```
[Out] 9/(32*(-1 + 2*x)) + (41*Log[1 - 2*x])/128 - (25*Log[3 + 2*x])/128
```

Maple [A] time = 0.007, size = 27, normalized size = 0.8

$$\frac{9}{64x-32}+\frac{41\ln(2x-1)}{128}-\frac{25\ln(3+2x)}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+3*x-4)/(2*x-1)^2/(3+2*x),x)
```

```
[Out] 9/32/(2*x-1)+41/128*ln(2*x-1)-25/128*ln(3+2*x)
```

Maxima [A] time = 1.05904, size = 35, normalized size = 1.09

$$\frac{9}{32(2x-1)}-\frac{25}{128}\log(2x+3)+\frac{41}{128}\log(2x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3*x-4)/(-1+2*x)^2/(3+2*x),x, algorithm="maxima")
```

```
[Out] 9/32/(2*x - 1) - 25/128*log(2*x + 3) + 41/128*log(2*x - 1)
```

Fricas [A] time = 1.73741, size = 107, normalized size = 3.34

$$-\frac{25(2x-1)\log(2x+3)-41(2x-1)\log(2x-1)-36}{128(2x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3*x-4)/(-1+2*x)^2/(3+2*x),x, algorithm="fricas")
```

```
[Out] -1/128*(25*(2*x - 1)*log(2*x + 3) - 41*(2*x - 1)*log(2*x - 1) - 36)/(2*x -
1)
```

Sympy [A] time = 0.122067, size = 26, normalized size = 0.81

$$\frac{41\log\left(x-\frac{1}{2}\right)}{128}-\frac{25\log\left(x+\frac{3}{2}\right)}{128}+\frac{9}{64x-32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+3*x-4)/(-1+2*x)**2/(3+2*x),x)
```

```
[Out] 41*log(x - 1/2)/128 - 25*log(x + 3/2)/128 + 9/(64*x - 32)
```

Giac [A] time = 1.11357, size = 58, normalized size = 1.81

$$\frac{9}{32\,(2\,x-1)}-\frac{1}{8}\,\log\left(\frac{|2\,x-1|}{2\,(2\,x-1)^2}\right)-\frac{25}{128}\,\log\left(\left|-\frac{4}{2\,x-1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3*x-4)/(-1+2*x)^2/(3+2*x),x, algorithm="giac")
```

```
[Out] 9/32/(2*x - 1) - 1/8*log(1/2*abs(2*x - 1)/(2*x - 1)^2) - 25/128*log(abs(-4/
(2*x - 1) - 1))
```

3.358 $\int \frac{5-4x+3x^2}{(-1+x)\left(1+x^2\right)} \, dx$

Optimal. Leaf size=23

$$\frac{1}{2}\log\left(x^2+1\right)+2\log(1-x)-3\tan^{-1}(x)$$

```
[Out] -3*ArcTan[x] + 2*Log[1 - x] + Log[1 + x^2]/2
```

Rubi [A] time = 0.033852, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {1629, 635, 203, 260}

$$\frac{1}{2}\log\left(x^2+1\right)+2\log(1-x)-3\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(5 - 4*x + 3*x^2)/((-1 + x)*(1 + x^2)),x]
```

```
[Out] -3*ArcTan[x] + 2*Log[1 - x] + Log[1 + x^2]/2
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{5-4x+3x^2}{(-1+x)\left(1+x^2\right)}\,dx &= \int \left(\frac{2}{-1+x}+\frac{-3+x}{1+x^2}\right)dx\\ &= 2\log(1-x)+\int \frac{-3+x}{1+x^2}\,dx\\ &= 2\log(1-x)-3\int \frac{1}{1+x^2}\,dx+\int \frac{x}{1+x^2}\,dx\\ &= -3\tan^{-1}(x)+2\log(1-x)+\frac{1}{2}\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.0080614, size = 28, normalized size = 1.22

$$\frac{1}{2}\log\left((x-1)^2+2(x-1)+2\right)+2\log(x-1)-3\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 - 4*x + 3*x^2)/((-1 + x)*(1 + x^2)),x]
```

```
[Out] -3*ArcTan[x] + Log[2 + 2*(-1 + x) + (-1 + x)^2]/2 + 2*Log[-1 + x]
```

Maple [A] time = 0.005, size = 20, normalized size = 0.9

$$\frac{\ln\left(x^2+1\right)}{2}-3\arctan(x)+2\ln(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2-4*x+5)/(x-1)/(x^2+1),x)
```

```
[Out] 1/2*ln(x^2+1)-3*arctan(x)+2*ln(x-1)
```

Maxima [A] time = 1.64397, size = 26, normalized size = 1.13

$$-3 \arctan(x) + \frac{1}{2} \log\left(x^2 + 1\right) + 2 \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-4*x+5)/(-1+x)/(x^2+1),x, algorithm="maxima")
```

```
[Out] -3*arctan(x) + 1/2*log(x^2 + 1) + 2*log(x - 1)
```

Fricas [A] time = 1.77813, size = 65, normalized size = 2.83

$$-3 \arctan(x) + \frac{1}{2} \log\left(x^2 + 1\right) + 2 \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-4*x+5)/(-1+x)/(x^2+1),x, algorithm="fricas")
```

```
[Out] -3*arctan(x) + 1/2*log(x^2 + 1) + 2*log(x - 1)
```

Sympy [A] time = 0.120955, size = 19, normalized size = 0.83

$$2 \log(x - 1) + \frac{\log\left(x^2 + 1\right)}{2} - 3 \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2-4*x+5)/(-1+x)/(x**2+1),x)
```

```
[Out] 2*log(x - 1) + log(x**2 + 1)/2 - 3*atan(x)
```

Giac [A] time = 1.24783, size = 27, normalized size = 1.17

$$-3 \arctan(x) + \frac{1}{2} \log\left(x^2 + 1\right) + 2 \log(|x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-4*x+5)/(-1+x)/(x^2+1),x, algorithm="giac")
```

```
[Out] -3*arctan(x) + 1/2*log(x^2 + 1) + 2*log(abs(x - 1))
```

3.359 $\int \frac{-1-2x+x^2}{(-1+x)^2(1+x^2)}\,dx$

Optimal. Leaf size=24

$$-\frac{1}{2}\log\left(x^2+1\right)+\frac{1}{x-1}+\log(1-x)+\tan^{-1}(x)$$

```
[Out] (-1 + x)^(-1) + ArcTan[x] + Log[1 - x] - Log[1 + x^2]/2
```

Rubi [A] time = 0.0345563, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {1629, 635, 203, 260}

$$-\frac{1}{2}\log\left(x^2+1\right)+\frac{1}{x-1}+\log(1-x)+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 - 2*x + x^2)/((-1 + x)^2*(1 + x^2)),x]
```

```
[Out] (-1 + x)^(-1) + ArcTan[x] + Log[1 - x] - Log[1 + x^2]/2
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{-1-2x+x^2}{(-1+x)^2\left(1+x^2\right)}\,dx &= \int \left(-\frac{1}{(-1+x)^2}+\frac{1}{-1+x}+\frac{1-x}{1+x^2}\right)dx\\
&= \frac{1}{-1+x}+\log(1-x)+\int \frac{1-x}{1+x^2}\,dx\\
&= \frac{1}{-1+x}+\log(1-x)+\int \frac{1}{1+x^2}\,dx-\int \frac{x}{1+x^2}\,dx\\
&= \frac{1}{-1+x}+\tan^{-1}(x)+\log(1-x)-\frac{1}{2}\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.014975, size = 22, normalized size = 0.92

$$-\frac{1}{2}\log\left(x^2+1\right)+\frac{1}{x-1}+\log(x-1)+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 - 2*x + x^2)/((-1 + x)^2*(1 + x^2)),x]
```

```
[Out] (-1 + x)^(-1) + ArcTan[x] + Log[-1 + x] - Log[1 + x^2]/2
```

Maple [A] time = 0.007, size = 21, normalized size = 0.9

$$-\frac{\ln\left(x^2+1\right)}{2}+\arctan(x)+\ln(x-1)+(x-1)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-2*x-1)/(x-1)^2/(x^2+1),x)
```

```
[Out] -1/2*ln(x^2+1)+arctan(x)+ln(x-1)+1/(x-1)
```

Maxima [A] time = 1.55428, size = 27, normalized size = 1.12

$$\frac{1}{x-1} + \arctan\left(x\right) - \frac{1}{2}\,\log\left(x^2+1\right) + \log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-2*x-1)/(-1+x)^2/(x^2+1),x, algorithm="maxima")
```

```
[Out] 1/(x - 1) + arctan(x) - 1/2*log(x^2 + 1) + log(x - 1)
```

Fricas [A] time = 1.40806, size = 115, normalized size = 4.79

$$\frac{2\,(x-1)\arctan\left(x\right) - (x-1)\log\left(x^2+1\right) + 2\,(x-1)\log\left(x-1\right) + 2}{2\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-2*x-1)/(-1+x)^2/(x^2+1),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(x - 1)*arctan(x) - (x - 1)*log(x^2 + 1) + 2*(x - 1)*log(x - 1) + 2)
/(x - 1)
```

Sympy [A] time = 0.123806, size = 20, normalized size = 0.83

$$\log\left(x-1\right) - \frac{\log\left(x^2+1\right)}{2} + \operatorname{atan}\left(x\right) + \frac{1}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-2*x-1)/(-1+x)**2/(x**2+1),x)
```

```
[Out] log(x - 1) - log(x**2 + 1)/2 + atan(x) + 1/(x - 1)
```

Giac [B] time = 1.12356, size = 63, normalized size = 2.62

$$\frac{1}{4}\,\pi - \pi\left\lfloor\frac{\pi + 4\,\arctan\left(x\right)}{4\,\pi} + \frac{1}{2}\right\rfloor + \frac{1}{x-1} + \arctan\left(x\right) - \frac{1}{2}\,\log\left(\frac{2}{x-1} + \frac{2}{(x-1)^2} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-2*x-1)/(-1+x)^2/(x^2+1),x, algorithm="giac")
```

```
[Out] 1/4*pi - pi*floor(1/4*(pi + 4*arctan(x))/pi + 1/2) + 1/(x - 1) + arctan(x)
- 1/2*log(2/(x - 1) + 2/(x - 1)^2 + 1)
```

3.360 $\int \frac{5+x^3}{(10-6x+x^2)\left(\frac{1}{2}-x+x^2\right)}\,dx$

Optimal. Leaf size=49

$$\frac{56}{221}\log\left(x^2-6x+10\right)+\frac{109}{442}\log\left(2x^2-2x+1\right)-\frac{261}{221}\tan^{-1}(1-2x)-\frac{1026}{221}\tan^{-1}(3-x)$$

```
[Out] (-261*ArcTan[1 - 2*x])/221 - (1026*ArcTan[3 - x])/221 + (56*Log[10 - 6*x +
x^2])/221 + (109*Log[1 - 2*x + 2*x^2])/442
```

Rubi [A] time = 0.142056, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {6728, 634, 618, 204, 628, 617}

$$\frac{56}{221}\log\left(x^2-6x+10\right)+\frac{109}{442}\log\left(2x^2-2x+1\right)-\frac{261}{221}\tan^{-1}(1-2x)-\frac{1026}{221}\tan^{-1}(3-x)$$

Antiderivative was successfully verified.

```
[In] Int[(5 + x^3)/((10 - 6*x + x^2)*(1/2 - x + x^2)),x]
```

```
[Out] (-261*ArcTan[1 - 2*x])/221 - (1026*ArcTan[3 - x])/221 + (56*Log[10 - 6*x +
x^2])/221 + (109*Log[1 - 2*x + 2*x^2])/442
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{5+x^3}{\left(10-6x+x^2\right)\left(\frac{1}{2}-x+x^2\right)}\,dx &= \int \left(\frac{2(345+56x)}{221\left(10-6x+x^2\right)}+\frac{2(76+109x)}{221\left(1-2x+2x^2\right)}\right)dx \\
&= \frac{2}{221}\int \frac{345+56x}{10-6x+x^2}\,dx+\frac{2}{221}\int \frac{76+109x}{1-2x+2x^2}\,dx \\
&= \frac{109}{442}\int \frac{-2+4x}{1-2x+2x^2}\,dx+\frac{56}{221}\int \frac{-6+2x}{10-6x+x^2}\,dx+\frac{261}{221}\int \frac{1}{1-2x+2x^2}\,dx+\frac{102}{221} \\
&= \frac{56}{221}\log\left(10-6x+x^2\right)+\frac{109}{442}\log\left(1-2x+2x^2\right)+\frac{261}{221}\,\mathrm{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1\right. \\
&= -\frac{261}{221}\tan^{-1}(1-2x)-\frac{1026}{221}\tan^{-1}(3-x)+\frac{56}{221}\log\left(10-6x+x^2\right)+\frac{109}{442}\log\left(1-\right.
\end{aligned}$$

Mathematica [A] time = 0.0147152, size = 49, normalized size = 1.

$$\frac{56}{221}\log\left(x^2-6x+10\right)+\frac{109}{442}\log\left(2x^2-2x+1\right)-\frac{261}{221}\tan^{-1}(1-2x)-\frac{1026}{221}\tan^{-1}(3-x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + x^3)/((10 - 6*x + x^2)*(1/2 - x + x^2)),x]
```

```
[Out] (-261*ArcTan[1 - 2*x])/221 - (1026*ArcTan[3 - x])/221 + (56*Log[10 - 6*x +
x^2])/221 + (109*Log[1 - 2*x + 2*x^2])/442
```

Maple [A] time = 0.005, size = 40, normalized size = 0.8

$$\frac{261\ \arctan\left(2\,x-1\right)}{221}+\frac{1026\ \arctan\left(-3+x\right)}{221}+\frac{56\ \ln\left(x^{2}-6\,x+10\right)}{221}+\frac{109\ \ln\left(2\,x^{2}-2\,x+1\right)}{442}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+5)/(x^2-6*x+10)/(1/2-x+x^2),x)
```

```
[Out] 261/221*arctan(2*x-1)+1026/221*arctan(-3+x)+56/221*ln(x^2-6*x+10)+109/442*l
n(2*x^2-2*x+1)
```

Maxima [A] time = 1.70234, size = 53, normalized size = 1.08

$$\frac{261}{221}\ \arctan\left(2\,x-1\right)+\frac{1026}{221}\ \arctan\left(x-3\right)+\frac{109}{442}\ \log\left(2\,x^{2}-2\,x+1\right)+\frac{56}{221}\ \log\left(x^{2}-6\,x+10\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+5)/(x^2-6*x+10)/(1/2-x+x^2),x, algorithm="maxima")
```

```
[Out] 261/221*arctan(2*x - 1) + 1026/221*arctan(x - 3) + 109/442*log(2*x^2 - 2*x
+ 1) + 56/221*log(x^2 - 6*x + 10)
```

Fricas [A] time = 1.50995, size = 146, normalized size = 2.98

$$\frac{261}{221}\ \arctan\left(2\,x-1\right)+\frac{1026}{221}\ \arctan\left(x-3\right)+\frac{109}{442}\ \log\left(x^{2}-x+\frac{1}{2}\right)+\frac{56}{221}\ \log\left(x^{2}-6\,x+10\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+5)/(x^2-6*x+10)/(1/2-x+x^2),x, algorithm="fricas")
```

```
[Out] 261/221*arctan(2*x - 1) + 1026/221*arctan(x - 3) + 109/442*log(x^2 - x + 1/
2) + 56/221*log(x^2 - 6*x + 10)
```

Sympy [A] time = 0.197824, size = 44, normalized size = 0.9

$$\frac{56\log\left(x^2-6x+10\right)}{221}+\frac{109\log\left(x^2-x+\frac{1}{2}\right)}{442}+\frac{1026\operatorname{atan}(x-3)}{221}+\frac{261\operatorname{atan}(2x-1)}{221}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+5)/(x**2-6*x+10)/(1/2-x+x**2),x)
```

```
[Out] 56*log(x**2 - 6*x + 10)/221 + 109*log(x**2 - x + 1/2)/442 + 1026*atan(x - 3
)/221 + 261*atan(2*x - 1)/221
```

Giac [A] time = 1.11396, size = 53, normalized size = 1.08

$$\frac{261}{221}\arctan\left(2x-1\right)+\frac{1026}{221}\arctan\left(x-3\right)+\frac{109}{442}\log\left(2x^2-2x+1\right)+\frac{56}{221}\log\left(x^2-6x+10\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+5)/(x^2-6*x+10)/(1/2-x+x^2),x, algorithm="giac")
```

```
[Out] 261/221*arctan(2*x - 1) + 1026/221*arctan(x - 3) + 109/442*log(2*x^2 - 2*x
+ 1) + 56/221*log(x^2 - 6*x + 10)
```

3.361 $\int \frac{4+3x+x^2}{(-3+x)(-2+x)(-1+x)}\,dx$

Optimal. Leaf size=25

$$4\log(1-x)-14\log(2-x)+11\log(3-x)$$

```
[Out] 4*Log[1 - x] - 14*Log[2 - x] + 11*Log[3 - x]
```

Rubi [A] time = 0.0545906, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {1612}

$$4\log(1-x)-14\log(2-x)+11\log(3-x)$$

Antiderivative was successfully verified.

```
[In] Int[(4 + 3*x + x^2)/((-3 + x)*(-2 + x)*(-1 + x)),x]
```

```
[Out] 4*Log[1 - x] - 14*Log[2 - x] + 11*Log[3 - x]
```

Rule 1612

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^
n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px,
 x] && IntegersQ[m, n]
```

Rubi steps

$$\begin{aligned}\int \frac{4+3x+x^2}{(-3+x)(-2+x)(-1+x)}\,dx &= \int \left(\frac{11}{-3+x}-\frac{14}{-2+x}+\frac{4}{-1+x}\right)dx\\ &= 4\log(1-x)-14\log(2-x)+11\log(3-x)\end{aligned}$$

Mathematica [A] time = 0.0082174, size = 19, normalized size = 0.76

$$11\log(x-3)-14\log(x-2)+4\log(x-1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + 3*x + x^2)/((-3 + x)*(-2 + x)*(-1 + x)),x]
```

```
[Out] 11*Log[-3 + x] - 14*Log[-2 + x] + 4*Log[-1 + x]
```

Maple [A] time = 0.005, size = 20, normalized size = 0.8

$$4 \ln (x-1)+11 \ln (-3+x)-14 \ln (-2+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+3*x+4)/(-3+x)/(-2+x)/(x-1),x)
```

```
[Out] 4*ln(x-1)+11*ln(-3+x)-14*ln(-2+x)
```

Maxima [A] time = 1.16459, size = 26, normalized size = 1.04

$$4 \log (x-1)-14 \log (x-2)+11 \log (x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3*x+4)/(-3+x)/(-2+x)/(-1+x),x, algorithm="maxima")
```

```
[Out] 4*log(x - 1) - 14*log(x - 2) + 11*log(x - 3)
```

Fricas [A] time = 1.51765, size = 62, normalized size = 2.48

$$4 \log (x-1)-14 \log (x-2)+11 \log (x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3*x+4)/(-3+x)/(-2+x)/(-1+x),x, algorithm="fricas")
```

```
[Out] 4*log(x - 1) - 14*log(x - 2) + 11*log(x - 3)
```

Sympy [A] time = 0.132669, size = 19, normalized size = 0.76

$$11\log(x-3) - 14\log(x-2) + 4\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+3*x+4)/(-3+x)/(-2+x)/(-1+x),x)
```

```
[Out] 11*log(x - 3) - 14*log(x - 2) + 4*log(x - 1)
```

Giac [A] time = 1.13396, size = 30, normalized size = 1.2

$$4\log(|x-1|) - 14\log(|x-2|) + 11\log(|x-3|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3*x+4)/(-3+x)/(-2+x)/(-1+x),x, algorithm="giac")
```

```
[Out] 4*log(abs(x - 1)) - 14*log(abs(x - 2)) + 11*log(abs(x - 3))
```

3.362 $\int \frac{1+16x}{(5+x)^2(-3+2x)\left(1+x+x^2\right)} \, dx$

Optimal. Leaf size=60

$$-\frac{481 \log \left(x^2+x+1\right)}{5586}-\frac{79}{273(x+5)}+\frac{200 \log(3-2x)}{3211}+\frac{2731 \log(x+5)}{24843}+\frac{451 \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2793\sqrt{3}}$$

```
[Out] -79/(273*(5 + x)) + (451*ArcTan[(1 + 2*x)/Sqrt[3]])/(2793*Sqrt[3]) + (200*L
og[3 - 2*x])/3211 + (2731*Log[5 + x])/24843 - (481*Log[1 + x + x^2])/5586
```

Rubi [A] time = 0.246448, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {6728, 634, 618, 204, 628}

$$-\frac{481 \log \left(x^2+x+1\right)}{5586}-\frac{79}{273(x+5)}+\frac{200 \log(3-2x)}{3211}+\frac{2731 \log(x+5)}{24843}+\frac{451 \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2793\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 16*x)/((5 + x)^2*(-3 + 2*x)*(1 + x + x^2)),x]
```

```
[Out] -79/(273*(5 + x)) + (451*ArcTan[(1 + 2*x)/Sqrt[3]])/(2793*Sqrt[3]) + (200*L
og[3 - 2*x])/3211 + (2731*Log[5 + x])/24843 - (481*Log[1 + x + x^2])/5586
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1+16x}{(5+x)^2(-3+2x)\left(1+x+x^2\right)}\,dx = \int \left(\frac{79}{273(5+x)^2}+\frac{2731}{24843(5+x)}+\frac{400}{3211(-3+2x)}+\frac{-15-481x}{2793\left(1+x+x^2\right)}\right)dx$$

$$= -\frac{79}{273(5+x)}+\frac{200\log(3-2x)}{3211}+\frac{2731\log(5+x)}{24843}+\frac{\int \frac{-15-481x}{1+x+x^2}\,dx}{2793}$$

$$= -\frac{79}{273(5+x)}+\frac{200\log(3-2x)}{3211}+\frac{2731\log(5+x)}{24843}+\frac{451\int \frac{1}{1+x+x^2}\,dx}{5586}-\frac{481\int \frac{1+}{1+x}}{558}$$

$$= -\frac{79}{273(5+x)}+\frac{200\log(3-2x)}{3211}+\frac{2731\log(5+x)}{24843}-\frac{481\log\left(1+x+x^2\right)}{5586}-\frac{451}{}$$

$$= -\frac{79}{273(5+x)}+\frac{451\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{2793\sqrt{3}}+\frac{200\log(3-2x)}{3211}+\frac{2731\log(5+x)}{24843}-\frac{481\log}{}$$

Mathematica [A] time = 0.0560955, size = 54, normalized size = 0.9

$$\frac{-243867\log\left(x^2+x+1\right)-\frac{819546}{x+5}+176400\log(3-2x)+311334\log(x+5)+152438\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2832102}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 16*x)/((5 + x)^2*(-3 + 2*x)*(1 + x + x^2)),x]
```

```
[Out] (-819546/(5 + x) + 152438*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + 176400*Log[3
- 2*x] + 311334*Log[5 + x] - 243867*Log[1 + x + x^2])/2832102
```

Maple [A] time = 0.01, size = 48, normalized size = 0.8

$$-\frac{79}{1365+273\,x}+\frac{2731\,\ln\left(5+x\right)}{24843}+\frac{200\,\ln\left(-3+2\,x\right)}{3211}-\frac{481\,\ln\left(x^{2}+x+1\right)}{5586}+\frac{451\,\sqrt{3}}{8379}\arctan\left(\frac{\left(1+2\,x\right)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+16*x)/(5+x)^2/(-3+2*x)/(x^2+x+1),x)
```

```
[Out] -79/273/(5+x)+2731/24843*ln(5+x)+200/3211*ln(-3+2*x)-481/5586*ln(x^2+x+1)+4
51/8379*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Maxima [A] time = 1.65074, size = 63, normalized size = 1.05

$$\frac{451}{8379}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x+1)\right)-\frac{79}{273\,(x+5)}-\frac{481}{5586}\log\left(x^{2}+x+1\right)+\frac{200}{3211}\log\left(2\,x-3\right)+\frac{2731}{24843}\log\left(x+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+16*x)/(5+x)^2/(-3+2*x)/(x^2+x+1),x, algorithm="maxima")
```

```
[Out] 451/8379*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 79/273/(x + 5) - 481/5586*
log(x^2 + x + 1) + 200/3211*log(2*x - 3) + 2731/24843*log(x + 5)
```

Fricas [A] time = 1.54704, size = 236, normalized size = 3.93

$$\frac{152438\,\sqrt{3}(x+5)\arctan\left(\frac{1}{3}\sqrt{3}(2\,x+1)\right)-243867\,(x+5)\log\left(x^{2}+x+1\right)+176400\,(x+5)\log\left(2\,x-3\right)+311334\,(x\cdots}{2832102\,(x+5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+16*x)/(5+x)^2/(-3+2*x)/(x^2+x+1),x, algorithm="fricas")
```

```
[Out] 1/2832102*(152438*sqrt(3)*(x + 5)*arctan(1/3*sqrt(3)*(2*x + 1)) - 243867*(x
 + 5)*log(x^2 + x + 1) + 176400*(x + 5)*log(2*x - 3) + 311334*(x + 5)*log(x
 + 5) - 819546)/(x + 5)
```

Sympy [A] time = 0.225335, size = 63, normalized size = 1.05

$$\frac{200\log\left(x-\frac{3}{2}\right)}{3211}+\frac{2731\log\left(x+5\right)}{24843}-\frac{481\log\left(x^2+x+1\right)}{5586}+\frac{451\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{8379}-\frac{79}{273x+1365}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+16*x)/(5+x)**2/(-3+2*x)/(x**2+x+1),x)
```

```
[Out] 200*log(x - 3/2)/3211 + 2731*log(x + 5)/24843 - 481*log(x**2 + x + 1)/5586
+ 451*sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/8379 - 79/(273*x + 1365)
```

Giac [A] time = 1.09722, size = 81, normalized size = 1.35

$$\frac{451}{8379}\sqrt{3}\arctan\left(-\sqrt{3}\left(\frac{14}{x+5}-3\right)\right)-\frac{79}{273\,(x+5)}-\frac{481}{5586}\log\left(-\frac{9}{x+5}+\frac{21}{(x+5)^2}+1\right)+\frac{200}{3211}\log\left(\left|-\frac{13}{x+5}+2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+16*x)/(5+x)^2/(-3+2*x)/(x^2+x+1),x, algorithm="giac")
```

```
[Out] 451/8379*sqrt(3)*arctan(-sqrt(3)*(14/(x + 5) - 3)) - 79/273/(x + 5) - 481/5
586*log(-9/(x + 5) + 21/(x + 5)^2 + 1) + 200/3211*log(abs(-13/(x + 5) + 2))
```

3.363 $\int \frac{-1+x^3}{1+x+x^2}\,dx$

Optimal. Leaf size=11

$$\frac{x^2}{2}-x$$

```
[Out] -x + x^2/2
```

Rubi [A] time = 0.0074939, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1586}

$$\frac{x^2}{2}-x$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^3)/(1 + x + x^2),x]
```

```
[Out] -x + x^2/2
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+x^3}{1+x+x^2}\,dx &= \int (-1+x)\,dx\\ &= -x+\frac{x^2}{2}\end{aligned}$$

Mathematica [A] time = 0.0004252, size = 11, normalized size = 1.

$$\frac{x^2}{2}-x$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^3)/(1 + x + x^2),x]
```

```
[Out] -x + x^2/2
```

Maple [A] time = 0., size = 10, normalized size = 0.9

$$-x+\frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-1)/(x^2+x+1),x)
```

```
[Out] -x+1/2*x^2
```

Maxima [A] time = 1.10546, size = 12, normalized size = 1.09

$$\frac{1}{2}\,x^2-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)/(x^2+x+1),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - x
```

Fricas [A] time = 1.36925, size = 18, normalized size = 1.64

$$\frac{1}{2}\,x^2-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)/(x^2+x+1),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 - x
```

Sympy [A] time = 0.054409, size = 5, normalized size = 0.45

$$\frac{x^2}{2} - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-1)/(x**2+x+1),x)
```

```
[Out] x**2/2 - x
```

Giac [A] time = 1.13676, size = 12, normalized size = 1.09

$$\frac{1}{2}x^2 - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)/(x^2+x+1),x, algorithm="giac")
```

```
[Out] 1/2*x^2 - x
```

3.364 $\int \frac{-3+x^3}{-7-6x+x^2}\,dx$

Optimal. Leaf size=29

$$\frac{x^2}{2}+6x+\frac{85}{2}\log(7-x)+\frac{1}{2}\log(x+1)$$

```
[Out] 6*x + x^2/2 + (85*Log[7 - x])/2 + Log[1 + x]/2
```

Rubi [A] time = 0.0161322, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1657, 632, 31}

$$\frac{x^2}{2}+6x+\frac{85}{2}\log(7-x)+\frac{1}{2}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(-3 + x^3)/(-7 - 6*x + x^2),x]
```

```
[Out] 6*x + x^2/2 + (85*Log[7 - x])/2 + Log[1 + x]/2
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{-3+x^3}{-7-6x+x^2}\,dx &= \int \left(6+x+\frac{39+43x}{-7-6x+x^2}\right)dx\\
&= 6x+\frac{x^2}{2}+\int \frac{39+43x}{-7-6x+x^2}\,dx\\
&= 6x+\frac{x^2}{2}+\frac{1}{2}\int \frac{1}{1+x}\,dx+\frac{85}{2}\int \frac{1}{-7+x}\,dx\\
&= 6x+\frac{x^2}{2}+\frac{85}{2}\log(7-x)+\frac{1}{2}\log(1+x)
\end{aligned}
$$

Mathematica [A] time = 0.0050677, size = 29, normalized size = 1.

$$\frac{x^2}{2}+6x+\frac{85}{2}\log(7-x)+\frac{1}{2}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 + x^3)/(-7 - 6*x + x^2),x]
```

```
[Out] 6*x + x^2/2 + (85*Log[7 - x])/2 + Log[1 + x]/2
```

Maple [A] time = 0.006, size = 22, normalized size = 0.8

$$\frac{x^2}{2}+6\,x+\frac{85\,\ln(x-7)}{2}+\frac{\ln(1+x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-3)/(x^2-6*x-7),x)
```

```
[Out] 1/2*x^2+6*x+85/2*ln(x-7)+1/2*ln(1+x)
```

Maxima [A] time = 1.05042, size = 28, normalized size = 0.97

$$\frac{1}{2}\,x^2+6\,x+\frac{1}{2}\,\log(x+1)+\frac{85}{2}\,\log(x-7)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-3)/(x^2-6*x-7),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 + 6*x + 1/2*log(x + 1) + 85/2*log(x - 7)
```

Fricas [A] time = 1.43253, size = 68, normalized size = 2.34

$$\frac{1}{2}x^2 + 6x + \frac{1}{2}\log(x+1) + \frac{85}{2}\log(x-7)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-3)/(x^2-6*x-7),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 + 6*x + 1/2*log(x + 1) + 85/2*log(x - 7)
```

Sympy [A] time = 0.095896, size = 22, normalized size = 0.76

$$\frac{x^2}{2} + 6x + \frac{85\log(x-7)}{2} + \frac{\log(x+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-3)/(x**2-6*x-7),x)
```

```
[Out] x**2/2 + 6*x + 85*log(x - 7)/2 + log(x + 1)/2
```

Giac [A] time = 1.10221, size = 31, normalized size = 1.07

$$\frac{1}{2}x^2 + 6x + \frac{1}{2}\log(|x+1|) + \frac{85}{2}\log(|x-7|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-3)/(x^2-6*x-7),x, algorithm="giac")
```

```
[Out] 1/2*x^2 + 6*x + 1/2*log(abs(x + 1)) + 85/2*log(abs(x - 7))
```

3.365 $\int \frac{1+x^3}{\left(13+4x+x^2\right)^2} \, dx$

Optimal. Leaf size=45

$$\frac{47x+67}{18\left(x^2+4x+13\right)}+\frac{1}{2}\log\left(x^2+4x+13\right)-\frac{61}{54}\tan^{-1}\left(\frac{x+2}{3}\right)$$

```
[Out] (67 + 47*x)/(18*(13 + 4*x + x^2)) - (61*ArcTan[(2 + x)/3])/54 + Log[13 + 4*
x + x^2]/2
```

Rubi [A] time = 0.0248464, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1660, 634, 618, 204, 628}

$$\frac{47x+67}{18\left(x^2+4x+13\right)}+\frac{1}{2}\log\left(x^2+4x+13\right)-\frac{61}{54}\tan^{-1}\left(\frac{x+2}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^3)/(13 + 4*x + x^2)^2,x]
```

```
[Out] (67 + 47*x)/(18*(13 + 4*x + x^2)) - (61*ArcTan[(2 + x)/3])/54 + Log[13 + 4*
x + x^2]/2
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x^3}{\left(13+4x+x^2\right)^2}\,dx &= \frac{67+47x}{18\left(13+4x+x^2\right)}+\frac{1}{36}\int \frac{-50+36x}{13+4x+x^2}\,dx\\
&= \frac{67+47x}{18\left(13+4x+x^2\right)}+\frac{1}{2}\int \frac{4+2x}{13+4x+x^2}\,dx-\frac{61}{18}\int \frac{1}{13+4x+x^2}\,dx\\
&= \frac{67+47x}{18\left(13+4x+x^2\right)}+\frac{1}{2}\log\left(13+4x+x^2\right)+\frac{61}{9}\,\mathrm{Subst}\left(\int \frac{1}{-36-x^2}\,dx,x,4+2x\right)\\
&= \frac{67+47x}{18\left(13+4x+x^2\right)}-\frac{61}{54}\tan^{-1}\left(\frac{2+x}{3}\right)+\frac{1}{2}\log\left(13+4x+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.0121463, size = 45, normalized size = 1.

$$\frac{47x+67}{18\left(x^2+4x+13\right)}+\frac{1}{2}\log\left(x^2+4x+13\right)-\frac{61}{54}\tan^{-1}\left(\frac{x+2}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^3)/(13 + 4*x + x^2)^2,x]
```

```
[Out] (67 + 47*x)/(18*(13 + 4*x + x^2)) - (61*ArcTan[(2 + x)/3])/54 + Log[13 + 4*
x + x^2]/2
```

Maple [A] time = 0.006, size = 37, normalized size = 0.8

$$\frac{1}{x^2+4x+13}\left(\frac{47x}{18}+\frac{67}{18}\right)+\frac{\ln\left(x^2+4x+13\right)}{2}-\frac{61}{54}\arctan\left(\frac{2}{3}+\frac{x}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+1)/(x^2+4*x+13)^2,x)
```

```
[Out] (47/18*x+67/18)/(x^2+4*x+13)+1/2*ln(x^2+4*x+13)-61/54*arctan(2/3+1/3*x)
```

Maxima [A] time = 1.6549, size = 50, normalized size = 1.11

$$\frac{47x+67}{18\left(x^2+4x+13\right)}-\frac{61}{54}\arctan\left(\frac{1}{3}x+\frac{2}{3}\right)+\frac{1}{2}\log\left(x^2+4x+13\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/(x^2+4*x+13)^2,x, algorithm="maxima")
```

```
[Out] 1/18*(47*x + 67)/(x^2 + 4*x + 13) - 61/54*arctan(1/3*x + 2/3) + 1/2*log(x^2
 + 4*x + 13)
```

Fricas [A] time = 1.40957, size = 165, normalized size = 3.67

$$-\frac{61\left(x^2+4x+13\right)\arctan\left(\frac{1}{3}x+\frac{2}{3}\right)-27\left(x^2+4x+13\right)\log\left(x^2+4x+13\right)-141x-201}{54\left(x^2+4x+13\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/(x^2+4*x+13)^2,x, algorithm="fricas")
```

```
[Out] -1/54*(61*(x^2 + 4*x + 13)*arctan(1/3*x + 2/3) - 27*(x^2 + 4*x + 13)*log(x^
2 + 4*x + 13) - 141*x - 201)/(x^2 + 4*x + 13)
```

Sympy [A] time = 0.128042, size = 37, normalized size = 0.82

$$\frac{47x+67}{18x^2+72x+234}+\frac{\log\left(x^2+4x+13\right)}{2}-\frac{61\,\mathrm{atan}\left(\frac{x}{3}+\frac{2}{3}\right)}{54}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+1)/(x**2+4*x+13)**2,x)
```

```
[Out] (47*x + 67)/(18*x**2 + 72*x + 234) + log(x**2 + 4*x + 13)/2 - 61*atan(x/3 +
 2/3)/54
```

Giac [A] time = 1.11191, size = 50, normalized size = 1.11

$$\frac{47\,x+67}{18\left(x^2+4\,x+13\right)}-\frac{61}{54}\,\arctan\left(\frac{1}{3}\,x+\frac{2}{3}\right)+\frac{1}{2}\,\log\left(x^2+4\,x+13\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/(x^2+4*x+13)^2,x, algorithm="giac")
```

```
[Out] 1/18*(47*x + 67)/(x^2 + 4*x + 13) - 61/54*arctan(1/3*x + 2/3) + 1/2*log(x^2
 + 4*x + 13)
```

3.366 $\int \frac{-32+36x-42x^2+21x^3-10x^4+3x^5}{x(1+x^2)(4+x^2)^2}\,dx$

Optimal. Leaf size=32

$$\frac{1}{x^2+4}+\log\left(x^2+4\right)-2\log(x)+\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)+2\tan^{-1}(x)$$

```
[Out] (4 + x^2)^(-1) + ArcTan[x/2]/2 + 2*ArcTan[x] - 2*Log[x] + Log[4 + x^2]
```

Rubi [A] time = 0.254368, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.116, Rules used = {6725, 203, 261, 635, 260}

$$\frac{1}{x^2+4}+\log\left(x^2+4\right)-2\log(x)+\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)+2\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-32 + 36*x - 42*x^2 + 21*x^3 - 10*x^4 + 3*x^5)/(x*(1 + x^2)*(4 + x^2)^
2),x]
```

```
[Out] (4 + x^2)^(-1) + ArcTan[x/2]/2 + 2*ArcTan[x] - 2*Log[x] + Log[4 + x^2]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{-32+36x-42x^2+21x^3-10x^4+3x^5}{x\left(1+x^2\right)\left(4+x^2\right)^2}\,dx &= \int \left(-\frac{2}{x}+\frac{2}{1+x^2}-\frac{2x}{\left(4+x^2\right)^2}+\frac{1+2x}{4+x^2}\right)dx \\
&= -2\log(x)+2\int \frac{1}{1+x^2}\,dx-2\int \frac{x}{\left(4+x^2\right)^2}\,dx+\int \frac{1+2x}{4+x^2}\,dx \\
&= \frac{1}{4+x^2}+2\tan^{-1}(x)-2\log(x)+2\int \frac{x}{4+x^2}\,dx+\int \frac{1}{4+x^2}\,dx \\
&= \frac{1}{4+x^2}+\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)+2\tan^{-1}(x)-2\log(x)+\log\left(4+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.017828, size = 32, normalized size = 1.

$$\frac{1}{x^2+4}+\log\left(x^2+4\right)-2\log(x)+\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)+2\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-32 + 36*x - 42*x^2 + 21*x^3 - 10*x^4 + 3*x^5)/(x*(1 + x^2)*(4 +
 x^2)^2),x]
```

```
[Out] (4 + x^2)^(-1) + ArcTan[x/2]/2 + 2*ArcTan[x] - 2*Log[x] + Log[4 + x^2]
```

Maple [A] time = 0.009, size = 29, normalized size = 0.9

$$\left(x^2+4\right)^{-1}+\frac{1}{2}\arctan\left(\frac{x}{2}\right)+2\arctan(x)-2\ln(x)+\ln\left(x^2+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^5-10*x^4+21*x^3-42*x^2+36*x-32)/x/(x^2+1)/(x^2+4)^2,x)
```

```
[Out] 1/(x^2+4)+1/2*arctan(1/2*x)+2*arctan(x)-2*ln(x)+ln(x^2+4)
```

Maxima [A] time = 1.63389, size = 38, normalized size = 1.19

$$\frac{1}{x^2+4}+\frac{1}{2}\arctan\left(\frac{1}{2}x\right)+2\arctan(x)+\log\left(x^2+4\right)-2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^5-10*x^4+21*x^3-42*x^2+36*x-32)/x/(x^2+1)/(x^2+4)^2,x, algor
ithm="maxima")
```

```
[Out] 1/(x^2 + 4) + 1/2*arctan(1/2*x) + 2*arctan(x) + log(x^2 + 4) - 2*log(x)
```

Fricas [A] time = 1.51326, size = 158, normalized size = 4.94

$$\frac{\left(x^2+4\right)\arctan\left(\frac{1}{2}x\right)+4\left(x^2+4\right)\arctan(x)+2\left(x^2+4\right)\log\left(x^2+4\right)-4\left(x^2+4\right)\log(x)+2}{2\left(x^2+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^5-10*x^4+21*x^3-42*x^2+36*x-32)/x/(x^2+1)/(x^2+4)^2,x, algor
ithm="fricas")
```

```
[Out] 1/2*((x^2 + 4)*arctan(1/2*x) + 4*(x^2 + 4)*arctan(x) + 2*(x^2 + 4)*log(x^2
 + 4) - 4*(x^2 + 4)*log(x) + 2)/(x^2 + 4)
```

Sympy [A] time = 0.221786, size = 29, normalized size = 0.91

$$-2\log(x)+\log\left(x^2+4\right)+\frac{\operatorname{atan}\left(\frac{x}{2}\right)}{2}+2\operatorname{atan}(x)+\frac{1}{x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**5-10*x**4+21*x**3-42*x**2+36*x-32)/x/(x**2+1)/(x**2+4)**2,x
)
```

```
[Out] -2*log(x) + log(x**2 + 4) + atan(x/2)/2 + 2*atan(x) + 1/(x**2 + 4)
```

Giac [A] time = 1.15844, size = 39, normalized size = 1.22

$$\frac{1}{x^2+4}+\frac{1}{2}\arctan\left(\frac{1}{2}x\right)+2\arctan(x)+\log\left(x^2+4\right)-2\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^5-10*x^4+21*x^3-42*x^2+36*x-32)/x/(x^2+1)/(x^2+4)^2,x, algor
ithm="giac")
```

```
[Out] 1/(x^2 + 4) + 1/2*arctan(1/2*x) + 2*arctan(x) + log(x^2 + 4) - 2*log(abs(x)
)
```

3.367 $\int \frac{-1+x^4+7x^5+x^9}{-7+6x^4+x^8} \, dx$

Optimal. Leaf size=148

$$\frac{x^2}{2}-\frac{\log\left(x^2-\sqrt{2}\sqrt[4]{7}x+\sqrt{7}\right)}{4\sqrt{2}7^{3/4}}+\frac{\log\left(x^2+\sqrt{2}\sqrt[4]{7}x+\sqrt{7}\right)}{4\sqrt{2}7^{3/4}}-\frac{1}{2}\tanh^{-1}\left(x^2\right)-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}x}{\sqrt[4]{7}}\right)}{2\sqrt{2}7^{3/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{7}}+1\right)}{2\sqrt{2}7^{3/4}}$$

```
[Out] x^2/2 - ArcTan[1 - (Sqrt[2]*x)/7^(1/4)]/(2*Sqrt[2]*7^(3/4)) + ArcTan[1 + (S
qrt[2]*x)/7^(1/4)]/(2*Sqrt[2]*7^(3/4)) - ArcTanh[x^2]/2 - Log[Sqrt[7] - Sqr
t[2]*7^(1/4)*x + x^2]/(4*Sqrt[2]*7^(3/4)) + Log[Sqrt[7] + Sqrt[2]*7^(1/4)*x
 + x^2]/(4*Sqrt[2]*7^(3/4))
```

Rubi [A] time = 0.135981, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 13, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {1790, 1403, 211, 1165, 628, 1162, 617, 204, 1584, 1478, 275, 321, 207}

$$\frac{x^2}{2}-\frac{\log\left(x^2-\sqrt{2}\sqrt[4]{7}x+\sqrt{7}\right)}{4\sqrt{2}7^{3/4}}+\frac{\log\left(x^2+\sqrt{2}\sqrt[4]{7}x+\sqrt{7}\right)}{4\sqrt{2}7^{3/4}}-\frac{1}{2}\tanh^{-1}\left(x^2\right)-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}x}{\sqrt[4]{7}}\right)}{2\sqrt{2}7^{3/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{7}}+1\right)}{2\sqrt{2}7^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^4 + 7*x^5 + x^9)/(-7 + 6*x^4 + x^8),x]
```

```
[Out] x^2/2 - ArcTan[1 - (Sqrt[2]*x)/7^(1/4)]/(2*Sqrt[2]*7^(3/4)) + ArcTan[1 + (S
qrt[2]*x)/7^(1/4)]/(2*Sqrt[2]*7^(3/4)) - ArcTanh[x^2]/2 - Log[Sqrt[7] - Sqr
t[2]*7^(1/4)*x + x^2]/(4*Sqrt[2]*7^(3/4)) + Log[Sqrt[7] + Sqrt[2]*7^(1/4)*x
 + x^2]/(4*Sqrt[2]*7^(3/4))
```

Rule 1790

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*n]*x^(k*n
), {k, 0, (q - j)/n + 1}]*(a + b*x^n + c*x^(2*n))^p, {j, 0, n - 1}], x]] /;
 FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
 0] && IGtQ[n, 0] &&  !PolyQ[Pq, x^n]
```

Rule 1403

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2
_))^(p_.), x_Symbol] :> Int[(d + e*x^n)^(p + q)*(a/d + (c*x^n)/e)^p, x] /;
```

```
FreeQ[{a, b, c, d, e, n, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && E
qQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1478

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (b_.)*(x_)^(n
_) + (c_.)*(x_)^(n2_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d + e*x^n)^(q + p)*
(a/d + (c*x^n)/e)^p, x] /; FreeQ[{a, b, c, d, e, f, m, n, q}, x] && EqQ[n2,
 2*n] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p
]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{-1+x^4+7x^5+x^9}{-7+6x^4+x^8}\,dx = \int \left(\frac{-1+x^4}{-7+6x^4+x^8} + \frac{x\left(7x^4+x^8\right)}{-7+6x^4+x^8}\right) dx$$
$$= \int \frac{-1+x^4}{-7+6x^4+x^8}\,dx + \int \frac{x\left(7x^4+x^8\right)}{-7+6x^4+x^8}\,dx$$
$$= \int \frac{1}{7+x^4}\,dx + \int \frac{x^5\left(7+x^4\right)}{-7+6x^4+x^8}\,dx$$
$$= \frac{\int \frac{\sqrt{7}-x^2}{7+x^4}\,dx}{2\sqrt{7}} + \frac{\int \frac{\sqrt{7}+x^2}{7+x^4}\,dx}{2\sqrt{7}} + \int \frac{x^5}{-1+x^4}\,dx$$
$$= \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, x^2\right) - \frac{\int \frac{\sqrt{2}\sqrt[4]{7}+2x}{-\sqrt{7}-\sqrt{2}\sqrt[4]{7}x-x^2}\,dx}{4\sqrt{2}7^{3/4}} - \frac{\int \frac{\sqrt{2}\sqrt[4]{7}-2x}{-\sqrt{7}+\sqrt{2}\sqrt[4]{7}x-x^2}\,dx}{4\sqrt{2}7^{3/4}} + \frac{\int \frac{1}{\sqrt{7}-\sqrt{2}\sqrt[4]{7}x+x^2}}{4\sqrt{7}}$$
$$= \frac{x^2}{2} - \frac{\log\left(\sqrt{7}-\sqrt{2}\sqrt[4]{7}x+x^2\right)}{4\sqrt{2}7^{3/4}} + \frac{\log\left(\sqrt{7}+\sqrt{2}\sqrt[4]{7}x+x^2\right)}{4\sqrt{2}7^{3/4}} + \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, x^2\right)$$
$$= \frac{x^2}{2} - \frac{\tan^{-1}\left(1-\frac{\sqrt{2}x}{\sqrt[4]{7}}\right)}{2\sqrt{2}7^{3/4}} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}x}{\sqrt[4]{7}}\right)}{2\sqrt{2}7^{3/4}} - \frac{1}{2}\tanh^{-1}\left(x^2\right) - \frac{\log\left(\sqrt{7}-\sqrt{2}\sqrt[4]{7}x+x^2\right)}{4\sqrt{2}7^{3/4}} + \log($$

Mathematica [A] time = 0.0685093, size = 159, normalized size = 1.07

$$\frac{1}{56}\left(28x^2 - 14\log\left(x^2+1\right) - \sqrt{2}\sqrt[4]{7}\log\left(\sqrt{7}x^2 - \sqrt{2}7^{3/4}x + 7\right) + \sqrt{2}\sqrt[4]{7}\log\left(\sqrt{7}x^2 + \sqrt{2}7^{3/4}x + 7\right) + 14\log(1-x) + 14\log\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^4 + 7*x^5 + x^9)/(-7 + 6*x^4 + x^8),x]
```

```
[Out] (28*x^2 - 2*Sqrt[2]*7^(1/4)*ArcTan[1 - (Sqrt[2]*x)/7^(1/4)] + 2*Sqrt[2]*7^(
1/4)*ArcTan[1 + (Sqrt[2]*x)/7^(1/4)] + 14*Log[1 - x] + 14*Log[1 + x] - 14*L
og[1 + x^2] - Sqrt[2]*7^(1/4)*Log[7 - Sqrt[2]*7^(3/4)*x + Sqrt[7]*x^2] + Sq
rt[2]*7^(1/4)*Log[7 + Sqrt[2]*7^(3/4)*x + Sqrt[7]*x^2])/56
```

Maple [A] time = 0.012, size = 110, normalized size = 0.7

$$\frac{x^2}{2} - \frac{\ln\left(x^2+1\right)}{4} + \frac{\ln(x-1)}{4} + \frac{\sqrt[4]{7}\sqrt{2}}{28}\arctan\left(-1 + \frac{x\sqrt{2}7^{\frac{3}{4}}}{7}\right) + \frac{\sqrt[4]{7}\sqrt{2}}{56}\ln\left(\frac{x^2+\sqrt[4]{7}x\sqrt{2}+\sqrt{7}}{x^2-\sqrt[4]{7}x\sqrt{2}+\sqrt{7}}\right) + \frac{\sqrt[4]{7}\sqrt{2}}{28}\arctan\left(1 + \frac{x\sqrt{}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^9+7*x^5+x^4-1)/(x^8+6*x^4-7),x)
```

```
[Out] 1/2*x^2-1/4*ln(x^2+1)+1/4*ln(x-1)+1/28*arctan(-1+1/7*x*2^(1/2)*7^(3/4))*7^(
1/4)*2^(1/2)+1/56*7^(1/4)*2^(1/2)*ln((x^2+7^(1/4)*x*2^(1/2)+7^(1/2))/(x^2-7
^(1/4)*x*2^(1/2)+7^(1/2)))+1/28*arctan(1+1/7*x*2^(1/2)*7^(3/4))*7^(1/4)*2^(
1/2)+1/4*ln(1+x)
```

Maxima [A] time = 1.68216, size = 178, normalized size = 1.2

$$\frac{1}{2}x^2+\frac{1}{28}\cdot 7^{\frac{1}{4}}\sqrt{2}\arctan\left(\frac{1}{14}\cdot 7^{\frac{3}{4}}\sqrt{2}\left(2x+7^{\frac{1}{4}}\sqrt{2}\right)\right)+\frac{1}{28}\cdot 7^{\frac{1}{4}}\sqrt{2}\arctan\left(\frac{1}{14}\cdot 7^{\frac{3}{4}}\sqrt{2}\left(2x-7^{\frac{1}{4}}\sqrt{2}\right)\right)+\frac{1}{56}\cdot 7^{\frac{1}{4}}\sqrt{2}\log\left(x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^9+7*x^5+x^4-1)/(x^8+6*x^4-7),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 + 1/28*7^(1/4)*sqrt(2)*arctan(1/14*7^(3/4)*sqrt(2)*(2*x + 7^(1/4)*s
qrt(2))) + 1/28*7^(1/4)*sqrt(2)*arctan(1/14*7^(3/4)*sqrt(2)*(2*x - 7^(1/4)*
sqrt(2))) + 1/56*7^(1/4)*sqrt(2)*log(x^2 + 7^(1/4)*sqrt(2)*x + sqrt(7)) - 1
/56*7^(1/4)*sqrt(2)*log(x^2 - 7^(1/4)*sqrt(2)*x + sqrt(7)) - 1/4*log(x^2 +
1) + 1/4*log(x + 1) + 1/4*log(x - 1)
```

Fricas [A] time = 1.58688, size = 635, normalized size = 4.29

$$-\frac{1}{686}\cdot 343^{\frac{3}{4}}\sqrt{2}\arctan\left(-\frac{1}{7}\cdot 343^{\frac{1}{4}}\sqrt{2}x+\frac{1}{49}\cdot 343^{\frac{1}{4}}\sqrt{2}\sqrt{343^{\frac{3}{4}}\sqrt{2}x+49x^2+49\sqrt{7}}-1\right)-\frac{1}{686}\cdot 343^{\frac{3}{4}}\sqrt{2}\arctan\left(-\frac{1}{7}\cdot\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^9+7*x^5+x^4-1)/(x^8+6*x^4-7),x, algorithm="fricas")
```

```
[Out] -1/686*343^(3/4)*sqrt(2)*arctan(-1/7*343^(1/4)*sqrt(2)*x + 1/49*343^(1/4)*s
qrt(2)*sqrt(343^(3/4)*sqrt(2)*x + 49*x^2 + 49*sqrt(7)) - 1) - 1/686*343^(3/
4)*sqrt(2)*arctan(-1/7*343^(1/4)*sqrt(2)*x + 1/49*343^(1/4)*sqrt(2)*sqrt(-3
43^(3/4)*sqrt(2)*x + 49*x^2 + 49*sqrt(7)) + 1) + 1/2744*343^(3/4)*sqrt(2)*l
og(343^(3/4)*sqrt(2)*x + 49*x^2 + 49*sqrt(7)) - 1/2744*343^(3/4)*sqrt(2)*lo
g(-343^(3/4)*sqrt(2)*x + 49*x^2 + 49*sqrt(7)) + 1/2*x^2 - 1/4*log(x^2 + 1)
```

```
+ 1/4*log(x^2 - 1)
```

Sympy [A] time = 0.420059, size = 146, normalized size = 0.99

$$\frac{x^2}{2} + \frac{\log(x^2-1)}{4} - \frac{\log(x^2+1)}{4} - \frac{\sqrt{2}\sqrt[4]{7}\log\left(x^2-\sqrt{2}\sqrt[4]{7}x+\sqrt{7}\right)}{56} + \frac{\sqrt{2}\sqrt[4]{7}\log\left(x^2+\sqrt{2}\sqrt[4]{7}x+\sqrt{7}\right)}{56} + \frac{\sqrt{2}\sqrt[4]{7}\operatorname{atan}\left(\frac{\sqrt{2}\cdot}{7}\right.}{28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**9+7*x**5+x**4-1)/(x**8+6*x**4-7),x)
```

```
[Out] x**2/2 + log(x**2 - 1)/4 - log(x**2 + 1)/4 - sqrt(2)*7**(1/4)*log(x**2 - sq
rt(2)*7**(1/4)*x + sqrt(7))/56 + sqrt(2)*7**(1/4)*log(x**2 + sqrt(2)*7**(1/
4)*x + sqrt(7))/56 + sqrt(2)*7**(1/4)*atan(sqrt(2)*7**(3/4)*x/7 - 1)/28 + s
qrt(2)*7**(1/4)*atan(sqrt(2)*7**(3/4)*x/7 + 1)/28
```

Giac [A] time = 1.33989, size = 165, normalized size = 1.11

$$\frac{1}{2}x^2 + \frac{1}{28}\cdot 28^{\frac{1}{4}}\arctan\left(\frac{1}{14}\cdot 7^{\frac{3}{4}}\sqrt{2}\left(2x+7^{\frac{1}{4}}\sqrt{2}\right)\right) + \frac{1}{28}\cdot 28^{\frac{1}{4}}\arctan\left(\frac{1}{14}\cdot 7^{\frac{3}{4}}\sqrt{2}\left(2x-7^{\frac{1}{4}}\sqrt{2}\right)\right) + \frac{1}{56}\cdot 28^{\frac{1}{4}}\log\left(x^2+7^{\frac{1}{4}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^9+7*x^5+x^4-1)/(x^8+6*x^4-7),x, algorithm="giac")
```

```
[Out] 1/2*x^2 + 1/28*28^(1/4)*arctan(1/14*7^(3/4)*sqrt(2)*(2*x + 7^(1/4)*sqrt(2))
) + 1/28*28^(1/4)*arctan(1/14*7^(3/4)*sqrt(2)*(2*x - 7^(1/4)*sqrt(2))) + 1/
56*28^(1/4)*log(x^2 + 7^(1/4)*sqrt(2)*x + sqrt(7)) - 1/56*28^(1/4)*log(x^2
- 7^(1/4)*sqrt(2)*x + sqrt(7)) - 1/4*log(x^2 + 1) + 1/4*log(abs(x + 1)) + 1
/4*log(abs(x - 1))
```

3.368 $\int \frac{1+x^3+x^6}{x+x^5}\,dx$

Optimal. Leaf size=112

$$\frac{x^2}{2}+\frac{\log\left(x^2-\sqrt{2}x+1\right)}{4\sqrt{2}}-\frac{\log\left(x^2+\sqrt{2}x+1\right)}{4\sqrt{2}}-\frac{1}{4}\log\left(x^4+1\right)-\frac{1}{2}\tan^{-1}\left(x^2\right)+\log(x)-\frac{\tan^{-1}\left(1-\sqrt{2}x\right)}{2\sqrt{2}}+\frac{\tan^{-1}(\ldots}{2\ldots}$$

```
[Out] x^2/2 - ArcTan[x^2]/2 - ArcTan[1 - Sqrt[2]*x]/(2*Sqrt[2]) + ArcTan[1 + Sqrt
[2]*x]/(2*Sqrt[2]) + Log[x] + Log[1 - Sqrt[2]*x + x^2]/(4*Sqrt[2]) - Log[1
+ Sqrt[2]*x + x^2]/(4*Sqrt[2]) - Log[1 + x^4]/4
```

Rubi [A] time = 0.114055, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 13, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.812, Rules used = {1593, 1833, 297, 1162, 617, 204, 1165, 628, 1834, 1248, 635, 203, 260}

$$\frac{x^2}{2}+\frac{\log\left(x^2-\sqrt{2}x+1\right)}{4\sqrt{2}}-\frac{\log\left(x^2+\sqrt{2}x+1\right)}{4\sqrt{2}}-\frac{1}{4}\log\left(x^4+1\right)-\frac{1}{2}\tan^{-1}\left(x^2\right)+\log(x)-\frac{\tan^{-1}\left(1-\sqrt{2}x\right)}{2\sqrt{2}}+\frac{\tan^{-1}(\ldots}{2\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^3 + x^6)/(x + x^5),x]
```

```
[Out] x^2/2 - ArcTan[x^2]/2 - ArcTan[1 - Sqrt[2]*x]/(2*Sqrt[2]) + ArcTan[1 + Sqrt
[2]*x]/(2*Sqrt[2]) + Log[x] + Log[1 - Sqrt[2]*x + x^2]/(4*Sqrt[2]) - Log[1
+ Sqrt[2]*x + x^2]/(4*Sqrt[2]) - Log[1 + x^4]/4
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1833

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j +
(k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1}]*(a + b*x^n)^p)/c^j, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
```

```
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x^3+x^6}{x+x^5}\,dx &= \int \frac{1+x^3+x^6}{x\left(1+x^4\right)}\,dx\\
&= \int \left(\frac{x^2}{1+x^4}+\frac{1+x^6}{x\left(1+x^4\right)}\right)dx\\
&= \int \frac{x^2}{1+x^4}\,dx+\int \frac{1+x^6}{x\left(1+x^4\right)}\,dx\\
&= -\left(\frac{1}{2}\int \frac{1-x^2}{1+x^4}\,dx\right)+\frac{1}{2}\int \frac{1+x^2}{1+x^4}\,dx+\int \left(\frac{1}{x}+x+\frac{x\left(-1-x^2\right)}{1+x^4}\right)dx\\
&= \frac{x^2}{2}+\log(x)+\frac{1}{4}\int \frac{1}{1-\sqrt{2}x+x^2}\,dx+\frac{1}{4}\int \frac{1}{1+\sqrt{2}x+x^2}\,dx+\frac{\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx}{4\sqrt{2}}+\frac{\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx}{4\sqrt{2}}+\\
&= \frac{x^2}{2}+\log(x)+\frac{\log\left(1-\sqrt{2}x+x^2\right)}{4\sqrt{2}}-\frac{\log\left(1+\sqrt{2}x+x^2\right)}{4\sqrt{2}}+\frac{1}{2}\operatorname{Subst}\left(\int \frac{-1-x}{1+x^2}\,dx,x,x^2\right)+\frac{\operatorname{Subst}\left(\right.}{}\\
&= \frac{x^2}{2}-\frac{\tan^{-1}\left(1-\sqrt{2}x\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(1+\sqrt{2}x\right)}{2\sqrt{2}}+\log(x)+\frac{\log\left(1-\sqrt{2}x+x^2\right)}{4\sqrt{2}}-\frac{\log\left(1+\sqrt{2}x+x^2\right)}{4\sqrt{2}}-\frac{1}{2}\\
&= \frac{x^2}{2}-\frac{1}{2}\tan^{-1}\left(x^2\right)-\frac{\tan^{-1}\left(1-\sqrt{2}x\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(1+\sqrt{2}x\right)}{2\sqrt{2}}+\log(x)+\frac{\log\left(1-\sqrt{2}x+x^2\right)}{4\sqrt{2}}-\frac{\log\left(1+\right.}{4}
\end{aligned}$$

Mathematica [A] time = 0.0507398, size = 101, normalized size = 0.9

$$\frac{1}{8}\left(4x^2+\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)-\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-2\log\left(x^4+1\right)+8\log(x)-2\left(\sqrt{2}-2\right)\tan^{-1}\left(1-\sqrt{2}x\right)+2\left(\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^3 + x^6)/(x + x^5),x]
```

```
[Out] (4*x^2 - 2*(-2 + Sqrt[2])*ArcTan[1 - Sqrt[2]*x] + 2*(2 + Sqrt[2])*ArcTan[1
+ Sqrt[2]*x] + 8*Log[x] + Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] - Sqrt[2]*Log[1
+ Sqrt[2]*x + x^2] - 2*Log[1 + x^4])/8
```

Maple [A] time = 0.006, size = 79, normalized size = 0.7

$$\frac{x^2}{2}+\ln(x)-\frac{\arctan\left(x^2\right)}{2}+\frac{\arctan\left(1+x\sqrt{2}\right)\sqrt{2}}{4}+\frac{\arctan\left(-1+x\sqrt{2}\right)\sqrt{2}}{4}+\frac{\sqrt{2}}{8}\ln\left(\frac{1+x^2-x\sqrt{2}}{1+x^2+x\sqrt{2}}\right)-\frac{\ln\left(x^4+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6+x^3+1)/(x^5+x),x)
```

```
[Out] 1/2*x^2+ln(x)-1/2*arctan(x^2)+1/4*arctan(1+x*2^(1/2))*2^(1/2)+1/4*arctan(-1
+x*2^(1/2))*2^(1/2)+1/8*2^(1/2)*ln((1+x^2-x*2^(1/2))/(1+x^2+x*2^(1/2)))-1/4
*ln(x^4+1)
```

Maxima [A] time = 1.71442, size = 134, normalized size = 1.2

$$\frac{1}{4}\sqrt{2}\left(\sqrt{2}+1\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)-\frac{1}{4}\sqrt{2}\left(\sqrt{2}-1\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)-\frac{1}{8}\sqrt{2}\left(\sqrt{2}+1\right)\log\left(x^2+\sqrt{2}x+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+x^3+1)/(x^5+x),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*(sqrt(2) + 1)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) - 1/4*sqrt(2)
*(sqrt(2) - 1)*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) - 1/8*sqrt(2)*(sqrt(2) +
 1)*log(x^2 + sqrt(2)*x + 1) - 1/8*sqrt(2)*(sqrt(2) - 1)*log(x^2 - sqrt(2)*
x + 1) + 1/2*x^2 + log(x)
```

Fricas [C] time = 9.76508, size = 2269, normalized size = 20.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+x^3+1)/(x^5+x),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 - 1/4*(2*sqrt(1/4*I) + I + 1)*log((2*sqrt(1/4*I) + I + 1)^3 - 5*(2*
sqrt(1/4*I) + I + 1)^2 + 3*x + 20*sqrt(1/4*I) + 10*I + 5) - 1/4*(2*sqrt(-1/
4*I) - I + 1)*log(-(2*sqrt(1/4*I) + I + 1)^3 - (2*sqrt(1/4*I) + I + 2)*(2*s
qrt(-1/4*I) - I + 1)^2 + 4*(2*sqrt(1/4*I) + I + 1)^2 - ((2*sqrt(1/4*I) + I
+ 1)^2 - 8*sqrt(1/4*I) - 4*I - 6)*(2*sqrt(-1/4*I) - I + 1) + 3*x - 16*sqrt(
1/4*I) - 8*I - 9) + 1/4*(sqrt(1/4*I) + sqrt(-1/4*I) - 2*sqrt(-3/16*(2*sqrt(
1/4*I) + I + 1)^2 - 1/8*(2*sqrt(1/4*I) + I - 3)*(2*sqrt(-1/4*I) - I + 1) -
3/16*(2*sqrt(-1/4*I) - I + 1)^2 + sqrt(1/4*I) + 1/2*I - 1/2) - 1)*log(1/2*(
2*sqrt(1/4*I) + I + 2)*(2*sqrt(-1/4*I) - I + 1)^2 + 1/2*(2*sqrt(1/4*I) + I
```

```
+ 1)^2 + 1/2*((2*sqrt(1/4*I) + I + 1)^2 - 8*sqrt(1/4*I) - 4*I - 6)*(2*sqrt(
-1/4*I) - I + 1) + 2*sqrt(-3/16*(2*sqrt(1/4*I) + I + 1)^2 - 1/8*(2*sqrt(1/4
*I) + I - 3)*(2*sqrt(-1/4*I) - I + 1) - 3/16*(2*sqrt(-1/4*I) - I + 1)^2 + s
qrt(1/4*I) + 1/2*I - 1/2)*((2*sqrt(1/4*I) + I + 2)*(2*sqrt(-1/4*I) - I + 1)
 + 2*sqrt(1/4*I) + I - 1) + 3*x - 2*sqrt(1/4*I) - I + 2) + 1/4*(sqrt(1/4*I)
 + sqrt(-1/4*I) + 2*sqrt(-3/16*(2*sqrt(1/4*I) + I + 1)^2 - 1/8*(2*sqrt(1/4*
I) + I - 3)*(2*sqrt(-1/4*I) - I + 1) - 3/16*(2*sqrt(-1/4*I) - I + 1)^2 + sq
rt(1/4*I) + 1/2*I - 1/2) - 1)*log(1/2*(2*sqrt(1/4*I) + I + 2)*(2*sqrt(-1/4*
I) - I + 1)^2 + 1/2*(2*sqrt(1/4*I) + I + 1)^2 + 1/2*((2*sqrt(1/4*I) + I + 1
)^2 - 8*sqrt(1/4*I) - 4*I - 6)*(2*sqrt(-1/4*I) - I + 1) - 2*sqrt(-3/16*(2*s
qrt(1/4*I) + I + 1)^2 - 1/8*(2*sqrt(1/4*I) + I - 3)*(2*sqrt(-1/4*I) - I + 1
) - 3/16*(2*sqrt(-1/4*I) - I + 1)^2 + sqrt(1/4*I) + 1/2*I - 1/2)*((2*sqrt(1
/4*I) + I + 2)*(2*sqrt(-1/4*I) - I + 1) + 2*sqrt(1/4*I) + I - 1) + 3*x - 2*
sqrt(1/4*I) - I + 2) + log(x)
```

Sympy [A] time = 0.782248, size = 61, normalized size = 0.54

$$\frac{x^2}{2} + \log(x) + \operatorname{RootSum}\left(256t^4 + 256t^3 + 128t^2 + 16t + 1, \left(t \mapsto t\log\left(\frac{1792t^4}{73} + \frac{704t^3}{219} - \frac{3152t^2}{219} - \frac{2584t}{219} + x - \frac{344}{219}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6+x**3+1)/(x**5+x),x)
```

```
[Out] x**2/2 + log(x) + RootSum(256*_t**4 + 256*_t**3 + 128*_t**2 + 16*_t + 1, La
mbda(_t, _t*log(1792*_t**4/73 + 704*_t**3/219 - 3152*_t**2/219 - 2584*_t/21
9 + x - 344/219)))
```

Giac [A] time = 1.28067, size = 124, normalized size = 1.11

$$\frac{1}{2}x^2 + \frac{1}{4}\left(\sqrt{2}+2\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right) + \frac{1}{4}\left(\sqrt{2}-2\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right) - \frac{1}{8}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right) + \frac{1}{8}\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+x^3+1)/(x^5+x),x, algorithm="giac")
```

```
[Out] 1/2*x^2 + 1/4*(sqrt(2) + 2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/4*(sqrt
(2) - 2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) - 1/8*sqrt(2)*log(x^2 + sqrt(2
)*x + 1) + 1/8*sqrt(2)*log(x^2 - sqrt(2)*x + 1) - 1/4*log(x^4 + 1) + log(ab
```

```
s(x))
```

3.369 $\int \frac{1+x^2}{-x+x^2} \, dx$

Optimal. Leaf size=14

$$x + 2\log(1 - x) - \log(x)$$

```
[Out] x + 2*Log[1 - x] - Log[x]
```

Rubi [A] time = 0.0163195, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1593, 894}

$$x + 2\log(1 - x) - \log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/(-x + x^2),x]
```

```
[Out] x + 2*Log[1 - x] - Log[x]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \frac{1+x^2}{-x+x^2}\,dx = \int \frac{1+x^2}{(-1+x)x}\,dx$$
$$= \int \left(1+\frac{2}{-1+x}-\frac{1}{x}\right)dx$$
$$= x+2\log(1-x)-\log(x)$$

Mathematica [A] time = 0.0030484, size = 14, normalized size = 1.

$$x+2\log(1-x)-\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/(-x + x^2),x]
```

```
[Out] x + 2*Log[1 - x] - Log[x]
```

Maple [A] time = 0.004, size = 13, normalized size = 0.9

$$x+2\ln(x-1)-\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(x^2-x),x)
```

```
[Out] x+2*ln(x-1)-ln(x)
```

Maxima [A] time = 1.09689, size = 16, normalized size = 1.14

$$x+2\log(x-1)-\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2-x),x, algorithm="maxima")
```

```
[Out] x + 2*log(x - 1) - log(x)
```

Fricas [A] time = 1.41249, size = 36, normalized size = 2.57

$$x + 2\log(x - 1) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2-x),x, algorithm="fricas")
```

```
[Out] x + 2*log(x - 1) - log(x)
```

Sympy [A] time = 0.089288, size = 10, normalized size = 0.71

$$x - \log(x) + 2\log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x**2-x),x)
```

```
[Out] x - log(x) + 2*log(x - 1)
```

Giac [A] time = 1.19812, size = 19, normalized size = 1.36

$$x + 2\log(|x - 1|) - \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2-x),x, algorithm="giac")
```

```
[Out] x + 2*log(abs(x - 1)) - log(abs(x))
```

3.370 $\int \frac{1+x^3}{-x+x^3}\,dx$

Optimal. Leaf size=12

$$x + \log(1 - x) - \log(x)$$

```
[Out] x + Log[1 - x] - Log[x]
```

Rubi [A] time = 0.0293969, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1593, 1802}

$$x + \log(1 - x) - \log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^3)/(-x + x^3),x]
```

```
[Out] x + Log[1 - x] - Log[x]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{1 + x^3}{-x + x^3}\,dx &= \int \frac{1 + x^3}{x\left(-1 + x^2\right)}\,dx \\
&= \int \left(1 + \frac{1}{-1 + x} - \frac{1}{x}\right) dx \\
&= x + \log(1 - x) - \log(x)
\end{aligned}$$

Mathematica [A] time = 0.0044697, size = 12, normalized size = 1.

$$x + \log(1 - x) - \log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^3)/(-x + x^3),x]
```

```
[Out] x + Log[1 - x] - Log[x]
```

Maple [A] time = 0.005, size = 11, normalized size = 0.9

$$x + \ln(x - 1) - \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+1)/(x^3-x),x)
```

```
[Out] x+ln(x-1)-ln(x)
```

Maxima [A] time = 1.05532, size = 14, normalized size = 1.17

$$x + \log(x - 1) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/(x^3-x),x, algorithm="maxima")
```

```
[Out] x + log(x - 1) - log(x)
```

Fricas [A] time = 1.71472, size = 34, normalized size = 2.83

$$x + \log(x - 1) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/(x^3-x),x, algorithm="fricas")
```

```
[Out] x + log(x - 1) - log(x)
```

Sympy [A] time = 0.089337, size = 8, normalized size = 0.67

$$x - \log(x) + \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+1)/(x**3-x),x)
```

```
[Out] x - log(x) + log(x - 1)
```

Giac [A] time = 1.12206, size = 16, normalized size = 1.33

$$x + \log(|x - 1|) - \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/(x^3-x),x, algorithm="giac")
```

```
[Out] x + log(abs(x - 1)) - log(abs(x))
```

3.371 $\int \frac{1+x^3}{-x^2+x^3} \, dx$

Optimal. Leaf size=17

$$x + \frac{1}{x} + 2\log(1-x) - \log(x)$$

```
[Out] x^(-1) + x + 2*Log[1 - x] - Log[x]
```

Rubi [A] time = 0.0295838, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1593, 1620}

$$x + \frac{1}{x} + 2\log(1-x) - \log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^3)/(-x^2 + x^3),x]
```

```
[Out] x^(-1) + x + 2*Log[1 - x] - Log[x]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\int \frac{1+x^3}{-x^2+x^3}\,dx = \int \frac{1+x^3}{(-1+x)x^2}\,dx$$
$$= \int \left(1+\frac{2}{-1+x}-\frac{1}{x^2}-\frac{1}{x}\right)dx$$
$$= \frac{1}{x}+x+2\log(1-x)-\log(x)$$

Mathematica [A] time = 0.0035802, size = 17, normalized size = 1.

$$x+\frac{1}{x}+2\log(1-x)-\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^3)/(-x^2 + x^3),x]
```

```
[Out] x^(-1) + x + 2*Log[1 - x] - Log[x]
```

Maple [A] time = 0.006, size = 16, normalized size = 0.9

$$x+2\,\ln(x-1)+x^{-1}-\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+1)/(x^3-x^2),x)
```

```
[Out] x+2*ln(x-1)+1/x-ln(x)
```

Maxima [A] time = 1.09659, size = 20, normalized size = 1.18

$$x+\frac{1}{x}+2\,\log(x-1)-\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/(x^3-x^2),x, algorithm="maxima")
```

```
[Out] x + 1/x + 2*log(x - 1) - log(x)
```

Fricas [A] time = 1.72478, size = 55, normalized size = 3.24

$$\frac{x^2 + 2\,x\log\left(x - 1\right) - x\log\left(x\right) + 1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/(x^3-x^2),x, algorithm="fricas")
```

```
[Out] (x^2 + 2*x*log(x - 1) - x*log(x) + 1)/x
```

Sympy [A] time = 0.09717, size = 14, normalized size = 0.82

$$x - \log\left(x\right) + 2\log\left(x - 1\right) + \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+1)/(x**3-x**2),x)
```

```
[Out] x - log(x) + 2*log(x - 1) + 1/x
```

Giac [A] time = 1.11114, size = 23, normalized size = 1.35

$$x + \frac{1}{x} + 2\,\log\left(|x - 1|\right) - \log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/(x^3-x^2),x, algorithm="giac")
```

```
[Out] x + 1/x + 2*log(abs(x - 1)) - log(abs(x))
```

3.372 $\int \frac{-1+x^5}{-x+x^3} \, dx$

Optimal. Leaf size=17

$$\frac{x^3}{3} + x + \log(x) - \log(x+1)$$

```
[Out] x + x^3/3 + Log[x] - Log[1 + x]
```

Rubi [A] time = 0.0293711, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1593, 1802}

$$\frac{x^3}{3} + x + \log(x) - \log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^5)/(-x + x^3),x]
```

```
[Out] x + x^3/3 + Log[x] - Log[1 + x]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{-1+x^5}{-x+x^3}\,dx = \int \frac{-1+x^5}{x\left(-1+x^2\right)}\,dx$$
$$= \int \left(1+\frac{1}{-1-x}+\frac{1}{x}+x^2\right)dx$$
$$= x+\frac{x^3}{3}+\log(x)-\log(1+x)$$

Mathematica [A] time = 0.0044405, size = 17, normalized size = 1.

$$\frac{x^3}{3}+x+\log(x)-\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^5)/(-x + x^3),x]
```

```
[Out] x + x^3/3 + Log[x] - Log[1 + x]
```

Maple [A] time = 0.004, size = 16, normalized size = 0.9

$$x+\frac{x^3}{3}+\ln(x)-\ln(1+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5-1)/(x^3-x),x)
```

```
[Out] x+1/3*x^3+ln(x)-ln(1+x)
```

Maxima [A] time = 1.06308, size = 20, normalized size = 1.18

$$\frac{1}{3}x^3+x-\log(x+1)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-1)/(x^3-x),x, algorithm="maxima")
```

```
[Out] 1/3*x^3 + x - log(x + 1) + log(x)
```

Fricas [A] time = 1.16111, size = 47, normalized size = 2.76

$$\frac{1}{3}x^3 + x - \log(x+1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-1)/(x^3-x),x, algorithm="fricas")
```

```
[Out] 1/3*x^3 + x - log(x + 1) + log(x)
```

Sympy [A] time = 0.089873, size = 14, normalized size = 0.82

$$\frac{x^3}{3} + x + \log(x) - \log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**5-1)/(x**3-x),x)
```

```
[Out] x**3/3 + x + log(x) - log(x + 1)
```

Giac [A] time = 1.1499, size = 23, normalized size = 1.35

$$\frac{1}{3}x^3 + x - \log(|x+1|) + \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-1)/(x^3-x),x, algorithm="giac")
```

```
[Out] 1/3*x^3 + x - log(abs(x + 1)) + log(abs(x))
```

3.373 $\int \frac{1+x^4}{x^3+x^5} \, dx$

Optimal. Leaf size=18

$$-\frac{1}{2x^2}+\log \left(x^2+1\right)-\log (x)$$

```
[Out] -1/(2*x^2) - Log[x] + Log[1 + x^2]
```

Rubi [A] time = 0.0298592, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1593, 1252, 894}

$$-\frac{1}{2x^2}+\log \left(x^2+1\right)-\log (x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^4)/(x^3 + x^5),x]
```

```
[Out] -1/(2*x^2) - Log[x] + Log[1 + x^2]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{1+x^4}{x^3+x^5}\,dx &= \int \frac{1+x^4}{x^3\left(1+x^2\right)}\,dx\\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{1+x^2}{x^2(1+x)}\,dx,x,x^2\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{1}{x^2}-\frac{1}{x}+\frac{2}{1+x}\right)dx,x,x^2\right)\\
&= -\frac{1}{2x^2}-\log(x)+\log\left(1+x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.0046253, size = 18, normalized size = 1.

$$-\frac{1}{2x^2}+\log\left(x^2+1\right)-\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^4)/(x^3 + x^5),x]
```

```
[Out] -1/(2*x^2) - Log[x] + Log[1 + x^2]
```

Maple [A] time = 0.005, size = 17, normalized size = 0.9

$$-\frac{1}{2\,x^2}-\ln(x)+\ln\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)/(x^5+x^3),x)
```

```
[Out] -1/2/x^2-ln(x)+ln(x^2+1)
```

Maxima [A] time = 1.65857, size = 22, normalized size = 1.22

$$-\frac{1}{2\,x^2}+\log\left(x^2+1\right)-\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^5+x^3),x, algorithm="maxima")
```

```
[Out] -1/2/x^2 + log(x^2 + 1) - log(x)
```

Fricas [A] time = 1.22791, size = 66, normalized size = 3.67

$$\frac{2x^2\log\left(x^2+1\right)-2x^2\log(x)-1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^5+x^3),x, algorithm="fricas")
```

```
[Out] 1/2*(2*x^2*log(x^2 + 1) - 2*x^2*log(x) - 1)/x^2
```

Sympy [A] time = 0.097069, size = 15, normalized size = 0.83

$$-\log(x)+\log\left(x^2+1\right)-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)/(x**5+x**3),x)
```

```
[Out] -log(x) + log(x**2 + 1) - 1/(2*x**2)
```

Giac [A] time = 1.12886, size = 31, normalized size = 1.72

$$\frac{x^2-1}{2x^2}+\log\left(x^2+1\right)-\frac{1}{2}\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^5+x^3),x, algorithm="giac")
```

```
[Out] 1/2*(x^2 - 1)/x^2 + log(x^2 + 1) - 1/2*log(x^2)
```

3.374 $\int \frac{1+x^2}{x+2x^2+x^3}\,dx$

Optimal. Leaf size=10

$$\frac{2}{x+1}+\log(x)$$

```
[Out] 2/(1 + x) + Log[x]
```

Rubi [A] time = 0.0186148, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1594, 27, 894}

$$\frac{2}{x+1}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/(x + 2*x^2 + x^3),x]
```

```
[Out] 2/(1 + x) + Log[x]
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{1+x^2}{x+2x^2+x^3}\,dx &= \int \frac{1+x^2}{x\left(1+2x+x^2\right)}\,dx\\ &= \int \frac{1+x^2}{x(1+x)^2}\,dx\\ &= \int \left(\frac{1}{x}-\frac{2}{(1+x)^2}\right)dx\\ &= \frac{2}{1+x}+\log(x)\end{aligned}$$

Mathematica [A] time = 0.0048352, size = 10, normalized size = 1.

$$\frac{2}{x+1}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/(x + 2*x^2 + x^3),x]
```

```
[Out] 2/(1 + x) + Log[x]
```

Maple [A] time = 0.005, size = 11, normalized size = 1.1

$$2\,(1+x)^{-1}+\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(x^3+2*x^2+x),x)
```

```
[Out] 2/(1+x)+ln(x)
```

Maxima [A] time = 1.09606, size = 14, normalized size = 1.4

$$\frac{2}{x+1}+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^3+2*x^2+x),x, algorithm="maxima")
```

```
[Out] 2/(x + 1) + log(x)
```

Fricas [A] time = 1.1844, size = 41, normalized size = 4.1

$$\frac{(x+1)\log(x)+2}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^3+2*x^2+x),x, algorithm="fricas")
```

```
[Out] ((x + 1)*log(x) + 2)/(x + 1)
```

Sympy [A] time = 0.081395, size = 7, normalized size = 0.7

$$\log(x)+\frac{2}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x**3+2*x**2+x),x)
```

```
[Out] log(x) + 2/(x + 1)
```

Giac [A] time = 1.14539, size = 15, normalized size = 1.5

$$\frac{2}{x+1}+\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^3+2*x^2+x),x, algorithm="giac")
```

```
[Out] 2/(x + 1) + log(abs(x))
```

3.375 $\int \frac{1+x^5}{-10x-3x^2+x^3}\,dx$

Optimal. Leaf size=42

$$\frac{x^3}{3}+\frac{3x^2}{2}+19x+\frac{3126}{35}\log(5-x)-\frac{\log(x)}{10}-\frac{31}{14}\log(x+2)$$

```
[Out] 19*x + (3*x^2)/2 + x^3/3 + (3126*Log[5 - x])/35 - Log[x]/10 - (31*Log[2 + x
])/14
```

Rubi [A] time = 0.0400056, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {1594, 1628}

$$\frac{x^3}{3}+\frac{3x^2}{2}+19x+\frac{3126}{35}\log(5-x)-\frac{\log(x)}{10}-\frac{31}{14}\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^5)/(-10*x - 3*x^2 + x^3),x]
```

```
[Out] 19*x + (3*x^2)/2 + x^3/3 + (3126*Log[5 - x])/35 - Log[x]/10 - (31*Log[2 + x
])/14
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{1+x^5}{-10x-3x^2+x^3}\,dx = \int \frac{1+x^5}{x\left(-10-3x+x^2\right)}\,dx$$
$$= \int \left(19+\frac{3126}{35(-5+x)}-\frac{1}{10x}+3x+x^2-\frac{31}{14(2+x)}\right)dx$$
$$= 19x+\frac{3x^2}{2}+\frac{x^3}{3}+\frac{3126}{35}\log(5-x)-\frac{\log(x)}{10}-\frac{31}{14}\log(2+x)$$

Mathematica [A] time = 0.0066278, size = 42, normalized size = 1.

$$\frac{x^3}{3}+\frac{3x^2}{2}+19x+\frac{3126}{35}\log(5-x)-\frac{\log(x)}{10}-\frac{31}{14}\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^5)/(-10*x - 3*x^2 + x^3),x]
```

```
[Out] 19*x + (3*x^2)/2 + x^3/3 + (3126*Log[5 - x])/35 - Log[x]/10 - (31*Log[2 + x
])/14
```

Maple [A] time = 0.009, size = 31, normalized size = 0.7

$$\frac{x^3}{3}+\frac{3\,x^2}{2}+19\,x-\frac{\ln(x)}{10}-\frac{31\,\ln(2+x)}{14}+\frac{3126\,\ln(-5+x)}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5+1)/(x^3-3*x^2-10*x),x)
```

```
[Out] 1/3*x^3+3/2*x^2+19*x-1/10*ln(x)-31/14*ln(2+x)+3126/35*ln(-5+x)
```

Maxima [A] time = 1.13029, size = 41, normalized size = 0.98

$$\frac{1}{3}x^3+\frac{3}{2}x^2+19\,x-\frac{31}{14}\log(x+2)+\frac{3126}{35}\log(x-5)-\frac{1}{10}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+1)/(x^3-3*x^2-10*x),x, algorithm="maxima")
```

```
[Out] 1/3*x^3 + 3/2*x^2 + 19*x - 31/14*log(x + 2) + 3126/35*log(x - 5) - 1/10*log
(x)
```

Fricas [A] time = 1.26431, size = 108, normalized size = 2.57

$$\frac{1}{3}x^3 + \frac{3}{2}x^2 + 19x - \frac{31}{14}\log(x+2) + \frac{3126}{35}\log(x-5) - \frac{1}{10}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+1)/(x^3-3*x^2-10*x),x, algorithm="fricas")
```

```
[Out] 1/3*x^3 + 3/2*x^2 + 19*x - 31/14*log(x + 2) + 3126/35*log(x - 5) - 1/10*log
(x)
```

Sympy [A] time = 0.130948, size = 36, normalized size = 0.86

$$\frac{x^3}{3} + \frac{3x^2}{2} + 19x - \frac{\log(x)}{10} + \frac{3126\log(x-5)}{35} - \frac{31\log(x+2)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**5+1)/(x**3-3*x**2-10*x),x)
```

```
[Out] x**3/3 + 3*x**2/2 + 19*x - log(x)/10 + 3126*log(x - 5)/35 - 31*log(x + 2)/1
4
```

Giac [A] time = 1.13175, size = 45, normalized size = 1.07

$$\frac{1}{3}x^3 + \frac{3}{2}x^2 + 19x - \frac{31}{14}\log(|x+2|) + \frac{3126}{35}\log(|x-5|) - \frac{1}{10}\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+1)/(x^3-3*x^2-10*x),x, algorithm="giac")
```

```
[Out] 1/3*x^3 + 3/2*x^2 + 19*x - 31/14*log(abs(x + 2)) + 3126/35*log(abs(x - 5))
- 1/10*log(abs(x))
```

3.376 $\int \frac{15-5x+x^2+x^3}{(5+x^2)(3+2x+x^2)}\,dx$

Optimal. Leaf size=46

$$\frac{1}{2}\log\left(x^2+2x+3\right)-\sqrt{5}\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)+\frac{5\tan^{-1}\left(\frac{x+1}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[Out] -(Sqrt[5]*ArcTan[x/Sqrt[5]]) + (5*ArcTan[(1 + x)/Sqrt[2]])/Sqrt[2] + Log[3
+ 2*x + x^2]/2
```

Rubi [A] time = 0.125426, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {6725, 203, 634, 618, 204, 628}

$$\frac{1}{2}\log\left(x^2+2x+3\right)-\sqrt{5}\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)+\frac{5\tan^{-1}\left(\frac{x+1}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(15 - 5*x + x^2 + x^3)/((5 + x^2)*(3 + 2*x + x^2)),x]
```

```
[Out] -(Sqrt[5]*ArcTan[x/Sqrt[5]]) + (5*ArcTan[(1 + x)/Sqrt[2]])/Sqrt[2] + Log[3
+ 2*x + x^2]/2
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{15-5x+x^2+x^3}{(5+x^2)(3+2x+x^2)}\,dx &= \int \left(-\frac{5}{5+x^2}+\frac{6+x}{3+2x+x^2}\right)dx \\
&= -\left(5\int \frac{1}{5+x^2}\,dx\right)+\int \frac{6+x}{3+2x+x^2}\,dx \\
&= -\sqrt{5}\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)+\frac{1}{2}\int \frac{2+2x}{3+2x+x^2}\,dx+5\int \frac{1}{3+2x+x^2}\,dx \\
&= -\sqrt{5}\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)+\frac{1}{2}\log\left(3+2x+x^2\right)-10\,\text{Subst}\left(\int \frac{1}{-8-x^2}\,dx,x,2+2x\right) \\
&= -\sqrt{5}\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)+\frac{5\tan^{-1}\left(\frac{1+x}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{1}{2}\log\left(3+2x+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.0167781, size = 46, normalized size = 1.

$$\frac{1}{2}\log\left(x^2+2x+3\right)-\sqrt{5}\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)+\frac{5\tan^{-1}\left(\frac{x+1}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(15 - 5*x + x^2 + x^3)/((5 + x^2)*(3 + 2*x + x^2)),x]
```

```
[Out] -(Sqrt[5]*ArcTan[x/Sqrt[5]]) + (5*ArcTan[(1 + x)/Sqrt[2]])/Sqrt[2] + Log[3
+ 2*x + x^2]/2
```

Maple [A] time = 0., size = 41, normalized size = 0.9

$$\frac{\ln\left(x^{2}+2x+3\right)}{2}+\frac{5\sqrt{2}}{2}\arctan\left(\frac{(2+2x)\sqrt{2}}{4}\right)-\arctan\left(\frac{x\sqrt{5}}{5}\right)\sqrt{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x^2-5*x+15)/(x^2+5)/(x^2+2*x+3),x)
```

```
[Out] 1/2*ln(x^2+2*x+3)+5/2*2^(1/2)*arctan(1/4*(2+2*x)*2^(1/2))-arctan(1/5*x*5^(1
/2))*5^(1/2)
```

Maxima [A] time = 1.67868, size = 51, normalized size = 1.11

$$\frac{5}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x+1)\right)-\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}x\right)+\frac{1}{2}\log\left(x^{2}+2x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2-5*x+15)/(x^2+5)/(x^2+2*x+3),x, algorithm="maxima")
```

```
[Out] 5/2*sqrt(2)*arctan(1/2*sqrt(2)*(x + 1)) - sqrt(5)*arctan(1/5*sqrt(5)*x) + 1
/2*log(x^2 + 2*x + 3)
```

Fricas [A] time = 1.22307, size = 132, normalized size = 2.87

$$\frac{5}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x+1)\right)-\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}x\right)+\frac{1}{2}\log\left(x^{2}+2x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2-5*x+15)/(x^2+5)/(x^2+2*x+3),x, algorithm="fricas")
```

```
[Out] 5/2*sqrt(2)*arctan(1/2*sqrt(2)*(x + 1)) - sqrt(5)*arctan(1/5*sqrt(5)*x) + 1
/2*log(x^2 + 2*x + 3)
```

Sympy [A] time = 0.181882, size = 51, normalized size = 1.11

$$\frac{\log\left(x^2+2x+3\right)}{2}-\sqrt{5}\,\mathrm{atan}\left(\frac{\sqrt{5}x}{5}\right)+\frac{5\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}+\frac{\sqrt{2}}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x**2-5*x+15)/(x**2+5)/(x**2+2*x+3),x)
```

```
[Out] log(x**2 + 2*x + 3)/2 - sqrt(5)*atan(sqrt(5)*x/5) + 5*sqrt(2)*atan(sqrt(2)*
x/2 + sqrt(2)/2)/2
```

Giac [A] time = 1.08551, size = 51, normalized size = 1.11

$$\frac{5}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x+1)\right)-\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}x\right)+\frac{1}{2}\log\left(x^2+2x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2-5*x+15)/(x^2+5)/(x^2+2*x+3),x, algorithm="giac")
```

```
[Out] 5/2*sqrt(2)*arctan(1/2*sqrt(2)*(x + 1)) - sqrt(5)*arctan(1/5*sqrt(5)*x) + 1
/2*log(x^2 + 2*x + 3)
```

3.377 $\int \frac{1}{(1+x^2)\left(3+\frac{10x}{1+x^2}\right)} dx$

Optimal. Leaf size=19

$$\frac{1}{8}\log(3x+1)-\frac{1}{8}\log(x+3)$$

```
[Out] -Log[3 + x]/8 + Log[1 + 3*x]/8
```

Rubi [A] time = 0.0618751, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6688, 616, 31}

$$\frac{1}{8}\log(3x+1)-\frac{1}{8}\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x^2)*(3 + (10*x)/(1 + x^2))),x]
```

```
[Out] -Log[3 + x]/8 + Log[1 + 3*x]/8
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{\left(1+x^2\right)\left(3+\frac{10x}{1+x^2}\right)}\,dx = \int \frac{1}{3+10x+3x^2}\,dx$$
$$= \frac{3}{8}\int \frac{1}{1+3x}\,dx - \frac{3}{8}\int \frac{1}{9+3x}\,dx$$
$$= -\frac{1}{8}\log(3+x) + \frac{1}{8}\log(1+3x)$$

Mathematica [A] time = 0.0031748, size = 19, normalized size = 1.

$$\frac{1}{8}\log(3x+1) - \frac{1}{8}\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x^2)*(3 + (10*x)/(1 + x^2))),x]
```

```
[Out] -Log[3 + x]/8 + Log[1 + 3*x]/8
```

Maple [A] time = 0.006, size = 16, normalized size = 0.8

$$-\frac{\ln(3+x)}{8} + \frac{\ln(1+3x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)/(3+10*x/(x^2+1)),x)
```

```
[Out] -1/8*ln(3+x)+1/8*ln(1+3*x)
```

Maxima [A] time = 1.08613, size = 20, normalized size = 1.05

$$\frac{1}{8}\log(3x+1) - \frac{1}{8}\log(x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/(3+10*x/(x^2+1)),x, algorithm="maxima")
```

```
[Out] 1/8*log(3*x + 1) - 1/8*log(x + 3)
```

Fricas [A] time = 1.23929, size = 47, normalized size = 2.47

$$\frac{1}{8}\log(3x+1) - \frac{1}{8}\log(x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/(3+10*x/(x^2+1)),x, algorithm="fricas")
```

```
[Out] 1/8*log(3*x + 1) - 1/8*log(x + 3)
```

Sympy [A] time = 0.098616, size = 14, normalized size = 0.74

$$\frac{\log\left(x+\frac{1}{3}\right)}{8} - \frac{\log(x+3)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)/(3+10*x/(x**2+1)),x)
```

```
[Out] log(x + 1/3)/8 - log(x + 3)/8
```

Giac [A] time = 1.15885, size = 23, normalized size = 1.21

$$\frac{1}{8}\log(|3x+1|) - \frac{1}{8}\log(|x+3|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/(3+10*x/(x^2+1)),x, algorithm="giac")
```

```
[Out] 1/8*log(abs(3*x + 1)) - 1/8*log(abs(x + 3))
```

3.378 $\int \frac{x^3}{13+\frac{2}{x}+15x} \, dx$

Optimal. Leaf size=40

$$\frac{x^3}{45}-\frac{13x^2}{450}+\frac{139x}{3375}-\frac{16}{567}\log(3x+2)+\frac{\log(5x+1)}{4375}$$

```
[Out] (139*x)/3375 - (13*x^2)/450 + x^3/45 - (16*Log[2 + 3*x])/567 + Log[1 + 5*x]
/4375
```

Rubi [A] time = 0.020967, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1386, 701, 632, 31}

$$\frac{x^3}{45}-\frac{13x^2}{450}+\frac{139x}{3375}-\frac{16}{567}\log(3x+2)+\frac{\log(5x+1)}{4375}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(13 + 2/x + 15*x),x]
```

```
[Out] (139*x)/3375 - (13*x^2)/450 + x^3/45 - (16*Log[2 + 3*x])/567 + Log[1 + 5*x]
/4375
```

Rule 1386

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n_.) + (b_.)*(x_)^(mn_))^(p_.), x_Symbol
] :> Int[x^(m - n*p)*(b + a*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n}
, x] && EqQ[mn, -n] && IntegerQ[p] && PosQ[n]
```

Rule 701

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[PolynomialDivide[(d + e*x)^m, a + b*x + c*x^2, x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 NeQ[2*c*d - b*e, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
```

```
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{13+\frac{2}{x}+15x}\,dx &= \int \frac{x^4}{2+13x+15x^2}\,dx\\
&= \int \left(\frac{139}{3375}-\frac{13x}{225}+\frac{x^2}{15}-\frac{278+1417x}{3375\left(2+13x+15x^2\right)}\right)dx\\
&= \frac{139x}{3375}-\frac{13x^2}{450}+\frac{x^3}{45}-\frac{\int \frac{278+1417x}{2+13x+15x^2}\,dx}{3375}\\
&= \frac{139x}{3375}-\frac{13x^2}{450}+\frac{x^3}{45}+\frac{3}{875}\int \frac{1}{3+15x}\,dx-\frac{80}{189}\int \frac{1}{10+15x}\,dx\\
&= \frac{139x}{3375}-\frac{13x^2}{450}+\frac{x^3}{45}-\frac{16}{567}\log(2+3x)+\frac{\log(1+5x)}{4375}
\end{aligned}$$

Mathematica [A] time = 0.0051427, size = 40, normalized size = 1.

$$\frac{x^3}{45}-\frac{13x^2}{450}+\frac{139x}{3375}-\frac{16}{567}\log(3x+2)+\frac{\log(5x+1)}{4375}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(13 + 2/x + 15*x),x]
```

```
[Out] (139*x)/3375 - (13*x^2)/450 + x^3/45 - (16*Log[2 + 3*x])/567 + Log[1 + 5*x]
/4375
```

Maple [A] time = 0.006, size = 31, normalized size = 0.8

$$\frac{139\,x}{3375}-\frac{13\,x^2}{450}+\frac{x^3}{45}-\frac{16\,\ln(2+3\,x)}{567}+\frac{\ln(1+5\,x)}{4375}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(13+2/x+15*x),x)
```

```
[Out] 139/3375*x-13/450*x^2+1/45*x^3-16/567*ln(2+3*x)+1/4375*ln(1+5*x)
```

Maxima [A] time = 1.2908, size = 41, normalized size = 1.02

$$\frac{1}{45}\,x^3 - \frac{13}{450}\,x^2 + \frac{139}{3375}\,x + \frac{1}{4375}\,\log\left(5\,x+1\right) - \frac{16}{567}\,\log\left(3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(13+2/x+15*x),x, algorithm="maxima")
```

```
[Out] 1/45*x^3 - 13/450*x^2 + 139/3375*x + 1/4375*log(5*x + 1) - 16/567*log(3*x +
 2)
```

Fricas [A] time = 1.18517, size = 108, normalized size = 2.7

$$\frac{1}{45}\,x^3 - \frac{13}{450}\,x^2 + \frac{139}{3375}\,x + \frac{1}{4375}\,\log\left(5\,x+1\right) - \frac{16}{567}\,\log\left(3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(13+2/x+15*x),x, algorithm="fricas")
```

```
[Out] 1/45*x^3 - 13/450*x^2 + 139/3375*x + 1/4375*log(5*x + 1) - 16/567*log(3*x +
 2)
```

Sympy [A] time = 0.10749, size = 34, normalized size = 0.85

$$\frac{x^3}{45} - \frac{13x^2}{450} + \frac{139x}{3375} + \frac{\log\left(x+\frac{1}{5}\right)}{4375} - \frac{16\log\left(x+\frac{2}{3}\right)}{567}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(13+2/x+15*x),x)
```

```
[Out] x**3/45 - 13*x**2/450 + 139*x/3375 + log(x + 1/5)/4375 - 16*log(x + 2/3)/56
7
```

Giac [A] time = 1.13038, size = 43, normalized size = 1.08

$$\frac{1}{45}x^3 - \frac{13}{450}x^2 + \frac{139}{3375}x + \frac{1}{4375}\log(|5x+1|) - \frac{16}{567}\log(|3x+2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(13+2/x+15*x),x, algorithm="giac")
```

```
[Out] 1/45*x^3 - 13/450*x^2 + 139/3375*x + 1/4375*log(abs(5*x + 1)) - 16/567*log(
abs(3*x + 2))
```

3.379 $\int \frac{x^2}{13+\frac{2}{x}+15x} \, dx$

Optimal. Leaf size=33

$$\frac{x^2}{30} - \frac{13x}{225} + \frac{8}{189}\log(3x+2) - \frac{1}{875}\log(5x+1)$$

```
[Out] (-13*x)/225 + x^2/30 + (8*Log[2 + 3*x])/189 - Log[1 + 5*x]/875
```

Rubi [A] time = 0.0203888, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1386, 701, 632, 31}

$$\frac{x^2}{30} - \frac{13x}{225} + \frac{8}{189}\log(3x+2) - \frac{1}{875}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(13 + 2/x + 15*x),x]
```

```
[Out] (-13*x)/225 + x^2/30 + (8*Log[2 + 3*x])/189 - Log[1 + 5*x]/875
```

Rule 1386

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n_.) + (b_.)*(x_)^(mn_))^(p_.), x_Symbol
] :> Int[x^(m - n*p)*(b + a*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n}
, x] && EqQ[mn, -n] && IntegerQ[p] && PosQ[n]
```

Rule 701

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[PolynomialDivide[(d + e*x)^m, a + b*x + c*x^2, x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 NeQ[2*c*d - b*e, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
```

```
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2}{13+\frac{2}{x}+15x}\,dx &= \int \frac{x^3}{2+13x+15x^2}\,dx \\
&= \int \left(-\frac{13}{225}+\frac{x}{15}+\frac{26+139x}{225\left(2+13x+15x^2\right)}\right)dx \\
&= -\frac{13x}{225}+\frac{x^2}{30}+\frac{1}{225}\int \frac{26+139x}{2+13x+15x^2}\,dx \\
&= -\frac{13x}{225}+\frac{x^2}{30}-\frac{3}{175}\int \frac{1}{3+15x}\,dx+\frac{40}{63}\int \frac{1}{10+15x}\,dx \\
&= -\frac{13x}{225}+\frac{x^2}{30}+\frac{8}{189}\log(2+3x)-\frac{1}{875}\log(1+5x)
\end{aligned}
$$

Mathematica [A] time = 0.0040892, size = 33, normalized size = 1.

$$\frac{x^2}{30}-\frac{13x}{225}+\frac{8}{189}\log(3x+2)-\frac{1}{875}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(13 + 2/x + 15*x),x]
```

```
[Out] (-13*x)/225 + x^2/30 + (8*Log[2 + 3*x])/189 - Log[1 + 5*x]/875
```

Maple [A] time = 0.005, size = 26, normalized size = 0.8

$$-\frac{13\,x}{225}+\frac{x^2}{30}+\frac{8\,\ln(2+3\,x)}{189}-\frac{\ln(1+5\,x)}{875}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(13+2/x+15*x),x)
```

```
[Out] -13/225*x+1/30*x^2+8/189*ln(2+3*x)-1/875*ln(1+5*x)
```

Maxima [A] time = 1.02046, size = 34, normalized size = 1.03

$$\frac{1}{30}\,x^2-\frac{13}{225}\,x-\frac{1}{875}\,\log\left(5\,x+1\right)+\frac{8}{189}\,\log\left(3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(13+2/x+15*x),x, algorithm="maxima")
```

```
[Out] 1/30*x^2 - 13/225*x - 1/875*log(5*x + 1) + 8/189*log(3*x + 2)
```

Fricas [A] time = 1.24822, size = 85, normalized size = 2.58

$$\frac{1}{30}\,x^2-\frac{13}{225}\,x-\frac{1}{875}\,\log\left(5\,x+1\right)+\frac{8}{189}\,\log\left(3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(13+2/x+15*x),x, algorithm="fricas")
```

```
[Out] 1/30*x^2 - 13/225*x - 1/875*log(5*x + 1) + 8/189*log(3*x + 2)
```

Sympy [A] time = 0.106369, size = 27, normalized size = 0.82

$$\frac{x^2}{30}-\frac{13x}{225}-\frac{\log\left(x+\frac{1}{5}\right)}{875}+\frac{8\log\left(x+\frac{2}{3}\right)}{189}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(13+2/x+15*x),x)
```

```
[Out] x**2/30 - 13*x/225 - log(x + 1/5)/875 + 8*log(x + 2/3)/189
```

Giac [A] time = 1.24597, size = 36, normalized size = 1.09

$$\frac{1}{30}\,x^2 - \frac{13}{225}\,x - \frac{1}{875}\,\log\left(|5\,x+1|\right) + \frac{8}{189}\,\log\left(|3\,x+2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(13+2/x+15*x),x, algorithm="giac")
```

```
[Out] 1/30*x^2 - 13/225*x - 1/875*log(abs(5*x + 1)) + 8/189*log(abs(3*x + 2))
```

3.380 $\int \frac{x}{13+\frac{2}{x}+15x} \, dx$

Optimal. Leaf size=26

$$\frac{x}{15}-\frac{4}{63}\log(3x+2)+\frac{1}{175}\log(5x+1)$$

```
[Out] x/15 - (4*Log[2 + 3*x])/63 + Log[1 + 5*x]/175
```

Rubi [A] time = 0.0147503, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1386, 703, 632, 31}

$$\frac{x}{15}-\frac{4}{63}\log(3x+2)+\frac{1}{175}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Int[x/(13 + 2/x + 15*x),x]
```

```
[Out] x/15 - (4*Log[2 + 3*x])/63 + Log[1 + 5*x]/175
```

Rule 1386

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n_.) + (b_.)*(x_)^(mn_))^(p_.), x_Symbol
] :> Int[x^(m - n*p)*(b + a*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n}
, x] && EqQ[mn, -n] && IntegerQ[p] && PosQ[n]
```

Rule 703

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^
(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
```

```
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x}{13+\frac{2}{x}+15x}\,dx &= \int \frac{x^2}{2+13x+15x^2}\,dx \\
&= \frac{x}{15}+\frac{1}{15}\int \frac{-2-13x}{2+13x+15x^2}\,dx \\
&= \frac{x}{15}+\frac{3}{35}\int \frac{1}{3+15x}\,dx-\frac{20}{21}\int \frac{1}{10+15x}\,dx \\
&= \frac{x}{15}-\frac{4}{63}\log(2+3x)+\frac{1}{175}\log(1+5x)
\end{aligned}
$$

Mathematica [A] time = 0.0044611, size = 26, normalized size = 1.

$$\frac{x}{15}-\frac{4}{63}\log(3x+2)+\frac{1}{175}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(13 + 2/x + 15*x),x]
```

```
[Out] x/15 - (4*Log[2 + 3*x])/63 + Log[1 + 5*x]/175
```

Maple [A] time = 0.007, size = 21, normalized size = 0.8

$$\frac{x}{15}-\frac{4\ln(2+3x)}{63}+\frac{\ln(1+5x)}{175}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(13+2/x+15*x),x)
```

```
[Out] 1/15*x-4/63*ln(2+3*x)+1/175*ln(1+5*x)
```

Maxima [A] time = 1.04901, size = 27, normalized size = 1.04

$$\frac{1}{15}\,x + \frac{1}{175}\,\log\left(5\,x + 1\right) - \frac{4}{63}\,\log\left(3\,x + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(13+2/x+15*x),x, algorithm="maxima")
```

```
[Out] 1/15*x + 1/175*log(5*x + 1) - 4/63*log(3*x + 2)
```

Fricas [A] time = 1.14777, size = 66, normalized size = 2.54

$$\frac{1}{15}\,x + \frac{1}{175}\,\log\left(5\,x + 1\right) - \frac{4}{63}\,\log\left(3\,x + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(13+2/x+15*x),x, algorithm="fricas")
```

```
[Out] 1/15*x + 1/175*log(5*x + 1) - 4/63*log(3*x + 2)
```

Sympy [A] time = 0.105634, size = 20, normalized size = 0.77

$$\frac{x}{15} + \frac{\log\left(x + \frac{1}{5}\right)}{175} - \frac{4\log\left(x + \frac{2}{3}\right)}{63}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(13+2/x+15*x),x)
```

```
[Out] x/15 + log(x + 1/5)/175 - 4*log(x + 2/3)/63
```

Giac [A] time = 1.28539, size = 30, normalized size = 1.15

$$\frac{1}{15}\,x + \frac{1}{175}\,\log\left(|5\,x + 1|\right) - \frac{4}{63}\,\log\left(|3\,x + 2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(13+2/x+15*x),x, algorithm="giac")
```

```
[Out] 1/15*x + 1/175*log(abs(5*x + 1)) - 4/63*log(abs(3*x + 2))
```

3.381 $\int \frac{1}{13+\frac{2}{x}+15x} \, dx$

Optimal. Leaf size=21

$$\frac{2}{21}\log(3x+2)-\frac{1}{35}\log(5x+1)$$

```
[Out] (2*Log[2 + 3*x])/21 - Log[1 + 5*x]/35
```

Rubi [A] time = 0.0073871, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1350, 632, 31}

$$\frac{2}{21}\log(3x+2)-\frac{1}{35}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(13 + 2/x + 15*x)^(-1),x]
```

```
[Out] (2*Log[2 + 3*x])/21 - Log[1 + 5*x]/35
```

Rule 1350

```
Int[((a_) + (c_.)*(x_)^(n_.) + (b_.)*(x_)^(mn_))^(p_.), x_Symbol] :> Int[(b
 + a*x^n + c*x^(2*n))^p/x^(n*p), x] /; FreeQ[{a, b, c, n}, x] && EqQ[mn, -n
] && IntegerQ[p] && PosQ[n]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{13+\frac{2}{x}+15x}\,dx &= \int \frac{x}{2+13x+15x^2}\,dx \\
&= -\left(\frac{3}{7}\int \frac{1}{3+15x}\,dx\right)+\frac{10}{7}\int \frac{1}{10+15x}\,dx \\
&= \frac{2}{21}\log(2+3x)-\frac{1}{35}\log(1+5x)
\end{aligned}
$$

Mathematica [A] time = 0.0029758, size = 21, normalized size = 1.

$$\frac{2}{21}\log(3x+2)-\frac{1}{35}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(13 + 2/x + 15*x)^(-1),x]
```

```
[Out] (2*Log[2 + 3*x])/21 - Log[1 + 5*x]/35
```

Maple [A] time = 0.006, size = 18, normalized size = 0.9

$$\frac{2\ln(2+3x)}{21}-\frac{\ln(1+5x)}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(13+2/x+15*x),x)
```

```
[Out] 2/21*ln(2+3*x)-1/35*ln(1+5*x)
```

Maxima [A] time = 0.998803, size = 23, normalized size = 1.1

$$-\frac{1}{35}\log(5x+1)+\frac{2}{21}\log(3x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(13+2/x+15*x),x, algorithm="maxima")
```

```
[Out] -1/35*log(5*x + 1) + 2/21*log(3*x + 2)
```

Fricas [A] time = 1.25802, size = 54, normalized size = 2.57

$$-\frac{1}{35}\log\left(5x+1\right)+\frac{2}{21}\log\left(3x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(13+2/x+15*x),x, algorithm="fricas")
```

```
[Out] -1/35*log(5*x + 1) + 2/21*log(3*x + 2)
```

Sympy [A] time = 0.098802, size = 17, normalized size = 0.81

$$-\frac{\log\left(x+\frac{1}{5}\right)}{35}+\frac{2\log\left(x+\frac{2}{3}\right)}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(13+2/x+15*x),x)
```

```
[Out] -log(x + 1/5)/35 + 2*log(x + 2/3)/21
```

Giac [A] time = 1.15851, size = 26, normalized size = 1.24

$$-\frac{1}{35}\log\left(|5x+1|\right)+\frac{2}{21}\log\left(|3x+2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(13+2/x+15*x),x, algorithm="giac")
```

```
[Out] -1/35*log(abs(5*x + 1)) + 2/21*log(abs(3*x + 2))
```

3.382 $\int \frac{1}{x\left(13+\frac{2}{x}+15x\right)}\,dx$

Optimal. Leaf size=21

$$\frac{1}{7}\log(5x+1)-\frac{1}{7}\log(3x+2)$$

```
[Out] -Log[2 + 3*x]/7 + Log[1 + 5*x]/7
```

Rubi [A] time = 0.0104293, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1386, 616, 31}

$$\frac{1}{7}\log(5x+1)-\frac{1}{7}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(13 + 2/x + 15*x)),x]
```

```
[Out] -Log[2 + 3*x]/7 + Log[1 + 5*x]/7
```

Rule 1386

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n_.) + (b_.)*(x_)^(mn_))^(p_.), x_Symbol
] :> Int[x^(m - n*p)*(b + a*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n}
, x] && EqQ[mn, -n] && IntegerQ[p] && PosQ[n]
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{x\left(13+\frac{2}{x}+15x\right)}\,dx = \int \frac{1}{2+13x+15x^2}\,dx$$
$$= \frac{15}{7}\int \frac{1}{3+15x}\,dx - \frac{15}{7}\int \frac{1}{10+15x}\,dx$$
$$= -\frac{1}{7}\log(2+3x) + \frac{1}{7}\log(1+5x)$$

Mathematica [A] time = 0.0026965, size = 21, normalized size = 1.

$$\frac{1}{7}\log(5x+1) - \frac{1}{7}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(13 + 2/x + 15*x)),x]
```

```
[Out] -Log[2 + 3*x]/7 + Log[1 + 5*x]/7
```

Maple [A] time = 0.004, size = 18, normalized size = 0.9

$$-\frac{\ln(2+3x)}{7} + \frac{\ln(1+5x)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(13+2/x+15*x),x)
```

```
[Out] -1/7*ln(2+3*x)+1/7*ln(1+5*x)
```

Maxima [A] time = 1.00946, size = 23, normalized size = 1.1

$$\frac{1}{7}\log(5x+1) - \frac{1}{7}\log(3x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(13+2/x+15*x),x, algorithm="maxima")
```

```
[Out] 1/7*log(5*x + 1) - 1/7*log(3*x + 2)
```

Fricas [A] time = 1.25171, size = 50, normalized size = 2.38

$$\frac{1}{7}\log\left(5x+1\right)-\frac{1}{7}\log\left(3x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(13+2/x+15*x),x, algorithm="fricas")
```

```
[Out] 1/7*log(5*x + 1) - 1/7*log(3*x + 2)
```

Sympy [A] time = 0.095938, size = 15, normalized size = 0.71

$$\frac{\log\left(x+\frac{1}{5}\right)}{7}-\frac{\log\left(x+\frac{2}{3}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(13+2/x+15*x),x)
```

```
[Out] log(x + 1/5)/7 - log(x + 2/3)/7
```

Giac [A] time = 1.1028, size = 26, normalized size = 1.24

$$\frac{1}{7}\log\left(|5x+1|\right)-\frac{1}{7}\log\left(|3x+2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(13+2/x+15*x),x, algorithm="giac")
```

```
[Out] 1/7*log(abs(5*x + 1)) - 1/7*log(abs(3*x + 2))
```

3.383 $\int \frac{1}{x^2\left(13+\frac{2}{x}+15x\right)}\,dx$

Optimal. Leaf size=27

$$\frac{\log(x)}{2}+\frac{3}{14}\log(3x+2)-\frac{5}{7}\log(5x+1)$$

```
[Out] Log[x]/2 + (3*Log[2 + 3*x])/14 - (5*Log[1 + 5*x])/7
```

Rubi [A] time = 0.0157206, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1386, 705, 29, 632, 31}

$$\frac{\log(x)}{2}+\frac{3}{14}\log(3x+2)-\frac{5}{7}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(13 + 2/x + 15*x)),x]
```

```
[Out] Log[x]/2 + (3*Log[2 + 3*x])/14 - (5*Log[1 + 5*x])/7
```

Rule 1386

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n_.) + (b_.)*(x_)^(mn_))^(p_.), x_Symbol
] :> Int[x^(m - n*p)*(b + a*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n}
, x] && EqQ[mn, -n] && IntegerQ[p] && PosQ[n]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2\left(13+\frac{2}{x}+15x\right)}\,dx &= \int \frac{1}{x\left(2+13x+15x^2\right)}\,dx\\
&= \frac{1}{2}\int \frac{1}{x}\,dx+\frac{1}{2}\int \frac{-13-15x}{2+13x+15x^2}\,dx\\
&= \frac{\log(x)}{2}+\frac{45}{14}\int \frac{1}{10+15x}\,dx-\frac{75}{7}\int \frac{1}{3+15x}\,dx\\
&= \frac{\log(x)}{2}+\frac{3}{14}\log(2+3x)-\frac{5}{7}\log(1+5x)
\end{aligned}$$

Mathematica [A] time = 0.0044054, size = 27, normalized size = 1.

$$\frac{\log(x)}{2}+\frac{3}{14}\log(3x+2)-\frac{5}{7}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(13 + 2/x + 15*x)),x]
```

```
[Out] Log[x]/2 + (3*Log[2 + 3*x])/14 - (5*Log[1 + 5*x])/7
```

Maple [A] time = 0.006, size = 22, normalized size = 0.8

$$\frac{\ln(x)}{2}+\frac{3\,\ln(2+3\,x)}{14}-\frac{5\,\ln(1+5\,x)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(13+2/x+15*x),x)
```

```
[Out] 1/2*ln(x)+3/14*ln(2+3*x)-5/7*ln(1+5*x)
```

Maxima [A] time = 0.997671, size = 28, normalized size = 1.04

$$-\frac{5}{7}\log(5x+1)+\frac{3}{14}\log(3x+2)+\frac{1}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(13+2/x+15*x),x, algorithm="maxima")
```

```
[Out] -5/7*log(5*x + 1) + 3/14*log(3*x + 2) + 1/2*log(x)
```

Fricas [A] time = 1.2219, size = 70, normalized size = 2.59

$$-\frac{5}{7}\log(5x+1)+\frac{3}{14}\log(3x+2)+\frac{1}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(13+2/x+15*x),x, algorithm="fricas")
```

```
[Out] -5/7*log(5*x + 1) + 3/14*log(3*x + 2) + 1/2*log(x)
```

Sympy [A] time = 0.127447, size = 24, normalized size = 0.89

$$\frac{\log(x)}{2}-\frac{5\log\left(x+\frac{1}{5}\right)}{7}+\frac{3\log\left(x+\frac{2}{3}\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(13+2/x+15*x),x)
```

```
[Out] log(x)/2 - 5*log(x + 1/5)/7 + 3*log(x + 2/3)/14
```

Giac [A] time = 1.14009, size = 32, normalized size = 1.19

$$-\frac{5}{7}\,\log\left(|5\,x+1|\right)+\frac{3}{14}\,\log\left(|3\,x+2|\right)+\frac{1}{2}\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(13+2/x+15*x),x, algorithm="giac")
```

```
[Out] -5/7*log(abs(5*x + 1)) + 3/14*log(abs(3*x + 2)) + 1/2*log(abs(x))
```

3.384 $\int \frac{1}{x^3\left(13+\frac{2}{x}+15x\right)}\,dx$

Optimal. Leaf size=34

$$-\frac{1}{2x}-\frac{13\log(x)}{4}-\frac{9}{28}\log(3x+2)+\frac{25}{7}\log(5x+1)$$

```
[Out] -1/(2*x) - (13*Log[x])/4 - (9*Log[2 + 3*x])/28 + (25*Log[1 + 5*x])/7
```

Rubi [A] time = 0.0297196, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1386, 709, 800}

$$-\frac{1}{2x}-\frac{13\log(x)}{4}-\frac{9}{28}\log(3x+2)+\frac{25}{7}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(13 + 2/x + 15*x)),x]
```

```
[Out] -1/(2*x) - (13*Log[x])/4 - (9*Log[2 + 3*x])/28 + (25*Log[1 + 5*x])/7
```

Rule 1386

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n_.) + (b_.)*(x_)^(mn_))^(p_.), x_Symbol
] :> Int[x^(m - n*p)*(b + a*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n}
, x] && EqQ[mn, -n] && IntegerQ[p] && PosQ[n]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
```

```
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\left(13+\frac{2}{x}+15x\right)}\,dx &= \int \frac{1}{x^2\left(2+13x+15x^2\right)}\,dx\\ &= -\frac{1}{2x}+\frac{1}{2}\int \frac{-13-15x}{x\left(2+13x+15x^2\right)}\,dx\\ &= -\frac{1}{2x}+\frac{1}{2}\int \left(-\frac{13}{2x}-\frac{27}{14(2+3x)}+\frac{250}{7(1+5x)}\right)dx\\ &= -\frac{1}{2x}-\frac{13\log(x)}{4}-\frac{9}{28}\log(2+3x)+\frac{25}{7}\log(1+5x)\end{aligned}$$

Mathematica [A] time = 0.0040524, size = 34, normalized size = 1.

$$-\frac{1}{2x}-\frac{13\log(x)}{4}-\frac{9}{28}\log(3x+2)+\frac{25}{7}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(13 + 2/x + 15*x)),x]
```

```
[Out] -1/(2*x) - (13*Log[x])/4 - (9*Log[2 + 3*x])/28 + (25*Log[1 + 5*x])/7
```

Maple [A] time = 0.006, size = 27, normalized size = 0.8

$$-\frac{1}{2\,x}-\frac{13\,\ln(x)}{4}-\frac{9\,\ln(2+3\,x)}{28}+\frac{25\,\ln(1+5\,x)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(13+2/x+15*x),x)
```

```
[Out] -1/2/x-13/4*ln(x)-9/28*ln(2+3*x)+25/7*ln(1+5*x)
```

Maxima [A] time = 0.99963, size = 35, normalized size = 1.03

$$-\frac{1}{2\,x}+\frac{25}{7}\,\log\left(5\,x+1\right)-\frac{9}{28}\,\log\left(3\,x+2\right)-\frac{13}{4}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(13+2/x+15*x),x, algorithm="maxima")
```

```
[Out] -1/2/x + 25/7*log(5*x + 1) - 9/28*log(3*x + 2) - 13/4*log(x)
```

Fricas [A] time = 1.2629, size = 90, normalized size = 2.65

$$\frac{100\,x\log\left(5\,x+1\right)-9\,x\log\left(3\,x+2\right)-91\,x\log\left(x\right)-14}{28\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(13+2/x+15*x),x, algorithm="fricas")
```

```
[Out] 1/28*(100*x*log(5*x + 1) - 9*x*log(3*x + 2) - 91*x*log(x) - 14)/x
```

Sympy [A] time = 0.14262, size = 31, normalized size = 0.91

$$-\frac{13\log\left(x\right)}{4}+\frac{25\log\left(x+\frac{1}{5}\right)}{7}-\frac{9\log\left(x+\frac{2}{3}\right)}{28}-\frac{1}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(13+2/x+15*x),x)
```

```
[Out] -13*log(x)/4 + 25*log(x + 1/5)/7 - 9*log(x + 2/3)/28 - 1/(2*x)
```

Giac [A] time = 1.12182, size = 39, normalized size = 1.15

$$-\frac{1}{2\,x}+\frac{25}{7}\,\log\left(|5\,x+1|\right)-\frac{9}{28}\,\log\left(|3\,x+2|\right)-\frac{13}{4}\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(13+2/x+15*x),x, algorithm="giac")
```

```
[Out] -1/2/x + 25/7*log(abs(5*x + 1)) - 9/28*log(abs(3*x + 2)) - 13/4*log(abs(x))
```

3.385 $\int \frac{1}{x^4\left(13+\frac{2}{x}+15x\right)} dx$

Optimal. Leaf size=41

$$-\frac{1}{4x^2}+\frac{13}{4x}+\frac{139\log(x)}{8}+\frac{27}{56}\log(3x+2)-\frac{125}{7}\log(5x+1)$$

```
[Out] -1/(4*x^2) + 13/(4*x) + (139*Log[x])/8 + (27*Log[2 + 3*x])/56 - (125*Log[1
+ 5*x])/7
```

Rubi [A] time = 0.0338796, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1386, 709, 800}

$$-\frac{1}{4x^2}+\frac{13}{4x}+\frac{139\log(x)}{8}+\frac{27}{56}\log(3x+2)-\frac{125}{7}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(13 + 2/x + 15*x)),x]
```

```
[Out] -1/(4*x^2) + 13/(4*x) + (139*Log[x])/8 + (27*Log[2 + 3*x])/56 - (125*Log[1
+ 5*x])/7
```

Rule 1386

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n_.) + (b_.)*(x_)^(mn_))^(p_.), x_Symbol
] :> Int[x^(m - n*p)*(b + a*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n}
, x] && EqQ[mn, -n] && IntegerQ[p] && PosQ[n]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^4\left(13+\frac{2}{x}+15x\right)}\,dx &= \int \frac{1}{x^3\left(2+13x+15x^2\right)}\,dx \\
&= -\frac{1}{4x^2}+\frac{1}{2}\int \frac{-13-15x}{x^2\left(2+13x+15x^2\right)}\,dx \\
&= -\frac{1}{4x^2}+\frac{1}{2}\int \left(-\frac{13}{2x^2}+\frac{139}{4x}+\frac{81}{28(2+3x)}-\frac{1250}{7(1+5x)}\right)dx \\
&= -\frac{1}{4x^2}+\frac{13}{4x}+\frac{139\log(x)}{8}+\frac{27}{56}\log(2+3x)-\frac{125}{7}\log(1+5x)
\end{aligned}
$$

Mathematica [A] time = 0.0045968, size = 41, normalized size = 1.

$$
-\frac{1}{4x^2}+\frac{13}{4x}+\frac{139\log(x)}{8}+\frac{27}{56}\log(3x+2)-\frac{125}{7}\log(5x+1)
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(13 + 2/x + 15*x)),x]
```

```
[Out] -1/(4*x^2) + 13/(4*x) + (139*Log[x])/8 + (27*Log[2 + 3*x])/56 - (125*Log[1
+ 5*x])/7
```

Maple [A] time = 0.008, size = 32, normalized size = 0.8

$$
-\frac{1}{4\,x^2}+\frac{13}{4\,x}+\frac{139\,\ln(x)}{8}+\frac{27\,\ln(2+3\,x)}{56}-\frac{125\,\ln(1+5\,x)}{7}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(13+2/x+15*x),x)
```

```
[Out] -1/4/x^2+13/4/x+139/8*ln(x)+27/56*ln(2+3*x)-125/7*ln(1+5*x)
```

Maxima [A] time = 0.9811, size = 42, normalized size = 1.02

$$\frac{13\,x-1}{4\,x^2}-\frac{125}{7}\,\log\left(5\,x+1\right)+\frac{27}{56}\,\log\left(3\,x+2\right)+\frac{139}{8}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(13+2/x+15*x),x, algorithm="maxima")
```

```
[Out] 1/4*(13*x - 1)/x^2 - 125/7*log(5*x + 1) + 27/56*log(3*x + 2) + 139/8*log(x)
```

Fricas [A] time = 1.25822, size = 117, normalized size = 2.85

$$-\frac{1000\,x^2\log\left(5\,x+1\right)-27\,x^2\log\left(3\,x+2\right)-973\,x^2\log\left(x\right)-182\,x+14}{56\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(13+2/x+15*x),x, algorithm="fricas")
```

```
[Out] -1/56*(1000*x^2*log(5*x + 1) - 27*x^2*log(3*x + 2) - 973*x^2*log(x) - 182*x
 + 14)/x^2
```

Sympy [A] time = 0.153319, size = 36, normalized size = 0.88

$$\frac{139\log\left(x\right)}{8}-\frac{125\log\left(x+\frac{1}{5}\right)}{7}+\frac{27\log\left(x+\frac{2}{3}\right)}{56}+\frac{13x-1}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(13+2/x+15*x),x)
```

```
[Out] 139*log(x)/8 - 125*log(x + 1/5)/7 + 27*log(x + 2/3)/56 + (13*x - 1)/(4*x**2
)
```

Giac [A] time = 1.14518, size = 46, normalized size = 1.12

$$\frac{13\,x-1}{4\,x^2}-\frac{125}{7}\,\log\left(|5\,x+1|\right)+\frac{27}{56}\,\log\left(|3\,x+2|\right)+\frac{139}{8}\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(13+2/x+15*x),x, algorithm="giac")
```

```
[Out] 1/4*(13*x - 1)/x^2 - 125/7*log(abs(5*x + 1)) + 27/56*log(abs(3*x + 2)) + 13
9/8*log(abs(x))
```

3.386 $\int \frac{1}{x^5\left(13+\frac{2}{x}+15x\right)}\,dx$

Optimal. Leaf size=48

$$\frac{13}{8x^2}-\frac{1}{6x^3}-\frac{139}{8x}-\frac{1417\log(x)}{16}-\frac{81}{112}\log(3x+2)+\frac{625}{7}\log(5x+1)$$

```
[Out] -1/(6*x^3) + 13/(8*x^2) - 139/(8*x) - (1417*Log[x])/16 - (81*Log[2 + 3*x])/
112 + (625*Log[1 + 5*x])/7
```

Rubi [A] time = 0.040875, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1386, 709, 800}

$$\frac{13}{8x^2}-\frac{1}{6x^3}-\frac{139}{8x}-\frac{1417\log(x)}{16}-\frac{81}{112}\log(3x+2)+\frac{625}{7}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(13 + 2/x + 15*x)),x]
```

```
[Out] -1/(6*x^3) + 13/(8*x^2) - 139/(8*x) - (1417*Log[x])/16 - (81*Log[2 + 3*x])/
112 + (625*Log[1 + 5*x])/7
```

Rule 1386

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n_.) + (b_.)*(x_)^(mn_))^(p_.), x_Symbol
] :> Int[x^(m - n*p)*(b + a*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n}
, x] && EqQ[mn, -n] && IntegerQ[p] && PosQ[n]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^5\left(13+\frac{2}{x}+15x\right)}\,dx &= \int \frac{1}{x^4\left(2+13x+15x^2\right)}\,dx\\
&= -\frac{1}{6x^3}+\frac{1}{2}\int \frac{-13-15x}{x^3\left(2+13x+15x^2\right)}\,dx\\
&= -\frac{1}{6x^3}+\frac{1}{2}\int \left(-\frac{13}{2x^3}+\frac{139}{4x^2}-\frac{1417}{8x}-\frac{243}{56(2+3x)}+\frac{6250}{7(1+5x)}\right)dx\\
&= -\frac{1}{6x^3}+\frac{13}{8x^2}-\frac{139}{8x}-\frac{1417\log(x)}{16}-\frac{81}{112}\log(2+3x)+\frac{625}{7}\log(1+5x)
\end{aligned}$$

Mathematica [A] time = 0.0046693, size = 48, normalized size = 1.

$$\frac{13}{8x^2}-\frac{1}{6x^3}-\frac{139}{8x}-\frac{1417\log(x)}{16}-\frac{81}{112}\log(3x+2)+\frac{625}{7}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(13 + 2/x + 15*x)),x]
```

```
[Out] -1/(6*x^3) + 13/(8*x^2) - 139/(8*x) - (1417*Log[x])/16 - (81*Log[2 + 3*x])/
112 + (625*Log[1 + 5*x])/7
```

Maple [A] time = 0.007, size = 37, normalized size = 0.8

$$-\frac{1}{6\,x^3}+\frac{13}{8\,x^2}-\frac{139}{8\,x}-\frac{1417\,\ln(x)}{16}-\frac{81\,\ln(2+3\,x)}{112}+\frac{625\,\ln(1+5\,x)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(13+2/x+15*x),x)
```

```
[Out] -1/6/x^3+13/8/x^2-139/8/x-1417/16*ln(x)-81/112*ln(2+3*x)+625/7*ln(1+5*x)
```

Maxima [A] time = 1.00942, size = 49, normalized size = 1.02

$$-\frac{417\,x^2-39\,x+4}{24\,x^3}+\frac{625}{7}\,\log\left(5\,x+1\right)-\frac{81}{112}\,\log\left(3\,x+2\right)-\frac{1417}{16}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(13+2/x+15*x),x, algorithm="maxima")
```

```
[Out] -1/24*(417*x^2 - 39*x + 4)/x^3 + 625/7*log(5*x + 1) - 81/112*log(3*x + 2) -
 1417/16*log(x)
```

Fricas [A] time = 1.30015, size = 138, normalized size = 2.88

$$\frac{30000\,x^3\log\left(5\,x+1\right)-243\,x^3\log\left(3\,x+2\right)-29757\,x^3\log\left(x\right)-5838\,x^2+546\,x-56}{336\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(13+2/x+15*x),x, algorithm="fricas")
```

```
[Out] 1/336*(30000*x^3*log(5*x + 1) - 243*x^3*log(3*x + 2) - 29757*x^3*log(x) - 5
838*x^2 + 546*x - 56)/x^3
```

Sympy [A] time = 0.16058, size = 41, normalized size = 0.85

$$-\frac{1417\log\left(x\right)}{16}+\frac{625\log\left(x+\frac{1}{5}\right)}{7}-\frac{81\log\left(x+\frac{2}{3}\right)}{112}-\frac{417x^2-39x+4}{24x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(13+2/x+15*x),x)
```

```
[Out] -1417*log(x)/16 + 625*log(x + 1/5)/7 - 81*log(x + 2/3)/112 - (417*x**2 - 39
*x + 4)/(24*x**3)
```

Giac [A] time = 1.24626, size = 53, normalized size = 1.1

$$-\frac{417\,x^2-39\,x+4}{24\,x^3}+\frac{625}{7}\,\log\left(|5\,x+1|\right)-\frac{81}{112}\,\log\left(|3\,x+2|\right)-\frac{1417}{16}\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(13+2/x+15*x),x, algorithm="giac")
```

```
[Out] -1/24*(417*x^2 - 39*x + 4)/x^3 + 625/7*log(abs(5*x + 1)) - 81/112*log(abs(3
*x + 2)) - 1417/16*log(abs(x))
```

3.387 $\int \frac{x^2}{2-(1+x^2)^4}\,dx$

Optimal. Leaf size=157

$$\frac{i\sqrt{1-i\sqrt[4]{2}}\tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt[4]{2}}}\right)}{4\,2^{3/4}}-\frac{i\sqrt{1+i\sqrt[4]{2}}\tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt[4]{2}}}\right)}{4\,2^{3/4}}-\frac{\sqrt{1+\sqrt[4]{2}}\tan^{-1}\left(\frac{x}{\sqrt{1+\sqrt[4]{2}}}\right)}{4\,2^{3/4}}+\frac{\sqrt{\sqrt[4]{2}-1}\tanh^{-1}\left(\frac{x}{\sqrt{\sqrt[4]{2}-1}}\right)}{4\,2^{3/4}}$$

```
[Out] ((I/4)*Sqrt[1 - I*2^(1/4)]*ArcTan[x/Sqrt[1 - I*2^(1/4)]])/2^(3/4) - ((I/4)*
Sqrt[1 + I*2^(1/4)]*ArcTan[x/Sqrt[1 + I*2^(1/4)]])/2^(3/4) - (Sqrt[1 + 2^(1
/4)]*ArcTan[x/Sqrt[1 + 2^(1/4)]])/(4*2^(3/4)) + (Sqrt[-1 + 2^(1/4)]*ArcTanh
[x/Sqrt[-1 + 2^(1/4)]])/(4*2^(3/4))
```

Rubi [A] time = 0.168297, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {6740, 206, 203, 1972, 205}

$$\frac{i\sqrt{1-i\sqrt[4]{2}}\tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt[4]{2}}}\right)}{4\,2^{3/4}}-\frac{i\sqrt{1+i\sqrt[4]{2}}\tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt[4]{2}}}\right)}{4\,2^{3/4}}-\frac{\sqrt{1+\sqrt[4]{2}}\tan^{-1}\left(\frac{x}{\sqrt{1+\sqrt[4]{2}}}\right)}{4\,2^{3/4}}+\frac{\sqrt{\sqrt[4]{2}-1}\tanh^{-1}\left(\frac{x}{\sqrt{\sqrt[4]{2}-1}}\right)}{4\,2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(2 - (1 + x^2)^4),x]
```

```
[Out] ((I/4)*Sqrt[1 - I*2^(1/4)]*ArcTan[x/Sqrt[1 - I*2^(1/4)]])/2^(3/4) - ((I/4)*
Sqrt[1 + I*2^(1/4)]*ArcTan[x/Sqrt[1 + I*2^(1/4)]])/2^(3/4) - (Sqrt[1 + 2^(1
/4)]*ArcTan[x/Sqrt[1 + 2^(1/4)]])/(4*2^(3/4)) + (Sqrt[-1 + 2^(1/4)]*ArcTanh
[x/Sqrt[-1 + 2^(1/4)]])/(4*2^(3/4))
```

Rule 6740

```
Int[(v_)/((a_) + (b_.)*(u_)^(n_.)), x_Symbol] :> Int[ExpandIntegrand[Polyno
mialInSubst[v, u, x]/(a + b*x^n), x] /. x -> u, x] /; FreeQ[{a, b}, x] && I
GtQ[n, 0] && PolynomialInQ[v, u, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1972

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && B
inomialQ[u, x] &&  !BinomialMatchQ[u, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^2}{2-\left(1+x^2\right)^4}\,dx = \int \left(\frac{-\sqrt[4]{2}+\sqrt{2}}{8\left(-1+\sqrt[4]{2}-x^2\right)}+\frac{-\sqrt[4]{2}-\sqrt{2}}{8\left(1+\sqrt[4]{2}+x^2\right)}+\frac{-\sqrt[4]{2}-i\sqrt{2}}{8\left(\sqrt[4]{2}-i\left(1+x^2\right)\right)}+\frac{-\sqrt[4]{2}+i\sqrt{2}}{8\left(\sqrt[4]{2}+i\left(1+x^2\right)\right)}\right)dx$$

$$= -\frac{\left(1-\sqrt[4]{2}\right)\int \frac{1}{-1+\sqrt[4]{2}-x^2}\,dx}{4\ 2^{3/4}}-\frac{\left(1-i\sqrt[4]{2}\right)\int \frac{1}{\sqrt[4]{2}+i\left(1+x^2\right)}\,dx}{4\ 2^{3/4}}-\frac{\left(1+i\sqrt[4]{2}\right)\int \frac{1}{\sqrt[4]{2}-i\left(1+x^2\right)}\,dx}{4\ 2^{3/4}}-\frac{\left(1+\sqrt[4]{2}\right)\int \cdots}{4\ 2^{3/4}}\cdots$$

$$= -\frac{\sqrt{1+\sqrt[4]{2}}\tan^{-1}\left(\frac{x}{\sqrt{1+\sqrt[4]{2}}}\right)}{4\ 2^{3/4}}+\frac{\sqrt{-1+\sqrt[4]{2}}\tanh^{-1}\left(\frac{x}{\sqrt{-1+\sqrt[4]{2}}}\right)}{4\ 2^{3/4}}-\frac{\left(1-i\sqrt[4]{2}\right)\int \frac{1}{i+\sqrt[4]{2}+ix^2}\,dx}{4\ 2^{3/4}}-\frac{\left(1+i\sqrt[4]{2}\right)\cdots}{\cdots}$$

$$= \frac{i\sqrt{1-i\sqrt[4]{2}}\tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt[4]{2}}}\right)}{4\ 2^{3/4}}-\frac{i\sqrt{1+i\sqrt[4]{2}}\tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt[4]{2}}}\right)}{4\ 2^{3/4}}-\frac{\sqrt{1+\sqrt[4]{2}}\tan^{-1}\left(\frac{x}{\sqrt{1+\sqrt[4]{2}}}\right)}{4\ 2^{3/4}}+\frac{\sqrt{-1+\sqrt[4]{2}}\cdots}{\cdots}$$

Mathematica [C] time = 0.0167993, size = 61, normalized size = 0.39

$$-\frac{1}{8}\text{RootSum}\left[\#1^8+4\#1^6+6\#1^4+4\#1^2-1\&,\frac{\#1\log(x-\#1)}{\#1^6+3\#1^4+3\#1^2+1}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(2 - (1 + x^2)^4),x]
```

```
[Out] -RootSum[-1 + 4*#1^2 + 6*#1^4 + 4*#1^6 + #1^8 & , (Log[x - #1]*#1)/(1 + 3*#
1^2 + 3*#1^4 + #1^6) & ]/8
```

Maple [C] time = 0.009, size = 54, normalized size = 0.3

$$-\frac{1}{8}\sum_{\_R=\mathrm{RootOf}\left(\_Z^8+4\_Z^6+6\_Z^4+4\_Z^2-1\right)}\frac{\_R^2\ln(x-\_R)}{\_R^7+3\_R^5+3\_R^3+\_R}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(2-(x^2+1)^4),x)
```

```
[Out] -1/8*sum(_R^2/(_R^7+3*_R^5+3*_R^3+_R)*ln(x-_R),_R=RootOf(_Z^8+4*_Z^6+6*_Z^4
+4*_Z^2-1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{x^2}{\left(x^2+1\right)^4-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(2-(x^2+1)^4),x, algorithm="maxima")
```

```
[Out] -integrate(x^2/((x^2 + 1)^4 - 2), x)
```

Fricas [B] time = 8.71275, size = 4887, normalized size = 31.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(2-(x^2+1)^4),x, algorithm="fricas")
```

```
[Out] -1/16*sqrt(2)*sqrt(1/2*sqrt(2) + sqrt(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2^
(3/4) + sqrt(2))*(2^(3/4) - sqrt(2)) - 3/16*(2^(3/4) - sqrt(2))^2 + 1))*log
```

```
(1/4*((sqrt(2)*(2^(3/4) + sqrt(2)) + sqrt(2))*(2^(3/4) - sqrt(2))^2 + sqrt(
2)*(2^(3/4) + sqrt(2))^2 - (sqrt(2)*(2^(3/4) + sqrt(2))^2 - 4*sqrt(2))*(2^(
3/4) - sqrt(2)) + 4*sqrt(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2^(3/4) + sqrt(
2))*(2^(3/4) - sqrt(2)) - 3/16*(2^(3/4) - sqrt(2))^2 + 1)*((sqrt(2)*(2^(3/4
) + sqrt(2)) + sqrt(2))*(2^(3/4) - sqrt(2)) - sqrt(2)*(2^(3/4) + sqrt(2)) -
 4*sqrt(2)) - 4*sqrt(2)*(2^(3/4) + sqrt(2)) + 4*sqrt(2))*sqrt(1/2*sqrt(2) +
 sqrt(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2^(3/4) + sqrt(2))*(2^(3/4) - sqrt
(2)) - 3/16*(2^(3/4) - sqrt(2))^2 + 1)) + 6*x) + 1/16*sqrt(2)*sqrt(1/2*sqrt
(2) + sqrt(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2^(3/4) + sqrt(2))*(2^(3/4) -
 sqrt(2)) - 3/16*(2^(3/4) - sqrt(2))^2 + 1))*log(-1/4*((sqrt(2)*(2^(3/4) +
sqrt(2)) + sqrt(2))*(2^(3/4) - sqrt(2))^2 + sqrt(2)*(2^(3/4) + sqrt(2))^2 -
 (sqrt(2)*(2^(3/4) + sqrt(2))^2 - 4*sqrt(2))*(2^(3/4) - sqrt(2)) + 4*sqrt(-
3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2^(3/4) + sqrt(2))*(2^(3/4) - sqrt(2)) -
3/16*(2^(3/4) - sqrt(2))^2 + 1)*((sqrt(2)*(2^(3/4) + sqrt(2)) + sqrt(2))*(2
^(3/4) - sqrt(2)) - sqrt(2)*(2^(3/4) + sqrt(2)) - 4*sqrt(2)) - 4*sqrt(2)*(2
^(3/4) + sqrt(2)) + 4*sqrt(2))*sqrt(1/2*sqrt(2) + sqrt(-3/16*(2^(3/4) + sqr
t(2))^2 + 1/8*(2^(3/4) + sqrt(2))*(2^(3/4) - sqrt(2)) - 3/16*(2^(3/4) - sqr
t(2))^2 + 1)) + 6*x) - 1/16*sqrt(2)*sqrt(1/2*sqrt(2) - sqrt(-3/16*(2^(3/4)
+ sqrt(2))^2 + 1/8*(2^(3/4) + sqrt(2))*(2^(3/4) - sqrt(2)) - 3/16*(2^(3/4)
- sqrt(2))^2 + 1))*log(1/4*((sqrt(2)*(2^(3/4) + sqrt(2)) + sqrt(2))*(2^(3/4
) - sqrt(2))^2 + sqrt(2)*(2^(3/4) + sqrt(2))^2 - (sqrt(2)*(2^(3/4) + sqrt(2
))^2 - 4*sqrt(2))*(2^(3/4) - sqrt(2)) - 4*sqrt(-3/16*(2^(3/4) + sqrt(2))^2
+ 1/8*(2^(3/4) + sqrt(2))*(2^(3/4) - sqrt(2)) - 3/16*(2^(3/4) - sqrt(2))^2
+ 1)*((sqrt(2)*(2^(3/4) + sqrt(2)) + sqrt(2))*(2^(3/4) - sqrt(2)) - sqrt(2)
*(2^(3/4) + sqrt(2)) - 4*sqrt(2)) - 4*sqrt(2)*(2^(3/4) + sqrt(2)) + 4*sqrt(
2))*sqrt(1/2*sqrt(2) - sqrt(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2^(3/4) + sq
rt(2))*(2^(3/4) - sqrt(2)) - 3/16*(2^(3/4) - sqrt(2))^2 + 1)) + 6*x) + 1/16
*sqrt(2)*sqrt(1/2*sqrt(2) - sqrt(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2^(3/4)
 + sqrt(2))*(2^(3/4) - sqrt(2)) - 3/16*(2^(3/4) - sqrt(2))^2 + 1))*log(-1/4
*((sqrt(2)*(2^(3/4) + sqrt(2)) + sqrt(2))*(2^(3/4) - sqrt(2))^2 + sqrt(2)*(
2^(3/4) + sqrt(2))^2 - (sqrt(2)*(2^(3/4) + sqrt(2))^2 - 4*sqrt(2))*(2^(3/4)
 - sqrt(2)) - 4*sqrt(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2^(3/4) + sqrt(2))*
(2^(3/4) - sqrt(2)) - 3/16*(2^(3/4) - sqrt(2))^2 + 1)*((sqrt(2)*(2^(3/4) +
sqrt(2)) + sqrt(2))*(2^(3/4) - sqrt(2)) - sqrt(2)*(2^(3/4) + sqrt(2)) - 4*s
qrt(2)) - 4*sqrt(2)*(2^(3/4) + sqrt(2)) + 4*sqrt(2))*sqrt(1/2*sqrt(2) - sqr
t(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2^(3/4) + sqrt(2))*(2^(3/4) - sqrt(2))
 - 3/16*(2^(3/4) - sqrt(2))^2 + 1)) + 6*x) + 1/16*sqrt(2^(3/4) - sqrt(2))*l
og(1/4*((2^(3/4) + sqrt(2))^3 + (2^(3/4) + sqrt(2) + 1)*(2^(3/4) - sqrt(2))
^2 - ((2^(3/4) + sqrt(2))^2 - 4)*(2^(3/4) - sqrt(2)) - 4*2^(3/4) - 4*sqrt(2
) - 6)*sqrt(2^(3/4) - sqrt(2)) + 3*x) - 1/16*sqrt(2^(3/4) - sqrt(2))*log(-1
/4*((2^(3/4) + sqrt(2))^3 + (2^(3/4) + sqrt(2) + 1)*(2^(3/4) - sqrt(2))^2 -
 ((2^(3/4) + sqrt(2))^2 - 4)*(2^(3/4) - sqrt(2)) - 4*2^(3/4) - 4*sqrt(2) -
6)*sqrt(2^(3/4) - sqrt(2)) + 3*x) - sqrt(-1/256*2^(3/4) - 1/256*sqrt(2))*lo
g(4*((2^(3/4) + sqrt(2))^3 - (2^(3/4) + sqrt(2))^2 - 10)*sqrt(-1/256*2^(3/4
) - 1/256*sqrt(2)) + 3*x) + sqrt(-1/256*2^(3/4) - 1/256*sqrt(2))*log(-4*((2
```

```
^(3/4) + sqrt(2))^3 - (2^(3/4) + sqrt(2))^2 - 10)*sqrt(-1/256*2^(3/4) - 1/2
56*sqrt(2)) + 3*x)
```

Sympy [A] time = 0.206149, size = 41, normalized size = 0.26

$$-\operatorname{RootSum}\left(1073741824t^8-65536t^4+1024t^2-1,\left(t\mapsto t\log\left(-\frac{67108864t^7}{3}-\frac{262144t^5}{3}-\frac{40t}{3}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(2-(x**2+1)**4),x)
```

```
[Out] -RootSum(1073741824*_t**8 - 65536*_t**4 + 1024*_t**2 - 1, Lambda(_t, _t*log
(-67108864*_t**7/3 - 262144*_t**5/3 - 40*_t/3 + x)))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2}{\left(x^2+1\right)^4-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(2-(x^2+1)^4),x, algorithm="giac")
```

```
[Out] integrate(-x^2/((x^2 + 1)^4 - 2), x)
```

3.388 $\int \frac{x^2}{2-\left(1-x^2\right)^4} \, dx$

Optimal. Leaf size=157

$$-\frac{\sqrt{\sqrt[4]{2}-1}\tan^{-1}\left(\frac{x}{\sqrt{\sqrt[4]{2}-1}}\right)}{4\ 2^{3/4}}-\frac{i\sqrt{1-i\sqrt[4]{2}}\tanh^{-1}\left(\frac{x}{\sqrt{1-i\sqrt[4]{2}}}\right)}{4\ 2^{3/4}}+\frac{i\sqrt{1+i\sqrt[4]{2}}\tanh^{-1}\left(\frac{x}{\sqrt{1+i\sqrt[4]{2}}}\right)}{4\ 2^{3/4}}+\frac{\sqrt{1+\sqrt[4]{2}}\tanh^{-1}\left(\frac{x}{\sqrt{1+\sqrt[4]{2}}}\right)}{4\ 2^{3/4}}$$

```
[Out] -(Sqrt[-1 + 2^(1/4)]*ArcTan[x/Sqrt[-1 + 2^(1/4)]])/(4*2^(3/4)) - ((I/4)*Sqr
t[1 - I*2^(1/4)]*ArcTanh[x/Sqrt[1 - I*2^(1/4)]])/2^(3/4) + ((I/4)*Sqrt[1 +
I*2^(1/4)]*ArcTanh[x/Sqrt[1 + I*2^(1/4)]])/2^(3/4) + (Sqrt[1 + 2^(1/4)]*Arc
Tanh[x/Sqrt[1 + 2^(1/4)]])/(4*2^(3/4))
```

Rubi [A] time = 0.111525, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {6740, 206, 203, 1972, 208}

$$-\frac{\sqrt{\sqrt[4]{2}-1}\tan^{-1}\left(\frac{x}{\sqrt{\sqrt[4]{2}-1}}\right)}{4\ 2^{3/4}}-\frac{i\sqrt{1-i\sqrt[4]{2}}\tanh^{-1}\left(\frac{x}{\sqrt{1-i\sqrt[4]{2}}}\right)}{4\ 2^{3/4}}+\frac{i\sqrt{1+i\sqrt[4]{2}}\tanh^{-1}\left(\frac{x}{\sqrt{1+i\sqrt[4]{2}}}\right)}{4\ 2^{3/4}}+\frac{\sqrt{1+\sqrt[4]{2}}\tanh^{-1}\left(\frac{x}{\sqrt{1+\sqrt[4]{2}}}\right)}{4\ 2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(2 - (1 - x^2)^4),x]
```

```
[Out] -(Sqrt[-1 + 2^(1/4)]*ArcTan[x/Sqrt[-1 + 2^(1/4)]])/(4*2^(3/4)) - ((I/4)*Sqr
t[1 - I*2^(1/4)]*ArcTanh[x/Sqrt[1 - I*2^(1/4)]])/2^(3/4) + ((I/4)*Sqrt[1 +
I*2^(1/4)]*ArcTanh[x/Sqrt[1 + I*2^(1/4)]])/2^(3/4) + (Sqrt[1 + 2^(1/4)]*Arc
Tanh[x/Sqrt[1 + 2^(1/4)]])/(4*2^(3/4))
```

Rule 6740

```
Int[(v_)/((a_) + (b_.)*(u_)^(n_.)), x_Symbol] :> Int[ExpandIntegrand[Polyno
mialInSubst[v, u, x]/(a + b*x^n), x] /. x -> u, x] /; FreeQ[{a, b}, x] && I
GtQ[n, 0] && PolynomialInQ[v, u, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1972

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && B
inomialQ[u, x] &&  !BinomialMatchQ[u, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x^2}{2-\left(1-x^2\right)^4}\,dx = \int \left(\frac{\sqrt[4]{2}+\sqrt{2}}{8\left(1+\sqrt[4]{2}-x^2\right)}+\frac{\sqrt[4]{2}-\sqrt{2}}{8\left(-1+\sqrt[4]{2}+x^2\right)}+\frac{\sqrt[4]{2}+i\sqrt{2}}{8\left(\sqrt[4]{2}-i\left(1-x^2\right)\right)}+\frac{\sqrt[4]{2}-i\sqrt{2}}{8\left(\sqrt[4]{2}+i\left(1-x^2\right)\right)}\right)dx$$

$$= \frac{\left(1-\sqrt[4]{2}\right)\int \frac{1}{-1+\sqrt[4]{2}+x^2}\,dx}{4\ 2^{3/4}}+\frac{\left(1-i\sqrt[4]{2}\right)\int \frac{1}{\sqrt[4]{2}+i\left(1-x^2\right)}\,dx}{4\ 2^{3/4}}+\frac{\left(1+i\sqrt[4]{2}\right)\int \frac{1}{\sqrt[4]{2}-i\left(1-x^2\right)}\,dx}{4\ 2^{3/4}}+\frac{\left(1+\sqrt[4]{2}\right)\int \frac{}{1}}{4\ 2^{3/}}$$

$$= -\frac{\sqrt{-1+\sqrt[4]{2}}\tan^{-1}\left(\frac{x}{\sqrt{-1+\sqrt[4]{2}}}\right)}{4\ 2^{3/4}}+\frac{\sqrt{1+\sqrt[4]{2}}\tanh^{-1}\left(\frac{x}{\sqrt{1+\sqrt[4]{2}}}\right)}{4\ 2^{3/4}}+\frac{\left(1-i\sqrt[4]{2}\right)\int \frac{1}{i+\sqrt[4]{2}-ix^2}\,dx}{4\ 2^{3/4}}+\frac{\left(1+i\sqrt[4]{2}\right.}{}$$

$$= -\frac{\sqrt{-1+\sqrt[4]{2}}\tan^{-1}\left(\frac{x}{\sqrt{-1+\sqrt[4]{2}}}\right)}{4\ 2^{3/4}}-\frac{i\sqrt{1-i\sqrt[4]{2}}\tanh^{-1}\left(\frac{x}{\sqrt{1-i\sqrt[4]{2}}}\right)}{4\ 2^{3/4}}+\frac{i\sqrt{1+i\sqrt[4]{2}}\tanh^{-1}\left(\frac{x}{\sqrt{1+i\sqrt[4]{2}}}\right)}{4\ 2^{3/4}}+\frac{\sqrt{}}{}$$

Mathematica [C] time = 0.0178763, size = 61, normalized size = 0.39

$$-\frac{1}{8}\text{RootSum}\left[\#1^8-4\#1^6+6\#1^4-4\#1^2-1\&,\frac{\#1\log(x-\#1)}{\#1^6-3\#1^4+3\#1^2-1}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(2 - (1 - x^2)^4),x]
```

```
[Out] -RootSum[-1 - 4*#1^2 + 6*#1^4 - 4*#1^6 + #1^8 & , (Log[x - #1]*#1)/(-1 + 3*
#1^2 - 3*#1^4 + #1^6) & ]/8
```

Maple [C] time = 0.009, size = 56, normalized size = 0.4

$$-\frac{1}{8}\sum_{\_R=\mathrm{RootOf}\left(\_Z^8-4\_Z^6+6\_Z^4-4\_Z^2-1\right)}\frac{\_R^2\ln\left(x-\_R\right)}{\_R^7-3\_R^5+3\_R^3-\_R}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(2-(-x^2+1)^4),x)
```

```
[Out] -1/8*sum(_R^2/(_R^7-3*_R^5+3*_R^3-_R)*ln(x-_R),_R=RootOf(_Z^8-4*_Z^6+6*_Z^4
-4*_Z^2-1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{x^2}{\left(x^2-1\right)^4-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(2-(-x^2+1)^4),x, algorithm="maxima")
```

```
[Out] -integrate(x^2/((x^2 - 1)^4 - 2), x)
```

Fricas [B] time = 8.6147, size = 4898, normalized size = 31.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(2-(-x^2+1)^4),x, algorithm="fricas")
```

```
[Out] -1/16*sqrt(2)*sqrt(-1/2*sqrt(2) + sqrt(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2
^(3/4) + sqrt(2))*(2^(3/4) - sqrt(2)) - 3/16*(2^(3/4) - sqrt(2))^2 + 1))*lo
```

```
g(1/4*((sqrt(2)*(2^(3/4) - sqrt(2)) - sqrt(2))*(2^(3/4) + sqrt(2))^2 - sqrt
(2)*(2^(3/4) - sqrt(2))^2 - (sqrt(2)*(2^(3/4) - sqrt(2))^2 - 4*sqrt(2))*(2^
(3/4) + sqrt(2)) + 4*((sqrt(2)*(2^(3/4) - sqrt(2)) - sqrt(2))*(2^(3/4) + sq
rt(2)) + sqrt(2)*(2^(3/4) - sqrt(2)) - 4*sqrt(2))*sqrt(-3/16*(2^(3/4) + sqr
t(2))^2 + 1/8*(2^(3/4) + sqrt(2))*(2^(3/4) - sqrt(2)) - 3/16*(2^(3/4) - sqr
t(2))^2 + 1) - 4*sqrt(2)*(2^(3/4) - sqrt(2)) - 4*sqrt(2))*sqrt(-1/2*sqrt(2)
 + sqrt(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2^(3/4) + sqrt(2))*(2^(3/4) - sq
rt(2)) - 3/16*(2^(3/4) - sqrt(2))^2 + 1)) + 6*x) + 1/16*sqrt(2)*sqrt(-1/2*s
qrt(2) + sqrt(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2^(3/4) + sqrt(2))*(2^(3/4
) - sqrt(2)) - 3/16*(2^(3/4) - sqrt(2))^2 + 1))*log(-1/4*((sqrt(2)*(2^(3/4)
 - sqrt(2)) - sqrt(2))*(2^(3/4) + sqrt(2))^2 - sqrt(2)*(2^(3/4) - sqrt(2))^
2 - (sqrt(2)*(2^(3/4) - sqrt(2))^2 - 4*sqrt(2))*(2^(3/4) + sqrt(2)) + 4*((s
qrt(2)*(2^(3/4) - sqrt(2)) - sqrt(2))*(2^(3/4) + sqrt(2)) + sqrt(2)*(2^(3/4
) - sqrt(2)) - 4*sqrt(2))*sqrt(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2^(3/4) +
 sqrt(2))*(2^(3/4) - sqrt(2)) - 3/16*(2^(3/4) - sqrt(2))^2 + 1) - 4*sqrt(2)
*(2^(3/4) - sqrt(2)) - 4*sqrt(2))*sqrt(-1/2*sqrt(2) + sqrt(-3/16*(2^(3/4) +
 sqrt(2))^2 + 1/8*(2^(3/4) + sqrt(2))*(2^(3/4) - sqrt(2)) - 3/16*(2^(3/4) -
 sqrt(2))^2 + 1)) + 6*x) - 1/16*sqrt(2)*sqrt(-1/2*sqrt(2) - sqrt(-3/16*(2^(
3/4) + sqrt(2))^2 + 1/8*(2^(3/4) + sqrt(2))*(2^(3/4) - sqrt(2)) - 3/16*(2^(
3/4) - sqrt(2))^2 + 1))*log(1/4*((sqrt(2)*(2^(3/4) - sqrt(2)) - sqrt(2))*(2
^(3/4) + sqrt(2))^2 - sqrt(2)*(2^(3/4) - sqrt(2))^2 - (sqrt(2)*(2^(3/4) - s
qrt(2))^2 - 4*sqrt(2))*(2^(3/4) + sqrt(2)) - 4*((sqrt(2)*(2^(3/4) - sqrt(2)
) - sqrt(2))*(2^(3/4) + sqrt(2)) + sqrt(2)*(2^(3/4) - sqrt(2)) - 4*sqrt(2))
*sqrt(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2^(3/4) + sqrt(2))*(2^(3/4) - sqrt
(2)) - 3/16*(2^(3/4) - sqrt(2))^2 + 1) - 4*sqrt(2)*(2^(3/4) - sqrt(2)) - 4*
sqrt(2))*sqrt(-1/2*sqrt(2) - sqrt(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2^(3/4
) + sqrt(2))*(2^(3/4) - sqrt(2)) - 3/16*(2^(3/4) - sqrt(2))^2 + 1)) + 6*x)
+ 1/16*sqrt(2)*sqrt(-1/2*sqrt(2) - sqrt(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(
2^(3/4) + sqrt(2))*(2^(3/4) - sqrt(2)) - 3/16*(2^(3/4) - sqrt(2))^2 + 1))*l
og(-1/4*((sqrt(2)*(2^(3/4) - sqrt(2)) - sqrt(2))*(2^(3/4) + sqrt(2))^2 - sq
rt(2)*(2^(3/4) - sqrt(2))^2 - (sqrt(2)*(2^(3/4) - sqrt(2))^2 - 4*sqrt(2))*(
2^(3/4) + sqrt(2)) - 4*((sqrt(2)*(2^(3/4) - sqrt(2)) - sqrt(2))*(2^(3/4) +
sqrt(2)) + sqrt(2)*(2^(3/4) - sqrt(2)) - 4*sqrt(2))*sqrt(-3/16*(2^(3/4) + s
qrt(2))^2 + 1/8*(2^(3/4) + sqrt(2))*(2^(3/4) - sqrt(2)) - 3/16*(2^(3/4) - s
qrt(2))^2 + 1) - 4*sqrt(2)*(2^(3/4) - sqrt(2)) - 4*sqrt(2))*sqrt(-1/2*sqrt(
2) - sqrt(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2^(3/4) + sqrt(2))*(2^(3/4) -
sqrt(2)) - 3/16*(2^(3/4) - sqrt(2))^2 + 1)) + 6*x) + 1/16*sqrt(2^(3/4) + sq
rt(2))*log(1/4*((2^(3/4) - sqrt(2))^3 + (2^(3/4) + sqrt(2))^2*(2^(3/4) - sq
rt(2) - 1) - ((2^(3/4) - sqrt(2))^2 - 4)*(2^(3/4) + sqrt(2)) - 4*2^(3/4) +
4*sqrt(2) + 6)*sqrt(2^(3/4) + sqrt(2)) + 3*x) - 1/16*sqrt(2^(3/4) + sqrt(2)
)*log(-1/4*((2^(3/4) - sqrt(2))^3 + (2^(3/4) + sqrt(2))^2*(2^(3/4) - sqrt(2
) - 1) - ((2^(3/4) - sqrt(2))^2 - 4)*(2^(3/4) + sqrt(2)) - 4*2^(3/4) + 4*sq
rt(2) + 6)*sqrt(2^(3/4) + sqrt(2)) + 3*x) - sqrt(-1/256*2^(3/4) + 1/256*sqr
t(2))*log(4*((2^(3/4) - sqrt(2))^3 + (2^(3/4) - sqrt(2))^2 + 10)*sqrt(-1/25
6*2^(3/4) + 1/256*sqrt(2)) + 3*x) + sqrt(-1/256*2^(3/4) + 1/256*sqrt(2))*lo
```

```
g(-4*((2^(3/4) - sqrt(2))^3 + (2^(3/4) - sqrt(2))^2 + 10)*sqrt(-1/256*2^(3/
4) + 1/256*sqrt(2)) + 3*x)
```

Sympy [A] time = 0.20163, size = 41, normalized size = 0.26

$$-\operatorname{RootSum}\left(1073741824t^8 - 65536t^4 - 1024t^2 - 1, \left(t \mapsto t\log\left(-\frac{67108864t^7}{3} + \frac{262144t^5}{3} + \frac{40t}{3} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(2-(-x**2+1)**4),x)
```

```
[Out] -RootSum(1073741824*_t**8 - 65536*_t**4 - 1024*_t**2 - 1, Lambda(_t, _t*log
(-67108864*_t**7/3 + 262144*_t**5/3 + 40*_t/3 + x)))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2}{\left(x^2-1\right)^4-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(2-(-x^2+1)^4),x, algorithm="giac")
```

```
[Out] integrate(-x^2/((x^2 - 1)^4 - 2), x)
```

3.389 $\int \frac{x^2}{2+(1+x^2)^4} \, dx$

Optimal. Leaf size=188

$$\frac{\sqrt[4]{-1}\sqrt{1-\sqrt[4]{-2}}\tan^{-1}\left(\frac{x}{\sqrt{1-\sqrt[4]{-2}}}\right)}{4\ 2^{3/4}}-\frac{(-1)^{3/4}\sqrt{1+i\sqrt[4]{-2}}\tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt[4]{-2}}}\right)}{4\ 2^{3/4}}-\frac{\sqrt[4]{-1}\sqrt{1+\sqrt[4]{-2}}\tan^{-1}\left(\frac{x}{\sqrt{1+\sqrt[4]{-2}}}\right)}{4\ 2^{3/4}}+\frac{1}{8}i\left(\sqrt[4]{-2}+\right.$$

```
[Out] ((-1)^(1/4)*Sqrt[1 - (-2)^(1/4)]*ArcTan[x/Sqrt[1 - (-2)^(1/4)]])/(4*2^(3/4)
) - ((-1)^(3/4)*Sqrt[1 + I*(-2)^(1/4)]*ArcTan[x/Sqrt[1 + I*(-2)^(1/4)]])/(4
*2^(3/4)) - ((-1)^(1/4)*Sqrt[1 + (-2)^(1/4)]*ArcTan[x/Sqrt[1 + (-2)^(1/4)]]
)/(4*2^(3/4)) + (I/8)*((-2)^(1/4) + Sqrt[2])*Sqrt[(1 + I)/((1 + I) + 2^(3/4
))]*ArcTan[Sqrt[(1 + I)/((1 + I) + 2^(3/4))]*x]
```

Rubi [A] time = 0.276079, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6740, 204, 203, 1972, 205}

$$\frac{\sqrt[4]{-1}\sqrt{1-\sqrt[4]{-2}}\tan^{-1}\left(\frac{x}{\sqrt{1-\sqrt[4]{-2}}}\right)}{4\ 2^{3/4}}-\frac{(-1)^{3/4}\sqrt{1+i\sqrt[4]{-2}}\tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt[4]{-2}}}\right)}{4\ 2^{3/4}}-\frac{\sqrt[4]{-1}\sqrt{1+\sqrt[4]{-2}}\tan^{-1}\left(\frac{x}{\sqrt{1+\sqrt[4]{-2}}}\right)}{4\ 2^{3/4}}+\frac{1}{8}i\left(\sqrt[4]{-2}+\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(2 + (1 + x^2)^4),x]
```

```
[Out] ((-1)^(1/4)*Sqrt[1 - (-2)^(1/4)]*ArcTan[x/Sqrt[1 - (-2)^(1/4)]])/(4*2^(3/4)
) - ((-1)^(3/4)*Sqrt[1 + I*(-2)^(1/4)]*ArcTan[x/Sqrt[1 + I*(-2)^(1/4)]])/(4
*2^(3/4)) - ((-1)^(1/4)*Sqrt[1 + (-2)^(1/4)]*ArcTan[x/Sqrt[1 + (-2)^(1/4)]]
)/(4*2^(3/4)) + (I/8)*((-2)^(1/4) + Sqrt[2])*Sqrt[(1 + I)/((1 + I) + 2^(3/4
))]*ArcTan[Sqrt[(1 + I)/((1 + I) + 2^(3/4))]*x]
```

Rule 6740

```
Int[(v_)/((a_) + (b_.)*(u_)^(n_.)), x_Symbol] :> Int[ExpandIntegrand[Polyno
mialInSubst[v, u, x]/(a + b*x^n), x] /. x -> u, x] /; FreeQ[{a, b}, x] && I
GtQ[n, 0] && PolynomialInQ[v, u, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1972

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && B
inomialQ[u, x] &&  !BinomialMatchQ[u, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{2+\left(1+x^2\right)^4}\,dx &= \int \left(\frac{-\sqrt[4]{-2}+i\sqrt{2}}{8\left(-1+\sqrt[4]{-2}-x^2\right)}+\frac{-\sqrt[4]{-2}-i\sqrt{2}}{8\left(1+\sqrt[4]{-2}+x^2\right)}+\frac{-\sqrt[4]{-2}+\sqrt{2}}{8\left(\sqrt[4]{-2}-i\left(1+x^2\right)\right)}+\frac{-\sqrt[4]{-2}-\sqrt{2}}{8\left(\sqrt[4]{-2}+i\left(1+x^2\right)\right)}\right)dx\\ &= \frac{1}{8}\left(-\sqrt[4]{-2}-\sqrt{2}\right)\int \frac{1}{\sqrt[4]{-2}+i\left(1+x^2\right)}\,dx+\frac{1}{8}\left(-\sqrt[4]{-2}-i\sqrt{2}\right)\int \frac{1}{1+\sqrt[4]{-2}+x^2}\,dx+\frac{1}{8}\left(-\sqrt[4]{-2}+i\sqrt{2}\right.\\ &= \frac{\sqrt[4]{-1}\sqrt{1-\sqrt[4]{-2}}\tan^{-1}\left(\frac{x}{\sqrt{1-\sqrt[4]{-2}}}\right)}{4\ 2^{3/4}}-\frac{\sqrt[4]{-1}\sqrt{1+\sqrt[4]{-2}}\tan^{-1}\left(\frac{x}{\sqrt{1+\sqrt[4]{-2}}}\right)}{4\ 2^{3/4}}+\frac{1}{8}\left(-\sqrt[4]{-2}-\sqrt{2}\right)\int \frac{1}{i+\sqrt[4]{-2}\ldots}\\ &= \frac{\sqrt[4]{-1}\sqrt{1-\sqrt[4]{-2}}\tan^{-1}\left(\frac{x}{\sqrt{1-\sqrt[4]{-2}}}\right)}{4\ 2^{3/4}}-\frac{(-1)^{3/4}\sqrt{1+i\sqrt[4]{-2}}\tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt[4]{-2}}}\right)}{4\ 2^{3/4}}-\frac{\sqrt[4]{-1}\sqrt{1+\sqrt[4]{-2}}\tan^{-1}\left(\ldots\right.}{4\ 2^{3/4}}\end{aligned}$$

Mathematica [C]   time = 0.0134079, size = 61, normalized size = 0.32

$$\frac{1}{8}\text{RootSum}\left[\#1^8+4\#1^6+6\#1^4+4\#1^2+3\&,\frac{\#1\log(x-\#1)}{\#1^6+3\#1^4+3\#1^2+1}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(2 + (1 + x^2)^4),x]
```

```
[Out] RootSum[3 + 4*#1^2 + 6*#1^4 + 4*#1^6 + #1^8 & , (Log[x - #1]*#1)/(1 + 3*#1^
2 + 3*#1^4 + #1^6) & ]/8
```

Maple [C] time = 0.007, size = 54, normalized size = 0.3

$$\frac{1}{8}\sum_{\_R=\mathrm{RootOf}\left(\_Z^8+4\_Z^6+6\_Z^4+4\_Z^2+3\right)}\frac{\_R^2\ln\left(x-\_R\right)}{\_R^7+3\_R^5+3\_R^3+\_R}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(2+(x^2+1)^4),x)
```

```
[Out] 1/8*sum(_R^2/(_R^7+3*_R^5+3*_R^3+_R)*ln(x-_R),_R=RootOf(_Z^8+4*_Z^6+6*_Z^4+
4*_Z^2+3))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(x^2+1\right)^4+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(2+(x^2+1)^4),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((x^2 + 1)^4 + 2), x)
```

Fricas [B] time = 9.79067, size = 8759, normalized size = 46.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(2+(x^2+1)^4),x, algorithm="fricas")
```

```
[Out] -1/16*sqrt(2)*sqrt(sqrt(-12288*(1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)
))^2 - 12288*(-1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 - 1/8*(I*sq
rt(2) + 128*sqrt(-1/8192*I*sqrt(2)))*(-I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2
))) - 1) + 32*sqrt(1/8192*I*sqrt(2)) + 32*sqrt(-1/8192*I*sqrt(2)))*log((163
84*sqrt(2)*(-1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2*(-I*sqrt(2) +
 128*sqrt(1/8192*I*sqrt(2))) + 16384*(sqrt(2)*(I*sqrt(2) + 128*sqrt(-1/8192
*I*sqrt(2))) - sqrt(2))*(1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 -
16384*sqrt(2)*(-1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 - sqrt(-12
288*(1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 12288*(-1/256*I*sqrt
(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*sqrt(-1/8192*I*
sqrt(2)))*(-I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2))) - 1)*((sqrt(2)*(I*sqrt(
2) + 128*sqrt(-1/8192*I*sqrt(2))) - sqrt(2))*(-I*sqrt(2) + 128*sqrt(1/8192*
I*sqrt(2))) - sqrt(2)*(I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2)))) + sqrt(2))
*sqrt(sqrt(-12288*(1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 12288*
(-1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*s
qrt(-1/8192*I*sqrt(2)))*(-I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2))) - 1) + 32
*sqrt(1/8192*I*sqrt(2)) + 32*sqrt(-1/8192*I*sqrt(2))) + 2*x) + 1/16*sqrt(2)
*sqrt(sqrt(-12288*(1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 12288*
(-1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*s
qrt(-1/8192*I*sqrt(2)))*(-I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2))) - 1) + 32
*sqrt(1/8192*I*sqrt(2)) + 32*sqrt(-1/8192*I*sqrt(2)))*log(-(16384*sqrt(2)*(
-1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2*(-I*sqrt(2) + 128*sqrt(1/
8192*I*sqrt(2))) + 16384*(sqrt(2)*(I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2)))
 - sqrt(2))*(1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 16384*sqrt(2
)*(-1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 - sqrt(-12288*(1/256*I
*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 12288*(-1/256*I*sqrt(2) - 1/2*sq
rt(-1/8192*I*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2)))*(-
I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2))) - 1)*((sqrt(2)*(I*sqrt(2) + 128*sqr
t(-1/8192*I*sqrt(2))) - sqrt(2))*(-I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2)))
- sqrt(2)*(I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2)))) + sqrt(2))*sqrt(sqrt(-
12288*(1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 12288*(-1/256*I*sq
rt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*sqrt(-1/8192*
I*sqrt(2)))*(-I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2))) - 1) + 32*sqrt(1/8192
*I*sqrt(2)) + 32*sqrt(-1/8192*I*sqrt(2))) + 2*x) - 1/16*sqrt(2)*sqrt(-sqrt(
-12288*(1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 12288*(-1/256*I*s
qrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*sqrt(-1/8192
*I*sqrt(2)))*(-I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2))) - 1) + 32*sqrt(1/819
2*I*sqrt(2)) + 32*sqrt(-1/8192*I*sqrt(2)))*log((16384*sqrt(2)*(-1/256*I*sqr
t(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2*(-I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(
2))) + 16384*(sqrt(2)*(I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2))) - sqrt(2))*
(1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 16384*sqrt(2)*(-1/256*I*
sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 + sqrt(-12288*(1/256*I*sqrt(2) - 1
/2*sqrt(1/8192*I*sqrt(2)))^2 - 12288*(-1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I
*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2)))*(-I*sqrt(2) +
128*sqrt(1/8192*I*sqrt(2))) - 1)*((sqrt(2)*(I*sqrt(2) + 128*sqrt(-1/8192*I*
```

```
sqrt(2))) - sqrt(2))*(-I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2))) - sqrt(2)*(I
*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2)))) + sqrt(2))*sqrt(-sqrt(-12288*(1/25
6*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 12288*(-1/256*I*sqrt(2) - 1/2
*sqrt(-1/8192*I*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2)))
*(-I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2))) - 1) + 32*sqrt(1/8192*I*sqrt(2))
 + 32*sqrt(-1/8192*I*sqrt(2))) + 2*x) + 1/16*sqrt(2)*sqrt(-sqrt(-12288*(1/2
56*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 12288*(-1/256*I*sqrt(2) - 1/
2*sqrt(-1/8192*I*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2))
)*(-I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2))) - 1) + 32*sqrt(1/8192*I*sqrt(2)
) + 32*sqrt(-1/8192*I*sqrt(2)))*log(-(16384*sqrt(2)*(-1/256*I*sqrt(2) - 1/2
*sqrt(-1/8192*I*sqrt(2)))^2*(-I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2))) + 163
84*(sqrt(2)*(I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2))) - sqrt(2))*(1/256*I*s
qrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 16384*sqrt(2)*(-1/256*I*sqrt(2) -
1/2*sqrt(-1/8192*I*sqrt(2)))^2 + sqrt(-12288*(1/256*I*sqrt(2) - 1/2*sqrt(1/
8192*I*sqrt(2)))^2 - 12288*(-1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))
^2 - 1/8*(I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2)))*(-I*sqrt(2) + 128*sqrt(1
/8192*I*sqrt(2))) - 1)*((sqrt(2)*(I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2)))
- sqrt(2))*(-I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2))) - sqrt(2)*(I*sqrt(2) +
 128*sqrt(-1/8192*I*sqrt(2)))) + sqrt(2))*sqrt(-sqrt(-12288*(1/256*I*sqrt(2
) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 12288*(-1/256*I*sqrt(2) - 1/2*sqrt(-1/8
192*I*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2)))*(-I*sqrt(
2) + 128*sqrt(1/8192*I*sqrt(2))) - 1) + 32*sqrt(1/8192*I*sqrt(2)) + 32*sqrt
(-1/8192*I*sqrt(2))) + 2*x) - sqrt(1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt
(2)))*log(8*(8388608*(-1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^3 - 3
2768*(-1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2*(-I*sqrt(2) + 128*s
qrt(1/8192*I*sqrt(2))) + 32768*(1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)
))^2*(-I*sqrt(2) - 128*sqrt(-1/8192*I*sqrt(2)) + 1) - 2*I*sqrt(2) - 256*sqr
t(-1/8192*I*sqrt(2)) + 3)*sqrt(1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2))
) + x) + sqrt(1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))*log(-8*(8388608
*(-1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^3 - 32768*(-1/256*I*sqrt(
2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2*(-I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2)
)) + 32768*(1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2*(-I*sqrt(2) - 1
28*sqrt(-1/8192*I*sqrt(2)) + 1) - 2*I*sqrt(2) - 256*sqrt(-1/8192*I*sqrt(2))
 + 3)*sqrt(1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2))) + x) + sqrt(-1/256
*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))*log(8*(8388608*(-1/256*I*sqrt(2)
- 1/2*sqrt(-1/8192*I*sqrt(2)))^3 - 32768*(-1/256*I*sqrt(2) - 1/2*sqrt(-1/81
92*I*sqrt(2)))^2 - 2*I*sqrt(2) - 256*sqrt(-1/8192*I*sqrt(2)) + 5)*sqrt(-1/2
56*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2))) + x) - sqrt(-1/256*I*sqrt(2) -
1/2*sqrt(-1/8192*I*sqrt(2)))*log(-8*(8388608*(-1/256*I*sqrt(2) - 1/2*sqrt(-
1/8192*I*sqrt(2)))^3 - 32768*(-1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)
))^2 - 2*I*sqrt(2) - 256*sqrt(-1/8192*I*sqrt(2)) + 5)*sqrt(-1/256*I*sqrt(2)
 - 1/2*sqrt(-1/8192*I*sqrt(2))) + x)
```

Sympy [A] time = 0.188685, size = 39, normalized size = 0.21

$$\operatorname{RootSum}\left(1073741824t^8 + 65536t^4 + 1024t^2 + 3, \left(t \mapsto t\log\left(67108864t^7 - 262144t^5 + 4096t^3 + 40t + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(2+(x**2+1)**4),x)
```

```
[Out] RootSum(1073741824*_t**8 + 65536*_t**4 + 1024*_t**2 + 3, Lambda(_t, _t*log(
67108864*_t**7 - 262144*_t**5 + 4096*_t**3 + 40*_t + x)))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(x^2+1\right)^4+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(2+(x^2+1)^4),x, algorithm="giac")
```

```
[Out] integrate(x^2/((x^2 + 1)^4 + 2), x)
```

3.390 $\int \frac{x^2}{2+\left(1-x^2\right)^4}\,dx$

Optimal. Leaf size=188

$$-\frac{\sqrt[4]{-1}\sqrt{1-\sqrt[4]{-2}}\tanh^{-1}\left(\frac{x}{\sqrt{1-\sqrt[4]{-2}}}\right)}{4\ 2^{3/4}}+\frac{(-1)^{3/4}\sqrt{1+i\sqrt[4]{-2}}\tanh^{-1}\left(\frac{x}{\sqrt{1+i\sqrt[4]{-2}}}\right)}{4\ 2^{3/4}}+\frac{\sqrt[4]{-1}\sqrt{1+\sqrt[4]{-2}}\tanh^{-1}\left(\frac{x}{\sqrt{1+\sqrt[4]{-2}}}\right)}{4\ 2^{3/4}}-\frac{1}{8}i\Big($$

```
[Out] -((-1)^(1/4)*Sqrt[1 - (-2)^(1/4)]*ArcTanh[x/Sqrt[1 - (-2)^(1/4)]])/(4*2^(3/
4)) + ((-1)^(3/4)*Sqrt[1 + I*(-2)^(1/4)]*ArcTanh[x/Sqrt[1 + I*(-2)^(1/4)]])
/(4*2^(3/4)) + ((-1)^(1/4)*Sqrt[1 + (-2)^(1/4)]*ArcTanh[x/Sqrt[1 + (-2)^(1/
4)]])/(4*2^(3/4)) - (I/8)*((-2)^(1/4) + Sqrt[2])*Sqrt[(1 + I)/((1 + I) + 2^
(3/4))]*ArcTanh[Sqrt[(1 + I)/((1 + I) + 2^(3/4))]*x]
```

Rubi [A] time = 0.155724, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {6740, 206, 207, 1972, 208}

$$-\frac{\sqrt[4]{-1}\sqrt{1-\sqrt[4]{-2}}\tanh^{-1}\left(\frac{x}{\sqrt{1-\sqrt[4]{-2}}}\right)}{4\ 2^{3/4}}+\frac{(-1)^{3/4}\sqrt{1+i\sqrt[4]{-2}}\tanh^{-1}\left(\frac{x}{\sqrt{1+i\sqrt[4]{-2}}}\right)}{4\ 2^{3/4}}+\frac{\sqrt[4]{-1}\sqrt{1+\sqrt[4]{-2}}\tanh^{-1}\left(\frac{x}{\sqrt{1+\sqrt[4]{-2}}}\right)}{4\ 2^{3/4}}-\frac{1}{8}i\Big($$

Antiderivative was successfully verified.

```
[In] Int[x^2/(2 + (1 - x^2)^4),x]
```

```
[Out] -((-1)^(1/4)*Sqrt[1 - (-2)^(1/4)]*ArcTanh[x/Sqrt[1 - (-2)^(1/4)]])/(4*2^(3/
4)) + ((-1)^(3/4)*Sqrt[1 + I*(-2)^(1/4)]*ArcTanh[x/Sqrt[1 + I*(-2)^(1/4)]])
/(4*2^(3/4)) + ((-1)^(1/4)*Sqrt[1 + (-2)^(1/4)]*ArcTanh[x/Sqrt[1 + (-2)^(1/
4)]])/(4*2^(3/4)) - (I/8)*((-2)^(1/4) + Sqrt[2])*Sqrt[(1 + I)/((1 + I) + 2^
(3/4))]*ArcTanh[Sqrt[(1 + I)/((1 + I) + 2^(3/4))]*x]
```

Rule 6740

```
Int[(v_)/((a_) + (b_.)*(u_)^(n_.)), x_Symbol] :> Int[ExpandIntegrand[Polyno
mialInSubst[v, u, x]/(a + b*x^n), x] /. x -> u, x] /; FreeQ[{a, b}, x] && I
GtQ[n, 0] && PolynomialInQ[v, u, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1972

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && B
inomialQ[u, x] &&  !BinomialMatchQ[u, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x^2}{2+\left(1-x^2\right)^4}\,dx = \int \left(\frac{\sqrt[4]{-2}+i\sqrt{2}}{8\left(1+\sqrt[4]{-2}-x^2\right)}+\frac{\sqrt[4]{-2}-i\sqrt{2}}{8\left(-1+\sqrt[4]{-2}+x^2\right)}+\frac{\sqrt[4]{-2}-\sqrt{2}}{8\left(\sqrt[4]{-2}-i\left(1-x^2\right)\right)}+\frac{\sqrt[4]{-2}+\sqrt{2}}{8\left(\sqrt[4]{-2}+i\left(1-x^2\right)\right)}\right)dx$$

$$= \frac{1}{8}\left(\sqrt[4]{-2}-\sqrt{2}\right)\int \frac{1}{\sqrt[4]{-2}-i\left(1-x^2\right)}\,dx+\frac{1}{8}\left(\sqrt[4]{-2}-i\sqrt{2}\right)\int \frac{1}{-1+\sqrt[4]{-2}+x^2}\,dx+\frac{1}{8}\left(\sqrt[4]{-2}+i\sqrt{2}\right)\int$$

$$= -\frac{\sqrt[4]{-1}\sqrt{1-\sqrt[4]{-2}}\tanh^{-1}\left(\frac{x}{\sqrt{1-\sqrt[4]{-2}}}\right)}{4\ 2^{3/4}}+\frac{\sqrt[4]{-1}\sqrt{1+\sqrt[4]{-2}}\tanh^{-1}\left(\frac{x}{\sqrt{1+\sqrt[4]{-2}}}\right)}{4\ 2^{3/4}}+\frac{1}{8}\left(\sqrt[4]{-2}-\sqrt{2}\right)\int \frac{}{-i+}$$

$$= -\frac{\sqrt[4]{-1}\sqrt{1-\sqrt[4]{-2}}\tanh^{-1}\left(\frac{x}{\sqrt{1-\sqrt[4]{-2}}}\right)}{4\ 2^{3/4}}+\frac{(-1)^{3/4}\sqrt{1+i\sqrt[4]{-2}}\tanh^{-1}\left(\frac{x}{\sqrt{1+i\sqrt[4]{-2}}}\right)}{4\ 2^{3/4}}+\frac{\sqrt[4]{-1}\sqrt{1+\sqrt[4]{-2}}\,\mathrm{tanh}}{4\ 2^{3/4}}$$

Mathematica [C] time = 0.0132833, size = 61, normalized size = 0.32

$$\frac{1}{8}\text{RootSum}\left[\#1^8-4\#1^6+6\#1^4-4\#1^2+3\&,\frac{\#1\log(x-\#1)}{\#1^6-3\#1^4+3\#1^2-1}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(2 + (1 - x^2)^4),x]
```

```
[Out] RootSum[3 - 4*#1^2 + 6*#1^4 - 4*#1^6 + #1^8 & , (Log[x - #1]*#1)/(-1 + 3*#1
^2 - 3*#1^4 + #1^6) & ]/8
```

Maple [C] time = 0.006, size = 56, normalized size = 0.3

$$\frac{1}{8}\sum_{\_R=\mathrm{RootOf}\left(\_Z^8-4\_Z^6+6\_Z^4-4\_Z^2+3\right)}\frac{\_R^2\ln\left(x-\_R\right)}{\_R^7-3\_R^5+3\_R^3-\_R}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(2+(-x^2+1)^4),x)
```

```
[Out] 1/8*sum(_R^2/(_R^7-3*_R^5+3*_R^3-_R)*ln(x-_R),_R=RootOf(_Z^8-4*_Z^6+6*_Z^4-
4*_Z^2+3))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(x^2-1\right)^4+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(2+(-x^2+1)^4),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((x^2 - 1)^4 + 2), x)
```

Fricas [B] time = 7.33124, size = 8735, normalized size = 46.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(2+(-x^2+1)^4),x, algorithm="fricas")
```

```
[Out] -1/16*sqrt(2)*sqrt(sqrt(-12288*(1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2
)))^2 - 12288*(-1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 1/8*(I*sq
rt(2) + 128*sqrt(1/8192*I*sqrt(2)))*(-I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2
))) - 1) + 32*sqrt(1/8192*I*sqrt(2)) + 32*sqrt(-1/8192*I*sqrt(2)))*log((163
84*sqrt(2)*(-1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2*(-I*sqrt(2) +
128*sqrt(-1/8192*I*sqrt(2))) + 16384*(sqrt(2)*(I*sqrt(2) + 128*sqrt(1/8192*
I*sqrt(2))) + sqrt(2))*(1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 +
16384*sqrt(2)*(-1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - sqrt(-122
88*(1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 - 12288*(-1/256*I*sqrt
(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*sqrt(1/8192*I*sq
rt(2)))*(-I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2))) - 1)*((sqrt(2)*(I*sqrt(2
) + 128*sqrt(1/8192*I*sqrt(2))) + sqrt(2))*(-I*sqrt(2) + 128*sqrt(-1/8192*I
*sqrt(2))) + sqrt(2)*(I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2)))) - sqrt(2))*s
qrt(sqrt(-12288*(1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 - 12288*(
-1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*sqr
t(1/8192*I*sqrt(2)))*(-I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2))) - 1) + 32*s
qrt(1/8192*I*sqrt(2)) + 32*sqrt(-1/8192*I*sqrt(2))) + 2*x) + 1/16*sqrt(2)*s
qrt(sqrt(-12288*(1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 - 12288*(
-1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*sqr
t(1/8192*I*sqrt(2)))*(-I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2))) - 1) + 32*s
qrt(1/8192*I*sqrt(2)) + 32*sqrt(-1/8192*I*sqrt(2)))*log(-(16384*sqrt(2)*(-1
/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2*(-I*sqrt(2) + 128*sqrt(-1/81
92*I*sqrt(2))) + 16384*(sqrt(2)*(I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2))) +
sqrt(2))*(1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 + 16384*sqrt(2)*
(-1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - sqrt(-12288*(1/256*I*sq
rt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 - 12288*(-1/256*I*sqrt(2) - 1/2*sqrt
(1/8192*I*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2)))*(-I*sq
rt(2) + 128*sqrt(-1/8192*I*sqrt(2))) - 1)*((sqrt(2)*(I*sqrt(2) + 128*sqrt(1
/8192*I*sqrt(2))) + sqrt(2))*(-I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2))) + s
qrt(2)*(I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2)))) - sqrt(2))*sqrt(sqrt(-1228
8*(1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 - 12288*(-1/256*I*sqrt(
2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*sqrt(1/8192*I*sqr
t(2)))*(-I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2))) - 1) + 32*sqrt(1/8192*I*s
qrt(2)) + 32*sqrt(-1/8192*I*sqrt(2))) + 2*x) - 1/16*sqrt(2)*sqrt(-sqrt(-122
88*(1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 - 12288*(-1/256*I*sqrt
(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*sqrt(1/8192*I*sq
rt(2)))*(-I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2))) - 1) + 32*sqrt(1/8192*I*
sqrt(2)) + 32*sqrt(-1/8192*I*sqrt(2)))*log((16384*sqrt(2)*(-1/256*I*sqrt(2)
 - 1/2*sqrt(1/8192*I*sqrt(2)))^2*(-I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2)))
 + 16384*(sqrt(2)*(I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2))) + sqrt(2))*(1/25
6*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 + 16384*sqrt(2)*(-1/256*I*sqrt
(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 + sqrt(-12288*(1/256*I*sqrt(2) - 1/2*sq
rt(-1/8192*I*sqrt(2)))^2 - 12288*(-1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt
(2)))^2 - 1/8*(I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2)))*(-I*sqrt(2) + 128*sq
rt(-1/8192*I*sqrt(2))) - 1)*((sqrt(2)*(I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2
```

```
))) + sqrt(2))*(-I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2))) + sqrt(2)*(I*sqrt
(2) + 128*sqrt(1/8192*I*sqrt(2)))) - sqrt(2))*sqrt(-sqrt(-12288*(1/256*I*sq
rt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 - 12288*(-1/256*I*sqrt(2) - 1/2*sqrt
(1/8192*I*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2)))*(-I*sq
rt(2) + 128*sqrt(-1/8192*I*sqrt(2))) - 1) + 32*sqrt(1/8192*I*sqrt(2)) + 32*
sqrt(-1/8192*I*sqrt(2))) + 2*x) + 1/16*sqrt(2)*sqrt(-sqrt(-12288*(1/256*I*s
qrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 - 12288*(-1/256*I*sqrt(2) - 1/2*sqr
t(1/8192*I*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2)))*(-I*s
qrt(2) + 128*sqrt(-1/8192*I*sqrt(2))) - 1) + 32*sqrt(1/8192*I*sqrt(2)) + 32
*sqrt(-1/8192*I*sqrt(2)))*log(-(16384*sqrt(2)*(-1/256*I*sqrt(2) - 1/2*sqrt(
1/8192*I*sqrt(2)))^2*(-I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2))) + 16384*(sq
rt(2)*(I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2))) + sqrt(2))*(1/256*I*sqrt(2)
 - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 + 16384*sqrt(2)*(-1/256*I*sqrt(2) - 1/2*sq
rt(1/8192*I*sqrt(2)))^2 + sqrt(-12288*(1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I
*sqrt(2)))^2 - 12288*(-1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 1/
8*(I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2)))*(-I*sqrt(2) + 128*sqrt(-1/8192*I
*sqrt(2))) - 1)*((sqrt(2)*(I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2))) + sqrt(2
))*(-I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2))) + sqrt(2)*(I*sqrt(2) + 128*sq
rt(1/8192*I*sqrt(2)))) - sqrt(2))*sqrt(-sqrt(-12288*(1/256*I*sqrt(2) - 1/2*
sqrt(-1/8192*I*sqrt(2)))^2 - 12288*(-1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sq
rt(2)))^2 - 1/8*(I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2)))*(-I*sqrt(2) + 128*
sqrt(-1/8192*I*sqrt(2))) - 1) + 32*sqrt(1/8192*I*sqrt(2)) + 32*sqrt(-1/8192
*I*sqrt(2))) + 2*x) - sqrt(1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))*l
og(8*(8388608*(-1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^3 + 32768*(1/
256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2*(-I*sqrt(2) - 128*sqrt(1/819
2*I*sqrt(2)) - 1) - 32768*(-1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2
*(-I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2))) - 2*I*sqrt(2) - 256*sqrt(1/8192
*I*sqrt(2)) - 3)*sqrt(1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2))) + x) +
 sqrt(1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))*log(-8*(8388608*(-1/25
6*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^3 + 32768*(1/256*I*sqrt(2) - 1/2*
sqrt(-1/8192*I*sqrt(2)))^2*(-I*sqrt(2) - 128*sqrt(1/8192*I*sqrt(2)) - 1) -
32768*(-1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2*(-I*sqrt(2) + 128*s
qrt(-1/8192*I*sqrt(2))) - 2*I*sqrt(2) - 256*sqrt(1/8192*I*sqrt(2)) - 3)*sqr
t(1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2))) + x) + sqrt(-1/256*I*sqrt(
2) - 1/2*sqrt(1/8192*I*sqrt(2)))*log(8*(8388608*(-1/256*I*sqrt(2) - 1/2*sqr
t(1/8192*I*sqrt(2)))^3 + 32768*(-1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2
)))^2 - 2*I*sqrt(2) - 256*sqrt(1/8192*I*sqrt(2)) - 5)*sqrt(-1/256*I*sqrt(2)
 - 1/2*sqrt(1/8192*I*sqrt(2))) + x) - sqrt(-1/256*I*sqrt(2) - 1/2*sqrt(1/81
92*I*sqrt(2)))*log(-8*(8388608*(-1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2
)))^3 + 32768*(-1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 2*I*sqrt(
2) - 256*sqrt(1/8192*I*sqrt(2)) - 5)*sqrt(-1/256*I*sqrt(2) - 1/2*sqrt(1/819
2*I*sqrt(2))) + x)
```

Sympy [A] time = 0.187626, size = 39, normalized size = 0.21

$$\mathrm{RootSum}\left(1073741824t^8 + 65536t^4 - 1024t^2 + 3, \left(t \mapsto t\log\left(67108864t^7 + 262144t^5 + 4096t^3 - 40t + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(2+(-x**2+1)**4),x)
```

```
[Out] RootSum(1073741824*_t**8 + 65536*_t**4 - 1024*_t**2 + 3, Lambda(_t, _t*log(
67108864*_t**7 + 262144*_t**5 + 4096*_t**3 - 40*_t + x)))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(x^2-1\right)^4+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(2+(-x^2+1)^4),x, algorithm="giac")
```

```
[Out] integrate(x^2/((x^2 - 1)^4 + 2), x)
```

3.391 $\int \frac{1-x^2}{a+b\left(1-x^2\right)^4}\,dx$

Optimal. Leaf size=663

$$\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\log\left(-\sqrt{2}\sqrt[8]{b}x\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}+\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}x^2\right)}{8\sqrt{2}\sqrt{-a}b^{3/8}\sqrt{\sqrt{-a}+\sqrt{b}}}-\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\log\left(\sqrt{2}\sqrt[8]{b}x\sqrt{\right.}{8\sqrt{2}\sqrt{-}$$

```
[Out] -ArcTan[(b^(1/8)*x)/Sqrt[(-a)^(1/4) - b^(1/4)]]/(4*Sqrt[-a]*Sqrt[(-a)^(1/4)
 - b^(1/4)]*b^(3/8)) - (Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] - b^(1/4)]*ArcTan[(Sq
rt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)] - Sqrt[2]*b^(1/8)*x)/Sqrt[Sqrt[Sqrt[
-a] + Sqrt[b]] - b^(1/4)]])/(4*Sqrt[2]*Sqrt[-a]*Sqrt[Sqrt[-a] + Sqrt[b]]*b^
(3/8)) + (Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] - b^(1/4)]*ArcTan[(Sqrt[Sqrt[Sqrt[-
a] + Sqrt[b]] + b^(1/4)] + Sqrt[2]*b^(1/8)*x)/Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]]
 - b^(1/4)]])/(4*Sqrt[2]*Sqrt[-a]*Sqrt[Sqrt[-a] + Sqrt[b]]*b^(3/8)) + ArcTa
nh[(b^(1/8)*x)/Sqrt[(-a)^(1/4) + b^(1/4)]]/(4*Sqrt[-a]*Sqrt[(-a)^(1/4) + b^
(1/4)]*b^(3/8)) + (Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)]*Log[Sqrt[Sqrt[-
a] + Sqrt[b]] - Sqrt[2]*Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)]*b^(1/8)*x
+ b^(1/4)*x^2])/(8*Sqrt[2]*Sqrt[-a]*Sqrt[Sqrt[-a] + Sqrt[b]]*b^(3/8)) - (Sq
rt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)]*Log[Sqrt[Sqrt[-a] + Sqrt[b]] + Sqrt[
2]*Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)]*b^(1/8)*x + b^(1/4)*x^2])/(8*Sq
rt[2]*Sqrt[-a]*Sqrt[Sqrt[-a] + Sqrt[b]]*b^(3/8))
```

Rubi [A] time = 1.10775, antiderivative size = 663, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {6740, 1990, 1166, 205, 208, 1169, 634, 618, 204, 628}

$$\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\log\left(-\sqrt{2}\sqrt[8]{b}x\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}+\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}x^2\right)}{8\sqrt{2}\sqrt{-a}b^{3/8}\sqrt{\sqrt{-a}+\sqrt{b}}}-\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\log\left(\sqrt{2}\sqrt[8]{b}x\sqrt{\right.}{8\sqrt{2}\sqrt{-}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^2)/(a + b*(1 - x^2)^4),x]
```

```
[Out] -ArcTan[(b^(1/8)*x)/Sqrt[(-a)^(1/4) - b^(1/4)]]/(4*Sqrt[-a]*Sqrt[(-a)^(1/4)
 - b^(1/4)]*b^(3/8)) - (Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] - b^(1/4)]*ArcTan[(Sq
```

```
rt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)] - Sqrt[2]*b^(1/8)*x)/Sqrt[Sqrt[Sqrt[
-a] + Sqrt[b]] - b^(1/4)]])/(4*Sqrt[2]*Sqrt[-a]*Sqrt[Sqrt[-a] + Sqrt[b]]*b^
(3/8)) + (Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] - b^(1/4)]*ArcTan[(Sqrt[Sqrt[Sqrt[-
a] + Sqrt[b]] + b^(1/4)] + Sqrt[2]*b^(1/8)*x)/Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]]
 - b^(1/4)]])/(4*Sqrt[2]*Sqrt[-a]*Sqrt[Sqrt[-a] + Sqrt[b]]*b^(3/8)) + ArcTa
nh[(b^(1/8)*x)/Sqrt[(-a)^(1/4) + b^(1/4)]]/(4*Sqrt[-a]*Sqrt[(-a)^(1/4) + b^
(1/4)]*b^(3/8)) + (Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)]*Log[Sqrt[Sqrt[-
a] + Sqrt[b]] - Sqrt[2]*Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)]*b^(1/8)*x
+ b^(1/4)*x^2])/(8*Sqrt[2]*Sqrt[-a]*Sqrt[Sqrt[-a] + Sqrt[b]]*b^(3/8)) - (Sq
rt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)]*Log[Sqrt[Sqrt[-a] + Sqrt[b]] + Sqrt[
2]*Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)]*b^(1/8)*x + b^(1/4)*x^2])/(8*Sq
rt[2]*Sqrt[-a]*Sqrt[Sqrt[-a] + Sqrt[b]]*b^(3/8))
```

Rule 6740

```
Int[(v_)/((a_) + (b_.)*(u_)^(n_.)), x_Symbol] :> Int[ExpandIntegrand[Polyno
mialInSubst[v, u, x]/(a + b*x^n), x] /. x -> u, x] /; FreeQ[{a, b}, x] && I
GtQ[n, 0] && PolynomialInQ[v, u, x]
```

Rule 1990

```
Int[(u_)^(q_.)*(v_)^(p_.), x_Symbol] :> Int[ExpandToSum[u, x]^q*ExpandToSum
[v, x]^p, x] /; FreeQ[{p, q}, x] && BinomialQ[u, x] && TrinomialQ[v, x] &&
 !(BinomialMatchQ[u, x] && TrinomialMatchQ[v, x])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1-x^2}{a+b\left(1-x^2\right)^4}\,dx = \int\left(-\frac{\sqrt{b}\left(1-x^2\right)}{2\sqrt{-a}\left(\sqrt{-a}\sqrt{b}-b\left(1-x^2\right)^2\right)}-\frac{\sqrt{b}\left(1-x^2\right)}{2\sqrt{-a}\left(\sqrt{-a}\sqrt{b}+b\left(1-x^2\right)^2\right)}\right)dx$$

$$= -\frac{\sqrt{b}\int\frac{1-x^2}{\sqrt{-a}\sqrt{b}-b\left(1-x^2\right)^2}\,dx}{2\sqrt{-a}} - \frac{\sqrt{b}\int\frac{1-x^2}{\sqrt{-a}\sqrt{b}+b\left(1-x^2\right)^2}\,dx}{2\sqrt{-a}}$$

$$= -\frac{\sqrt{b}\int\frac{1-x^2}{\left(\sqrt{-a}-\sqrt{b}\right)\sqrt{b}+2bx^2-bx^4}\,dx}{2\sqrt{-a}} - \frac{\sqrt{b}\int\frac{1-x^2}{\left(\sqrt{-a}+\sqrt{b}\right)\sqrt{b}-2bx^2+bx^4}\,dx}{2\sqrt{-a}}$$

$$= -\frac{\int\frac{\frac{\sqrt{2}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}}{\sqrt[8]{b}}-\left(1+\frac{\sqrt{\sqrt{-a}+\sqrt{b}}}{\sqrt[4]{b}}\right)x}{\frac{\sqrt{\sqrt{-a}+\sqrt{b}}}{\sqrt[4]{b}}-\frac{\sqrt{2}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\,x}{\sqrt[8]{b}}+x^2}\,dx}{4\sqrt{2}\sqrt{-a}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\sqrt{\sqrt{-a}+\sqrt{b}}\sqrt[8]{b}} - \frac{\int\frac{\frac{\sqrt{2}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}}{\sqrt[8]{b}}+\left(1+\frac{\sqrt{\sqrt{-a}+\sqrt{b}}}{\sqrt[4]{b}}\right)x}{\frac{\sqrt{\sqrt{-a}+\sqrt{b}}}{\sqrt[4]{b}}+\frac{\sqrt{2}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\,x}{\sqrt[8]{b}}+x^2}\,dx}{4\sqrt{2}\sqrt{-a}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\sqrt{\sqrt{-a}+\sqrt{b}}\sqrt[8]{b}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt[8]{b}x}{\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}}\right)}{4\sqrt{-a}\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}b^{3/8}} + \frac{\tanh^{-1}\left(\frac{\sqrt[8]{b}x}{\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}}\right)}{4\sqrt{-a}\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}b^{3/8}} + \frac{\left(1-\frac{\sqrt[4]{b}}{\sqrt{\sqrt{-a}+\sqrt{b}}}\right)\int\frac{1}{\frac{\sqrt{\sqrt{-a}+\sqrt{b}}}{\sqrt[4]{b}}-\frac{\sqrt{2}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\,x}{\sqrt[8]{b}}+x^2}\cdots}{8\sqrt{-a}\sqrt{b}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt[8]{b}x}{\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}}\right)}{4\sqrt{-a}\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}b^{3/8}} + \frac{\tanh^{-1}\left(\frac{\sqrt[8]{b}x}{\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}}\right)}{4\sqrt{-a}\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}b^{3/8}} + \frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\log\left(\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt{2}\cdots\right.}{8\sqrt{2}\sqrt{-a}\sqrt{\sqrt{-a}+\cdots}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt[8]{b}x}{\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}}\right)}{4\sqrt{-a}\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}b^{3/8}} - \frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}\tan^{-1}\left(\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}-\sqrt{2}\sqrt[8]{b}x}{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}}\right)}{4\sqrt{2}\sqrt{-a}\sqrt{\sqrt{-a}+\sqrt{b}}b^{3/8}} + \frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}\cdots}}{4\sqrt{\cdots}}$$

Mathematica [C] time = 0.0384053, size = 63, normalized size = 0.1

$$-\frac{\text{RootSum}\left[\#1^8 b-4\#1^6 b+6\#1^4 b-4\#1^2 b+a+b\&,\frac{\log(x-\#1)}{\#1^5-2\#1^3+\#1}\&\right]}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^2)/(a + b*(1 - x^2)^4),x]
```

```
[Out] -RootSum[a + b - 4*b*#1^2 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & , Log[x - #1]/(#
1 - 2*#1^3 + #1^5) & ]/(8*b)
```

Maple [C] time = 0.072, size = 69, normalized size = 0.1

$$\frac{1}{8b} \sum_{\_R=\mathrm{RootOf}\left(b\_Z^8-4b\_Z^6+6b\_Z^4-4b\_Z^2+a+b\right)} \frac{\left(-\_R^2+1\right)\ln\left(x-\_R\right)}{\_R^7-3\_R^5+3\_R^3-\_R}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)/(a+b*(-x^2+1)^4),x)
```

```
[Out] 1/8/b*sum((-_R^2+1)/(_R^7-3*_R^5+3*_R^3-_R)*ln(x-_R),_R=RootOf(_Z^8*b-4*_Z^
6*b+6*_Z^4*b-4*_Z^2*b+a+b))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2-1}{\left(x^2-1\right)^4 b + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(a+b*(-x^2+1)^4),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 1)/((x^2 - 1)^4*b + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(a+b*(-x^2+1)^4),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 2.98831, size = 133, normalized size = 0.2

$$-\operatorname{RootSum}\left(t^8\left(16777216a^5b^3+16777216a^4b^4\right)+1048576t^6a^3b^3+24576t^4a^2b^2+256t^2ab+1,\left(t\mapsto t\log\left(-6291456t^7\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)/(a+b*(-x**2+1)**4),x)
```

```
[Out] -RootSum(_t**8*(16777216*a**5*b**3 + 16777216*a**4*b**4) + 1048576*_t**6*a*
*3*b**3 + 24576*_t**4*a**2*b**2 + 256*_t**2*a*b + 1, Lambda(_t, _t*log(-629
1456*_t**7*a**4*b**3 - 6291456*_t**7*a**3*b**4 + 65536*_t**5*a**3*b**2 - 32
7680*_t**5*a**2*b**3 - 512*_t**3*a**2*b - 5632*_t**3*a*b**2 - 32*_t*b + x))
)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2-1}{\left(x^2-1\right)^4b+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(a+b*(-x^2+1)^4),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 - 1)/((x^2 - 1)^4*b + a), x)
```

3.392 $\int \frac{1-x^2}{a+b\left(-1+x^2\right)^4}\,dx$

Optimal. Leaf size=663

$$\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\log\left(-\sqrt{2}\sqrt[8]{b}x\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}+\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}x^2\right)}{8\sqrt{2}\sqrt{-a}b^{3/8}\sqrt{\sqrt{-a}+\sqrt{b}}}-\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\log\left(\sqrt{2}\sqrt[8]{b}x\right.}{8\sqrt{2}\sqrt{-}}$$

```
[Out] -ArcTan[(b^(1/8)*x)/Sqrt[(-a)^(1/4) - b^(1/4)]]/(4*Sqrt[-a]*Sqrt[(-a)^(1/4)
 - b^(1/4)]*b^(3/8)) - (Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] - b^(1/4)]*ArcTan[(Sq
rt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)] - Sqrt[2]*b^(1/8)*x)/Sqrt[Sqrt[Sqrt[
-a] + Sqrt[b]] - b^(1/4)]])/(4*Sqrt[2]*Sqrt[-a]*Sqrt[Sqrt[-a] + Sqrt[b]]*b^
(3/8)) + (Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] - b^(1/4)]*ArcTan[(Sqrt[Sqrt[Sqrt[-
a] + Sqrt[b]] + b^(1/4)] + Sqrt[2]*b^(1/8)*x)/Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]]
 - b^(1/4)]])/(4*Sqrt[2]*Sqrt[-a]*Sqrt[Sqrt[-a] + Sqrt[b]]*b^(3/8)) + ArcTa
nh[(b^(1/8)*x)/Sqrt[(-a)^(1/4) + b^(1/4)]]/(4*Sqrt[-a]*Sqrt[(-a)^(1/4) + b^
(1/4)]*b^(3/8)) + (Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)]*Log[Sqrt[Sqrt[-
a] + Sqrt[b]] - Sqrt[2]*Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)]*b^(1/8)*x
+ b^(1/4)*x^2])/(8*Sqrt[2]*Sqrt[-a]*Sqrt[Sqrt[-a] + Sqrt[b]]*b^(3/8)) - (Sq
rt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)]*Log[Sqrt[Sqrt[-a] + Sqrt[b]] + Sqrt[
2]*Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)]*b^(1/8)*x + b^(1/4)*x^2])/(8*Sq
rt[2]*Sqrt[-a]*Sqrt[Sqrt[-a] + Sqrt[b]]*b^(3/8))
```

Rubi [A] time = 0.648222, antiderivative size = 663, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {6740, 1990, 1166, 205, 208, 1169, 634, 618, 204, 628}

$$\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\log\left(-\sqrt{2}\sqrt[8]{b}x\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}+\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}x^2\right)}{8\sqrt{2}\sqrt{-a}b^{3/8}\sqrt{\sqrt{-a}+\sqrt{b}}}-\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\log\left(\sqrt{2}\sqrt[8]{b}x\right.}{8\sqrt{2}\sqrt{-}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^2)/(a + b*(-1 + x^2)^4),x]
```

```
[Out] -ArcTan[(b^(1/8)*x)/Sqrt[(-a)^(1/4) - b^(1/4)]]/(4*Sqrt[-a]*Sqrt[(-a)^(1/4)
 - b^(1/4)]*b^(3/8)) - (Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] - b^(1/4)]*ArcTan[(Sq
```

```
rt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)] - Sqrt[2]*b^(1/8)*x)/Sqrt[Sqrt[Sqrt[
-a] + Sqrt[b]] - b^(1/4)]])/(4*Sqrt[2]*Sqrt[-a]*Sqrt[Sqrt[-a] + Sqrt[b]]*b^
(3/8)) + (Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] - b^(1/4)]*ArcTan[(Sqrt[Sqrt[Sqrt[-
a] + Sqrt[b]] + b^(1/4)] + Sqrt[2]*b^(1/8)*x)/Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]]
 - b^(1/4)]])/(4*Sqrt[2]*Sqrt[-a]*Sqrt[Sqrt[-a] + Sqrt[b]]*b^(3/8)) + ArcTa
nh[(b^(1/8)*x)/Sqrt[(-a)^(1/4) + b^(1/4)]]/(4*Sqrt[-a]*Sqrt[(-a)^(1/4) + b^
(1/4)]*b^(3/8)) + (Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)]*Log[Sqrt[Sqrt[-
a] + Sqrt[b]] - Sqrt[2]*Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)]*b^(1/8)*x
+ b^(1/4)*x^2])/(8*Sqrt[2]*Sqrt[-a]*Sqrt[Sqrt[-a] + Sqrt[b]]*b^(3/8)) - (Sq
rt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)]*Log[Sqrt[Sqrt[-a] + Sqrt[b]] + Sqrt[
2]*Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)]*b^(1/8)*x + b^(1/4)*x^2])/(8*Sq
rt[2]*Sqrt[-a]*Sqrt[Sqrt[-a] + Sqrt[b]]*b^(3/8))
```

Rule 6740

```
Int[(v_)/((a_) + (b_.)*(u_)^(n_.)), x_Symbol] :> Int[ExpandIntegrand[Polyno
mialInSubst[v, u, x]/(a + b*x^n), x] /. x -> u, x] /; FreeQ[{a, b}, x] && I
GtQ[n, 0] && PolynomialInQ[v, u, x]
```

Rule 1990

```
Int[(u_)^(q_.)*(v_)^(p_.), x_Symbol] :> Int[ExpandToSum[u, x]^q*ExpandToSum
[v, x]^p, x] /; FreeQ[{p, q}, x] && BinomialQ[u, x] && TrinomialQ[v, x] &&
 !(BinomialMatchQ[u, x] && TrinomialMatchQ[v, x])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1-x^2}{a+b\left(-1+x^2\right)^4}\,dx = -\int \frac{-1+x^2}{a+b\left(-1+x^2\right)^4}\,dx$$

$$= -\int \left(-\frac{\sqrt{b}\left(-1+x^2\right)}{2\sqrt{-a}\left(\sqrt{-a}\sqrt{b}-b\left(-1+x^2\right)^2\right)} - \frac{\sqrt{b}\left(-1+x^2\right)}{2\sqrt{-a}\left(\sqrt{-a}\sqrt{b}+b\left(-1+x^2\right)^2\right)}\right)dx$$

$$= \frac{\sqrt{b}\int \frac{-1+x^2}{\sqrt{-a}\sqrt{b}-b\left(-1+x^2\right)^2}\,dx}{2\sqrt{-a}} + \frac{\sqrt{b}\int \frac{-1+x^2}{\sqrt{-a}\sqrt{b}+b\left(-1+x^2\right)^2}\,dx}{2\sqrt{-a}}$$

$$= \frac{\sqrt{b}\int \frac{-1+x^2}{\left(\sqrt{-a}-\sqrt{b}\right)\sqrt{b}+2bx^2-bx^4}\,dx}{2\sqrt{-a}} + \frac{\sqrt{b}\int \frac{-1+x^2}{\left(\sqrt{-a}+\sqrt{b}\right)\sqrt{b}-2bx^2+bx^4}\,dx}{2\sqrt{-a}}$$

$$= \frac{\int \frac{-\frac{\sqrt{2}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}}{\sqrt[8]{b}}-\left(-1-\frac{\sqrt{\sqrt{-a}+\sqrt{b}}}{\sqrt[4]{b}}\right)x}{\frac{\sqrt{\sqrt{-a}+\sqrt{b}}}{\sqrt[4]{b}}-\frac{\sqrt{2}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}x}{\sqrt[8]{b}}+x^2}\,dx}{4\sqrt{2}\sqrt{-a}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\sqrt{\sqrt{-a}+\sqrt{b}}\sqrt[8]{b}} + \frac{\int \frac{-\frac{\sqrt{2}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}}{\sqrt[8]{b}}+\left(-1-\frac{\sqrt{\sqrt{-a}+\sqrt{b}}}{\sqrt[4]{b}}\right)x}{\frac{\sqrt{\sqrt{-a}+\sqrt{b}}}{\sqrt[4]{b}}+\frac{\sqrt{2}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}x}{\sqrt[8]{b}}+x^2}\,dx}{4\sqrt{2}\sqrt{-a}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\sqrt{\sqrt{-a}+\sqrt{b}}\sqrt[8]{b}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt[8]{b}x}{\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}}\right)}{4\sqrt{-a}\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}b^{3/8}} + \frac{\tanh^{-1}\left(\frac{\sqrt[8]{b}x}{\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}}\right)}{4\sqrt{-a}\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}b^{3/8}} + \frac{\left(1-\frac{\sqrt[4]{b}}{\sqrt{\sqrt{-a}+\sqrt{b}}}\right)\int \frac{1}{\frac{\sqrt{\sqrt{-a}+\sqrt{b}}}{\sqrt[4]{b}}-\frac{\sqrt{2}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}x}{\sqrt[8]{b}}+x \cdots}}{8\sqrt{-a}\sqrt{b}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt[8]{b}x}{\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}}\right)}{4\sqrt{-a}\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}b^{3/8}} + \frac{\tanh^{-1}\left(\frac{\sqrt[8]{b}x}{\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}}\right)}{4\sqrt{-a}\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}b^{3/8}} + \frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\log\left(\sqrt{\sqrt{-a}+\sqrt{b}}-\cdots\right.}{8\sqrt{2}\sqrt{-a}\sqrt{-a \cdots}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt[8]{b}x}{\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}}\right)}{4\sqrt{-a}\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}b^{3/8}} - \frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}\tan^{-1}\left(\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}-\sqrt{2}\sqrt[8]{b}x}{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}}\right)}{4\sqrt{2}\sqrt{-a}\sqrt{\sqrt{-a}+\sqrt{b}}b^{3/8}} + \frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{\cdots}}}}{4\cdots}$$

Mathematica [C] time = 0.0316148, size = 63, normalized size = 0.1

$$-\frac{\text{RootSum}\left[\#1^8b-4\#1^6b+6\#1^4b-4\#1^2b+a+b\&,\frac{\log(x-\#1)}{\#1^5-2\#1^3+\#1}\&\right]}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^2)/(a + b*(-1 + x^2)^4),x]
```

```
[Out] -RootSum[a + b - 4*b*#1^2 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & , Log[x - #1]/(#
1 - 2*#1^3 + #1^5) & ]/(8*b)
```

Maple [C] time = 0.002, size = 69, normalized size = 0.1

$$\frac{1}{8b}\sum_{\_R=\mathrm{RootOf}\left(\_Z^8 b-4\_Z^6 b+6\_Z^4 b-4\_Z^2 b+a+b\right)}\frac{\left(-\_R^2+1\right)\ln\left(x-\_R\right)}{\_R^7-3\_R^5+3\_R^3-\_R}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)/(a+b*(x^2-1)^4),x)
```

```
[Out] 1/8/b*sum((-_R^2+1)/(_R^7-3*_R^5+3*_R^3-_R)*ln(x-_R),_R=RootOf(_Z^8*b-4*_Z^
6*b+6*_Z^4*b-4*_Z^2*b+a+b))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2-1}{\left(x^2-1\right)^4 b+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(a+b*(x^2-1)^4),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 1)/((x^2 - 1)^4*b + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(a+b*(x^2-1)^4),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 3.01024, size = 133, normalized size = 0.2

$$-\operatorname{RootSum}\left(t^8\left(16777216a^5b^3+16777216a^4b^4\right)+1048576t^6a^3b^3+24576t^4a^2b^2+256t^2ab+1,\left(t\mapsto t\log\left(-6291456t^7\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)/(a+b*(x**2-1)**4),x)
```

```
[Out] -RootSum(_t**8*(16777216*a**5*b**3 + 16777216*a**4*b**4) + 1048576*_t**6*a*
*3*b**3 + 24576*_t**4*a**2*b**2 + 256*_t**2*a*b + 1, Lambda(_t, _t*log(-629
1456*_t**7*a**4*b**3 - 6291456*_t**7*a**3*b**4 + 65536*_t**5*a**3*b**2 - 32
7680*_t**5*a**2*b**3 - 512*_t**3*a**2*b - 5632*_t**3*a*b**2 - 32*_t*b + x))
)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2-1}{\left(x^2-1\right)^4 b+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(a+b*(x^2-1)^4),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 - 1)/((x^2 - 1)^4*b + a), x)
```

3.393 $\int \frac{(1+x^2)^2}{ax^6+b(1+x^2)^3} \, dx$

Optimal. Leaf size=168

$$\frac{\tan^{-1}\left(\frac{x\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}}{\sqrt[6]{b}}\right)}{3b^{5/6}\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}}+\frac{\tan^{-1}\left(\frac{x\sqrt{\sqrt[3]{b}-\sqrt[3]{-1}\sqrt[3]{a}}}{\sqrt[6]{b}}\right)}{3b^{5/6}\sqrt{\sqrt[3]{b}-\sqrt[3]{-1}\sqrt[3]{a}}}+\frac{\tan^{-1}\left(\frac{x\sqrt{(-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}}}{\sqrt[6]{b}}\right)}{3b^{5/6}\sqrt{(-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}}}$$

```
[Out] ArcTan[(Sqrt[a^(1/3) + b^(1/3)]*x)/b^(1/6)]/(3*Sqrt[a^(1/3) + b^(1/3)]*b^(5
/6)) + ArcTan[(Sqrt[-((-1)^(1/3)*a^(1/3)) + b^(1/3)]*x)/b^(1/6)]/(3*Sqrt[-(
(-1)^(1/3)*a^(1/3)) + b^(1/3)]*b^(5/6)) + ArcTan[(Sqrt[(-1)^(2/3)*a^(1/3) +
 b^(1/3)]*x)/b^(1/6)]/(3*Sqrt[(-1)^(2/3)*a^(1/3) + b^(1/3)]*b^(5/6))
```

Rubi [F] time = 0.380003, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{(1+x^2)^2}{ax^6+b(1+x^2)^3} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 + x^2)^2/(a*x^6 + b*(1 + x^2)^3),x]
```

```
[Out] Defer[Int][(a*x^6 + b*(1 + x^2)^3)^(-1), x] + 2*Defer[Int][x^2/(a*x^6 + b*(
1 + x^2)^3), x] + Defer[Int][x^4/(a*x^6 + b*(1 + x^2)^3), x]
```

Rubi steps

$$\int \frac{(1+x^2)^2}{ax^6+b(1+x^2)^3}\,dx = \int \left(\frac{1}{b+3bx^2+3bx^4+a\left(1+\frac{b}{a}\right)x^6} + \frac{2x^2}{b+3bx^2+3bx^4+a\left(1+\frac{b}{a}\right)x^6} + \frac{x^4}{b+3bx^2+3bx^4+a\left(1\right.}\right.$$

$$= 2\int \frac{x^2}{b+3bx^2+3bx^4+a\left(1+\frac{b}{a}\right)x^6}\,dx + \int \frac{1}{b+3bx^2+3bx^4+a\left(1+\frac{b}{a}\right)x^6}\,dx + \int \frac{}{b+3bx^2+}$$

$$= 2\int \frac{x^2}{ax^6+b(1+x^2)^3}\,dx + \int \frac{1}{ax^6+b(1+x^2)^3}\,dx + \int \frac{x^4}{ax^6+b(1+x^2)^3}\,dx$$

Mathematica [C] time = 0.0681047, size = 95, normalized size = 0.57

$$\frac{1}{6}\text{RootSum}\left[\#1^6a+\#1^6b+3\#1^4b+3\#1^2b+b\&,\frac{\#1^4\log(x-\#1)+2\#1^2\log(x-\#1)+\log(x-\#1)}{\#1^5a+\#1^5b+2\#1^3b+\#1b}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)^2/(a*x^6 + b*(1 + x^2)^3),x]
```

```
[Out] RootSum[b + 3*b*#1^2 + 3*b*#1^4 + a*#1^6 + b*#1^6 & , (Log[x - #1] + 2*Log[
x - #1]*#1^2 + Log[x - #1]*#1^4)/(b*#1 + 2*b*#1^3 + a*#1^5 + b*#1^5) & ]/6
```

Maple [C] time = 0.253, size = 67, normalized size = 0.4

$$\frac{1}{6}\sum_{\_R=\text{RootOf}\left((a+b)\_Z^6+3b\_Z^4+3b\_Z^2+b\right)}\frac{\left(\_R^4+2\_R^2+1\right)\ln(x-\_R)}{\_R^5a+\_R^5b+2\_R^3b+\_Rb}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)^2/(a*x^6+b*(x^2+1)^3),x)
```

```
[Out] 1/6*sum((_R^4+2*_R^2+1)/(_R^5*a+_R^5*b+2*_R^3*b+_R*b)*ln(x-_R),_R=RootOf((a
+b)*_Z^6+3*b*_Z^4+3*b*_Z^2+b))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x^2+1)^2}{ax^6+(x^2+1)^3 b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^2/(a*x^6+b*(x^2+1)^3),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + 1)^2/(a*x^6 + (x^2 + 1)^3*b), x)
```

Fricas [C] time = 10.2428, size = 14804, normalized size = 88.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^2/(a*x^6+b*(x^2+1)^3),x, algorithm="fricas")
```

```
[Out] 1/36*sqrt(1/2)*sqrt((-I*sqrt(3) + 1)*(1/(a*b^3 + b^4) - 1/(a*b + b^2)^2)/(-
1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b
+ b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 1296*(I*sqrt(3) + 1)*(-1/9331
2/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)
^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 72/(a*b + b^2))*log(1/6*sqrt(1/2)*s
qrt((-I*sqrt(3) + 1)*(1/(a*b^3 + b^4) - 1/(a*b + b^2)^2)/(-1/93312/(a*b^5 +
 b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/933
12*a/((a + b)^2*b^5))^(1/3) - 1296*(I*sqrt(3) + 1)*(-1/93312/(a*b^5 + b^6)
+ 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/(
(a + b)^2*b^5))^(1/3) - 72/(a*b + b^2))*b + x) - 1/36*sqrt(1/2)*sqrt((-I*sq
rt(3) + 1)*(1/(a*b^3 + b^4) - 1/(a*b + b^2)^2)/(-1/93312/(a*b^5 + b^6) + 1/
31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a +
 b)^2*b^5))^(1/3) - 1296*(I*sqrt(3) + 1)*(-1/93312/(a*b^5 + b^6) + 1/31104/
((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*
b^5))^(1/3) - 72/(a*b + b^2))*log(-1/6*sqrt(1/2)*sqrt((-I*sqrt(3) + 1)*(1/(
a*b^3 + b^4) - 1/(a*b + b^2)^2)/(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 +
 b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/
3) - 1296*(I*sqrt(3) + 1)*(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*
(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 7
2/(a*b + b^2))*b + x) + 1/72*sqrt(-((a*b + b^2)*((-I*sqrt(3) + 1)*(1/(a*b^3
 + b^4) - 1/(a*b + b^2)^2)/(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)
*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) -
```

1296*(I*sqrt(3) + 1)*(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 72/(a*b + b^2)) + 3*sqrt(1/3)*(a*b + b^2)*sqrt(-((a^2*b^3 + 2*a*b^4 + b^5)*((-I*sqrt(3) + 1)*(1/(a*b^3 + b^4) - 1/(a*b + b^2)^2)/(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 1296*(I*sqrt(3) + 1)*(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 72/(a*b + b^2))^2 + 144*(a*b^2 + b^3)*((-I*sqrt(3) + 1)*(1/(a*b^3 + b^4) - 1/(a*b + b^2)^2)/(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 1296*(I*sqrt(3) + 1)*(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 72/(a*b + b^2)) + 20736*a + 5184*b)/(a^2*b^3 + 2*a*b^4 + b^5)) + 216)/(a*b + b^2))*log(1/12*b*sqrt(-((a*b + b^2)*((-I*sqrt(3) + 1)*(1/(a*b^3 + b^4) - 1/(a*b + b^2)^2)/(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 1296*(I*sqrt(3) + 1)*(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 72/(a*b + b^2)) + 3*sqrt(1/3)*(a*b + b^2)*sqrt(-((a^2*b^3 + 2*a*b^4 + b^5)*((-I*sqrt(3) + 1)*(1/(a*b^3 + b^4) - 1/(a*b + b^2)^2)/(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 1296*(I*sqrt(3) + 1)*(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 72/(a*b + b^2))^2 + 144*(a*b^2 + b^3)*((-I*sqrt(3) + 1)*(1/(a*b^3 + b^4) - 1/(a*b + b^2)^2)/(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 1296*(I*sqrt(3) + 1)*(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 72/(a*b + b^2)) + 20736*a + 5184*b)/(a^2*b^3 + 2*a*b^4 + b^5)) + 216)/(a*b + b^2)) + x) - 1/72*sqrt(-((a*b + b^2)*((-I*sqrt(3) + 1)*(1/(a*b^3 + b^4) - 1/(a*b + b^2)^2)/(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 1296*(I*sqrt(3) + 1)*(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 72/(a*b + b^2)) + 3*sqrt(1/3)*(a*b + b^2)*sqrt(-((a^2*b^3 + 2*a*b^4 + b^5)*((-I*sqrt(3) + 1)*(1/(a*b^3 + b^4) - 1/(a*b + b^2)^2)/(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 1296*(I*sqrt(3) + 1)*(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 72/(a*b + b^2))^2 + 144*(a*b^2 + b^3)*((-I*sqrt(3) + 1)*(1/(a*b^3 + b^4) - 1/(a*b + b^2)^2)/(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 1296*(I*sqrt(3) + 1)*(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 72/(a*b + b^2)) + 20736*a + 5184*b)/(a^2*b^3 + 2*a*b^4 + b^5)) + 216)/(a*b + b^2))*log(-1/12*b*sq

rt(-((a*b + b^2)*((-I*sqrt(3) + 1)*(1/(a*b^3 + b^4) - 1/(a*b + b^2)^2)/(-1/
93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b +
b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 1296*(I*sqrt(3) + 1)*(-1/93312/
(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3
 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 72/(a*b + b^2)) + 3*sqrt(1/3)*(a*b +
b^2)*sqrt(-((a^2*b^3 + 2*a*b^4 + b^5)*((-I*sqrt(3) + 1)*(1/(a*b^3 + b^4) -
1/(a*b + b^2)^2)/(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^
2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 1296*(I*sq
rt(3) + 1)*(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) -
1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 72/(a*b + b^2))^
2 + 144*(a*b^2 + b^3)*((-I*sqrt(3) + 1)*(1/(a*b^3 + b^4) - 1/(a*b + b^2)^2)
/(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a
*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 1296*(I*sqrt(3) + 1)*(-1/9
3312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b
^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 72/(a*b + b^2)) + 20736*a + 5184
*b)/(a^2*b^3 + 2*a*b^4 + b^5)) + 216)/(a*b + b^2)) + x) + 1/72*sqrt(-((a*b
 + b^2)*((-I*sqrt(3) + 1)*(1/(a*b^3 + b^4) - 1/(a*b + b^2)^2)/(-1/93312/(a*b
^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1
/93312*a/((a + b)^2*b^5))^(1/3) - 1296*(I*sqrt(3) + 1)*(-1/93312/(a*b^5 + b
^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312
*a/((a + b)^2*b^5))^(1/3) - 72/(a*b + b^2)) - 3*sqrt(1/3)*(a*b + b^2)*sqrt(
-((a^2*b^3 + 2*a*b^4 + b^5)*((-I*sqrt(3) + 1)*(1/(a*b^3 + b^4) - 1/(a*b + b
^2)^2)/(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46
656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 1296*(I*sqrt(3) + 1)
*(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a
*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 72/(a*b + b^2))^2 + 144*(a
b^2 + b^3)((-I*sqrt(3) + 1)*(1/(a*b^3 + b^4) - 1/(a*b + b^2)^2)/(-1/93312
/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^
3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 1296*(I*sqrt(3) + 1)*(-1/93312/(a*b^
5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/
93312*a/((a + b)^2*b^5))^(1/3) - 72/(a*b + b^2)) + 20736*a + 5184*b)/(a^2*b
^3 + 2*a*b^4 + b^5)) + 216)/(a*b + b^2))*log(1/12*b*sqrt(-((a*b + b^2)*((-I
sqrt(3) + 1)(1/(a*b^3 + b^4) - 1/(a*b + b^2)^2)/(-1/93312/(a*b^5 + b^6) +
 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((
a + b)^2*b^5))^(1/3) - 1296*(I*sqrt(3) + 1)*(-1/93312/(a*b^5 + b^6) + 1/311
04/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)
^2*b^5))^(1/3) - 72/(a*b + b^2)) - 3*sqrt(1/3)*(a*b + b^2)*sqrt(-((a^2*b^3
 + 2*a*b^4 + b^5)*((-I*sqrt(3) + 1)*(1/(a*b^3 + b^4) - 1/(a*b + b^2)^2)/(-1/
93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b +
b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 1296*(I*sqrt(3) + 1)*(-1/93312/
(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3
 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 72/(a*b + b^2))^2 + 144*(a*b^2 + b^3)
*((-I*sqrt(3) + 1)*(1/(a*b^3 + b^4) - 1/(a*b + b^2)^2)/(-1/93312/(a*b^5 + b
^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312
*a/((a + b)^2*b^5))^(1/3) - 1296*(I*sqrt(3) + 1)*(-1/93312/(a*b^5 + b^6) +

```
1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a
 + b)^2*b^5))^(1/3) - 72/(a*b + b^2)) + 20736*a + 5184*b)/(a^2*b^3 + 2*a*b^
4 + b^5)) + 216)/(a*b + b^2)) + x) - 1/72*sqrt(-((a*b + b^2)*((-I*sqrt(3) +
 1)*(1/(a*b^3 + b^4) - 1/(a*b + b^2)^2)/(-1/93312/(a*b^5 + b^6) + 1/31104/(
(a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b
^5))^(1/3) - 1296*(I*sqrt(3) + 1)*(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3
 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(
1/3) - 72/(a*b + b^2)) - 3*sqrt(1/3)*(a*b + b^2)*sqrt(-((a^2*b^3 + 2*a*b^4
+ b^5)*((-I*sqrt(3) + 1)*(1/(a*b^3 + b^4) - 1/(a*b + b^2)^2)/(-1/93312/(a*b
^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1
/93312*a/((a + b)^2*b^5))^(1/3) - 1296*(I*sqrt(3) + 1)*(-1/93312/(a*b^5 + b
^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312
*a/((a + b)^2*b^5))^(1/3) - 72/(a*b + b^2))^2 + 144*(a*b^2 + b^3)*((-I*sqrt
(3) + 1)*(1/(a*b^3 + b^4) - 1/(a*b + b^2)^2)/(-1/93312/(a*b^5 + b^6) + 1/31
104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b
)^2*b^5))^(1/3) - 1296*(I*sqrt(3) + 1)*(-1/93312/(a*b^5 + b^6) + 1/31104/((
a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^
5))^(1/3) - 72/(a*b + b^2)) + 20736*a + 5184*b)/(a^2*b^3 + 2*a*b^4 + b^5))
+ 216)/(a*b + b^2))*log(-1/12*b*sqrt(-((a*b + b^2)*((-I*sqrt(3) + 1)*(1/(a*
b^3 + b^4) - 1/(a*b + b^2)^2)/(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b
^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3)
 - 1296*(I*sqrt(3) + 1)*(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a
*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 72/
(a*b + b^2)) - 3*sqrt(1/3)*(a*b + b^2)*sqrt(-((a^2*b^3 + 2*a*b^4 + b^5)*((-
I*sqrt(3) + 1)*(1/(a*b^3 + b^4) - 1/(a*b + b^2)^2)/(-1/93312/(a*b^5 + b^6)
+ 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/(
(a + b)^2*b^5))^(1/3) - 1296*(I*sqrt(3) + 1)*(-1/93312/(a*b^5 + b^6) + 1/31
104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b
)^2*b^5))^(1/3) - 72/(a*b + b^2))^2 + 144*(a*b^2 + b^3)*((-I*sqrt(3) + 1)*(
1/(a*b^3 + b^4) - 1/(a*b + b^2)^2)/(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^
3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^
(1/3) - 1296*(I*sqrt(3) + 1)*(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^
4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3)
- 72/(a*b + b^2)) + 20736*a + 5184*b)/(a^2*b^3 + 2*a*b^4 + b^5)) + 216)/(a*
b + b^2)) + x)
```

Sympy [A] time = 1.67096, size = 42, normalized size = 0.25

$$\operatorname{RootSum}\left(t^{6}\left(46656ab^{5}+46656b^{6}\right)+3888t^{4}b^{4}+108t^{2}b^{2}+1,\left(t\mapsto t\log\left(6tb+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)**2/(a*x**6+b*(x**2+1)**3),x)
```

```
[Out] RootSum(_t**6*(46656*a*b**5 + 46656*b**6) + 3888*_t**4*b**4 + 108*_t**2*b**
2 + 1, Lambda(_t, _t*log(6*_t*b + x)))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2+1\right)^2}{ax^6+\left(x^2+1\right)^3 b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^2/(a*x^6+b*(x^2+1)^3),x, algorithm="giac")
```

```
[Out] integrate((x^2 + 1)^2/(a*x^6 + (x^2 + 1)^3*b), x)
```

3.394 $\int \frac{(d+ex)^3}{a+cx^4} \, dx$

Optimal. Leaf size=320

$$-\frac{d\left(\sqrt{c}d^2-3\sqrt{a}e^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{3/4}}+\frac{d\left(\sqrt{c}d^2-3\sqrt{a}e^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{3/4}}-\frac{d\left(3\sqrt{a}e^2+\sqrt{c}\right.}{2}$$

```
[Out] (3*d^2*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[c]) - (d*(Sqrt[c]*d
^2 + 3*Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(
3/4)*c^(3/4)) + (d*(Sqrt[c]*d^2 + 3*Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4
)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(3/4)) - (d*(Sqrt[c]*d^2 - 3*Sqrt[a]*e^
2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/
4)*c^(3/4)) + (d*(Sqrt[c]*d^2 - 3*Sqrt[a]*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4
)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(3/4)) + (e^3*Log[a + c*x^
4])/(4*c)
```

Rubi [A] time = 0.256101, antiderivative size = 320, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 11, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.647, Rules used = {1876, 1168, 1162, 617, 204, 1165, 628, 1248, 635, 205, 260}

$$-\frac{d\left(\sqrt{c}d^2-3\sqrt{a}e^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{3/4}}+\frac{d\left(\sqrt{c}d^2-3\sqrt{a}e^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{3/4}}-\frac{d\left(3\sqrt{a}e^2+\sqrt{c}\right.}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(a + c*x^4),x]
```


```
[Out] (3*d^2*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[c]) - (d*(Sqrt[c]*d
^2 + 3*Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(
3/4)*c^(3/4)) + (d*(Sqrt[c]*d^2 + 3*Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4
)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(3/4)) - (d*(Sqrt[c]*d^2 - 3*Sqrt[a]*e^
2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/
4)*c^(3/4)) + (d*(Sqrt[c]*d^2 - 3*Sqrt[a]*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4
)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(3/4)) + (e^3*Log[a + c*x^
4])/(4*c)
```


Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
```

```
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{(d+ex)^3}{a+cx^4}\,dx = \int \left(\frac{d^3+3de^2x^2}{a+cx^4}+\frac{x\left(3d^2e+e^3x^2\right)}{a+cx^4}\right)dx$$

$$= \int \frac{d^3+3de^2x^2}{a+cx^4}\,dx + \int \frac{x\left(3d^2e+e^3x^2\right)}{a+cx^4}\,dx$$

$$= \frac{1}{2}\,\text{Subst}\left(\int \frac{3d^2e+e^3x}{a+cx^2}\,dx, x, x^2\right) + \frac{\left(d\left(\frac{\sqrt{c}d^2}{\sqrt{a}}-3e^2\right)\right)\int \frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4}\,dx}{2c} + \frac{\left(d\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+3e^2\right)\right)\int \frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4}\,dx}{2c}$$

$$= \frac{1}{2}\left(3d^2e\right)\text{Subst}\left(\int \frac{1}{a+cx^2}\,dx, x, x^2\right) + \frac{1}{2}e^3\,\text{Subst}\left(\int \frac{x}{a+cx^2}\,dx, x, x^2\right) + \frac{\left(d\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+3e^2\right)\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}\cdots}}{4c}$$

$$= \frac{3d^2e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{c}} - \frac{d\left(\sqrt{c}d^2-3\sqrt{a}e^2\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{3/4}} + \frac{d\left(\sqrt{c}d^2-3\sqrt{a}e^2\right)\log\left(\sqrt{a}+\cdots\right.}{4\sqrt{2}a^{3/4}c^{3/4}}$$

$$= \frac{3d^2e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{c}} - \frac{d\left(\sqrt{c}d^2+3\sqrt{a}e^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}c^{3/4}} + \frac{d\left(\sqrt{c}d^2+3\sqrt{a}e^2\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}c^{3/4}} - \frac{d\cdots}{}$$

Mathematica [A] time = 0.242222, size = 322, normalized size = 1.01

$$\frac{-\sqrt{2}\sqrt[4]{c}\left(\sqrt[4]{a}\sqrt{c}d^3-3a^{3/4}de^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)+\sqrt{2}\sqrt[4]{c}\left(\sqrt[4]{a}\sqrt{c}d^3-3a^{3/4}de^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(a + c*x^4),x]
```

```
[Out] (-2*a^(1/4)*c^(1/4)*d*(Sqrt[2]*Sqrt[c]*d^2 + 6*a^(1/4)*c^(1/4)*d*e + 3*Sqrt
[2]*Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*a^(1/4)*c^(1/4
)*d*(Sqrt[2]*Sqrt[c]*d^2 - 6*a^(1/4)*c^(1/4)*d*e + 3*Sqrt[2]*Sqrt[a]*e^2)*A
rcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] - Sqrt[2]*c^(1/4)*(a^(1/4)*Sqrt[c]*d
^3 - 3*a^(3/4)*d*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2
] + Sqrt[2]*c^(1/4)*(a^(1/4)*Sqrt[c]*d^3 - 3*a^(3/4)*d*e^2)*Log[Sqrt[a] + S
qrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + 2*a*e^3*Log[a + c*x^4])/(8*a*c)
```

Maple [A] time = 0.009, size = 314, normalized size = 1.

$$\frac{d^3\sqrt{2}}{8a}\sqrt[4]{\frac{a}{c}}\ln\left(\left(x^2+\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)\left(x^2-\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)^{-1}\right)+\frac{d^3\sqrt{2}}{4a}\sqrt[4]{\frac{a}{c}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{c}}}+1\right)+\frac{d^3\sqrt{2}}{4a}\sqrt[4]{\frac{a}{c}}\arctan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(c*x^4+a),x)
```

```
[Out] 1/8*d^3*(a/c)^(1/4)/a*2^(1/2)*ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x
^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+1/4*d^3*(a/c)^(1/4)/a*2^(1/2)*arctan
(2^(1/2)/(a/c)^(1/4)*x+1)+1/4*d^3*(a/c)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/c
)^(1/4)*x-1)+3/2*e*d^2/(a*c)^(1/2)*arctan(x^2*(1/a*c)^(1/2))+3/8*d*e^2/c/(a
/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c)^(1/
4)*x*2^(1/2)+(a/c)^(1/2)))+3/4*d*e^2/c/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(
a/c)^(1/4)*x+1)+3/4*d*e^2/c/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*
x-1)+1/4*e^3*ln(c*x^4+a)/c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 3.697, size = 384, normalized size = 1.2

$$\operatorname{RootSum}\left(256t^4a^3c^4 - 256t^3a^3c^3e^3 + t^2\left(96a^3c^2e^6 + 480a^2c^3d^4e^2\right) + t\left(-16a^3ce^9 + 192a^2c^2d^4e^5 - 48ac^3d^8e\right) + a^3e^{12} + 3a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(c*x**4+a),x)
```

```
[Out] RootSum(256*_t**4*a**3*c**4 - 256*_t**3*a**3*c**3*e**3 + _t**2*(96*a**3*c**
2*e**6 + 480*a**2*c**3*d**4*e**2) + _t*(-16*a**3*c*e**9 + 192*a**2*c**2*d**
4*e**5 - 48*a*c**3*d**8*e) + a**3*e**12 + 3*a**2*c*d**4*e**8 + 3*a*c**2*d**
8*e**4 + c**3*d**12, Lambda(_t, _t*log(x + (1728*_t**3*a**4*c**3*e**6 + 960
*_t**3*a**3*c**4*d**4*e**2 - 1296*_t**2*a**4*c**2*e**9 - 2016*_t**2*a**3*c*
*3*d**4*e**5 + 48*_t**2*a**2*c**4*d**8*e + 324*_t*a**4*c*e**12 + 4716*_t*a*
*3*c**2*d**4*e**8 + 1452*_t*a**2*c**3*d**8*e**4 + 4*_t*a*c**4*d**12 - 27*a*
*4*e**15 + 1119*a**3*c*d**4*e**11 - 609*a**2*c**2*d**8*e**7 - 91*a*c**3*d**
12*e**3)/(729*a**3*c*d**3*e**12 - 1053*a**2*c**2*d**7*e**8 - 117*a*c**3*d**
11*e**4 + c**4*d**15))))
```

Giac [A] time = 1.2005, size = 420, normalized size = 1.31

$$\frac{e^3 \log\left(\left|cx^4+a\right|\right)}{4\,c}+\frac{\sqrt{2}\left(3\,\sqrt{2}\sqrt{ac}c^2d^2e+\left(ac^3\right)^{\frac{1}{4}}c^2d^3+3\,\left(ac^3\right)^{\frac{3}{4}}de^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac^3}+\frac{\sqrt{2}\left(3\,\sqrt{2}\sqrt{ac}c^2d^2e+\left(a\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*e^3*log(abs(c*x^4 + a))/c + 1/4*sqrt(2)*(3*sqrt(2)*sqrt(a*c)*c^2*d^2*e
+ (a*c^3)^(1/4)*c^2*d^3 + 3*(a*c^3)^(3/4)*d*e^2)*arctan(1/2*sqrt(2)*(2*x +
sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^3) + 1/4*sqrt(2)*(3*sqrt(2)*sqrt(a*c
)*c^2*d^2*e + (a*c^3)^(1/4)*c^2*d^3 + 3*(a*c^3)^(3/4)*d*e^2)*arctan(1/2*sqr
t(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^3) + 1/8*sqrt(2)*((a*c^3
)^(1/4)*c^2*d^3 - 3*(a*c^3)^(3/4)*d*e^2)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) +
sqrt(a/c))/(a*c^3) - 1/8*sqrt(2)*((a*c^3)^(1/4)*c^2*d^3 - 3*(a*c^3)^(3/4)*d
*e^2)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c^3)
```

3.395 $\int \frac{(d+ex)^2}{a+cx^4}\,dx$

Optimal. Leaf size=291

$$-\frac{\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{3/4}}+\frac{\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{3/4}}-\frac{\left(\sqrt{a}e^2+\sqrt{c}d^2\right)\tan^{-}}{2\sqrt{2}a^{3/4}c}$$

```
[Out] (d*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(Sqrt[a]*Sqrt[c]) - ((Sqrt[c]*d^2 + Sqr
t[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(3/
4)) + ((Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])
/(2*Sqrt[2]*a^(3/4)*c^(3/4)) - ((Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] - S
qrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(3/4)) + ((Sq
rt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*
x^2])/(4*Sqrt[2]*a^(3/4)*c^(3/4))
```

Rubi [A] time = 0.198006, antiderivative size = 291, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{3/4}}+\frac{\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{3/4}}-\frac{\left(\sqrt{a}e^2+\sqrt{c}d^2\right)\tan^{-}}{2\sqrt{2}a^{3/4}c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(a + c*x^4),x]
```

```
[Out] (d*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(Sqrt[a]*Sqrt[c]) - ((Sqrt[c]*d^2 + Sqr
t[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(3/
4)) + ((Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])
/(2*Sqrt[2]*a^(3/4)*c^(3/4)) - ((Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] - S
qrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(3/4)) + ((Sq
rt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*
x^2])/(4*Sqrt[2]*a^(3/4)*c^(3/4))
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
```

```
0] && Expon[Pq, x] < n
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(d+ex)^2}{a+cx^4}\,dx = \int \left(\frac{2dex}{a+cx^4}+\frac{d^2+e^2x^2}{a+cx^4}\right)dx$$

$$= (2de)\int \frac{x}{a+cx^4}\,dx + \int \frac{d^2+e^2x^2}{a+cx^4}\,dx$$

$$= (de)\,\mathrm{Subst}\left(\int \frac{1}{a+cx^2}\,dx, x, x^2\right) + \frac{\left(\frac{\sqrt{c}d^2}{\sqrt{a}}-e^2\right)\int \frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4}\,dx}{2c} + \frac{\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)\int \frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4}\,dx}{2c}$$

$$= \frac{de\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{c}} + \frac{\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4c} + \frac{\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4c} - \frac{\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\int -}{4\sqrt{2}a^{3/4}}$$

$$= \frac{de\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{c}} - \frac{\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{3/4}} + \frac{\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\right.}{4\sqrt{2}a^{3/4}c^{3/4}}$$

$$= \frac{de\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{c}} - \frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}c^{3/4}} + \frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}c^{3/4}} - \frac{\left(\sqrt{c}d^2-\right.}{}$$

Mathematica [A] time = 0.103288, size = 243, normalized size = 0.84

$$\frac{-\sqrt{2}\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\left(\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)-\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)\right)-2\left(4\sqrt[4]{a}\sqrt[4]{c}de+\sqrt{2}\sqrt{a}e^2+\sqrt{2}\sqrt{c}\right.}{8a^{3/4}c^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(a + c*x^4),x]
```

```
[Out] (-2*(Sqrt[2]*Sqrt[c]*d^2 + 4*a^(1/4)*c^(1/4)*d*e + Sqrt[2]*Sqrt[a]*e^2)*Arc
Tan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*(Sqrt[2]*Sqrt[c]*d^2 - 4*a^(1/4)*c
```

```
^(1/4)*d*e + Sqrt[2]*Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] -
 Sqrt[2]*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*(Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)
*x + Sqrt[c]*x^2] - Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])
)/(8*a^(3/4)*c^(3/4))
```

Maple [A] time = 0.003, size = 292, normalized size = 1.

$$\frac{d^2\sqrt{2}}{8a}\sqrt[4]{\frac{a}{c}}\ln\left(\left(x^2+\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)\left(x^2-\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)^{-1}\right)+\frac{d^2\sqrt{2}}{4a}\sqrt[4]{\frac{a}{c}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{c}}}+1\right)+\frac{d^2\sqrt{2}}{4a}\sqrt[4]{\frac{a}{c}}\arctan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(c*x^4+a),x)
```

```
[Out] 1/8*d^2*(a/c)^(1/4)/a*2^(1/2)*ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x
^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+1/4*d^2*(a/c)^(1/4)/a*2^(1/2)*arctan
(2^(1/2)/(a/c)^(1/4)*x+1)+1/4*d^2*(a/c)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/c
)^(1/4)*x-1)+d*e/(a*c)^(1/2)*arctan(x^2*(1/a*c)^(1/2))+1/8*e^2/c/(a/c)^(1/4
)*2^(1/2)*ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*x*2^(
1/2)+(a/c)^(1/2)))+1/4*e^2/c/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)
*x+1)+1/4*e^2/c/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 2.56115, size = 277, normalized size = 0.95

$$\operatorname{RootSum}\left(256t^4a^3c^3 + 192t^2a^2c^2d^2e^2 + t\left(32a^2cde^5 - 32ac^2d^5e\right) + a^2e^8 + 2acd^4e^4 + c^2d^8, \left(t \mapsto t\log\left(x + \frac{64t^3a^4c^2e^6 + 4\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(c*x**4+a),x)
```

```
[Out] RootSum(256*_t**4*a**3*c**3 + 192*_t**2*a**2*c**2*d**2*e**2 + _t*(32*a**2*c
*d*e**5 - 32*a*c**2*d**5*e) + a**2*e**8 + 2*a*c*d**4*e**4 + c**2*d**8, Lamb
da(_t, _t*log(x + (64*_t**3*a**4*c**2*e**6 + 448*_t**3*a**3*c**3*d**4*e**2
- 160*_t**2*a**3*c**2*d**3*e**5 + 32*_t**2*a**2*c**3*d**7*e + 60*_t*a**3*c*
d**2*e**8 + 256*_t*a**2*c**2*d**6*e**4 + 4*_t*a*c**3*d**10 + 6*a**3*d*e**11
 - 24*a**2*c*d**5*e**7 - 30*a*c**2*d**9*e**3)/(a**3*e**12 - 33*a**2*c*d**4*
e**8 - 33*a*c**2*d**8*e**4 + c**3*d**12))))
```

Giac [A] time = 1.11494, size = 385, normalized size = 1.32

$$\frac{\sqrt{2}\left(2\sqrt{2}\sqrt{ac}c^2de + \left(ac^3\right)^{\frac{1}{4}}c^2d^2 + \left(ac^3\right)^{\frac{3}{4}}e^2\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4ac^3} + \frac{\sqrt{2}\left(2\sqrt{2}\sqrt{ac}c^2de + \left(ac^3\right)^{\frac{1}{4}}c^2d^2 + \left(ac^3\right)^{\frac{3}{4}}e^2\right)\operatorname{arct}}{4ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*(2*sqrt(2)*sqrt(a*c)*c^2*d*e + (a*c^3)^(1/4)*c^2*d^2 + (a*c^3)^
(3/4)*e^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c
^3) + 1/4*sqrt(2)*(2*sqrt(2)*sqrt(a*c)*c^2*d*e + (a*c^3)^(1/4)*c^2*d^2 + (a
*c^3)^(3/4)*e^2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4)
)/(a*c^3) + 1/8*sqrt(2)*((a*c^3)^(1/4)*c^2*d^2 - (a*c^3)^(3/4)*e^2)*log(x^2
 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c^3) - 1/8*sqrt(2)*((a*c^3)^(1/4)*
c^2*d^2 - (a*c^3)^(3/4)*e^2)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(
```

```
a*c^3)
```

3.396 $\int \frac{d+ex}{a+cx^4}\,dx$

Optimal. Leaf size=219

$$-\frac{d\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}}+\frac{d\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}}-\frac{d\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}}+\frac{d\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}}+\frac{e}{-}$$

```
[Out] (e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[c]) - (d*ArcTan[1 - (Sqrt
[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(1/4)) + (d*ArcTan[1 + (Sqrt[
2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(1/4)) - (d*Log[Sqrt[a] - Sqrt
[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(1/4)) + (d*Log[
Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(1
/4))
```

Rubi [A] time = 0.173949, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {1876, 211, 1165, 628, 1162, 617, 204, 275, 205}

$$-\frac{d\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}}+\frac{d\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}}-\frac{d\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}}+\frac{d\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}}+\frac{e}{-}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(a + c*x^4),x]
```



```
[Out] (e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[c]) - (d*ArcTan[1 - (Sqrt
[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(1/4)) + (d*ArcTan[1 + (Sqrt[
2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(1/4)) - (d*Log[Sqrt[a] - Sqrt
[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(1/4)) + (d*Log[
Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(1
/4))
```


Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
```

```
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{d+ex}{a+cx^4}\,dx &= \int \left(\frac{d}{a+cx^4}+\frac{ex}{a+cx^4}\right)dx\\
&= d\int \frac{1}{a+cx^4}\,dx+e\int \frac{x}{a+cx^4}\,dx\\
&= \frac{d\int \frac{\sqrt{a}-\sqrt{c}x^2}{a+cx^4}\,dx}{2\sqrt{a}}+\frac{d\int \frac{\sqrt{a}+\sqrt{c}x^2}{a+cx^4}\,dx}{2\sqrt{a}}+\frac{1}{2}e\,\text{Subst}\left(\int \frac{1}{a+cx^2}\,dx,x,x^2\right)\\
&= \frac{e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{c}}+\frac{d\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4\sqrt{a}\sqrt{c}}+\frac{d\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4\sqrt{a}\sqrt{c}}-\frac{d\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}a^{3/4}\sqrt[4]{c}}-\frac{d\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,d
\\
&= \frac{e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{c}}-\frac{d\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}}+\frac{d\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}}+\frac{d\,\text{Subst}\left(\int \frac{1}{-1-x^2}\right.}{2\sqrt{2}a}\\
&= \frac{e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{c}}-\frac{d\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}}+\frac{d\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}}-\frac{d\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}}+\frac{d\log
\end{aligned}$$

Mathematica [A] time = 0.057691, size = 184, normalized size = 0.84

$$\frac{-2\left(2\sqrt[4]{a}e+\sqrt{2}\sqrt[4]{c}d\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)+2\left(\sqrt{2}\sqrt[4]{c}d-2\sqrt[4]{a}e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)+\sqrt{2}\sqrt[4]{c}d\left(\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right.\right.}{8a^{3/4}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(a + c*x^4),x]
```

```
[Out] (-2*(Sqrt[2]*c^(1/4)*d + 2*a^(1/4)*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4
)] + 2*(Sqrt[2]*c^(1/4)*d - 2*a^(1/4)*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(
1/4)] + Sqrt[2]*c^(1/4)*d*(-Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[
```

```
c]*x^2] + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]))/(8*a^(3/
4)*Sqrt[c])
```

Maple [A] time = 0.003, size = 151, normalized size = 0.7

$$\frac{d\sqrt{2}}{8a}\sqrt[4]{\frac{a}{c}}\ln\left(\left(x^2+\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)\left(x^2-\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)^{-1}\right)+\frac{d\sqrt{2}}{4a}\sqrt[4]{\frac{a}{c}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{c}}}+1\right)+\frac{d\sqrt{2}}{4a}\sqrt[4]{\frac{a}{c}}\arctan\left(x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(c*x^4+a),x)
```

```
[Out] 1/8*d*(a/c)^(1/4)/a*2^(1/2)*ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2
-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+1/4*d*(a/c)^(1/4)/a*2^(1/2)*arctan(2^(
1/2)/(a/c)^(1/4)*x+1)+1/4*d*(a/c)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4
)*x-1)+1/2*e/(a*c)^(1/2)*arctan(x^2*(1/a*c)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 0.791425, size = 124, normalized size = 0.57

$$\operatorname{RootSum}\left(256t^4a^3c^2 + 32t^2a^2ce^2 - 16tacd^2e + ae^4 + cd^4, \left(t \mapsto t\log\left(x + \frac{-128t^3a^3ce^2 - 16t^2a^2cd^2e - 8ta^2e^4 - 4tacd^4 +}{4ade^4 - cd^5}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x**4+a),x)
```

```
[Out] RootSum(256*_t**4*a**3*c**2 + 32*_t**2*a**2*c*e**2 - 16*_t*a*c*d**2*e + a*e
**4 + c*d**4, Lambda(_t, _t*log(x + (-128*_t**3*a**3*c*e**2 - 16*_t**2*a**2
*c*d**2*e - 8*_t*a**2*e**4 - 4*_t*a*c*d**4 + 5*a*d**2*e**3)/(4*a*d*e**4 - c
*d**5))))
```

Giac [A] time = 1.18158, size = 290, normalized size = 1.32

$$\frac{\sqrt{2}\left(ac^3\right)^{\frac{1}{4}} d\log\left(x^2 + \sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{8\,ac} - \frac{\sqrt{2}\left(ac^3\right)^{\frac{1}{4}} d\log\left(x^2 - \sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{8\,ac} - \frac{\sqrt{2}\left(\sqrt{2}\sqrt{ac}ce - \left(ac^3\right)^{\frac{1}{4}} cd\right)\arctan\left(\right.}{4\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*(a*c^3)^(1/4)*d*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a
*c) - 1/8*sqrt(2)*(a*c^3)^(1/4)*d*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/
c))/(a*c) - 1/4*sqrt(2)*(sqrt(2)*sqrt(a*c)*c*e - (a*c^3)^(1/4)*c*d)*arctan(
1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^2) - 1/4*sqrt(2)*
(sqrt(2)*sqrt(a*c)*c*e - (a*c^3)^(1/4)*c*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(
2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^2)
```

3.397 $\int \frac{1}{a+cx^4} dx$

Optimal. Leaf size=185

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}}$$

```
[Out] -ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(2*Sqrt[2]*a^(3/4)*c^(1/4)) + ArcT
an[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(2*Sqrt[2]*a^(3/4)*c^(1/4)) - Log[Sqrt[
a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/(4*Sqrt[2]*a^(3/4)*c^(1/4)) +
 Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/(4*Sqrt[2]*a^(3/4)*
c^(1/4))
```

Rubi [A] time = 0.0983566, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {211, 1165, 628, 1162, 617, 204}

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^4)^(-1),x]
```

```
[Out] -ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(2*Sqrt[2]*a^(3/4)*c^(1/4)) + ArcT
an[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(2*Sqrt[2]*a^(3/4)*c^(1/4)) - Log[Sqrt[
a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/(4*Sqrt[2]*a^(3/4)*c^(1/4)) +
 Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/(4*Sqrt[2]*a^(3/4)*
c^(1/4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{a+cx^4}\,dx = \frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{a+cx^4}\,dx}{2\sqrt{a}} + \frac{\int \frac{\sqrt{a}+\sqrt{c}x^2}{a+cx^4}\,dx}{2\sqrt{a}}$$

$$= \frac{\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4\sqrt{a}\sqrt{c}} + \frac{\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4\sqrt{a}\sqrt{c}} - \frac{\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}a^{3/4}\sqrt[4]{c}} - \frac{\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}a^{3/4}\sqrt[4]{c}}$$

$$= -\frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}} + \frac{\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}} -$$

$$= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}} - \frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}} + \frac{\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\right.}{4\sqrt{2}a^{3/4}\sqrt[4]{c}}$$

Mathematica [A] time = 0.0182744, size = 134, normalized size = 0.72

$$\frac{-\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)+\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)-2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)+2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^4)^(-1),x]
```

```
[Out] (-2*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*c^(1/4)
*x)/a^(1/4)] - Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + Log
[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(
1/4))
```

Maple [A] time = 0.002, size = 128, normalized size = 0.7

$$\frac{\sqrt{2}}{8a}\sqrt[4]{\frac{a}{c}}\ln\left(\left(x^2+\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)\left(x^2-\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)^{-1}\right)+\frac{\sqrt{2}}{4a}\sqrt[4]{\frac{a}{c}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{c}}}+1\right)+\frac{\sqrt{2}}{4a}\sqrt[4]{\frac{a}{c}}\arctan\left(x\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+a),x)
```

```
[Out] 1/8*(a/c)^(1/4)/a*2^(1/2)*ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(
a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+1/4*(a/c)^(1/4)/a*2^(1/2)*arctan(2^(1/2)
/(a/c)^(1/4)*x+1)+1/4*(a/c)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.41322, size = 306, normalized size = 1.65

$$\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}} \arctan\left(-a^2cx\left(-\frac{1}{a^3c}\right)^{\frac{3}{4}} + \sqrt{a^2\sqrt{-\frac{1}{a^3c}} + x^2}a^2c\left(-\frac{1}{a^3c}\right)^{\frac{3}{4}}\right) + \frac{1}{4}\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}} \log\left(a\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}} + x\right) - \frac{1}{4}\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}} \log\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] (-1/(a^3*c))^(1/4)*arctan(-a^2*c*x*(-1/(a^3*c))^(3/4) + sqrt(a^2*sqrt(-1/(a
^3*c)) + x^2)*a^2*c*(-1/(a^3*c))^(3/4)) + 1/4*(-1/(a^3*c))^(1/4)*log(a*(-1/
(a^3*c))^(1/4) + x) - 1/4*(-1/(a^3*c))^(1/4)*log(-a*(-1/(a^3*c))^(1/4) + x)
```

Sympy [A] time = 0.144521, size = 20, normalized size = 0.11

$$\operatorname{RootSum}\left(256t^4a^3c + 1, \left(t \mapsto t\log\left(4ta + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+a),x)
```

```
[Out] RootSum(256*_t**4*a**3*c + 1, Lambda(_t, _t*log(4*_t*a + x)))
```

Giac [A] time = 1.21046, size = 242, normalized size = 1.31

$$\frac{\sqrt{2}\left(ac^{3}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4ac}+\frac{\sqrt{2}\left(ac^{3}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4ac}+\frac{\sqrt{2}\left(ac^{3}\right)^{\frac{1}{4}}\log\left(x^{2}+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8ac}-\frac{\sqrt{2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a
/c)^(1/4))/(a*c) + 1/4*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x - sqrt
(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c) + 1/8*sqrt(2)*(a*c^3)^(1/4)*log(x^2 + s
qrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c) - 1/8*sqrt(2)*(a*c^3)^(1/4)*log(x^2
 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c)
```

3.398 $\int \frac{1}{(d+ex)\left(a+cx^4\right)}\,dx$

Optimal. Leaf size=416

$$-\frac{\sqrt[4]{c}d\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^4+cd^4\right)}+\frac{\sqrt[4]{c}d\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^4+cd^4\right)}-\frac{\sqrt[4]{c}d\left(\sqrt{a}e^2\right.}{2}$$

```
[Out] -(Sqrt[c]*d^2*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*Sqrt[a]*(c*d^4 + a*e^4))
- (c^(1/4)*d*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(
1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)) + (c^(1/4)*d*(Sqrt[c]*d^2 + Sqrt
[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4
 + a*e^4)) + (e^3*Log[d + e*x])/(c*d^4 + a*e^4) - (c^(1/4)*d*(Sqrt[c]*d^2 -
 Sqrt[a]*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sq
rt[2]*a^(3/4)*(c*d^4 + a*e^4)) + (c^(1/4)*d*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log
[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*
d^4 + a*e^4)) - (e^3*Log[a + c*x^4])/(4*(c*d^4 + a*e^4))
```

Rubi [A] time = 0.428482, antiderivative size = 416, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 12, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.706, Rules used = {6725, 1876, 1168, 1162, 617, 204, 1165, 628, 1248, 635, 205, 260}

$$-\frac{\sqrt[4]{c}d\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^4+cd^4\right)}+\frac{\sqrt[4]{c}d\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^4+cd^4\right)}-\frac{\sqrt[4]{c}d\left(\sqrt{a}e^2\right.}{2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(a + c*x^4)),x]
```

```
[Out] -(Sqrt[c]*d^2*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*Sqrt[a]*(c*d^4 + a*e^4))
- (c^(1/4)*d*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(
1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)) + (c^(1/4)*d*(Sqrt[c]*d^2 + Sqrt
[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4
 + a*e^4)) + (e^3*Log[d + e*x])/(c*d^4 + a*e^4) - (c^(1/4)*d*(Sqrt[c]*d^2 -
 Sqrt[a]*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sq
rt[2]*a^(3/4)*(c*d^4 + a*e^4)) + (c^(1/4)*d*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log
[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*
d^4 + a*e^4)) - (e^3*Log[a + c*x^4])/(4*(c*d^4 + a*e^4))
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{1}{(d+ex)\left(a+cx^4\right)}\,dx = \int \left(\frac{e^4}{\left(cd^4+ae^4\right)(d+ex)} + \frac{c\left(d^3-d^2ex+de^2x^2-e^3x^3\right)}{\left(cd^4+ae^4\right)\left(a+cx^4\right)}\right)dx$$

$$= \frac{e^3\log(d+ex)}{cd^4+ae^4} + \frac{c\int \frac{d^3-d^2ex+de^2x^2-e^3x^3}{a+cx^4}\,dx}{cd^4+ae^4}$$

$$= \frac{e^3\log(d+ex)}{cd^4+ae^4} + \frac{c\int \left(\frac{d^3+de^2x^2}{a+cx^4} + \frac{x\left(-d^2e-e^3x^2\right)}{a+cx^4}\right)dx}{cd^4+ae^4}$$

$$= \frac{e^3\log(d+ex)}{cd^4+ae^4} + \frac{c\int \frac{d^3+de^2x^2}{a+cx^4}\,dx}{cd^4+ae^4} + \frac{c\int \frac{x\left(-d^2e-e^3x^2\right)}{a+cx^4}\,dx}{cd^4+ae^4}$$

$$= \frac{e^3\log(d+ex)}{cd^4+ae^4} + \frac{c\,\mathrm{Subst}\left(\int \frac{-d^2e-e^3x}{a+cx^2}\,dx,x,x^2\right)}{2\left(cd^4+ae^4\right)} + \frac{\left(d\left(\frac{\sqrt{c}d^2}{\sqrt{a}}-e^2\right)\right)\int \frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4}\,dx}{2\left(cd^4+ae^4\right)} + \frac{\left(d\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+\right.\right.}{2\left(\right.}$$

$$= \frac{e^3\log(d+ex)}{cd^4+ae^4} - \frac{\left(cd^2e\right)\mathrm{Subst}\left(\int \frac{1}{a+cx^2}\,dx,x,x^2\right)}{2\left(cd^4+ae^4\right)} - \frac{\left(ce^3\right)\mathrm{Subst}\left(\int \frac{x}{a+cx^2}\,dx,x,x^2\right)}{2\left(cd^4+ae^4\right)} + \frac{\left(d\left(\frac{\sqrt{c}d^2}{\sqrt{a}}\right.\right.}{}$$

$$= -\frac{\sqrt{c}d^2e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\left(cd^4+ae^4\right)} + \frac{e^3\log(d+ex)}{cd^4+ae^4} - \frac{\sqrt[4]{c}d\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(cd^4+ae^4\right)}$$

$$= -\frac{\sqrt{c}d^2e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\left(cd^4+ae^4\right)} - \frac{\sqrt[4]{c}d\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(cd^4+ae^4\right)} + \frac{\sqrt[4]{c}d\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\tan^{-1}}{2\sqrt{2}a^{3/4}\left(cd^4+a\right.}$$

Mathematica [A] time = 0.153133, size = 404, normalized size = 0.97

$$\frac{-2a^{3/4}e^3\log\left(a+cx^4\right)+8a^{3/4}e^3\log(d+ex)-\sqrt{2}c^{3/4}d^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)+\sqrt{2}c^{3/4}d^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(a + c*x^4)),x]
```

```
[Out] (-2*c^(1/4)*d*(Sqrt[2]*Sqrt[c]*d^2 - 2*a^(1/4)*c^(1/4)*d*e + Sqrt[2]*Sqrt[a
]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*c^(1/4)*d*(Sqrt[2]*Sqrt[
c]*d^2 + 2*a^(1/4)*c^(1/4)*d*e + Sqrt[2]*Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c
^(1/4)*x)/a^(1/4)] + 8*a^(3/4)*e^3*Log[d + e*x] - Sqrt[2]*c^(3/4)*d^3*Log[S
```

```
qrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + Sqrt[2]*Sqrt[a]*c^(1/4)
*d*e^2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + Sqrt[2]*c^(
3/4)*d^3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] - Sqrt[2]*S
qrt[a]*c^(1/4)*d*e^2*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]
 - 2*a^(3/4)*e^3*Log[a + c*x^4])/(8*a^(3/4)*(c*d^4 + a*e^4))
```

Maple [A] time = 0.007, size = 433, normalized size = 1.

$$\frac{e^3 \ln(ex+d)}{ae^4+cd^4} + \frac{cd^3\sqrt{2}}{\left(8\,ae^4+8\,cd^4\right)a}\sqrt[4]{\frac{a}{c}}\ln\left(\left(x^2+\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)\left(x^2-\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)^{-1}\right) + \frac{cd^3\sqrt{2}}{\left(4\,ae^4+4\,cd^4\right)a}\sqrt[4]{\frac{a}{c}}\,\mathrm{arctan}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(c*x^4+a),x)
```

```
[Out] e^3*ln(e*x+d)/(a*e^4+c*d^4)+1/8*c/(a*e^4+c*d^4)*d^3*(a/c)^(1/4)/a*2^(1/2)*l
n((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^
(1/2)))+1/4*c/(a*e^4+c*d^4)*d^3*(a/c)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/c)^
(1/4)*x+1)+1/4*c/(a*e^4+c*d^4)*d^3*(a/c)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/
c)^(1/4)*x-1)-1/2*c/(a*e^4+c*d^4)*e*d^2/(a*c)^(1/2)*arctan(x^2*(1/a*c)^(1/2
))+1/8/(a*e^4+c*d^4)*d*e^2/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*x*2^(1/2
)+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+1/4/(a*e^4+c*d^4)*d
*e^2/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+1/4/(a*e^4+c*d^4)*
d*e^2/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)-1/4*e^3*ln(c*x^4+
a)/(a*e^4+c*d^4)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17897, size = 517, normalized size = 1.24

$$\frac{\left(ac^3\right)^{\frac{1}{4}}c^2d\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}ac^3d^2-2\left(ac^3\right)^{\frac{1}{4}}ac^2de+\sqrt{2}\sqrt{ac}ac^2e^2\right)}+\frac{\left(ac^3\right)^{\frac{1}{4}}c^2d\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}ac^3d^2+2\left(ac^3\right)^{\frac{1}{4}}ac^2de+\sqrt{2}\sqrt{ac}ac^2e^2\right)}+\frac{\left(\left(ac^3\right)^{\frac{1}{4}}c^2d^3-\left(ac^3\right)^{\frac{3}{4}}\right.}{4\left(\sqrt{2}ac\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/2*(a*c^3)^(1/4)*c^2*d*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c
)^(1/4))/(sqrt(2)*a*c^3*d^2 - 2*(a*c^3)^(1/4)*a*c^2*d*e + sqrt(2)*sqrt(a*c)
*a*c^2*e^2) + 1/2*(a*c^3)^(1/4)*c^2*d*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/
c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a*c^3*d^2 + 2*(a*c^3)^(1/4)*a*c^2*d*e + sqr
t(2)*sqrt(a*c)*a*c^2*e^2) + 1/4*((a*c^3)^(1/4)*c^2*d^3 - (a*c^3)^(3/4)*d*e^
2)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a*c^3*d^4 + sqrt(2
```

```
)*a^2*c^2*e^4) - 1/4*((a*c^3)^(1/4)*c^2*d^3 - (a*c^3)^(3/4)*d*e^2)*log(x^2
- sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a*c^3*d^4 + sqrt(2)*a^2*c^2*e
^4) - 1/4*e^3*log(abs(c*x^4 + a))/(c*d^4 + a*e^4) + e^4*log(abs(x*e + d))/(
c*d^4*e + a*e^5)
```

3.399 $\int \frac{1}{(d+ex)^2(a+cx^4)}\,dx$

Optimal. Leaf size=552

$$-\frac{\sqrt[4]{c}\left(\sqrt{c}d^2\left(cd^4-3ae^4\right)-\sqrt{a}e^2\left(3cd^4-ae^4\right)\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^4+cd^4\right)^2}+\frac{\sqrt[4]{c}\left(\sqrt{c}d^2\left(cd^4-3ae^4\right)-\sqrt{a}e^2\left(3cd^4-\right.\right.}{4\sqrt{2}a^{3/4}\left(ae\right.}$$

```
[Out] -(e^3/((c*d^4 + a*e^4)*(d + e*x))) - (Sqrt[c]*d*e*(c*d^4 - a*e^4)*ArcTan[(S
qrt[c]*x^2)/Sqrt[a]])/(Sqrt[a]*(c*d^4 + a*e^4)^2) - (c^(1/4)*(Sqrt[c]*d^2*(
c*d^4 - 3*a*e^4) + Sqrt[a]*e^2*(3*c*d^4 - a*e^4))*ArcTan[1 - (Sqrt[2]*c^(1/
4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^2) + (c^(1/4)*(Sqrt[c]*d
^2*(c*d^4 - 3*a*e^4) + Sqrt[a]*e^2*(3*c*d^4 - a*e^4))*ArcTan[1 + (Sqrt[2]*c
^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^2) + (4*c*d^3*e^3*Lo
g[d + e*x])/(c*d^4 + a*e^4)^2 - (c^(1/4)*(Sqrt[c]*d^2*(c*d^4 - 3*a*e^4) - S
qrt[a]*e^2*(3*c*d^4 - a*e^4))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqr
t[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^2) + (c^(1/4)*(Sqrt[c]*d^2*(c
*d^4 - 3*a*e^4) - Sqrt[a]*e^2*(3*c*d^4 - a*e^4))*Log[Sqrt[a] + Sqrt[2]*a^(1
/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^2) - (c*d^
3*e^3*Log[a + c*x^4])/(c*d^4 + a*e^4)^2
```

Rubi [A] time = 0.808323, antiderivative size = 552, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 12, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.706, Rules used = {6725, 1876, 1248, 635, 205, 260, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{\sqrt[4]{c}\left(\sqrt{c}d^2\left(cd^4-3ae^4\right)-\sqrt{a}e^2\left(3cd^4-ae^4\right)\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^4+cd^4\right)^2}+\frac{\sqrt[4]{c}\left(\sqrt{c}d^2\left(cd^4-3ae^4\right)-\sqrt{a}e^2\left(3cd^4-\right.\right.}{4\sqrt{2}a^{3/4}\left(ae\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^2*(a + c*x^4)),x]
```

```
[Out] -(e^3/((c*d^4 + a*e^4)*(d + e*x))) - (Sqrt[c]*d*e*(c*d^4 - a*e^4)*ArcTan[(S
qrt[c]*x^2)/Sqrt[a]])/(Sqrt[a]*(c*d^4 + a*e^4)^2) - (c^(1/4)*(Sqrt[c]*d^2*(
c*d^4 - 3*a*e^4) + Sqrt[a]*e^2*(3*c*d^4 - a*e^4))*ArcTan[1 - (Sqrt[2]*c^(1/
4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^2) + (c^(1/4)*(Sqrt[c]*d
^2*(c*d^4 - 3*a*e^4) + Sqrt[a]*e^2*(3*c*d^4 - a*e^4))*ArcTan[1 + (Sqrt[2]*c
^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^2) + (4*c*d^3*e^3*Lo
g[d + e*x])/(c*d^4 + a*e^4)^2 - (c^(1/4)*(Sqrt[c]*d^2*(c*d^4 - 3*a*e^4) - S
```

```
qrt[a]*e^2*(3*c*d^4 - a*e^4))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqr
t[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^2) + (c^(1/4)*(Sqrt[c]*d^2*(c
*d^4 - 3*a*e^4) - Sqrt[a]*e^2*(3*c*d^4 - a*e^4))*Log[Sqrt[a] + Sqrt[2]*a^(1
/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^2) - (c*d^
3*e^3*Log[a + c*x^4])/(c*d^4 + a*e^4)^2
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
```

```
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex)^2\left(a+cx^4\right)}\,dx = \int\left(\frac{e^4}{\left(cd^4+ae^4\right)(d+ex)^2}+\frac{4cd^3e^4}{\left(cd^4+ae^4\right)^2(d+ex)}+\frac{c\left(d^2\left(cd^4-3ae^4\right)-2de\left(cd^4-ae^4\right)x+e^2\right.}{\left(cd^4+ae^4\right)^2\left(a+\right.}\right.$$

$$=-\frac{e^3}{\left(cd^4+ae^4\right)(d+ex)}+\frac{4cd^3e^3\log(d+ex)}{\left(cd^4+ae^4\right)^2}+\frac{c\int\frac{d^2\left(cd^4-3ae^4\right)-2de\left(cd^4-ae^4\right)x+e^2\left(3cd^4-ae^4\right)x^2-4cd^3e^3x^3}{a+cx^4}}{\left(cd^4+ae^4\right)^2}$$

$$=-\frac{e^3}{\left(cd^4+ae^4\right)(d+ex)}+\frac{4cd^3e^3\log(d+ex)}{\left(cd^4+ae^4\right)^2}+\frac{c\int\left(\frac{x\left(-2de\left(cd^4-ae^4\right)-4cd^3e^3x^2\right)}{a+cx^4}+\frac{d^2\left(cd^4-3ae^4\right)+e^2\left(3cd^4-\right.}{a+cx^4}\right.}{\left(cd^4+ae^4\right)^2}$$

$$=-\frac{e^3}{\left(cd^4+ae^4\right)(d+ex)}+\frac{4cd^3e^3\log(d+ex)}{\left(cd^4+ae^4\right)^2}+\frac{c\int\frac{x\left(-2de\left(cd^4-ae^4\right)-4cd^3e^3x^2\right)}{a+cx^4}\,dx}{\left(cd^4+ae^4\right)^2}+\frac{c\int\frac{d^2\left(cd^4-3ae^4\right)+}{a+}}{\left(cd^4+\right.}$$

$$=-\frac{e^3}{\left(cd^4+ae^4\right)(d+ex)}+\frac{4cd^3e^3\log(d+ex)}{\left(cd^4+ae^4\right)^2}+\frac{c\,\mathrm{Subst}\left(\int\frac{-2de\left(cd^4-ae^4\right)-4cd^3e^3x}{a+cx^2}\,dx,x,x^2\right)}{2\left(cd^4+ae^4\right)^2}-\frac{\left(3c\right.}{}$$

$$=-\frac{e^3}{\left(cd^4+ae^4\right)(d+ex)}+\frac{4cd^3e^3\log(d+ex)}{\left(cd^4+ae^4\right)^2}-\frac{\left(2c^2d^3e^3\right)\mathrm{Subst}\left(\int\frac{x}{a+cx^2}\,dx,x,x^2\right)}{\left(cd^4+ae^4\right)^2}-\frac{\left(cde\left(cd^4\right.\right.}{}$$

$$=-\frac{e^3}{\left(cd^4+ae^4\right)(d+ex)}-\frac{\sqrt{c}de\left(cd^4-ae^4\right)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{\sqrt{a}\left(cd^4+ae^4\right)^2}+\frac{4cd^3e^3\log(d+ex)}{\left(cd^4+ae^4\right)^2}+\frac{\sqrt[4]{c}\left(3cd^4e^2-\right.}{}$$

$$=-\frac{e^3}{\left(cd^4+ae^4\right)(d+ex)}-\frac{\sqrt{c}de\left(cd^4-ae^4\right)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{\sqrt{a}\left(cd^4+ae^4\right)^2}-\frac{\sqrt[4]{c}\left(3cd^4e^2-ae^6+\frac{\sqrt{c}d^2\left(cd^4-3ae^4\right)}{\sqrt{a}}\right)\mathrm{ta}}{2\sqrt{2}\sqrt[4]{a}\left(cd^4+ae^4\right)^2}$$

<u>Mathematica</u> [A] time = 0.706478, size = 524, normalized size = 0.95

$$\frac{-\frac{\sqrt{2}\sqrt[4]{c}\left(a^{3/2}e^6-3\sqrt{a}cd^4e^2-3a\sqrt{c}d^2e^4+c^{3/2}d^6\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{a^{3/4}}+\frac{\sqrt{2}\sqrt[4]{c}\left(a^{3/2}e^6-3\sqrt{a}cd^4e^2-3a\sqrt{c}d^2e^4+c^{3/2}d^6\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{a^{3/4}}+\frac{2\sqrt[4]{c}\left(\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^2*(a + c*x^4)),x]
```

```
[Out] ((-8*e^3*(c*d^4 + a*e^4))/(d + e*x) + (2*c^(1/4)*(-(Sqrt[c]*d^2) + Sqrt[a]*
e^2)*(Sqrt[2]*c*d^4 - 4*a^(1/4)*c^(3/4)*d^3*e + 4*Sqrt[2]*Sqrt[a]*Sqrt[c]*d
^2*e^2 - 4*a^(3/4)*c^(1/4)*d*e^3 + Sqrt[2]*a*e^4)*ArcTan[1 - (Sqrt[2]*c^(1/
4)*x)/a^(1/4)])/a^(3/4) + (2*c^(1/4)*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*(Sqrt[2]*c
*d^4 + 4*a^(1/4)*c^(3/4)*d^3*e + 4*Sqrt[2]*Sqrt[a]*Sqrt[c]*d^2*e^2 + 4*a^(3
/4)*c^(1/4)*d*e^3 + Sqrt[2]*a*e^4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])
/a^(3/4) + 32*c*d^3*e^3*Log[d + e*x] - (Sqrt[2]*c^(1/4)*(c^(3/2)*d^6 - 3*Sq
rt[a]*c*d^4*e^2 - 3*a*Sqrt[c]*d^2*e^4 + a^(3/2)*e^6)*Log[Sqrt[a] - Sqrt[2]*
a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(3/4) + (Sqrt[2]*c^(1/4)*(c^(3/2)*d^6 -
 3*Sqrt[a]*c*d^4*e^2 - 3*a*Sqrt[c]*d^2*e^4 + a^(3/2)*e^6)*Log[Sqrt[a] + Sqr
t[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(3/4) - 8*c*d^3*e^3*Log[a + c*x^4]
)/(8*(c*d^4 + a*e^4)^2)
```

Maple [A] time = 0.01, size = 866, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(c*x^4+a),x)
```

```
[Out] -e^3/(a*e^4+c*d^4)/(e*x+d)+4*c*d^3*e^3*ln(e*x+d)/(a*e^4+c*d^4)^2-3/4*c/(a*e
^4+c*d^4)^2*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^2*e^4+1/4
*c^2/(a*e^4+c*d^4)^2*(a/c)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*
d^6-3/8*c/(a*e^4+c*d^4)^2*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*x*2^(1/2)
+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))*d^2*e^4+1/8*c^2/(a*e
^4+c*d^4)^2*(a/c)^(1/4)/a*2^(1/2)*ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)
)/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))*d^6-3/4*c/(a*e^4+c*d^4)^2*(a/c)^
(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^2*e^4+1/4*c^2/(a*e^4+c*d^4)
^2*(a/c)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^6+c/(a*e^4+c*d^4
)^2/(a*c)^(1/2)*arctan(x^2*(1/a*c)^(1/2))*a*d*e^5-c^2/(a*e^4+c*d^4)^2/(a*c)
^(1/2)*arctan(x^2*(1/a*c)^(1/2))*d^5*e-1/8/(a*e^4+c*d^4)^2/(a/c)^(1/4)*2^(1
/2)*ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(
a/c)^(1/2)))*a*e^6+3/8*c/(a*e^4+c*d^4)^2/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^
(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))*d^4*e
^2-1/4/(a*e^4+c*d^4)^2/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*
a*e^6+3/4*c/(a*e^4+c*d^4)^2/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*
x-1)*d^4*e^2-1/4/(a*e^4+c*d^4)^2/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(
1/4)*x+1)*a*e^6+3/4*c/(a*e^4+c*d^4)^2/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a
/c)^(1/4)*x+1)*d^4*e^2-c*d^3*e^3*ln(c*x^4+a)/(a*e^4+c*d^4)^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(c*x**4+a),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^4+a),x, algorithm="giac")
```

```
[Out] Timed out
```

3.400 $\int \frac{1}{(d+ex)^3\left(a+cx^4\right)}\,dx$

Optimal. Leaf size=680

$$-\frac{c^{3/4}d\left(3a^2e^8-12acd^4e^4-2\sqrt{a}\sqrt{c}d^2e^2\left(3cd^4-5ae^4\right)+c^2d^8\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^4+cd^4\right)^3}+\frac{c^{3/4}d\left(3a^2e^8-12acd^4e^4\right.}{}$$

```
[Out] -e^3/(2*(c*d^4 + a*e^4)*(d + e*x)^2) - (4*c*d^3*e^3)/((c*d^4 + a*e^4)^2*(d
+ e*x)) - (Sqrt[c]*e*(3*c^2*d^8 - 12*a*c*d^4*e^4 + a^2*e^8)*ArcTan[(Sqrt[c]
*x^2)/Sqrt[a]])/(2*Sqrt[a]*(c*d^4 + a*e^4)^3) - (c^(3/4)*d*(c^2*d^8 - 12*a*
c*d^4*e^4 + 3*a^2*e^8 + 2*Sqrt[a]*Sqrt[c]*d^2*e^2*(3*c*d^4 - 5*a*e^4))*ArcT
an[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^3)
+ (c^(3/4)*d*(c^2*d^8 - 12*a*c*d^4*e^4 + 3*a^2*e^8 + 2*Sqrt[a]*Sqrt[c]*d^2*
e^2*(3*c*d^4 - 5*a*e^4))*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2
]*a^(3/4)*(c*d^4 + a*e^4)^3) + (2*c*d^2*e^3*(5*c*d^4 - 3*a*e^4)*Log[d + e*x
])/(c*d^4 + a*e^4)^3 - (c^(3/4)*d*(c^2*d^8 - 12*a*c*d^4*e^4 + 3*a^2*e^8 - 2
*Sqrt[a]*Sqrt[c]*d^2*e^2*(3*c*d^4 - 5*a*e^4))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)
*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^3) + (c^(3/4)
*d*(c^2*d^8 - 12*a*c*d^4*e^4 + 3*a^2*e^8 - 2*Sqrt[a]*Sqrt[c]*d^2*e^2*(3*c*d
^4 - 5*a*e^4))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*S
qrt[2]*a^(3/4)*(c*d^4 + a*e^4)^3) - (c*d^2*e^3*(5*c*d^4 - 3*a*e^4)*Log[a +
c*x^4])/(2*(c*d^4 + a*e^4)^3)
```

Rubi [A] time = 0.949867, antiderivative size = 680, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 12, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.706, Rules used = {6725, 1876, 1168, 1162, 617, 204, 1165, 628, 1248, 635, 205, 260}

$$-\frac{c^{3/4}d\left(3a^2e^8-12acd^4e^4-2\sqrt{a}\sqrt{c}d^2e^2\left(3cd^4-5ae^4\right)+c^2d^8\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^4+cd^4\right)^3}+\frac{c^{3/4}d\left(3a^2e^8-12acd^4e^4\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^3*(a + c*x^4)),x]
```

```
[Out] -e^3/(2*(c*d^4 + a*e^4)*(d + e*x)^2) - (4*c*d^3*e^3)/((c*d^4 + a*e^4)^2*(d
+ e*x)) - (Sqrt[c]*e*(3*c^2*d^8 - 12*a*c*d^4*e^4 + a^2*e^8)*ArcTan[(Sqrt[c]
*x^2)/Sqrt[a]])/(2*Sqrt[a]*(c*d^4 + a*e^4)^3) - (c^(3/4)*d*(c^2*d^8 - 12*a*
c*d^4*e^4 + 3*a^2*e^8 + 2*Sqrt[a]*Sqrt[c]*d^2*e^2*(3*c*d^4 - 5*a*e^4))*ArcT
```

```
an[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^3)
+ (c^(3/4)*d*(c^2*d^8 - 12*a*c*d^4*e^4 + 3*a^2*e^8 + 2*Sqrt[a]*Sqrt[c]*d^2*
e^2*(3*c*d^4 - 5*a*e^4))*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2
]*a^(3/4)*(c*d^4 + a*e^4)^3) + (2*c*d^2*e^3*(5*c*d^4 - 3*a*e^4)*Log[d + e*x
])/(c*d^4 + a*e^4)^3 - (c^(3/4)*d*(c^2*d^8 - 12*a*c*d^4*e^4 + 3*a^2*e^8 - 2
*Sqrt[a]*Sqrt[c]*d^2*e^2*(3*c*d^4 - 5*a*e^4))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)
*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^3) + (c^(3/4)
*d*(c^2*d^8 - 12*a*c*d^4*e^4 + 3*a^2*e^8 - 2*Sqrt[a]*Sqrt[c]*d^2*e^2*(3*c*d
^4 - 5*a*e^4))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*S
qrt[2]*a^(3/4)*(c*d^4 + a*e^4)^3) - (c*d^2*e^3*(5*c*d^4 - 3*a*e^4)*Log[a +
c*x^4])/(2*(c*d^4 + a*e^4)^3)
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{1}{(d+ex)^3\left(a+cx^4\right)}\,dx = \int\left(\frac{e^4}{\left(cd^4+ae^4\right)(d+ex)^3}+\frac{4cd^3e^4}{\left(cd^4+ae^4\right)^2(d+ex)^2}+\frac{2cd^2e^4\left(5cd^4-3ae^4\right)}{\left(cd^4+ae^4\right)^3(d+ex)}+\frac{c\left(d\left(c^2d^8-12ac\right.\right.}{}\right.$$

$$= -\frac{e^3}{2\left(cd^4+ae^4\right)(d+ex)^2}-\frac{4cd^3e^3}{\left(cd^4+ae^4\right)^2(d+ex)}+\frac{2cd^2e^3\left(5cd^4-3ae^4\right)\log(d+ex)}{\left(cd^4+ae^4\right)^3}+\frac{c\int\frac{d\left(c^2\right.}{}}{}$$

$$= -\frac{e^3}{2\left(cd^4+ae^4\right)(d+ex)^2}-\frac{4cd^3e^3}{\left(cd^4+ae^4\right)^2(d+ex)}+\frac{2cd^2e^3\left(5cd^4-3ae^4\right)\log(d+ex)}{\left(cd^4+ae^4\right)^3}+\frac{c\int\left(\frac{d\left(\right.}{}\right.}{}$$

$$= -\frac{e^3}{2\left(cd^4+ae^4\right)(d+ex)^2}-\frac{4cd^3e^3}{\left(cd^4+ae^4\right)^2(d+ex)}+\frac{2cd^2e^3\left(5cd^4-3ae^4\right)\log(d+ex)}{\left(cd^4+ae^4\right)^3}+\frac{c\int\frac{d\left(c^2\right.}{}}{}$$

$$= -\frac{e^3}{2\left(cd^4+ae^4\right)(d+ex)^2}-\frac{4cd^3e^3}{\left(cd^4+ae^4\right)^2(d+ex)}+\frac{2cd^2e^3\left(5cd^4-3ae^4\right)\log(d+ex)}{\left(cd^4+ae^4\right)^3}+\frac{c\,\mathrm{Subs}}{}$$

$$= -\frac{e^3}{2\left(cd^4+ae^4\right)(d+ex)^2}-\frac{4cd^3e^3}{\left(cd^4+ae^4\right)^2(d+ex)}+\frac{2cd^2e^3\left(5cd^4-3ae^4\right)\log(d+ex)}{\left(cd^4+ae^4\right)^3}-\frac{\left(c^2d^2e^3\right.}{}$$

$$= -\frac{e^3}{2\left(cd^4+ae^4\right)(d+ex)^2}-\frac{4cd^3e^3}{\left(cd^4+ae^4\right)^2(d+ex)}-\frac{\sqrt{c}e\left(3c^2d^8-12acd^4e^4+a^2e^8\right)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right.}{2\sqrt{a}\left(cd^4+ae^4\right)^3}$$

$$= -\frac{e^3}{2\left(cd^4+ae^4\right)(d+ex)^2}-\frac{4cd^3e^3}{\left(cd^4+ae^4\right)^2(d+ex)}-\frac{\sqrt{c}e\left(3c^2d^8-12acd^4e^4+a^2e^8\right)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right.}{2\sqrt{a}\left(cd^4+ae^4\right)^3}$$

Mathematica [A] time = 0.986304, size = 738, normalized size = 1.09

$$\frac{-\sqrt{2}c^{3/4}d(d+ex)^2\left(10a^{3/2}\sqrt{c}d^2e^6+3a^2e^8-6\sqrt{a}c^{3/2}d^6e^2-12acd^4e^4+c^2d^8\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)+\sqrt{2}c^{3/4}d(d}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^3*(a + c*x^4)),x]
```

```
[Out] (-4*a^(3/4)*e^3*(c*d^4 + a*e^4)^2 - 32*a^(3/4)*c*d^3*e^3*(c*d^4 + a*e^4)*(d
 + e*x) - 2*Sqrt[c]*(Sqrt[2]*c^(9/4)*d^9 - 6*a^(1/4)*c^2*d^8*e + 6*Sqrt[2]*
Sqrt[a]*c^(7/4)*d^7*e^2 - 12*Sqrt[2]*a*c^(5/4)*d^5*e^4 + 24*a^(5/4)*c*d^4*e
^5 - 10*Sqrt[2]*a^(3/2)*c^(3/4)*d^3*e^6 + 3*Sqrt[2]*a^2*c^(1/4)*d*e^8 - 2*a
^(9/4)*e^9)*(d + e*x)^2*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*Sqrt[c]
*(Sqrt[2]*c^(9/4)*d^9 + 6*a^(1/4)*c^2*d^8*e + 6*Sqrt[2]*Sqrt[a]*c^(7/4)*d^7
*e^2 - 12*Sqrt[2]*a*c^(5/4)*d^5*e^4 - 24*a^(5/4)*c*d^4*e^5 - 10*Sqrt[2]*a^(
3/2)*c^(3/4)*d^3*e^6 + 3*Sqrt[2]*a^2*c^(1/4)*d*e^8 + 2*a^(9/4)*e^9)*(d + e*
x)^2*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 16*a^(3/4)*c*d^2*e^3*(5*c*d^
4 - 3*a*e^4)*(d + e*x)^2*Log[d + e*x] - Sqrt[2]*c^(3/4)*d*(c^2*d^8 - 6*Sqrt
[a]*c^(3/2)*d^6*e^2 - 12*a*c*d^4*e^4 + 10*a^(3/2)*Sqrt[c]*d^2*e^6 + 3*a^2*e
^8)*(d + e*x)^2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + Sq
rt[2]*c^(3/4)*d*(c^2*d^8 - 6*Sqrt[a]*c^(3/2)*d^6*e^2 - 12*a*c*d^4*e^4 + 10*
a^(3/2)*Sqrt[c]*d^2*e^6 + 3*a^2*e^8)*(d + e*x)^2*Log[Sqrt[a] + Sqrt[2]*a^(1
/4)*c^(1/4)*x + Sqrt[c]*x^2] + 4*a^(3/4)*c*d^2*e^3*(-5*c*d^4 + 3*a*e^4)*(d
+ e*x)^2*Log[a + c*x^4])/(8*a^(3/4)*(c*d^4 + a*e^4)^3*(d + e*x)^2)
```

Maple [B] time = 0.013, size = 1201, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^3/(c*x^4+a),x)
```

```
[Out] -1/2*e^3/(a*e^4+c*d^4)/(e*x+d)^2-4*c*d^3*e^3/(a*e^4+c*d^4)^2/(e*x+d)-6*e^7*
c*d^2/(a*e^4+c*d^4)^3*ln(e*x+d)*a+10*e^3*c^2*d^6/(a*e^4+c*d^4)^3*ln(e*x+d)+
3/4*c/(a*e^4+c*d^4)^3*(a/c)^(1/4)*a*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)
*d*e^8-3*c^2/(a*e^4+c*d^4)^3*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)
*x+1)*d^5*e^4+1/4*c^3/(a*e^4+c*d^4)^3*(a/c)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/
(a/c)^(1/4)*x+1)*d^9+3/4*c/(a*e^4+c*d^4)^3*(a/c)^(1/4)*a*2^(1/2)*arctan(2^(
1/2)/(a/c)^(1/4)*x-1)*d*e^8-3*c^2/(a*e^4+c*d^4)^3*(a/c)^(1/4)*2^(1/2)*arcta
n(2^(1/2)/(a/c)^(1/4)*x-1)*d^5*e^4+1/4*c^3/(a*e^4+c*d^4)^3*(a/c)^(1/4)/a*2^
(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^9+3/8*c/(a*e^4+c*d^4)^3*(a/c)^(1/4)
*a*2^(1/2)*ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^
(1/2)+(a/c)^(1/2)))*d*e^8-3/2*c^2/(a*e^4+c*d^4)^3*(a/c)^(1/4)*2^(1/2)*ln((x
^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2
)))*d^5*e^4+1/8*c^3/(a*e^4+c*d^4)^3*(a/c)^(1/4)/a*2^(1/2)*ln((x^2+(a/c)^(1/
4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))*d^9-1/2*
c/(a*e^4+c*d^4)^3/(a*c)^(1/2)*arctan(x^2*(1/a*c)^(1/2))*a^2*e^9+6*c^2/(a*e^
4+c*d^4)^3/(a*c)^(1/2)*arctan(x^2*(1/a*c)^(1/2))*a*d^4*e^5-3/2*c^3/(a*e^4+c
*d^4)^3/(a*c)^(1/2)*arctan(x^2*(1/a*c)^(1/2))*d^8*e-5/4*c/(a*e^4+c*d^4)^3/(
```

```
a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c)^(1
/4)*x*2^(1/2)+(a/c)^(1/2)))*a*d^3*e^6+3/4*c^2/(a*e^4+c*d^4)^3/(a/c)^(1/4)*2
^(1/2)*ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2
)+(a/c)^(1/2)))*d^7*e^2-5/2*c/(a*e^4+c*d^4)^3/(a/c)^(1/4)*2^(1/2)*arctan(2^
(1/2)/(a/c)^(1/4)*x+1)*a*d^3*e^6+3/2*c^2/(a*e^4+c*d^4)^3/(a/c)^(1/4)*2^(1/2
)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^7*e^2-5/2*c/(a*e^4+c*d^4)^3/(a/c)^(1/4)
*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*a*d^3*e^6+3/2*c^2/(a*e^4+c*d^4)^3/
(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^7*e^2+3/2*c/(a*e^4+c*
d^4)^3*ln(c*x^4+a)*a*d^2*e^7-5/2*c^2/(a*e^4+c*d^4)^3*ln(c*x^4+a)*d^6*e^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**3/(c*x**4+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.60219, size = 1266, normalized size = 1.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/2*((a*c^3)^(1/4)*a*c^4*d^3 + sqrt(2)*a^2*c^4*e^3 - 3*(a*c^3)^(3/4)*a*c^2*
d*e^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)
*a^2*c^5*d^6 - 6*(a*c^3)^(1/4)*a^2*c^4*d^5*e + 9*sqrt(2)*sqrt(a*c)*a^2*c^4*
d^4*e^2 + 9*sqrt(2)*a^3*c^4*d^2*e^4 - 16*(a*c^3)^(3/4)*a^2*c^2*d^3*e^3 - 6*
(a*c^3)^(1/4)*a^3*c^3*d*e^5 + sqrt(2)*sqrt(a*c)*a^3*c^3*e^6) + 1/2*((a*c^3)
^(1/4)*a*c^4*d^3 - sqrt(2)*a^2*c^4*e^3 - 3*(a*c^3)^(3/4)*a*c^2*d*e^2)*arcta
n(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^2*c^5*d^6
 + 6*(a*c^3)^(1/4)*a^2*c^4*d^5*e + 9*sqrt(2)*sqrt(a*c)*a^2*c^4*d^4*e^2 + 9*
sqrt(2)*a^3*c^4*d^2*e^4 + 16*(a*c^3)^(3/4)*a^2*c^2*d^3*e^3 + 6*(a*c^3)^(1/4
)*a^3*c^3*d*e^5 + sqrt(2)*sqrt(a*c)*a^3*c^3*e^6) + 1/4*((a*c^3)^(1/4)*c^3*d
^9 - 6*(a*c^3)^(3/4)*c*d^7*e^2 - 12*(a*c^3)^(1/4)*a*c^2*d^5*e^4 + 10*(a*c^3
)^(3/4)*a*d^3*e^6 + 3*(a*c^3)^(1/4)*a^2*c*d*e^8)*log(x^2 + sqrt(2)*x*(a/c)^
(1/4) + sqrt(a/c))/(sqrt(2)*a*c^4*d^12 + 3*sqrt(2)*a^2*c^3*d^8*e^4 + 3*sqrt
(2)*a^3*c^2*d^4*e^8 + sqrt(2)*a^4*c*e^12) - 1/4*((a*c^3)^(1/4)*c^3*d^9 - 6*
(a*c^3)^(3/4)*c*d^7*e^2 - 12*(a*c^3)^(1/4)*a*c^2*d^5*e^4 + 10*(a*c^3)^(3/4)
*a*d^3*e^6 + 3*(a*c^3)^(1/4)*a^2*c*d*e^8)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) +
 sqrt(a/c))/(sqrt(2)*a*c^4*d^12 + 3*sqrt(2)*a^2*c^3*d^8*e^4 + 3*sqrt(2)*a^3
*c^2*d^4*e^8 + sqrt(2)*a^4*c*e^12) - 1/2*(5*c^2*d^6*e^3 - 3*a*c*d^2*e^7)*lo
g(abs(c*x^4 + a))/(c^3*d^12 + 3*a*c^2*d^8*e^4 + 3*a^2*c*d^4*e^8 + a^3*e^12)
 + 2*(5*c^2*d^6*e^4 - 3*a*c*d^2*e^8)*log(abs(x*e + d))/(c^3*d^12*e + 3*a*c^
2*d^8*e^5 + 3*a^2*c*d^4*e^9 + a^3*e^13) - 1/2*(9*c^2*d^8*e^3 + 10*a*c*d^4*e
^7 + a^2*e^11 + 8*(c^2*d^7*e^4 + a*c*d^3*e^8)*x)/((c*d^4 + a*e^4)^3*(x*e +
d)^2)
```

3.401 $\int \frac{(d+ex)^3}{\left(a+cx^4\right)^2}\,dx$

Optimal. Leaf size=349

$$-\frac{3d\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{3/4}}+\frac{3d\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{3/4}}-\frac{3d\left(\sqrt{a}e^2+\sqrt{c}\right.}{8}$$

```
[Out] -(a*e^3 - c*x*(d^3 + 3*d^2*e*x + 3*d*e^2*x^2))/(4*a*c*(a + c*x^4)) + (3*d^2
*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*a^(3/2)*Sqrt[c]) - (3*d*(Sqrt[c]*d^2 +
 Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*c
^(3/4)) + (3*d*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a
^(1/4)])/(8*Sqrt[2]*a^(7/4)*c^(3/4)) - (3*d*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log
[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^
(3/4)) + (3*d*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(
1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^(3/4))
```

Rubi [A] time = 0.298368, antiderivative size = 349, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 12, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.706, Rules used = {1854, 27, 12, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{3d\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{3/4}}+\frac{3d\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{3/4}}-\frac{3d\left(\sqrt{a}e^2+\sqrt{c}\right.}{8}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(a + c*x^4)^2,x]
```

```
[Out] -(a*e^3 - c*x*(d^3 + 3*d^2*e*x + 3*d*e^2*x^2))/(4*a*c*(a + c*x^4)) + (3*d^2
*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*a^(3/2)*Sqrt[c]) - (3*d*(Sqrt[c]*d^2 +
 Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*c
^(3/4)) + (3*d*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a
^(1/4)])/(8*Sqrt[2]*a^(7/4)*c^(3/4)) - (3*d*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log
[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^
(3/4)) + (3*d*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(
1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^(3/4))
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(d+ex)^3}{\left(a+cx^4\right)^2}\,dx = -\frac{ae^3-cx\left(d^3+3d^2ex+3de^2x^2\right)}{4ac\left(a+cx^4\right)} - \frac{\int \frac{-3d^3-6d^2ex-3de^2x^2}{a+cx^4}\,dx}{4a}$$

$$= -\frac{ae^3-cx\left(d^3+3d^2ex+3de^2x^2\right)}{4ac\left(a+cx^4\right)} - \frac{\int -\frac{3d(d+ex)^2}{a+cx^4}\,dx}{4a}$$

$$= -\frac{ae^3-cx\left(d^3+3d^2ex+3de^2x^2\right)}{4ac\left(a+cx^4\right)} + \frac{(3d)\int \frac{(d+ex)^2}{a+cx^4}\,dx}{4a}$$

$$= -\frac{ae^3-cx\left(d^3+3d^2ex+3de^2x^2\right)}{4ac\left(a+cx^4\right)} + \frac{(3d)\int \left(\frac{2dex}{a+cx^4}+\frac{d^2+e^2x^2}{a+cx^4}\right)dx}{4a}$$

$$= -\frac{ae^3-cx\left(d^3+3d^2ex+3de^2x^2\right)}{4ac\left(a+cx^4\right)} + \frac{(3d)\int \frac{d^2+e^2x^2}{a+cx^4}\,dx}{4a} + \frac{\left(3d^2e\right)\int \frac{x}{a+cx^4}\,dx}{2a}$$

$$= -\frac{ae^3-cx\left(d^3+3d^2ex+3de^2x^2\right)}{4ac\left(a+cx^4\right)} + \frac{\left(3d^2e\right)\text{Subst}\left(\int \frac{1}{a+cx^2}\,dx, x, x^2\right)}{4a} + \frac{\left(3d\left(\frac{\sqrt{c}d^2}{\sqrt{a}}-e^2\right)\right)\int \frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4}\,d\cdots}{8ac}$$

$$= -\frac{ae^3-cx\left(d^3+3d^2ex+3de^2x^2\right)}{4ac\left(a+cx^4\right)} + \frac{3d^2e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{c}} + \frac{\left(3d\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{16ac} + \frac{\left(3d\left(\cdots\right.\right.}{\cdots}$$

$$= -\frac{ae^3-cx\left(d^3+3d^2ex+3de^2x^2\right)}{4ac\left(a+cx^4\right)} + \frac{3d^2e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{c}} - \frac{3d\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\cdots\right.}{16\sqrt{2}a^{7/4}c^{3/4}}\cdots$$

$$= -\frac{ae^3-cx\left(d^3+3d^2ex+3de^2x^2\right)}{4ac\left(a+cx^4\right)} + \frac{3d^2e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{c}} - \frac{3d\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}c^{3/4}} + \frac{3d\cdots}{\cdots}$$

<u>Mathematica</u> [A] time = 0.386398, size = 347, normalized size = 0.99

$$\frac{3\sqrt{2}\sqrt[4]{c}\left(a^{3/4}de^2-\sqrt[4]{a}\sqrt{c}d^3\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)+3\sqrt{2}\sqrt[4]{c}\left(\sqrt[4]{a}\sqrt{c}d^3-a^{3/4}de^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(a + c*x^4)^2,x]
```

```
[Out] ((-8*a*(a*e^3 - c*d*x*(d^2 + 3*d*e*x + 3*e^2*x^2)))/(a + c*x^4) - 6*a^(1/4)
*c^(1/4)*d*(Sqrt[2]*Sqrt[c]*d^2 + 4*a^(1/4)*c^(1/4)*d*e + Sqrt[2]*Sqrt[a]*e
^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 6*a^(1/4)*c^(1/4)*d*(Sqrt[2]*
Sqrt[c]*d^2 - 4*a^(1/4)*c^(1/4)*d*e + Sqrt[2]*Sqrt[a]*e^2)*ArcTan[1 + (Sqrt
[2]*c^(1/4)*x)/a^(1/4)] + 3*Sqrt[2]*c^(1/4)*(-(a^(1/4)*Sqrt[c]*d^3) + a^(3/
4)*d*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + 3*Sqrt[2
]*c^(1/4)*(a^(1/4)*Sqrt[c]*d^3 - a^(3/4)*d*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/
4)*c^(1/4)*x + Sqrt[c]*x^2])/(32*a^2*c)
```

Maple [A] time = 0.005, size = 390, normalized size = 1.1

$$\frac{d^3x}{4a\left(cx^4+a\right)}+\frac{3d^3\sqrt{2}}{32a^2}\sqrt[4]{\frac{a}{c}}\ln\left(\left(x^2+\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)\left(x^2-\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)^{-1}\right)+\frac{3d^3\sqrt{2}}{16a^2}\sqrt[4]{\frac{a}{c}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{c}}}+1\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(c*x^4+a)^2,x)
```

```
[Out] 1/4*d^3*x/a/(c*x^4+a)+3/32*d^3/a^2*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*
x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+3/16*d^3/a^
2*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+3/16*d^3/a^2*(a/c)^(1
/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)+3/4*e*d^2*x^2/a/(c*x^4+a)+3/4*e
*d^2/a/(a*c)^(1/2)*arctan(x^2*(1/a*c)^(1/2))+3/4*d*e^2*x^3/a/(c*x^4+a)+3/32
*d*e^2/a/c/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(
x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+3/16*d*e^2/a/c/(a/c)^(1/4)*2^(1/2)*
arctan(2^(1/2)/(a/c)^(1/4)*x+1)+3/16*d*e^2/a/c/(a/c)^(1/4)*2^(1/2)*arctan(2
^(1/2)/(a/c)^(1/4)*x-1)+1/4*e^3*x^4/a/(c*x^4+a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 6.90911, size = 350, normalized size = 1.

$$\mathrm{RootSum}\left(65536t^4a^7c^3 + 27648t^2a^4c^2d^4e^2 + t\left(3456a^3cd^4e^5 - 3456a^2c^2d^8e\right) + 81a^2d^4e^8 + 162acd^8e^4 + 81c^2d^{12}, \left(t \mapsto t\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(c*x**4+a)**2,x)
```

```
[Out] RootSum(65536*_t**4*a**7*c**3 + 27648*_t**2*a**4*c**2*d**4*e**2 + _t*(3456*
a**3*c*d**4*e**5 - 3456*a**2*c**2*d**8*e) + 81*a**2*d**4*e**8 + 162*a*c*d**
8*e**4 + 81*c**2*d**12, Lambda(_t, _t*log(x + (4096*_t**3*a**7*c**2*e**6 +
28672*_t**3*a**6*c**3*d**4*e**2 - 7680*_t**2*a**5*c**2*d**4*e**5 + 1536*_t*
*2*a**4*c**3*d**8*e + 2160*_t*a**4*c*d**4*e**8 + 9216*_t*a**3*c**2*d**8*e**
4 + 144*_t*a**2*c**3*d**12 + 162*a**3*d**4*e**11 - 648*a**2*c*d**8*e**7 - 8
10*a*c**2*d**12*e**3)/(27*a**3*d**3*e**12 - 891*a**2*c*d**7*e**8 - 891*a*c*
*2*d**11*e**4 + 27*c**3*d**15))) + (-a*e**3 + c*d**3*x + 3*c*d**2*e*x**2 +
 3*c*d*e**2*x**3)/(4*a**2*c + 4*a*c**2*x**4)
```

Giac [A] time = 1.27711, size = 462, normalized size = 1.32

$$\frac{3\,cdx^3e^2 + 3\,cd^2x^2e + cd^3x - ae^3}{4\left(cx^4+a\right)ac} + \frac{3\,\sqrt{2}\left(2\,\sqrt{2}\sqrt{ac}c^2d^2e + \left(ac^3\right)^{\frac{1}{4}}c^2d^3 + \left(ac^3\right)^{\frac{3}{4}}de^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^2c^3} + \frac{3\,\sqrt{2}\left(2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*(3*c*d*x^3*e^2 + 3*c*d^2*x^2*e + c*d^3*x - a*e^3)/((c*x^4 + a)*a*c) + 3
/16*sqrt(2)*(2*sqrt(2)*sqrt(a*c)*c^2*d^2*e + (a*c^3)^(1/4)*c^2*d^3 + (a*c^3
)^(3/4)*d*e^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/
(a^2*c^3) + 3/16*sqrt(2)*(2*sqrt(2)*sqrt(a*c)*c^2*d^2*e + (a*c^3)^(1/4)*c^2
*d^3 + (a*c^3)^(3/4)*d*e^2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/
(a/c)^(1/4))/(a^2*c^3) + 3/32*sqrt(2)*((a*c^3)^(1/4)*c^2*d^3 - (a*c^3)^(3/4
)*d*e^2)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c^3) - 3/32*sqrt
(2)*((a*c^3)^(1/4)*c^2*d^3 - (a*c^3)^(3/4)*d*e^2)*log(x^2 - sqrt(2)*x*(a/c)
^(1/4) + sqrt(a/c))/(a^2*c^3)
```

3.402 $\int \frac{(d+ex)^2}{\left(a+cx^4\right)^2}\,dx$

Optimal. Leaf size=322

$$-\frac{\left(3\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{3/4}}+\frac{\left(3\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{3/4}}-\frac{\left(\sqrt{a}e^2+3\sqrt{c}d^2\right)}{8\sqrt{2}}$$

```
[Out] (x*(d + e*x)^2)/(4*a*(a + c*x^4)) + (d*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*
a^(3/2)*Sqrt[c]) - ((3*Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/
4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*c^(3/4)) + ((3*Sqrt[c]*d^2 + Sqrt[a]*e^2
)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*c^(3/4)) - ((
3*Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt
[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^(3/4)) + ((3*Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log
[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^
(3/4))
```

Rubi [A] time = 0.269161, antiderivative size = 322, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.588, Rules used = {1855, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{\left(3\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{3/4}}+\frac{\left(3\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{3/4}}-\frac{\left(\sqrt{a}e^2+3\sqrt{c}d^2\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(a + c*x^4)^2,x]
```

```
[Out] (x*(d + e*x)^2)/(4*a*(a + c*x^4)) + (d*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*
a^(3/2)*Sqrt[c]) - ((3*Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/
4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*c^(3/4)) + ((3*Sqrt[c]*d^2 + Sqrt[a]*e^2
)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*c^(3/4)) - ((
3*Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt
[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^(3/4)) + ((3*Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log
[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^
(3/4))
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(d+ex)^2}{\left(a+cx^4\right)^2}\,dx = \frac{x(d+ex)^2}{4a\left(a+cx^4\right)} - \frac{\int \frac{-3d^2-4dex-e^2x^2}{a+cx^4}\,dx}{4a}$$

$$= \frac{x(d+ex)^2}{4a\left(a+cx^4\right)} - \frac{\int \left(-\frac{4dex}{a+cx^4} + \frac{-3d^2-e^2x^2}{a+cx^4}\right)dx}{4a}$$

$$= \frac{x(d+ex)^2}{4a\left(a+cx^4\right)} - \frac{\int \frac{-3d^2-e^2x^2}{a+cx^4}\,dx}{4a} + \frac{(de)\int \frac{x}{a+cx^4}\,dx}{a}$$

$$= \frac{x(d+ex)^2}{4a\left(a+cx^4\right)} + \frac{(de)\,\text{Subst}\left(\int \frac{1}{a+cx^2}\,dx, x, x^2\right)}{2a} + \frac{\left(\frac{3\sqrt{c}d^2}{\sqrt{a}} - e^2\right)\int \frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4}\,dx}{8ac} + \frac{\left(\frac{3\sqrt{c}d^2}{\sqrt{a}} + e^2\right)\int \frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4}}{8ac}$$

$$= \frac{x(d+ex)^2}{4a\left(a+cx^4\right)} + \frac{de\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{c}} + \frac{\left(\frac{3\sqrt{c}d^2}{\sqrt{a}} + e^2\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}} + x^2}\,dx}{16ac} + \frac{\left(\frac{3\sqrt{c}d^2}{\sqrt{a}} + e^2\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} + \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}} + x^2}\,dx}{16ac} -$$

$$= \frac{x(d+ex)^2}{4a\left(a+cx^4\right)} + \frac{de\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{c}} - \frac{\left(3\sqrt{c}d^2 - \sqrt{a}e^2\right)\log\left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{3/4}} + \frac{\left(3\sqrt{c}d^2 - \sqrt{a}e^2\right)\mathrm{l}}{}$$

$$= \frac{x(d+ex)^2}{4a\left(a+cx^4\right)} + \frac{de\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{c}} - \frac{\left(3\sqrt{c}d^2 + \sqrt{a}e^2\right)\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}c^{3/4}} + \frac{\left(3\sqrt{c}d^2 + \sqrt{a}e^2\right)\tan^{-1}\left(1 + \right.}{8\sqrt{2}a^{7/4}c^{3/4}}$$

Mathematica [A] time = 0.325009, size = 321, normalized size = 1.

$$\frac{\frac{\sqrt{2}\left(a^{3/4}e^2 - 3\sqrt[4]{a}\sqrt{c}d^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2\right)}{c^{3/4}} + \frac{\sqrt{2}\left(3\sqrt[4]{a}\sqrt{c}d^2 - a^{3/4}e^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2\right)}{c^{3/4}} - \frac{2\sqrt[4]{a}\left(8\sqrt[4]{a}\sqrt[4]{c}de + \sqrt{2}\sqrt{a}e^2 + 3\sqrt{2}\sqrt{c}d^2\right)\tan^{-1}\left(1 - \right.}{c^{3/4}}}{32a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(a + c*x^4)^2,x]
```

```
[Out] ((8*a*x*(d + e*x)^2)/(a + c*x^4) - (2*a^(1/4)*(3*Sqrt[2]*Sqrt[c]*d^2 + 8*a^
(1/4)*c^(1/4)*d*e + Sqrt[2]*Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(
1/4)])/c^(3/4) + (2*a^(1/4)*(3*Sqrt[2]*Sqrt[c]*d^2 - 8*a^(1/4)*c^(1/4)*d*e
+ Sqrt[2]*Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/c^(3/4) + (
Sqrt[2]*(-3*a^(1/4)*Sqrt[c]*d^2 + a^(3/4)*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4
)*c^(1/4)*x + Sqrt[c]*x^2])/c^(3/4) + (Sqrt[2]*(3*a^(1/4)*Sqrt[c]*d^2 - a^(
```

```
3/4)*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(3/4))/
(32*a^2)
```

Maple [A] time = 0.004, size = 362, normalized size = 1.1

$$\frac{d^2x}{4a(cx^4+a)}+\frac{3d^2\sqrt{2}}{32a^2}\sqrt[4]{\frac{a}{c}}\ln\left(\left(x^2+\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)\left(x^2-\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)^{-1}\right)+\frac{3d^2\sqrt{2}}{16a^2}\sqrt[4]{\frac{a}{c}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{c}}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(c*x^4+a)^2,x)
```

```
[Out] 1/4*d^2*x/a/(c*x^4+a)+3/32*d^2/a^2*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*
x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+3/16*d^2/a^
2*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+3/16*d^2/a^2*(a/c)^(1
/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)+1/2*d*e*x^2/a/(c*x^4+a)+1/2*d*e
/a/(a*c)^(1/2)*arctan(x^2*(1/a*c)^(1/2))+1/4*e^2*x^3/a/(c*x^4+a)+1/32*e^2/a
/c/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c
)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+1/16*e^2/a/c/(a/c)^(1/4)*2^(1/2)*arctan(2^(
1/2)/(a/c)^(1/4)*x+1)+1/16*e^2/a/c/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)
^(1/4)*x-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 3.24013, size = 318, normalized size = 0.99

$$\operatorname{RootSum}\left(65536t^4a^7c^3 + 11264t^2a^4c^2d^2e^2 + t\left(256a^3cde^5 - 2304a^2c^2d^5e\right) + a^2e^8 + 82acd^4e^4 + 81c^2d^8, \left(t \mapsto t\log\left(x + \frac{4\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(c*x**4+a)**2,x)
```

```
[Out] RootSum(65536*_t**4*a**7*c**3 + 11264*_t**2*a**4*c**2*d**2*e**2 + _t*(256*a
**3*c*d*e**5 - 2304*a**2*c**2*d**5*e) + a**2*e**8 + 82*a*c*d**4*e**4 + 81*c
**2*d**8, Lambda(_t, _t*log(x + (4096*_t**3*a**7*c**2*e**6 + 356352*_t**3*a
**6*c**3*d**4*e**2 - 23552*_t**2*a**5*c**2*d**3*e**5 + 27648*_t**2*a**4*c**
3*d**7*e + 912*_t*a**4*c*d**2*e**8 + 43584*_t*a**3*c**2*d**6*e**4 + 3888*_t
*a**2*c**3*d**10 + 12*a**3*d*e**11 - 1088*a**2*c*d**5*e**7 - 7020*a*c**2*d*
*9*e**3)/(a**3*e**12 - 649*a**2*c*d**4*e**8 - 5841*a*c**2*d**8*e**4 + 729*c
**3*d**12)))) + (d**2*x + 2*d*e*x**2 + e**2*x**3)/(4*a**2 + 4*a*c*x**4)
```

Giac [A] time = 1.15916, size = 436, normalized size = 1.35

$$\frac{x^3e^2 + 2\,dx^2e + d^2x}{4\left(cx^4 + a\right)a} + \frac{\sqrt{2}\left(4\sqrt{2}\sqrt{ac}c^2de + 3\left(ac^3\right)^{\frac{1}{4}}c^2d^2 + \left(ac^3\right)^{\frac{3}{4}}e^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x + \sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^2c^3} + \frac{\sqrt{2}\left(4\sqrt{2}\sqrt{ac}c^2de + 3\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*(x^3*e^2 + 2*d*x^2*e + d^2*x)/((c*x^4 + a)*a) + 1/16*sqrt(2)*(4*sqrt(2)
*sqrt(a*c)*c^2*d*e + 3*(a*c^3)^(1/4)*c^2*d^2 + (a*c^3)^(3/4)*e^2)*arctan(1/
2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c^3) + 1/16*sqrt(2)
*(4*sqrt(2)*sqrt(a*c)*c^2*d*e + 3*(a*c^3)^(1/4)*c^2*d^2 + (a*c^3)^(3/4)*e^2
)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c^3) + 1
```

```
/32*sqrt(2)*(3*(a*c^3)^(1/4)*c^2*d^2 - (a*c^3)^(3/4)*e^2)*log(x^2 + sqrt(2)
*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c^3) - 1/32*sqrt(2)*(3*(a*c^3)^(1/4)*c^2*d
^2 - (a*c^3)^(3/4)*e^2)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c
^3)
```

3.403 $\int \frac{d+ex}{\left(a+cx^4\right)^2}\,dx$

Optimal. Leaf size=241

$$-\frac{3d\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{c}}+\frac{3d\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{c}}-\frac{3d\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}}+\frac{3d\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}}$$

```
[Out] (x*(d + e*x))/(4*a*(a + c*x^4)) + (e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*a^(3
/2)*Sqrt[c]) - (3*d*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(
7/4)*c^(1/4)) + (3*d*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^
(7/4)*c^(1/4)) - (3*d*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2
])/(16*Sqrt[2]*a^(7/4)*c^(1/4)) + (3*d*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4
)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^(1/4))
```

Rubi [A] time = 0.189006, antiderivative size = 241, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {1855, 1876, 211, 1165, 628, 1162, 617, 204, 275, 205}

$$-\frac{3d\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{c}}+\frac{3d\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{c}}-\frac{3d\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}}+\frac{3d\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(a + c*x^4)^2,x]
```

```
[Out] (x*(d + e*x))/(4*a*(a + c*x^4)) + (e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*a^(3
/2)*Sqrt[c]) - (3*d*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(
7/4)*c^(1/4)) + (3*d*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^
(7/4)*c^(1/4)) - (3*d*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2
])/(16*Sqrt[2]*a^(7/4)*c^(1/4)) + (3*d*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4
)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^(1/4))
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
```

```
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{d+ex}{\left(a+cx^4\right)^2}\,dx = \frac{x(d+ex)}{4a\left(a+cx^4\right)} - \frac{\int \frac{-3d-2ex}{a+cx^4}\,dx}{4a}$$

$$= \frac{x(d+ex)}{4a\left(a+cx^4\right)} - \frac{\int \left(-\frac{3d}{a+cx^4} - \frac{2ex}{a+cx^4}\right)dx}{4a}$$

$$= \frac{x(d+ex)}{4a\left(a+cx^4\right)} + \frac{(3d)\int \frac{1}{a+cx^4}\,dx}{4a} + \frac{e\int \frac{x}{a+cx^4}\,dx}{2a}$$

$$= \frac{x(d+ex)}{4a\left(a+cx^4\right)} + \frac{(3d)\int \frac{\sqrt{a}-\sqrt{c}x^2}{a+cx^4}\,dx}{8a^{3/2}} + \frac{(3d)\int \frac{\sqrt{a}+\sqrt{c}x^2}{a+cx^4}\,dx}{8a^{3/2}} + \frac{e\,\mathrm{Subst}\left(\int \frac{1}{a+cx^2}\,dx, x, x^2\right)}{4a}$$

$$= \frac{x(d+ex)}{4a\left(a+cx^4\right)} + \frac{e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{c}} + \frac{(3d)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{16a^{3/2}\sqrt{c}} + \frac{(3d)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{16a^{3/2}\sqrt{c}} - \frac{(3d)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-}}{16\sqrt{2}a^{7/4}\sqrt[4]{c}}$$

$$= \frac{x(d+ex)}{4a\left(a+cx^4\right)} + \frac{e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{c}} - \frac{3d\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{c}} + \frac{3d\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right.}{16\sqrt{2}a^{7/4}\sqrt[4]{c}}$$

$$= \frac{x(d+ex)}{4a\left(a+cx^4\right)} + \frac{e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{c}} - \frac{3d\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}} + \frac{3d\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}} - \frac{3d\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\right.}{16\sqrt{2}a^{7/4}}$$

Mathematica [A] time = 0.199239, size = 224, normalized size = 0.93

$$\frac{\frac{8a^{3/4}x(d+ex)}{a+cx^4}-\frac{2\left(4\sqrt[4]{a}e+3\sqrt{2}\sqrt[4]{c}d\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{c}}+\frac{2\left(3\sqrt{2}\sqrt[4]{c}d-4\sqrt[4]{a}e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{c}}-\frac{3\sqrt{2}d\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{\sqrt[4]{c}}+\frac{3\sqrt{2}d\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{\cdots}\right.}{\sqrt[4]{c}}}{32a^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(a + c*x^4)^2,x]
```

```
[Out] ((8*a^(3/4)*x*(d + e*x))/(a + c*x^4) - (2*(3*Sqrt[2]*c^(1/4)*d + 4*a^(1/4)*
e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/Sqrt[c] + (2*(3*Sqrt[2]*c^(1/4)
*d - 4*a^(1/4)*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/Sqrt[c] - (3*Sqr
t[2]*d*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(1/4) + (3
*Sqrt[2]*d*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(1/4))
/(32*a^(7/4))
```

Maple [A] time = 0.004, size = 188, normalized size = 0.8

$$\frac{dx}{4a\left(cx^4+a\right)}+\frac{3d\sqrt{2}}{32a^2}\sqrt[4]{\frac{a}{c}}\ln\left(\left(x^2+\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)\left(x^2-\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)^{-1}\right)+\frac{3d\sqrt{2}}{16a^2}\sqrt[4]{\frac{a}{c}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{c}}}+1\right)+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(c*x^4+a)^2,x)
```

```
[Out] 1/4*d*x/a/(c*x^4+a)+3/32*d/a^2*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*x*2^
(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+3/16*d/a^2*(a/c
)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+3/16*d/a^2*(a/c)^(1/4)*2^(1
/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)+1/4*e*x^2/a/(c*x^4+a)+1/4*e/a/(a*c)^(1/
2)*arctan(x^2*(1/a*c)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 1.28393, size = 155, normalized size = 0.64

$$\text{RootSum}\left(65536t^4a^7c^2 + 2048t^2a^4ce^2 - 1152ta^2cd^2e + 16ae^4 + 81cd^4, \left(t \mapsto t\log\left(x + \frac{-32768t^3a^6ce^2 - 4608t^2a^4cd^2e - }{192ade^4}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x**4+a)**2,x)
```

```
[Out] RootSum(65536*_t**4*a**7*c**2 + 2048*_t**2*a**4*c*e**2 - 1152*_t*a**2*c*d**
2*e + 16*a*e**4 + 81*c*d**4, Lambda(_t, _t*log(x + (-32768*_t**3*a**6*c*e**
2 - 4608*_t**2*a**4*c*d**2*e - 512*_t*a**3*e**4 - 1296*_t*a**2*c*d**4 + 360
*a*d**2*e**3)/(192*a*d*e**4 - 243*c*d**5)))) + (d*x + e*x**2)/(4*a**2 + 4*a
*c*x**4)
```

Giac [A] time = 1.11144, size = 325, normalized size = 1.35

$$\frac{3\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}d\log\left(x^2 + \sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{32\,a^2c} - \frac{3\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}d\log\left(x^2 - \sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{32\,a^2c} + \frac{x^2e + dx}{4\left(cx^4 + a\right)a} + \frac{\sqrt{2}\left(2\sqrt{2}\sqrt{ac}ce + \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 3/32*sqrt(2)*(a*c^3)^(1/4)*d*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(
a^2*c) - 3/32*sqrt(2)*(a*c^3)^(1/4)*d*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqr
t(a/c))/(a^2*c) + 1/4*(x^2*e + d*x)/((c*x^4 + a)*a) + 1/16*sqrt(2)*(2*sqrt(
2)*sqrt(a*c)*c*e + 3*(a*c^3)^(1/4)*c*d)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(
a/c)^(1/4))/(a/c)^(1/4))/(a^2*c^2) + 1/16*sqrt(2)*(2*sqrt(2)*sqrt(a*c)*c*e
+ 3*(a*c^3)^(1/4)*c*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)
^(1/4))/(a^2*c^2)
```

3.404 $\int \frac{1}{\left(a+cx^4\right)^2}\,dx$

Optimal. Leaf size=202

$$-\frac{3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{c}}+\frac{3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{c}}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}}+\frac{3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}}+\frac{\ }{4}$$

```
[Out] x/(4*a*(a + c*x^4)) - (3*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2
]*a^(7/4)*c^(1/4)) + (3*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]
*a^(7/4)*c^(1/4)) - (3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^
2])/(16*Sqrt[2]*a^(7/4)*c^(1/4)) + (3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)
*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^(1/4))
```

Rubi [A] time = 0.115676, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.778, Rules used = {199, 211, 1165, 628, 1162, 617, 204}

$$-\frac{3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{c}}+\frac{3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{c}}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}}+\frac{3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}}+\frac{\ }{4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^4)^(-2),x]
```

```
[Out] x/(4*a*(a + c*x^4)) - (3*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2
]*a^(7/4)*c^(1/4)) + (3*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]
*a^(7/4)*c^(1/4)) - (3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^
2])/(16*Sqrt[2]*a^(7/4)*c^(1/4)) + (3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)
*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^(1/4))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(a+cx^4)^2}\,dx = \frac{x}{4a(a+cx^4)} + \frac{3\int \frac{1}{a+cx^4}\,dx}{4a}$$

$$= \frac{x}{4a(a+cx^4)} + \frac{3\int \frac{\sqrt{a}-\sqrt{c}x^2}{a+cx^4}\,dx}{8a^{3/2}} + \frac{3\int \frac{\sqrt{a}+\sqrt{c}x^2}{a+cx^4}\,dx}{8a^{3/2}}$$

$$= \frac{x}{4a(a+cx^4)} + \frac{3\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{16a^{3/2}\sqrt{c}} + \frac{3\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{16a^{3/2}\sqrt{c}} - \frac{3\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{16\sqrt{2}a^{7/4}\sqrt[4]{c}} - \frac{3\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}}{16\sqrt{2}a^{7/4}\sqrt[4]{c}}$$

$$= \frac{x}{4a(a+cx^4)} - \frac{3\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{c}} + \frac{3\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{c}} + \frac{3\,\mathrm{Subst}\left(\int \frac{1}{-1-x}\right.}{8\sqrt{2}}$$

$$= \frac{x}{4a(a+cx^4)} - \frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}} + \frac{3\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}} - \frac{3\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{c}} + 3\log$$

Mathematica [A] time = 0.10742, size = 183, normalized size = 0.91

$$\frac{\frac{8a^{3/4}x}{a+cx^4} - \frac{3\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{\sqrt[4]{c}} + \frac{3\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{\sqrt[4]{c}} - \frac{6\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt[4]{c}} + \frac{6\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt[4]{c}}}{32a^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^4)^(-2),x]
```

```
[Out] ((8*a^(3/4)*x)/(a + c*x^4) - (6*Sqrt[2]*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1
/4)])/c^(1/4) + (6*Sqrt[2]*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/c^(1/4)
 - (3*Sqrt[2]*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(1/
4) + (3*Sqrt[2]*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(
1/4))/(32*a^(7/4))
```

Maple [A] time = 0.004, size = 143, normalized size = 0.7

$$\frac{x}{4a(cx^4+a)} + \frac{3\sqrt{2}}{32a^2}\sqrt[4]{\frac{a}{c}}\ln\left(\left(x^2+\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)\left(x^2-\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)^{-1}\right) + \frac{3\sqrt{2}}{16a^2}\sqrt[4]{\frac{a}{c}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{c}}}+1\right) + \frac{3\sqrt{}}{16a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+a)^2,x)
```

```
[Out] 1/4*x/a/(c*x^4+a)+3/32/a^2*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*x*2^(1/2
)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+3/16/a^2*(a/c)^(1/4
)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+3/16/a^2*(a/c)^(1/4)*2^(1/2)*arct
an(2^(1/2)/(a/c)^(1/4)*x-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.996213, size = 414, normalized size = 2.05

$$\frac{12\left(acx^4+a^2\right)\left(-\frac{1}{a^7c}\right)^{\frac{1}{4}}\arctan\left(-a^5cx\left(-\frac{1}{a^7c}\right)^{\frac{3}{4}}+\sqrt{a^4\sqrt{-\frac{1}{a^7c}}+x^2}\,a^5c\left(-\frac{1}{a^7c}\right)^{\frac{3}{4}}\right)+3\left(acx^4+a^2\right)\left(-\frac{1}{a^7c}\right)^{\frac{1}{4}}\log\left(a^2\left(-\frac{1}{a^7c}\right)^{\frac{1}{4}}\right.}{16\left(acx^4+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(12*(a*c*x^4 + a^2)*(-1/(a^7*c))^(1/4)*arctan(-a^5*c*x*(-1/(a^7*c))^(3
/4) + sqrt(a^4*sqrt(-1/(a^7*c)) + x^2)*a^5*c*(-1/(a^7*c))^(3/4)) + 3*(a*c*x
^4 + a^2)*(-1/(a^7*c))^(1/4)*log(a^2*(-1/(a^7*c))^(1/4) + x) - 3*(a*c*x^4 +
 a^2)*(-1/(a^7*c))^(1/4)*log(-a^2*(-1/(a^7*c))^(1/4) + x) + 4*x)/(a*c*x^4 +
 a^2)
```

Sympy [A] time = 0.469475, size = 39, normalized size = 0.19

$$\frac{x}{4a^2+4acx^4} + \text{RootSum}\left(65536t^4a^7c+81, \left(t \mapsto t\log\left(\frac{16ta^2}{3}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+a)**2,x)
```

```
[Out] x/(4*a**2 + 4*a*c*x**4) + RootSum(65536*_t**4*a**7*c + 81, Lambda(_t, _t*lo
g(16*_t*a**2/3 + x)))
```

Giac [A] time = 1.11891, size = 262, normalized size = 1.3

$$\frac{x}{4\left(cx^4+a\right)a} + \frac{3\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16a^2c} + \frac{3\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16a^2c} + \frac{3\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}\log\left(x^2+\sqrt{2}x\right.}{32a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*x/((c*x^4 + a)*a) + 3/16*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x
+ sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c) + 3/16*sqrt(2)*(a*c^3)^(1/4)*ar
ctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c) + 3/32*sq
rt(2)*(a*c^3)^(1/4)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c) -
3/32*sqrt(2)*(a*c^3)^(1/4)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^
2*c)
```

3.405 $\int \frac{1}{(d+ex)\left(a+cx^4\right)^2}\,dx$

Optimal. Leaf size=855

$$\frac{\log(d+ex)e^7}{\left(cd^4+ae^4\right)^2}-\frac{\log\left(cx^4+a\right)e^7}{4\left(cd^4+ae^4\right)^2}-\frac{\sqrt{c}d^2\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)e^5}{2\sqrt{a}\left(cd^4+ae^4\right)^2}-\frac{\sqrt[4]{c}d\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)e^4}{2\sqrt{2}a^{3/4}\left(cd^4+ae^4\right)^2}+\frac{\sqrt[4]{c}d\left(\sqrt{c}d^2+\sqrt{a}\cdots\right.}{2\sqrt{2}a^{\cdots}}$$

```
[Out] (a*e^3 + c*x*(d^3 - d^2*e*x + d*e^2*x^2))/(4*a*(c*d^4 + a*e^4)*(a + c*x^4))
 - (Sqrt[c]*d^2*e^5*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*Sqrt[a]*(c*d^4 + a*e^
4)^2) - (Sqrt[c]*d^2*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*a^(3/2)*(c*d^4 + a
*e^4)) - (c^(1/4)*d*e^4*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(
1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^2) - (c^(1/4)*d*(3*Sqr
t[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]
*a^(7/4)*(c*d^4 + a*e^4)) + (c^(1/4)*d*e^4*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcT
an[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^2)
+ (c^(1/4)*d*(3*Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a
^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)) + (e^7*Log[d + e*x])/(c*d^4 +
a*e^4)^2 - (c^(1/4)*d*e^4*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] - Sqrt[2]
*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^2) -
(c^(1/4)*d*(3*Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1
/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)) + (c^(1/4)*d*e^4
*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt
[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^2) + (c^(1/4)*d*(3*Sqrt[c]*d^2
 - Sqrt[a]*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16
*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)) - (e^7*Log[a + c*x^4])/(4*(c*d^4 + a*e^4)
^2)
```

Rubi [A] time = 0.849351, antiderivative size = 855, normalized size of antiderivative = 1., number of steps used = 31, number of rules used = 14, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.824, Rules used = {6742, 1854, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628, 1248, 635, 260}

$$\frac{\log(d+ex)e^7}{\left(cd^4+ae^4\right)^2}-\frac{\log\left(cx^4+a\right)e^7}{4\left(cd^4+ae^4\right)^2}-\frac{\sqrt{c}d^2\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)e^5}{2\sqrt{a}\left(cd^4+ae^4\right)^2}-\frac{\sqrt[4]{c}d\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)e^4}{2\sqrt{2}a^{3/4}\left(cd^4+ae^4\right)^2}+\frac{\sqrt[4]{c}d\left(\sqrt{c}d^2+\sqrt{a}\cdots\right.}{2\sqrt{2}a^{\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(a + c*x^4)^2),x]
```

```
[Out] (a*e^3 + c*x*(d^3 - d^2*e*x + d*e^2*x^2))/(4*a*(c*d^4 + a*e^4)*(a + c*x^4))
 - (Sqrt[c]*d^2*e^5*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*Sqrt[a]*(c*d^4 + a*e^
4)^2) - (Sqrt[c]*d^2*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*a^(3/2)*(c*d^4 + a
*e^4)) - (c^(1/4)*d*e^4*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(
1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^2) - (c^(1/4)*d*(3*Sqr
t[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]
*a^(7/4)*(c*d^4 + a*e^4)) + (c^(1/4)*d*e^4*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcT
an[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^2)
+ (c^(1/4)*d*(3*Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a
^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)) + (e^7*Log[d + e*x])/(c*d^4 +
a*e^4)^2 - (c^(1/4)*d*e^4*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] - Sqrt[2]
*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^2) -
(c^(1/4)*d*(3*Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1
/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)) + (c^(1/4)*d*e^4
*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt
[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^2) + (c^(1/4)*d*(3*Sqrt[c]*d^2
 - Sqrt[a]*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16
*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)) - (e^7*Log[a + c*x^4])/(4*(c*d^4 + a*e^4)
^2)
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
```

```
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{1}{(d+ex)\left(a+cx^4\right)^2}\,dx = \int\left(\frac{e^8}{\left(cd^4+ae^4\right)^2(d+ex)} + \frac{c\left(d^3-d^2ex+de^2x^2-e^3x^3\right)}{\left(cd^4+ae^4\right)\left(a+cx^4\right)^2} - \frac{ce^4\left(-d^3+d^2ex-de^2x^2+e^3x^3\right)}{\left(cd^4+ae^4\right)^2\left(a+cx^4\right)}\right.$$

$$= \frac{e^7\log(d+ex)}{\left(cd^4+ae^4\right)^2} - \frac{\left(ce^4\right)\int\frac{-d^3+d^2ex-de^2x^2+e^3x^3}{a+cx^4}\,dx}{\left(cd^4+ae^4\right)^2} + \frac{c\int\frac{d^3-d^2ex+de^2x^2-e^3x^3}{\left(a+cx^4\right)^2}\,dx}{cd^4+ae^4}$$

$$= \frac{ae^3+cx\left(d^3-d^2ex+de^2x^2\right)}{4a\left(cd^4+ae^4\right)\left(a+cx^4\right)} + \frac{e^7\log(d+ex)}{\left(cd^4+ae^4\right)^2} - \frac{\left(ce^4\right)\int\left(\frac{-d^3-de^2x^2}{a+cx^4}+\frac{x\left(d^2e+e^3x^2\right)}{a+cx^4}\right)dx}{\left(cd^4+ae^4\right)^2} - \frac{c\int\frac{-3}{}}{4a}$$

$$= \frac{ae^3+cx\left(d^3-d^2ex+de^2x^2\right)}{4a\left(cd^4+ae^4\right)\left(a+cx^4\right)} + \frac{e^7\log(d+ex)}{\left(cd^4+ae^4\right)^2} - \frac{\left(ce^4\right)\int\frac{-d^3-de^2x^2}{a+cx^4}\,dx}{\left(cd^4+ae^4\right)^2} - \frac{\left(ce^4\right)\int\frac{x\left(d^2e+e^3x^2\right)}{a+cx^4}\,dx}{\left(cd^4+ae^4\right)^2}$$

$$= \frac{ae^3+cx\left(d^3-d^2ex+de^2x^2\right)}{4a\left(cd^4+ae^4\right)\left(a+cx^4\right)} + \frac{e^7\log(d+ex)}{\left(cd^4+ae^4\right)^2} - \frac{\left(ce^4\right)\text{Subst}\left(\int\frac{d^2e+e^3x}{a+cx^2}\,dx,x,x^2\right)}{2\left(cd^4+ae^4\right)^2} + \frac{\left(de^4\left(\sqrt{c}\right.\right.}{}$$

$$= \frac{ae^3+cx\left(d^3-d^2ex+de^2x^2\right)}{4a\left(cd^4+ae^4\right)\left(a+cx^4\right)} + \frac{e^7\log(d+ex)}{\left(cd^4+ae^4\right)^2} - \frac{\left(cd^2e^5\right)\text{Subst}\left(\int\frac{1}{a+cx^2}\,dx,x,x^2\right)}{2\left(cd^4+ae^4\right)^2} - \left(ce^7\right)\text{S}$$

$$= \frac{ae^3+cx\left(d^3-d^2ex+de^2x^2\right)}{4a\left(cd^4+ae^4\right)\left(a+cx^4\right)} - \frac{\sqrt{c}d^2e^5\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\left(cd^4+ae^4\right)^2} - \frac{\sqrt{c}d^2e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{3/2}\left(cd^4+ae^4\right)} + \frac{e^7\log(d+ex)}{\left(cd^4+ae^4\right)^2}$$

$$= \frac{ae^3+cx\left(d^3-d^2ex+de^2x^2\right)}{4a\left(cd^4+ae^4\right)\left(a+cx^4\right)} - \frac{\sqrt{c}d^2e^5\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\left(cd^4+ae^4\right)^2} - \frac{\sqrt{c}d^2e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{3/2}\left(cd^4+ae^4\right)} - \frac{\sqrt[4]{c}de^4\left(\sqrt{c}d^2\right.}{2\sqrt{2}}$$

$$= \frac{ae^3+cx\left(d^3-d^2ex+de^2x^2\right)}{4a\left(cd^4+ae^4\right)\left(a+cx^4\right)} - \frac{\sqrt{c}d^2e^5\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\left(cd^4+ae^4\right)^2} - \frac{\sqrt{c}d^2e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{3/2}\left(cd^4+ae^4\right)} - \frac{\sqrt[4]{c}de^4\left(\sqrt{c}d^2\right.}{2\sqrt{2}}$$

Mathematica [A] time = 0.421641, size = 558, normalized size = 0.65

$$\frac{\frac{\sqrt{2}\sqrt[4]{c}\left(5a^{3/2}de^6+\sqrt{a}cd^5e^2-7a\sqrt{c}d^3e^4-3c^{3/2}d^7\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{a^{7/4}} + \frac{\sqrt{2}\sqrt[4]{c}\left(-5a^{3/2}de^6-\sqrt{a}cd^5e^2+7a\sqrt{c}d^3e^4+3c^{3/2}d^7\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{a^{7/4}} - }{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(a + c*x^4)^2),x]
```

```
[Out] ((8*(c*d^4 + a*e^4)*(a*e^3 + c*d*x*(d^2 - d*e*x + e^2*x^2)))/(a*(a + c*x^4)
) - (2*c^(1/4)*d*(3*Sqrt[2]*c^(3/2)*d^6 - 4*a^(1/4)*c^(5/4)*d^5*e + Sqrt[2]
*Sqrt[a]*c*d^4*e^2 + 7*Sqrt[2]*a*Sqrt[c]*d^2*e^4 - 12*a^(5/4)*c^(1/4)*d*e^5
 + 5*Sqrt[2]*a^(3/2)*e^6)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(7/4)
+ (2*c^(1/4)*d*(3*Sqrt[2]*c^(3/2)*d^6 + 4*a^(1/4)*c^(5/4)*d^5*e + Sqrt[2]*S
qrt[a]*c*d^4*e^2 + 7*Sqrt[2]*a*Sqrt[c]*d^2*e^4 + 12*a^(5/4)*c^(1/4)*d*e^5 +
 5*Sqrt[2]*a^(3/2)*e^6)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(7/4) +
32*e^7*Log[d + e*x] + (Sqrt[2]*c^(1/4)*(-3*c^(3/2)*d^7 + Sqrt[a]*c*d^5*e^2
- 7*a*Sqrt[c]*d^3*e^4 + 5*a^(3/2)*d*e^6)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1
/4)*x + Sqrt[c]*x^2])/a^(7/4) + (Sqrt[2]*c^(1/4)*(3*c^(3/2)*d^7 - Sqrt[a]*c
*d^5*e^2 + 7*a*Sqrt[c]*d^3*e^4 - 5*a^(3/2)*d*e^6)*Log[Sqrt[a] + Sqrt[2]*a^(
1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(7/4) - 8*e^7*Log[a + c*x^4])/(32*(c*d^4 +
 a*e^4)^2)
```

Maple [A] time = 0.025, size = 1122, normalized size = 1.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(c*x^4+a)^2,x)
```

```
[Out] e^7*ln(e*x+d)/(a*e^4+c*d^4)^2+1/4*c/(a*e^4+c*d^4)^2/(c*x^4+a)*d*e^6*x^3+1/4
*c^2/(a*e^4+c*d^4)^2/(c*x^4+a)*d^5*e^2/a*x^3-1/4*c/(a*e^4+c*d^4)^2/(c*x^4+a
)*e^5*d^2*x^2-1/4*c^2/(a*e^4+c*d^4)^2/(c*x^4+a)*e*d^6/a*x^2+1/4*c/(a*e^4+c*
d^4)^2/(c*x^4+a)*d^3*x*e^4+1/4*c^2/(a*e^4+c*d^4)^2/(c*x^4+a)*d^7/a*x+1/4/(a
*e^4+c*d^4)^2/(c*x^4+a)*a*e^7+1/4*c/(a*e^4+c*d^4)^2/(c*x^4+a)*e^3*d^4+7/16*
c/(a*e^4+c*d^4)^2/a*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^3
*e^4+3/16*c^2/(a*e^4+c*d^4)^2/a^2*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^
(1/4)*x+1)*d^7+7/16*c/(a*e^4+c*d^4)^2/a*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/
(a/c)^(1/4)*x-1)*d^3*e^4+3/16*c^2/(a*e^4+c*d^4)^2/a^2*(a/c)^(1/4)*2^(1/2)*a
rctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^7+7/32*c/(a*e^4+c*d^4)^2/a*(a/c)^(1/4)*2^(
1/2)*ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+
(a/c)^(1/2)))*d^3*e^4+3/32*c^2/(a*e^4+c*d^4)^2/a^2*(a/c)^(1/4)*2^(1/2)*ln((
x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/
2)))*d^7-3/4*c/(a*e^4+c*d^4)^2/(a*c)^(1/2)*arctan(x^2*(1/a*c)^(1/2))*d^2*e^
5-1/4*c^2/(a*e^4+c*d^4)^2/a/(a*c)^(1/2)*arctan(x^2*(1/a*c)^(1/2))*d^6*e+5/3
2/(a*e^4+c*d^4)^2/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(
```

```
1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))*d*e^6+1/32*c/(a*e^4+c*d^4)^2
/a/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c
)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))*d^5*e^2+5/16/(a*e^4+c*d^4)^2/(a/c)^(1/4)*2^
(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d*e^6+1/16*c/(a*e^4+c*d^4)^2/a/(a/c)^
(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^5*e^2+5/16/(a*e^4+c*d^4)^2/
(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d*e^6+1/16*c/(a*e^4+c*d
^4)^2/a/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^5*e^2-1/4*e^7
*ln(c*x^4+a)/(a*e^4+c*d^4)^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^4+a)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^4+a)^2,x, algorithm="fricas")

[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.29737, size = 1038, normalized size = 1.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/8*(4*sqrt(2)*sqrt(a*c)*c^3*d^2*e + 3*(a*c^3)^(1/4)*c^3*d^3 + 5*(a*c^3)^(3
/4)*c*d*e^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(s
qrt(2)*a^2*c^4*d^4 - 4*(a*c^3)^(1/4)*a^2*c^3*d^3*e + 4*sqrt(2)*sqrt(a*c)*a^
2*c^3*d^2*e^2 + sqrt(2)*a^3*c^3*e^4 - 4*(a*c^3)^(3/4)*a^2*c*d*e^3) + 1/8*(4
*sqrt(2)*sqrt(a*c)*c^3*d^2*e + 3*(a*c^3)^(1/4)*c^3*d^3 + 5*(a*c^3)^(3/4)*c*
d*e^2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)
*a^2*c^4*d^4 + 4*(a*c^3)^(1/4)*a^2*c^3*d^3*e + 4*sqrt(2)*sqrt(a*c)*a^2*c^3*
d^2*e^2 + sqrt(2)*a^3*c^3*e^4 + 4*(a*c^3)^(3/4)*a^2*c*d*e^3) + 1/16*(3*(a*c
^3)^(1/4)*c^3*d^7 - (a*c^3)^(3/4)*c*d^5*e^2 + 7*(a*c^3)^(1/4)*a*c^2*d^3*e^4
 - 5*(a*c^3)^(3/4)*a*d*e^6)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(s
qrt(2)*a^2*c^4*d^8 + 2*sqrt(2)*a^3*c^3*d^4*e^4 + sqrt(2)*a^4*c^2*e^8) - 1/1
6*(3*(a*c^3)^(1/4)*c^3*d^7 - (a*c^3)^(3/4)*c*d^5*e^2 + 7*(a*c^3)^(1/4)*a*c^
2*d^3*e^4 - 5*(a*c^3)^(3/4)*a*d*e^6)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt
(a/c))/(sqrt(2)*a^2*c^4*d^8 + 2*sqrt(2)*a^3*c^3*d^4*e^4 + sqrt(2)*a^4*c^2*e
^8) - 1/4*e^7*log(abs(c*x^4 + a))/(c^2*d^8 + 2*a*c*d^4*e^4 + a^2*e^8) + e^8
*log(abs(x*e + d))/(c^2*d^8*e + 2*a*c*d^4*e^5 + a^2*e^9) + 1/4*(a*c*d^4*e^3
 + (c^2*d^5*e^2 + a*c*d*e^6)*x^3 - (c^2*d^6*e + a*c*d^2*e^5)*x^2 + a^2*e^7
+ (c^2*d^7 + a*c*d^3*e^4)*x)/((c*d^4 + a*e^4)^2*(c*x^4 + a)*a)
```

3.406 $\int \frac{1}{(d+ex)^2\left(a+cx^4\right)^2}\,dx$

Optimal. Leaf size=1141

result too large to display

```
[Out] -(e^7/((c*d^4 + a*e^4)^2*(d + e*x))) + (c*(4*a*d^3*e^3 + x*(d^2*(c*d^4 - 3*
a*e^4) - 2*d*e*(c*d^4 - a*e^4)*x + e^2*(3*c*d^4 - a*e^4)*x^2)))/(4*a*(c*d^4
 + a*e^4)^2*(a + c*x^4)) - (Sqrt[c]*d*e^5*(3*c*d^4 - a*e^4)*ArcTan[(Sqrt[c]
*x^2)/Sqrt[a]])/(Sqrt[a]*(c*d^4 + a*e^4)^3) - (Sqrt[c]*d*e*(c*d^4 - a*e^4)*
ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*a^(3/2)*(c*d^4 + a*e^4)^2) - (c^(1/4)*(3*
Sqrt[c]*d^2*(c*d^4 - 3*a*e^4) + Sqrt[a]*e^2*(3*c*d^4 - a*e^4))*ArcTan[1 - (
Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^2) - (c^(1/
4)*e^4*(Sqrt[c]*d^2*(5*c*d^4 - 3*a*e^4) + Sqrt[a]*e^2*(7*c*d^4 - a*e^4))*Ar
cTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^3
) + (c^(1/4)*(3*Sqrt[c]*d^2*(c*d^4 - 3*a*e^4) + Sqrt[a]*e^2*(3*c*d^4 - a*e^
4))*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^4 + a*
e^4)^2) + (c^(1/4)*e^4*(Sqrt[c]*d^2*(5*c*d^4 - 3*a*e^4) + Sqrt[a]*e^2*(7*c*
d^4 - a*e^4))*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(
c*d^4 + a*e^4)^3) + (8*c*d^3*e^7*Log[d + e*x])/(c*d^4 + a*e^4)^3 - (c^(1/4)
*(3*Sqrt[c]*d^2*(c*d^4 - 3*a*e^4) - Sqrt[a]*e^2*(3*c*d^4 - a*e^4))*Log[Sqrt
[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^4
+ a*e^4)^2) - (c^(1/4)*e^4*(Sqrt[c]*d^2*(5*c*d^4 - 3*a*e^4) - Sqrt[a]*e^2*(
7*c*d^4 - a*e^4))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(
4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^3) + (c^(1/4)*(3*Sqrt[c]*d^2*(c*d^4 - 3*a
*e^4) - Sqrt[a]*e^2*(3*c*d^4 - a*e^4))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4
)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^2) + (c^(1/4)*e^4*(
Sqrt[c]*d^2*(5*c*d^4 - 3*a*e^4) - Sqrt[a]*e^2*(7*c*d^4 - a*e^4))*Log[Sqrt[a
] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a
*e^4)^3) - (2*c*d^3*e^7*Log[a + c*x^4])/(c*d^4 + a*e^4)^3
```

Rubi [A] time = 1.65882, antiderivative size = 1141, normalized size of antiderivative = 1., number of steps used = 31, number of rules used = 14, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.824, Rules used = {6742, 1854, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628, 1248, 635, 260}

$$\frac{8cd^3\log(d+ex)e^7}{\left(cd^4+ae^4\right)^3}-\frac{2cd^3\log\left(cx^4+a\right)e^7}{\left(cd^4+ae^4\right)^3}-\frac{e^7}{\left(cd^4+ae^4\right)^2(d+ex)}-\frac{\sqrt{c}d\left(3cd^4-ae^4\right)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)e^5}{\sqrt{a}\left(cd^4+ae^4\right)^3}-\frac{\sqrt[4]{c}\left(\sqrt{c}\left(5cd^4-\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^2*(a + c*x^4)^2),x]
```

```
[Out] -(e^7/((c*d^4 + a*e^4)^2*(d + e*x))) + (c*(4*a*d^3*e^3 + x*(d^2*(c*d^4 - 3*
a*e^4) - 2*d*e*(c*d^4 - a*e^4)*x + e^2*(3*c*d^4 - a*e^4)*x^2)))/(4*a*(c*d^4
 + a*e^4)^2*(a + c*x^4)) - (Sqrt[c]*d*e^5*(3*c*d^4 - a*e^4)*ArcTan[(Sqrt[c]
*x^2)/Sqrt[a]])/(Sqrt[a]*(c*d^4 + a*e^4)^3) - (Sqrt[c]*d*e*(c*d^4 - a*e^4)*
ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*a^(3/2)*(c*d^4 + a*e^4)^2) - (c^(1/4)*(3*
Sqrt[c]*d^2*(c*d^4 - 3*a*e^4) + Sqrt[a]*e^2*(3*c*d^4 - a*e^4))*ArcTan[1 - (
Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^2) - (c^(1/
4)*e^4*(Sqrt[c]*d^2*(5*c*d^4 - 3*a*e^4) + Sqrt[a]*e^2*(7*c*d^4 - a*e^4))*Ar
cTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^3
) + (c^(1/4)*(3*Sqrt[c]*d^2*(c*d^4 - 3*a*e^4) + Sqrt[a]*e^2*(3*c*d^4 - a*e^
4))*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^4 + a*
e^4)^2) + (c^(1/4)*e^4*(Sqrt[c]*d^2*(5*c*d^4 - 3*a*e^4) + Sqrt[a]*e^2*(7*c*
d^4 - a*e^4))*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(
c*d^4 + a*e^4)^3) + (8*c*d^3*e^7*Log[d + e*x])/(c*d^4 + a*e^4)^3 - (c^(1/4)
*(3*Sqrt[c]*d^2*(c*d^4 - 3*a*e^4) - Sqrt[a]*e^2*(3*c*d^4 - a*e^4))*Log[Sqrt
[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^4
+ a*e^4)^2) - (c^(1/4)*e^4*(Sqrt[c]*d^2*(5*c*d^4 - 3*a*e^4) - Sqrt[a]*e^2*(
7*c*d^4 - a*e^4))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(
4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^3) + (c^(1/4)*(3*Sqrt[c]*d^2*(c*d^4 - 3*a
*e^4) - Sqrt[a]*e^2*(3*c*d^4 - a*e^4))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4
)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^2) + (c^(1/4)*e^4*(
Sqrt[c]*d^2*(5*c*d^4 - 3*a*e^4) - Sqrt[a]*e^2*(7*c*d^4 - a*e^4))*Log[Sqrt[a
] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a
*e^4)^3) - (2*c*d^3*e^7*Log[a + c*x^4])/(c*d^4 + a*e^4)^3
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
```

```
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{1}{(d+ex)^2\left(a+cx^4\right)^2}\,dx = \int\left(\frac{e^8}{\left(cd^4+ae^4\right)^2(d+ex)^2} + \frac{8cd^3e^8}{\left(cd^4+ae^4\right)^3(d+ex)} + \frac{c\left(d^2\left(cd^4-3ae^4\right)-2de\left(cd^4-ae^4\right)x \cdots\right.}{\left(cd^4+ae^4\right)^2\left(a \cdots\right.}\right.$$

$$= -\frac{e^7}{\left(cd^4+ae^4\right)^2(d+ex)} + \frac{8cd^3e^7\log(d+ex)}{\left(cd^4+ae^4\right)^3} + \frac{\left(ce^4\right)\int \frac{d^2\left(5cd^4-3ae^4\right)-2de\left(3cd^4-ae^4\right)x+e^2\left(7cd^4-ae^4\right)x \cdots}{a+cx^4}}{\left(cd^4+ae^4\right)^3}$$

$$= -\frac{e^7}{\left(cd^4+ae^4\right)^2(d+ex)} + \frac{c\left(4ad^3e^3+x\left(d^2\left(cd^4-3ae^4\right)-2de\left(cd^4-ae^4\right)x+e^2\left(3cd^4-ae^4 \cdots\right.\right.\right.}{4a\left(cd^4+ae^4\right)^2\left(a+cx^4\right)}$$

$$= -\frac{e^7}{\left(cd^4+ae^4\right)^2(d+ex)} + \frac{c\left(4ad^3e^3+x\left(d^2\left(cd^4-3ae^4\right)-2de\left(cd^4-ae^4\right)x+e^2\left(3cd^4-ae^4 \cdots\right.\right.\right.}{4a\left(cd^4+ae^4\right)^2\left(a+cx^4\right)}$$

$$= -\frac{e^7}{\left(cd^4+ae^4\right)^2(d+ex)} + \frac{c\left(4ad^3e^3+x\left(d^2\left(cd^4-3ae^4\right)-2de\left(cd^4-ae^4\right)x+e^2\left(3cd^4-ae^4 \cdots\right.\right.\right.}{4a\left(cd^4+ae^4\right)^2\left(a+cx^4\right)}$$

$$= -\frac{e^7}{\left(cd^4+ae^4\right)^2(d+ex)} + \frac{c\left(4ad^3e^3+x\left(d^2\left(cd^4-3ae^4\right)-2de\left(cd^4-ae^4\right)x+e^2\left(3cd^4-ae^4 \cdots\right.\right.\right.}{4a\left(cd^4+ae^4\right)^2\left(a+cx^4\right)}$$

$$= -\frac{e^7}{\left(cd^4+ae^4\right)^2(d+ex)} + \frac{c\left(4ad^3e^3+x\left(d^2\left(cd^4-3ae^4\right)-2de\left(cd^4-ae^4\right)x+e^2\left(3cd^4-ae^4 \cdots\right.\right.\right.}{4a\left(cd^4+ae^4\right)^2\left(a+cx^4\right)}$$

$$= -\frac{e^7}{\left(cd^4+ae^4\right)^2(d+ex)} + \frac{c\left(4ad^3e^3+x\left(d^2\left(cd^4-3ae^4\right)-2de\left(cd^4-ae^4\right)x+e^2\left(3cd^4-ae^4 \cdots\right.\right.\right.}{4a\left(cd^4+ae^4\right)^2\left(a+cx^4\right)}$$

$$= -\frac{e^7}{\left(cd^4+ae^4\right)^2(d+ex)} + \frac{c\left(4ad^3e^3+x\left(d^2\left(cd^4-3ae^4\right)-2de\left(cd^4-ae^4\right)x+e^2\left(3cd^4-ae^4 \cdots\right.\right.\right.}{4a\left(cd^4+ae^4\right)^2\left(a+cx^4\right)}$$

Mathematica [A] time = 0.983025, size = 807, normalized size = 0.71

$$256cd^3\log(d+ex)e^7 - 64cd^3\log\left(cx^4+a\right)e^7 - \frac{32\left(cd^4+ae^4\right)e^7}{d+ex} + \frac{8c\left(cd^4+ae^4\right)\left(cx\left(d^2-2exd+3e^2x^2\right)d^4+ae^3\left(4d^3-3exd^2+2e^2x^2d-e^3x^3\right)\right)}{a\left(cx^4+a\right)} + \frac{2\sqrt[4]{\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^2*(a + c*x^4)^2),x]
```

```
[Out] ((-32*e^7*(c*d^4 + a*e^4))/(d + e*x) + (8*c*(c*d^4 + a*e^4)*(c*d^4*x*(d^2 -
 2*d*e*x + 3*e^2*x^2) + a*e^3*(4*d^3 - 3*d^2*e*x + 2*d*e^2*x^2 - e^3*x^3)))
/(a*(a + c*x^4)) + (2*c^(1/4)*(-3*Sqrt[2]*c^(5/2)*d^10 + 8*a^(1/4)*c^(9/4)*
d^9*e - 3*Sqrt[2]*Sqrt[a]*c^2*d^8*e^2 - 14*Sqrt[2]*a*c^(3/2)*d^6*e^4 + 48*a
^(5/4)*c^(5/4)*d^5*e^5 - 30*Sqrt[2]*a^(3/2)*c*d^4*e^6 + 21*Sqrt[2]*a^2*Sqrt
[c]*d^2*e^8 - 24*a^(9/4)*c^(1/4)*d*e^9 + 5*Sqrt[2]*a^(5/2)*e^10)*ArcTan[1 -
 (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(7/4) + (2*c^(1/4)*(3*Sqrt[2]*c^(5/2)*d^10
 + 8*a^(1/4)*c^(9/4)*d^9*e + 3*Sqrt[2]*Sqrt[a]*c^2*d^8*e^2 + 14*Sqrt[2]*a*c
^(3/2)*d^6*e^4 + 48*a^(5/4)*c^(5/4)*d^5*e^5 + 30*Sqrt[2]*a^(3/2)*c*d^4*e^6
- 21*Sqrt[2]*a^2*Sqrt[c]*d^2*e^8 - 24*a^(9/4)*c^(1/4)*d*e^9 - 5*Sqrt[2]*a^(
5/2)*e^10)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(7/4) + 256*c*d^3*e^7
*Log[d + e*x] - (Sqrt[2]*c^(1/4)*(3*c^(5/2)*d^10 - 3*Sqrt[a]*c^2*d^8*e^2 +
14*a*c^(3/2)*d^6*e^4 - 30*a^(3/2)*c*d^4*e^6 - 21*a^2*Sqrt[c]*d^2*e^8 + 5*a^
(5/2)*e^10)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(7/4)
 + (Sqrt[2]*c^(1/4)*(3*c^(5/2)*d^10 - 3*Sqrt[a]*c^2*d^8*e^2 + 14*a*c^(3/2)*
d^6*e^4 - 30*a^(3/2)*c*d^4*e^6 - 21*a^2*Sqrt[c]*d^2*e^8 + 5*a^(5/2)*e^10)*L
og[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(7/4) - 64*c*d^3*e
^7*Log[a + c*x^4])/(32*(c*d^4 + a*e^4)^3)
```

Maple [A] time = 0.022, size = 1636, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(c*x^4+a)^2,x)
```

```
[Out] 7/8*c^2/(a*e^4+c*d^4)^3/a*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+
1)*d^6*e^4+7/8*c^2/(a*e^4+c*d^4)^3/a*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/
c)^(1/4)*x-1)*d^6*e^4+7/16*c^2/(a*e^4+c*d^4)^3/a*(a/c)^(1/4)*2^(1/2)*ln((x^
2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)
))*d^6*e^4+3/32*c^2/(a*e^4+c*d^4)^3/a/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/
4)*x*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))*d^8*e^2+
3/16*c^2/(a*e^4+c*d^4)^3/a/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x
+1)*d^8*e^2+3/16*c^2/(a*e^4+c*d^4)^3/a/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(
a/c)^(1/4)*x-1)*d^8*e^2+c^2/(a*e^4+c*d^4)^3/(c*x^4+a)*d^7*e^3+1/4*c^3/(a*e^
4+c*d^4)^3/(c*x^4+a)*d^10/a*x-1/4*c/(a*e^4+c*d^4)^3/(c*x^4+a)*e^10*a*x^3+1/
2*c^2/(a*e^4+c*d^4)^3/(c*x^4+a)*e^6*x^3*d^4-1/2*c^2/(a*e^4+c*d^4)^3/(c*x^4+
```

```
a)*d^6*x*e^4+c/(a*e^4+c*d^4)^3/(c*x^4+a)*a*d^3*e^7-3*c^2/(a*e^4+c*d^4)^3/(a
*c)^(1/2)*arctan(x^2*(1/a*c)^(1/2))*d^5*e^5-5/16/(a*e^4+c*d^4)^3*a/(a/c)^(1
/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*e^10-5/16/(a*e^4+c*d^4)^3*a/(a/
c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*e^10-5/32/(a*e^4+c*d^4)^3*
a/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c)
^(1/4)*x*2^(1/2)+(a/c)^(1/2)))*e^10+8*c*d^3*e^7*ln(e*x+d)/(a*e^4+c*d^4)^3-2
*c*d^3*e^7*ln(c*x^4+a)/(a*e^4+c*d^4)^3-1/2*c^3/(a*e^4+c*d^4)^3/(c*x^4+a)*d^
9*e/a*x^2+15/8*c/(a*e^4+c*d^4)^3/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(
1/4)*x-1)*d^4*e^6+3/32*c^3/(a*e^4+c*d^4)^3/a^2*(a/c)^(1/4)*2^(1/2)*ln((x^2+
(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))
*d^10-1/2*c^3/(a*e^4+c*d^4)^3/a/(a*c)^(1/2)*arctan(x^2*(1/a*c)^(1/2))*d^9*e
+15/16*c/(a*e^4+c*d^4)^3/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*x*2^(1/2)+
(a/c)^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))*d^4*e^6-21/16*c/(a*e^
4+c*d^4)^3*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^2*e^8+3/16
*c^3/(a*e^4+c*d^4)^3/a^2*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1
)*d^10-21/16*c/(a*e^4+c*d^4)^3*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/
4)*x-1)*d^2*e^8+3/16*c^3/(a*e^4+c*d^4)^3/a^2*(a/c)^(1/4)*2^(1/2)*arctan(2^(
1/2)/(a/c)^(1/4)*x-1)*d^10-21/32*c/(a*e^4+c*d^4)^3*(a/c)^(1/4)*2^(1/2)*ln((
x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/
2)))*d^2*e^8+1/2*c/(a*e^4+c*d^4)^3/(c*x^4+a)*d*e^9*a*x^2-3/4*c/(a*e^4+c*d^4
)^3/(c*x^4+a)*d^2*a*x*e^8+3/4*c^3/(a*e^4+c*d^4)^3/(c*x^4+a)*e^2/a*x^3*d^8+3
/2*c/(a*e^4+c*d^4)^3*a/(a*c)^(1/2)*arctan(x^2*(1/a*c)^(1/2))*e^9*d+15/8*c/(
a*e^4+c*d^4)^3/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^4*e^6-
e^7/(a*e^4+c*d^4)^2/(e*x+d)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.407 $\int \frac{1}{(d+ex)^3 (a+cx^4)^2} \, dx$

Optimal. Leaf size=1384

result too large to display

```
[Out] -e^7/(2*(c*d^4 + a*e^4)^2*(d + e*x)^2) - (8*c*d^3*e^7)/((c*d^4 + a*e^4)^3*(
d + e*x)) + (c*(2*a*d^2*e^3*(5*c*d^4 - 3*a*e^4) + x*(d*(c^2*d^8 - 12*a*c*d^
4*e^4 + 3*a^2*e^8) - e*(3*c^2*d^8 - 12*a*c*d^4*e^4 + a^2*e^8)*x + 2*c*d^3*e
^2*(3*c*d^4 - 5*a*e^4)*x^2)))/(4*a*(c*d^4 + a*e^4)^3*(a + c*x^4)) - (Sqrt[c
]*e^5*(21*c^2*d^8 - 26*a*c*d^4*e^4 + a^2*e^8)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]]
)/(2*Sqrt[a]*(c*d^4 + a*e^4)^4) - (Sqrt[c]*e*(3*c^2*d^8 - 12*a*c*d^4*e^4 +
a^2*e^8)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*a^(3/2)*(c*d^4 + a*e^4)^3) - (c^
(3/4)*d*(3*c^2*d^8 - 36*a*c*d^4*e^4 + 9*a^2*e^8 + 2*Sqrt[a]*Sqrt[c]*d^2*e^2
*(3*c*d^4 - 5*a*e^4))*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a
^(7/4)*(c*d^4 + a*e^4)^3) - (c^(3/4)*d*e^4*(4*Sqrt[a]*Sqrt[c]*d^2*e^2*(7*c*
d^4 - 5*a*e^4) + 3*(5*c^2*d^8 - 10*a*c*d^4*e^4 + a^2*e^8))*ArcTan[1 - (Sqrt
[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^4) + (c^(3/4)*d
*(3*c^2*d^8 - 36*a*c*d^4*e^4 + 9*a^2*e^8 + 2*Sqrt[a]*Sqrt[c]*d^2*e^2*(3*c*d
^4 - 5*a*e^4))*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*
(c*d^4 + a*e^4)^3) + (c^(3/4)*d*e^4*(4*Sqrt[a]*Sqrt[c]*d^2*e^2*(7*c*d^4 - 5
*a*e^4) + 3*(5*c^2*d^8 - 10*a*c*d^4*e^4 + a^2*e^8))*ArcTan[1 + (Sqrt[2]*c^(
1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^4) + (12*c*d^2*e^7*(3*
c*d^4 - a*e^4)*Log[d + e*x])/(c*d^4 + a*e^4)^4 - (c^(3/4)*d*(3*c^2*d^8 - 36
*a*c*d^4*e^4 + 9*a^2*e^8 - 2*Sqrt[a]*Sqrt[c]*d^2*e^2*(3*c*d^4 - 5*a*e^4))*L
og[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*
(c*d^4 + a*e^4)^3) + (c^(3/4)*d*e^4*(4*Sqrt[a]*Sqrt[c]*d^2*e^2*(7*c*d^4 - 5
*a*e^4) - 3*(5*c^2*d^8 - 10*a*c*d^4*e^4 + a^2*e^8))*Log[Sqrt[a] - Sqrt[2]*a
^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^4) + (c
^(3/4)*d*(3*c^2*d^8 - 36*a*c*d^4*e^4 + 9*a^2*e^8 - 2*Sqrt[a]*Sqrt[c]*d^2*e^
2*(3*c*d^4 - 5*a*e^4))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^
2])/(16*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^3) - (c^(3/4)*d*e^4*(4*Sqrt[a]*Sqrt
[c]*d^2*e^2*(7*c*d^4 - 5*a*e^4) - 3*(5*c^2*d^8 - 10*a*c*d^4*e^4 + a^2*e^8))
*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)
*(c*d^4 + a*e^4)^4) - (3*c*d^2*e^7*(3*c*d^4 - a*e^4)*Log[a + c*x^4])/(c*d^4
 + a*e^4)^4
```

Rubi [A] time = 1.96275, antiderivative size = 1384, normalized size of antiderivative = 1., number of steps used = 31, number of rules used = 14, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.824, Rules used = {6742, 1854, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628, 1248, 635, 260}

$$\frac{12cd^2\left(3cd^4-ae^4\right)\log(d+ex)e^7}{\left(cd^4+ae^4\right)^4}-\frac{3cd^2\left(3cd^4-ae^4\right)\log\left(cx^4+a\right)e^7}{\left(cd^4+ae^4\right)^4}-\frac{8cd^3e^7}{\left(cd^4+ae^4\right)^3(d+ex)}-\frac{e^7}{2\left(cd^4+ae^4\right)^2(d+ex)^2}-$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^3*(a + c*x^4)^2),x]
```

```
[Out] -e^7/(2*(c*d^4 + a*e^4)^2*(d + e*x)^2) - (8*c*d^3*e^7)/((c*d^4 + a*e^4)^3*(
d + e*x)) + (c*(2*a*d^2*e^3*(5*c*d^4 - 3*a*e^4) + x*(d*(c^2*d^8 - 12*a*c*d^
4*e^4 + 3*a^2*e^8) - e*(3*c^2*d^8 - 12*a*c*d^4*e^4 + a^2*e^8)*x + 2*c*d^3*e
^2*(3*c*d^4 - 5*a*e^4)*x^2)))/(4*a*(c*d^4 + a*e^4)^3*(a + c*x^4)) - (Sqrt[c
]*e^5*(21*c^2*d^8 - 26*a*c*d^4*e^4 + a^2*e^8)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]]
)/(2*Sqrt[a]*(c*d^4 + a*e^4)^4) - (Sqrt[c]*e*(3*c^2*d^8 - 12*a*c*d^4*e^4 +
a^2*e^8)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*a^(3/2)*(c*d^4 + a*e^4)^3) - (c^
(3/4)*d*(3*c^2*d^8 - 36*a*c*d^4*e^4 + 9*a^2*e^8 + 2*Sqrt[a]*Sqrt[c]*d^2*e^2
*(3*c*d^4 - 5*a*e^4))*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a
^(7/4)*(c*d^4 + a*e^4)^3) - (c^(3/4)*d*e^4*(4*Sqrt[a]*Sqrt[c]*d^2*e^2*(7*c*
d^4 - 5*a*e^4) + 3*(5*c^2*d^8 - 10*a*c*d^4*e^4 + a^2*e^8))*ArcTan[1 - (Sqrt
[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^4) + (c^(3/4)*d
*(3*c^2*d^8 - 36*a*c*d^4*e^4 + 9*a^2*e^8 + 2*Sqrt[a]*Sqrt[c]*d^2*e^2*(3*c*d
^4 - 5*a*e^4))*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*
(c*d^4 + a*e^4)^3) + (c^(3/4)*d*e^4*(4*Sqrt[a]*Sqrt[c]*d^2*e^2*(7*c*d^4 - 5
*a*e^4) + 3*(5*c^2*d^8 - 10*a*c*d^4*e^4 + a^2*e^8))*ArcTan[1 + (Sqrt[2]*c^(
1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^4) + (12*c*d^2*e^7*(3*
c*d^4 - a*e^4)*Log[d + e*x])/(c*d^4 + a*e^4)^4 - (c^(3/4)*d*(3*c^2*d^8 - 36
*a*c*d^4*e^4 + 9*a^2*e^8 - 2*Sqrt[a]*Sqrt[c]*d^2*e^2*(3*c*d^4 - 5*a*e^4))*L
og[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*
(c*d^4 + a*e^4)^3) + (c^(3/4)*d*e^4*(4*Sqrt[a]*Sqrt[c]*d^2*e^2*(7*c*d^4 - 5
*a*e^4) - 3*(5*c^2*d^8 - 10*a*c*d^4*e^4 + a^2*e^8))*Log[Sqrt[a] - Sqrt[2]*a
^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^4) + (c
^(3/4)*d*(3*c^2*d^8 - 36*a*c*d^4*e^4 + 9*a^2*e^8 - 2*Sqrt[a]*Sqrt[c]*d^2*e^
2*(3*c*d^4 - 5*a*e^4))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^
2])/(16*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^3) - (c^(3/4)*d*e^4*(4*Sqrt[a]*Sqrt
[c]*d^2*e^2*(7*c*d^4 - 5*a*e^4) - 3*(5*c^2*d^8 - 10*a*c*d^4*e^4 + a^2*e^8))
*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)
*(c*d^4 + a*e^4)^4) - (3*c*d^2*e^7*(3*c*d^4 - a*e^4)*Log[a + c*x^4])/(c*d^4
 + a*e^4)^4
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{1}{(d+ex)^3\left(a+cx^4\right)^2}\,dx = \int\Bigg(\frac{e^8}{\left(cd^4+ae^4\right)^2(d+ex)^3} + \frac{8cd^3e^8}{\left(cd^4+ae^4\right)^3(d+ex)^2} + \frac{12cd^2e^8\left(3cd^4-ae^4\right)}{\left(cd^4+ae^4\right)^4(d+ex)} + \frac{c\left(d\left(c^2d^8-1\right.\right.}{}$$

$$= -\frac{e^7}{2\left(cd^4+ae^4\right)^2(d+ex)^2} - \frac{8cd^3e^7}{\left(cd^4+ae^4\right)^3(d+ex)} + \frac{12cd^2e^7\left(3cd^4-ae^4\right)\log(d+ex)}{\left(cd^4+ae^4\right)^4} + \frac{(ce}{}$$

$$= -\frac{e^7}{2\left(cd^4+ae^4\right)^2(d+ex)^2} - \frac{8cd^3e^7}{\left(cd^4+ae^4\right)^3(d+ex)} + \frac{c\left(2ad^2e^3\left(5cd^4-3ae^4\right)+x\left(d\left(c^2d^8-1\right.\right.\right.}{}$$

$$= -\frac{e^7}{2\left(cd^4+ae^4\right)^2(d+ex)^2} - \frac{8cd^3e^7}{\left(cd^4+ae^4\right)^3(d+ex)} + \frac{c\left(2ad^2e^3\left(5cd^4-3ae^4\right)+x\left(d\left(c^2d^8-1\right.\right.\right.}{}$$

$$= -\frac{e^7}{2\left(cd^4+ae^4\right)^2(d+ex)^2} - \frac{8cd^3e^7}{\left(cd^4+ae^4\right)^3(d+ex)} + \frac{c\left(2ad^2e^3\left(5cd^4-3ae^4\right)+x\left(d\left(c^2d^8-1\right.\right.\right.}{}$$

$$= -\frac{e^7}{2\left(cd^4+ae^4\right)^2(d+ex)^2} - \frac{8cd^3e^7}{\left(cd^4+ae^4\right)^3(d+ex)} + \frac{c\left(2ad^2e^3\left(5cd^4-3ae^4\right)+x\left(d\left(c^2d^8-1\right.\right.\right.}{}$$

$$= -\frac{e^7}{2\left(cd^4+ae^4\right)^2(d+ex)^2} - \frac{8cd^3e^7}{\left(cd^4+ae^4\right)^3(d+ex)} + \frac{c\left(2ad^2e^3\left(5cd^4-3ae^4\right)+x\left(d\left(c^2d^8-1\right.\right.\right.}{}$$

$$= -\frac{e^7}{2\left(cd^4+ae^4\right)^2(d+ex)^2} - \frac{8cd^3e^7}{\left(cd^4+ae^4\right)^3(d+ex)} + \frac{c\left(2ad^2e^3\left(5cd^4-3ae^4\right)+x\left(d\left(c^2d^8-1\right.\right.\right.}{}$$

$$= -\frac{e^7}{2\left(cd^4+ae^4\right)^2(d+ex)^2} - \frac{8cd^3e^7}{\left(cd^4+ae^4\right)^3(d+ex)} + \frac{c\left(2ad^2e^3\left(5cd^4-3ae^4\right)+x\left(d\left(c^2d^8-1\right.\right.\right.}{}$$

Mathematica [A] time = 1.41356, size = 996, normalized size = 0.72

$$384cd^2\left(3cd^4-ae^4\right)\log(d+ex)e^7 - 96cd^2\left(3cd^4-ae^4\right)\log\left(cx^4+a\right)e^7 - \frac{256cd^3\left(cd^4+ae^4\right)e^7}{d+ex} - \frac{16\left(cd^4+ae^4\right)^2e^7}{(d+ex)^2} + \frac{8c\left(cd^4+ae^4\right)\left(c^2x\left(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^3*(a + c*x^4)^2),x]
```

```
[Out] ((-16*e^7*(c*d^4 + a*e^4)^2)/(d + e*x)^2 - (256*c*d^3*e^7*(c*d^4 + a*e^4))/
(d + e*x) + (8*c*(c*d^4 + a*e^4)*(-(a^2*e^7*(6*d^2 - 3*d*e*x + e^2*x^2)) +
c^2*d^7*x*(d^2 - 3*d*e*x + 6*e^2*x^2) + 2*a*c*d^3*e^3*(5*d^3 - 6*d^2*e*x +
6*d*e^2*x^2 - 5*e^3*x^3)))/(a*(a + c*x^4)) - (6*Sqrt[c]*(Sqrt[2]*c^(13/4)*d
^13 - 4*a^(1/4)*c^3*d^12*e + 2*Sqrt[2]*Sqrt[a]*c^(11/4)*d^11*e^2 + 9*Sqrt[2
]*a*c^(9/4)*d^9*e^4 - 44*a^(5/4)*c^2*d^8*e^5 + 36*Sqrt[2]*a^(3/2)*c^(7/4)*d
^7*e^6 - 49*Sqrt[2]*a^2*c^(5/4)*d^5*e^8 + 84*a^(9/4)*c*d^4*e^9 - 30*Sqrt[2]
*a^(5/2)*c^(3/4)*d^3*e^10 + 7*Sqrt[2]*a^3*c^(1/4)*d*e^12 - 4*a^(13/4)*e^13)
*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(7/4) + (6*Sqrt[c]*(Sqrt[2]*c^(
13/4)*d^13 + 4*a^(1/4)*c^3*d^12*e + 2*Sqrt[2]*Sqrt[a]*c^(11/4)*d^11*e^2 + 9
*Sqrt[2]*a*c^(9/4)*d^9*e^4 + 44*a^(5/4)*c^2*d^8*e^5 + 36*Sqrt[2]*a^(3/2)*c^
(7/4)*d^7*e^6 - 49*Sqrt[2]*a^2*c^(5/4)*d^5*e^8 - 84*a^(9/4)*c*d^4*e^9 - 30*
Sqrt[2]*a^(5/2)*c^(3/4)*d^3*e^10 + 7*Sqrt[2]*a^3*c^(1/4)*d*e^12 + 4*a^(13/4
)*e^13)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(7/4) + 384*c*d^2*e^7*(3
*c*d^4 - a*e^4)*Log[d + e*x] - (3*Sqrt[2]*c^(3/4)*(c^3*d^13 - 2*Sqrt[a]*c^(
5/2)*d^11*e^2 + 9*a*c^2*d^9*e^4 - 36*a^(3/2)*c^(3/2)*d^7*e^6 - 49*a^2*c*d^5
*e^8 + 30*a^(5/2)*Sqrt[c]*d^3*e^10 + 7*a^3*d*e^12)*Log[Sqrt[a] - Sqrt[2]*a^
(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(7/4) + (3*Sqrt[2]*c^(3/4)*(c^3*d^13 - 2*
Sqrt[a]*c^(5/2)*d^11*e^2 + 9*a*c^2*d^9*e^4 - 36*a^(3/2)*c^(3/2)*d^7*e^6 - 4
9*a^2*c*d^5*e^8 + 30*a^(5/2)*Sqrt[c]*d^3*e^10 + 7*a^3*d*e^12)*Log[Sqrt[a] +
 Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(7/4) - 96*c*d^2*e^7*(3*c*d^4
- a*e^4)*Log[a + c*x^4])/(32*(c*d^4 + a*e^4)^4)
```

Maple [A] time = 0.026, size = 2121, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^3/(c*x^4+a)^2,x)
```

```
[Out] 36*e^7*c^2*d^6/(a*e^4+c*d^4)^4*ln(e*x+d)+5/2*c^3/(a*e^4+c*d^4)^4/(c*x^4+a)*
d^10*e^3-9*c^2/(a*e^4+c*d^4)^4*ln(c*x^4+a)*d^6*e^7-33/4*c^3/(a*e^4+c*d^4)^4
/(a*c)^(1/2)*arctan(x^2*(1/a*c)^(1/2))*d^8*e^5+3*c/(a*e^4+c*d^4)^4*a*ln(c*x
^4+a)*d^2*e^11-3/4*c/(a*e^4+c*d^4)^4*a^2/(a*c)^(1/2)*arctan(x^2*(1/a*c)^(1/
2))*e^13-12*e^11*c*d^2/(a*e^4+c*d^4)^4*ln(e*x+d)*a-c^3/(a*e^4+c*d^4)^4/(c*x
^4+a)*d^7*e^6*x^3+9/4*c^3/(a*e^4+c*d^4)^4/(c*x^4+a)*e^5*x^2*d^8-11/4*c^3/(a
*e^4+c*d^4)^4/(c*x^4+a)*d^9*x*e^4+1/4*c^4/(a*e^4+c*d^4)^4/(c*x^4+a)*d^13/a*
```

```
x+c^2/(a*e^4+c*d^4)^4/(c*x^4+a)*a*d^6*e^7-1/4*c/(a*e^4+c*d^4)^4/(c*x^4+a)*e
^13*a^2*x^2-3/2*c/(a*e^4+c*d^4)^4/(c*x^4+a)*a^2*d^2*e^11-8*c*d^3*e^7/(a*e^4
+c*d^4)^3/(e*x+d)+63/4*c^2/(a*e^4+c*d^4)^4*a/(a*c)^(1/2)*arctan(x^2*(1/a*c)
^(1/2))*d^4*e^9+3/16*c^4/(a*e^4+c*d^4)^4/a^2*(a/c)^(1/4)*2^(1/2)*arctan(2^(
1/2)/(a/c)^(1/4)*x+1)*d^13+3/16*c^4/(a*e^4+c*d^4)^4/a^2*(a/c)^(1/4)*2^(1/2)
*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^13+3/32*c^4/(a*e^4+c*d^4)^4/a^2*(a/c)^(1
/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2
^(1/2)+(a/c)^(1/2)))*d^13-3/4*c^4/(a*e^4+c*d^4)^4/a/(a*c)^(1/2)*arctan(x^2*
(1/a*c)^(1/2))*d^12*e+27/8*c^2/(a*e^4+c*d^4)^4/(a/c)^(1/4)*2^(1/2)*ln((x^2-
(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))
*d^7*e^6+27/4*c^2/(a*e^4+c*d^4)^4/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^
(1/4)*x+1)*d^7*e^6+27/4*c^2/(a*e^4+c*d^4)^4/(a/c)^(1/4)*2^(1/2)*arctan(2^(1
/2)/(a/c)^(1/4)*x-1)*d^7*e^6-147/16*c^2/(a*e^4+c*d^4)^4*(a/c)^(1/4)*2^(1/2)
*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^5*e^8-147/16*c^2/(a*e^4+c*d^4)^4*(a/c)^(
1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^5*e^8-147/32*c^2/(a*e^4+c*d^
4)^4*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a
/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))*d^5*e^8+3/4*c/(a*e^4+c*d^4)^4/(c*x^4+a)*d
*a^2*x*e^12+3/2*c^4/(a*e^4+c*d^4)^4/(c*x^4+a)*d^11*e^2/a*x^3+11/4*c^2/(a*e^
4+c*d^4)^4/(c*x^4+a)*e^9*a*x^2*d^4-3/4*c^4/(a*e^4+c*d^4)^4/(c*x^4+a)*e/a*x^
2*d^12-9/4*c^2/(a*e^4+c*d^4)^4/(c*x^4+a)*d^5*a*x*e^8-5/2*c^2/(a*e^4+c*d^4)^
4/(c*x^4+a)*d^3*e^10*a*x^3+3/8*c^3/(a*e^4+c*d^4)^4/a/(a/c)^(1/4)*2^(1/2)*ar
ctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^11*e^2+3/8*c^3/(a*e^4+c*d^4)^4/a/(a/c)^(1/4
)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^11*e^2+27/16*c^3/(a*e^4+c*d^4)^
4/a*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^9*e^4+27/16*c^3/(
a*e^4+c*d^4)^4/a*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^9*e^
4+21/16*c/(a*e^4+c*d^4)^4*a*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*
x+1)*d*e^12+21/16*c/(a*e^4+c*d^4)^4*a*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a
/c)^(1/4)*x-1)*d*e^12+21/32*c/(a*e^4+c*d^4)^4*a*(a/c)^(1/4)*2^(1/2)*ln((x^2
+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))
)*d*e^12-45/16*c/(a*e^4+c*d^4)^4*a/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*
x*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))*d^3*e^10-1/
2*e^7/(a*e^4+c*d^4)^2/(e*x+d)^2-45/8*c/(a*e^4+c*d^4)^4*a/(a/c)^(1/4)*2^(1/2
)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^3*e^10-45/8*c/(a*e^4+c*d^4)^4*a/(a/c)^(
1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^3*e^10+27/32*c^3/(a*e^4+c*d^
4)^4/a*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-
(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))*d^9*e^4+3/16*c^3/(a*e^4+c*d^4)^4/a/(a/c
)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c)^(1/4)
*x*2^(1/2)+(a/c)^(1/2)))*d^11*e^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**3/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.70529, size = 1993, normalized size = 1.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 3/8*(2*sqrt(2)*sqrt(a*c)*c^3*d^4*e + (a*c^3)^(1/4)*c^3*d^5 + 4*sqrt(2)*a*c^
3*d^2*e^3 + 2*(a*c^3)^(3/4)*c*d^3*e^2 - 9*(a*c^3)^(1/4)*a*c^2*d*e^4 + 2*sqr
```

```
t(2)*sqrt(a*c)*a*c^2*e^5)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a
/c)^(1/4))/(sqrt(2)*a^2*c^4*d^8 - 8*(a*c^3)^(1/4)*a^2*c^3*d^7*e + 16*sqrt(2
)*sqrt(a*c)*a^2*c^3*d^6*e^2 + 34*sqrt(2)*a^3*c^3*d^4*e^4 - 40*(a*c^3)^(3/4)
*a^2*c*d^5*e^3 - 40*(a*c^3)^(1/4)*a^3*c^2*d^3*e^5 + 16*sqrt(2)*sqrt(a*c)*a^
3*c^2*d^2*e^6 + sqrt(2)*a^4*c^2*e^8 - 8*(a*c^3)^(3/4)*a^3*d*e^7) + 3/8*(2*s
qrt(2)*sqrt(a*c)*c^3*d^4*e + (a*c^3)^(1/4)*c^3*d^5 - 4*sqrt(2)*a*c^3*d^2*e^
3 + 2*(a*c^3)^(3/4)*c*d^3*e^2 - 9*(a*c^3)^(1/4)*a*c^2*d*e^4 + 2*sqrt(2)*sqr
t(a*c)*a*c^2*e^5)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4
))/(sqrt(2)*a^2*c^4*d^8 + 8*(a*c^3)^(1/4)*a^2*c^3*d^7*e + 16*sqrt(2)*sqrt(a
*c)*a^2*c^3*d^6*e^2 + 34*sqrt(2)*a^3*c^3*d^4*e^4 + 40*(a*c^3)^(3/4)*a^2*c*d
^5*e^3 + 40*(a*c^3)^(1/4)*a^3*c^2*d^3*e^5 + 16*sqrt(2)*sqrt(a*c)*a^3*c^2*d^
2*e^6 + sqrt(2)*a^4*c^2*e^8 + 8*(a*c^3)^(3/4)*a^3*d*e^7) + 3/16*((a*c^3)^(1
/4)*c^4*d^13 - 2*(a*c^3)^(3/4)*c^2*d^11*e^2 + 9*(a*c^3)^(1/4)*a*c^3*d^9*e^4
 - 36*(a*c^3)^(3/4)*a*c*d^7*e^6 - 49*(a*c^3)^(1/4)*a^2*c^2*d^5*e^8 + 30*(a*
c^3)^(3/4)*a^2*d^3*e^10 + 7*(a*c^3)^(1/4)*a^3*c*d*e^12)*log(x^2 + sqrt(2)*x
*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a^2*c^5*d^16 + 4*sqrt(2)*a^3*c^4*d^12*e^
4 + 6*sqrt(2)*a^4*c^3*d^8*e^8 + 4*sqrt(2)*a^5*c^2*d^4*e^12 + sqrt(2)*a^6*c*
e^16) - 3/16*((a*c^3)^(1/4)*c^4*d^13 - 2*(a*c^3)^(3/4)*c^2*d^11*e^2 + 9*(a*
c^3)^(1/4)*a*c^3*d^9*e^4 - 36*(a*c^3)^(3/4)*a*c*d^7*e^6 - 49*(a*c^3)^(1/4)*
a^2*c^2*d^5*e^8 + 30*(a*c^3)^(3/4)*a^2*d^3*e^10 + 7*(a*c^3)^(1/4)*a^3*c*d*e
^12)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a^2*c^5*d^16 + 4
*sqrt(2)*a^3*c^4*d^12*e^4 + 6*sqrt(2)*a^4*c^3*d^8*e^8 + 4*sqrt(2)*a^5*c^2*d
^4*e^12 + sqrt(2)*a^6*c*e^16) - 3*(3*c^2*d^6*e^7 - a*c*d^2*e^11)*log(abs(c*
x^4 + a))/(c^4*d^16 + 4*a*c^3*d^12*e^4 + 6*a^2*c^2*d^8*e^8 + 4*a^3*c*d^4*e^
12 + a^4*e^16) + 12*(3*c^2*d^6*e^8 - a*c*d^2*e^12)*log(abs(x*e + d))/(c^4*d
^16*e + 4*a*c^3*d^12*e^5 + 6*a^2*c^2*d^8*e^9 + 4*a^3*c*d^4*e^13 + a^4*e^17)
 + 1/4*(10*a*c^3*d^12*e^3 - 30*a^2*c^2*d^8*e^7 - 42*a^3*c*d^4*e^11 + 6*(c^4
*d^11*e^4 - 6*a*c^3*d^7*e^8 - 7*a^2*c^2*d^3*e^12)*x^5 + 3*(3*c^4*d^12*e^3 -
 11*a*c^3*d^8*e^7 - 15*a^2*c^2*d^4*e^11 - a^3*c*e^15)*x^4 - 2*a^4*e^15 + (c
^4*d^13*e^2 + 3*a*c^3*d^9*e^6 + 3*a^2*c^2*d^5*e^10 + a^3*c*d*e^14)*x^3 - (c
^4*d^14*e + 3*a*c^3*d^10*e^5 + 3*a^2*c^2*d^6*e^9 + a^3*c*d^2*e^13)*x^2 + (c
^4*d^15 + 9*a*c^3*d^11*e^4 - 33*a^2*c^2*d^7*e^8 - 41*a^3*c*d^3*e^12)*x)/((c
*d^4 + a*e^4)^4*(c*x^4 + a)*(x*e + d)^2*a)
```

3.408 $\int \frac{(d+ex)^3}{\left(a+cx^4\right)^3}\,dx$

Optimal. Leaf size=394

$$-\frac{3d\left(7\sqrt{c}d^2-5\sqrt{a}e^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{128\sqrt{2}a^{11/4}c^{3/4}}+\frac{3d\left(7\sqrt{c}d^2-5\sqrt{a}e^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{128\sqrt{2}a^{11/4}c^{3/4}}-3d\left(5\sqrt{a}e^2\right.$$

```
[Out] (x*(7*d^3 + 18*d^2*e*x + 15*d*e^2*x^2))/(32*a^2*(a + c*x^4)) - (a*e^3 - c*x
*(d^3 + 3*d^2*e*x + 3*d*e^2*x^2))/(8*a*c*(a + c*x^4)^2) + (9*d^2*e*ArcTan[(
Sqrt[c]*x^2)/Sqrt[a]])/(16*a^(5/2)*Sqrt[c]) - (3*d*(7*Sqrt[c]*d^2 + 5*Sqrt[
a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*c^(3/
4)) + (3*d*(7*Sqrt[c]*d^2 + 5*Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a
^(1/4)])/(64*Sqrt[2]*a^(11/4)*c^(3/4)) - (3*d*(7*Sqrt[c]*d^2 - 5*Sqrt[a]*e^
2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^(
11/4)*c^(3/4)) + (3*d*(7*Sqrt[c]*d^2 - 5*Sqrt[a]*e^2)*Log[Sqrt[a] + Sqrt[2]
*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^(11/4)*c^(3/4))
```

Rubi [A] time = 0.346238, antiderivative size = 394, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 11, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.647, Rules used = {1854, 1855, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{3d\left(7\sqrt{c}d^2-5\sqrt{a}e^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{128\sqrt{2}a^{11/4}c^{3/4}}+\frac{3d\left(7\sqrt{c}d^2-5\sqrt{a}e^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{128\sqrt{2}a^{11/4}c^{3/4}}-3d\left(5\sqrt{a}e^2\right.$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(a + c*x^4)^3,x]
```


```
[Out] (x*(7*d^3 + 18*d^2*e*x + 15*d*e^2*x^2))/(32*a^2*(a + c*x^4)) - (a*e^3 - c*x
*(d^3 + 3*d^2*e*x + 3*d*e^2*x^2))/(8*a*c*(a + c*x^4)^2) + (9*d^2*e*ArcTan[(
Sqrt[c]*x^2)/Sqrt[a]])/(16*a^(5/2)*Sqrt[c]) - (3*d*(7*Sqrt[c]*d^2 + 5*Sqrt[
a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*c^(3/
4)) + (3*d*(7*Sqrt[c]*d^2 + 5*Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a
^(1/4)])/(64*Sqrt[2]*a^(11/4)*c^(3/4)) - (3*d*(7*Sqrt[c]*d^2 - 5*Sqrt[a]*e^
2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^(
11/4)*c^(3/4)) + (3*d*(7*Sqrt[c]*d^2 - 5*Sqrt[a]*e^2)*Log[Sqrt[a] + Sqrt[2]
*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^(11/4)*c^(3/4))
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
```

```
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(d+ex)^3}{\left(a+cx^4\right)^3}\,dx = -\frac{ae^3-cx\left(d^3+3d^2ex+3de^2x^2\right)}{8ac\left(a+cx^4\right)^2} - \frac{\int \frac{-7d^3-18d^2ex-15de^2x^2}{\left(a+cx^4\right)^2}\,dx}{8a}$$

$$= \frac{x\left(7d^3+18d^2ex+15de^2x^2\right)}{32a^2\left(a+cx^4\right)} - \frac{ae^3-cx\left(d^3+3d^2ex+3de^2x^2\right)}{8ac\left(a+cx^4\right)^2} + \frac{\int \frac{21d^3+36d^2ex+15de^2x^2}{a+cx^4}\,dx}{32a^2}$$

$$= \frac{x\left(7d^3+18d^2ex+15de^2x^2\right)}{32a^2\left(a+cx^4\right)} - \frac{ae^3-cx\left(d^3+3d^2ex+3de^2x^2\right)}{8ac\left(a+cx^4\right)^2} + \frac{\int \left(\frac{36d^2ex}{a+cx^4}+\frac{21d^3+15de^2x^2}{a+cx^4}\right)dx}{32a^2}$$

$$= \frac{x\left(7d^3+18d^2ex+15de^2x^2\right)}{32a^2\left(a+cx^4\right)} - \frac{ae^3-cx\left(d^3+3d^2ex+3de^2x^2\right)}{8ac\left(a+cx^4\right)^2} + \frac{\int \frac{21d^3+15de^2x^2}{a+cx^4}\,dx}{32a^2} + \frac{\left(9d^2e\right)\int \frac{x}{a+cx^4}\,d}{8a^2}$$

$$= \frac{x\left(7d^3+18d^2ex+15de^2x^2\right)}{32a^2\left(a+cx^4\right)} - \frac{ae^3-cx\left(d^3+3d^2ex+3de^2x^2\right)}{8ac\left(a+cx^4\right)^2} + \frac{\left(9d^2e\right)\mathrm{Subst}\left(\int \frac{1}{a+cx^2}\,dx,x,x^2\right)}{16a^2} +$$

$$= \frac{x\left(7d^3+18d^2ex+15de^2x^2\right)}{32a^2\left(a+cx^4\right)} - \frac{ae^3-cx\left(d^3+3d^2ex+3de^2x^2\right)}{8ac\left(a+cx^4\right)^2} + \frac{9d^2e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{c}} - \frac{\left(3d\left(\frac{7\sqrt{c}d^2}{\sqrt{a}}-5\right.\right.}{12}$$

$$= \frac{x\left(7d^3+18d^2ex+15de^2x^2\right)}{32a^2\left(a+cx^4\right)} - \frac{ae^3-cx\left(d^3+3d^2ex+3de^2x^2\right)}{8ac\left(a+cx^4\right)^2} + \frac{9d^2e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{c}} - \frac{3d\left(\frac{7\sqrt{c}d^2}{\sqrt{a}}-5e\right.}{}$$

$$= \frac{x\left(7d^3+18d^2ex+15de^2x^2\right)}{32a^2\left(a+cx^4\right)} - \frac{ae^3-cx\left(d^3+3d^2ex+3de^2x^2\right)}{8ac\left(a+cx^4\right)^2} + \frac{9d^2e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{c}} - \frac{3d\left(7\sqrt{c}d^2+5\right.}{6}$$

Mathematica [A] time = 0.35593, size = 388, normalized size = 0.98

$$\frac{\frac{3\sqrt{2}\left(5a^{3/4}de^2-7\sqrt[4]{a}\sqrt{c}d^3\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{c^{3/4}} + \frac{3\sqrt{2}\left(7\sqrt[4]{a}\sqrt{c}d^3-5a^{3/4}de^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{c^{3/4}} - \frac{32a^2\left(ae^3-cdx\left(d^2+3dex+3e^2x^2\right)\right)}{c\left(a+cx^4\right)^2} - \frac{6\sqrt[4]{a}}{}}{256a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(a + c*x^4)^3,x]
```

```
[Out] ((8*a*d*x*(7*d^2 + 18*d*e*x + 15*e^2*x^2))/(a + c*x^4) - (32*a^2*(a*e^3 - c
*d*x*(d^2 + 3*d*e*x + 3*e^2*x^2)))/(c*(a + c*x^4)^2) - (6*a^(1/4)*d*(7*Sqrt
```

```
[2]*Sqrt[c]*d^2 + 24*a^(1/4)*c^(1/4)*d*e + 5*Sqrt[2]*Sqrt[a]*e^2)*ArcTan[1
- (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/c^(3/4) + (6*a^(1/4)*d*(7*Sqrt[2]*Sqrt[c]*d
^2 - 24*a^(1/4)*c^(1/4)*d*e + 5*Sqrt[2]*Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^
(1/4)*x)/a^(1/4)])/c^(3/4) + (3*Sqrt[2]*(-7*a^(1/4)*Sqrt[c]*d^3 + 5*a^(3/4)
*d*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(3/4) + (
3*Sqrt[2]*(7*a^(1/4)*Sqrt[c]*d^3 - 5*a^(3/4)*d*e^2)*Log[Sqrt[a] + Sqrt[2]*a
^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(3/4))/(256*a^3)
```

Maple [A] time = 0.006, size = 470, normalized size = 1.2

$$\frac{d^3x}{8\,a\left(cx^4+a\right)^2}+\frac{7\,d^3x}{32\,a^2\left(cx^4+a\right)}+\frac{21\,d^3\sqrt{2}}{256\,a^3}\sqrt[4]{\frac{a}{c}}\ln\left(\left(x^2+\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)\left(x^2-\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)^{-1}\right)+\frac{21\,d^3\sqrt{2}}{128\,a^3}\sqrt[4]{\frac{a}{c}}\,\mathrm{arct}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(c*x^4+a)^3,x)
```

```
[Out] 1/8*d^3*x/a/(c*x^4+a)^2+7/32*d^3/a^2*x/(c*x^4+a)+21/256*d^3/a^3*(a/c)^(1/4)
*2^(1/2)*ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1
/2)+(a/c)^(1/2)))+21/128*d^3/a^3*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(
1/4)*x+1)+21/128*d^3/a^3*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1
)+3/8*e*d^2*x^2/a/(c*x^4+a)^2+9/16*e*d^2/a^2*x^2/(c*x^4+a)+9/16*e*d^2/a^2/(
a*c)^(1/2)*arctan(x^2*(1/a*c)^(1/2))+3/8*d*e^2*x^3/a/(c*x^4+a)^2+15/32*d*e^
2/a^2*x^3/(c*x^4+a)+15/256*d*e^2/a^2/c/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1
/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+15/128*
d*e^2/a^2/c/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+15/128*d*e^
2/a^2/c/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)+1/8*e^3*x^4/a/(
c*x^4+a)^2+1/8*e^3/a^2*x^4/(c*x^4+a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 9.52886, size = 413, normalized size = 1.05

$$\mathrm{RootSum}\left(268435456t^4a^{11}c^3 + 63111168t^2a^6c^2d^4e^2 + t\left(4147200a^4cd^4e^5 - 8128512a^3c^2d^8e\right) + 50625a^2d^4e^8 + 245106a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(c*x**4+a)**3,x)
```

```
[Out] RootSum(268435456*_t**4*a**11*c**3 + 63111168*_t**2*a**6*c**2*d**4*e**2 + _
t*(4147200*a**4*c*d**4*e**5 - 8128512*a**3*c**2*d**8*e) + 50625*a**2*d**4*e
**8 + 245106*a*c*d**8*e**4 + 194481*c**2*d**12, Lambda(_t, _t*log(x + (2621
44000*_t**3*a**10*c**2*e**6 + 3714056192*_t**3*a**9*c**3*d**4*e**2 - 539688
960*_t**2*a**7*c**2*d**4*e**5 + 202309632*_t**2*a**6*c**3*d**8*e + 77328000
*_t*a**5*c*d**4*e**8 + 660699648*_t*a**4*c**2*d**8*e**4 + 19361664*_t*a**3*
c**3*d**12 + 3037500*a**3*d**4*e**11 - 26360640*a**2*c*d**8*e**7 - 60566940
*a*c**2*d**12*e**3)/(421875*a**3*d**3*e**12 - 29598075*a**2*c*d**7*e**8 - 5
8012227*a*c**2*d**11*e**4 + 3176523*c**3*d**15)))) + (-4*a**2*e**3 + 11*a*c
*d**3*x + 30*a*c*d**2*e*x**2 + 27*a*c*d*e**2*x**3 + 7*c**2*d**3*x**5 + 18*c
**2*d**2*e*x**6 + 15*c**2*d*e**2*x**7)/(32*a**4*c + 64*a**3*c**2*x**4 + 32*
a**2*c**3*x**8)
```

Giac [A] time = 1.211, size = 525, normalized size = 1.33

$$\frac{3\sqrt{2}\left(12\sqrt{2}\sqrt{ac}c^2d^2e + 7\left(ac^3\right)^{\frac{1}{4}}c^2d^3 + 5\left(ac^3\right)^{\frac{3}{4}}de^2\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{128\,a^3c^3} + \frac{3\sqrt{2}\left(12\sqrt{2}\sqrt{ac}c^2d^2e + 7\left(ac^3\right)^{\frac{1}{4}}c^2d^3\right.}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^4+a)^3,x, algorithm="giac")
```

```
[Out] 3/128*sqrt(2)*(12*sqrt(2)*sqrt(a*c)*c^2*d^2*e + 7*(a*c^3)^(1/4)*c^2*d^3 + 5
*(a*c^3)^(3/4)*d*e^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^
(1/4))/(a^3*c^3) + 3/128*sqrt(2)*(12*sqrt(2)*sqrt(a*c)*c^2*d^2*e + 7*(a*c^3
)^(1/4)*c^2*d^3 + 5*(a*c^3)^(3/4)*d*e^2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*
(a/c)^(1/4))/(a/c)^(1/4))/(a^3*c^3) + 3/256*sqrt(2)*(7*(a*c^3)^(1/4)*c^2*d^
3 - 5*(a*c^3)^(3/4)*d*e^2)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^
3*c^3) - 3/256*sqrt(2)*(7*(a*c^3)^(1/4)*c^2*d^3 - 5*(a*c^3)^(3/4)*d*e^2)*lo
g(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^3*c^3) + 1/32*(15*c^2*d*x^7*e
^2 + 18*c^2*d^2*x^6*e + 7*c^2*d^3*x^5 + 27*a*c*d*x^3*e^2 + 30*a*c*d^2*x^2*e
 + 11*a*c*d^3*x - 4*a^2*e^3)/((c*x^4 + a)^2*a^2*c)
```

3.409 $\int \frac{(d+ex)^2}{(a+cx^4)^3}\,dx$

Optimal. Leaf size=360

$$-\frac{\left(21\sqrt{c}d^2-5\sqrt{a}e^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{128\sqrt{2}a^{11/4}c^{3/4}}+\frac{\left(21\sqrt{c}d^2-5\sqrt{a}e^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{128\sqrt{2}a^{11/4}c^{3/4}}-\frac{\left(5\sqrt{a}e^2+2\right.}{}$$

```
[Out] (x*(d + e*x)^2)/(8*a*(a + c*x^4)^2) + (x*(7*d^2 + 12*d*e*x + 5*e^2*x^2))/(3
2*a^2*(a + c*x^4)) + (3*d*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(8*a^(5/2)*Sqrt[
c]) - ((21*Sqrt[c]*d^2 + 5*Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1
/4)])/(64*Sqrt[2]*a^(11/4)*c^(3/4)) + ((21*Sqrt[c]*d^2 + 5*Sqrt[a]*e^2)*Arc
Tan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*c^(3/4)) - ((21*
Sqrt[c]*d^2 - 5*Sqrt[a]*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt
[c]*x^2])/(128*Sqrt[2]*a^(11/4)*c^(3/4)) + ((21*Sqrt[c]*d^2 - 5*Sqrt[a]*e^2
)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^(1
1/4)*c^(3/4))
```

Rubi [A] time = 0.326419, antiderivative size = 360, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 10, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.588, Rules used = {1855, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{\left(21\sqrt{c}d^2-5\sqrt{a}e^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{128\sqrt{2}a^{11/4}c^{3/4}}+\frac{\left(21\sqrt{c}d^2-5\sqrt{a}e^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{128\sqrt{2}a^{11/4}c^{3/4}}-\frac{\left(5\sqrt{a}e^2+2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(a + c*x^4)^3,x]
```


```
[Out] (x*(d + e*x)^2)/(8*a*(a + c*x^4)^2) + (x*(7*d^2 + 12*d*e*x + 5*e^2*x^2))/(3
2*a^2*(a + c*x^4)) + (3*d*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(8*a^(5/2)*Sqrt[
c]) - ((21*Sqrt[c]*d^2 + 5*Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1
/4)])/(64*Sqrt[2]*a^(11/4)*c^(3/4)) + ((21*Sqrt[c]*d^2 + 5*Sqrt[a]*e^2)*Arc
Tan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*c^(3/4)) - ((21*
Sqrt[c]*d^2 - 5*Sqrt[a]*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt
[c]*x^2])/(128*Sqrt[2]*a^(11/4)*c^(3/4)) + ((21*Sqrt[c]*d^2 - 5*Sqrt[a]*e^2
)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^(1
1/4)*c^(3/4))
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(d+ex)^2}{\left(a+cx^4\right)^3}\,dx = \frac{x(d+ex)^2}{8a\left(a+cx^4\right)^2} - \frac{\int \frac{-7d^2-12dex-5e^2x^2}{\left(a+cx^4\right)^2}\,dx}{8a}$$

$$= \frac{x(d+ex)^2}{8a\left(a+cx^4\right)^2} + \frac{x\left(7d^2+12dex+5e^2x^2\right)}{32a^2\left(a+cx^4\right)} + \frac{\int \frac{21d^2+24dex+5e^2x^2}{a+cx^4}\,dx}{32a^2}$$

$$= \frac{x(d+ex)^2}{8a\left(a+cx^4\right)^2} + \frac{x\left(7d^2+12dex+5e^2x^2\right)}{32a^2\left(a+cx^4\right)} + \frac{\int \left(\frac{24dex}{a+cx^4}+\frac{21d^2+5e^2x^2}{a+cx^4}\right)dx}{32a^2}$$

$$= \frac{x(d+ex)^2}{8a\left(a+cx^4\right)^2} + \frac{x\left(7d^2+12dex+5e^2x^2\right)}{32a^2\left(a+cx^4\right)} + \frac{\int \frac{21d^2+5e^2x^2}{a+cx^4}\,dx}{32a^2} + \frac{(3de)\int \frac{x}{a+cx^4}\,dx}{4a^2}$$

$$= \frac{x(d+ex)^2}{8a\left(a+cx^4\right)^2} + \frac{x\left(7d^2+12dex+5e^2x^2\right)}{32a^2\left(a+cx^4\right)} + \frac{(3de)\,\text{Subst}\left(\int \frac{1}{a+cx^2}\,dx, x, x^2\right)}{8a^2} + \frac{\left(\frac{21\sqrt{c}d^2}{\sqrt{a}}-5e^2\right)\int \frac{\sqrt{a}\sqrt{c}-c\cdots}{a+cx^4}\cdots}{64a^2c}\cdots$$

$$= \frac{x(d+ex)^2}{8a\left(a+cx^4\right)^2} + \frac{x\left(7d^2+12dex+5e^2x^2\right)}{32a^2\left(a+cx^4\right)} + \frac{3de\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{c}} - \frac{\left(\frac{21\sqrt{c}d^2}{\sqrt{a}}-5e^2\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{128\sqrt{2}a^{9/4}c^{3/4}} - \cdots$$

$$= \frac{x(d+ex)^2}{8a\left(a+cx^4\right)^2} + \frac{x\left(7d^2+12dex+5e^2x^2\right)}{32a^2\left(a+cx^4\right)} + \frac{3de\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{c}} - \frac{\left(\frac{21\sqrt{c}d^2}{\sqrt{a}}-5e^2\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\cdots\right.}{128\sqrt{2}a^{9/4}c^{3/4}}\cdots$$

$$= \frac{x(d+ex)^2}{8a\left(a+cx^4\right)^2} + \frac{x\left(7d^2+12dex+5e^2x^2\right)}{32a^2\left(a+cx^4\right)} + \frac{3de\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{c}} - \frac{\left(21\sqrt{c}d^2+5\sqrt{a}e^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}c^{3/4}}\cdots$$

Mathematica [A] time = 0.298442, size = 358, normalized size = 0.99

$$\frac{\frac{\sqrt{2}\left(5a^{3/4}e^2-21\sqrt[4]{a}\sqrt{c}d^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{c^{3/4}} + \frac{\sqrt{2}\left(21\sqrt[4]{a}\sqrt{c}d^2-5a^{3/4}e^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{c^{3/4}} + \frac{32a^2x(d+ex)^2}{\left(a+cx^4\right)^2} - \frac{2\sqrt[4]{a}\left(48\sqrt[4]{a}\sqrt[4]{c}de+5\sqrt{2}\sqrt{a}e^2\cdots\right.}{\cdots}\cdots}{256a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(a + c*x^4)^3,x]
```

```
[Out] ((32*a^2*x*(d + e*x)^2)/(a + c*x^4)^2 + (8*a*x*(7*d^2 + 12*d*e*x + 5*e^2*x^
2))/(a + c*x^4) - (2*a^(1/4)*(21*Sqrt[2]*Sqrt[c]*d^2 + 48*a^(1/4)*c^(1/4)*d
```

```
*e + 5*Sqrt[2]*Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/c^(3/4
) + (2*a^(1/4)*(21*Sqrt[2]*Sqrt[c]*d^2 - 48*a^(1/4)*c^(1/4)*d*e + 5*Sqrt[2]
*Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/c^(3/4) + (Sqrt[2]*(
-21*a^(1/4)*Sqrt[c]*d^2 + 5*a^(3/4)*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1
/4)*x + Sqrt[c]*x^2])/c^(3/4) + (Sqrt[2]*(21*a^(1/4)*Sqrt[c]*d^2 - 5*a^(3/4
)*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(3/4))/(25
6*a^3)
```

Maple [A] time = 0.005, size = 419, normalized size = 1.2

$$\frac{d^2x}{8\,a\left(cx^4+a\right)^2}+\frac{7\,d^2x}{32\,a^2\left(cx^4+a\right)}+\frac{21\,d^2\sqrt{2}}{256\,a^3}\sqrt[4]{\frac{a}{c}}\ln\left(\left(x^2+\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)\left(x^2-\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)^{-1}\right)+\frac{21\,d^2\sqrt{2}}{128\,a^3}\sqrt[4]{\frac{a}{c}}\,\mathrm{ar}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(c*x^4+a)^3,x)
```

```
[Out] 1/8*d^2*x/a/(c*x^4+a)^2+7/32*d^2/a^2*x/(c*x^4+a)+21/256*d^2/a^3*(a/c)^(1/4)
*2^(1/2)*ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1
/2)+(a/c)^(1/2)))+21/128*d^2/a^3*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(
1/4)*x+1)+21/128*d^2/a^3*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1
)+1/4*d*e*x^2/a/(c*x^4+a)^2+3/8*d*e/a^2*x^2/(c*x^4+a)+3/8*d*e/a^2/(a*c)^(1/
2)*arctan(x^2*(1/a*c)^(1/2))+1/8*e^2*x^3/a/(c*x^4+a)^2+5/32*e^2/a^2*x^3/(c*
x^4+a)+5/256*e^2/a^2/c/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a
/c)^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+5/128*e^2/a^2/c/(a/c)^(
1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+5/128*e^2/a^2/c/(a/c)^(1/4)*2^
(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 4.51826, size = 374, normalized size = 1.04

$$\operatorname{RootSum}\left(268435456t^4a^{11}c^3 + 25755648t^2a^6c^2d^2e^2 + t\left(307200a^4cde^5 - 5419008a^3c^2d^5e\right) + 625a^2e^8 + 111906acd^4e^4 +\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(c*x**4+a)**3,x)
```

```
[Out] RootSum(268435456*_t**4*a**11*c**3 + 25755648*_t**2*a**6*c**2*d**2*e**2 + _
t*(307200*a**4*c*d*e**5 - 5419008*a**3*c**2*d**5*e) + 625*a**2*e**8 + 11190
6*a*c*d**4*e**4 + 194481*c**2*d**8, Lambda(_t, _t*log(x + (262144000*_t**3*
a**10*c**2*e**6 + 46110081024*_t**3*a**9*c**3*d**4*e**2 - 1645608960*_t**2*
a**7*c**2*d**3*e**5 + 3641573376*_t**2*a**6*c**3*d**7*e + 32688000*_t*a**5*
c*d**2*e**8 + 3128219136*_t*a**4*c**2*d**6*e**4 + 522764928*_t*a**3*c**3*d*
*10 + 225000*a**3*d*e**11 - 43338240*a**2*c*d**5*e**7 - 523431720*a*c**2*d*
*9*e**3)/(15625*a**3*e**12 - 21357225*a**2*c*d**4*e**8 - 376741449*a*c**2*d
**8*e**4 + 85766121*c**3*d**12)))) + (11*a*d**2*x + 20*a*d*e*x**2 + 9*a*e**
2*x**3 + 7*c*d**2*x**5 + 12*c*d*e*x**6 + 5*c*e**2*x**7)/(32*a**4 + 64*a**3*
c*x**4 + 32*a**2*c**2*x**8)
```

Giac [A] time = 1.26103, size = 481, normalized size = 1.34

$$\frac{5\,cx^7e^2 + 12\,cdx^6e + 7\,cd^2x^5 + 9\,ax^3e^2 + 20\,adx^2e + 11\,ad^2x}{32\left(cx^4+a\right)^2a^2} + \frac{\sqrt{2}\left(24\,\sqrt{2}\sqrt{ac}c^2de + 21\,\left(ac^3\right)^{\frac{1}{4}}c^2d^2 + 5\,\left(ac^3\right)^{\frac{3}{4}}e^2\right)\arctan}{128\,a^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^4+a)^3,x, algorithm="giac")
```

```
[Out] 1/32*(5*c*x^7*e^2 + 12*c*d*x^6*e + 7*c*d^2*x^5 + 9*a*x^3*e^2 + 20*a*d*x^2*e
 + 11*a*d^2*x)/((c*x^4 + a)^2*a^2) + 1/128*sqrt(2)*(24*sqrt(2)*sqrt(a*c)*c^
2*d*e + 21*(a*c^3)^(1/4)*c^2*d^2 + 5*(a*c^3)^(3/4)*e^2)*arctan(1/2*sqrt(2)*
(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^3*c^3) + 1/128*sqrt(2)*(24*sqrt
(2)*sqrt(a*c)*c^2*d*e + 21*(a*c^3)^(1/4)*c^2*d^2 + 5*(a*c^3)^(3/4)*e^2)*arc
tan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^3*c^3) + 1/256*
sqrt(2)*(21*(a*c^3)^(1/4)*c^2*d^2 - 5*(a*c^3)^(3/4)*e^2)*log(x^2 + sqrt(2)*
x*(a/c)^(1/4) + sqrt(a/c))/(a^3*c^3) - 1/256*sqrt(2)*(21*(a*c^3)^(1/4)*c^2*
d^2 - 5*(a*c^3)^(3/4)*e^2)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^
3*c^3)
```

3.410 $\int \frac{d+ex}{(a+cx^4)^3} \, dx$

Optimal. Leaf size=266

$$\frac{x(7d+6ex)}{32a^2\left(a+cx^4\right)} - \frac{21d\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{128\sqrt{2}a^{11/4}\sqrt[4]{c}} + \frac{21d\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{128\sqrt{2}a^{11/4}\sqrt[4]{c}} - \frac{21d\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{c}} + \cdots$$

```
[Out] (x*(d + e*x))/(8*a*(a + c*x^4)^2) + (x*(7*d + 6*e*x))/(32*a^2*(a + c*x^4))
+ (3*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(16*a^(5/2)*Sqrt[c]) - (21*d*ArcTan[1
 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*c^(1/4)) + (21*d*ArcT
an[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*c^(1/4)) - (21*d*
Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^(11/
4)*c^(1/4)) + (21*d*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])
/(128*Sqrt[2]*a^(11/4)*c^(1/4))
```

Rubi [A] time = 0.249303, antiderivative size = 266, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 10, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {1855, 1876, 211, 1165, 628, 1162, 617, 204, 275, 205}

$$\frac{x(7d+6ex)}{32a^2\left(a+cx^4\right)} - \frac{21d\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{128\sqrt{2}a^{11/4}\sqrt[4]{c}} + \frac{21d\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{128\sqrt{2}a^{11/4}\sqrt[4]{c}} - \frac{21d\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{c}} + \cdots$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(a + c*x^4)^3,x]
```

```
[Out] (x*(d + e*x))/(8*a*(a + c*x^4)^2) + (x*(7*d + 6*e*x))/(32*a^2*(a + c*x^4))
+ (3*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(16*a^(5/2)*Sqrt[c]) - (21*d*ArcTan[1
 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*c^(1/4)) + (21*d*ArcT
an[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*c^(1/4)) - (21*d*
Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^(11/
4)*c^(1/4)) + (21*d*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])
/(128*Sqrt[2]*a^(11/4)*c^(1/4))
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
```

```
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{d+ex}{\left(a+cx^4\right)^3}\,dx = \frac{x(d+ex)}{8a\left(a+cx^4\right)^2} - \frac{\int \frac{-7d-6ex}{\left(a+cx^4\right)^2}\,dx}{8a}$$

$$= \frac{x(d+ex)}{8a\left(a+cx^4\right)^2} + \frac{x(7d+6ex)}{32a^2\left(a+cx^4\right)} + \frac{\int \frac{21d+12ex}{a+cx^4}\,dx}{32a^2}$$

$$= \frac{x(d+ex)}{8a\left(a+cx^4\right)^2} + \frac{x(7d+6ex)}{32a^2\left(a+cx^4\right)} + \frac{\int \left(\frac{21d}{a+cx^4} + \frac{12ex}{a+cx^4}\right)dx}{32a^2}$$

$$= \frac{x(d+ex)}{8a\left(a+cx^4\right)^2} + \frac{x(7d+6ex)}{32a^2\left(a+cx^4\right)} + \frac{(21d)\int \frac{1}{a+cx^4}\,dx}{32a^2} + \frac{(3e)\int \frac{x}{a+cx^4}\,dx}{8a^2}$$

$$= \frac{x(d+ex)}{8a\left(a+cx^4\right)^2} + \frac{x(7d+6ex)}{32a^2\left(a+cx^4\right)} + \frac{(21d)\int \frac{\sqrt{a}-\sqrt{c}x^2}{a+cx^4}\,dx}{64a^{5/2}} + \frac{(21d)\int \frac{\sqrt{a}+\sqrt{c}x^2}{a+cx^4}\,dx}{64a^{5/2}} + \frac{(3e)\,\text{Subst}\left(\int \frac{1}{a+cx^2}\,dx\right.}{16a^2}$$

$$= \frac{x(d+ex)}{8a\left(a+cx^4\right)^2} + \frac{x(7d+6ex)}{32a^2\left(a+cx^4\right)} + \frac{3e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{c}} + \frac{(21d)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{128a^{5/2}\sqrt{c}} + \frac{(21d)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}}{128a^{5/2}\sqrt{c}}$$

$$= \frac{x(d+ex)}{8a\left(a+cx^4\right)^2} + \frac{x(7d+6ex)}{32a^2\left(a+cx^4\right)} + \frac{3e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{c}} - \frac{21d\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{128\sqrt{2}a^{11/4}\sqrt[4]{c}} + \frac{21d\log\left(\cdots\right.}{}$$

$$= \frac{x(d+ex)}{8a\left(a+cx^4\right)^2} + \frac{x(7d+6ex)}{32a^2\left(a+cx^4\right)} + \frac{3e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{c}} - \frac{21d\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{c}} + \frac{21d\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right.}{64\sqrt{2}a^{11/4}\sqrt[4]{c}}$$

Mathematica [A] time = 0.19068, size = 249, normalized size = 0.94

$$\frac{\frac{32a^{7/4}x(d+ex)}{\left(a+cx^4\right)^2} + \frac{8a^{3/4}x(7d+6ex)}{a+cx^4} - \frac{6\left(8\sqrt[4]{a}e+7\sqrt{2}\sqrt[4]{c}d\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{c}} + \frac{6\left(7\sqrt{2}\sqrt[4]{c}d-8\sqrt[4]{a}e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{c}} - \frac{21\sqrt{2}d\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{\sqrt[4]{c}} \cdots}{256a^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(a + c*x^4)^3,x]
```

```
[Out] ((32*a^(7/4)*x*(d + e*x))/(a + c*x^4)^2 + (8*a^(3/4)*x*(7*d + 6*e*x))/(a +
c*x^4) - (6*(7*Sqrt[2]*c^(1/4)*d + 8*a^(1/4)*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)
```

```
*x)/a^(1/4)])/Sqrt[c] + (6*(7*Sqrt[2]*c^(1/4)*d - 8*a^(1/4)*e)*ArcTan[1 + (
Sqrt[2]*c^(1/4)*x)/a^(1/4)])/Sqrt[c] - (21*Sqrt[2]*d*Log[Sqrt[a] - Sqrt[2]*
a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(1/4) + (21*Sqrt[2]*d*Log[Sqrt[a] + Sqr
t[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(1/4))/(256*a^(11/4))
```

Maple [A] time = 0.006, size = 222, normalized size = 0.8

$$\frac{dx}{8a\left(cx^4+a\right)^2}+\frac{7dx}{32a^2\left(cx^4+a\right)}+\frac{21d\sqrt{2}}{256a^3}\sqrt[4]{\frac{a}{c}}\ln\left(\left(x^2+\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)\left(x^2-\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)^{-1}\right)+\frac{21d\sqrt{2}}{128a^3}\sqrt[4]{\frac{a}{c}}\,\mathrm{arcta}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(c*x^4+a)^3,x)
```

```
[Out] 1/8*d*x/a/(c*x^4+a)^2+7/32*d/a^2*x/(c*x^4+a)+21/256*d/a^3*(a/c)^(1/4)*2^(1/
2)*ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a
/c)^(1/2)))+21/128*d/a^3*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1
)+21/128*d/a^3*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)+1/8*e*x^
2/a/(c*x^4+a)^2+3/16*e/a^2*x^2/(c*x^4+a)+3/16*e/a^2/(a*c)^(1/2)*arctan(x^2*
(1/a*c)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 2.29965, size = 192, normalized size = 0.72

$$\text{RootSum}\left(268435456t^4a^{11}c^2 + 4718592t^2a^6ce^2 - 2709504ta^3cd^2e + 20736ae^4 + 194481cd^4, \left(t \mapsto t\log\left(x + \frac{-6710886\ldots}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x**4+a)**3,x)
```

```
[Out] RootSum(268435456*_t**4*a**11*c**2 + 4718592*_t**2*a**6*c*e**2 - 2709504*_t
*a**3*c*d**2*e + 20736*a*e**4 + 194481*c*d**4, Lambda(_t, _t*log(x + (-6710
8864*_t**3*a**9*c*e**2 - 9633792*_t**2*a**6*c*d**2*e - 589824*_t*a**4*e**4
- 2765952*_t*a**3*c*d**4 + 423360*a*d**2*e**3)/(193536*a*d*e**4 - 453789*c*
d**5)))) + (11*a*d*x + 10*a*e*x**2 + 7*c*d*x**5 + 6*c*e*x**6)/(32*a**4 + 64
*a**3*c*x**4 + 32*a**2*c**2*x**8)
```

Giac [A] time = 1.21667, size = 351, normalized size = 1.32

$$\frac{21\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}d\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{256\,a^3c} - \frac{21\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}d\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{256\,a^3c} + \frac{3\sqrt{2}\left(4\sqrt{2}\sqrt{ac}ce+7\left(ac^3\right)^{\frac{1}{4}}\ldots\right.}{128\,a\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^4+a)^3,x, algorithm="giac")
```

```
[Out] 21/256*sqrt(2)*(a*c^3)^(1/4)*d*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))
/(a^3*c) - 21/256*sqrt(2)*(a*c^3)^(1/4)*d*log(x^2 - sqrt(2)*x*(a/c)^(1/4) +
 sqrt(a/c))/(a^3*c) + 3/128*sqrt(2)*(4*sqrt(2)*sqrt(a*c)*c*e + 7*(a*c^3)^(1
/4)*c*d)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^3*c
^2) + 3/128*sqrt(2)*(4*sqrt(2)*sqrt(a*c)*c*e + 7*(a*c^3)^(1/4)*c*d)*arctan(
1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^3*c^2) + 1/32*(6*c*
x^6*e + 7*c*d*x^5 + 10*a*x^2*e + 11*a*d*x)/((c*x^4 + a)^2*a^2)
```

3.411 $\int \frac{1}{(a+cx^4)^3} dx$

Optimal. Leaf size=219

$$\frac{7x}{32a^2(a+cx^4)} - \frac{21\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{128\sqrt{2}a^{11/4}\sqrt[4]{c}} + \frac{21\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{128\sqrt{2}a^{11/4}\sqrt[4]{c}} - \frac{21\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{c}} + \frac{21\text{ta}}{6}$$

```
[Out] x/(8*a*(a + c*x^4)^2) + (7*x)/(32*a^2*(a + c*x^4)) - (21*ArcTan[1 - (Sqrt[2
]*c^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*c^(1/4)) + (21*ArcTan[1 + (Sqrt
[2]*c^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*c^(1/4)) - (21*Log[Sqrt[a] -
Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^(11/4)*c^(1/4)) +
(21*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^
(11/4)*c^(1/4))
```

Rubi [A] time = 0.141574, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.778, Rules used = {199, 211, 1165, 628, 1162, 617, 204}

$$\frac{7x}{32a^2(a+cx^4)} - \frac{21\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{128\sqrt{2}a^{11/4}\sqrt[4]{c}} + \frac{21\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{128\sqrt{2}a^{11/4}\sqrt[4]{c}} - \frac{21\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{c}} + \frac{21\text{ta}}{6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^4)^(-3),x]
```


```
[Out] x/(8*a*(a + c*x^4)^2) + (7*x)/(32*a^2*(a + c*x^4)) - (21*ArcTan[1 - (Sqrt[2
]*c^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*c^(1/4)) + (21*ArcTan[1 + (Sqrt
[2]*c^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*c^(1/4)) - (21*Log[Sqrt[a] -
Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^(11/4)*c^(1/4)) +
(21*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^
(11/4)*c^(1/4))
```


Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
```

```
ator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(a+cx^4)^3}\,dx = \frac{x}{8a(a+cx^4)^2} + \frac{7\int \frac{1}{(a+cx^4)^2}\,dx}{8a}$$

$$= \frac{x}{8a(a+cx^4)^2} + \frac{7x}{32a^2(a+cx^4)} + \frac{21\int \frac{1}{a+cx^4}\,dx}{32a^2}$$

$$= \frac{x}{8a(a+cx^4)^2} + \frac{7x}{32a^2(a+cx^4)} + \frac{21\int \frac{\sqrt{a}-\sqrt{c}x^2}{a+cx^4}\,dx}{64a^{5/2}} + \frac{21\int \frac{\sqrt{a}+\sqrt{c}x^2}{a+cx^4}\,dx}{64a^{5/2}}$$

$$= \frac{x}{8a(a+cx^4)^2} + \frac{7x}{32a^2(a+cx^4)} + \frac{21\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{128a^{5/2}\sqrt{c}} + \frac{21\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{128a^{5/2}\sqrt{c}} - \frac{21\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,d}{128\sqrt{2}a^{11/4}\sqrt[4]{c}}$$

$$= \frac{x}{8a(a+cx^4)^2} + \frac{7x}{32a^2(a+cx^4)} - \frac{21\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{128\sqrt{2}a^{11/4}\sqrt[4]{c}} + \frac{21\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x\right.}{128\sqrt{2}a^{11/4}\sqrt[4]{c}}$$

$$= \frac{x}{8a(a+cx^4)^2} + \frac{7x}{32a^2(a+cx^4)} - \frac{21\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{c}} + \frac{21\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{c}} - \frac{21\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{}\right.}{128\sqrt{2}a^{1}}$$

Mathematica [A] time = 0.0836012, size = 200, normalized size = 0.91

$$\frac{\frac{32a^{7/4}x}{(a+cx^4)^2} + \frac{56a^{3/4}x}{a+cx^4} - \frac{21\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{\sqrt[4]{c}} + \frac{21\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{\sqrt[4]{c}} - \frac{42\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt[4]{c}} + \frac{42\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt[4]{c}}}{256a^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^4)^(-3),x]
```

```
[Out] ((32*a^(7/4)*x)/(a + c*x^4)^2 + (56*a^(3/4)*x)/(a + c*x^4) - (42*Sqrt[2]*Ar
cTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/c^(1/4) + (42*Sqrt[2]*ArcTan[1 + (Sq
rt[2]*c^(1/4)*x)/a^(1/4)])/c^(1/4) - (21*Sqrt[2]*Log[Sqrt[a] - Sqrt[2]*a^(1
/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(1/4) + (21*Sqrt[2]*Log[Sqrt[a] + Sqrt[2]*a
^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(1/4))/(256*a^(11/4))
```

Maple [A] time = 0.004, size = 158, normalized size = 0.7

$$\frac{x}{8\,a\left(cx^{4}+a\right)^{2}}+\frac{7\,x}{32\,a^{2}\left(cx^{4}+a\right)}+\frac{21\,\sqrt{2}}{256\,a^{3}}\sqrt[4]{\frac{a}{c}}\ln\left(\left(x^{2}+\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)\left(x^{2}-\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)^{-1}\right)+\frac{21\,\sqrt{2}}{128\,a^{3}}\sqrt[4]{\frac{a}{c}}\,\mathrm{arcta}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+a)^3,x)
```

```
[Out] 1/8*x/a/(c*x^4+a)^2+7/32*x/a^2/(c*x^4+a)+21/256/a^3*(a/c)^(1/4)*2^(1/2)*ln(
(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1
/2)))+21/128/a^3*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+21/128
/a^3*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.0135, size = 545, normalized size = 2.49

$$\frac{28\,cx^{5}+84\left(a^{2}c^{2}x^{8}+2\,a^{3}cx^{4}+a^{4}\right)\left(-\frac{1}{a^{11}c}\right)^{\frac{1}{4}}\arctan\left(-a^{8}cx\left(-\frac{1}{a^{11}c}\right)^{\frac{3}{4}}+\sqrt{a^{6}\sqrt{-\frac{1}{a^{11}c}}+x^{2}}\,a^{8}c\left(-\frac{1}{a^{11}c}\right)^{\frac{3}{4}}\right)+21\left(a^{2}c^{2}x^{8}+2\right.}{128\left(a^{2}c^{2}x^{8}+2\,a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] 1/128*(28*c*x^5 + 84*(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4)*(-1/(a^11*c))^(1/4)*
arctan(-a^8*c*x*(-1/(a^11*c))^(3/4) + sqrt(a^6*sqrt(-1/(a^11*c)) + x^2)*a^8
*c*(-1/(a^11*c))^(3/4)) + 21*(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4)*(-1/(a^11*c)
```

```
)^(1/4)*log(a^3*(-1/(a^11*c))^(1/4) + x) - 21*(a^2*c^2*x^8 + 2*a^3*c*x^4 +
a^4)*(-1/(a^11*c))^(1/4)*log(-a^3*(-1/(a^11*c))^(1/4) + x) + 44*a*x)/(a^2*c
^2*x^8 + 2*a^3*c*x^4 + a^4)
```

Sympy [A] time = 1.22615, size = 63, normalized size = 0.29

$$\frac{11ax + 7cx^5}{32a^4 + 64a^3cx^4 + 32a^2c^2x^8} + \operatorname{RootSum}\left(268435456t^4a^{11}c + 194481, \left(t \mapsto t\log\left(\frac{128ta^3}{21} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+a)**3,x)
```

```
[Out] (11*a*x + 7*c*x**5)/(32*a**4 + 64*a**3*c*x**4 + 32*a**2*c**2*x**8) + RootSu
m(268435456*_t**4*a**11*c + 194481, Lambda(_t, _t*log(128*_t*a**3/21 + x)))
```

Giac [A] time = 1.14943, size = 275, normalized size = 1.26

$$\frac{21\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{128\,a^3c} + \frac{21\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{128\,a^3c} + \frac{21\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}\log\left(x^2 + \sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{256\,a^3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a)^3,x, algorithm="giac")
```

```
[Out] 21/128*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))
/(a/c)^(1/4))/(a^3*c) + 21/128*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*
x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^3*c) + 21/256*sqrt(2)*(a*c^3)^(1/4
)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^3*c) - 21/256*sqrt(2)*(a*
c^3)^(1/4)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^3*c) + 1/32*(7*c
*x^5 + 11*a*x)/((c*x^4 + a)^2*a^2)
```

3.412 $\int \frac{1}{(d+ex)(a+cx^4)^3} dx$

Optimal. Leaf size=1352

result too large to display

```
[Out] (c*x*(7*d^3 - 6*d^2*e*x + 5*d*e^2*x^2))/(32*a^2*(c*d^4 + a*e^4)*(a + c*x^4)
) + (a*e^3 + c*x*(d^3 - d^2*e*x + d*e^2*x^2))/(8*a*(c*d^4 + a*e^4)*(a + c*x
^4)^2) + (e^4*(a*e^3 + c*x*(d^3 - d^2*e*x + d*e^2*x^2)))/(4*a*(c*d^4 + a*e^
4)^2*(a + c*x^4)) - (Sqrt[c]*d^2*e^9*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*Sqrt
[a]*(c*d^4 + a*e^4)^3) - (Sqrt[c]*d^2*e^5*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4
*a^(3/2)*(c*d^4 + a*e^4)^2) - (3*Sqrt[c]*d^2*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]
])/(16*a^(5/2)*(c*d^4 + a*e^4)) - (c^(1/4)*d*e^8*(Sqrt[c]*d^2 + Sqrt[a]*e^2
)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^
4)^3) - (c^(1/4)*d*e^4*(3*Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^
(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^2) - (c^(1/4)*d*(21*S
qrt[c]*d^2 + 5*Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(64*Sq
rt[2]*a^(11/4)*(c*d^4 + a*e^4)) + (c^(1/4)*d*e^8*(Sqrt[c]*d^2 + Sqrt[a]*e^2
)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^
4)^3) + (c^(1/4)*d*e^4*(3*Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^
(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^2) + (c^(1/4)*d*(21*S
qrt[c]*d^2 + 5*Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(64*Sq
rt[2]*a^(11/4)*(c*d^4 + a*e^4)) + (e^11*Log[d + e*x])/(c*d^4 + a*e^4)^3 - (
c^(1/4)*d*e^8*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(
1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^3) - (c^(1/4)*d*e
^4*(3*Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x +
Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^2) - (c^(1/4)*d*(21*Sqrt[
c]*d^2 - 5*Sqrt[a]*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x
^2])/(128*Sqrt[2]*a^(11/4)*(c*d^4 + a*e^4)) + (c^(1/4)*d*e^8*(Sqrt[c]*d^2 -
 Sqrt[a]*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sq
rt[2]*a^(3/4)*(c*d^4 + a*e^4)^3) + (c^(1/4)*d*e^4*(3*Sqrt[c]*d^2 - Sqrt[a]*
e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^
(7/4)*(c*d^4 + a*e^4)^2) + (c^(1/4)*d*(21*Sqrt[c]*d^2 - 5*Sqrt[a]*e^2)*Log[
Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^(11/4)*(
c*d^4 + a*e^4)) - (e^11*Log[a + c*x^4])/(4*(c*d^4 + a*e^4)^3)
```

Rubi [A] time = 1.41165, antiderivative size = 1352, normalized size of antiderivative = 1., number of steps used = 46, number of rules used = 15, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.882, Rules used = {6742, 1854, 1855, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628, 1248, 635, 260}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(a + c*x^4)^3),x]
```

```
[Out] (c*x*(7*d^3 - 6*d^2*e*x + 5*d*e^2*x^2))/(32*a^2*(c*d^4 + a*e^4)*(a + c*x^4)
) + (a*e^3 + c*x*(d^3 - d^2*e*x + d*e^2*x^2))/(8*a*(c*d^4 + a*e^4)*(a + c*x
^4)^2) + (e^4*(a*e^3 + c*x*(d^3 - d^2*e*x + d*e^2*x^2)))/(4*a*(c*d^4 + a*e^
4)^2*(a + c*x^4)) - (Sqrt[c]*d^2*e^9*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*Sqrt
[a]*(c*d^4 + a*e^4)^3) - (Sqrt[c]*d^2*e^5*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4
*a^(3/2)*(c*d^4 + a*e^4)^2) - (3*Sqrt[c]*d^2*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]
])/(16*a^(5/2)*(c*d^4 + a*e^4)) - (c^(1/4)*d*e^8*(Sqrt[c]*d^2 + Sqrt[a]*e^2
)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^
4)^3) - (c^(1/4)*d*e^4*(3*Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^
(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^2) - (c^(1/4)*d*(21*S
qrt[c]*d^2 + 5*Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(64*Sq
rt[2]*a^(11/4)*(c*d^4 + a*e^4)) + (c^(1/4)*d*e^8*(Sqrt[c]*d^2 + Sqrt[a]*e^2
)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^
4)^3) + (c^(1/4)*d*e^4*(3*Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^
(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^2) + (c^(1/4)*d*(21*S
qrt[c]*d^2 + 5*Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(64*Sq
rt[2]*a^(11/4)*(c*d^4 + a*e^4)) + (e^11*Log[d + e*x])/(c*d^4 + a*e^4)^3 - (
c^(1/4)*d*e^8*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(
1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^3) - (c^(1/4)*d*e
^4*(3*Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x +
Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^2) - (c^(1/4)*d*(21*Sqrt[
c]*d^2 - 5*Sqrt[a]*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x
^2])/(128*Sqrt[2]*a^(11/4)*(c*d^4 + a*e^4)) + (c^(1/4)*d*e^8*(Sqrt[c]*d^2 -
 Sqrt[a]*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sq
rt[2]*a^(3/4)*(c*d^4 + a*e^4)^3) + (c^(1/4)*d*e^4*(3*Sqrt[c]*d^2 - Sqrt[a]*
e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^
(7/4)*(c*d^4 + a*e^4)^2) + (c^(1/4)*d*(21*Sqrt[c]*d^2 - 5*Sqrt[a]*e^2)*Log[
Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^(11/4)*(
c*d^4 + a*e^4)) - (e^11*Log[a + c*x^4])/(4*(c*d^4 + a*e^4)^3)
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{1}{(d+ex)\left(a+cx^4\right)^3}\,dx = \int \left(\frac{e^{12}}{\left(cd^4+ae^4\right)^3(d+ex)} + \frac{c\left(d^3-d^2ex+de^2x^2-e^3x^3\right)}{\left(cd^4+ae^4\right)\left(a+cx^4\right)^3} - \frac{ce^4\left(-d^3+d^2ex-de^2x^2+e^3x^3\right)}{\left(cd^4+ae^4\right)^2\left(a+cx^4\right)^2} \cdots\right.$$

$$= \frac{e^{11}\log(d+ex)}{\left(cd^4+ae^4\right)^3} - \frac{\left(ce^8\right)\int \frac{-d^3+d^2ex-de^2x^2+e^3x^3}{a+cx^4}\,dx}{\left(cd^4+ae^4\right)^3} - \frac{\left(ce^4\right)\int \frac{-d^3+d^2ex-de^2x^2+e^3x^3}{\left(a+cx^4\right)^2}\,dx}{\left(cd^4+ae^4\right)^2} + \frac{c\int \frac{d^3-d^2ex\cdots}{(a\cdots}}{cd^4\cdots}$$

$$= \frac{ae^3+cx\left(d^3-d^2ex+de^2x^2\right)}{8a\left(cd^4+ae^4\right)\left(a+cx^4\right)^2} + \frac{e^4\left(ae^3+cx\left(d^3-d^2ex+de^2x^2\right)\right)}{4a\left(cd^4+ae^4\right)^2\left(a+cx^4\right)} + \frac{e^{11}\log(d+ex)}{\left(cd^4+ae^4\right)^3} - \frac{\left(ce^8\right)\cdots}{\cdots}$$

$$= \frac{cx\left(7d^3-6d^2ex+5de^2x^2\right)}{32a^2\left(cd^4+ae^4\right)\left(a+cx^4\right)} + \frac{ae^3+cx\left(d^3-d^2ex+de^2x^2\right)}{8a\left(cd^4+ae^4\right)\left(a+cx^4\right)^2} + \frac{e^4\left(ae^3+cx\left(d^3-d^2ex+de^2x^2\cdots\right.\right.}{4a\left(cd^4+ae^4\right)^2\left(a+cx^4\right)}$$

$$= \frac{cx\left(7d^3-6d^2ex+5de^2x^2\right)}{32a^2\left(cd^4+ae^4\right)\left(a+cx^4\right)} + \frac{ae^3+cx\left(d^3-d^2ex+de^2x^2\right)}{8a\left(cd^4+ae^4\right)\left(a+cx^4\right)^2} + \frac{e^4\left(ae^3+cx\left(d^3-d^2ex+de^2x^2\cdots\right.\right.}{4a\left(cd^4+ae^4\right)^2\left(a+cx^4\right)}$$

$$= \frac{cx\left(7d^3-6d^2ex+5de^2x^2\right)}{32a^2\left(cd^4+ae^4\right)\left(a+cx^4\right)} + \frac{ae^3+cx\left(d^3-d^2ex+de^2x^2\right)}{8a\left(cd^4+ae^4\right)\left(a+cx^4\right)^2} + \frac{e^4\left(ae^3+cx\left(d^3-d^2ex+de^2x^2\cdots\right.\right.}{4a\left(cd^4+ae^4\right)^2\left(a+cx^4\right)}$$

$$= \frac{cx\left(7d^3-6d^2ex+5de^2x^2\right)}{32a^2\left(cd^4+ae^4\right)\left(a+cx^4\right)} + \frac{ae^3+cx\left(d^3-d^2ex+de^2x^2\right)}{8a\left(cd^4+ae^4\right)\left(a+cx^4\right)^2} + \frac{e^4\left(ae^3+cx\left(d^3-d^2ex+de^2x^2\cdots\right.\right.}{4a\left(cd^4+ae^4\right)^2\left(a+cx^4\right)}$$

$$= \frac{cx\left(7d^3-6d^2ex+5de^2x^2\right)}{32a^2\left(cd^4+ae^4\right)\left(a+cx^4\right)} + \frac{ae^3+cx\left(d^3-d^2ex+de^2x^2\right)}{8a\left(cd^4+ae^4\right)\left(a+cx^4\right)^2} + \frac{e^4\left(ae^3+cx\left(d^3-d^2ex+de^2x^2\cdots\right.\right.}{4a\left(cd^4+ae^4\right)^2\left(a+cx^4\right)}$$

$$= \frac{cx\left(7d^3-6d^2ex+5de^2x^2\right)}{32a^2\left(cd^4+ae^4\right)\left(a+cx^4\right)} + \frac{ae^3+cx\left(d^3-d^2ex+de^2x^2\right)}{8a\left(cd^4+ae^4\right)\left(a+cx^4\right)^2} + \frac{e^4\left(ae^3+cx\left(d^3-d^2ex+de^2x^2\cdots\right.\right.}{4a\left(cd^4+ae^4\right)^2\left(a+cx^4\right)}$$

$$= \frac{cx\left(7d^3-6d^2ex+5de^2x^2\right)}{32a^2\left(cd^4+ae^4\right)\left(a+cx^4\right)} + \frac{ae^3+cx\left(d^3-d^2ex+de^2x^2\right)}{8a\left(cd^4+ae^4\right)\left(a+cx^4\right)^2} + \frac{e^4\left(ae^3+cx\left(d^3-d^2ex+de^2x^2\cdots\right.\right.}{4a\left(cd^4+ae^4\right)^2\left(a+cx^4\right)}$$

Mathematica [A] time = 0.762644, size = 835, normalized size = 0.62

$$256\log(d+ex)e^{11}-64\log\left(cx^4+a\right)e^{11}+\frac{32\left(cd^4+ae^4\right)^2\left(ae^3+cdx\left(d^2-exd+e^2x^2\right)\right)}{a\left(cx^4+a\right)^2}+\frac{8\left(cd^4+ae^4\right)\left(8a^2e^7+acdx\left(15d^2-14exd+13e^2x^2\right)e^4+c^2d^5x\left(7d^2\right.\right.}{a^2\left(cx^4+a\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(a + c*x^4)^3),x]
```

```
[Out] ((32*(c*d^4 + a*e^4)^2*(a*e^3 + c*d*x*(d^2 - d*e*x + e^2*x^2)))/(a*(a + c*x
^4)^2) + (8*(c*d^4 + a*e^4)*(8*a^2*e^7 + c^2*d^5*x*(7*d^2 - 6*d*e*x + 5*e^2
*x^2) + a*c*d*e^4*x*(15*d^2 - 14*d*e*x + 13*e^2*x^2)))/(a^2*(a + c*x^4)) -
(2*c^(1/4)*d*(21*Sqrt[2]*c^(5/2)*d^10 - 24*a^(1/4)*c^(9/4)*d^9*e + 5*Sqrt[2
]*Sqrt[a]*c^2*d^8*e^2 + 66*Sqrt[2]*a*c^(3/2)*d^6*e^4 - 80*a^(5/4)*c^(5/4)*d
^5*e^5 + 18*Sqrt[2]*a^(3/2)*c*d^4*e^6 + 77*Sqrt[2]*a^2*Sqrt[c]*d^2*e^8 - 12
0*a^(9/4)*c^(1/4)*d*e^9 + 45*Sqrt[2]*a^(5/2)*e^10)*ArcTan[1 - (Sqrt[2]*c^(1
/4)*x)/a^(1/4)])/a^(11/4) + (2*c^(1/4)*d*(21*Sqrt[2]*c^(5/2)*d^10 + 24*a^(1
/4)*c^(9/4)*d^9*e + 5*Sqrt[2]*Sqrt[a]*c^2*d^8*e^2 + 66*Sqrt[2]*a*c^(3/2)*d^
6*e^4 + 80*a^(5/4)*c^(5/4)*d^5*e^5 + 18*Sqrt[2]*a^(3/2)*c*d^4*e^6 + 77*Sqrt
[2]*a^2*Sqrt[c]*d^2*e^8 + 120*a^(9/4)*c^(1/4)*d*e^9 + 45*Sqrt[2]*a^(5/2)*e^
10)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(11/4) + 256*e^11*Log[d + e*
x] + (Sqrt[2]*c^(1/4)*(-21*c^(5/2)*d^11 + 5*Sqrt[a]*c^2*d^9*e^2 - 66*a*c^(3
/2)*d^7*e^4 + 18*a^(3/2)*c*d^5*e^6 - 77*a^2*Sqrt[c]*d^3*e^8 + 45*a^(5/2)*d*
e^10)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(11/4) + (S
qrt[2]*c^(1/4)*(21*c^(5/2)*d^11 - 5*Sqrt[a]*c^2*d^9*e^2 + 66*a*c^(3/2)*d^7*
e^4 - 18*a^(3/2)*c*d^5*e^6 + 77*a^2*Sqrt[c]*d^3*e^8 - 45*a^(5/2)*d*e^10)*Lo
g[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(11/4) - 64*e^11*Lo
g[a + c*x^4])/(256*(c*d^4 + a*e^4)^3)
```

Maple [A] time = 0.023, size = 2098, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(c*x^4+a)^3,x)
```

```
[Out] 1/8*c^2/(a*e^4+c*d^4)^3/(c*x^4+a)^2*e^3*d^8+1/4*c^2/(a*e^4+c*d^4)^3/(c*x^4+
a)^2*x^4*d^4*e^7+13/16*c^2/(a*e^4+c*d^4)^3/(c*x^4+a)^2*d^5*e^6*x^3-7/8*c^2/
```

(a*e^4+c*d^4)^3/(c*x^4+a)^2*d^6*e^5*x^2+15/16*c^2/(a*e^4+c*d^4)^3/(c*x^4+a)
^2*d^7*x*e^4+11/32*c^3/(a*e^4+c*d^4)^3/(c*x^4+a)^2*d^11/a*x+1/4*c/(a*e^4+c*
d^4)^3/(c*x^4+a)^2*x^4*a*e^11+1/2*c/(a*e^4+c*d^4)^3/(c*x^4+a)^2*a*e^7*d^4-1
5/16*c/(a*e^4+c*d^4)^3/(a*c)^(1/2)*arctan(x^2*(1/a*c)^(1/2))*d^2*e^9+45/256
/(a*e^4+c*d^4)^3/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1
/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))*d*e^10+45/128/(a*e^4+c*d^4)^3
/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d*e^10+45/128/(a*e^4+c
*d^4)^3/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d*e^10+13/32*c^
2/(a*e^4+c*d^4)^3/(c*x^4+a)^2*d*e^10*x^7-7/16*c^2/(a*e^4+c*d^4)^3/(c*x^4+a)
^2*d^2*e^9*x^6+15/32*c^2/(a*e^4+c*d^4)^3/(c*x^4+a)^2*d^3*x^5*e^8+7/32*c^4/(
a*e^4+c*d^4)^3/(c*x^4+a)^2*d^11/a^2*x^5+3/8/(a*e^4+c*d^4)^3/(c*x^4+a)^2*a^2
*e^11-3/16*c^3/(a*e^4+c*d^4)^3/a^2/(a*c)^(1/2)*arctan(x^2*(1/a*c)^(1/2))*d^
10*e+21/128*c^3/(a*e^4+c*d^4)^3/a^3*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c
)^(1/4)*x-1)*d^11+21/256*c^3/(a*e^4+c*d^4)^3/a^3*(a/c)^(1/4)*2^(1/2)*ln((x^
2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)
))*d^11+17/32*c/(a*e^4+c*d^4)^3/(c*x^4+a)^2*d*e^10*a*x^3-9/16*c/(a*e^4+c*d^
4)^3/(c*x^4+a)^2*d^2*e^9*a*x^2+19/32*c/(a*e^4+c*d^4)^3/(c*x^4+a)^2*d^3*a*x*
e^8+9/16*c^3/(a*e^4+c*d^4)^3/(c*x^4+a)^2*d^5*e^6/a*x^7+5/32*c^4/(a*e^4+c*d^
4)^3/(c*x^4+a)^2*d^9*e^2/a^2*x^7-5/8*c^3/(a*e^4+c*d^4)^3/(c*x^4+a)^2*d^6*e^
5/a*x^6-3/16*c^4/(a*e^4+c*d^4)^3/(c*x^4+a)^2*d^10*e/a^2*x^6+11/16*c^3/(a*e^
4+c*d^4)^3/(c*x^4+a)^2*d^7/a*x^5*e^4+9/32*c^3/(a*e^4+c*d^4)^3/(c*x^4+a)^2*d
^9*e^2/a*x^3-5/16*c^3/(a*e^4+c*d^4)^3/(c*x^4+a)^2*d^10*e/a*x^2+21/128*c^3/(
a*e^4+c*d^4)^3/a^3*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^11
-5/8*c^2/(a*e^4+c*d^4)^3/a/(a*c)^(1/2)*arctan(x^2*(1/a*c)^(1/2))*d^6*e^5+77
/128*c/(a*e^4+c*d^4)^3/a*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1
)*d^3*e^8+9/128*c/(a*e^4+c*d^4)^3/a/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)
*x*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))*d^5*e^6+9/
64*c/(a*e^4+c*d^4)^3/a/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*
d^5*e^6+e^11*ln(e*x+d)/(a*e^4+c*d^4)^3-1/4*e^11*ln(c*x^4+a)/(a*e^4+c*d^4)^3
+33/128*c^2/(a*e^4+c*d^4)^3/a^2*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*x*2
^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))*d^7*e^4+33/64*
c^2/(a*e^4+c*d^4)^3/a^2*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)
*d^7*e^4+33/64*c^2/(a*e^4+c*d^4)^3/a^2*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(
a/c)^(1/4)*x-1)*d^7*e^4+9/64*c/(a*e^4+c*d^4)^3/a/(a/c)^(1/4)*2^(1/2)*arctan
(2^(1/2)/(a/c)^(1/4)*x+1)*d^5*e^6+77/128*c/(a*e^4+c*d^4)^3/a*(a/c)^(1/4)*2^
(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^3*e^8+77/256*c/(a*e^4+c*d^4)^3/a*(a
/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/
4)*x*2^(1/2)+(a/c)^(1/2)))*d^3*e^8+5/256*c^2/(a*e^4+c*d^4)^3/a^2/(a/c)^(1/4
)*2^(1/2)*ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*x*2^(
1/2)+(a/c)^(1/2)))*d^9*e^2+5/128*c^2/(a*e^4+c*d^4)^3/a^2/(a/c)^(1/4)*2^(1/2
)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^9*e^2+5/128*c^2/(a*e^4+c*d^4)^3/a^2/(a/
c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^9*e^2

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x**4+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.37036, size = 1705, normalized size = 1.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^4+a)^3,x, algorithm="giac")
```

```
[Out] 1/64*(51*sqrt(2)*sqrt(a*c)*c^3*d^4*e + 21*(a*c^3)^(1/4)*c^3*d^5 - 75*sqrt(2
)*a*c^3*d^2*e^3 + 122*(a*c^3)^(3/4)*c*d^3*e^2 + 45*(a*c^3)^(1/4)*a*c^2*d*e^
4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^3
*c^4*d^6 - 6*(a*c^3)^(1/4)*a^3*c^3*d^5*e + 9*sqrt(2)*sqrt(a*c)*a^3*c^3*d^4*
e^2 + 9*sqrt(2)*a^4*c^3*d^2*e^4 - 16*(a*c^3)^(3/4)*a^3*c*d^3*e^3 - 6*(a*c^3
)^(1/4)*a^4*c^2*d*e^5 + sqrt(2)*sqrt(a*c)*a^4*c^2*e^6) + 1/64*(51*sqrt(2)*s
qrt(a*c)*c^3*d^4*e + 21*(a*c^3)^(1/4)*c^3*d^5 + 75*sqrt(2)*a*c^3*d^2*e^3 +
122*(a*c^3)^(3/4)*c*d^3*e^2 + 45*(a*c^3)^(1/4)*a*c^2*d*e^4)*arctan(1/2*sqrt
(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^3*c^4*d^6 + 6*(a*c^
3)^(1/4)*a^3*c^3*d^5*e + 9*sqrt(2)*sqrt(a*c)*a^3*c^3*d^4*e^2 + 9*sqrt(2)*a^
4*c^3*d^2*e^4 + 16*(a*c^3)^(3/4)*a^3*c*d^3*e^3 + 6*(a*c^3)^(1/4)*a^4*c^2*d*
e^5 + sqrt(2)*sqrt(a*c)*a^4*c^2*e^6) + 1/128*(21*(a*c^3)^(1/4)*c^4*d^11 - 5
*(a*c^3)^(3/4)*c^2*d^9*e^2 + 66*(a*c^3)^(1/4)*a*c^3*d^7*e^4 - 18*(a*c^3)^(3
/4)*a*c*d^5*e^6 + 77*(a*c^3)^(1/4)*a^2*c^2*d^3*e^8 - 45*(a*c^3)^(3/4)*a^2*d
*e^10)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a^3*c^5*d^12 +
 3*sqrt(2)*a^4*c^4*d^8*e^4 + 3*sqrt(2)*a^5*c^3*d^4*e^8 + sqrt(2)*a^6*c^2*e^
12) - 1/128*(21*(a*c^3)^(1/4)*c^4*d^11 - 5*(a*c^3)^(3/4)*c^2*d^9*e^2 + 66*(
a*c^3)^(1/4)*a*c^3*d^7*e^4 - 18*(a*c^3)^(3/4)*a*c*d^5*e^6 + 77*(a*c^3)^(1/4
)*a^2*c^2*d^3*e^8 - 45*(a*c^3)^(3/4)*a^2*d*e^10)*log(x^2 - sqrt(2)*x*(a/c)^
(1/4) + sqrt(a/c))/(sqrt(2)*a^3*c^5*d^12 + 3*sqrt(2)*a^4*c^4*d^8*e^4 + 3*sq
rt(2)*a^5*c^3*d^4*e^8 + sqrt(2)*a^6*c^2*e^12) - 1/4*e^11*log(abs(c*x^4 + a)
)/(c^3*d^12 + 3*a*c^2*d^8*e^4 + 3*a^2*c*d^4*e^8 + a^3*e^12) + e^12*log(abs(
x*e + d))/(c^3*d^12*e + 3*a*c^2*d^8*e^5 + 3*a^2*c*d^4*e^9 + a^3*e^13) + 1/3
2*(4*a^2*c^2*d^8*e^3 + 16*a^3*c*d^4*e^7 + (5*c^4*d^9*e^2 + 18*a*c^3*d^5*e^6
 + 13*a^2*c^2*d*e^10)*x^7 - 2*(3*c^4*d^10*e + 10*a*c^3*d^6*e^5 + 7*a^2*c^2*
d^2*e^9)*x^6 + (7*c^4*d^11 + 22*a*c^3*d^7*e^4 + 15*a^2*c^2*d^3*e^8)*x^5 + 8
*(a^2*c^2*d^4*e^7 + a^3*c*e^11)*x^4 + 12*a^4*e^11 + (9*a*c^3*d^9*e^2 + 26*a
^2*c^2*d^5*e^6 + 17*a^3*c*d*e^10)*x^3 - 2*(5*a*c^3*d^10*e + 14*a^2*c^2*d^6*
e^5 + 9*a^3*c*d^2*e^9)*x^2 + (11*a*c^3*d^11 + 30*a^2*c^2*d^7*e^4 + 19*a^3*c
*d^3*e^8)*x)/((c*d^4 + a*e^4)^3*(c*x^4 + a)^2*a^2)
```

3.413 $\int \frac{1}{(d+ex)^2\left(a+cx^4\right)^3}\,dx$

Optimal. Leaf size=1830

result too large to display

```
[Out] -(e^11/((c*d^4 + a*e^4)^3*(d + e*x))) + (c*x*(7*d^2*(c*d^4 - 3*a*e^4) - 12*
d*e*(c*d^4 - a*e^4)*x + 5*e^2*(3*c*d^4 - a*e^4)*x^2))/(32*a^2*(c*d^4 + a*e^
4)^2*(a + c*x^4)) + (c*(4*a*d^3*e^3 + x*(d^2*(c*d^4 - 3*a*e^4) - 2*d*e*(c*d
^4 - a*e^4)*x + e^2*(3*c*d^4 - a*e^4)*x^2)))/(8*a*(c*d^4 + a*e^4)^2*(a + c*
x^4)^2) + (c*e^4*(8*a*d^3*e^3 + x*(d^2*(5*c*d^4 - 3*a*e^4) - 2*d*e*(3*c*d^4
 - a*e^4)*x + e^2*(7*c*d^4 - a*e^4)*x^2)))/(4*a*(c*d^4 + a*e^4)^3*(a + c*x^
4)) - (Sqrt[c]*d*e^9*(5*c*d^4 - a*e^4)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(Sqrt
[a]*(c*d^4 + a*e^4)^4) - (Sqrt[c]*d*e^5*(3*c*d^4 - a*e^4)*ArcTan[(Sqrt[c]*x
^2)/Sqrt[a]])/(2*a^(3/2)*(c*d^4 + a*e^4)^3) - (3*Sqrt[c]*d*e*(c*d^4 - a*e^4
)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(8*a^(5/2)*(c*d^4 + a*e^4)^2) - (c^(1/4)*(
21*Sqrt[c]*d^2*(c*d^4 - 3*a*e^4) + 5*Sqrt[a]*e^2*(3*c*d^4 - a*e^4))*ArcTan[
1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*(c*d^4 + a*e^4)^2) -
 (c^(1/4)*e^4*(3*Sqrt[c]*d^2*(5*c*d^4 - 3*a*e^4) + Sqrt[a]*e^2*(7*c*d^4 - a
*e^4))*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^4 +
 a*e^4)^3) - (c^(1/4)*e^8*(3*Sqrt[c]*d^2*(3*c*d^4 - a*e^4) + Sqrt[a]*e^2*(1
1*c*d^4 - a*e^4))*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/
4)*(c*d^4 + a*e^4)^4) + (c^(1/4)*(21*Sqrt[c]*d^2*(c*d^4 - 3*a*e^4) + 5*Sqrt
[a]*e^2*(3*c*d^4 - a*e^4))*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(64*Sqr
t[2]*a^(11/4)*(c*d^4 + a*e^4)^2) + (c^(1/4)*e^4*(3*Sqrt[c]*d^2*(5*c*d^4 - 3
*a*e^4) + Sqrt[a]*e^2*(7*c*d^4 - a*e^4))*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(
1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^3) + (c^(1/4)*e^8*(3*Sqrt[c]*d^2*
(3*c*d^4 - a*e^4) + Sqrt[a]*e^2*(11*c*d^4 - a*e^4))*ArcTan[1 + (Sqrt[2]*c^(
1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^4) + (12*c*d^3*e^11*Lo
g[d + e*x])/(c*d^4 + a*e^4)^4 - (c^(1/4)*e^8*(9*c^(3/2)*d^6 - 11*Sqrt[a]*c*
d^4*e^2 - 3*a*Sqrt[c]*d^2*e^4 + a^(3/2)*e^6)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*
c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^4) - (c^(1/4)*
(21*Sqrt[c]*d^2*(c*d^4 - 3*a*e^4) - 5*Sqrt[a]*e^2*(3*c*d^4 - a*e^4))*Log[Sq
rt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^(11/4)*(c*
d^4 + a*e^4)^2) - (c^(1/4)*e^4*(3*Sqrt[c]*d^2*(5*c*d^4 - 3*a*e^4) - Sqrt[a]
*e^2*(7*c*d^4 - a*e^4))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x
^2])/(16*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^3) + (c^(1/4)*e^8*(9*c^(3/2)*d^6 -
 11*Sqrt[a]*c*d^4*e^2 - 3*a*Sqrt[c]*d^2*e^4 + a^(3/2)*e^6)*Log[Sqrt[a] + Sq
rt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^
4) + (c^(1/4)*(21*Sqrt[c]*d^2*(c*d^4 - 3*a*e^4) - 5*Sqrt[a]*e^2*(3*c*d^4 -
a*e^4))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2
]*a^(11/4)*(c*d^4 + a*e^4)^2) + (c^(1/4)*e^4*(3*Sqrt[c]*d^2*(5*c*d^4 - 3*a*
e^4) - Sqrt[a]*e^2*(7*c*d^4 - a*e^4))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)
```

```
*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^3) - (3*c*d^3*e^11*L
og[a + c*x^4])/(c*d^4 + a*e^4)^4
```

Rubi [A] time = 2.78142, antiderivative size = 1830, normalized size of antiderivative = 1., number of steps used = 46, number of rules used = 15, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.882, Rules used = {6742, 1854, 1855, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628, 1248, 635, 260}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^2*(a + c*x^4)^3),x]
```

```
[Out] -(e^11/((c*d^4 + a*e^4)^3*(d + e*x))) + (c*x*(7*d^2*(c*d^4 - 3*a*e^4) - 12*
d*e*(c*d^4 - a*e^4)*x + 5*e^2*(3*c*d^4 - a*e^4)*x^2))/(32*a^2*(c*d^4 + a*e^
4)^2*(a + c*x^4)) + (c*(4*a*d^3*e^3 + x*(d^2*(c*d^4 - 3*a*e^4) - 2*d*e*(c*d
^4 - a*e^4)*x + e^2*(3*c*d^4 - a*e^4)*x^2)))/(8*a*(c*d^4 + a*e^4)^2*(a + c*
x^4)^2) + (c*e^4*(8*a*d^3*e^3 + x*(d^2*(5*c*d^4 - 3*a*e^4) - 2*d*e*(3*c*d^4
 - a*e^4)*x + e^2*(7*c*d^4 - a*e^4)*x^2)))/(4*a*(c*d^4 + a*e^4)^3*(a + c*x^
4)) - (Sqrt[c]*d*e^9*(5*c*d^4 - a*e^4)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(Sqrt
[a]*(c*d^4 + a*e^4)^4) - (Sqrt[c]*d*e^5*(3*c*d^4 - a*e^4)*ArcTan[(Sqrt[c]*x
^2)/Sqrt[a]])/(2*a^(3/2)*(c*d^4 + a*e^4)^3) - (3*Sqrt[c]*d*e*(c*d^4 - a*e^4
)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(8*a^(5/2)*(c*d^4 + a*e^4)^2) - (c^(1/4)*(
21*Sqrt[c]*d^2*(c*d^4 - 3*a*e^4) + 5*Sqrt[a]*e^2*(3*c*d^4 - a*e^4))*ArcTan[
1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*(c*d^4 + a*e^4)^2) -
 (c^(1/4)*e^4*(3*Sqrt[c]*d^2*(5*c*d^4 - 3*a*e^4) + Sqrt[a]*e^2*(7*c*d^4 - a
*e^4))*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^4 +
 a*e^4)^3) - (c^(1/4)*e^8*(3*Sqrt[c]*d^2*(3*c*d^4 - a*e^4) + Sqrt[a]*e^2*(1
1*c*d^4 - a*e^4))*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/
4)*(c*d^4 + a*e^4)^4) + (c^(1/4)*(21*Sqrt[c]*d^2*(c*d^4 - 3*a*e^4) + 5*Sqrt
[a]*e^2*(3*c*d^4 - a*e^4))*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(64*Sqr
t[2]*a^(11/4)*(c*d^4 + a*e^4)^2) + (c^(1/4)*e^4*(3*Sqrt[c]*d^2*(5*c*d^4 - 3
*a*e^4) + Sqrt[a]*e^2*(7*c*d^4 - a*e^4))*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(
1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^3) + (c^(1/4)*e^8*(3*Sqrt[c]*d^2*
(3*c*d^4 - a*e^4) + Sqrt[a]*e^2*(11*c*d^4 - a*e^4))*ArcTan[1 + (Sqrt[2]*c^(
1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^4) + (12*c*d^3*e^11*Lo
g[d + e*x])/(c*d^4 + a*e^4)^4 - (c^(1/4)*e^8*(9*c^(3/2)*d^6 - 11*Sqrt[a]*c*
d^4*e^2 - 3*a*Sqrt[c]*d^2*e^4 + a^(3/2)*e^6)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*
c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^4) - (c^(1/4)*
(21*Sqrt[c]*d^2*(c*d^4 - 3*a*e^4) - 5*Sqrt[a]*e^2*(3*c*d^4 - a*e^4))*Log[Sq
rt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^(11/4)*(c*
```

```
d^4 + a*e^4)^2) - (c^(1/4)*e^4*(3*Sqrt[c]*d^2*(5*c*d^4 - 3*a*e^4) - Sqrt[a]
*e^2*(7*c*d^4 - a*e^4))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x
^2])/(16*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^3) + (c^(1/4)*e^8*(9*c^(3/2)*d^6 -
 11*Sqrt[a]*c*d^4*e^2 - 3*a*Sqrt[c]*d^2*e^4 + a^(3/2)*e^6)*Log[Sqrt[a] + Sq
rt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^
4) + (c^(1/4)*(21*Sqrt[c]*d^2*(c*d^4 - 3*a*e^4) - 5*Sqrt[a]*e^2*(3*c*d^4 -
a*e^4))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2
]*a^(11/4)*(c*d^4 + a*e^4)^2) + (c^(1/4)*e^4*(3*Sqrt[c]*d^2*(5*c*d^4 - 3*a*
e^4) - Sqrt[a]*e^2*(7*c*d^4 - a*e^4))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)
*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^3) - (3*c*d^3*e^11*L
og[a + c*x^4])/(c*d^4 + a*e^4)^4
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
```

```
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{1}{(d+ex)^2\left(a+cx^4\right)^3}\,dx = \int\left(\frac{e^{12}}{\left(cd^4+ae^4\right)^3(d+ex)^2}+\frac{12cd^3e^{12}}{\left(cd^4+ae^4\right)^4(d+ex)}+\frac{c\left(d^2\left(cd^4-3ae^4\right)-2de\left(cd^4-ae^4\right)x\right.}{\left(cd^4+ae^4\right)^2\left(a\right.}\right.$$

$$= -\frac{e^{11}}{\left(cd^4+ae^4\right)^3(d+ex)}+\frac{12cd^3e^{11}\log(d+ex)}{\left(cd^4+ae^4\right)^4}+\frac{\left(ce^8\right)\int\frac{3d^2\left(3cd^4-ae^4\right)-2de\left(5cd^4-ae^4\right)x+e^2\left(11cd^4-a\right.}{a+cx^4}}{\left(cd^4+ae^4\right)^4}$$

$$= -\frac{e^{11}}{\left(cd^4+ae^4\right)^3(d+ex)}+\frac{c\left(4ad^3e^3+x\left(d^2\left(cd^4-3ae^4\right)-2de\left(cd^4-ae^4\right)x+e^2\left(3cd^4-ae^4\right.\right.\right.}{8a\left(cd^4+ae^4\right)^2\left(a+cx^4\right)^2}$$

$$= -\frac{e^{11}}{\left(cd^4+ae^4\right)^3(d+ex)}+\frac{cx\left(7d^2\left(cd^4-3ae^4\right)-12de\left(cd^4-ae^4\right)x+5e^2\left(3cd^4-ae^4\right)x^2\right)}{32a^2\left(cd^4+ae^4\right)^2\left(a+cx^4\right)}+$$

$$= -\frac{e^{11}}{\left(cd^4+ae^4\right)^3(d+ex)}+\frac{cx\left(7d^2\left(cd^4-3ae^4\right)-12de\left(cd^4-ae^4\right)x+5e^2\left(3cd^4-ae^4\right)x^2\right)}{32a^2\left(cd^4+ae^4\right)^2\left(a+cx^4\right)}+$$

$$= -\frac{e^{11}}{\left(cd^4+ae^4\right)^3(d+ex)}+\frac{cx\left(7d^2\left(cd^4-3ae^4\right)-12de\left(cd^4-ae^4\right)x+5e^2\left(3cd^4-ae^4\right)x^2\right)}{32a^2\left(cd^4+ae^4\right)^2\left(a+cx^4\right)}+$$

$$= -\frac{e^{11}}{\left(cd^4+ae^4\right)^3(d+ex)}+\frac{cx\left(7d^2\left(cd^4-3ae^4\right)-12de\left(cd^4-ae^4\right)x+5e^2\left(3cd^4-ae^4\right)x^2\right)}{32a^2\left(cd^4+ae^4\right)^2\left(a+cx^4\right)}+$$

$$= -\frac{e^{11}}{\left(cd^4+ae^4\right)^3(d+ex)}+\frac{cx\left(7d^2\left(cd^4-3ae^4\right)-12de\left(cd^4-ae^4\right)x+5e^2\left(3cd^4-ae^4\right)x^2\right)}{32a^2\left(cd^4+ae^4\right)^2\left(a+cx^4\right)}+$$

$$= -\frac{e^{11}}{\left(cd^4+ae^4\right)^3(d+ex)}+\frac{cx\left(7d^2\left(cd^4-3ae^4\right)-12de\left(cd^4-ae^4\right)x+5e^2\left(3cd^4-ae^4\right)x^2\right)}{32a^2\left(cd^4+ae^4\right)^2\left(a+cx^4\right)}+$$

$$= -\frac{e^{11}}{\left(cd^4+ae^4\right)^3(d+ex)}+\frac{cx\left(7d^2\left(cd^4-3ae^4\right)-12de\left(cd^4-ae^4\right)x+5e^2\left(3cd^4-ae^4\right)x^2\right)}{32a^2\left(cd^4+ae^4\right)^2\left(a+cx^4\right)}+$$

Mathematica [A] time = 1.69329, size = 1115, normalized size = 0.61

$$3072cd^3\log(d+ex)e^{11}-768cd^3\log\left(cx^4+a\right)e^{11}-\frac{256\left(cd^4+ae^4\right)e^{11}}{d+ex}+\frac{8c\left(cd^4+ae^4\right)\left(c^2x\left(7d^2-12exd+15e^2x^2\right)d^8+2ace^4x\left(13d^2-24exd+33e^2x^2\right)\right.}{a^2\left(cx^4+a\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^2*(a + c*x^4)^3),x]
```

```
[Out] ((-256*e^11*(c*d^4 + a*e^4))/(d + e*x) + (8*c*(c*d^4 + a*e^4)*(c^2*d^8*x*(7
*d^2 - 12*d*e*x + 15*e^2*x^2) + 2*a*c*d^4*e^4*x*(13*d^2 - 24*d*e*x + 33*e^2
*x^2) + a^2*e^7*(64*d^3 - 45*d^2*e*x + 28*d*e^2*x^2 - 13*e^3*x^3)))/(a^2*(a
 + c*x^4)) + (32*c*(c*d^4 + a*e^4)^2*(c*d^4*x*(d^2 - 2*d*e*x + 3*e^2*x^2) +
 a*e^3*(4*d^3 - 3*d^2*e*x + 2*d*e^2*x^2 - e^3*x^3)))/(a*(a + c*x^4)^2) - (6
*c^(1/4)*(7*Sqrt[2]*c^(7/2)*d^14 - 16*a^(1/4)*c^(13/4)*d^13*e + 5*Sqrt[2]*S
qrt[a]*c^3*d^12*e^2 + 33*Sqrt[2]*a*c^(5/2)*d^10*e^4 - 80*a^(5/4)*c^(9/4)*d^
9*e^5 + 27*Sqrt[2]*a^(3/2)*c^2*d^8*e^6 + 77*Sqrt[2]*a^2*c^(3/2)*d^6*e^8 - 2
40*a^(9/4)*c^(5/4)*d^5*e^9 + 135*Sqrt[2]*a^(5/2)*c*d^4*e^10 - 77*Sqrt[2]*a^
3*Sqrt[c]*d^2*e^12 + 80*a^(13/4)*c^(1/4)*d*e^13 - 15*Sqrt[2]*a^(7/2)*e^14)*
ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(11/4) + (6*c^(1/4)*(7*Sqrt[2]*c
^(7/2)*d^14 + 16*a^(1/4)*c^(13/4)*d^13*e + 5*Sqrt[2]*Sqrt[a]*c^3*d^12*e^2 +
 33*Sqrt[2]*a*c^(5/2)*d^10*e^4 + 80*a^(5/4)*c^(9/4)*d^9*e^5 + 27*Sqrt[2]*a^
(3/2)*c^2*d^8*e^6 + 77*Sqrt[2]*a^2*c^(3/2)*d^6*e^8 + 240*a^(9/4)*c^(5/4)*d^
5*e^9 + 135*Sqrt[2]*a^(5/2)*c*d^4*e^10 - 77*Sqrt[2]*a^3*Sqrt[c]*d^2*e^12 -
80*a^(13/4)*c^(1/4)*d*e^13 - 15*Sqrt[2]*a^(7/2)*e^14)*ArcTan[1 + (Sqrt[2]*c
^(1/4)*x)/a^(1/4)])/a^(11/4) + 3072*c*d^3*e^11*Log[d + e*x] - (3*Sqrt[2]*c^
(1/4)*(7*c^(7/2)*d^14 - 5*Sqrt[a]*c^3*d^12*e^2 + 33*a*c^(5/2)*d^10*e^4 - 27
*a^(3/2)*c^2*d^8*e^6 + 77*a^2*c^(3/2)*d^6*e^8 - 135*a^(5/2)*c*d^4*e^10 - 77
*a^3*Sqrt[c]*d^2*e^12 + 15*a^(7/2)*e^14)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1
/4)*x + Sqrt[c]*x^2])/a^(11/4) + (3*Sqrt[2]*c^(1/4)*(7*c^(7/2)*d^14 - 5*Sqr
t[a]*c^3*d^12*e^2 + 33*a*c^(5/2)*d^10*e^4 - 27*a^(3/2)*c^2*d^8*e^6 + 77*a^2
*c^(3/2)*d^6*e^8 - 135*a^(5/2)*c*d^4*e^10 - 77*a^3*Sqrt[c]*d^2*e^12 + 15*a^
(7/2)*e^14)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(11/4
) - 768*c*d^3*e^11*Log[a + c*x^4])/(256*(c*d^4 + a*e^4)^4)
```

Maple [A] time = 0.027, size = 2769, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(c*x^4+a)^3,x)
```

```
[Out] 1/2*c^3/(a*e^4+c*d^4)^4/(c*x^4+a)^2*d^11*e^3+5/2*c/(a*e^4+c*d^4)^4/(c*x^4+a
)^2*a^2*d^3*e^11-17/32*c/(a*e^4+c*d^4)^4/(c*x^4+a)^2*e^14*a^2*x^3-45/8*c^2/
(a*e^4+c*d^4)^4/(a*c)^(1/2)*arctan(x^2*(1/a*c)^(1/2))*d^5*e^9-45/256/(a*e^4
+c*d^4)^4*a/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/
(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))*e^14-45/128/(a*e^4+c*d^4)^4*a/(a/c
)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*e^14-45/128/(a*e^4+c*d^4)^4
*a/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*e^14+53/32*c^3/(a*e^
4+c*d^4)^4/(c*x^4+a)^2*e^10*x^7*d^4-5/8*c^3/(a*e^4+c*d^4)^4/(c*x^4+a)^2*d^5
*e^9*x^6-19/32*c^3/(a*e^4+c*d^4)^4/(c*x^4+a)^2*d^6*x^5*e^8+101/32*c^3/(a*e^
4+c*d^4)^4/(c*x^4+a)^2*e^6*x^3*d^8-17/8*c^3/(a*e^4+c*d^4)^4/(c*x^4+a)^2*d^9
*e^5*x^2+29/32*c^3/(a*e^4+c*d^4)^4/(c*x^4+a)^2*d^10*x*e^4-13/32*c^2/(a*e^4+
c*d^4)^4/(c*x^4+a)^2*e^14*a*x^7+7/32*c^5/(a*e^4+c*d^4)^4/(c*x^4+a)^2*d^14/a
^2*x^5+11/32*c^4/(a*e^4+c*d^4)^4/(c*x^4+a)^2*d^14/a*x+2*c^3/(a*e^4+c*d^4)^4
/(c*x^4+a)^2*x^4*d^7*e^7+3*c^2/(a*e^4+c*d^4)^4/(c*x^4+a)^2*a*d^7*e^7+12*c*d
^3*e^11*ln(e*x+d)/(a*e^4+c*d^4)^4-3*c*d^3*e^11*ln(c*x^4+a)/(a*e^4+c*d^4)^4+
9/8*c/(a*e^4+c*d^4)^4/(c*x^4+a)^2*d*e^13*a^2*x^2-57/32*c/(a*e^4+c*d^4)^4/(c
*x^4+a)^2*d^2*a^2*x*e^12+2*c^2/(a*e^4+c*d^4)^4/(c*x^4+a)^2*x^4*a*d^3*e^11-3
/8*c^5/(a*e^4+c*d^4)^4/(c*x^4+a)^2*d^13*e/a^2*x^6+81/32*c^4/(a*e^4+c*d^4)^4
/(c*x^4+a)^2*e^6/a*x^7*d^8+15/32*c^5/(a*e^4+c*d^4)^4/(c*x^4+a)^2*e^2/a^2*x^
7*d^12+7/8*c^2/(a*e^4+c*d^4)^4/(c*x^4+a)^2*d*e^13*a*x^6-15/8*c^4/(a*e^4+c*d
^4)^4/(c*x^4+a)^2*d^9*e^5/a*x^6-45/32*c^2/(a*e^4+c*d^4)^4/(c*x^4+a)^2*d^2*a
*x^5*e^12+33/32*c^4/(a*e^4+c*d^4)^4/(c*x^4+a)^2*d^10/a*x^5*e^4+57/32*c^2/(a
*e^4+c*d^4)^4/(c*x^4+a)^2*e^10*a*x^3*d^4+27/32*c^4/(a*e^4+c*d^4)^4/(c*x^4+a
)^2*e^2/a*x^3*d^12-3/8*c^2/(a*e^4+c*d^4)^4/(c*x^4+a)^2*d^5*e^9*a*x^2-5/8*c^
4/(a*e^4+c*d^4)^4/(c*x^4+a)^2*d^13*e/a*x^2-39/32*c^2/(a*e^4+c*d^4)^4/(c*x^4
+a)^2*d^6*a*x*e^8+405/128*c/(a*e^4+c*d^4)^4/(a/c)^(1/4)*2^(1/2)*arctan(2^(1
/2)/(a/c)^(1/4)*x-1)*d^4*e^10+405/256*c/(a*e^4+c*d^4)^4/(a/c)^(1/4)*2^(1/2)
*ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c
)^(1/2)))*d^4*e^10+405/128*c/(a*e^4+c*d^4)^4/(a/c)^(1/4)*2^(1/2)*arctan(2^(
1/2)/(a/c)^(1/4)*x+1)*d^4*e^10+15/8*c/(a*e^4+c*d^4)^4*a/(a*c)^(1/2)*arctan(
x^2*(1/a*c)^(1/2))*d*e^13-3/8*c^4/(a*e^4+c*d^4)^4/a^2/(a*c)^(1/2)*arctan(x^
2*(1/a*c)^(1/2))*d^13*e-15/8*c^3/(a*e^4+c*d^4)^4/a/(a*c)^(1/2)*arctan(x^2*(
1/a*c)^(1/2))*d^9*e^5-231/128*c/(a*e^4+c*d^4)^4*(a/c)^(1/4)*2^(1/2)*arctan(
2^(1/2)/(a/c)^(1/4)*x+1)*d^2*e^12+21/128*c^4/(a*e^4+c*d^4)^4/a^3*(a/c)^(1/4
)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^14-231/128*c/(a*e^4+c*d^4)^4*(a
/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^2*e^12+21/128*c^4/(a*e^
4+c*d^4)^4/a^3*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^14-231
/256*c/(a*e^4+c*d^4)^4*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a
/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))*d^2*e^12+21/256*c^4/(a*
e^4+c*d^4)^4/a^3*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1
/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))*d^14-e^11/(a*e^4+c*d^4)^3/(e*
x+d)+15/128*c^3/(a*e^4+c*d^4)^4/a^2/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c
```

```
)^(1/4)*x-1)*d^12*e^2+81/128*c^2/(a*e^4+c*d^4)^4/a/(a/c)^(1/4)*2^(1/2)*arct
an(2^(1/2)/(a/c)^(1/4)*x+1)*d^8*e^6+231/256*c^2/(a*e^4+c*d^4)^4/a*(a/c)^(1/
4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^
(1/2)+(a/c)^(1/2)))*d^6*e^8+99/256*c^3/(a*e^4+c*d^4)^4/a^2*(a/c)^(1/4)*2^(1
/2)*ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(
a/c)^(1/2)))*d^10*e^4+81/256*c^2/(a*e^4+c*d^4)^4/a/(a/c)^(1/4)*2^(1/2)*ln((
x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/
2)))*d^8*e^6+15/256*c^3/(a*e^4+c*d^4)^4/a^2/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/
c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))*d^
12*e^2+15/128*c^3/(a*e^4+c*d^4)^4/a^2/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a
/c)^(1/4)*x+1)*d^12*e^2+81/128*c^2/(a*e^4+c*d^4)^4/a/(a/c)^(1/4)*2^(1/2)*ar
ctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^8*e^6+231/128*c^2/(a*e^4+c*d^4)^4/a*(a/c)^(
1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^6*e^8+99/128*c^3/(a*e^4+c*d^
4)^4/a^2*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^10*e^4+231/1
28*c^2/(a*e^4+c*d^4)^4/a*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1
)*d^6*e^8+99/128*c^3/(a*e^4+c*d^4)^4/a^2*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)
/(a/c)^(1/4)*x-1)*d^10*e^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(c*x**4+a)**3,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^4+a)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.414 $\int \frac{1}{(d+ex)^3\left(a+cx^4\right)^3}\,dx$

Optimal. Leaf size=2204

result too large to display

```
[Out] -e^11/(2*(c*d^4 + a*e^4)^3*(d + e*x)^2) - (12*c*d^3*e^11)/((c*d^4 + a*e^4)^
4*(d + e*x)) + (c*x*(7*d*(c^2*d^8 - 12*a*c*d^4*e^4 + 3*a^2*e^8) - 6*e*(3*c^
2*d^8 - 12*a*c*d^4*e^4 + a^2*e^8)*x + 10*c*d^3*e^2*(3*c*d^4 - 5*a*e^4)*x^2)
)/(32*a^2*(c*d^4 + a*e^4)^3*(a + c*x^4)) + (c*(2*a*d^2*e^3*(5*c*d^4 - 3*a*e
^4) + x*(d*(c^2*d^8 - 12*a*c*d^4*e^4 + 3*a^2*e^8) - e*(3*c^2*d^8 - 12*a*c*d
^4*e^4 + a^2*e^8)*x + 2*c*d^3*e^2*(3*c*d^4 - 5*a*e^4)*x^2)))/(8*a*(c*d^4 +
a*e^4)^3*(a + c*x^4)^2) + (c*e^4*(12*a*d^2*e^3*(3*c*d^4 - a*e^4) + x*(3*d*(
5*c^2*d^8 - 10*a*c*d^4*e^4 + a^2*e^8) - e*(21*c^2*d^8 - 26*a*c*d^4*e^4 + a^
2*e^8)*x + 4*c*d^3*e^2*(7*c*d^4 - 5*a*e^4)*x^2)))/(4*a*(c*d^4 + a*e^4)^4*(a
 + c*x^4)) - (Sqrt[c]*e^9*(55*c^2*d^8 - 40*a*c*d^4*e^4 + a^2*e^8)*ArcTan[(S
qrt[c]*x^2)/Sqrt[a]])/(2*Sqrt[a]*(c*d^4 + a*e^4)^5) - (Sqrt[c]*e^5*(21*c^2*
d^8 - 26*a*c*d^4*e^4 + a^2*e^8)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*a^(3/2)*(
c*d^4 + a*e^4)^4) - (3*Sqrt[c]*e*(3*c^2*d^8 - 12*a*c*d^4*e^4 + a^2*e^8)*Arc
Tan[(Sqrt[c]*x^2)/Sqrt[a]])/(16*a^(5/2)*(c*d^4 + a*e^4)^3) - (3*c^(3/4)*d*e
^8*(15*c^2*d^8 - 16*a*c*d^4*e^4 + a^2*e^8 + 2*Sqrt[a]*Sqrt[c]*d^2*e^2*(11*c
*d^4 - 5*a*e^4))*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4
)*(c*d^4 + a*e^4)^5) - (c^(3/4)*d*e^4*(4*Sqrt[a]*Sqrt[c]*d^2*e^2*(7*c*d^4 -
 5*a*e^4) + 9*(5*c^2*d^8 - 10*a*c*d^4*e^4 + a^2*e^8))*ArcTan[1 - (Sqrt[2]*c
^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^4) - (c^(3/4)*d*(10*
Sqrt[a]*Sqrt[c]*d^2*e^2*(3*c*d^4 - 5*a*e^4) + 21*(c^2*d^8 - 12*a*c*d^4*e^4
+ 3*a^2*e^8))*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)
*(c*d^4 + a*e^4)^3) + (3*c^(3/4)*d*e^8*(15*c^2*d^8 - 16*a*c*d^4*e^4 + a^2*e
^8 + 2*Sqrt[a]*Sqrt[c]*d^2*e^2*(11*c*d^4 - 5*a*e^4))*ArcTan[1 + (Sqrt[2]*c^
(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^5) + (c^(3/4)*d*e^4*(
4*Sqrt[a]*Sqrt[c]*d^2*e^2*(7*c*d^4 - 5*a*e^4) + 9*(5*c^2*d^8 - 10*a*c*d^4*e
^4 + a^2*e^8))*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*
(c*d^4 + a*e^4)^4) + (c^(3/4)*d*(10*Sqrt[a]*Sqrt[c]*d^2*e^2*(3*c*d^4 - 5*a*
e^4) + 21*(c^2*d^8 - 12*a*c*d^4*e^4 + 3*a^2*e^8))*ArcTan[1 + (Sqrt[2]*c^(1/
4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*(c*d^4 + a*e^4)^3) + (6*c*d^2*e^11*(13
*c*d^4 - 3*a*e^4)*Log[d + e*x])/(c*d^4 + a*e^4)^5 - (3*c^(3/4)*d*e^8*(15*c^
2*d^8 - 16*a*c*d^4*e^4 + a^2*e^8 - 2*Sqrt[a]*Sqrt[c]*d^2*e^2*(11*c*d^4 - 5*
a*e^4))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*
a^(3/4)*(c*d^4 + a*e^4)^5) + (c^(3/4)*d*e^4*(4*Sqrt[a]*Sqrt[c]*d^2*e^2*(7*c
*d^4 - 5*a*e^4) - 9*(5*c^2*d^8 - 10*a*c*d^4*e^4 + a^2*e^8))*Log[Sqrt[a] - S
qrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4
)^4) + (c^(3/4)*d*(10*Sqrt[a]*Sqrt[c]*d^2*e^2*(3*c*d^4 - 5*a*e^4) - 21*(c^2
*d^8 - 12*a*c*d^4*e^4 + 3*a^2*e^8))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x
```

```
 + Sqrt[c]*x^2])/(128*Sqrt[2]*a^(11/4)*(c*d^4 + a*e^4)^3) + (3*c^(3/4)*d*e^
8*(15*c^2*d^8 - 16*a*c*d^4*e^4 + a^2*e^8 - 2*Sqrt[a]*Sqrt[c]*d^2*e^2*(11*c*
d^4 - 5*a*e^4))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*
Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^5) - (c^(3/4)*d*e^4*(4*Sqrt[a]*Sqrt[c]*d^2*
e^2*(7*c*d^4 - 5*a*e^4) - 9*(5*c^2*d^8 - 10*a*c*d^4*e^4 + a^2*e^8))*Log[Sqr
t[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^4
 + a*e^4)^4) - (c^(3/4)*d*(10*Sqrt[a]*Sqrt[c]*d^2*e^2*(3*c*d^4 - 5*a*e^4) -
 21*(c^2*d^8 - 12*a*c*d^4*e^4 + 3*a^2*e^8))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c
^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^(11/4)*(c*d^4 + a*e^4)^3) - (3*c*d^
2*e^11*(13*c*d^4 - 3*a*e^4)*Log[a + c*x^4])/(2*(c*d^4 + a*e^4)^5)
```

Rubi [A] time = 3.1648, antiderivative size = 2204, normalized size of antiderivative = 1., number of steps used = 46, number of rules used = 15, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.882, Rules used = {6742, 1854, 1855, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628, 1248, 635, 260}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^3*(a + c*x^4)^3),x]
```

```
[Out] -e^11/(2*(c*d^4 + a*e^4)^3*(d + e*x)^2) - (12*c*d^3*e^11)/((c*d^4 + a*e^4)^
4*(d + e*x)) + (c*x*(7*d*(c^2*d^8 - 12*a*c*d^4*e^4 + 3*a^2*e^8) - 6*e*(3*c^
2*d^8 - 12*a*c*d^4*e^4 + a^2*e^8)*x + 10*c*d^3*e^2*(3*c*d^4 - 5*a*e^4)*x^2)
)/(32*a^2*(c*d^4 + a*e^4)^3*(a + c*x^4)) + (c*(2*a*d^2*e^3*(5*c*d^4 - 3*a*e
^4) + x*(d*(c^2*d^8 - 12*a*c*d^4*e^4 + 3*a^2*e^8) - e*(3*c^2*d^8 - 12*a*c*d
^4*e^4 + a^2*e^8)*x + 2*c*d^3*e^2*(3*c*d^4 - 5*a*e^4)*x^2)))/(8*a*(c*d^4 +
a*e^4)^3*(a + c*x^4)^2) + (c*e^4*(12*a*d^2*e^3*(3*c*d^4 - a*e^4) + x*(3*d*(
5*c^2*d^8 - 10*a*c*d^4*e^4 + a^2*e^8) - e*(21*c^2*d^8 - 26*a*c*d^4*e^4 + a^
2*e^8)*x + 4*c*d^3*e^2*(7*c*d^4 - 5*a*e^4)*x^2)))/(4*a*(c*d^4 + a*e^4)^4*(a
 + c*x^4)) - (Sqrt[c]*e^9*(55*c^2*d^8 - 40*a*c*d^4*e^4 + a^2*e^8)*ArcTan[(S
qrt[c]*x^2)/Sqrt[a]])/(2*Sqrt[a]*(c*d^4 + a*e^4)^5) - (Sqrt[c]*e^5*(21*c^2*
d^8 - 26*a*c*d^4*e^4 + a^2*e^8)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*a^(3/2)*(
c*d^4 + a*e^4)^4) - (3*Sqrt[c]*e*(3*c^2*d^8 - 12*a*c*d^4*e^4 + a^2*e^8)*Arc
Tan[(Sqrt[c]*x^2)/Sqrt[a]])/(16*a^(5/2)*(c*d^4 + a*e^4)^3) - (3*c^(3/4)*d*e
^8*(15*c^2*d^8 - 16*a*c*d^4*e^4 + a^2*e^8 + 2*Sqrt[a]*Sqrt[c]*d^2*e^2*(11*c
*d^4 - 5*a*e^4))*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4
)*(c*d^4 + a*e^4)^5) - (c^(3/4)*d*e^4*(4*Sqrt[a]*Sqrt[c]*d^2*e^2*(7*c*d^4 -
 5*a*e^4) + 9*(5*c^2*d^8 - 10*a*c*d^4*e^4 + a^2*e^8))*ArcTan[1 - (Sqrt[2]*c
^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^4) - (c^(3/4)*d*(10*
Sqrt[a]*Sqrt[c]*d^2*e^2*(3*c*d^4 - 5*a*e^4) + 21*(c^2*d^8 - 12*a*c*d^4*e^4
```

```
+ 3*a^2*e^8))*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)
*(c*d^4 + a*e^4)^3) + (3*c^(3/4)*d*e^8*(15*c^2*d^8 - 16*a*c*d^4*e^4 + a^2*e
^8 + 2*Sqrt[a]*Sqrt[c]*d^2*e^2*(11*c*d^4 - 5*a*e^4))*ArcTan[1 + (Sqrt[2]*c^
(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^5) + (c^(3/4)*d*e^4*(
4*Sqrt[a]*Sqrt[c]*d^2*e^2*(7*c*d^4 - 5*a*e^4) + 9*(5*c^2*d^8 - 10*a*c*d^4*e
^4 + a^2*e^8))*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*
(c*d^4 + a*e^4)^4) + (c^(3/4)*d*(10*Sqrt[a]*Sqrt[c]*d^2*e^2*(3*c*d^4 - 5*a*
e^4) + 21*(c^2*d^8 - 12*a*c*d^4*e^4 + 3*a^2*e^8))*ArcTan[1 + (Sqrt[2]*c^(1/
4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*(c*d^4 + a*e^4)^3) + (6*c*d^2*e^11*(13
*c*d^4 - 3*a*e^4)*Log[d + e*x])/(c*d^4 + a*e^4)^5 - (3*c^(3/4)*d*e^8*(15*c^
2*d^8 - 16*a*c*d^4*e^4 + a^2*e^8 - 2*Sqrt[a]*Sqrt[c]*d^2*e^2*(11*c*d^4 - 5*
a*e^4))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*
a^(3/4)*(c*d^4 + a*e^4)^5) + (c^(3/4)*d*e^4*(4*Sqrt[a]*Sqrt[c]*d^2*e^2*(7*c
*d^4 - 5*a*e^4) - 9*(5*c^2*d^8 - 10*a*c*d^4*e^4 + a^2*e^8))*Log[Sqrt[a] - S
qrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4
)^4) + (c^(3/4)*d*(10*Sqrt[a]*Sqrt[c]*d^2*e^2*(3*c*d^4 - 5*a*e^4) - 21*(c^2
*d^8 - 12*a*c*d^4*e^4 + 3*a^2*e^8))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x
 + Sqrt[c]*x^2])/(128*Sqrt[2]*a^(11/4)*(c*d^4 + a*e^4)^3) + (3*c^(3/4)*d*e^
8*(15*c^2*d^8 - 16*a*c*d^4*e^4 + a^2*e^8 - 2*Sqrt[a]*Sqrt[c]*d^2*e^2*(11*c*
d^4 - 5*a*e^4))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*
Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^5) - (c^(3/4)*d*e^4*(4*Sqrt[a]*Sqrt[c]*d^2*
e^2*(7*c*d^4 - 5*a*e^4) - 9*(5*c^2*d^8 - 10*a*c*d^4*e^4 + a^2*e^8))*Log[Sqr
t[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^4
 + a*e^4)^4) - (c^(3/4)*d*(10*Sqrt[a]*Sqrt[c]*d^2*e^2*(3*c*d^4 - 5*a*e^4) -
 21*(c^2*d^8 - 12*a*c*d^4*e^4 + 3*a^2*e^8))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c
^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^(11/4)*(c*d^4 + a*e^4)^3) - (3*c*d^
2*e^11*(13*c*d^4 - 3*a*e^4)*Log[a + c*x^4])/(2*(c*d^4 + a*e^4)^5)
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
```

```
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{1}{(d+ex)^3\left(a+cx^4\right)^3}\,dx = \int\Bigg(\frac{e^{12}}{\left(cd^4+ae^4\right)^3(d+ex)^3} + \frac{12cd^3e^{12}}{\left(cd^4+ae^4\right)^4(d+ex)^2} + \frac{6cd^2e^{12}\left(13cd^4-3ae^4\right)}{\left(cd^4+ae^4\right)^5(d+ex)} + \frac{c\left(d\left(c^2d^8 - \cdots\right.\right.}{\cdots}$$

$$= -\frac{e^{11}}{2\left(cd^4+ae^4\right)^3(d+ex)^2} - \frac{12cd^3e^{11}}{\left(cd^4+ae^4\right)^4(d+ex)} + \frac{6cd^2e^{11}\left(13cd^4-3ae^4\right)\log(d+ex)}{\left(cd^4+ae^4\right)^5} + \cdots$$

$$= -\frac{e^{11}}{2\left(cd^4+ae^4\right)^3(d+ex)^2} - \frac{12cd^3e^{11}}{\left(cd^4+ae^4\right)^4(d+ex)} + \frac{c\left(2ad^2e^3\left(5cd^4-3ae^4\right)+x\left(d\left(c^2d^8-\cdots\right.\right.\right.}{\cdots}$$

$$= -\frac{e^{11}}{2\left(cd^4+ae^4\right)^3(d+ex)^2} - \frac{12cd^3e^{11}}{\left(cd^4+ae^4\right)^4(d+ex)} + \frac{cx\left(7d\left(c^2d^8-12acd^4e^4+3a^2e^8\right)-6e\left(\cdots\right.\right.}{32a^2\cdots}$$

$$= -\frac{e^{11}}{2\left(cd^4+ae^4\right)^3(d+ex)^2} - \frac{12cd^3e^{11}}{\left(cd^4+ae^4\right)^4(d+ex)} + \frac{cx\left(7d\left(c^2d^8-12acd^4e^4+3a^2e^8\right)-6e\left(\cdots\right.\right.}{32a^2\cdots}$$

$$= -\frac{e^{11}}{2\left(cd^4+ae^4\right)^3(d+ex)^2} - \frac{12cd^3e^{11}}{\left(cd^4+ae^4\right)^4(d+ex)} + \frac{cx\left(7d\left(c^2d^8-12acd^4e^4+3a^2e^8\right)-6e\left(\cdots\right.\right.}{32a^2\cdots}$$

$$= -\frac{e^{11}}{2\left(cd^4+ae^4\right)^3(d+ex)^2} - \frac{12cd^3e^{11}}{\left(cd^4+ae^4\right)^4(d+ex)} + \frac{cx\left(7d\left(c^2d^8-12acd^4e^4+3a^2e^8\right)-6e\left(\cdots\right.\right.}{32a^2\cdots}$$

$$= -\frac{e^{11}}{2\left(cd^4+ae^4\right)^3(d+ex)^2} - \frac{12cd^3e^{11}}{\left(cd^4+ae^4\right)^4(d+ex)} + \frac{cx\left(7d\left(c^2d^8-12acd^4e^4+3a^2e^8\right)-6e\left(\cdots\right.\right.}{32a^2\cdots}$$

$$= -\frac{e^{11}}{2\left(cd^4+ae^4\right)^3(d+ex)^2} - \frac{12cd^3e^{11}}{\left(cd^4+ae^4\right)^4(d+ex)} + \frac{cx\left(7d\left(c^2d^8-12acd^4e^4+3a^2e^8\right)-6e\left(\cdots\right.\right.}{32a^2\cdots}$$

$$= -\frac{e^{11}}{2\left(cd^4+ae^4\right)^3(d+ex)^2} - \frac{12cd^3e^{11}}{\left(cd^4+ae^4\right)^4(d+ex)} + \frac{cx\left(7d\left(c^2d^8-12acd^4e^4+3a^2e^8\right)-6e\left(\cdots\right.\right.}{32a^2\cdots}$$

Mathematica [A] time = 3.04755, size = 1338, normalized size = 0.61

$$1536cd^2\left(13cd^4-3ae^4\right)\log(d+ex)e^{11}-384cd^2\left(13cd^4-3ae^4\right)\log\left(cx^4+a\right)e^{11}-\frac{3072cd^3\left(cd^4+ae^4\right)e^{11}}{d+ex}-\frac{128\left(cd^4+ae^4\right)^2e^{11}}{(d+ex)^2}+\frac{8c(c}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^3*(a + c*x^4)^3),x]
```

```
[Out] ((-128*e^11*(c*d^4 + a*e^4)^2)/(d + e*x)^2 - (3072*c*d^3*e^11*(c*d^4 + a*e^
4))/(d + e*x) + (8*c*(c*d^4 + a*e^4)*(a^3*e^11*(-96*d^2 + 45*d*e*x - 14*e^2
*x^2) + c^3*d^11*x*(7*d^2 - 18*d*e*x + 30*e^2*x^2) + a*c^2*d^7*e^4*x*(43*d^
2 - 114*d*e*x + 204*e^2*x^2) + a^2*c*d^3*e^7*(288*d^3 - 303*d^2*e*x + 274*d
*e^2*x^2 - 210*e^3*x^3)))/(a^2*(a + c*x^4)) + (32*c*(c*d^4 + a*e^4)^2*(-(a^
2*e^7*(6*d^2 - 3*d*e*x + e^2*x^2)) + c^2*d^7*x*(d^2 - 3*d*e*x + 6*e^2*x^2)
+ 2*a*c*d^3*e^3*(5*d^3 - 6*d^2*e*x + 6*d*e^2*x^2 - 5*e^3*x^3)))/(a*(a + c*x
^4)^2) - (6*Sqrt[c]*(7*Sqrt[2]*c^(17/4)*d^17 - 24*a^(1/4)*c^4*d^16*e + 10*S
qrt[2]*Sqrt[a]*c^(15/4)*d^15*e^2 + 50*Sqrt[2]*a*c^(13/4)*d^13*e^4 - 176*a^(
5/4)*c^3*d^12*e^5 + 78*Sqrt[2]*a^(3/2)*c^(11/4)*d^11*e^6 + 220*Sqrt[2]*a^2*
c^(9/4)*d^9*e^8 - 960*a^(9/4)*c^2*d^8*e^9 + 702*Sqrt[2]*a^(5/2)*c^(7/4)*d^7
*e^10 - 770*Sqrt[2]*a^3*c^(5/4)*d^5*e^12 + 1200*a^(13/4)*c*d^4*e^13 - 390*S
qrt[2]*a^(7/2)*c^(3/4)*d^3*e^14 + 77*Sqrt[2]*a^4*c^(1/4)*d*e^16 - 40*a^(17/
4)*e^17)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(11/4) + (6*Sqrt[c]*(7*
Sqrt[2]*c^(17/4)*d^17 + 24*a^(1/4)*c^4*d^16*e + 10*Sqrt[2]*Sqrt[a]*c^(15/4)
*d^15*e^2 + 50*Sqrt[2]*a*c^(13/4)*d^13*e^4 + 176*a^(5/4)*c^3*d^12*e^5 + 78*
Sqrt[2]*a^(3/2)*c^(11/4)*d^11*e^6 + 220*Sqrt[2]*a^2*c^(9/4)*d^9*e^8 + 960*a
^(9/4)*c^2*d^8*e^9 + 702*Sqrt[2]*a^(5/2)*c^(7/4)*d^7*e^10 - 770*Sqrt[2]*a^3
*c^(5/4)*d^5*e^12 - 1200*a^(13/4)*c*d^4*e^13 - 390*Sqrt[2]*a^(7/2)*c^(3/4)*
d^3*e^14 + 77*Sqrt[2]*a^4*c^(1/4)*d*e^16 + 40*a^(17/4)*e^17)*ArcTan[1 + (Sq
rt[2]*c^(1/4)*x)/a^(1/4)])/a^(11/4) + 1536*c*d^2*e^11*(13*c*d^4 - 3*a*e^4)*
Log[d + e*x] - (3*Sqrt[2]*c^(3/4)*(7*c^4*d^17 - 10*Sqrt[a]*c^(7/2)*d^15*e^2
 + 50*a*c^3*d^13*e^4 - 78*a^(3/2)*c^(5/2)*d^11*e^6 + 220*a^2*c^2*d^9*e^8 -
702*a^(5/2)*c^(3/2)*d^7*e^10 - 770*a^3*c*d^5*e^12 + 390*a^(7/2)*Sqrt[c]*d^3
*e^14 + 77*a^4*d*e^16)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^
2])/a^(11/4) + (3*Sqrt[2]*c^(3/4)*(7*c^4*d^17 - 10*Sqrt[a]*c^(7/2)*d^15*e^2
 + 50*a*c^3*d^13*e^4 - 78*a^(3/2)*c^(5/2)*d^11*e^6 + 220*a^2*c^2*d^9*e^8 -
702*a^(5/2)*c^(3/2)*d^7*e^10 - 770*a^3*c*d^5*e^12 + 390*a^(7/2)*Sqrt[c]*d^3
*e^14 + 77*a^4*d*e^16)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^
2])/a^(11/4) - 384*c*d^2*e^11*(13*c*d^4 - 3*a*e^4)*Log[a + c*x^4])/(256*(c*
d^4 + a*e^4)^5)
```

Maple [A] time = 0.035, size = 3334, normalized size = 1.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^3/(c*x^4+a)^3,x)
```

```
[Out] 57/32*c/(a*e^4+c*d^4)^5/(c*x^4+a)^2*d*a^3*x*e^16+25/16*c^5/(a*e^4+c*d^4)^5/
(c*x^4+a)^2*d^13/a*x^5*e^4-129/16*c^3/(a*e^4+c*d^4)^5/(c*x^4+a)^2*d^5*a*x^5
*e^12-1155/64*c^2/(a*e^4+c*d^4)^5*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^
(1/4)*x+1)*d^5*e^12-1155/64*c^2/(a*e^4+c*d^4)^5*(a/c)^(1/4)*2^(1/2)*arctan(
2^(1/2)/(a/c)^(1/4)*x-1)*d^5*e^12-1155/128*c^2/(a*e^4+c*d^4)^5*(a/c)^(1/4)*
2^(1/2)*ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/
2)+(a/c)^(1/2)))*d^5*e^12+1053/64*c^2/(a*e^4+c*d^4)^5/(a/c)^(1/4)*2^(1/2)*a
rctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^7*e^10+1053/64*c^2/(a*e^4+c*d^4)^5/(a/c)^(
1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^7*e^10+1053/128*c^2/(a*e^4+c
*d^4)^5/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2
+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))*d^7*e^10-3*c^2/(a*e^4+c*d^4)^5/(c*x^4+
a)^2*x^4*a^2*d^2*e^15+6*c^3/(a*e^4+c*d^4)^5/(c*x^4+a)^2*x^4*a*d^6*e^11+45/3
2*c^2/(a*e^4+c*d^4)^5/(c*x^4+a)^2*d*a^2*x^5*e^16-85/8*c^3/(a*e^4+c*d^4)^5/(
c*x^4+a)^2*d^9*a*x*e^8+117/16*c^5/(a*e^4+c*d^4)^5/(c*x^4+a)^2*d^11*e^6/a*x^
7+15/16*c^6/(a*e^4+c*d^4)^5/(c*x^4+a)^2*d^15*e^2/a^2*x^7-105/16*c^3/(a*e^4+
c*d^4)^5/(c*x^4+a)^2*d^3*e^14*a*x^7-125/16*c^2/(a*e^4+c*d^4)^5/(c*x^4+a)^2*
d^3*e^14*a^2*x^3-31/16*c^3/(a*e^4+c*d^4)^5/(c*x^4+a)^2*d^7*e^10*a*x^3+27/16
*c^5/(a*e^4+c*d^4)^5/(c*x^4+a)^2*d^15*e^2/a*x^3+78*e^11*c^2*d^6/(a*e^4+c*d^
4)^5*ln(e*x+d)+5/4*c^4/(a*e^4+c*d^4)^5/(c*x^4+a)^2*d^14*e^3-39/2*c^2/(a*e^4
+c*d^4)^5*ln(c*x^4+a)*d^6*e^11+11/32*c^5/(a*e^4+c*d^4)^5/(c*x^4+a)^2*d^17/a
*x-27/8*c^4/(a*e^4+c*d^4)^5/(c*x^4+a)^2*e^5*x^2*d^12+121/16*c^4/(a*e^4+c*d^
4)^5/(c*x^4+a)^2*d^11*e^6*x^3-12*c*d^3*e^11/(a*e^4+c*d^4)^4/(e*x+d)-18*e^15
*c*d^2/(a*e^4+c*d^4)^5*ln(e*x+d)*a+9*c^4/(a*e^4+c*d^4)^5/(c*x^4+a)^2*x^4*d^
10*e^7+9/2*c/(a*e^4+c*d^4)^5*a*ln(c*x^4+a)*d^2*e^15-15/16*c/(a*e^4+c*d^4)^5
*a^2/(a*c)^(1/2)*arctan(x^2*(1/a*c)^(1/2))*e^17-9/16*c/(a*e^4+c*d^4)^5/(c*x
^4+a)^2*e^17*a^3*x^2-15/4*c/(a*e^4+c*d^4)^5/(c*x^4+a)^2*a^3*d^2*e^15+43/4*c
^3/(a*e^4+c*d^4)^5/(c*x^4+a)^2*a*d^10*e^7+23/4*c^2/(a*e^4+c*d^4)^5/(c*x^4+a
)^2*a^2*d^6*e^11+5/16*c^4/(a*e^4+c*d^4)^5/(c*x^4+a)^2*d^13*x*e^4-65/8*c^4/(
a*e^4+c*d^4)^5/(c*x^4+a)^2*d^9*x^5*e^8-45/2*c^3/(a*e^4+c*d^4)^5/(a*c)^(1/2)
*arctan(x^2*(1/a*c)^(1/2))*d^8*e^9+5*c^4/(a*e^4+c*d^4)^5/(c*x^4+a)^2*e^9*x^
6*d^8-3/16*c^4/(a*e^4+c*d^4)^5/(c*x^4+a)^2*d^7*e^10*x^7-7/16*c^2/(a*e^4+c*d
^4)^5/(c*x^4+a)^2*e^17*a^2*x^6+7/32*c^6/(a*e^4+c*d^4)^5/(c*x^4+a)^2*d^17/a^
2*x^5-9/16*c^6/(a*e^4+c*d^4)^5/(c*x^4+a)^2*e/a^2*x^6*d^16-33/8*c^5/(a*e^4+c
*d^4)^5/(c*x^4+a)^2*e^5/a*x^6*d^12+65/8*c^3/(a*e^4+c*d^4)^5/(c*x^4+a)^2*e^1
3*a*x^6*d^4+75/8*c^2/(a*e^4+c*d^4)^5/(c*x^4+a)^2*e^13*a^2*x^2*d^4+15/2*c^3/
(a*e^4+c*d^4)^5/(c*x^4+a)^2*e^9*a*x^2*d^8-15/16*c^5/(a*e^4+c*d^4)^5/(c*x^4+
```

```
a)^2*e/a*x^2*d^16-141/16*c^2/(a*e^4+c*d^4)^5/(c*x^4+a)^2*d^5*a^2*x*e^12+21/
256*c^5/(a*e^4+c*d^4)^5/a^3*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*x*2^(1/
2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))*d^17+225/8*c^2/(a*
e^4+c*d^4)^5*a/(a*c)^(1/2)*arctan(x^2*(1/a*c)^(1/2))*d^4*e^13-33/8*c^4/(a*e
^4+c*d^4)^5/a/(a*c)^(1/2)*arctan(x^2*(1/a*c)^(1/2))*d^12*e^5+21/128*c^5/(a*
e^4+c*d^4)^5/a^3*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^17-9
/16*c^5/(a*e^4+c*d^4)^5/a^2/(a*c)^(1/2)*arctan(x^2*(1/a*c)^(1/2))*d^16*e+21
/128*c^5/(a*e^4+c*d^4)^5/a^3*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)
*x+1)*d^17+231/128*c/(a*e^4+c*d^4)^5*a*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(
a/c)^(1/4)*x-1)*d*e^16+231/256*c/(a*e^4+c*d^4)^5*a*(a/c)^(1/4)*2^(1/2)*ln((
x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/
2)))*d*e^16+231/128*c/(a*e^4+c*d^4)^5*a*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/
(a/c)^(1/4)*x+1)*d*e^16+15/64*c^4/(a*e^4+c*d^4)^5/a^2/(a/c)^(1/4)*2^(1/2)*a
rctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^15*e^2+15/64*c^4/(a*e^4+c*d^4)^5/a^2/(a/c)
^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^15*e^2+15/128*c^4/(a*e^4+c
*d^4)^5/a^2/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/
(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))*d^15*e^2-585/64*c/(a*e^4+c*d^4)^5*
a/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^3*e^14-585/128*c/(a
*e^4+c*d^4)^5*a/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/
2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))*d^3*e^14+117/64*c^3/(a*e^4+c*d
^4)^5/a/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^11*e^6+75/128
*c^4/(a*e^4+c*d^4)^5/a^2*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*x*2^(1/2)+
(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))*d^13*e^4+165/64*c^3/(
a*e^4+c*d^4)^5/a*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1
/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))*d^9*e^8+75/64*c^4/(a*e^4+c*d^
4)^5/a^2*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^13*e^4+117/6
4*c^3/(a*e^4+c*d^4)^5/a/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)
*d^11*e^6+117/128*c^3/(a*e^4+c*d^4)^5/a/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(
1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))*d^11*e
^6-585/64*c/(a*e^4+c*d^4)^5*a/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4
)*x+1)*d^3*e^14+165/32*c^3/(a*e^4+c*d^4)^5/a*(a/c)^(1/4)*2^(1/2)*arctan(2^(
1/2)/(a/c)^(1/4)*x+1)*d^9*e^8+75/64*c^4/(a*e^4+c*d^4)^5/a^2*(a/c)^(1/4)*2^(
1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^13*e^4+165/32*c^3/(a*e^4+c*d^4)^5/a*
(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^9*e^8-1/2*e^11/(a*e^4
+c*d^4)^3/(e*x+d)^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**3/(c*x**4+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.83335, size = 2869, normalized size = 1.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*x^4+a)^3,x, algorithm="giac")
```

```
[Out] -3/64*(23*sqrt(2)*sqrt(a*c)*a*c^5*d^6*e - 7*(a*c^3)^(1/4)*a*c^5*d^7 + 30*sq
rt(2)*a^2*c^5*d^4*e^3 - 65*(a*c^3)^(3/4)*a*c^3*d^5*e^2 + 65*(a*c^3)^(1/4)*a
^2*c^4*d^3*e^4 + 115*sqrt(2)*sqrt(a*c)*a^2*c^4*d^2*e^5 - 20*sqrt(2)*a^3*c^4
*e^7 + 123*(a*c^3)^(3/4)*a^2*c^2*d*e^6)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(
a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^4*c^6*d^10 - 10*(a*c^3)^(1/4)*a^4*c^5*d
```

```
^9*e - 25*sqrt(2)*sqrt(a*c)*a^4*c^5*d^8*e^2 + 90*sqrt(2)*a^5*c^5*d^6*e^4 -
80*(a*c^3)^(3/4)*a^4*c^3*d^7*e^3 - 148*(a*c^3)^(1/4)*a^5*c^4*d^5*e^5 - 90*s
qrt(2)*sqrt(a*c)*a^5*c^4*d^4*e^6 + 25*sqrt(2)*a^6*c^4*d^2*e^8 - 80*(a*c^3)^
(3/4)*a^5*c^2*d^3*e^7 - 10*(a*c^3)^(1/4)*a^6*c^3*d*e^9 - sqrt(2)*sqrt(a*c)*
a^6*c^3*e^10) - 3/64*(23*sqrt(2)*sqrt(a*c)*a*c^5*d^6*e - 7*(a*c^3)^(1/4)*a*
c^5*d^7 - 30*sqrt(2)*a^2*c^5*d^4*e^3 - 65*(a*c^3)^(3/4)*a*c^3*d^5*e^2 + 65*
(a*c^3)^(1/4)*a^2*c^4*d^3*e^4 + 115*sqrt(2)*sqrt(a*c)*a^2*c^4*d^2*e^5 + 20*
sqrt(2)*a^3*c^4*e^7 + 123*(a*c^3)^(3/4)*a^2*c^2*d*e^6)*arctan(1/2*sqrt(2)*(
2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^4*c^6*d^10 + 10*(a*c^3)^
(1/4)*a^4*c^5*d^9*e - 25*sqrt(2)*sqrt(a*c)*a^4*c^5*d^8*e^2 + 90*sqrt(2)*a^5
*c^5*d^6*e^4 + 80*(a*c^3)^(3/4)*a^4*c^3*d^7*e^3 + 148*(a*c^3)^(1/4)*a^5*c^4
*d^5*e^5 - 90*sqrt(2)*sqrt(a*c)*a^5*c^4*d^4*e^6 + 25*sqrt(2)*a^6*c^4*d^2*e^
8 + 80*(a*c^3)^(3/4)*a^5*c^2*d^3*e^7 + 10*(a*c^3)^(1/4)*a^6*c^3*d*e^9 - sqr
t(2)*sqrt(a*c)*a^6*c^3*e^10) + 3/128*(7*(a*c^3)^(1/4)*c^5*d^17 - 10*(a*c^3)
^(3/4)*c^3*d^15*e^2 + 50*(a*c^3)^(1/4)*a*c^4*d^13*e^4 - 78*(a*c^3)^(3/4)*a*
c^2*d^11*e^6 + 220*(a*c^3)^(1/4)*a^2*c^3*d^9*e^8 - 702*(a*c^3)^(3/4)*a^2*c*
d^7*e^10 - 770*(a*c^3)^(1/4)*a^3*c^2*d^5*e^12 + 390*(a*c^3)^(3/4)*a^3*d^3*e
^14 + 77*(a*c^3)^(1/4)*a^4*c*d*e^16)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt
(a/c))/(sqrt(2)*a^3*c^6*d^20 + 5*sqrt(2)*a^4*c^5*d^16*e^4 + 10*sqrt(2)*a^5*
c^4*d^12*e^8 + 10*sqrt(2)*a^6*c^3*d^8*e^12 + 5*sqrt(2)*a^7*c^2*d^4*e^16 + s
qrt(2)*a^8*c*e^20) - 3/128*(7*(a*c^3)^(1/4)*c^5*d^17 - 10*(a*c^3)^(3/4)*c^3
*d^15*e^2 + 50*(a*c^3)^(1/4)*a*c^4*d^13*e^4 - 78*(a*c^3)^(3/4)*a*c^2*d^11*e
^6 + 220*(a*c^3)^(1/4)*a^2*c^3*d^9*e^8 - 702*(a*c^3)^(3/4)*a^2*c*d^7*e^10 -
 770*(a*c^3)^(1/4)*a^3*c^2*d^5*e^12 + 390*(a*c^3)^(3/4)*a^3*d^3*e^14 + 77*(
a*c^3)^(1/4)*a^4*c*d*e^16)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sq
rt(2)*a^3*c^6*d^20 + 5*sqrt(2)*a^4*c^5*d^16*e^4 + 10*sqrt(2)*a^5*c^4*d^12*e
^8 + 10*sqrt(2)*a^6*c^3*d^8*e^12 + 5*sqrt(2)*a^7*c^2*d^4*e^16 + sqrt(2)*a^8
*c*e^20) - 3/2*(13*c^2*d^6*e^11 - 3*a*c*d^2*e^15)*log(abs(c*x^4 + a))/(c^5*
d^20 + 5*a*c^4*d^16*e^4 + 10*a^2*c^3*d^12*e^8 + 10*a^3*c^2*d^8*e^12 + 5*a^4
*c*d^4*e^16 + a^5*e^20) + 6*(13*c^2*d^6*e^12 - 3*a*c*d^2*e^16)*log(abs(x*e
+ d))/(c^5*d^20*e + 5*a*c^4*d^16*e^5 + 10*a^2*c^3*d^12*e^9 + 10*a^3*c^2*d^8
*e^13 + 5*a^4*c*d^4*e^17 + a^5*e^21) + 1/32*(30*c^5*d^11*x^9*e^4 + 42*c^5*d
^12*x^8*e^3 + c^5*d^13*x^7*e^2 - 4*c^5*d^14*x^6*e + 7*c^5*d^15*x^5 + 204*a*
c^4*d^7*x^9*e^8 + 294*a*c^4*d^8*x^8*e^7 + 19*a*c^4*d^9*x^7*e^6 - 28*a*c^4*d
^10*x^6*e^5 + 97*a*c^4*d^11*x^5*e^4 + 78*a*c^4*d^12*x^4*e^3 + 5*a*c^4*d^13*
x^3*e^2 - 8*a*c^4*d^14*x^2*e + 11*a*c^4*d^15*x - 594*a^2*c^3*d^3*x^9*e^12 -
 546*a^2*c^3*d^4*x^8*e^11 + 35*a^2*c^3*d^5*x^7*e^10 - 44*a^2*c^3*d^6*x^6*e^
9 + 461*a^2*c^3*d^7*x^5*e^8 + 586*a^2*c^3*d^8*x^4*e^7 + 31*a^2*c^3*d^9*x^3*
e^6 - 40*a^2*c^3*d^10*x^2*e^5 + 79*a^2*c^3*d^11*x*e^4 + 40*a^2*c^3*d^12*e^3
 - 30*a^3*c^2*x^8*e^15 + 17*a^3*c^2*d*x^7*e^14 - 20*a^3*c^2*d^2*x^6*e^13 -
1165*a^3*c^2*d^3*x^5*e^12 - 1078*a^3*c^2*d^4*x^4*e^11 + 47*a^3*c^2*d^5*x^3*
e^10 - 56*a^3*c^2*d^6*x^2*e^9 + 269*a^3*c^2*d^7*x*e^8 + 304*a^3*c^2*d^8*e^7
 - 50*a^4*c*x^4*e^15 + 21*a^4*c*d*x^3*e^14 - 24*a^4*c*d^2*x^2*e^13 - 567*a^
4*c*d^3*x*e^12 - 520*a^4*c*d^4*e^11 - 16*a^5*e^15)/((a^2*c^4*d^16 + 4*a^3*c
^3*d^12*e^4 + 6*a^4*c^2*d^8*e^8 + 4*a^5*c*d^4*e^12 + a^6*e^16)*(c*x^5*e + c
```

```
*d*x^4 + a*x*e + a*d)^2)
```

3.415 $\int \frac{-1+x}{1-x+x^2} dx$

Optimal. Leaf size=32

$$\frac{1}{2}\log\left(x^2-x+1\right)+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 - x + x^2]/2
```

Rubi [A] time = 0.0191008, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {634, 618, 204, 628}

$$\frac{1}{2}\log\left(x^2-x+1\right)+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x)/(1 - x + x^2),x]
```

```
[Out] ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 - x + x^2]/2
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{-1+x}{1-x+x^2}\,dx &= -\left(\frac{1}{2}\int \frac{1}{1-x+x^2}\,dx\right)+\frac{1}{2}\int \frac{-1+2x}{1-x+x^2}\,dx\\
&= \frac{1}{2}\log\left(1-x+x^2\right)+\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right)\\
&= -\frac{\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}\log\left(1-x+x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.0094068, size = 33, normalized size = 1.03

$$\frac{1}{2}\log\left(x^2-x+1\right)-\frac{\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x)/(1 - x + x^2),x]
```

```
[Out] -(ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3]) + Log[1 - x + x^2]/2
```

Maple [A] time = 0.003, size = 29, normalized size = 0.9

$$\frac{\ln\left(x^2-x+1\right)}{2}-\frac{\sqrt{3}}{3}\arctan\left(\frac{(2\,x-1)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-1)/(x^2-x+1),x)
```

```
[Out] 1/2*ln(x^2-x+1)-1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Maxima [A] time = 1.46493, size = 38, normalized size = 1.19

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{2}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(x^2-x+1),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*log(x^2 - x + 1)
```

Fricas [A] time = 1.44267, size = 90, normalized size = 2.81

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{2}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(x^2-x+1),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*log(x^2 - x + 1)
```

Sympy [A] time = 0.101236, size = 34, normalized size = 1.06

$$\frac{\log\left(x^2-x+1\right)}{2}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(x**2-x+1),x)
```

```
[Out] log(x**2 - x + 1)/2 - sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/3
```

Giac [A] time = 1.11259, size = 38, normalized size = 1.19

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{2}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(x^2-x+1),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*log(x^2 - x + 1)
```

3.416 $\int \frac{-1+x^2}{1+x^3} dx$

Optimal. Leaf size=32

$$\frac{1}{2}\log\left(x^2-x+1\right)+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 - x + x^2]/2
```

Rubi [A] time = 0.029412, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1872, 634, 618, 204, 628}

$$\frac{1}{2}\log\left(x^2-x+1\right)+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^2)/(1 + x^3),x]
```

```
[Out] ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 - x + x^2]/2
```

Rule 1872

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (a/b)^(1/3)}, Dist[q^2/a
, Int[(A + C*q*x)/(q^2 - q*x + x^2), x], x]] /; EqQ[A - B*(a/b)^(1/3) + C*(
a/b)^(2/3), 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{-1+x^2}{1+x^3}\,dx &= \int \frac{-1+x}{1-x+x^2}\,dx \\
&= -\left(\frac{1}{2}\int \frac{1}{1-x+x^2}\,dx\right)+\frac{1}{2}\int \frac{-1+2x}{1-x+x^2}\,dx \\
&= \frac{1}{2}\log\left(1-x+x^2\right)+\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right) \\
&= -\frac{\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}\log\left(1-x+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.005401, size = 33, normalized size = 1.03

$$\frac{1}{2}\log\left(x^2-x+1\right)-\frac{\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^2)/(1 + x^3),x]
```

```
[Out] -(ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3]) + Log[1 - x + x^2]/2
```

Maple [A] time = 0.003, size = 29, normalized size = 0.9

$$\frac{\ln\left(x^2-x+1\right)}{2}-\frac{\sqrt{3}}{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)/(x^3+1),x)
```

```
[Out] 1/2*ln(x^2-x+1)-1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Maxima [A] time = 1.45092, size = 38, normalized size = 1.19

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{2}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^3+1),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*log(x^2 - x + 1)
```

Fricas [A] time = 1.40583, size = 90, normalized size = 2.81

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{2}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*log(x^2 - x + 1)
```

Sympy [A] time = 0.101599, size = 34, normalized size = 1.06

$$\frac{\log\left(x^2-x+1\right)}{2}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)/(x**3+1),x)
```

```
[Out] log(x**2 - x + 1)/2 - sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/3
```

Giac [A] time = 1.1297, size = 38, normalized size = 1.19

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{2}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^3+1),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*log(x^2 - x + 1)
```

3.417 $\int \frac{-4+3x}{4-2x+x^2} dx$

Optimal. Leaf size=32

$$\frac{3}{2}\log\left(x^2-2x+4\right)+\frac{\tan^{-1}\left(\frac{1-x}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] ArcTan[(1 - x)/Sqrt[3]]/Sqrt[3] + (3*Log[4 - 2*x + x^2])/2
```

Rubi [A] time = 0.0168806, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {634, 618, 204, 628}

$$\frac{3}{2}\log\left(x^2-2x+4\right)+\frac{\tan^{-1}\left(\frac{1-x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-4 + 3*x)/(4 - 2*x + x^2),x]
```

```
[Out] ArcTan[(1 - x)/Sqrt[3]]/Sqrt[3] + (3*Log[4 - 2*x + x^2])/2
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{-4+3x}{4-2x+x^2}\,dx &= \frac{3}{2}\int \frac{-2+2x}{4-2x+x^2}\,dx-\int \frac{1}{4-2x+x^2}\,dx\\ &= \frac{3}{2}\log\left(4-2x+x^2\right)+2\,\mathrm{Subst}\left(\int \frac{1}{-12-x^2}\,dx,x,-2+2x\right)\\ &= \frac{\tan^{-1}\left(\frac{1-x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{3}{2}\log\left(4-2x+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.0085422, size = 31, normalized size = 0.97

$$\frac{3}{2}\log\left(x^2-2x+4\right)-\frac{\tan^{-1}\left(\frac{x-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-4 + 3*x)/(4 - 2*x + x^2),x]
```

```
[Out] -(ArcTan[(-1 + x)/Sqrt[3]]/Sqrt[3]) + (3*Log[4 - 2*x + x^2])/2
```

Maple [A] time = 0.003, size = 29, normalized size = 0.9

$$\frac{3\,\ln\left(x^2-2\,x+4\right)}{2}-\frac{\sqrt{3}}{3}\arctan\left(\frac{(2\,x-2)\,\sqrt{3}}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4+3*x)/(x^2-2*x+4),x)
```

```
[Out] 3/2*ln(x^2-2*x+4)-1/3*3^(1/2)*arctan(1/6*(2*x-2)*3^(1/2))
```

Maxima [A] time = 1.45734, size = 35, normalized size = 1.09

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(x-1)\right)+\frac{3}{2}\log\left(x^2-2x+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4+3*x)/(x^2-2*x+4),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(x - 1)) + 3/2*log(x^2 - 2*x + 4)
```

Fricas [A] time = 0.985448, size = 90, normalized size = 2.81

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(x-1)\right)+\frac{3}{2}\log\left(x^2-2x+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4+3*x)/(x^2-2*x+4),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(x - 1)) + 3/2*log(x^2 - 2*x + 4)
```

Sympy [A] time = 0.103575, size = 36, normalized size = 1.12

$$\frac{3\log\left(x^2-2x+4\right)}{2}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4+3*x)/(x**2-2*x+4),x)
```

```
[Out] 3*log(x**2 - 2*x + 4)/2 - sqrt(3)*atan(sqrt(3)*x/3 - sqrt(3)/3)/3
```

Giac [A] time = 1.1138, size = 35, normalized size = 1.09

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(x-1)\right)+\frac{3}{2}\log\left(x^2-2x+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4+3*x)/(x^2-2*x+4),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(x - 1)) + 3/2*log(x^2 - 2*x + 4)
```

3.418 $\int \frac{-8+2x+3x^2}{8+x^3}\,dx$

Optimal. Leaf size=32

$$\frac{3}{2}\log\left(x^2-2x+4\right)+\frac{\tan^{-1}\left(\frac{1-x}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] ArcTan[(1 - x)/Sqrt[3]]/Sqrt[3] + (3*Log[4 - 2*x + x^2])/2
```

Rubi [A] time = 0.035154, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1872, 634, 618, 204, 628}

$$\frac{3}{2}\log\left(x^2-2x+4\right)+\frac{\tan^{-1}\left(\frac{1-x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-8 + 2*x + 3*x^2)/(8 + x^3),x]
```

```
[Out] ArcTan[(1 - x)/Sqrt[3]]/Sqrt[3] + (3*Log[4 - 2*x + x^2])/2
```

Rule 1872

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (a/b)^(1/3)}, Dist[q^2/a
, Int[(A + C*q*x)/(q^2 - q*x + x^2), x], x]] /; EqQ[A - B*(a/b)^(1/3) + C*(
a/b)^(2/3), 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{-8+2x+3x^2}{8+x^3}\,dx &= \frac{1}{2}\int \frac{-8+6x}{4-2x+x^2}\,dx \\
&= \frac{3}{2}\int \frac{-2+2x}{4-2x+x^2}\,dx - \int \frac{1}{4-2x+x^2}\,dx \\
&= \frac{3}{2}\log\left(4-2x+x^2\right) + 2\,\mathrm{Subst}\left(\int \frac{1}{-12-x^2}\,dx, x, -2+2x\right) \\
&= \frac{\tan^{-1}\left(\frac{1-x}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{3}{2}\log\left(4-2x+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.0050206, size = 31, normalized size = 0.97

$$\frac{3}{2}\log\left(x^2-2x+4\right) - \frac{\tan^{-1}\left(\frac{x-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-8 + 2*x + 3*x^2)/(8 + x^3),x]
```

```
[Out] -(ArcTan[(-1 + x)/Sqrt[3]]/Sqrt[3]) + (3*Log[4 - 2*x + x^2])/2
```

Maple [A] time = 0.001, size = 29, normalized size = 0.9

$$\frac{3\,\ln\left(x^{2}-2\,x+4\right)}{2}-\frac{\sqrt{3}}{3}\arctan\left(\frac{\left(2\,x-2\right)\sqrt{3}}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2*x-8)/(x^3+8),x)
```

```
[Out] 3/2*ln(x^2-2*x+4)-1/3*3^(1/2)*arctan(1/6*(2*x-2)*3^(1/2))
```

Maxima [A] time = 1.46215, size = 35, normalized size = 1.09

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(x-1)\right)+\frac{3}{2}\log\left(x^{2}-2\,x+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2*x-8)/(x^3+8),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(x - 1)) + 3/2*log(x^2 - 2*x + 4)
```

Fricas [A] time = 0.976078, size = 90, normalized size = 2.81

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(x-1)\right)+\frac{3}{2}\log\left(x^{2}-2\,x+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2*x-8)/(x^3+8),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(x - 1)) + 3/2*log(x^2 - 2*x + 4)
```

Sympy [A] time = 0.10787, size = 36, normalized size = 1.12

$$\frac{3\log\left(x^{2}-2x+4\right)}{2}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2*x-8)/(x**3+8),x)
```

```
[Out] 3*log(x**2 - 2*x + 4)/2 - sqrt(3)*atan(sqrt(3)*x/3 - sqrt(3)/3)/3
```

Giac [A] time = 1.13532, size = 35, normalized size = 1.09

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(x-1)\right)+\frac{3}{2}\log\left(x^2-2x+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2*x-8)/(x^3+8),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(x - 1)) + 3/2*log(x^2 - 2*x + 4)
```

3.419 $\int \frac{2+x}{-1+2x+x^2} dx$

Optimal. Leaf size=45

$$\frac{1}{4}\left(2+\sqrt{2}\right)\log\left(x-\sqrt{2}+1\right)+\frac{1}{4}\left(2-\sqrt{2}\right)\log\left(x+\sqrt{2}+1\right)$$

```
[Out] ((2 + Sqrt[2])*Log[1 - Sqrt[2] + x])/4 + ((2 - Sqrt[2])*Log[1 + Sqrt[2] + x
])/4
```

Rubi [A] time = 0.0119036, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {632, 31}

$$\frac{1}{4}\left(2+\sqrt{2}\right)\log\left(x-\sqrt{2}+1\right)+\frac{1}{4}\left(2-\sqrt{2}\right)\log\left(x+\sqrt{2}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x)/(-1 + 2*x + x^2),x]
```

```
[Out] ((2 + Sqrt[2])*Log[1 - Sqrt[2] + x])/4 + ((2 - Sqrt[2])*Log[1 + Sqrt[2] + x
])/4
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{2+x}{-1+2x+x^2}\,dx = -\left(\frac{1}{4}\left(-2+\sqrt{2}\right)\int \frac{1}{1+\sqrt{2}+x}\,dx\right) + \frac{1}{4}\left(2+\sqrt{2}\right)\int \frac{1}{1-\sqrt{2}+x}\,dx$$
$$= \frac{1}{4}\left(2+\sqrt{2}\right)\log\left(1-\sqrt{2}+x\right) + \frac{1}{4}\left(2-\sqrt{2}\right)\log\left(1+\sqrt{2}+x\right)$$

Mathematica [A] time = 0.0202686, size = 42, normalized size = 0.93

$$\frac{1}{4}\left(\left(2+\sqrt{2}\right)\log\left(-x+\sqrt{2}-1\right)-\left(\sqrt{2}-2\right)\log\left(x+\sqrt{2}+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x)/(-1 + 2*x + x^2),x]
```

```
[Out] ((2 + Sqrt[2])*Log[-1 + Sqrt[2] - x] - (-2 + Sqrt[2])*Log[1 + Sqrt[2] + x])
/4
```

Maple [A] time = 0.003, size = 29, normalized size = 0.6

$$\frac{\ln\left(x^2+2x-1\right)}{2}-\frac{\sqrt{2}}{2}\mathrm{Artanh}\left(\frac{(2+2x)\sqrt{2}}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+x)/(x^2+2*x-1),x)
```

```
[Out] 1/2*ln(x^2+2*x-1)-1/2*2^(1/2)*arctanh(1/4*(2+2*x)*2^(1/2))
```

Maxima [A] time = 1.48366, size = 47, normalized size = 1.04

$$\frac{1}{4}\sqrt{2}\log\left(\frac{x-\sqrt{2}+1}{x+\sqrt{2}+1}\right)+\frac{1}{2}\log\left(x^2+2x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x^2+2*x-1),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*log((x - sqrt(2) + 1)/(x + sqrt(2) + 1)) + 1/2*log(x^2 + 2*x -
1)
```

Fricas [A] time = 0.982831, size = 128, normalized size = 2.84

$$\frac{1}{4}\sqrt{2}\log\left(\frac{x^2-2\sqrt{2}(x+1)+2x+3}{x^2+2x-1}\right)+\frac{1}{2}\log\left(x^2+2x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x^2+2*x-1),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log((x^2 - 2*sqrt(2)*(x + 1) + 2*x + 3)/(x^2 + 2*x - 1)) + 1/2*
log(x^2 + 2*x - 1)
```

Sympy [A] time = 0.100667, size = 39, normalized size = 0.87

$$\left(\frac{1}{2}-\frac{\sqrt{2}}{4}\right)\log\left(x+1+\sqrt{2}\right)+\left(\frac{\sqrt{2}}{4}+\frac{1}{2}\right)\log\left(x-\sqrt{2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x**2+2*x-1),x)
```

```
[Out] (1/2 - sqrt(2)/4)*log(x + 1 + sqrt(2)) + (sqrt(2)/4 + 1/2)*log(x - sqrt(2)
+ 1)
```

Giac [A] time = 1.1336, size = 59, normalized size = 1.31

$$\frac{1}{4}\sqrt{2}\log\left(\frac{\left|2x-2\sqrt{2}+2\right|}{\left|2x+2\sqrt{2}+2\right|}\right)+\frac{1}{2}\log\left(\left|x^2+2x-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x^2+2*x-1),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*log(abs(2*x - 2*sqrt(2) + 2)/abs(2*x + 2*sqrt(2) + 2)) + 1/2*lo
g(abs(x^2 + 2*x - 1))
```

3.420 $\int \frac{-4+x^2}{2-5x+x^3}\,dx$

Optimal. Leaf size=45

$$\frac{1}{4}\left(2+\sqrt{2}\right)\log\left(x-\sqrt{2}+1\right)+\frac{1}{4}\left(2-\sqrt{2}\right)\log\left(x+\sqrt{2}+1\right)$$

```
[Out] ((2 + Sqrt[2])*Log[1 - Sqrt[2] + x])/4 + ((2 - Sqrt[2])*Log[1 + Sqrt[2] + x
])/4
```

Rubi [A] time = 0.0209227, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2055, 632, 31}

$$\frac{1}{4}\left(2+\sqrt{2}\right)\log\left(x-\sqrt{2}+1\right)+\frac{1}{4}\left(2-\sqrt{2}\right)\log\left(x+\sqrt{2}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-4 + x^2)/(2 - 5*x + x^3),x]
```

```
[Out] ((2 + Sqrt[2])*Log[1 - Sqrt[2] + x])/4 + ((2 - Sqrt[2])*Log[1 + Sqrt[2] + x
])/4
```

Rule 2055

```
Int[(u_.)*(P_)*(Q_)^(q_), x_Symbol] :> Module[{gcd = PolyGCD[P, Q, x]}, Int
[u*gcd^(q + 1)*PolynomialQuotient[P, gcd, x]*PolynomialQuotient[Q, gcd, x]^
q, x] /; NeQ[gcd, 1]] /; ILtQ[q, 0] && PolyQ[P, x] && PolyQ[Q, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{-4+x^2}{2-5x+x^3}\,dx=\int \frac{2+x}{-1+2x+x^2}\,dx$$
$$=-\left(\frac{1}{4}\left(-2+\sqrt{2}\right)\int \frac{1}{1+\sqrt{2}+x}\,dx\right)+\frac{1}{4}\left(2+\sqrt{2}\right)\int \frac{1}{1-\sqrt{2}+x}\,dx$$
$$=\frac{1}{4}\left(2+\sqrt{2}\right)\log\left(1-\sqrt{2}+x\right)+\frac{1}{4}\left(2-\sqrt{2}\right)\log\left(1+\sqrt{2}+x\right)$$

Mathematica [A] time = 0.0051534, size = 42, normalized size = 0.93

$$\frac{1}{4}\left(\left(2+\sqrt{2}\right)\log\left(-x+\sqrt{2}-1\right)-\left(\sqrt{2}-2\right)\log\left(x+\sqrt{2}+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-4 + x^2)/(2 - 5*x + x^3),x]
```

```
[Out] ((2 + Sqrt[2])*Log[-1 + Sqrt[2] - x] - (-2 + Sqrt[2])*Log[1 + Sqrt[2] + x])
/4
```

Maple [A] time = 0.003, size = 29, normalized size = 0.6

$$\frac{\ln\left(x^2+2x-1\right)}{2}-\frac{\sqrt{2}}{2}\mathrm{Artanh}\left(\frac{(2+2x)\sqrt{2}}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-4)/(x^3-5*x+2),x)
```

```
[Out] 1/2*ln(x^2+2*x-1)-1/2*2^(1/2)*arctanh(1/4*(2+2*x)*2^(1/2))
```

Maxima [A] time = 1.45851, size = 47, normalized size = 1.04

$$\frac{1}{4}\sqrt{2}\log\left(\frac{x-\sqrt{2}+1}{x+\sqrt{2}+1}\right)+\frac{1}{2}\log\left(x^2+2x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-4)/(x^3-5*x+2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*log((x - sqrt(2) + 1)/(x + sqrt(2) + 1)) + 1/2*log(x^2 + 2*x -
1)
```

Fricas [A] time = 0.959254, size = 128, normalized size = 2.84

$$\frac{1}{4}\sqrt{2}\log\left(\frac{x^2-2\sqrt{2}(x+1)+2x+3}{x^2+2x-1}\right)+\frac{1}{2}\log\left(x^2+2x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-4)/(x^3-5*x+2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log((x^2 - 2*sqrt(2)*(x + 1) + 2*x + 3)/(x^2 + 2*x - 1)) + 1/2*
log(x^2 + 2*x - 1)
```

Sympy [A] time = 0.108295, size = 39, normalized size = 0.87

$$\left(\frac{1}{2}-\frac{\sqrt{2}}{4}\right)\log\left(x+1+\sqrt{2}\right)+\left(\frac{\sqrt{2}}{4}+\frac{1}{2}\right)\log\left(x-\sqrt{2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-4)/(x**3-5*x+2),x)
```

```
[Out] (1/2 - sqrt(2)/4)*log(x + 1 + sqrt(2)) + (sqrt(2)/4 + 1/2)*log(x - sqrt(2)
+ 1)
```

Giac [A] time = 1.11646, size = 59, normalized size = 1.31

$$\frac{1}{4}\sqrt{2}\log\left(\frac{\left|2x-2\sqrt{2}+2\right|}{\left|2x+2\sqrt{2}+2\right|}\right)+\frac{1}{2}\log\left(\left|x^2+2x-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-4)/(x^3-5*x+2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*log(abs(2*x - 2*sqrt(2) + 2)/abs(2*x + 2*sqrt(2) + 2)) + 1/2*lo
g(abs(x^2 + 2*x - 1))
```

3.421 $\int \frac{2}{-1+4x^2} dx$

Optimal. Leaf size=6

$$-\tanh^{-1}(2x)$$

```
[Out] -ArcTanh[2*x]
```

Rubi [A] time = 0.0024724, antiderivative size = 6, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {12, 207}

$$-\tanh^{-1}(2x)$$

Antiderivative was successfully verified.

```
[In] Int[2/(-1 + 4*x^2),x]
```

```
[Out] -ArcTanh[2*x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{2}{-1+4x^2} dx &= 2\int \frac{1}{-1+4x^2} dx\\ &= -\tanh^{-1}(2x)\end{aligned}$$

Mathematica [B] time = 0.0029948, size = 23, normalized size = 3.83

$$2\left(\frac{1}{4}\log(1-2x)-\frac{1}{4}\log(2x+1)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[2/(-1 + 4*x^2),x]
```

```
[Out] 2*(Log[1 - 2*x]/4 - Log[1 + 2*x]/4)
```

Maple [B] time = 0.004, size = 18, normalized size = 3.

$$\frac{\ln(2x-1)}{2}-\frac{\ln(1+2x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2/(4*x^2-1),x)
```

```
[Out] 1/2*ln(2*x-1)-1/2*ln(1+2*x)
```

Maxima [B] time = 0.957874, size = 23, normalized size = 3.83

$$-\frac{1}{2}\log(2x+1)+\frac{1}{2}\log(2x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/(4*x^2-1),x, algorithm="maxima")
```

```
[Out] -1/2*log(2*x + 1) + 1/2*log(2*x - 1)
```

Fricas [B] time = 0.96732, size = 51, normalized size = 8.5

$$-\frac{1}{2}\log(2x+1)+\frac{1}{2}\log(2x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/(4*x^2-1),x, algorithm="fricas")
```

```
[Out] -1/2*log(2*x + 1) + 1/2*log(2*x - 1)
```

Sympy [B] time = 0.0836, size = 15, normalized size = 2.5

$$\frac{\log\left(x-\frac{1}{2}\right)}{2}-\frac{\log\left(x+\frac{1}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/(4*x**2-1),x)
```

```
[Out] log(x - 1/2)/2 - log(x + 1/2)/2
```

Giac [B] time = 1.13055, size = 20, normalized size = 3.33

$$-\frac{1}{2}\log\left(\left|x+\frac{1}{2}\right|\right)+\frac{1}{2}\log\left(\left|x-\frac{1}{2}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/(4*x^2-1),x, algorithm="giac")
```

```
[Out] -1/2*log(abs(x + 1/2)) + 1/2*log(abs(x - 1/2))
```

3.422 $\int \left(\frac{1}{-1+2x}-\frac{1}{1+2x}\right) dx$

Optimal. Leaf size=21

$$\frac{1}{2}\log(1-2x)-\frac{1}{2}\log(2x+1)$$

```
[Out] Log[1 - 2*x]/2 - Log[1 + 2*x]/2
```

Rubi [A] time = 0.0035305, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\frac{1}{2}\log(1-2x)-\frac{1}{2}\log(2x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 2*x)^(-1) - (1 + 2*x)^(-1),x]
```

```
[Out] Log[1 - 2*x]/2 - Log[1 + 2*x]/2
```

Rubi steps

$$\int \left(\frac{1}{-1+2x}-\frac{1}{1+2x}\right) dx=\frac{1}{2}\log(1-2x)-\frac{1}{2}\log(1+2x)$$

Mathematica [A] time = 0.0024301, size = 23, normalized size = 1.1

$$2\left(\frac{1}{4}\log(1-2x)-\frac{1}{4}\log(2x+1)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + 2*x)^(-1) - (1 + 2*x)^(-1),x]
```

```
[Out] 2*(Log[1 - 2*x]/4 - Log[1 + 2*x]/4)
```

Maple [A] time = 0.003, size = 18, normalized size = 0.9

$$\frac{\ln(2x-1)}{2}-\frac{\ln(1+2x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x-1)-1/(1+2*x),x)
```

```
[Out] 1/2*ln(2*x-1)-1/2*ln(1+2*x)
```

Maxima [A] time = 0.964697, size = 23, normalized size = 1.1

$$-\frac{1}{2}\log(2x+1)+\frac{1}{2}\log(2x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+2*x)-1/(1+2*x),x, algorithm="maxima")
```

```
[Out] -1/2*log(2*x + 1) + 1/2*log(2*x - 1)
```

Fricas [A] time = 0.964978, size = 51, normalized size = 2.43

$$-\frac{1}{2}\log(2x+1)+\frac{1}{2}\log(2x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+2*x)-1/(1+2*x),x, algorithm="fricas")
```

```
[Out] -1/2*log(2*x + 1) + 1/2*log(2*x - 1)
```

Sympy [A] time = 0.089191, size = 15, normalized size = 0.71

$$\frac{\log\left(x-\frac{1}{2}\right)}{2}-\frac{\log\left(x+\frac{1}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+2*x)-1/(1+2*x),x)
```

```
[Out] log(x - 1/2)/2 - log(x + 1/2)/2
```

Giac [A] time = 1.11539, size = 26, normalized size = 1.24

$$-\frac{1}{2}\log\left(|2x+1|\right)+\frac{1}{2}\log\left(|2x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+2*x)-1/(1+2*x),x, algorithm="giac")
```

```
[Out] -1/2*log(abs(2*x + 1)) + 1/2*log(abs(2*x - 1))
```

3.423 $\int \frac{x}{(1-x^2)^5} \, dx$

Optimal. Leaf size=13

$$\frac{1}{8\left(1-x^2\right)^4}$$

```
[Out] 1/(8*(1 - x^2)^4)
```

Rubi [A] time = 0.0021836, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {261}

$$\frac{1}{8\left(1-x^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Int[x/(1 - x^2)^5,x]

[Out] 1/(8*(1 - x^2)^4)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{\left(1-x^2\right)^5} \, dx = \frac{1}{8\left(1-x^2\right)^4}$$

Mathematica [A] time = 0.0024871, size = 11, normalized size = 0.85

$$\frac{1}{8\left(x^2-1\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(1 - x^2)^5,x]
```

```
[Out] 1/(8*(-1 + x^2)^4)
```

Maple [A] time = 0.002, size = 10, normalized size = 0.8

$$\frac{1}{8\left(x^{2}-1\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^2+1)^5,x)
```

```
[Out] 1/8/(x^2-1)^4
```

Maxima [A] time = 0.95232, size = 12, normalized size = 0.92

$$\frac{1}{8\left(x^{2}-1\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+1)^5,x, algorithm="maxima")
```

```
[Out] 1/8/(x^2 - 1)^4
```

Fricas [B] time = 0.923981, size = 53, normalized size = 4.08

$$\frac{1}{8\left(x^{8}-4x^{6}+6x^{4}-4x^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+1)^5,x, algorithm="fricas")
```

```
[Out] 1/8/(x^8 - 4*x^6 + 6*x^4 - 4*x^2 + 1)
```

Sympy [B] time = 0.120779, size = 22, normalized size = 1.69

$$\frac{1}{8x^8 - 32x^6 + 48x^4 - 32x^2 + 8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**2+1)**5,x)
```

```
[Out] 1/(8*x**8 - 32*x**6 + 48*x**4 - 32*x**2 + 8)
```

Giac [A] time = 1.12121, size = 12, normalized size = 0.92

$$\frac{1}{8\left(x^2 - 1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+1)^5,x, algorithm="giac")
```

```
[Out] 1/8/(x^2 - 1)^4
```

3.424 $\int \left(-\frac{1}{32(-1+x)^5}+\frac{3}{64(-1+x)^4}-\frac{5}{128(-1+x)^3}+\frac{5}{256(-1+x)^2}-\frac{1}{32(1+x)^5}-\right.$

Optimal. Leaf size=13

$$\frac{1}{8\left(1-x^2\right)^4}$$

[Out] 1/(8*(1 - x^2)^4)

Rubi [B] time = 0.0118387, antiderivative size = 81, normalized size of antiderivative = 6.23, number of steps used = 1, number of rules used = 0, integrand size = 73, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\frac{5}{256(x+1)}+\frac{5}{256(x+1)^2}+\frac{1}{64(x+1)^3}+\frac{1}{128(x+1)^4}+\frac{5}{256(1-x)}+\frac{5}{256(1-x)^2}+\frac{1}{64(1-x)^3}+\frac{1}{128(1-x)^4}$$

Antiderivative was successfully verified.

```
[In] Int[-1/(32*(-1 + x)^5) + 3/(64*(-1 + x)^4) - 5/(128*(-1 + x)^3) + 5/(256*(-
1 + x)^2) - 1/(32*(1 + x)^5) - 3/(64*(1 + x)^4) - 5/(128*(1 + x)^3) - 5/(25
6*(1 + x)^2),x]
```

```
[Out] 1/(128*(1 - x)^4) + 1/(64*(1 - x)^3) + 5/(256*(1 - x)^2) + 5/(256*(1 - x))
+ 1/(128*(1 + x)^4) + 1/(64*(1 + x)^3) + 5/(256*(1 + x)^2) + 5/(256*(1 + x)
)
```

Rubi steps

$$\int \left(-\frac{1}{32(-1+x)^5}+\frac{3}{64(-1+x)^4}-\frac{5}{128(-1+x)^3}+\frac{5}{256(-1+x)^2}-\frac{1}{32(1+x)^5}-\frac{3}{64(1+x)^4}-\frac{5}{128(1+x)^3}-\frac{5}{256(1+x\right.}$$

Mathematica [A] time = 0.0015173, size = 11, normalized size = 0.85

$$\frac{1}{8\left(x^2-1\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[-1/(32*(-1 + x)^5) + 3/(64*(-1 + x)^4) - 5/(128*(-1 + x)^3) + 5/(
256*(-1 + x)^2) - 1/(32*(1 + x)^5) - 3/(64*(1 + x)^4) - 5/(128*(1 + x)^3) -
 5/(256*(1 + x)^2),x]
```

```
[Out] 1/(8*(-1 + x^2)^4)
```

Maple [B] time = 0.003, size = 58, normalized size = 4.5

$$\frac{1}{128\,(x-1)^4}-\frac{1}{64\,(x-1)^3}+\frac{5}{256\,(x-1)^2}-\frac{5}{256x-256}+\frac{1}{128\,(1+x)^4}+\frac{1}{64\,(1+x)^3}+\frac{5}{256\,(1+x)^2}+\frac{5}{256+256x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/32/(x-1)^5+3/64/(x-1)^4-5/128/(x-1)^3+5/256/(x-1)^2-1/32/(1+x)^5-3/6
4/(1+x)^4-5/128/(1+x)^3-5/256/(1+x)^2,x)
```

```
[Out] 1/128/(x-1)^4-1/64/(x-1)^3+5/256/(x-1)^2-5/256/(x-1)+1/128/(1+x)^4+1/64/(1+
x)^3+5/256/(1+x)^2+5/256/(1+x)
```

Maxima [B] time = 0.965265, size = 77, normalized size = 5.92

$$\frac{5}{256\,(x+1)}-\frac{5}{256\,(x-1)}+\frac{5}{256\,(x+1)^2}+\frac{5}{256\,(x-1)^2}+\frac{1}{64\,(x+1)^3}-\frac{1}{64\,(x-1)^3}+\frac{1}{128\,(x+1)^4}+\frac{1}{128\,(x-1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/32/(-1+x)^5+3/64/(-1+x)^4-5/128/(-1+x)^3+5/256/(-1+x)^2-1/32/(
1+x)^5-3/64/(1+x)^4-5/128/(1+x)^3-5/256/(1+x)^2,x, algorithm="maxima")
```

```
[Out] 5/256/(x + 1) - 5/256/(x - 1) + 5/256/(x + 1)^2 + 5/256/(x - 1)^2 + 1/64/(x
 + 1)^3 - 1/64/(x - 1)^3 + 1/128/(x + 1)^4 + 1/128/(x - 1)^4
```

Fricas [B] time = 1.01074, size = 53, normalized size = 4.08

$$\frac{1}{8\left(x^8-4x^6+6x^4-4x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/32/(-1+x)^5+3/64/(-1+x)^4-5/128/(-1+x)^3+5/256/(-1+x)^2-1/32/(
1+x)^5-3/64/(1+x)^4-5/128/(1+x)^3-5/256/(1+x)^2,x, algorithm="fricas")
```

```
[Out] 1/8/(x^8 - 4*x^6 + 6*x^4 - 4*x^2 + 1)
```

Sympy [B] time = 0.276525, size = 22, normalized size = 1.69

$$\frac{1}{8x^8 - 32x^6 + 48x^4 - 32x^2 + 8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/32/(-1+x)**5+3/64/(-1+x)**4-5/128/(-1+x)**3+5/256/(-1+x)**2-1/
32/(1+x)**5-3/64/(1+x)**4-5/128/(1+x)**3-5/256/(1+x)**2,x)
```

```
[Out] 1/(8*x**8 - 32*x**6 + 48*x**4 - 32*x**2 + 8)
```

Giac [B] time = 1.10273, size = 77, normalized size = 5.92

$$\frac{5}{256\,(x+1)} - \frac{5}{256\,(x-1)} + \frac{5}{256\,(x+1)^2} + \frac{5}{256\,(x-1)^2} + \frac{1}{64\,(x+1)^3} - \frac{1}{64\,(x-1)^3} + \frac{1}{128\,(x+1)^4} + \frac{1}{128\,(x-1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/32/(-1+x)^5+3/64/(-1+x)^4-5/128/(-1+x)^3+5/256/(-1+x)^2-1/32/(
1+x)^5-3/64/(1+x)^4-5/128/(1+x)^3-5/256/(1+x)^2,x, algorithm="giac")
```

```
[Out] 5/256/(x + 1) - 5/256/(x - 1) + 5/256/(x + 1)^2 + 5/256/(x - 1)^2 + 1/64/(x
 + 1)^3 - 1/64/(x - 1)^3 + 1/128/(x + 1)^4 + 1/128/(x - 1)^4
```

3.425 $\int \frac{1+x^6}{-1+x^6} \, dx$

Optimal. Leaf size=69

$$\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{6}\log\left(x^2+x+1\right)+x+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{2}{3}\tanh^{-1}(x)$$

```
[Out] x + ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] - ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] -
 (2*ArcTanh[x])/3 + Log[1 - x + x^2]/6 - Log[1 + x + x^2]/6
```

Rubi [A] time = 0.11063, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {388, 210, 634, 618, 204, 628, 206}

$$\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{6}\log\left(x^2+x+1\right)+x+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{2}{3}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^6)/(-1 + x^6),x]
```

```
[Out] x + ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] - ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] -
 (2*ArcTanh[x])/3 + Log[1 - x + x^2]/6 - Log[1 + x + x^2]/6
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1+x^6}{-1+x^6}\,dx &= x+2\int \frac{1}{-1+x^6}\,dx\\
&= x-\frac{2}{3}\int \frac{1}{1-x^2}\,dx-\frac{2}{3}\int \frac{1-\frac{x}{2}}{1-x+x^2}\,dx-\frac{2}{3}\int \frac{1+\frac{x}{2}}{1+x+x^2}\,dx\\
&= x-\frac{2}{3}\tanh^{-1}(x)+\frac{1}{6}\int \frac{-1+2x}{1-x+x^2}\,dx-\frac{1}{6}\int \frac{1+2x}{1+x+x^2}\,dx-\frac{1}{2}\int \frac{1}{1-x+x^2}\,dx-\frac{1}{2}\int \frac{1}{1+x+x^2}\,dx\\
&= x-\frac{2}{3}\tanh^{-1}(x)+\frac{1}{6}\log\left(1-x+x^2\right)-\frac{1}{6}\log\left(1+x+x^2\right)+\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right)+\text{Subst}\\
&= x-\frac{\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{2}{3}\tanh^{-1}(x)+\frac{1}{6}\log\left(1-x+x^2\right)-\frac{1}{6}\log\left(1+x+x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.0135999, size = 78, normalized size = 1.13

$$
\frac{1}{6}\left(\log\left(x^2-x+1\right)-\log\left(x^2+x+1\right)+6x+2\log(1-x)-2\log(x+1)-2\sqrt{3}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)-2\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^6)/(-1 + x^6),x]
```

```
[Out] (6*x - 2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sq
rt[3]] + 2*Log[1 - x] - 2*Log[1 + x] + Log[1 - x + x^2] - Log[1 + x + x^2])
/6
```

Maple [A] time = 0.01, size = 67, normalized size = 1.

$$
x+\frac{\ln(x-1)}{3}-\frac{\ln\left(x^2+x+1\right)}{6}-\frac{\sqrt{3}}{3}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)+\frac{\ln\left(x^2-x+1\right)}{6}-\frac{\sqrt{3}}{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)-\frac{\ln(1+x)}{3}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6+1)/(x^6-1),x)
```

```
[Out] x+1/3*ln(x-1)-1/6*ln(x^2+x+1)-1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)+1/6*l
n(x^2-x+1)-1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))-1/3*ln(1+x)
```

Maxima [A] time = 1.51663, size = 89, normalized size = 1.29

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+x-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{3}\log($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)/(x^6-1),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/3*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) + x - 1/6*log(x^2 + x + 1) + 1/6*log(x^2 - x + 1) - 1/3*log(x +
 1) + 1/3*log(x - 1)
```

Fricas [A] time = 1.02847, size = 232, normalized size = 3.36

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+x-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{3}\log($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)/(x^6-1),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/3*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) + x - 1/6*log(x^2 + x + 1) + 1/6*log(x^2 - x + 1) - 1/3*log(x +
 1) + 1/3*log(x - 1)
```

Sympy [A] time = 0.221452, size = 85, normalized size = 1.23

$$x+\frac{\log(x-1)}{3}-\frac{\log(x+1)}{3}+\frac{\log\left(x^2-x+1\right)}{6}-\frac{\log\left(x^2+x+1\right)}{6}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6+1)/(x**6-1),x)
```

```
[Out] x + log(x - 1)/3 - log(x + 1)/3 + log(x**2 - x + 1)/6 - log(x**2 + x + 1)/6
 - sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/3 - sqrt(3)*atan(2*sqrt(3)*x/3 +
 sqrt(3)/3)/3
```

Giac [A] time = 1.11525, size = 92, normalized size = 1.33

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+x-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{3}\log\left(|x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)/(x^6-1),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/3*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) + x - 1/6*log(x^2 + x + 1) + 1/6*log(x^2 - x + 1) - 1/3*log(abs
(x + 1)) + 1/3*log(abs(x - 1))
```

3.426 $\int \frac{\frac{1}{x^3}+x^3}{-\frac{1}{x^3}+x^3}\,dx$

Optimal. Leaf size=69

$$\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{6}\log\left(x^2+x+1\right)+x+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{2}{3}\tanh^{-1}(x)$$

```
[Out] x + ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] - ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] -
 (2*ArcTanh[x])/3 + Log[1 - x + x^2]/6 - Log[1 + x + x^2]/6
```

Rubi [A] time = 0.123517, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {1593, 1584, 388, 210, 634, 618, 204, 628, 206}

$$\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{6}\log\left(x^2+x+1\right)+x+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{2}{3}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-3) + x^3)/(-x^(-3) + x^3),x]
```

```
[Out] x + ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] - ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] -
 (2*ArcTanh[x])/3 + Log[1 - x + x^2]/6 - Log[1 + x + x^2]/6
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\frac{1}{x^3}+x^3}{-\frac{1}{x^3}+x^3}\,dx &= \int \frac{x^3\left(\frac{1}{x^3}+x^3\right)}{-1+x^6}\,dx\\
&= \int \frac{1+x^6}{-1+x^6}\,dx\\
&= x+2\int \frac{1}{-1+x^6}\,dx\\
&= x-\frac{2}{3}\int \frac{1}{1-x^2}\,dx-\frac{2}{3}\int \frac{1-\frac{x}{2}}{1-x+x^2}\,dx-\frac{2}{3}\int \frac{1+\frac{x}{2}}{1+x+x^2}\,dx\\
&= x-\frac{2}{3}\tanh^{-1}(x)+\frac{1}{6}\int \frac{-1+2x}{1-x+x^2}\,dx-\frac{1}{6}\int \frac{1+2x}{1+x+x^2}\,dx-\frac{1}{2}\int \frac{1}{1-x+x^2}\,dx-\frac{1}{2}\int \frac{1}{1+x+x^2}\,dx\\
&= x-\frac{2}{3}\tanh^{-1}(x)+\frac{1}{6}\log\left(1-x+x^2\right)-\frac{1}{6}\log\left(1+x+x^2\right)+\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right)+\text{Sub}\\
&= x-\frac{\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{2}{3}\tanh^{-1}(x)+\frac{1}{6}\log\left(1-x+x^2\right)-\frac{1}{6}\log\left(1+x+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.0056237, size = 78, normalized size = 1.13

$$\frac{1}{6}\left(\log\left(x^2-x+1\right)-\log\left(x^2+x+1\right)+6x+2\log(1-x)-2\log(x+1)-2\sqrt{3}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)-2\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-3) + x^3)/(-x^(-3) + x^3),x]
```

```
[Out] (6*x - 2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sq
rt[3]] + 2*Log[1 - x] - 2*Log[1 + x] + Log[1 - x + x^2] - Log[1 + x + x^2])
/6
```

Maple [A] time = 0.002, size = 67, normalized size = 1.

$$x+\frac{\ln(x-1)}{3}-\frac{\ln\left(x^2+x+1\right)}{6}-\frac{\sqrt{3}}{3}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)+\frac{\ln\left(x^2-x+1\right)}{6}-\frac{\sqrt{3}}{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)-\frac{\ln(1+x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/x^3+x^3)/(-1/x^3+x^3),x)
```

```
[Out] x+1/3*ln(x-1)-1/6*ln(x^2+x+1)-1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)+1/6*l
n(x^2-x+1)-1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))-1/3*ln(1+x)
```

Maxima [A] time = 1.45959, size = 89, normalized size = 1.29

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+x-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{3}\log(x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/x^3+x^3)/(-1/x^3+x^3),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/3*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) + x - 1/6*log(x^2 + x + 1) + 1/6*log(x^2 - x + 1) - 1/3*log(x +
 1) + 1/3*log(x - 1)
```

Fricas [A] time = 0.985839, size = 232, normalized size = 3.36

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+x-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{3}\log(x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/x^3+x^3)/(-1/x^3+x^3),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/3*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) + x - 1/6*log(x^2 + x + 1) + 1/6*log(x^2 - x + 1) - 1/3*log(x +
 1) + 1/3*log(x - 1)
```

Sympy [A] time = 0.226974, size = 85, normalized size = 1.23

$$x+\frac{\log(x-1)}{3}-\frac{\log(x+1)}{3}+\frac{\log\left(x^2-x+1\right)}{6}-\frac{\log\left(x^2+x+1\right)}{6}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/x**3+x**3)/(-1/x**3+x**3),x)
```

```
[Out] x + log(x - 1)/3 - log(x + 1)/3 + log(x**2 - x + 1)/6 - log(x**2 + x + 1)/6
 - sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/3 - sqrt(3)*atan(2*sqrt(3)*x/3 +
 sqrt(3)/3)/3
```

Giac [A] time = 1.10676, size = 92, normalized size = 1.33

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+x-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{3}\log($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/x^3+x^3)/(-1/x^3+x^3),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/3*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) + x - 1/6*log(x^2 + x + 1) + 1/6*log(x^2 - x + 1) - 1/3*log(abs
(x + 1)) + 1/3*log(abs(x - 1))
```

3.427 $\int \frac{-x+x^3}{6+2x} \, dx$

Optimal. Leaf size=24

$$\frac{x^3}{6}-\frac{3x^2}{4}+4x-12\log(x+3)$$

```
[Out] 4*x - (3*x^2)/4 + x^3/6 - 12*Log[3 + x]
```

Rubi [A] time = 0.0188889, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1593, 772}

$$\frac{x^3}{6}-\frac{3x^2}{4}+4x-12\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Int[(-x + x^3)/(6 + 2*x),x]

[Out] 4*x - (3*x^2)/4 + x^3/6 - 12*Log[3 + x]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{-x+x^3}{6+2x}\,dx = \int \frac{x\left(-1+x^2\right)}{6+2x}\,dx$$
$$= \int \left(4-\frac{3x}{2}+\frac{x^2}{2}-\frac{12}{3+x}\right)dx$$
$$= 4x-\frac{3x^2}{4}+\frac{x^3}{6}-12\log(3+x)$$

Mathematica [A] time = 0.0058096, size = 31, normalized size = 1.29

$$\frac{1}{2}\left(\frac{x^3}{3}-\frac{3x^2}{2}+8x-24\log(x+3)+\frac{93}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x + x^3)/(6 + 2*x),x]
```

```
[Out] (93/2 + 8*x - (3*x^2)/2 + x^3/3 - 24*Log[3 + x])/2
```

Maple [A] time = 0.003, size = 21, normalized size = 0.9

$$4x-\frac{3x^2}{4}+\frac{x^3}{6}-12\ln(3+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-x)/(6+2*x),x)
```

```
[Out] 4*x-3/4*x^2+1/6*x^3-12*ln(3+x)
```

Maxima [A] time = 1.04315, size = 27, normalized size = 1.12

$$\frac{1}{6}x^3-\frac{3}{4}x^2+4x-12\log(x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x)/(6+2*x),x, algorithm="maxima")
```

```
[Out] 1/6*x^3 - 3/4*x^2 + 4*x - 12*log(x + 3)
```

Fricas [A] time = 0.965709, size = 55, normalized size = 2.29

$$\frac{1}{6}x^3 - \frac{3}{4}x^2 + 4x - 12\log(x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x)/(6+2*x),x, algorithm="fricas")
```

```
[Out] 1/6*x^3 - 3/4*x^2 + 4*x - 12*log(x + 3)
```

Sympy [A] time = 0.072741, size = 20, normalized size = 0.83

$$\frac{x^3}{6} - \frac{3x^2}{4} + 4x - 12\log(x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-x)/(6+2*x),x)
```

```
[Out] x**3/6 - 3*x**2/4 + 4*x - 12*log(x + 3)
```

Giac [A] time = 1.15991, size = 28, normalized size = 1.17

$$\frac{1}{6}x^3 - \frac{3}{4}x^2 + 4x - 12\log(|x + 3|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x)/(6+2*x),x, algorithm="giac")
```

```
[Out] 1/6*x^3 - 3/4*x^2 + 4*x - 12*log(abs(x + 3))
```

3.428 $\int \frac{x+x^3}{-1+x} dx$

Optimal. Leaf size=26

$$\frac{x^3}{3} + \frac{x^2}{2} + 2x + 2\log(1-x)$$

```
[Out] 2*x + x^2/2 + x^3/3 + 2*Log[1 - x]
```

Rubi [A] time = 0.0163843, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1593, 772}

$$\frac{x^3}{3} + \frac{x^2}{2} + 2x + 2\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[(x + x^3)/(-1 + x),x]
```

```
[Out] 2*x + x^2/2 + x^3/3 + 2*Log[1 - x]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{x+x^3}{-1+x}\,dx = \int \frac{x\left(1+x^2\right)}{-1+x}\,dx$$
$$= \int \left(2+\frac{2}{-1+x}+x+x^2\right)dx$$
$$= 2x+\frac{x^2}{2}+\frac{x^3}{3}+2\log(1-x)$$

Mathematica [A] time = 0.0040439, size = 25, normalized size = 0.96

$$\frac{1}{6}\left(2x^3+3x^2+12x+12\log(x-1)-17\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + x^3)/(-1 + x),x]
```

```
[Out] (-17 + 12*x + 3*x^2 + 2*x^3 + 12*Log[-1 + x])/6
```

Maple [A] time = 0.002, size = 21, normalized size = 0.8

$$\frac{x^3}{3}+\frac{x^2}{2}+2x+2\ln(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x)/(x-1),x)
```

```
[Out] 1/3*x^3+1/2*x^2+2*x+2*ln(x-1)
```

Maxima [A] time = 1.01431, size = 27, normalized size = 1.04

$$\frac{1}{3}x^3+\frac{1}{2}x^2+2x+2\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x)/(-1+x),x, algorithm="maxima")
```

```
[Out] 1/3*x^3 + 1/2*x^2 + 2*x + 2*log(x - 1)
```

Fricas [A] time = 0.963695, size = 54, normalized size = 2.08

$$\frac{1}{3}x^3 + \frac{1}{2}x^2 + 2x + 2\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x)/(-1+x),x, algorithm="fricas")
```

```
[Out] 1/3*x^3 + 1/2*x^2 + 2*x + 2*log(x - 1)
```

Sympy [A] time = 0.07033, size = 19, normalized size = 0.73

$$\frac{x^3}{3} + \frac{x^2}{2} + 2x + 2\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x)/(-1+x),x)
```

```
[Out] x**3/3 + x**2/2 + 2*x + 2*log(x - 1)
```

Giac [A] time = 1.08493, size = 28, normalized size = 1.08

$$\frac{1}{3}x^3 + \frac{1}{2}x^2 + 2x + 2\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x)/(-1+x),x, algorithm="giac")
```

```
[Out] 1/3*x^3 + 1/2*x^2 + 2*x + 2*log(abs(x - 1))
```

3.429 $\int (ac+(bc+d)x)\,dx$

Optimal. Leaf size=17

$$acx+\frac{1}{2}x^2(bc+d)$$

```
[Out] a*c*x + ((b*c + d)*x^2)/2
```

Rubi [A] time = 0.0058426, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$acx+\frac{1}{2}x^2(bc+d)$$

Antiderivative was successfully verified.

```
[In] Int[a*c + (b*c + d)*x,x]
```

```
[Out] a*c*x + ((b*c + d)*x^2)/2
```

Rubi steps

$$\int (ac+(bc+d)x)\,dx=acx+\frac{1}{2}(bc+d)x^2$$

Mathematica [A] time = 0.0000349, size = 22, normalized size = 1.29

$$acx+\frac{1}{2}bcx^2+\frac{dx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[a*c + (b*c + d)*x,x]
```

```
[Out] a*c*x + (b*c*x^2)/2 + (d*x^2)/2
```

Maple [A] time = 0.002, size = 16, normalized size = 0.9

$$acx + \frac{(bc + d)\, x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a*c+(b*c+d)*x,x)
```

```
[Out] a*c*x+1/2*(b*c+d)*x^2
```

Maxima [A] time = 0.968542, size = 20, normalized size = 1.18

$$acx + \frac{1}{2}\,(bc + d)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*c+(b*c+d)*x,x, algorithm="maxima")
```

```
[Out] a*c*x + 1/2*(b*c + d)*x^2
```

Fricas [A] time = 0.901043, size = 45, normalized size = 2.65

$$\frac{1}{2}x^2cb + \frac{1}{2}x^2d + xca$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*c+(b*c+d)*x,x, algorithm="fricas")
```

```
[Out] 1/2*x^2*c*b + 1/2*x^2*d + x*c*a
```

Sympy [A] time = 0.052922, size = 15, normalized size = 0.88

$$acx + x^2\left(\frac{bc}{2} + \frac{d}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*c+(b*c+d)*x,x)
```

```
[Out] a*c*x + x**2*(b*c/2 + d/2)
```

Giac [A] time = 1.13595, size = 20, normalized size = 1.18

$$acx + \frac{1}{2}(bc + d)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*c+(b*c+d)*x,x, algorithm="giac")
```

```
[Out] a*c*x + 1/2*(b*c + d)*x^2
```

3.430 $\int (dx + c(a + bx))\,dx$

Optimal. Leaf size=24

$$\frac{c(a+bx)^2}{2b}+\frac{dx^2}{2}$$

```
[Out] (d*x^2)/2 + (c*(a + b*x)^2)/(2*b)
```

Rubi [A] time = 0.004717, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\frac{c(a+bx)^2}{2b}+\frac{dx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[d*x + c*(a + b*x),x]

[Out] (d*x^2)/2 + (c*(a + b*x)^2)/(2*b)
```

Rubi steps

$$\int (dx+c(a+bx))\,dx=\frac{dx^2}{2}+\frac{c(a+bx)^2}{2b}$$

Mathematica [A] time = 0.0011913, size = 22, normalized size = 0.92

$$acx+\frac{1}{2}bcx^2+\frac{dx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[d*x + c*(a + b*x),x]

[Out] a*c*x + (b*c*x^2)/2 + (d*x^2)/2
```

Maple [A] time = 0., size = 20, normalized size = 0.8

$$\frac{dx^2}{2} + c\left(ax + \frac{bx^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(d*x+c*(b*x+a),x)
```

```
[Out] 1/2*d*x^2+c*(a*x+1/2*b*x^2)
```

Maxima [A] time = 0.985245, size = 27, normalized size = 1.12

$$\frac{1}{2}\,dx^2 + \frac{1}{2}\left(bx^2 + 2\,ax\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d*x+c*(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*d*x^2 + 1/2*(b*x^2 + 2*a*x)*c
```

Fricas [A] time = 0.878348, size = 45, normalized size = 1.88

$$\frac{1}{2}x^2cb + \frac{1}{2}x^2d + xca$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d*x+c*(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*x^2*c*b + 1/2*x^2*d + x*c*a
```

Sympy [A] time = 0.05406, size = 15, normalized size = 0.62

$$acx + x^2\left(\frac{bc}{2} + \frac{d}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d*x+c*(b*x+a),x)
```

```
[Out] a*c*x + x**2*(b*c/2 + d/2)
```

Giac [A] time = 1.10687, size = 27, normalized size = 1.12

$$\frac{1}{2}\,dx^2 + \frac{1}{2}\left(bx^2 + 2\,ax\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d*x+c*(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*d*x^2 + 1/2*(b*x^2 + 2*a*x)*c
```

3.431 $\int \frac{4+4x}{x^2(1+x^2)} \, dx$

Optimal. Leaf size=22

$$-2\log\left(x^2+1\right)-\frac{4}{x}+4\log(x)-4\tan^{-1}(x)$$

```
[Out] -4/x - 4*ArcTan[x] + 4*Log[x] - 2*Log[1 + x^2]
```

Rubi [A] time = 0.0189116, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {801, 635, 203, 260}

$$-2\log\left(x^2+1\right)-\frac{4}{x}+4\log(x)-4\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(4 + 4*x)/(x^2*(1 + x^2)),x]
```

```
[Out] -4/x - 4*ArcTan[x] + 4*Log[x] - 2*Log[1 + x^2]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{4+4x}{x^2\left(1+x^2\right)}\,dx &= \int \left(\frac{4}{x^2}+\frac{4}{x}-\frac{4(1+x)}{1+x^2}\right)dx\\ &= -\frac{4}{x}+4\log(x)-4\int \frac{1+x}{1+x^2}\,dx\\ &= -\frac{4}{x}+4\log(x)-4\int \frac{1}{1+x^2}\,dx-4\int \frac{x}{1+x^2}\,dx\\ &= -\frac{4}{x}-4\tan^{-1}(x)+4\log(x)-2\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.0054922, size = 24, normalized size = 1.09

$$4\left(-\frac{1}{2}\log\left(x^2+1\right)-\frac{1}{x}+\log(x)-\tan^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + 4*x)/(x^2*(1 + x^2)),x]
```

```
[Out] 4*(-x^(-1) - ArcTan[x] + Log[x] - Log[1 + x^2]/2)
```

Maple [A] time = 0.006, size = 23, normalized size = 1.1

$$-4\,x^{-1}-4\,\arctan(x)+4\,\ln(x)-2\,\ln\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4+4*x)/x^2/(x^2+1),x)
```

```
[Out] -4/x-4*arctan(x)+4*ln(x)-2*ln(x^2+1)
```

Maxima [A] time = 1.48236, size = 30, normalized size = 1.36

$$-\frac{4}{x} - 4\arctan(x) - 2\log\left(x^2+1\right) + 4\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+4*x)/x^2/(x^2+1),x, algorithm="maxima")
```

```
[Out] -4/x - 4*arctan(x) - 2*log(x^2 + 1) + 4*log(x)
```

Fricas [A] time = 0.999919, size = 76, normalized size = 3.45

$$-\frac{2\left(2x\arctan(x) + x\log\left(x^2+1\right) - 2x\log(x) + 2\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+4*x)/x^2/(x^2+1),x, algorithm="fricas")
```

```
[Out] -2*(2*x*arctan(x) + x*log(x^2 + 1) - 2*x*log(x) + 2)/x
```

Sympy [A] time = 0.117294, size = 20, normalized size = 0.91

$$4\log(x) - 2\log\left(x^2+1\right) - 4\,\mathrm{atan}(x) - \frac{4}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+4*x)/x**2/(x**2+1),x)
```

```
[Out] 4*log(x) - 2*log(x**2 + 1) - 4*atan(x) - 4/x
```

Giac [A] time = 1.1062, size = 31, normalized size = 1.41

$$-\frac{4}{x} - 4\arctan(x) - 2\log\left(x^2+1\right) + 4\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+4*x)/x^2/(x^2+1),x, algorithm="giac")
```

```
[Out] -4/x - 4*arctan(x) - 2*log(x^2 + 1) + 4*log(abs(x))
```

3.432 $\int \frac{24+8x}{x(-4+x^2)} \, dx$

Optimal. Leaf size=17

$$5\log(2-x)-6\log(x)+\log(x+2)$$

```
[Out] 5*Log[2 - x] - 6*Log[x] + Log[2 + x]
```

Rubi [A] time = 0.0128238, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {801}

$$5\log(2-x)-6\log(x)+\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[(24 + 8*x)/(x*(-4 + x^2)),x]
```

```
[Out] 5*Log[2 - x] - 6*Log[x] + Log[2 + x]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{24+8x}{x\left(-4+x^2\right)} \, dx &= \int \left(\frac{5}{-2+x}-\frac{6}{x}+\frac{1}{2+x}\right) dx\\ &= 5\log(2-x)-6\log(x)+\log(2+x)\end{aligned}$$

Mathematica [A] time = 0.0044864, size = 27, normalized size = 1.59

$$8\left(\frac{5}{8}\log(2-x)-\frac{3\log(x)}{4}+\frac{1}{8}\log(x+2)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(24 + 8*x)/(x*(-4 + x^2)),x]
```

```
[Out] 8*((5*Log[2 - x])/8 - (3*Log[x])/4 + Log[2 + x]/8)
```

Maple [A] time = 0.007, size = 16, normalized size = 0.9

$$-6\ln(x) + \ln(2 + x) + 5\ln(-2 + x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((24+8*x)/x/(x^2-4),x)
```

```
[Out] -6*ln(x)+ln(2+x)+5*ln(-2+x)
```

Maxima [A] time = 0.973539, size = 20, normalized size = 1.18

$$\log(x + 2) + 5\log(x - 2) - 6\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((24+8*x)/x/(x^2-4),x, algorithm="maxima")
```

```
[Out] log(x + 2) + 5*log(x - 2) - 6*log(x)
```

Fricas [A] time = 1.01461, size = 51, normalized size = 3.

$$\log(x + 2) + 5\log(x - 2) - 6\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((24+8*x)/x/(x^2-4),x, algorithm="fricas")
```

```
[Out] log(x + 2) + 5*log(x - 2) - 6*log(x)
```

Sympy [A] time = 0.117198, size = 15, normalized size = 0.88

$$-6\log(x) + 5\log(x-2) + \log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((24+8*x)/x/(x**2-4),x)
```

```
[Out] -6*log(x) + 5*log(x - 2) + log(x + 2)
```

Giac [A] time = 1.10588, size = 24, normalized size = 1.41

$$\log(|x+2|) + 5\log(|x-2|) - 6\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((24+8*x)/x/(x^2-4),x, algorithm="giac")
```

```
[Out] log(abs(x + 2)) + 5*log(abs(x - 2)) - 6*log(abs(x))
```

3.433 $\int \frac{-1+x^2}{-2x+x^3}\,dx$

Optimal. Leaf size=19

$$\frac{1}{4}\log\left(2-x^2\right)+\frac{\log(x)}{2}$$

```
[Out] Log[x]/2 + Log[2 - x^2]/4
```

Rubi [A] time = 0.0210393, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1593, 446, 72}

$$\frac{1}{4}\log\left(2-x^2\right)+\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^2)/(-2*x + x^3),x]
```

```
[Out] Log[x]/2 + Log[2 - x^2]/4
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+x^2}{-2x+x^3}\,dx &= \int \frac{-1+x^2}{x\left(-2+x^2\right)}\,dx\\ &= \frac{1}{2}\,\text{Subst}\left(\int \frac{-1+x}{(-2+x)x}\,dx,x,x^2\right)\\ &= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{1}{2(-2+x)}+\frac{1}{2x}\right)dx,x,x^2\right)\\ &= \frac{\log(x)}{2}+\frac{1}{4}\log\left(2-x^2\right)\end{aligned}$$

Mathematica [A] time = 0.0036094, size = 19, normalized size = 1.

$$\frac{1}{4}\log\left(2-x^2\right)+\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^2)/(-2*x + x^3),x]
```

```
[Out] Log[x]/2 + Log[2 - x^2]/4
```

Maple [A] time = 0.004, size = 14, normalized size = 0.7

$$\frac{\ln(x)}{2}+\frac{\ln\left(x^2-2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)/(x^3-2*x),x)
```

```
[Out] 1/2*ln(x)+1/4*ln(x^2-2)
```

Maxima [A] time = 1.0539, size = 18, normalized size = 0.95

$$\frac{1}{4}\,\log\left(x^2-2\right)+\frac{1}{2}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^3-2*x),x, algorithm="maxima")
```

```
[Out] 1/4*log(x^2 - 2) + 1/2*log(x)
```

Fricas [A] time = 0.960073, size = 42, normalized size = 2.21

$$\frac{1}{4}\log\left(x^2-2\right)+\frac{1}{2}\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^3-2*x),x, algorithm="fricas")
```

```
[Out] 1/4*log(x^2 - 2) + 1/2*log(x)
```

Sympy [A] time = 0.087442, size = 12, normalized size = 0.63

$$\frac{\log\left(x\right)}{2}+\frac{\log\left(x^2-2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)/(x**3-2*x),x)
```

```
[Out] log(x)/2 + log(x**2 - 2)/4
```

Giac [A] time = 1.10821, size = 22, normalized size = 1.16

$$\frac{1}{4}\log\left(x^2\right)+\frac{1}{4}\log\left(\left|x^2-2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^3-2*x),x, algorithm="giac")
```

```
[Out] 1/4*log(x^2) + 1/4*log(abs(x^2 - 2))
```

3.434 $\int \frac{1+x^2}{3x+x^3} \, dx$

Optimal. Leaf size=12

$$\frac{1}{3}\log\left(x^3+3x\right)$$

```
[Out] Log[3*x + x^3]/3
```

Rubi [A] time = 0.006125, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1587}

$$\frac{1}{3}\log\left(x^3+3x\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/(3*x + x^3),x]
```

```
[Out] Log[3*x + x^3]/3
```

Rule 1587

```
Int[(Pp_)/(Qq_), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Si
mp[(Coeff[Pp, x, p]*Log[RemoveContent[Qq, x]])/(q*Coeff[Qq, x, q]), x] /; E
qQ[p, q - 1] && EqQ[Pp, Simplify[(Coeff[Pp, x, p]*D[Qq, x])/(q*Coeff[Qq, x,
 q])]]] /; PolyQ[Pp, x] && PolyQ[Qq, x]
```

Rubi steps

$$\int \frac{1+x^2}{3x+x^3} \, dx = \frac{1}{3}\log\left(3x+x^3\right)$$

Mathematica [A] time = 0.0034966, size = 17, normalized size = 1.42

$$\frac{1}{3}\log\left(x^2+3\right)+\frac{\log(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/(3*x + x^3),x]
```

```
[Out] Log[x]/3 + Log[3 + x^2]/3
```

Maple [A] time = 0.002, size = 11, normalized size = 0.9

$$\frac{\ln\left(x\left(x^2+3\right)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(x^3+3*x),x)
```

```
[Out] 1/3*ln(x*(x^2+3))
```

Maxima [A] time = 0.993105, size = 14, normalized size = 1.17

$$\frac{1}{3}\log\left(x^3+3x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^3+3*x),x, algorithm="maxima")
```

```
[Out] 1/3*log(x^3 + 3*x)
```

Fricas [A] time = 0.948328, size = 27, normalized size = 2.25

$$\frac{1}{3}\log\left(x^3+3x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^3+3*x),x, algorithm="fricas")
```

```
[Out] 1/3*log(x^3 + 3*x)
```

Sympy [A] time = 0.083129, size = 8, normalized size = 0.67

$$\frac{\log\left(x^3+3x\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x**3+3*x),x)
```

```
[Out] log(x**3 + 3*x)/3
```

Giac [A] time = 1.1117, size = 20, normalized size = 1.67

$$\frac{1}{3}\,\log\left(x^2+3\right)+\frac{1}{6}\,\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^3+3*x),x, algorithm="giac")
```

```
[Out] 1/3*log(x^2 + 3) + 1/6*log(x^2)
```

3.435 $\int \frac{a+3bx^2}{ax+bx^3}\,dx$

Optimal. Leaf size=10

$$\log\left(ax+bx^3\right)$$

```
[Out] Log[a*x + b*x^3]
```

Rubi [A] time = 0.0090229, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {1587}

$$\log\left(ax+bx^3\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + 3*b*x^2)/(a*x + b*x^3),x]

[Out] Log[a*x + b*x^3]
```

Rule 1587

```
Int[(Pp_)/(Qq_), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Si
mp[(Coeff[Pp, x, p]*Log[RemoveContent[Qq, x]])/(q*Coeff[Qq, x, q]), x] /; E
qQ[p, q - 1] && EqQ[Pp, Simplify[(Coeff[Pp, x, p]*D[Qq, x])/(q*Coeff[Qq, x,
 q])]]] /; PolyQ[Pp, x] && PolyQ[Qq, x]
```

Rubi steps

$$\int \frac{a+3bx^2}{ax+bx^3}\,dx = \log\left(ax+bx^3\right)$$

Mathematica [A] time = 0.0059126, size = 11, normalized size = 1.1

$$\log\left(a+bx^2\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + 3*b*x^2)/(a*x + b*x^3),x]
```

```
[Out] Log[x] + Log[a + b*x^2]
```

Maple [A] time = 0.002, size = 11, normalized size = 1.1

$$\ln\left(x\left(bx^2+a\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*b*x^2+a)/(b*x^3+a*x),x)
```

```
[Out] ln(x*(b*x^2+a))
```

Maxima [A] time = 0.965325, size = 14, normalized size = 1.4

$$\log\left(bx^3+ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*b*x^2+a)/(b*x^3+a*x),x, algorithm="maxima")
```

```
[Out] log(b*x^3 + a*x)
```

Fricas [A] time = 0.978201, size = 24, normalized size = 2.4

$$\log\left(bx^3+ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*b*x^2+a)/(b*x^3+a*x),x, algorithm="fricas")
```

```
[Out] log(b*x^3 + a*x)
```

Sympy [A] time = 0.27362, size = 8, normalized size = 0.8

$$\log\left(ax + bx^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*b*x**2+a)/(b*x**3+a*x),x)
```

```
[Out] log(a*x + b*x**3)
```

Giac [A] time = 1.09781, size = 22, normalized size = 2.2

$$\frac{1}{2}\log\left(x^2\right) + \log\left(\left|bx^2 + a\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*b*x^2+a)/(b*x^3+a*x),x, algorithm="giac")
```

```
[Out] 1/2*log(x^2) + log(abs(b*x^2 + a))
```

3.436 $\int \frac{-2+4x}{-x+x^3} dx$

Optimal. Leaf size=17

$$\log(1-x)+2\log(x)-3\log(x+1)$$

```
[Out] Log[1 - x] + 2*Log[x] - 3*Log[1 + x]
```

Rubi [A] time = 0.0181711, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1593, 801}

$$\log(1-x)+2\log(x)-3\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(-2 + 4*x)/(-x + x^3),x]
```

```
[Out] Log[1 - x] + 2*Log[x] - 3*Log[1 + x]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{-2+4x}{-x+x^3}\,dx &= \int \frac{-2+4x}{x\left(-1+x^2\right)}\,dx\\
&= \int \left(\frac{1}{-1+x}+\frac{2}{x}-\frac{3}{1+x}\right) dx\\
&= \log(1-x)+2\log(x)-3\log(1+x)
\end{aligned}$$

Mathematica [A] time = 0.0055575, size = 17, normalized size = 1.

$$\log(1-x)+2\log(x)-3\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 + 4*x)/(-x + x^3),x]
```

```
[Out] Log[1 - x] + 2*Log[x] - 3*Log[1 + x]
```

Maple [A] time = 0.004, size = 16, normalized size = 0.9

$$\ln(x-1)+2\ln(x)-3\ln(1+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2+4*x)/(x^3-x),x)
```

```
[Out] ln(x-1)+2*ln(x)-3*ln(1+x)
```

Maxima [A] time = 0.97238, size = 20, normalized size = 1.18

$$-3\log(x+1)+\log(x-1)+2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+4*x)/(x^3-x),x, algorithm="maxima")
```

```
[Out] -3*log(x + 1) + log(x - 1) + 2*log(x)
```

Fricas [A] time = 0.999472, size = 53, normalized size = 3.12

$$-3\log(x+1)+\log(x-1)+2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+4*x)/(x^3-x),x, algorithm="fricas")
```

```
[Out] -3*log(x + 1) + log(x - 1) + 2*log(x)
```

Sympy [A] time = 0.110978, size = 15, normalized size = 0.88

$$2\log(x) + \log(x-1) - 3\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+4*x)/(x**3-x),x)
```

```
[Out] 2*log(x) + log(x - 1) - 3*log(x + 1)
```

Giac [A] time = 1.18553, size = 24, normalized size = 1.41

$$-3\log(|x+1|) + \log(|x-1|) + 2\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+4*x)/(x^3-x),x, algorithm="giac")
```

```
[Out] -3*log(abs(x + 1)) + log(abs(x - 1)) + 2*log(abs(x))
```

3.437 $\int \frac{4+x}{4x+x^3}\,dx$

Optimal. Leaf size=23

$$-\frac{1}{2}\log\left(x^2+4\right)+\log(x)+\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)$$

```
[Out] ArcTan[x/2]/2 + Log[x] - Log[4 + x^2]/2
```

Rubi [A] time = 0.020751, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1593, 801, 635, 203, 260}

$$-\frac{1}{2}\log\left(x^2+4\right)+\log(x)+\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x)/(4*x + x^3),x]
```

```
[Out] ArcTan[x/2]/2 + Log[x] - Log[4 + x^2]/2
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{4+x}{4x+x^3}\,dx &= \int \frac{4+x}{x\left(4+x^2\right)}\,dx\\
&= \int \left(\frac{1}{x}+\frac{1-x}{4+x^2}\right)dx\\
&= \log(x)+\int \frac{1-x}{4+x^2}\,dx\\
&= \log(x)+\int \frac{1}{4+x^2}\,dx-\int \frac{x}{4+x^2}\,dx\\
&= \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)+\log(x)-\frac{1}{2}\log\left(4+x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.0042777, size = 23, normalized size = 1.

$$-\frac{1}{2}\log\left(x^2+4\right)+\log(x)+\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x)/(4*x + x^3),x]
```

```
[Out] ArcTan[x/2]/2 + Log[x] - Log[4 + x^2]/2
```

Maple [A] time = 0.004, size = 18, normalized size = 0.8

$$\frac{1}{2}\arctan\left(\frac{x}{2}\right)+\ln(x)-\frac{\ln\left(x^2+4\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4+x)/(x^3+4*x),x)
```

```
[Out] 1/2*arctan(1/2*x)+ln(x)-1/2*ln(x^2+4)
```

Maxima [A] time = 1.46112, size = 23, normalized size = 1.

$$\frac{1}{2}\arctan\left(\frac{1}{2}x\right)-\frac{1}{2}\log\left(x^2+4\right)+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+x)/(x^3+4*x),x, algorithm="maxima")
```

```
[Out] 1/2*arctan(1/2*x) - 1/2*log(x^2 + 4) + log(x)
```

Fricas [A] time = 1.01504, size = 63, normalized size = 2.74

$$\frac{1}{2}\arctan\left(\frac{1}{2}x\right)-\frac{1}{2}\log\left(x^2+4\right)+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+x)/(x^3+4*x),x, algorithm="fricas")
```

```
[Out] 1/2*arctan(1/2*x) - 1/2*log(x^2 + 4) + log(x)
```

Sympy [A] time = 0.114241, size = 17, normalized size = 0.74

$$\log\left(x\right)-\frac{\log\left(x^2+4\right)}{2}+\frac{\operatorname{atan}\left(\frac{x}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+x)/(x**3+4*x),x)
```

```
[Out] log(x) - log(x**2 + 4)/2 + atan(x/2)/2
```

Giac [A] time = 1.14473, size = 24, normalized size = 1.04

$$\frac{1}{2}\arctan\left(\frac{1}{2}x\right)-\frac{1}{2}\log\left(x^2+4\right)+\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+x)/(x^3+4*x),x, algorithm="giac")
```

```
[Out] 1/2*arctan(1/2*x) - 1/2*log(x^2 + 4) + log(abs(x))
```

3.438 $\int \frac{-x+2x^3}{1-x^2+x^4} \, dx$

Optimal. Leaf size=15

$$\frac{1}{2}\log\left(x^4-x^2+1\right)$$

```
[Out] Log[1 - x^2 + x^4]/2
```

Rubi [A] time = 0.0089732, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1587}

$$\frac{1}{2}\log\left(x^4-x^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-x + 2*x^3)/(1 - x^2 + x^4),x]
```

```
[Out] Log[1 - x^2 + x^4]/2
```

Rule 1587

```
Int[(Pp_)/(Qq_), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Si
mp[(Coeff[Pp, x, p]*Log[RemoveContent[Qq, x]])/(q*Coeff[Qq, x, q]), x] /; E
qQ[p, q - 1] && EqQ[Pp, Simplify[(Coeff[Pp, x, p]*D[Qq, x])/(q*Coeff[Qq, x,
 q])]]] /; PolyQ[Pp, x] && PolyQ[Qq, x]
```

Rubi steps

$$\int \frac{-x+2x^3}{1-x^2+x^4} \, dx = \frac{1}{2}\log\left(1-x^2+x^4\right)$$

Mathematica [A] time = 0.0041534, size = 15, normalized size = 1.

$$\frac{1}{2}\log\left(x^4-x^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x + 2*x^3)/(1 - x^2 + x^4),x]
```

```
[Out] Log[1 - x^2 + x^4]/2
```

Maple [A] time = 0.002, size = 14, normalized size = 0.9

$$\frac{\ln\left(x^4 - x^2 + 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3-x)/(x^4-x^2+1),x)
```

```
[Out] 1/2*ln(x^4-x^2+1)
```

Maxima [A] time = 0.961715, size = 18, normalized size = 1.2

$$\frac{1}{2}\log\left(x^4 - x^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3-x)/(x^4-x^2+1),x, algorithm="maxima")
```

```
[Out] 1/2*log(x^4 - x^2 + 1)
```

Fricas [A] time = 0.969938, size = 32, normalized size = 2.13

$$\frac{1}{2}\log\left(x^4 - x^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3-x)/(x^4-x^2+1),x, algorithm="fricas")
```

```
[Out] 1/2*log(x^4 - x^2 + 1)
```

Sympy [A] time = 0.086494, size = 10, normalized size = 0.67

$$\frac{\log\left(x^4 - x^2 + 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**3-x)/(x**4-x**2+1),x)
```

```
[Out] log(x**4 - x**2 + 1)/2
```

Giac [A] time = 1.11217, size = 18, normalized size = 1.2

$$\frac{1}{2} \log\left(x^4 - x^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3-x)/(x^4-x^2+1),x, algorithm="giac")
```

```
[Out] 1/2*log(x^4 - x^2 + 1)
```

3.439 $\int \frac{-3+x}{2x+3x^2+x^3}\,dx$

Optimal. Leaf size=21

$$-\frac{3\log(x)}{2}+4\log(x+1)-\frac{5}{2}\log(x+2)$$

```
[Out] (-3*Log[x])/2 + 4*Log[1 + x] - (5*Log[2 + x])/2
```

Rubi [A] time = 0.0213803, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1594, 800}

$$-\frac{3\log(x)}{2}+4\log(x+1)-\frac{5}{2}\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[(-3 + x)/(2*x + 3*x^2 + x^3),x]

[Out] (-3*Log[x])/2 + 4*Log[1 + x] - (5*Log[2 + x])/2
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{-3+x}{2x+3x^2+x^3}\,dx = \int \frac{-3+x}{x(2+3x+x^2)}\,dx$$
$$= \int \left(-\frac{3}{2x}+\frac{4}{1+x}-\frac{5}{2(2+x)}\right)dx$$
$$= -\frac{3\log(x)}{2}+4\log(1+x)-\frac{5}{2}\log(2+x)$$

Mathematica [A] time = 0.0054153, size = 21, normalized size = 1.

$$-\frac{3\log(x)}{2}+4\log(x+1)-\frac{5}{2}\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 + x)/(2*x + 3*x^2 + x^3),x]
```

```
[Out] (-3*Log[x])/2 + 4*Log[1 + x] - (5*Log[2 + x])/2
```

Maple [A] time = 0.006, size = 18, normalized size = 0.9

$$-\frac{3\ln(x)}{2}+4\ln(1+x)-\frac{5\ln(2+x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3+x)/(x^3+3*x^2+2*x),x)
```

```
[Out] -3/2*ln(x)+4*ln(1+x)-5/2*ln(2+x)
```

Maxima [A] time = 0.975493, size = 23, normalized size = 1.1

$$-\frac{5}{2}\log(x+2)+4\log(x+1)-\frac{3}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+x)/(x^3+3*x^2+2*x),x, algorithm="maxima")
```

```
[Out] -5/2*log(x + 2) + 4*log(x + 1) - 3/2*log(x)
```

Fricas [A] time = 0.962911, size = 61, normalized size = 2.9

$$-\frac{5}{2}\,\log\left(x+2\right)+4\,\log\left(x+1\right)-\frac{3}{2}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+x)/(x^3+3*x^2+2*x),x, algorithm="fricas")
```

```
[Out] -5/2*log(x + 2) + 4*log(x + 1) - 3/2*log(x)
```

Sympy [A] time = 0.11801, size = 20, normalized size = 0.95

$$-\frac{3\log\left(x\right)}{2}+4\log\left(x+1\right)-\frac{5\log\left(x+2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+x)/(x**3+3*x**2+2*x),x)
```

```
[Out] -3*log(x)/2 + 4*log(x + 1) - 5*log(x + 2)/2
```

Giac [A] time = 1.10726, size = 27, normalized size = 1.29

$$-\frac{5}{2}\,\log\left(|x+2|\right)+4\,\log\left(|x+1|\right)-\frac{3}{2}\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+x)/(x^3+3*x^2+2*x),x, algorithm="giac")
```

```
[Out] -5/2*log(abs(x + 2)) + 4*log(abs(x + 1)) - 3/2*log(abs(x))
```

3.440 $\int \frac{2+4x}{x^2+2x^3+x^4}\,dx$

Optimal. Leaf size=10

$$-\frac{2}{x(x+1)}$$

```
[Out] -2/(x*(1 + x))
```

Rubi [A] time = 0.0103235, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {1594, 27, 74}

$$-\frac{2}{x(x+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 4*x)/(x^2 + 2*x^3 + x^4),x]
```

```
[Out] -2/(x*(1 + x))
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rubi steps

$$\int \frac{2+4x}{x^2+2x^3+x^4}\,dx = \int \frac{2+4x}{x^2\left(1+2x+x^2\right)}\,dx$$
$$= \int \frac{2+4x}{x^2(1+x)^2}\,dx$$
$$= -\frac{2}{x(1+x)}$$

Mathematica [A] time = 0.005241, size = 9, normalized size = 0.9

$$-\frac{2}{x^2+x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 4*x)/(x^2 + 2*x^3 + x^4),x]
```

```
[Out] -2/(x + x^2)
```

Maple [A] time = 0.004, size = 14, normalized size = 1.4

$$2\,(1+x)^{-1}-2\,x^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+4*x)/(x^4+2*x^3+x^2),x)
```

```
[Out] 2/(1+x)-2/x
```

Maxima [A] time = 0.976928, size = 12, normalized size = 1.2

$$-\frac{2}{x^2+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+4*x)/(x^4+2*x^3+x^2),x, algorithm="maxima")
```

```
[Out] -2/(x^2 + x)
```

Fricas [A] time = 0.940711, size = 19, normalized size = 1.9

$$-\frac{2}{x^2+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+4*x)/(x^4+2*x^3+x^2),x, algorithm="fricas")
```

```
[Out] -2/(x^2 + x)
```

Sympy [A] time = 0.081098, size = 7, normalized size = 0.7

$$-\frac{2}{x^2+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+4*x)/(x**4+2*x**3+x**2),x)
```

```
[Out] -2/(x**2 + x)
```

Giac [A] time = 1.10217, size = 12, normalized size = 1.2

$$-\frac{2}{x^2+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+4*x)/(x^4+2*x^3+x^2),x, algorithm="giac")
```

```
[Out] -2/(x^2 + x)
```

3.441 $\int \frac{1+x}{-6x+x^2+x^3}\,dx$

Optimal. Leaf size=25

$$\frac{3}{10}\log(2-x)-\frac{\log(x)}{6}-\frac{2}{15}\log(x+3)$$

```
[Out] (3*Log[2 - x])/10 - Log[x]/6 - (2*Log[3 + x])/15
```

Rubi [A] time = 0.0208231, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1594, 800}

$$\frac{3}{10}\log(2-x)-\frac{\log(x)}{6}-\frac{2}{15}\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)/(-6*x + x^2 + x^3),x]

[Out] (3*Log[2 - x])/10 - Log[x]/6 - (2*Log[3 + x])/15
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{1+x}{-6x+x^2+x^3}\,dx = \int \frac{1+x}{x\left(-6+x+x^2\right)}\,dx$$
$$= \int \left(\frac{3}{10(-2+x)} - \frac{1}{6x} - \frac{2}{15(3+x)}\right) dx$$
$$= \frac{3}{10}\log(2-x) - \frac{\log(x)}{6} - \frac{2}{15}\log(3+x)$$

Mathematica [A] time = 0.0051272, size = 25, normalized size = 1.

$$\frac{3}{10}\log(2-x) - \frac{\log(x)}{6} - \frac{2}{15}\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)/(-6*x + x^2 + x^3),x]
```

```
[Out] (3*Log[2 - x])/10 - Log[x]/6 - (2*Log[3 + x])/15
```

Maple [A] time = 0.005, size = 18, normalized size = 0.7

$$-\frac{\ln(x)}{6} - \frac{2\ln(3+x)}{15} + \frac{3\ln(-2+x)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(x^3+x^2-6*x),x)
```

```
[Out] -1/6*ln(x)-2/15*ln(3+x)+3/10*ln(-2+x)
```

Maxima [A] time = 0.962201, size = 23, normalized size = 0.92

$$-\frac{2}{15}\log(x+3) + \frac{3}{10}\log(x-2) - \frac{1}{6}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^3+x^2-6*x),x, algorithm="maxima")
```

```
[Out] -2/15*log(x + 3) + 3/10*log(x - 2) - 1/6*log(x)
```

Fricas [A] time = 0.976519, size = 66, normalized size = 2.64

$$-\frac{2}{15}\log\left(x+3\right)+\frac{3}{10}\log\left(x-2\right)-\frac{1}{6}\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^3+x^2-6*x),x, algorithm="fricas")
```

```
[Out] -2/15*log(x + 3) + 3/10*log(x - 2) - 1/6*log(x)
```

Sympy [A] time = 0.119831, size = 20, normalized size = 0.8

$$-\frac{\log\left(x\right)}{6}+\frac{3\log\left(x-2\right)}{10}-\frac{2\log\left(x+3\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x**3+x**2-6*x),x)
```

```
[Out] -log(x)/6 + 3*log(x - 2)/10 - 2*log(x + 3)/15
```

Giac [A] time = 1.16003, size = 27, normalized size = 1.08

$$-\frac{2}{15}\log\left(|x+3|\right)+\frac{3}{10}\log\left(|x-2|\right)-\frac{1}{6}\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^3+x^2-6*x),x, algorithm="giac")
```

```
[Out] -2/15*log(abs(x + 3)) + 3/10*log(abs(x - 2)) - 1/6*log(abs(x))
```

3.442 $\int \frac{4x^2+x^3}{x+x^3} dx$

Optimal. Leaf size=14

$$2\log\left(x^2+1\right)+x-\tan^{-1}(x)$$

```
[Out] x - ArcTan[x] + 2*Log[1 + x^2]
```

Rubi [A] time = 0.0248439, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {1593, 1584, 774, 635, 203, 260}

$$2\log\left(x^2+1\right)+x-\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(4*x^2 + x^3)/(x + x^3),x]
```

```
[Out] x - ArcTan[x] + 2*Log[1 + x^2]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 774

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x
)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
```

```
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{4x^2+x^3}{x+x^3}\,dx &= \int \frac{4x^2+x^3}{x\left(1+x^2\right)}\,dx\\ &= \int \frac{x(4+x)}{1+x^2}\,dx\\ &= x+\int \frac{-1+4x}{1+x^2}\,dx\\ &= x+4\int \frac{x}{1+x^2}\,dx-\int \frac{1}{1+x^2}\,dx\\ &= x-\tan^{-1}(x)+2\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.0040059, size = 14, normalized size = 1.

$$2\log\left(x^2+1\right)+x-\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4*x^2 + x^3)/(x + x^3),x]
```

```
[Out] x - ArcTan[x] + 2*Log[1 + x^2]
```

Maple [A] time = 0.001, size = 15, normalized size = 1.1

$$x-\arctan(x)+2\ln\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+4*x^2)/(x^3+x),x)
```

```
[Out] x-arctan(x)+2*ln(x^2+1)
```

Maxima [A] time = 1.47022, size = 19, normalized size = 1.36

$$x - \arctan(x) + 2 \log\left(x^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+4*x^2)/(x^3+x),x, algorithm="maxima")
```

```
[Out] x - arctan(x) + 2*log(x^2 + 1)
```

Fricas [A] time = 0.976394, size = 43, normalized size = 3.07

$$x - \arctan(x) + 2 \log\left(x^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+4*x^2)/(x^3+x),x, algorithm="fricas")
```

```
[Out] x - arctan(x) + 2*log(x^2 + 1)
```

Sympy [A] time = 0.09272, size = 12, normalized size = 0.86

$$x + 2\log\left(x^2 + 1\right) - \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+4*x**2)/(x**3+x),x)
```

```
[Out] x + 2*log(x**2 + 1) - atan(x)
```

Giac [A] time = 1.13637, size = 19, normalized size = 1.36

$$x - \arctan(x) + 2 \log\left(x^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+4*x^2)/(x^3+x),x, algorithm="giac")
```

```
[Out] x - arctan(x) + 2*log(x^2 + 1)
```

3.443 $\int \frac{x+2x^3}{\left(x^2+x^4\right)^3}\,dx$

Optimal. Leaf size=13

$$-\frac{1}{4\left(x^4+x^2\right)^2}$$

```
[Out] -1/(4*(x^2 + x^4)^2)
```

Rubi [A] time = 0.0070267, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {1588}

$$-\frac{1}{4\left(x^4+x^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x + 2*x^3)/(x^2 + x^4)^3,x]
```

```
[Out] -1/(4*(x^2 + x^4)^2)
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x+2x^3}{\left(x^2+x^4\right)^3}\,dx=-\frac{1}{4\left(x^2+x^4\right)^2}$$

Mathematica [A] time = 0.0065667, size = 14, normalized size = 1.08

$$-\frac{1}{4x^4\left(x^2+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + 2*x^3)/(x^2 + x^4)^3,x]
```

```
[Out] -1/(4*x^4*(1 + x^2)^2)
```

Maple [B] time = 0.01, size = 30, normalized size = 2.3

$$-\frac{1}{4\left(x^2+1\right)^2}-\frac{1}{2x^2+2}-\frac{1}{4x^4}+\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3+x)/(x^4+x^2)^3,x)
```

```
[Out] -1/4/(x^2+1)^2-1/2/(x^2+1)-1/4/x^4+1/2/x^2
```

Maxima [A] time = 0.967609, size = 15, normalized size = 1.15

$$-\frac{1}{4\left(x^4+x^2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+x)/(x^4+x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4/(x^4 + x^2)^2
```

Fricas [A] time = 0.937095, size = 35, normalized size = 2.69

$$-\frac{1}{4\left(x^8+2x^6+x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+x)/(x^4+x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/4/(x^8 + 2*x^6 + x^4)
```

Sympy [A] time = 0.130826, size = 17, normalized size = 1.31

$$-\frac{1}{4x^8 + 8x^6 + 4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**3+x)/(x**4+x**2)**3,x)
```

```
[Out] -1/(4*x**8 + 8*x**6 + 4*x**4)
```

Giac [A] time = 1.20607, size = 15, normalized size = 1.15

$$-\frac{1}{4\left(x^4 + x^2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+x)/(x^4+x^2)^3,x, algorithm="giac")
```

```
[Out] -1/4/(x^4 + x^2)^2
```

3.444 $\int \frac{ax^2+bx^3}{cx^2+dx^3} dx$

Optimal. Leaf size=26

$$\frac{bx}{d} - \frac{(bc - ad)\log(c + dx)}{d^2}$$

```
[Out] (b*x)/d - ((b*c - a*d)*Log[c + d*x])/d^2
```

Rubi [A] time = 0.0452603, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {1593, 1584, 43}

$$\frac{bx}{d} - \frac{(bc - ad)\log(c + dx)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3)/(c*x^2 + d*x^3),x]
```

```
[Out] (b*x)/d - ((b*c - a*d)*Log[c + d*x])/d^2
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{ax^2+bx^3}{cx^2+dx^3}\,dx &= \int \frac{x^2(a+bx)}{cx^2+dx^3}\,dx\\ &= \int \frac{a+bx}{c+dx}\,dx\\ &= \int \left(\frac{b}{d}+\frac{-bc+ad}{d(c+dx)}\right)dx\\ &= \frac{bx}{d}-\frac{(bc-ad)\log(c+dx)}{d^2}\end{aligned}$$

Mathematica [A] time = 0.0075781, size = 25, normalized size = 0.96

$$\frac{(ad-bc)\log(c+dx)}{d^2}+\frac{bx}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3)/(c*x^2 + d*x^3),x]
```

```
[Out] (b*x)/d + ((-(b*c) + a*d)*Log[c + d*x])/d^2
```

Maple [A] time = 0.003, size = 32, normalized size = 1.2

$$\frac{bx}{d}+\frac{\ln(dx+c)\,a}{d}-\frac{\ln(dx+c)\,bc}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)/(d*x^3+c*x^2),x)
```

```
[Out] b*x/d+1/d*ln(d*x+c)*a-1/d^2*ln(d*x+c)*b*c
```

Maxima [A] time = 0.963804, size = 35, normalized size = 1.35

$$\frac{bx}{d}-\frac{(bc-ad)\log(dx+c)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)/(d*x^3+c*x^2),x, algorithm="maxima")
```

```
[Out] b*x/d - (b*c - a*d)*log(d*x + c)/d^2
```

Fricas [A] time = 0.962319, size = 54, normalized size = 2.08

$$\frac{bdx - (bc - ad)\log(dx + c)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)/(d*x^3+c*x^2),x, algorithm="fricas")
```

```
[Out] (b*d*x - (b*c - a*d)*log(d*x + c))/d^2
```

Sympy [A] time = 0.301285, size = 20, normalized size = 0.77

$$\frac{bx}{d} + \frac{(ad - bc)\log(c + dx)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x**2)/(d*x**3+c*x**2),x)
```

```
[Out] b*x/d + (a*d - b*c)*log(c + d*x)/d**2
```

Giac [A] time = 1.13029, size = 36, normalized size = 1.38

$$\frac{bx}{d} - \frac{(bc - ad)\log(|dx + c|)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)/(d*x^3+c*x^2),x, algorithm="giac")
```

```
[Out] b*x/d - (b*c - a*d)*log(abs(d*x + c))/d^2
```

3.445 $\int \frac{x+x^2}{-2x-x^2+x^3}\,dx$

Optimal. Leaf size=6

$$\log(2-x)$$

[Out] Log[2 - x]

Rubi [A] time = 0.0079106, antiderivative size = 6, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {1586, 31}

$$\log(2-x)$$

Antiderivative was successfully verified.

```
[In] Int[(x + x^2)/(-2*x - x^2 + x^3),x]
```

```
[Out] Log[2 - x]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x+x^2}{-2x-x^2+x^3}\,dx &= \int \frac{1}{-2+x}\,dx\\ &= \log(2-x)\end{aligned}$$

Mathematica [A] time = 0.0007617, size = 4, normalized size = 0.67

$$\log(x-2)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + x^2)/(-2*x - x^2 + x^3),x]
```

```
[Out] Log[-2 + x]
```

Maple [A] time = 0., size = 5, normalized size = 0.8

$$\ln(-2+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+x)/(x^3-x^2-2*x),x)
```

```
[Out] ln(-2+x)
```

Maxima [A] time = 1.00519, size = 5, normalized size = 0.83

$$\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x)/(x^3-x^2-2*x),x, algorithm="maxima")
```

```
[Out] log(x - 2)
```

Fricas [A] time = 0.954969, size = 16, normalized size = 2.67

$$\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x)/(x^3-x^2-2*x),x, algorithm="fricas")
```

```
[Out] log(x - 2)
```

Sympy [A] time = 0.058289, size = 3, normalized size = 0.5

$$\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x)/(x**3-x**2-2*x),x)
```

```
[Out] log(x - 2)
```

Giac [A] time = 1.104, size = 7, normalized size = 1.17

$$\log(|x-2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x)/(x^3-x^2-2*x),x, algorithm="giac")
```

```
[Out] log(abs(x - 2))
```

3.446 $\int \frac{1-5x^2}{x^3(1+x^2)} dx$

Optimal. Leaf size=20

$$-\frac{1}{2x^2} + 3\log\left(x^2+1\right) - 6\log(x)$$

```
[Out] -1/(2*x^2) - 6*Log[x] + 3*Log[1 + x^2]
```

Rubi [A] time = 0.0146122, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {446, 77}

$$-\frac{1}{2x^2} + 3\log\left(x^2+1\right) - 6\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 5*x^2)/(x^3*(1 + x^2)),x]

[Out] -1/(2*x^2) - 6*Log[x] + 3*Log[1 + x^2]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int \frac{1-5x^2}{x^3\left(1+x^2\right)}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1-5x}{x^2(1+x)}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{1}{x^2}-\frac{6}{x}+\frac{6}{1+x}\right)dx, x, x^2\right)$$
$$= -\frac{1}{2x^2} - 6\log(x) + 3\log\left(1+x^2\right)$$

Mathematica [A] time = 0.0046499, size = 20, normalized size = 1.

$$-\frac{1}{2x^2} + 3\log\left(x^2+1\right) - 6\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 5*x^2)/(x^3*(1 + x^2)),x]
```

```
[Out] -1/(2*x^2) - 6*Log[x] + 3*Log[1 + x^2]
```

Maple [A] time = 0.006, size = 19, normalized size = 1.

$$-\frac{1}{2\,x^2} - 6\,\ln(x) + 3\,\ln\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-5*x^2+1)/x^3/(x^2+1),x)
```

```
[Out] -1/2/x^2-6*ln(x)+3*ln(x^2+1)
```

Maxima [A] time = 0.981652, size = 27, normalized size = 1.35

$$-\frac{1}{2\,x^2} + 3\,\log\left(x^2+1\right) - 3\,\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5*x^2+1)/x^3/(x^2+1),x, algorithm="maxima")
```

```
[Out] -1/2/x^2 + 3*log(x^2 + 1) - 3*log(x^2)
```

Fricas [A] time = 0.969082, size = 68, normalized size = 3.4

$$\frac{6\,x^2\log\left(x^2+1\right)-12\,x^2\log\left(x\right)-1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5*x^2+1)/x^3/(x^2+1),x, algorithm="fricas")
```

```
[Out] 1/2*(6*x^2*log(x^2 + 1) - 12*x^2*log(x) - 1)/x^2
```

Sympy [A] time = 0.097832, size = 19, normalized size = 0.95

$$-6\log\left(x\right)+3\log\left(x^2+1\right)-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5*x**2+1)/x**3/(x**2+1),x)
```

```
[Out] -6*log(x) + 3*log(x**2 + 1) - 1/(2*x**2)
```

Giac [A] time = 1.11641, size = 36, normalized size = 1.8

$$\frac{6\,x^2-1}{2\,x^2}+3\,\log\left(x^2+1\right)-3\,\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5*x^2+1)/x^3/(x^2+1),x, algorithm="giac")
```

```
[Out] 1/2*(6*x^2 - 1)/x^2 + 3*log(x^2 + 1) - 3*log(x^2)
```

3.447 $\int \frac{2x}{(-1+x)(5+x^2)}\,dx$

Optimal. Leaf size=38

$$-\frac{1}{6}\log\left(x^2+5\right)+\frac{1}{3}\log(1-x)+\frac{1}{3}\sqrt{5}\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)$$

```
[Out] (Sqrt[5]*ArcTan[x/Sqrt[5]])/3 + Log[1 - x]/3 - Log[5 + x^2]/6
```

Rubi [A] time = 0.0254795, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {12, 801, 635, 203, 260}

$$-\frac{1}{6}\log\left(x^2+5\right)+\frac{1}{3}\log(1-x)+\frac{1}{3}\sqrt{5}\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2*x)/((-1 + x)*(5 + x^2)),x]
```

```
[Out] (Sqrt[5]*ArcTan[x/Sqrt[5]])/3 + Log[1 - x]/3 - Log[5 + x^2]/6
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{2x}{(-1+x)\left(5+x^2\right)}\,dx &= 2\int \frac{x}{(-1+x)\left(5+x^2\right)}\,dx \\
&= 2\int \left(\frac{1}{6(-1+x)}+\frac{5-x}{6\left(5+x^2\right)}\right)dx \\
&= \frac{1}{3}\log(1-x)+\frac{1}{3}\int \frac{5-x}{5+x^2}\,dx \\
&= \frac{1}{3}\log(1-x)-\frac{1}{3}\int \frac{x}{5+x^2}\,dx+\frac{5}{3}\int \frac{1}{5+x^2}\,dx \\
&= \frac{1}{3}\sqrt{5}\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)+\frac{1}{3}\log(1-x)-\frac{1}{6}\log\left(5+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.0098961, size = 40, normalized size = 1.05

$$2\left(-\frac{1}{12}\log\left(x^2+5\right)+\frac{1}{6}\log(1-x)+\frac{1}{6}\sqrt{5}\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*x)/((-1 + x)*(5 + x^2)),x]
```

```
[Out] 2*((Sqrt[5]*ArcTan[x/Sqrt[5]])/6 + Log[1 - x]/6 - Log[5 + x^2]/12)
```

Maple [A] time = 0.007, size = 28, normalized size = 0.7

$$\frac{\ln(x-1)}{3}-\frac{\ln\left(x^2+5\right)}{6}+\frac{\sqrt{5}}{3}\arctan\left(\frac{x\sqrt{5}}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2*x/(x-1)/(x^2+5),x)
```

```
[Out] 1/3*ln(x-1)-1/6*ln(x^2+5)+1/3*arctan(1/5*x*5^(1/2))*5^(1/2)
```

Maxima [A] time = 1.48105, size = 36, normalized size = 0.95

$$\frac{1}{3}\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}x\right)-\frac{1}{6}\log\left(x^2+5\right)+\frac{1}{3}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*x/(-1+x)/(x^2+5),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(5)*arctan(1/5*sqrt(5)*x) - 1/6*log(x^2 + 5) + 1/3*log(x - 1)
```

Fricas [A] time = 0.98477, size = 96, normalized size = 2.53

$$\frac{1}{3}\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}x\right)-\frac{1}{6}\log\left(x^2+5\right)+\frac{1}{3}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*x/(-1+x)/(x^2+5),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(5)*arctan(1/5*sqrt(5)*x) - 1/6*log(x^2 + 5) + 1/3*log(x - 1)
```

Sympy [A] time = 0.12578, size = 31, normalized size = 0.82

$$\frac{\log\left(x-1\right)}{3}-\frac{\log\left(x^2+5\right)}{6}+\frac{\sqrt{5}\operatorname{atan}\left(\frac{\sqrt{5}x}{5}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*x/(-1+x)/(x**2+5),x)
```

```
[Out] log(x - 1)/3 - log(x**2 + 5)/6 + sqrt(5)*atan(sqrt(5)*x/5)/3
```

Giac [A] time = 1.12193, size = 38, normalized size = 1.

$$\frac{1}{3}\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}x\right)-\frac{1}{6}\log\left(x^2+5\right)+\frac{1}{3}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*x/(-1+x)/(x^2+5),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(5)*arctan(1/5*sqrt(5)*x) - 1/6*log(x^2 + 5) + 1/3*log(abs(x - 1))
```

3.448 $\int \frac{2+x^2}{2+x} \, dx$

Optimal. Leaf size=17

$$\frac{x^2}{2} - 2x + 6\log(x+2)$$

```
[Out] -2*x + x^2/2 + 6*Log[2 + x]
```

Rubi [A] time = 0.0082738, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {697}

$$\frac{x^2}{2} - 2x + 6\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x^2)/(2 + x),x]
```

```
[Out] -2*x + x^2/2 + 6*Log[2 + x]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{2+x^2}{2+x} \, dx &= \int \left(-2+x+\frac{6}{2+x}\right) \, dx \\ &= -2x+\frac{x^2}{2}+6\log(2+x) \end{aligned}$$

Mathematica [A] time = 0.0034374, size = 18, normalized size = 1.06

$$\frac{x^2}{2} - 2x + 6\log(x+2) - 6$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x^2)/(2 + x),x]
```

```
[Out] -6 - 2*x + x^2/2 + 6*Log[2 + x]
```

Maple [A] time = 0.002, size = 16, normalized size = 0.9

$$-2x+\frac{x^2}{2}+6\ln(2+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+2)/(2+x),x)
```

```
[Out] -2*x+1/2*x^2+6*ln(2+x)
```

Maxima [A] time = 0.968912, size = 20, normalized size = 1.18

$$\frac{1}{2}x^2-2x+6\log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2)/(2+x),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - 2*x + 6*log(x + 2)
```

Fricas [A] time = 0.964359, size = 41, normalized size = 2.41

$$\frac{1}{2}x^2-2x+6\log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2)/(2+x),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 - 2*x + 6*log(x + 2)
```

Sympy [A] time = 0.070396, size = 14, normalized size = 0.82

$$\frac{x^2}{2} - 2x + 6\log(x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+2)/(2+x),x)
```

```
[Out] x**2/2 - 2*x + 6*log(x + 2)
```

Giac [A] time = 1.13121, size = 22, normalized size = 1.29

$$\frac{1}{2}x^2 - 2x + 6\log(|x + 2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2)/(2+x),x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 2*x + 6*log(abs(x + 2))
```

3.449 $\int \frac{1}{(-3+x)(4+x^2)}\,dx$

Optimal. Leaf size=31

$$-\frac{1}{26}\log\left(x^2+4\right)+\frac{1}{13}\log(3-x)-\frac{3}{26}\tan^{-1}\left(\frac{x}{2}\right)$$

```
[Out] (-3*ArcTan[x/2])/26 + Log[3 - x]/13 - Log[4 + x^2]/26
```

Rubi [A] time = 0.0120286, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {706, 31, 635, 203, 260}

$$-\frac{1}{26}\log\left(x^2+4\right)+\frac{1}{13}\log(3-x)-\frac{3}{26}\tan^{-1}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/((-3 + x)*(4 + x^2)),x]
```

```
[Out] (-3*ArcTan[x/2])/26 + Log[3 - x]/13 - Log[4 + x^2]/26
```

Rule 706

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(-3+x)\left(4+x^2\right)}\,dx &= \frac{1}{13}\int \frac{1}{-3+x}\,dx+\frac{1}{13}\int \frac{-3-x}{4+x^2}\,dx\\ &= \frac{1}{13}\log(3-x)-\frac{1}{13}\int \frac{x}{4+x^2}\,dx-\frac{3}{13}\int \frac{1}{4+x^2}\,dx\\ &= -\frac{3}{26}\tan^{-1}\left(\frac{x}{2}\right)+\frac{1}{13}\log(3-x)-\frac{1}{26}\log\left(4+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.0051457, size = 36, normalized size = 1.16

$$-\frac{1}{26}\log\left((x-3)^2+6(x-3)+13\right)+\frac{1}{13}\log(x-3)-\frac{3}{26}\tan^{-1}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((-3 + x)*(4 + x^2)),x]
```

```
[Out] (-3*ArcTan[x/2])/26 - Log[13 + 6*(-3 + x) + (-3 + x)^2]/26 + Log[-3 + x]/13
```

Maple [A] time = 0.004, size = 22, normalized size = 0.7

$$\frac{\ln(-3+x)}{13}-\frac{\ln\left(x^2+4\right)}{26}-\frac{3}{26}\arctan\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3+x)/(x^2+4),x)
```

```
[Out] 1/13*ln(-3+x)-1/26*ln(x^2+4)-3/26*arctan(1/2*x)
```

Maxima [A] time = 1.47843, size = 28, normalized size = 0.9

$$-\frac{3}{26}\arctan\left(\frac{1}{2}x\right)-\frac{1}{26}\log\left(x^2+4\right)+\frac{1}{13}\log\left(x-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3+x)/(x^2+4),x, algorithm="maxima")
```

```
[Out] -3/26*arctan(1/2*x) - 1/26*log(x^2 + 4) + 1/13*log(x - 3)
```

Fricas [A] time = 1.00476, size = 80, normalized size = 2.58

$$-\frac{3}{26}\arctan\left(\frac{1}{2}x\right)-\frac{1}{26}\log\left(x^2+4\right)+\frac{1}{13}\log\left(x-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3+x)/(x^2+4),x, algorithm="fricas")
```

```
[Out] -3/26*arctan(1/2*x) - 1/26*log(x^2 + 4) + 1/13*log(x - 3)
```

Sympy [A] time = 0.137788, size = 22, normalized size = 0.71

$$\frac{\log\left(x-3\right)}{13}-\frac{\log\left(x^2+4\right)}{26}-\frac{3\,\mathrm{atan}\left(\frac{x}{2}\right)}{26}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3+x)/(x**2+4),x)
```

```
[Out] log(x - 3)/13 - log(x**2 + 4)/26 - 3*atan(x/2)/26
```

Giac [A] time = 1.11134, size = 30, normalized size = 0.97

$$-\frac{3}{26}\arctan\left(\frac{1}{2}x\right)-\frac{1}{26}\log\left(x^2+4\right)+\frac{1}{13}\log\left(|x-3|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3+x)/(x^2+4),x, algorithm="giac")
```

```
[Out] -3/26*arctan(1/2*x) - 1/26*log(x^2 + 4) + 1/13*log(abs(x - 3))
```

3.450 $\int \frac{-2+3x^6}{x(5+2x^6)}\,dx$

Optimal. Leaf size=19

$$\frac{19}{60}\log\left(2x^6+5\right)-\frac{2\log(x)}{5}$$

```
[Out] (-2*Log[x])/5 + (19*Log[5 + 2*x^6])/60
```

Rubi [A] time = 0.0167028, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {446, 72}

$$\frac{19}{60}\log\left(2x^6+5\right)-\frac{2\log(x)}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(-2 + 3*x^6)/(x*(5 + 2*x^6)),x]
```

```
[Out] (-2*Log[x])/5 + (19*Log[5 + 2*x^6])/60
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \frac{-2+3x^6}{x\left(5+2x^6\right)}\,dx = \frac{1}{6}\,\text{Subst}\left(\int \frac{-2+3x}{x(5+2x)}\,dx, x, x^6\right)$$
$$= \frac{1}{6}\,\text{Subst}\left(\int \left(-\frac{2}{5x}+\frac{19}{5(5+2x)}\right)dx, x, x^6\right)$$
$$= -\frac{2\log(x)}{5}+\frac{19}{60}\log\left(5+2x^6\right)$$

Mathematica [A] time = 0.0055687, size = 19, normalized size = 1.

$$\frac{19}{60}\log\left(2x^6+5\right)-\frac{2\log(x)}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 + 3*x^6)/(x*(5 + 2*x^6)),x]
```

```
[Out] (-2*Log[x])/5 + (19*Log[5 + 2*x^6])/60
```

Maple [A] time = 0.005, size = 16, normalized size = 0.8

$$-\frac{2\,\ln(x)}{5}+\frac{19\,\ln\left(2\,x^6+5\right)}{60}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^6-2)/x/(2*x^6+5),x)
```

```
[Out] -2/5*ln(x)+19/60*ln(2*x^6+5)
```

Maxima [A] time = 0.980774, size = 23, normalized size = 1.21

$$\frac{19}{60}\,\log\left(2\,x^6+5\right)-\frac{1}{15}\,\log\left(x^6\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^6-2)/x/(2*x^6+5),x, algorithm="maxima")
```

```
[Out] 19/60*log(2*x^6 + 5) - 1/15*log(x^6)
```

Fricas [A] time = 0.966827, size = 47, normalized size = 2.47

$$\frac{19}{60} \log\left(2x^6 + 5\right) - \frac{2}{5} \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^6-2)/x/(2*x^6+5),x, algorithm="fricas")
```

```
[Out] 19/60*log(2*x^6 + 5) - 2/5*log(x)
```

Sympy [A] time = 0.106576, size = 17, normalized size = 0.89

$$-\frac{2\log(x)}{5} + \frac{19\log\left(2x^6 + 5\right)}{60}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**6-2)/x/(2*x**6+5),x)
```

```
[Out] -2*log(x)/5 + 19*log(2*x**6 + 5)/60
```

Giac [A] time = 1.14415, size = 23, normalized size = 1.21

$$\frac{19}{60} \log\left(2x^6 + 5\right) - \frac{1}{15} \log\left(x^6\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^6-2)/x/(2*x^6+5),x, algorithm="giac")
```

```
[Out] 19/60*log(2*x^6 + 5) - 1/15*log(x^6)
```

3.451 $\int \frac{3+2x}{(-2+x)(5+x)}\,dx$

Optimal. Leaf size=11

$$\log(2-x)+\log(x+5)$$

```
[Out] Log[2 - x] + Log[5 + x]
```

Rubi [A] time = 0.0049503, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {72}

$$\log(2-x)+\log(x+5)$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 2*x)/((-2 + x)*(5 + x)),x]
```

```
[Out] Log[2 - x] + Log[5 + x]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{3+2x}{(-2+x)(5+x)}\,dx &= \int \left(\frac{1}{-2+x}+\frac{1}{5+x}\right) dx\\ &= \log(2-x)+\log(5+x)\end{aligned}$$

Mathematica [A] time = 0.0037906, size = 9, normalized size = 0.82

$$\log(x-2)+\log(x+5)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 2*x)/((-2 + x)*(5 + x)),x]
```

```
[Out] Log[-2 + x] + Log[5 + x]
```

Maple [A] time = 0.001, size = 9, normalized size = 0.8

$$\ln\left((-2+x)(5+x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3+2*x)/(-2+x)/(5+x),x)
```

```
[Out] ln((-2+x)*(5+x))
```

Maxima [A] time = 1.00099, size = 12, normalized size = 1.09

$$\log\left(x+5\right)+\log\left(x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(-2+x)/(5+x),x, algorithm="maxima")
```

```
[Out] log(x + 5) + log(x - 2)
```

Fricas [A] time = 0.973725, size = 28, normalized size = 2.55

$$\log\left(x^{2}+3\,x-10\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(-2+x)/(5+x),x, algorithm="fricas")
```

```
[Out] log(x^2 + 3*x - 10)
```

Sympy [A] time = 0.08384, size = 8, normalized size = 0.73

$$\log\left(x^2+3x-10\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(-2+x)/(5+x),x)
```

```
[Out] log(x**2 + 3*x - 10)
```

Giac [A] time = 1.12288, size = 15, normalized size = 1.36

$$\log\left(|x+5|\right)+\log\left(|x-2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(-2+x)/(5+x),x, algorithm="giac")
```

```
[Out] log(abs(x + 5)) + log(abs(x - 2))
```

3.452 $\int \frac{x^4}{4+5x^2+x^4} dx$

Optimal. Leaf size=18

$$x - \frac{8}{3}\tan^{-1}\left(\frac{x}{2}\right) + \frac{1}{3}\tan^{-1}(x)$$

```
[Out] x - (8*ArcTan[x/2])/3 + ArcTan[x]/3
```

Rubi [A] time = 0.0140661, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1122, 1166, 203}

$$x - \frac{8}{3}\tan^{-1}\left(\frac{x}{2}\right) + \frac{1}{3}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(4 + 5*x^2 + x^4),x]
```

```
[Out] x - (8*ArcTan[x/2])/3 + ArcTan[x]/3
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{4+5x^2+x^4}\,dx &= x-\int \frac{4+5x^2}{4+5x^2+x^4}\,dx\\ &= x+\frac{1}{3}\int \frac{1}{1+x^2}\,dx-\frac{16}{3}\int \frac{1}{4+x^2}\,dx\\ &= x-\frac{8}{3}\tan^{-1}\left(\frac{x}{2}\right)+\frac{1}{3}\tan^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.0088147, size = 18, normalized size = 1.

$$x+\frac{8}{3}\tan^{-1}\left(\frac{2}{x}\right)+\frac{1}{3}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(4 + 5*x^2 + x^4),x]
```

```
[Out] x + (8*ArcTan[2/x])/3 + ArcTan[x]/3
```

Maple [A] time = 0.006, size = 13, normalized size = 0.7

$$x-\frac{8}{3}\arctan\left(\frac{x}{2}\right)+\frac{\arctan(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^4+5*x^2+4),x)
```

```
[Out] x-8/3*arctan(1/2*x)+1/3*arctan(x)
```

Maxima [A] time = 1.50497, size = 16, normalized size = 0.89

$$x-\frac{8}{3}\arctan\left(\frac{1}{2}x\right)+\frac{1}{3}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^4+5*x^2+4),x, algorithm="maxima")
```

```
[Out] x - 8/3*arctan(1/2*x) + 1/3*arctan(x)
```

Fricas [A] time = 1.00811, size = 53, normalized size = 2.94

$$x - \frac{8}{3}\arctan\left(\frac{1}{2}x\right) + \frac{1}{3}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^4+5*x^2+4),x, algorithm="fricas")
```

```
[Out] x - 8/3*arctan(1/2*x) + 1/3*arctan(x)
```

Sympy [A] time = 0.128109, size = 14, normalized size = 0.78

$$x - \frac{8\,\mathrm{atan}\left(\frac{x}{2}\right)}{3} + \frac{\mathrm{atan}(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(x**4+5*x**2+4),x)
```

```
[Out] x - 8*atan(x/2)/3 + atan(x)/3
```

Giac [A] time = 1.13935, size = 16, normalized size = 0.89

$$x - \frac{8}{3}\arctan\left(\frac{1}{2}x\right) + \frac{1}{3}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^4+5*x^2+4),x, algorithm="giac")
```

```
[Out] x - 8/3*arctan(1/2*x) + 1/3*arctan(x)
```

3.453 $\int \frac{1}{(1+x)(2+x)^2(3+x)^3} \, dx$

Optimal. Leaf size=46

$$\frac{1}{x+2}+\frac{5}{4(x+3)}+\frac{1}{4(x+3)^2}+\frac{1}{8}\log(x+1)+2\log(x+2)-\frac{17}{8}\log(x+3)$$

```
[Out] (2 + x)^(-1) + 1/(4*(3 + x)^2) + 5/(4*(3 + x)) + Log[1 + x]/8 + 2*Log[2 + x
] - (17*Log[3 + x])/8
```

Rubi [A] time = 0.0234159, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {88}

$$\frac{1}{x+2}+\frac{5}{4(x+3)}+\frac{1}{4(x+3)^2}+\frac{1}{8}\log(x+1)+2\log(x+2)-\frac{17}{8}\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x)*(2 + x)^2*(3 + x)^3),x]
```

```
[Out] (2 + x)^(-1) + 1/(4*(3 + x)^2) + 5/(4*(3 + x)) + Log[1 + x]/8 + 2*Log[2 + x
] - (17*Log[3 + x])/8
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1+x)(2+x)^2(3+x)^3} \, dx &= \int \left(\frac{1}{8(1+x)}-\frac{1}{(2+x)^2}+\frac{2}{2+x}-\frac{1}{2(3+x)^3}-\frac{5}{4(3+x)^2}-\frac{17}{8(3+x)}\right) dx\\ &= \frac{1}{2+x}+\frac{1}{4(3+x)^2}+\frac{5}{4(3+x)}+\frac{1}{8}\log(1+x)+2\log(2+x)-\frac{17}{8}\log(3+x)\end{aligned}$$

Mathematica [A] time = 0.016478, size = 44, normalized size = 0.96

$$\frac{1}{8}\left(\frac{8}{x+2}+\frac{10}{x+3}+\frac{2}{(x+3)^2}+\log(-x-1)+16\log(x+2)-17\log(x+3)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x)*(2 + x)^2*(3 + x)^3),x]
```

```
[Out] (8/(2 + x) + 2/(3 + x)^2 + 10/(3 + x) + Log[-1 - x] + 16*Log[2 + x] - 17*Lo
g[3 + x])/8
```

Maple [A] time = 0.01, size = 39, normalized size = 0.9

$$(2+x)^{-1}+\frac{1}{4\,(3+x)^2}+\frac{5}{12+4\,x}+\frac{\ln(1+x)}{8}+2\,\ln(2+x)-\frac{17\,\ln(3+x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+x)/(2+x)^2/(3+x)^3,x)
```

```
[Out] 1/(2+x)+1/4/(3+x)^2+5/4/(3+x)+1/8*ln(1+x)+2*ln(2+x)-17/8*ln(3+x)
```

Maxima [A] time = 1.02433, size = 62, normalized size = 1.35

$$\frac{9\,x^2+50\,x+68}{4\left(x^3+8\,x^2+21\,x+18\right)}-\frac{17}{8}\,\log(x+3)+2\,\log(x+2)+\frac{1}{8}\,\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(2+x)^2/(3+x)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(9*x^2 + 50*x + 68)/(x^3 + 8*x^2 + 21*x + 18) - 17/8*log(x + 3) + 2*log
(x + 2) + 1/8*log(x + 1)
```

Fricas [B] time = 0.988787, size = 239, normalized size = 5.2

$$\frac{18\,x^2 - 17\left(x^3 + 8\,x^2 + 21\,x + 18\right)\log\left(x + 3\right) + 16\left(x^3 + 8\,x^2 + 21\,x + 18\right)\log\left(x + 2\right) + \left(x^3 + 8\,x^2 + 21\,x + 18\right)\log\left(x + 1\right.}{8\left(x^3 + 8\,x^2 + 21\,x + 18\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(2+x)^2/(3+x)^3,x, algorithm="fricas")
```

```
[Out] 1/8*(18*x^2 - 17*(x^3 + 8*x^2 + 21*x + 18)*log(x + 3) + 16*(x^3 + 8*x^2 + 2
1*x + 18)*log(x + 2) + (x^3 + 8*x^2 + 21*x + 18)*log(x + 1) + 100*x + 136)/
(x^3 + 8*x^2 + 21*x + 18)
```

Sympy [A] time = 0.17561, size = 46, normalized size = 1.

$$\frac{9x^2 + 50x + 68}{4x^3 + 32x^2 + 84x + 72} + \frac{\log\left(x + 1\right)}{8} + 2\log\left(x + 2\right) - \frac{17\log\left(x + 3\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(2+x)**2/(3+x)**3,x)
```

```
[Out] (9*x**2 + 50*x + 68)/(4*x**3 + 32*x**2 + 84*x + 72) + log(x + 1)/8 + 2*log(
x + 2) - 17*log(x + 3)/8
```

Giac [A] time = 1.1537, size = 70, normalized size = 1.52

$$\frac{1}{x + 2} - \frac{\frac{7}{x+2} + 6}{4\left(\frac{1}{x+2} + 1\right)^2} + \frac{1}{8}\,\log\left(\left|-\frac{1}{x + 2} + 1\right|\right) - \frac{17}{8}\,\log\left(\left|-\frac{1}{x + 2} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(2+x)^2/(3+x)^3,x, algorithm="giac")
```

```
[Out] 1/(x + 2) - 1/4*(7/(x + 2) + 6)/(1/(x + 2) + 1)^2 + 1/8*log(abs(-1/(x + 2)
+ 1)) - 17/8*log(abs(-1/(x + 2) - 1))
```

3.454 $\int \frac{x}{-1+x^2} dx$

Optimal. Leaf size=12

$$\frac{1}{2}\log\left(1-x^2\right)$$

```
[Out] Log[1 - x^2]/2
```

Rubi [A] time = 0.0021809, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {260}

$$\frac{1}{2}\log\left(1-x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/(-1 + x^2),x]
```

```
[Out] Log[1 - x^2]/2
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{x}{-1+x^2}\,dx = \frac{1}{2}\log\left(1-x^2\right)$$

Mathematica [A] time = 0.0013138, size = 10, normalized size = 0.83

$$\frac{1}{2}\log\left(x^2-1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(-1 + x^2),x]
```

```
[Out] Log[-1 + x^2]/2
```

Maple [A] time = 0.002, size = 14, normalized size = 1.2

$$\frac{\ln(x-1)}{2}+\frac{\ln(1+x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2-1),x)
```

```
[Out] 1/2*ln(x-1)+1/2*ln(1+x)
```

Maxima [A] time = 0.970928, size = 11, normalized size = 0.92

$$\frac{1}{2}\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2-1),x, algorithm="maxima")
```

```
[Out] 1/2*log(x^2 - 1)
```

Fricas [A] time = 0.93465, size = 24, normalized size = 2.

$$\frac{1}{2}\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2-1),x, algorithm="fricas")
```

```
[Out] 1/2*log(x^2 - 1)
```

Sympy [A] time = 0.072212, size = 7, normalized size = 0.58

$$\frac{\log(x^2-1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**2-1),x)
```

```
[Out] log(x**2 - 1)/2
```

Giac [A] time = 1.14657, size = 12, normalized size = 1.

$$\frac{1}{2}\log\left(|x^2-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2-1),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(x^2 - 1))
```

3.455 $\int \frac{1}{\left(-1+x^2\right)^2} \, dx$

Optimal. Leaf size=21

$$\frac{x}{2\left(1-x^2\right)}+\frac{1}{2}\tanh^{-1}(x)$$

```
[Out] x/(2*(1 - x^2)) + ArcTanh[x]/2
```

Rubi [A] time = 0.0028625, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {199, 207}

$$\frac{x}{2\left(1-x^2\right)}+\frac{1}{2}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^2)^(-2),x]
```

```
[Out] x/(2*(1 - x^2)) + ArcTanh[x]/2
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(-1+x^2\right)^2}\,dx = \frac{x}{2\left(1-x^2\right)} - \frac{1}{2}\int \frac{1}{-1+x^2}\,dx$$
$$= \frac{x}{2\left(1-x^2\right)} + \frac{1}{2}\tanh^{-1}(x)$$

Mathematica [A] time = 0.0073377, size = 27, normalized size = 1.29

$$\frac{1}{4}\left(-\frac{2x}{x^2-1} - \log(1-x) + \log(x+1)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^2)^(-2),x]
```

```
[Out] ((-2*x)/(-1 + x^2) - Log[1 - x] + Log[1 + x])/4
```

Maple [A] time = 0.008, size = 28, normalized size = 1.3

$$-\frac{1}{4x-4} - \frac{\ln(x-1)}{4} - \frac{1}{4+4x} + \frac{\ln(1+x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2-1)^2,x)
```

```
[Out] -1/4/(x-1)-1/4*ln(x-1)-1/4/(1+x)+1/4*ln(1+x)
```

Maxima [A] time = 1.06343, size = 31, normalized size = 1.48

$$-\frac{x}{2\left(x^2-1\right)} + \frac{1}{4}\log(x+1) - \frac{1}{4}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^2,x, algorithm="maxima")
```

```
[Out] -1/2*x/(x^2 - 1) + 1/4*log(x + 1) - 1/4*log(x - 1)
```

Fricas [B] time = 0.952458, size = 90, normalized size = 4.29

$$\frac{\left(x^2-1\right)\log\left(x+1\right)-\left(x^2-1\right)\log\left(x-1\right)-2\,x}{4\left(x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^2,x, algorithm="fricas")
```

```
[Out] 1/4*((x^2 - 1)*log(x + 1) - (x^2 - 1)*log(x - 1) - 2*x)/(x^2 - 1)
```

Sympy [A] time = 0.099548, size = 20, normalized size = 0.95

$$-\frac{x}{2x^2-2}-\frac{\log\left(x-1\right)}{4}+\frac{\log\left(x+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2-1)**2,x)
```

```
[Out] -x/(2*x**2 - 2) - log(x - 1)/4 + log(x + 1)/4
```

Giac [A] time = 1.18725, size = 34, normalized size = 1.62

$$-\frac{x}{2\left(x^2-1\right)}+\frac{1}{4}\log\left(|x+1|\right)-\frac{1}{4}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^2,x, algorithm="giac")
```

```
[Out] -1/2*x/(x^2 - 1) + 1/4*log(abs(x + 1)) - 1/4*log(abs(x - 1))
```

3.456 $\int \frac{x^2}{(1+x^2)^2} \, dx$

Optimal. Leaf size=19

$$\frac{1}{2}\tan^{-1}(x) - \frac{x}{2(x^2+1)}$$

```
[Out] -x/(2*(1 + x^2)) + ArcTan[x]/2
```

Rubi [A] time = 0.0037943, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {288, 203}

$$\frac{1}{2}\tan^{-1}(x) - \frac{x}{2(x^2+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(1 + x^2)^2,x]

[Out] -x/(2*(1 + x^2)) + ArcTan[x]/2
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{x^2}{\left(1+x^2\right)^2}\,dx = -\frac{x}{2\left(1+x^2\right)} + \frac{1}{2}\int \frac{1}{1+x^2}\,dx$$
$$= -\frac{x}{2\left(1+x^2\right)} + \frac{1}{2}\tan^{-1}(x)$$

Mathematica [A] time = 0.0078803, size = 19, normalized size = 1.

$$\frac{1}{2}\tan^{-1}(x) - \frac{x}{2\left(x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(1 + x^2)^2,x]
```

```
[Out] -x/(2*(1 + x^2)) + ArcTan[x]/2
```

Maple [A] time = 0.004, size = 16, normalized size = 0.8

$$-\frac{x}{2x^2+2} + \frac{\arctan(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^2+1)^2,x)
```

```
[Out] -1/2*x/(x^2+1)+1/2*arctan(x)
```

Maxima [A] time = 1.5673, size = 20, normalized size = 1.05

$$-\frac{x}{2\left(x^2+1\right)} + \frac{1}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] -1/2*x/(x^2 + 1) + 1/2*arctan(x)
```

Fricas [A] time = 0.948795, size = 55, normalized size = 2.89

$$\frac{(x^2 + 1)\arctan(x) - x}{2(x^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] 1/2*((x^2 + 1)*arctan(x) - x)/(x^2 + 1)
```

Sympy [A] time = 0.093209, size = 12, normalized size = 0.63

$$-\frac{x}{2x^2 + 2} + \frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**2+1)**2,x)
```

```
[Out] -x/(2*x**2 + 2) + atan(x)/2
```

Giac [A] time = 1.22345, size = 20, normalized size = 1.05

$$-\frac{x}{2(x^2 + 1)} + \frac{1}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] -1/2*x/(x^2 + 1) + 1/2*arctan(x)
```

3.457 $\int \frac{1}{2+3x} dx$

Optimal. Leaf size=10

$$\frac{1}{3}\log(3x+2)$$

```
[Out] Log[2 + 3*x]/3
```

Rubi [A] time = 0.0009813, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {31}

$$\frac{1}{3}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x)^(-1),x]
```

```
[Out] Log[2 + 3*x]/3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{2+3x} dx = \frac{1}{3}\log(2+3x)$$

Mathematica [A] time = 0.0008833, size = 10, normalized size = 1.

$$\frac{1}{3}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x)^(-1),x]
```

```
[Out] Log[2 + 3*x]/3
```

Maple [A] time = 0., size = 9, normalized size = 0.9

$$\frac{\ln(2+3x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2+3*x),x)
```

```
[Out] 1/3*ln(2+3*x)
```

Maxima [A] time = 1.13284, size = 11, normalized size = 1.1

$$\frac{1}{3}\log(3x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*x),x, algorithm="maxima")
```

```
[Out] 1/3*log(3*x + 2)
```

Fricas [A] time = 1.19238, size = 24, normalized size = 2.4

$$\frac{1}{3}\log(3x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*x),x, algorithm="fricas")
```

```
[Out] 1/3*log(3*x + 2)
```

Sympy [A] time = 0.052239, size = 7, normalized size = 0.7

$$\frac{\log(3x+2)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*x),x)
```

```
[Out] log(3*x + 2)/3
```

Giac [A] time = 1.23442, size = 12, normalized size = 1.2

$$\frac{1}{3}\log(|3x+2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*x),x, algorithm="giac")
```

```
[Out] 1/3*log(abs(3*x + 2))
```

3.458 $\int \frac{1}{a^2+x^2}\,dx$

Optimal. Leaf size=10

$$\frac{\tan^{-1}\left(\frac{x}{a}\right)}{a}$$

```
[Out] ArcTan[x/a]/a
```

Rubi [A] time = 0.0025235, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {203}

$$\frac{\tan^{-1}\left(\frac{x}{a}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + x^2)^(-1),x]
```

```
[Out] ArcTan[x/a]/a
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{a^2+x^2}\,dx=\frac{\tan^{-1}\left(\frac{x}{a}\right)}{a}$$

Mathematica [A] time = 0.0022225, size = 10, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{x}{a}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + x^2)^(-1),x]
```

```
[Out] ArcTan[x/a]/a
```

Maple [A] time = 0.003, size = 11, normalized size = 1.1

$$\frac{1}{a}\arctan\left(\frac{x}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2+x^2),x)
```

```
[Out] arctan(x/a)/a
```

Maxima [A] time = 1.68187, size = 14, normalized size = 1.4

$$\frac{\arctan\left(\frac{x}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+x^2),x, algorithm="maxima")
```

```
[Out] arctan(x/a)/a
```

Fricas [A] time = 1.31858, size = 20, normalized size = 2.

$$\frac{\arctan\left(\frac{x}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+x^2),x, algorithm="fricas")
```

```
[Out] arctan(x/a)/a
```

Sympy [C] time = 0.101606, size = 20, normalized size = 2.

$$\frac{-\frac{i\log(-ia+x)}{2}+\frac{i\log(ia+x)}{2}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2+x**2),x)
```

```
[Out] (-I*log(-I*a + x)/2 + I*log(I*a + x)/2)/a
```

Giac [A] time = 1.2022, size = 14, normalized size = 1.4

$$\frac{\arctan\left(\frac{x}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+x^2),x, algorithm="giac")
```

```
[Out] arctan(x/a)/a
```

3.459 $\int \frac{1}{a+bx^2}\,dx$

Optimal. Leaf size=24

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

```
[Out] ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b])
```

Rubi [A] time = 0.0068038, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{a+bx^2}\,dx = \frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Mathematica [A] time = 0.0044932, size = 24, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b])
```

Maple [A] time = 0.002, size = 16, normalized size = 0.7

$$\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a),x)
```

```
[Out] 1/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.28787, size = 151, normalized size = 6.29

$$\left[-\frac{\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}x-a}{bx^2+a}\right)}{2\,ab}, \frac{\sqrt{ab}\arctan\left(\frac{\sqrt{ab}x}{a}\right)}{ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a))/(a*b), sqrt(
a*b)*arctan(sqrt(a*b)*x/a)/(a*b)]
```

Sympy [B] time = 0.121123, size = 53, normalized size = 2.21

$$-\frac{\sqrt{-\frac{1}{ab}}\log\left(-a\sqrt{-\frac{1}{ab}}+x\right)}{2}+\frac{\sqrt{-\frac{1}{ab}}\log\left(a\sqrt{-\frac{1}{ab}}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a),x)
```

```
[Out] -sqrt(-1/(a*b))*log(-a*sqrt(-1/(a*b)) + x)/2 + sqrt(-1/(a*b))*log(a*sqrt(-1
/(a*b)) + x)/2
```

Giac [A] time = 1.1088, size = 20, normalized size = 0.83

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a),x, algorithm="giac")
```

```
[Out] arctan(b*x/sqrt(a*b))/sqrt(a*b)
```

3.460 $\int \frac{1}{2-x+x^2}\,dx$

Optimal. Leaf size=19

$$-\frac{2\tan^{-1}\left(\frac{1-2x}{\sqrt{7}}\right)}{\sqrt{7}}$$

```
[Out] (-2*ArcTan[(1 - 2*x)/Sqrt[7]])/Sqrt[7]
```

Rubi [A] time = 0.0111904, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {618, 204}

$$-\frac{2\tan^{-1}\left(\frac{1-2x}{\sqrt{7}}\right)}{\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 - x + x^2)^(-1),x]
```

```
[Out] (-2*ArcTan[(1 - 2*x)/Sqrt[7]])/Sqrt[7]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{2-x+x^2}\,dx = -\left(2\,\text{Subst}\left(\int \frac{1}{-7-x^2}\,dx, x, -1+2x\right)\right)$$
$$= -\frac{2\tan^{-1}\left(\frac{1-2x}{\sqrt{7}}\right)}{\sqrt{7}}$$

Mathematica [A] time = 0.0062284, size = 19, normalized size = 1.

$$\frac{2\tan^{-1}\left(\frac{2x-1}{\sqrt{7}}\right)}{\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - x + x^2)^(-1),x]
```

```
[Out] (2*ArcTan[(-1 + 2*x)/Sqrt[7]])/Sqrt[7]
```

Maple [A] time = 0.003, size = 17, normalized size = 0.9

$$\frac{2\sqrt{7}}{7}\arctan\left(\frac{(2x-1)\sqrt{7}}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2-x+2),x)
```

```
[Out] 2/7*7^(1/2)*arctan(1/7*(2*x-1)*7^(1/2))
```

Maxima [A] time = 1.60247, size = 22, normalized size = 1.16

$$\frac{2}{7}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(2x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-x+2),x, algorithm="maxima")
```

```
[Out] 2/7*sqrt(7)*arctan(1/7*sqrt(7)*(2*x - 1))
```

Fricas [A] time = 1.23032, size = 58, normalized size = 3.05

$$\frac{2}{7}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(2x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-x+2),x, algorithm="fricas")
```

```
[Out] 2/7*sqrt(7)*arctan(1/7*sqrt(7)*(2*x - 1))
```

Sympy [A] time = 0.096842, size = 26, normalized size = 1.37

$$\frac{2\sqrt{7}\operatorname{atan}\left(\frac{2\sqrt{7}x}{7}-\frac{\sqrt{7}}{7}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2-x+2),x)
```

```
[Out] 2*sqrt(7)*atan(2*sqrt(7)*x/7 - sqrt(7)/7)/7
```

Giac [A] time = 1.12003, size = 22, normalized size = 1.16

$$\frac{2}{7}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(2x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-x+2),x, algorithm="giac")
```

```
[Out] 2/7*sqrt(7)*arctan(1/7*sqrt(7)*(2*x - 1))
```

3.461 $\int x^2 \left(4-x^2\right)^2 \, dx$

Optimal. Leaf size=22

$$\frac{x^7}{7}-\frac{8x^5}{5}+\frac{16x^3}{3}$$

```
[Out] (16*x^3)/3 - (8*x^5)/5 + x^7/7
```

Rubi [A] time = 0.0068336, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$\frac{x^7}{7}-\frac{8x^5}{5}+\frac{16x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(4 - x^2)^2,x]
```

```
[Out] (16*x^3)/3 - (8*x^5)/5 + x^7/7
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^2 \left(4-x^2\right)^2 \, dx &= \int \left(16x^2-8x^4+x^6\right) dx\\ &= \frac{16x^3}{3}-\frac{8x^5}{5}+\frac{x^7}{7}\end{aligned}$$

Mathematica [A] time = 0.0006652, size = 22, normalized size = 1.

$$\frac{x^7}{7}-\frac{8x^5}{5}+\frac{16x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(4 - x^2)^2,x]
```

```
[Out] (16*x^3)/3 - (8*x^5)/5 + x^7/7
```

Maple [A] time = 0., size = 17, normalized size = 0.8

$$\frac{16\,x^{3}}{3}-\frac{8\,x^{5}}{5}+\frac{x^{7}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-x^2+4)^2,x)
```

```
[Out] 16/3*x^3-8/5*x^5+1/7*x^7
```

Maxima [A] time = 1.04056, size = 22, normalized size = 1.

$$\frac{1}{7}\,x^{7}-\frac{8}{5}\,x^{5}+\frac{16}{3}\,x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^2+4)^2,x, algorithm="maxima")
```

```
[Out] 1/7*x^7 - 8/5*x^5 + 16/3*x^3
```

Fricas [A] time = 1.1137, size = 41, normalized size = 1.86

$$\frac{1}{7}x^{7}-\frac{8}{5}x^{5}+\frac{16}{3}x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^2+4)^2,x, algorithm="fricas")
```

```
[Out] 1/7*x^7 - 8/5*x^5 + 16/3*x^3
```

Sympy [A] time = 0.052004, size = 17, normalized size = 0.77

$$\frac{x^7}{7} - \frac{8x^5}{5} + \frac{16x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-x**2+4)**2,x)
```

```
[Out] x**7/7 - 8*x**5/5 + 16*x**3/3
```

Giac [A] time = 1.11976, size = 22, normalized size = 1.

$$\frac{1}{7}\,x^7 - \frac{8}{5}\,x^5 + \frac{16}{3}\,x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^2+4)^2,x, algorithm="giac")
```

```
[Out] 1/7*x^7 - 8/5*x^5 + 16/3*x^3
```

3.462 $\int x\left(1-x^3\right)^2\,dx$

Optimal. Leaf size=22

$$\frac{x^8}{8}-\frac{2x^5}{5}+\frac{x^2}{2}$$

```
[Out] x^2/2 - (2*x^5)/5 + x^8/8
```

Rubi [A] time = 0.0053139, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {270}

$$\frac{x^8}{8}-\frac{2x^5}{5}+\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(1 - x^3)^2,x]
```

```
[Out] x^2/2 - (2*x^5)/5 + x^8/8
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x\left(1-x^3\right)^2\,dx &= \int \left(x-2x^4+x^7\right)\,dx\\ &= \frac{x^2}{2}-\frac{2x^5}{5}+\frac{x^8}{8}\end{aligned}$$

Mathematica [A] time = 0.0007037, size = 22, normalized size = 1.

$$\frac{x^8}{8}-\frac{2x^5}{5}+\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(1 - x^3)^2,x]
```

```
[Out] x^2/2 - (2*x^5)/5 + x^8/8
```

Maple [A] time = 0., size = 17, normalized size = 0.8

$$\frac{x^2}{2} - \frac{2\,x^5}{5} + \frac{x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-x^3+1)^2,x)
```

```
[Out] 1/2*x^2-2/5*x^5+1/8*x^8
```

Maxima [A] time = 1.1042, size = 22, normalized size = 1.

$$\frac{1}{8}\,x^8 - \frac{2}{5}\,x^5 + \frac{1}{2}\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^3+1)^2,x, algorithm="maxima")
```

```
[Out] 1/8*x^8 - 2/5*x^5 + 1/2*x^2
```

Fricas [A] time = 1.03894, size = 39, normalized size = 1.77

$$\frac{1}{8}x^8 - \frac{2}{5}x^5 + \frac{1}{2}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^3+1)^2,x, algorithm="fricas")
```

```
[Out] 1/8*x^8 - 2/5*x^5 + 1/2*x^2
```

Sympy [A] time = 0.051905, size = 15, normalized size = 0.68

$$\frac{x^8}{8} - \frac{2x^5}{5} + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x**3+1)**2,x)
```

```
[Out] x**8/8 - 2*x**5/5 + x**2/2
```

Giac [A] time = 1.10453, size = 22, normalized size = 1.

$$\frac{1}{8}\,x^8 - \frac{2}{5}\,x^5 + \frac{1}{2}\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^3+1)^2,x, algorithm="giac")
```

```
[Out] 1/8*x^8 - 2/5*x^5 + 1/2*x^2
```

3.463 $\int \frac{-4+5x^2+x^3}{x^2} \, dx$

Optimal. Leaf size=16

$$\frac{x^2}{2} + 5x + \frac{4}{x}$$

```
[Out] 4/x + 5*x + x^2/2
```

Rubi [A] time = 0.0050837, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {14}

$$\frac{x^2}{2} + 5x + \frac{4}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(-4 + 5*x^2 + x^3)/x^2,x]
```

```
[Out] 4/x + 5*x + x^2/2
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{-4+5x^2+x^3}{x^2} \, dx &= \int \left(5 - \frac{4}{x^2} + x\right) dx \\ &= \frac{4}{x} + 5x + \frac{x^2}{2}\end{aligned}$$

Mathematica [A] time = 0.0009091, size = 16, normalized size = 1.

$$\frac{x^2}{2} + 5x + \frac{4}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-4 + 5*x^2 + x^3)/x^2,x]
```

```
[Out] 4/x + 5*x + x^2/2
```

Maple [A] time = 0.003, size = 15, normalized size = 0.9

$$4\,x^{-1}+5\,x+\frac{x^{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+5*x^2-4)/x^2,x)
```

```
[Out] 4/x+5*x+1/2*x^2
```

Maxima [A] time = 1.04251, size = 19, normalized size = 1.19

$$\frac{1}{2}\,x^{2}+5\,x+\frac{4}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+5*x^2-4)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*x^2 + 5*x + 4/x
```

Fricas [A] time = 1.20301, size = 35, normalized size = 2.19

$$\frac{x^{3}+10\,x^{2}+8}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+5*x^2-4)/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*(x^3 + 10*x^2 + 8)/x
```

Sympy [A] time = 0.065589, size = 10, normalized size = 0.62

$$\frac{x^2}{2} + 5x + \frac{4}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+5*x**2-4)/x**2,x)
```

```
[Out] x**2/2 + 5*x + 4/x
```

Giac [A] time = 1.10936, size = 19, normalized size = 1.19

$$\frac{1}{2}\,x^2 + 5\,x + \frac{4}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+5*x^2-4)/x^2,x, algorithm="giac")
```

```
[Out] 1/2*x^2 + 5*x + 4/x
```

3.464 $\int \frac{-1+x}{3-4x+3x^2} \, dx$

Optimal. Leaf size=37

$$\frac{1}{6}\log\left(3x^2-4x+3\right)+\frac{\tan^{-1}\left(\frac{2-3x}{\sqrt{5}}\right)}{3\sqrt{5}}$$

```
[Out] ArcTan[(2 - 3*x)/Sqrt[5]]/(3*Sqrt[5]) + Log[3 - 4*x + 3*x^2]/6
```

Rubi [A] time = 0.0208234, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {634, 618, 204, 628}

$$\frac{1}{6}\log\left(3x^2-4x+3\right)+\frac{\tan^{-1}\left(\frac{2-3x}{\sqrt{5}}\right)}{3\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x)/(3 - 4*x + 3*x^2),x]
```

```
[Out] ArcTan[(2 - 3*x)/Sqrt[5]]/(3*Sqrt[5]) + Log[3 - 4*x + 3*x^2]/6
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+x}{3-4x+3x^2}\,dx &= \frac{1}{6}\int \frac{-4+6x}{3-4x+3x^2}\,dx-\frac{1}{3}\int \frac{1}{3-4x+3x^2}\,dx\\ &= \frac{1}{6}\log\left(3-4x+3x^2\right)+\frac{2}{3}\,\text{Subst}\left(\int \frac{1}{-20-x^2}\,dx,x,-4+6x\right)\\ &= \frac{\tan^{-1}\left(\frac{2-3x}{\sqrt{5}}\right)}{3\sqrt{5}}+\frac{1}{6}\log\left(3-4x+3x^2\right)\end{aligned}$$

Mathematica [A] time = 0.0100448, size = 37, normalized size = 1.

$$\frac{1}{6}\log\left(3x^2-4x+3\right)-\frac{\tan^{-1}\left(\frac{3x-2}{\sqrt{5}}\right)}{3\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x)/(3 - 4*x + 3*x^2),x]
```

```
[Out] -ArcTan[(-2 + 3*x)/Sqrt[5]]/(3*Sqrt[5]) + Log[3 - 4*x + 3*x^2]/6
```

Maple [A] time = 0.004, size = 31, normalized size = 0.8

$$\frac{\ln\left(3\,x^2-4\,x+3\right)}{6}-\frac{\sqrt{5}}{15}\arctan\left(\frac{(6\,x-4)\sqrt{5}}{10}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-1)/(3*x^2-4*x+3),x)
```

```
[Out] 1/6*ln(3*x^2-4*x+3)-1/15*5^(1/2)*arctan(1/10*(6*x-4)*5^(1/2))
```

Maxima [A] time = 1.52259, size = 41, normalized size = 1.11

$$-\frac{1}{15}\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}(3x-2)\right)+\frac{1}{6}\log\left(3x^2-4x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(3*x^2-4*x+3),x, algorithm="maxima")
```

```
[Out] -1/15*sqrt(5)*arctan(1/5*sqrt(5)*(3*x - 2)) + 1/6*log(3*x^2 - 4*x + 3)
```

Fricas [A] time = 1.15898, size = 97, normalized size = 2.62

$$-\frac{1}{15}\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}(3x-2)\right)+\frac{1}{6}\log\left(3x^2-4x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(3*x^2-4*x+3),x, algorithm="fricas")
```

```
[Out] -1/15*sqrt(5)*arctan(1/5*sqrt(5)*(3*x - 2)) + 1/6*log(3*x^2 - 4*x + 3)
```

Sympy [A] time = 0.106238, size = 39, normalized size = 1.05

$$\frac{\log\left(x^2-\frac{4x}{3}+1\right)}{6}-\frac{\sqrt{5}\operatorname{atan}\left(\frac{3\sqrt{5}x}{5}-\frac{2\sqrt{5}}{5}\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(3*x**2-4*x+3),x)
```

```
[Out] log(x**2 - 4*x/3 + 1)/6 - sqrt(5)*atan(3*sqrt(5)*x/5 - 2*sqrt(5)/5)/15
```

<u>Giac</u> [A] time = 1.08268, size = 41, normalized size = 1.11

$$-\frac{1}{15}\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}(3x-2)\right)+\frac{1}{6}\log\left(3x^2-4x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(3*x^2-4*x+3),x, algorithm="giac")
```

```
[Out] -1/15*sqrt(5)*arctan(1/5*sqrt(5)*(3*x - 2)) + 1/6*log(3*x^2 - 4*x + 3)
```

3.465 $\int (2+x^3)^2\,dx$

Optimal. Leaf size=14

$$\frac{x^7}{7}+x^4+4x$$

```
[Out] 4*x + x^4 + x^7/7
```

Rubi [A] time = 0.0036466, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {194}

$$\frac{x^7}{7}+x^4+4x$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x^3)^2,x]
```

```
[Out] 4*x + x^4 + x^7/7
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int (2+x^3)^2\,dx &= \int (4+4x^3+x^6)\,dx\\ &= 4x+x^4+\frac{x^7}{7}\end{aligned}$$

Mathematica [A] time = 0.0004188, size = 14, normalized size = 1.

$$\frac{x^7}{7}+x^4+4x$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x^3)^2,x]
```

```
[Out] 4*x + x^4 + x^7/7
```

Maple [A] time = 0.001, size = 13, normalized size = 0.9

$$4x + x^4 + \frac{x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+2)^2,x)
```

```
[Out] 4*x+x^4+1/7*x^7
```

Maxima [A] time = 1.04816, size = 16, normalized size = 1.14

$$\frac{1}{7}x^7 + x^4 + 4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+2)^2,x, algorithm="maxima")
```

```
[Out] 1/7*x^7 + x^4 + 4*x
```

Fricas [A] time = 1.0112, size = 28, normalized size = 2.

$$\frac{1}{7}x^7 + x^4 + 4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+2)^2,x, algorithm="fricas")
```

```
[Out] 1/7*x^7 + x^4 + 4*x
```

Sympy [A] time = 0.049103, size = 10, normalized size = 0.71

$$\frac{x^7}{7} + x^4 + 4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+2)**2,x)
```

```
[Out] x**7/7 + x**4 + 4*x
```

Giac [A] time = 1.12614, size = 16, normalized size = 1.14

$$\frac{1}{7}\,x^7 + x^4 + 4\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+2)^2,x, algorithm="giac")
```

```
[Out] 1/7*x^7 + x^4 + 4*x
```

3.466 $\int \frac{-4+x^2}{2+x} \, dx$

Optimal. Leaf size=11

$$\frac{x^2}{2} - 2x$$

```
[Out] -2*x + x^2/2
```

Rubi [A] time = 0.0037104, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {627}

$$\frac{x^2}{2} - 2x$$

Antiderivative was successfully verified.

```
[In] Int[(-4 + x^2)/(2 + x),x]
```

```
[Out] -2*x + x^2/2
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rubi steps

$$\begin{aligned} \int \frac{-4 + x^2}{2 + x} \, dx &= \int (-2 + x) \, dx \\ &= -2x + \frac{x^2}{2} \end{aligned}$$

Mathematica [A] time = 0.0003356, size = 11, normalized size = 1.

$$\frac{x^2}{2} - 2x$$

Antiderivative was successfully verified.

```
[In] Integrate[(-4 + x^2)/(2 + x),x]
```

```
[Out] -2*x + x^2/2
```

Maple [A] time = 0., size = 10, normalized size = 0.9

$$-2x+\frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-4)/(2+x),x)
```

```
[Out] -2*x+1/2*x^2
```

Maxima [A] time = 1.05379, size = 12, normalized size = 1.09

$$\frac{1}{2}x^2-2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-4)/(2+x),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - 2*x
```

Fricas [A] time = 1.13777, size = 20, normalized size = 1.82

$$\frac{1}{2}x^2-2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-4)/(2+x),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 - 2*x
```

Sympy [A] time = 0.053813, size = 7, normalized size = 0.64

$$\frac{x^2}{2} - 2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-4)/(2+x),x)
```

```
[Out] x**2/2 - 2*x
```

Giac [A] time = 1.10592, size = 12, normalized size = 1.09

$$\frac{1}{2}\,x^2 - 2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-4)/(2+x),x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 2*x
```

3.467 $\int \frac{1}{(2+x)(1+x^2)}\,dx$

Optimal. Leaf size=25

$$-\frac{1}{10}\log\left(x^2+1\right)+\frac{1}{5}\log(x+2)+\frac{2}{5}\tan^{-1}(x)$$

```
[Out] (2*ArcTan[x])/5 + Log[2 + x]/5 - Log[1 + x^2]/10
```

Rubi [A] time = 0.0096625, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {706, 31, 635, 203, 260}

$$-\frac{1}{10}\log\left(x^2+1\right)+\frac{1}{5}\log(x+2)+\frac{2}{5}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/((2 + x)*(1 + x^2)),x]

[Out] (2*ArcTan[x])/5 + Log[2 + x]/5 - Log[1 + x^2]/10
```

Rule 706

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(2+x)\left(1+x^2\right)}\,dx &= \frac{1}{5}\int \frac{1}{2+x}\,dx+\frac{1}{5}\int \frac{2-x}{1+x^2}\,dx\\
&= \frac{1}{5}\log(2+x)-\frac{1}{5}\int \frac{x}{1+x^2}\,dx+\frac{2}{5}\int \frac{1}{1+x^2}\,dx\\
&= \frac{2}{5}\tan^{-1}(x)+\frac{1}{5}\log(2+x)-\frac{1}{10}\log\left(1+x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.0059501, size = 25, normalized size = 1.

$$-\frac{1}{10}\log\left(x^2+1\right)+\frac{1}{5}\log(x+2)+\frac{2}{5}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((2 + x)*(1 + x^2)),x]
```

```
[Out] (2*ArcTan[x])/5 + Log[2 + x]/5 - Log[1 + x^2]/10
```

Maple [A] time = 0.004, size = 20, normalized size = 0.8

$$\frac{2\arctan(x)}{5}+\frac{\ln(2+x)}{5}-\frac{\ln\left(x^2+1\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2+x)/(x^2+1),x)
```

```
[Out] 2/5*arctan(x)+1/5*ln(2+x)-1/10*ln(x^2+1)
```

Maxima [A] time = 1.60552, size = 26, normalized size = 1.04

$$\frac{2}{5}\arctan(x) - \frac{1}{10}\log\left(x^2+1\right) + \frac{1}{5}\log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+x)/(x^2+1),x, algorithm="maxima")
```

```
[Out] 2/5*arctan(x) - 1/10*log(x^2 + 1) + 1/5*log(x + 2)
```

Fricas [A] time = 1.36182, size = 70, normalized size = 2.8

$$\frac{2}{5}\arctan(x) - \frac{1}{10}\log\left(x^2+1\right) + \frac{1}{5}\log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+x)/(x^2+1),x, algorithm="fricas")
```

```
[Out] 2/5*arctan(x) - 1/10*log(x^2 + 1) + 1/5*log(x + 2)
```

Sympy [A] time = 0.11747, size = 20, normalized size = 0.8

$$\frac{\log(x+2)}{5} - \frac{\log\left(x^2+1\right)}{10} + \frac{2\,\mathrm{atan}(x)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+x)/(x**2+1),x)
```

```
[Out] log(x + 2)/5 - log(x**2 + 1)/10 + 2*atan(x)/5
```

Giac [A] time = 1.15265, size = 27, normalized size = 1.08

$$\frac{2}{5}\arctan\left(x\right) - \frac{1}{10}\log\left(x^{2} + 1\right) + \frac{1}{5}\log\left(|x + 2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+x)/(x^2+1),x, algorithm="giac")
```

```
[Out] 2/5*arctan(x) - 1/10*log(x^2 + 1) + 1/5*log(abs(x + 2))
```

3.468 $\int \frac{1}{(1+x)(1+x^2)} dx$

Optimal. Leaf size=25

$$-\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{2}\log(x+1)+\frac{1}{2}\tan^{-1}(x)$$

```
[Out] ArcTan[x]/2 + Log[1 + x]/2 - Log[1 + x^2]/4
```

Rubi [A] time = 0.0101712, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {706, 31, 635, 203, 260}

$$-\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{2}\log(x+1)+\frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x)*(1 + x^2)),x]
```

```
[Out] ArcTan[x]/2 + Log[1 + x]/2 - Log[1 + x^2]/4
```

Rule 706

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1+x)\left(1+x^2\right)}\,dx &= \frac{1}{2}\int \frac{1}{1+x}\,dx + \frac{1}{2}\int \frac{1-x}{1+x^2}\,dx\\ &= \frac{1}{2}\log(1+x) + \frac{1}{2}\int \frac{1}{1+x^2}\,dx - \frac{1}{2}\int \frac{x}{1+x^2}\,dx\\ &= \frac{1}{2}\tan^{-1}(x) + \frac{1}{2}\log(1+x) - \frac{1}{4}\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.005603, size = 25, normalized size = 1.

$$-\frac{1}{4}\log\left(x^2+1\right) + \frac{1}{2}\log(x+1) + \frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x)*(1 + x^2)),x]
```

```
[Out] ArcTan[x]/2 + Log[1 + x]/2 - Log[1 + x^2]/4
```

Maple [A] time = 0.005, size = 20, normalized size = 0.8

$$\frac{\arctan(x)}{2} + \frac{\ln(1+x)}{2} - \frac{\ln\left(x^2+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+x)/(x^2+1),x)
```

```
[Out] 1/2*arctan(x)+1/2*ln(1+x)-1/4*ln(x^2+1)
```

Maxima [A] time = 1.62019, size = 26, normalized size = 1.04

$$\frac{1}{2}\arctan(x)-\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{2}\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x^2+1),x, algorithm="maxima")
```

```
[Out] 1/2*arctan(x) - 1/4*log(x^2 + 1) + 1/2*log(x + 1)
```

Fricas [A] time = 1.17692, size = 69, normalized size = 2.76

$$\frac{1}{2}\arctan(x)-\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{2}\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x^2+1),x, algorithm="fricas")
```

```
[Out] 1/2*arctan(x) - 1/4*log(x^2 + 1) + 1/2*log(x + 1)
```

Sympy [A] time = 0.112937, size = 19, normalized size = 0.76

$$\frac{\log(x+1)}{2}-\frac{\log\left(x^2+1\right)}{4}+\frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x**2+1),x)
```

```
[Out] log(x + 1)/2 - log(x**2 + 1)/4 + atan(x)/2
```

Giac [A] time = 1.10645, size = 27, normalized size = 1.08

$$\frac{1}{2}\arctan(x) - \frac{1}{4}\log\left(x^2 + 1\right) + \frac{1}{2}\log\left(|x + 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x^2+1),x, algorithm="giac")
```

```
[Out] 1/2*arctan(x) - 1/4*log(x^2 + 1) + 1/2*log(abs(x + 1))
```

3.469 $\int \frac{x}{(1+x)(1+x^2)} \, dx$

Optimal. Leaf size=25

$$\frac{1}{4}\log\left(x^2+1\right)-\frac{1}{2}\log(x+1)+\frac{1}{2}\tan^{-1}(x)$$

```
[Out] ArcTan[x]/2 - Log[1 + x]/2 + Log[1 + x^2]/4
```

Rubi [A] time = 0.0170756, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {801, 635, 203, 260}

$$\frac{1}{4}\log\left(x^2+1\right)-\frac{1}{2}\log(x+1)+\frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 + x)*(1 + x^2)),x]
```

```
[Out] ArcTan[x]/2 - Log[1 + x]/2 + Log[1 + x^2]/4
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{(1+x)\left(1+x^2\right)}\,dx &= \int \left(-\frac{1}{2(1+x)}+\frac{1+x}{2\left(1+x^2\right)}\right)dx \\
&= -\frac{1}{2}\log(1+x)+\frac{1}{2}\int \frac{1+x}{1+x^2}\,dx \\
&= -\frac{1}{2}\log(1+x)+\frac{1}{2}\int \frac{1}{1+x^2}\,dx+\frac{1}{2}\int \frac{x}{1+x^2}\,dx \\
&= \frac{1}{2}\tan^{-1}(x)-\frac{1}{2}\log(1+x)+\frac{1}{4}\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.0058424, size = 25, normalized size = 1.

$$\frac{1}{4}\log\left(x^2+1\right)-\frac{1}{2}\log(x+1)+\frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 + x)*(1 + x^2)),x]
```

```
[Out] ArcTan[x]/2 - Log[1 + x]/2 + Log[1 + x^2]/4
```

Maple [A] time = 0.004, size = 20, normalized size = 0.8

$$\frac{\arctan(x)}{2}-\frac{\ln(1+x)}{2}+\frac{\ln\left(x^2+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1+x)/(x^2+1),x)
```

```
[Out] 1/2*arctan(x)-1/2*ln(1+x)+1/4*ln(x^2+1)
```

Maxima [A] time = 1.65667, size = 26, normalized size = 1.04

$$\frac{1}{2}\arctan(x) + \frac{1}{4}\log\left(x^2+1\right) - \frac{1}{2}\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(x^2+1),x, algorithm="maxima")
```

```
[Out] 1/2*arctan(x) + 1/4*log(x^2 + 1) - 1/2*log(x + 1)
```

Fricas [A] time = 1.27835, size = 69, normalized size = 2.76

$$\frac{1}{2}\arctan(x) + \frac{1}{4}\log\left(x^2+1\right) - \frac{1}{2}\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(x^2+1),x, algorithm="fricas")
```

```
[Out] 1/2*arctan(x) + 1/4*log(x^2 + 1) - 1/2*log(x + 1)
```

Sympy [A] time = 0.109413, size = 19, normalized size = 0.76

$$-\frac{\log(x+1)}{2} + \frac{\log\left(x^2+1\right)}{4} + \frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(x**2+1),x)
```

```
[Out] -log(x + 1)/2 + log(x**2 + 1)/4 + atan(x)/2
```

Giac [A] time = 1.11849, size = 27, normalized size = 1.08

$$\frac{1}{2}\arctan(x) + \frac{1}{4}\log\left(x^2+1\right) - \frac{1}{2}\log(|x+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(x^2+1),x, algorithm="giac")
```

```
[Out] 1/2*arctan(x) + 1/4*log(x^2 + 1) - 1/2*log(abs(x + 1))
```

3.470 $\int \frac{2x+x^2}{(1+x)^2} \, dx$

Optimal. Leaf size=9

$$\frac{x^2}{x+1}$$

```
[Out] x^2/(1 + x)
```

Rubi [A] time = 0.0059376, antiderivative size = 7, normalized size of antiderivative = 0.78, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {683}

$$x+\frac{1}{x+1}$$

Antiderivative was successfully verified.

```
[In] Int[(2*x + x^2)/(1 + x)^2,x]
```

```
[Out] x + (1 + x)^(-1)
```

Rule 683

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] &&  !(EqQ[m, 3] && NeQ[p, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{2x+x^2}{(1+x)^2} \, dx &= \int \left(1-\frac{1}{(1+x)^2}\right) \, dx \\
&= x+\frac{1}{1+x}
\end{aligned}$$

Mathematica [A] time = 0.0028421, size = 7, normalized size = 0.78

$$x+\frac{1}{x+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*x + x^2)/(1 + x)^2,x]
```

```
[Out] x + (1 + x)^(-1)
```

Maple [A] time = 0.003, size = 8, normalized size = 0.9

$$x + (1 + x)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+2*x)/(1+x)^2,x)
```

```
[Out] x+1/(1+x)
```

Maxima [A] time = 1.1354, size = 9, normalized size = 1.

$$x + \frac{1}{x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x)/(1+x)^2,x, algorithm="maxima")
```

```
[Out] x + 1/(x + 1)
```

Fricas [A] time = 1.17235, size = 31, normalized size = 3.44

$$\frac{x^2 + x + 1}{x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x)/(1+x)^2,x, algorithm="fricas")
```

```
[Out] (x^2 + x + 1)/(x + 1)
```

Sympy [A] time = 0.069685, size = 5, normalized size = 0.56

$$x + \frac{1}{x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+2*x)/(1+x)**2,x)
```

```
[Out] x + 1/(x + 1)
```

Giac [A] time = 1.1462, size = 11, normalized size = 1.22

$$x + \frac{1}{x + 1} + 1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x)/(1+x)^2,x, algorithm="giac")
```

```
[Out] x + 1/(x + 1) + 1
```

3.471 $\int \frac{-10+x^2}{4+9x^2+2x^4}\,dx$

Optimal. Leaf size=22

$$\tan^{-1}\left(\frac{x}{2}\right)-\frac{3\tan^{-1}\left(\sqrt{2}x\right)}{\sqrt{2}}$$

```
[Out] ArcTan[x/2] - (3*ArcTan[Sqrt[2]*x])/Sqrt[2]
```

Rubi [A] time = 0.0106643, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {1166, 203}

$$\tan^{-1}\left(\frac{x}{2}\right)-\frac{3\tan^{-1}\left(\sqrt{2}x\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-10 + x^2)/(4 + 9*x^2 + 2*x^4),x]
```

```
[Out] ArcTan[x/2] - (3*ArcTan[Sqrt[2]*x])/Sqrt[2]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{-10+x^2}{4+9x^2+2x^4}\,dx = -\left(3\int \frac{1}{1+2x^2}\,dx\right)+4\int \frac{1}{8+2x^2}\,dx$$
$$= \tan^{-1}\left(\frac{x}{2}\right)-\frac{3\tan^{-1}\left(\sqrt{2}x\right)}{\sqrt{2}}$$

Mathematica [A] time = 0.0124791, size = 22, normalized size = 1.

$$\tan^{-1}\left(\frac{x}{2}\right)-\frac{3\tan^{-1}\left(\sqrt{2}x\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-10 + x^2)/(4 + 9*x^2 + 2*x^4),x]
```

```
[Out] ArcTan[x/2] - (3*ArcTan[Sqrt[2]*x])/Sqrt[2]
```

Maple [A] time = 0.007, size = 17, normalized size = 0.8

$$\arctan\left(\frac{x}{2}\right)-\frac{3\arctan\left(x\sqrt{2}\right)\sqrt{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-10)/(2*x^4+9*x^2+4),x)
```

```
[Out] arctan(1/2*x)-3/2*arctan(x*2^(1/2))*2^(1/2)
```

Maxima [A] time = 1.64696, size = 22, normalized size = 1.

$$-\frac{3}{2}\sqrt{2}\arctan\left(\sqrt{2}x\right)+\arctan\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-10)/(2*x^4+9*x^2+4),x, algorithm="maxima")
```

```
[Out] -3/2*sqrt(2)*arctan(sqrt(2)*x) + arctan(1/2*x)
```

Fricas [A] time = 1.27442, size = 65, normalized size = 2.95

$$-\frac{3}{2}\sqrt{2}\arctan\left(\sqrt{2}x\right)+\arctan\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-10)/(2*x^4+9*x^2+4),x, algorithm="fricas")
```

```
[Out] -3/2*sqrt(2)*arctan(sqrt(2)*x) + arctan(1/2*x)
```

Sympy [A] time = 0.132663, size = 20, normalized size = 0.91

$$\operatorname{atan}\left(\frac{x}{2}\right)-\frac{3\sqrt{2}\operatorname{atan}\left(\sqrt{2}x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-10)/(2*x**4+9*x**2+4),x)
```

```
[Out] atan(x/2) - 3*sqrt(2)*atan(sqrt(2)*x)/2
```

Giac [A] time = 1.1067, size = 22, normalized size = 1.

$$-\frac{3}{2}\sqrt{2}\arctan\left(\sqrt{2}x\right)+\arctan\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-10)/(2*x^4+9*x^2+4),x, algorithm="giac")
```

```
[Out] -3/2*sqrt(2)*arctan(sqrt(2)*x) + arctan(1/2*x)
```

3.472 $\int \frac{31+5x}{11-4x+3x^2} \, dx$

Optimal. Leaf size=37

$$\frac{5}{6}\log\left(3x^2-4x+11\right)-\frac{103\tan^{-1}\left(\frac{2-3x}{\sqrt{29}}\right)}{3\sqrt{29}}$$

```
[Out] (-103*ArcTan[(2 - 3*x)/Sqrt[29]])/(3*Sqrt[29]) + (5*Log[11 - 4*x + 3*x^2])/
6
```

Rubi [A] time = 0.0223993, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {634, 618, 204, 628}

$$\frac{5}{6}\log\left(3x^2-4x+11\right)-\frac{103\tan^{-1}\left(\frac{2-3x}{\sqrt{29}}\right)}{3\sqrt{29}}$$

Antiderivative was successfully verified.

```
[In] Int[(31 + 5*x)/(11 - 4*x + 3*x^2),x]
```

```
[Out] (-103*ArcTan[(2 - 3*x)/Sqrt[29]])/(3*Sqrt[29]) + (5*Log[11 - 4*x + 3*x^2])/
6
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{31+5x}{11-4x+3x^2}\,dx &= \frac{5}{6}\int \frac{-4+6x}{11-4x+3x^2}\,dx + \frac{103}{3}\int \frac{1}{11-4x+3x^2}\,dx\\ &= \frac{5}{6}\log\left(11-4x+3x^2\right) - \frac{206}{3}\operatorname{Subst}\left(\int \frac{1}{-116-x^2}\,dx, x, -4+6x\right)\\ &= -\frac{103\tan^{-1}\left(\frac{2-3x}{\sqrt{29}}\right)}{3\sqrt{29}} + \frac{5}{6}\log\left(11-4x+3x^2\right)\end{aligned}$$

Mathematica [A] time = 0.0136187, size = 37, normalized size = 1.

$$\frac{5}{6}\log\left(3x^2-4x+11\right)+\frac{103\tan^{-1}\left(\frac{3x-2}{\sqrt{29}}\right)}{3\sqrt{29}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(31 + 5*x)/(11 - 4*x + 3*x^2),x]
```

```
[Out] (103*ArcTan[(-2 + 3*x)/Sqrt[29]])/(3*Sqrt[29]) + (5*Log[11 - 4*x + 3*x^2])/
6
```

Maple [A] time = 0.004, size = 31, normalized size = 0.8

$$\frac{5\ln\left(3x^2-4x+11\right)}{6}+\frac{103\sqrt{29}}{87}\arctan\left(\frac{(6x-4)\sqrt{29}}{58}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((31+5*x)/(3*x^2-4*x+11),x)
```

```
[Out] 5/6*ln(3*x^2-4*x+11)+103/87*29^(1/2)*arctan(1/58*(6*x-4)*29^(1/2))
```

Maxima [A] time = 1.65553, size = 41, normalized size = 1.11

$$\frac{103}{87}\sqrt{29}\arctan\left(\frac{1}{29}\sqrt{29}(3x-2)\right)+\frac{5}{6}\log\left(3x^2-4x+11\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((31+5*x)/(3*x^2-4*x+11),x, algorithm="maxima")
```

```
[Out] 103/87*sqrt(29)*arctan(1/29*sqrt(29)*(3*x - 2)) + 5/6*log(3*x^2 - 4*x + 11)
```

Fricas [A] time = 1.16902, size = 104, normalized size = 2.81

$$\frac{103}{87}\sqrt{29}\arctan\left(\frac{1}{29}\sqrt{29}(3x-2)\right)+\frac{5}{6}\log\left(3x^2-4x+11\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((31+5*x)/(3*x^2-4*x+11),x, algorithm="fricas")
```

```
[Out] 103/87*sqrt(29)*arctan(1/29*sqrt(29)*(3*x - 2)) + 5/6*log(3*x^2 - 4*x + 11)
```

Sympy [A] time = 0.111909, size = 44, normalized size = 1.19

$$\frac{5\log\left(x^2-\frac{4x}{3}+\frac{11}{3}\right)}{6}+\frac{103\sqrt{29}\operatorname{atan}\left(\frac{3\sqrt{29}x}{29}-\frac{2\sqrt{29}}{29}\right)}{87}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((31+5*x)/(3*x**2-4*x+11),x)
```

```
[Out] 5*log(x**2 - 4*x/3 + 11/3)/6 + 103*sqrt(29)*atan(3*sqrt(29)*x/29 - 2*sqrt(2
9)/29)/87
```

Giac [A] time = 1.13874, size = 41, normalized size = 1.11

$$\frac{103}{87}\sqrt{29}\arctan\left(\frac{1}{29}\sqrt{29}(3x-2)\right)+\frac{5}{6}\log\left(3x^2-4x+11\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((31+5*x)/(3*x^2-4*x+11),x, algorithm="giac")
```

```
[Out] 103/87*sqrt(29)*arctan(1/29*sqrt(29)*(3*x - 2)) + 5/6*log(3*x^2 - 4*x + 11)
```

3.473 $\int \frac{-2+x^2+x^3}{x^4} \, dx$

Optimal. Leaf size=15

$$\frac{2}{3x^3} - \frac{1}{x} + \log(x)$$

```
[Out] 2/(3*x^3) - x^(-1) + Log[x]
```

Rubi [A] time = 0.0039203, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {14}

$$\frac{2}{3x^3} - \frac{1}{x} + \log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-2 + x^2 + x^3)/x^4,x]

[Out] 2/(3*x^3) - x^(-1) + Log[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int \frac{-2+x^2+x^3}{x^4} \, dx &= \int \left(-\frac{2}{x^4}+\frac{1}{x^2}+\frac{1}{x}\right) \, dx \\ &= \frac{2}{3x^3}-\frac{1}{x}+\log(x) \end{aligned}$$

Mathematica [A] time = 0.0018698, size = 15, normalized size = 1.

$$\frac{2}{3x^3} - \frac{1}{x} + \log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 + x^2 + x^3)/x^4,x]
```

```
[Out] 2/(3*x^3) - x^(-1) + Log[x]
```

Maple [A] time = 0.006, size = 14, normalized size = 0.9

$$\frac{2}{3\,x^{3}}-x^{-1}+\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x^2-2)/x^4,x)
```

```
[Out] 2/3/x^3-1/x+ln(x)
```

Maxima [A] time = 1.10812, size = 20, normalized size = 1.33

$$-\frac{3\,x^{2}-2}{3\,x^{3}}+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2-2)/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*(3*x^2 - 2)/x^3 + log(x)
```

Fricas [A] time = 1.1896, size = 49, normalized size = 3.27

$$\frac{3\,x^{3}\log\left(x\right)-3\,x^{2}+2}{3\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2-2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*x^3*log(x) - 3*x^2 + 2)/x^3
```

Sympy [A] time = 0.084129, size = 14, normalized size = 0.93

$$\log(x) - \frac{3x^2 - 2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x**2-2)/x**4,x)
```

```
[Out] log(x) - (3*x**2 - 2)/(3*x**3)
```

Giac [A] time = 1.16263, size = 22, normalized size = 1.47

$$-\frac{3\,x^2 - 2}{3\,x^3} + \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2-2)/x^4,x, algorithm="giac")
```

```
[Out] -1/3*(3*x^2 - 2)/x^3 + log(abs(x))
```

3.474 $\int \frac{1+x+x^3}{x^2}\,dx$

Optimal. Leaf size=15

$$\frac{x^2}{2}-\frac{1}{x}+\log(x)$$

```
[Out] -x^(-1) + x^2/2 + Log[x]
```

Rubi [A] time = 0.0035479, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {14}

$$\frac{x^2}{2}-\frac{1}{x}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + x^3)/x^2,x]
```

```
[Out] -x^(-1) + x^2/2 + Log[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x+x^3}{x^2}\,dx &= \int \left(\frac{1}{x^2}+\frac{1}{x}+x\right)dx\\ &= -\frac{1}{x}+\frac{x^2}{2}+\log(x)\end{aligned}$$

Mathematica [A] time = 0.0007965, size = 15, normalized size = 1.

$$\frac{x^2}{2}-\frac{1}{x}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + x^3)/x^2,x]
```

```
[Out] -x^(-1) + x^2/2 + Log[x]
```

Maple [A] time = 0.006, size = 14, normalized size = 0.9

$$-x^{-1}+\frac{x^{2}}{2}+\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x+1)/x^2,x)
```

```
[Out] -1/x+1/2*x^2+ln(x)
```

Maxima [A] time = 1.09369, size = 18, normalized size = 1.2

$$\frac{1}{2}\,x^{2}-\frac{1}{x}+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x+1)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - 1/x + log(x)
```

Fricas [A] time = 1.20762, size = 41, normalized size = 2.73

$$\frac{x^{3}+2\,x\log\left(x\right)-2}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x+1)/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*(x^3 + 2*x*log(x) - 2)/x
```

Sympy [A] time = 0.070176, size = 10, normalized size = 0.67

$$\frac{x^2}{2} + \log(x) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x+1)/x**2,x)
```

```
[Out] x**2/2 + log(x) - 1/x
```

Giac [A] time = 1.14995, size = 19, normalized size = 1.27

$$\frac{1}{2}x^2 - \frac{1}{x} + \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x+1)/x^2,x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 1/x + log(abs(x))
```

3.475 $\int \frac{-2+x^2}{x\left(2+x^2\right)}\,dx$

Optimal. Leaf size=11

$$\log\left(x^2+2\right)-\log(x)$$

```
[Out] -Log[x] + Log[2 + x^2]
```

Rubi [A] time = 0.0112308, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {446, 72}

$$\log\left(x^2+2\right)-\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-2 + x^2)/(x*(2 + x^2)),x]
```

```
[Out] -Log[x] + Log[2 + x^2]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \frac{-2+x^2}{x\left(2+x^2\right)} dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{-2+x}{x(2+x)} dx, x, x^2\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \left(-\frac{1}{x}+\frac{2}{2+x}\right) dx, x, x^2\right)$$
$$= -\log(x)+\log\left(2+x^2\right)$$

Mathematica [A] time = 0.0031582, size = 11, normalized size = 1.

$$\log\left(x^2+2\right)-\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 + x^2)/(x*(2 + x^2)),x]
```

```
[Out] -Log[x] + Log[2 + x^2]
```

Maple [A] time = 0.004, size = 12, normalized size = 1.1

$$-\ln(x)+\ln\left(x^2+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-2)/x/(x^2+2),x)
```

```
[Out] -ln(x)+ln(x^2+2)
```

Maxima [A] time = 1.08186, size = 18, normalized size = 1.64

$$\log\left(x^2+2\right)-\frac{1}{2}\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-2)/x/(x^2+2),x, algorithm="maxima")
```

```
[Out] log(x^2 + 2) - 1/2*log(x^2)
```

Fricas [A] time = 1.24861, size = 31, normalized size = 2.82

$$\log\left(x^2+2\right)-\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-2)/x/(x^2+2),x, algorithm="fricas")
```

```
[Out] log(x^2 + 2) - log(x)
```

Sympy [A] time = 0.088362, size = 8, normalized size = 0.73

$$-\log\left(x\right)+\log\left(x^2+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-2)/x/(x**2+2),x)
```

```
[Out] -log(x) + log(x**2 + 2)
```

Giac [A] time = 1.1265, size = 18, normalized size = 1.64

$$\log\left(x^2+2\right)-\frac{1}{2}\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-2)/x/(x^2+2),x, algorithm="giac")
```

```
[Out] log(x^2 + 2) - 1/2*log(x^2)
```

3.476 $\int (-3+x)\left(-7+4x^2\right)\,dx$

Optimal. Leaf size=22

$$-4x^3+\frac{1}{16}\left(7-4x^2\right)^2+21x$$

```
[Out] 21*x - 4*x^3 + (7 - 4*x^2)^2/16
```

Rubi [A] time = 0.0042643, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {641}

$$-4x^3+\frac{1}{16}\left(7-4x^2\right)^2+21x$$

Antiderivative was successfully verified.

```
[In] Int[(-3 + x)*(-7 + 4*x^2),x]
```

```
[Out] 21*x - 4*x^3 + (7 - 4*x^2)^2/16
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int (-3+x)\left(-7+4x^2\right)\,dx &= \frac{1}{16}\left(7-4x^2\right)^2-3\int \left(-7+4x^2\right)\,dx\\ &= 21x-4x^3+\frac{1}{16}\left(7-4x^2\right)^2\end{aligned}$$

Mathematica [A] time = 0.0009434, size = 19, normalized size = 0.86

$$x^4-4x^3-\frac{7x^2}{2}+21x$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 + x)*(-7 + 4*x^2),x]
```

```
[Out] 21*x - (7*x^2)/2 - 4*x^3 + x^4
```

Maple [A] time = 0., size = 18, normalized size = 0.8

$$x^4 - 4x^3 - \frac{7x^2}{2} + 21x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3+x)*(4*x^2-7),x)
```

```
[Out] x^4-4*x^3-7/2*x^2+21*x
```

Maxima [A] time = 1.11595, size = 23, normalized size = 1.05

$$x^4 - 4x^3 - \frac{7}{2}x^2 + 21x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+x)*(4*x^2-7),x, algorithm="maxima")
```

```
[Out] x^4 - 4*x^3 - 7/2*x^2 + 21*x
```

Fricas [A] time = 1.13045, size = 41, normalized size = 1.86

$$x^4 - 4x^3 - \frac{7}{2}x^2 + 21x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+x)*(4*x^2-7),x, algorithm="fricas")
```

```
[Out] x^4 - 4*x^3 - 7/2*x^2 + 21*x
```

Sympy [A] time = 0.051037, size = 17, normalized size = 0.77

$$x^4 - 4x^3 - \frac{7x^2}{2} + 21x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+x)*(4*x**2-7),x)
```

```
[Out] x**4 - 4*x**3 - 7*x**2/2 + 21*x
```

Giac [A] time = 1.14309, size = 23, normalized size = 1.05

$$x^4 - 4\,x^3 - \frac{7}{2}\,x^2 + 21\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+x)*(4*x^2-7),x, algorithm="giac")
```

```
[Out] x^4 - 4*x^3 - 7/2*x^2 + 21*x
```

3.477 $\int (-2+7x)^3\,dx$

Optimal. Leaf size=11

$$\frac{1}{28}(2-7x)^4$$

```
[Out] (2 - 7*x)^4/28
```

Rubi [A] time = 0.0010748, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {32}

$$\frac{1}{28}(2-7x)^4$$

Antiderivative was successfully verified.

```
[In] Int[(-2 + 7*x)^3,x]
```

```
[Out] (2 - 7*x)^4/28
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (-2+7x)^3\,dx=\frac{1}{28}(2-7x)^4$$

Mathematica [A] time = 0.0014178, size = 11, normalized size = 1.

$$\frac{1}{28}(7x-2)^4$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 + 7*x)^3,x]
```

```
[Out] (-2 + 7*x)^4/28
```

Maple [A] time = 0.002, size = 10, normalized size = 0.9

$$\frac{(-2+7x)^4}{28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2+7*x)^3,x)
```

```
[Out] 1/28*(-2+7*x)^4
```

Maxima [B] time = 1.10926, size = 26, normalized size = 2.36

$$\frac{343}{4}x^4 - 98x^3 + 42x^2 - 8x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+7*x)^3,x, algorithm="maxima")
```

```
[Out] 343/4*x^4 - 98*x^3 + 42*x^2 - 8*x
```

Fricas [B] time = 1.06116, size = 47, normalized size = 4.27

$$\frac{343}{4}x^4 - 98x^3 + 42x^2 - 8x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+7*x)^3,x, algorithm="fricas")
```

```
[Out] 343/4*x^4 - 98*x^3 + 42*x^2 - 8*x
```

Sympy [B] time = 0.052643, size = 19, normalized size = 1.73

$$\frac{343x^4}{4} - 98x^3 + 42x^2 - 8x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+7*x)**3,x)
```

```
[Out] 343*x**4/4 - 98*x**3 + 42*x**2 - 8*x
```

Giac [A] time = 1.10219, size = 12, normalized size = 1.09

$$\frac{1}{28}\,(7\,x - 2)^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+7*x)^3,x, algorithm="giac")
```

```
[Out] 1/28*(7*x - 2)^4
```

3.478 $\int \frac{-7+4x^2}{3+2x}\,dx$

Optimal. Leaf size=13

$$x^2-3x+\log(2x+3)$$

```
[Out] -3*x + x^2 + Log[3 + 2*x]
```

Rubi [A] time = 0.010276, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {697}

$$x^2-3x+\log(2x+3)$$

Antiderivative was successfully verified.

```
[In] Int[(-7 + 4*x^2)/(3 + 2*x),x]
```

```
[Out] -3*x + x^2 + Log[3 + 2*x]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{-7+4x^2}{3+2x}\,dx &= \int \left(-3+2x+\frac{2}{3+2x}\right)dx\\ &= -3x+x^2+\log(3+2x)\end{aligned}$$

Mathematica [A] time = 0.0033279, size = 16, normalized size = 1.23

$$x^2-3x+\log(2x+3)-\frac{27}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-7 + 4*x^2)/(3 + 2*x),x]
```

```
[Out] -27/4 - 3*x + x^2 + Log[3 + 2*x]
```

Maple [A] time = 0.002, size = 14, normalized size = 1.1

$$-3x + x^2 + \ln(3 + 2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2-7)/(3+2*x),x)
```

```
[Out] -3*x+x^2+ln(3+2*x)
```

Maxima [A] time = 1.15183, size = 18, normalized size = 1.38

$$x^2 - 3x + \log(2x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-7)/(3+2*x),x, algorithm="maxima")
```

```
[Out] x^2 - 3*x + log(2*x + 3)
```

Fricas [A] time = 1.22185, size = 35, normalized size = 2.69

$$x^2 - 3x + \log(2x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-7)/(3+2*x),x, algorithm="fricas")
```

```
[Out] x^2 - 3*x + log(2*x + 3)
```

Sympy [A] time = 0.071488, size = 12, normalized size = 0.92

$$x^2 - 3x + \log(2x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2-7)/(3+2*x),x)
```

```
[Out] x**2 - 3*x + log(2*x + 3)
```

Giac [A] time = 1.12069, size = 19, normalized size = 1.46

$$x^2 - 3\,x + \log(|2\,x + 3|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-7)/(3+2*x),x, algorithm="giac")
```

```
[Out] x^2 - 3*x + log(abs(2*x + 3))
```

3.479 $\int \frac{1+x}{(-1+x)x^2} dx$

Optimal. Leaf size=16

$$\frac{1}{x} + 2\log(1-x) - 2\log(x)$$

```
[Out] x^(-1) + 2*Log[1 - x] - 2*Log[x]
```

Rubi [A] time = 0.0062877, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {77}

$$\frac{1}{x} + 2\log(1-x) - 2\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)/((-1 + x)*x^2),x]
```

```
[Out] x^(-1) + 2*Log[1 - x] - 2*Log[x]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned} \int \frac{1+x}{(-1+x)x^2} dx &= \int \left(\frac{2}{-1+x} - \frac{1}{x^2} - \frac{2}{x}\right) dx \\ &= \frac{1}{x} + 2\log(1-x) - 2\log(x) \end{aligned}$$

Mathematica [A] time = 0.002357, size = 16, normalized size = 1.

$$\frac{1}{x}+2\log(1-x)-2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)/((-1 + x)*x^2),x]
```

```
[Out] x^(-1) + 2*Log[1 - x] - 2*Log[x]
```

Maple [A] time = 0.006, size = 15, normalized size = 0.9

$$2\ln(x-1)+x^{-1}-2\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(x-1)/x^2,x)
```

```
[Out] 2*ln(x-1)+1/x-2*ln(x)
```

Maxima [A] time = 1.11391, size = 19, normalized size = 1.19

$$\frac{1}{x}+2\log(x-1)-2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-1+x)/x^2,x, algorithm="maxima")
```

```
[Out] 1/x + 2*log(x - 1) - 2*log(x)
```

Fricas [A] time = 1.20865, size = 50, normalized size = 3.12

$$\frac{2x\log(x-1)-2x\log(x)+1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-1+x)/x^2,x, algorithm="fricas")
```

```
[Out] (2*x*log(x - 1) - 2*x*log(x) + 1)/x
```

Sympy [A] time = 0.09292, size = 14, normalized size = 0.88

$$-2\log(x) + 2\log(x-1) + \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-1+x)/x**2,x)
```

```
[Out] -2*log(x) + 2*log(x - 1) + 1/x
```

Giac [A] time = 1.08165, size = 22, normalized size = 1.38

$$\frac{1}{x} + 2\log(|x-1|) - 2\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-1+x)/x^2,x, algorithm="giac")
```

```
[Out] 1/x + 2*log(abs(x - 1)) - 2*log(abs(x))
```

3.480 $\int \frac{1}{4x^2+4x^3+x^4} dx$

Optimal. Leaf size=27

$$\frac{x+1}{2\left(1-(x+1)^2\right)}+\frac{1}{2}\tanh^{-1}(x+1)$$

```
[Out] (1 + x)/(2*(1 - (1 + x)^2)) + ArcTanh[1 + x]/2
```

Rubi [A] time = 0.0107282, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1106, 199, 206}

$$\frac{x+1}{2\left(1-(x+1)^2\right)}+\frac{1}{2}\tanh^{-1}(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(4*x^2 + 4*x^3 + x^4)^(-1),x]
```

```
[Out] (1 + x)/(2*(1 - (1 + x)^2)) + ArcTanh[1 + x]/2
```

Rule 1106

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{4x^2+4x^3+x^4}\,dx &= \text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^2}\,dx,x,1+x\right)\\ &= \frac{1+x}{2\left(1-(1+x)^2\right)}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,1+x\right)\\ &= \frac{1+x}{2\left(1-(1+x)^2\right)}+\frac{1}{2}\tanh^{-1}(1+x)\end{aligned}$$

Mathematica [A] time = 0.0142848, size = 26, normalized size = 0.96

$$\frac{1}{4}\left(-\frac{2(x+1)}{x(x+2)}-\log(x)+\log(x+2)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4*x^2 + 4*x^3 + x^4)^(-1),x]
```

```
[Out] ((-2*(1 + x))/(x*(2 + x)) - Log[x] + Log[2 + x])/4
```

Maple [A] time = 0.009, size = 24, normalized size = 0.9

$$-\frac{1}{4\,x}-\frac{\ln(x)}{4}-\frac{1}{8+4\,x}+\frac{\ln(2+x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+4*x^3+4*x^2),x)
```

```
[Out] -1/4/x-1/4*ln(x)-1/4/(2+x)+1/4*ln(2+x)
```

Maxima [A] time = 1.08473, size = 34, normalized size = 1.26

$$-\frac{x+1}{2\left(x^2+2\,x\right)}+\frac{1}{4}\,\log(x+2)-\frac{1}{4}\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+4*x^3+4*x^2),x, algorithm="maxima")
```

```
[Out] -1/2*(x + 1)/(x^2 + 2*x) + 1/4*log(x + 2) - 1/4*log(x)
```

Fricas [A] time = 1.23551, size = 99, normalized size = 3.67

$$\frac{\left(x^2+2\,x\right)\log\left(x+2\right)-\left(x^2+2\,x\right)\log\left(x\right)-2\,x-2}{4\left(x^2+2\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+4*x^3+4*x^2),x, algorithm="fricas")
```

```
[Out] 1/4*((x^2 + 2*x)*log(x + 2) - (x^2 + 2*x)*log(x) - 2*x - 2)/(x^2 + 2*x)
```

Sympy [A] time = 0.101413, size = 22, normalized size = 0.81

$$-\frac{x+1}{2x^2+4x}-\frac{\log\left(x\right)}{4}+\frac{\log\left(x+2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+4*x**3+4*x**2),x)
```

```
[Out] -(x + 1)/(2*x**2 + 4*x) - log(x)/4 + log(x + 2)/4
```

Giac [A] time = 1.08426, size = 36, normalized size = 1.33

$$-\frac{x+1}{2\left(x^2+2\,x\right)}+\frac{1}{4}\,\log\left(|x+2|\right)-\frac{1}{4}\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+4*x^3+4*x^2),x, algorithm="giac")
```

```
[Out] -1/2*(x + 1)/(x^2 + 2*x) + 1/4*log(abs(x + 2)) - 1/4*log(abs(x))
```

3.481 $\int \frac{1+x^2}{1+x} \, dx$

Optimal. Leaf size=17

$$\frac{x^2}{2} - x + 2\log(x+1)$$

```
[Out] -x + x^2/2 + 2*Log[1 + x]
```

Rubi [A] time = 0.0083317, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {697}

$$\frac{x^2}{2} - x + 2\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/(1 + x),x]
```

```
[Out] -x + x^2/2 + 2*Log[1 + x]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1+x^2}{1+x} \, dx &= \int \left(-1+x+\frac{2}{1+x}\right) \, dx \\ &= -x+\frac{x^2}{2}+2\log(1+x) \end{aligned}$$

Mathematica [A] time = 0.0027629, size = 18, normalized size = 1.06

$$\frac{1}{2}\left(x^2-2x+4\log(x+1)-3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/(1 + x),x]
```

```
[Out] (-3 - 2*x + x^2 + 4*Log[1 + x])/2
```

Maple [A] time = 0.001, size = 16, normalized size = 0.9

$$-x+\frac{x^2}{2}+2\ln(1+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(1+x),x)
```

```
[Out] -x+1/2*x^2+2*ln(1+x)
```

Maxima [A] time = 1.09184, size = 20, normalized size = 1.18

$$\frac{1}{2}x^2-x+2\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(1+x),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - x + 2*log(x + 1)
```

Fricas [A] time = 1.20878, size = 38, normalized size = 2.24

$$\frac{1}{2}x^2-x+2\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(1+x),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 - x + 2*log(x + 1)
```

Sympy [A] time = 0.069357, size = 12, normalized size = 0.71

$$\frac{x^2}{2} - x + 2\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(1+x),x)
```

```
[Out] x**2/2 - x + 2*log(x + 1)
```

Giac [A] time = 1.09614, size = 22, normalized size = 1.29

$$\frac{1}{2}x^2 - x + 2\log(|x+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(1+x),x, algorithm="giac")
```

```
[Out] 1/2*x^2 - x + 2*log(abs(x + 1))
```

3.482 $\int \frac{-1+3x-3x^2+x^3}{x^2} \, dx$

Optimal. Leaf size=18

$$\frac{x^2}{2} - 3x + \frac{1}{x} + 3\log(x)$$

```
[Out] x^(-1) - 3*x + x^2/2 + 3*Log[x]
```

Rubi [A] time = 0.0060952, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {14}

$$\frac{x^2}{2} - 3x + \frac{1}{x} + 3\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 3*x - 3*x^2 + x^3)/x^2,x]
```

```
[Out] x^(-1) - 3*x + x^2/2 + 3*Log[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int \frac{-1+3x-3x^2+x^3}{x^2} \, dx &= \int \left(-3 - \frac{1}{x^2} + \frac{3}{x} + x\right) dx \\ &= \frac{1}{x} - 3x + \frac{x^2}{2} + 3\log(x) \end{aligned}$$

Mathematica [A] time = 0.0009972, size = 18, normalized size = 1.

$$\frac{x^2}{2} - 3x + \frac{1}{x} + 3\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + 3*x - 3*x^2 + x^3)/x^2,x]
```

```
[Out] x^(-1) - 3*x + x^2/2 + 3*Log[x]
```

Maple [A] time = 0.004, size = 17, normalized size = 0.9

$$x^{-1} - 3\,x + \frac{x^2}{2} + 3\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-3*x^2+3*x-1)/x^2,x)
```

```
[Out] 1/x-3*x+1/2*x^2+3*ln(x)
```

Maxima [A] time = 1.08976, size = 22, normalized size = 1.22

$$\frac{1}{2}\,x^2 - 3\,x + \frac{1}{x} + 3\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-3*x^2+3*x-1)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - 3*x + 1/x + 3*log(x)
```

Fricas [A] time = 1.29276, size = 51, normalized size = 2.83

$$\frac{x^3 - 6\,x^2 + 6\,x\log(x) + 2}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-3*x^2+3*x-1)/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*(x^3 - 6*x^2 + 6*x*log(x) + 2)/x
```

Sympy [A] time = 0.073847, size = 15, normalized size = 0.83

$$\frac{x^2}{2} - 3x + 3\log(x) + \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-3*x**2+3*x-1)/x**2,x)
```

```
[Out] x**2/2 - 3*x + 3*log(x) + 1/x
```

Giac [A] time = 1.10959, size = 23, normalized size = 1.28

$$\frac{1}{2}x^2 - 3x + \frac{1}{x} + 3\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-3*x^2+3*x-1)/x^2,x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 3*x + 1/x + 3*log(abs(x))
```

3.483 $\int \left(\frac{1}{2}\left(3-\sqrt{37}\right)+x\right)\left(\frac{1}{2}\left(3+\sqrt{37}\right)+x\right)\,dx$

Optimal. Leaf size=18

$$\frac{x^3}{3}+\frac{3x^2}{2}-7x$$

```
[Out] -7*x + (3*x^2)/2 + x^3/3
```

Rubi [A] time = 0.0103565, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {43}

$$\frac{x^3}{3}+\frac{3x^2}{2}-7x$$

Antiderivative was successfully verified.

```
[In] Int[((3 - Sqrt[37])/2 + x)*((3 + Sqrt[37])/2 + x),x]
```

```
[Out] -7*x + (3*x^2)/2 + x^3/3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \left(\frac{1}{2}\left(3-\sqrt{37}\right)+x\right)\left(\frac{1}{2}\left(3+\sqrt{37}\right)+x\right)\,dx &= \int \left(-7+3x+x^2\right)\,dx\\ &= -7x+\frac{3x^2}{2}+\frac{x^3}{3}\end{aligned}$$

Mathematica [A] time = 0.0006199, size = 18, normalized size = 1.

$$\frac{x^3}{3}+\frac{3x^2}{2}-7x$$

Antiderivative was successfully verified.

```
[In] Integrate[((3 - Sqrt[37])/2 + x)*((3 + Sqrt[37])/2 + x),x]
```

```
[Out] -7*x + (3*x^2)/2 + x^3/3
```

Maple [A] time = 0., size = 28, normalized size = 1.6

$$\frac{x^3}{3}+\frac{3\,x^2}{2}+\left(\frac{3}{2}-\frac{\sqrt{37}}{2}\right)\left(\frac{3}{2}+\frac{\sqrt{37}}{2}\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+3/2-1/2*37^(1/2))*(x+3/2+1/2*37^(1/2)),x)
```

```
[Out] 1/3*x^3+3/2*x^2+(3/2-1/2*37^(1/2))*(3/2+1/2*37^(1/2))*x
```

Maxima [A] time = 1.64185, size = 19, normalized size = 1.06

$$\frac{1}{3}\,x^3+\frac{3}{2}\,x^2-7\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+3/2-1/2*37^(1/2))*(x+3/2+1/2*37^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3 + 3/2*x^2 - 7*x
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+3/2-1/2*37^(1/2))*(x+3/2+1/2*37^(1/2)),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0.051822, size = 14, normalized size = 0.78

$$\frac{x^3}{3} + \frac{3x^2}{2} - 7x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+3/2-1/2*37**(1/2))*(x+3/2+1/2*37**(1/2)),x)
```

```
[Out] x**3/3 + 3*x**2/2 - 7*x
```

Giac [A] time = 1.12858, size = 19, normalized size = 1.06

$$\frac{1}{3}\,x^3 + \frac{3}{2}\,x^2 - 7\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+3/2-1/2*37^(1/2))*(x+3/2+1/2*37^(1/2)),x, algorithm="giac")
```

```
[Out] 1/3*x^3 + 3/2*x^2 - 7*x
```

3.484 $\int \frac{4+3x^2+2x^3}{(1+x)^4} dx$

Optimal. Leaf size=23

$$\frac{3}{x+1} - \frac{5}{3(x+1)^3} + 2\log(x+1)$$

```
[Out] -5/(3*(1 + x)^3) + 3/(1 + x) + 2*Log[1 + x]
```

Rubi [A] time = 0.0177378, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1850}

$$\frac{3}{x+1} - \frac{5}{3(x+1)^3} + 2\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(4 + 3*x^2 + 2*x^3)/(1 + x)^4,x]
```

```
[Out] -5/(3*(1 + x)^3) + 3/(1 + x) + 2*Log[1 + x]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{4+3x^2+2x^3}{(1+x)^4}\,dx &= \int \left(\frac{5}{(1+x)^4} - \frac{3}{(1+x)^2} + \frac{2}{1+x}\right) dx\\ &= -\frac{5}{3(1+x)^3} + \frac{3}{1+x} + 2\log(1+x)\end{aligned}$$

Mathematica [A] time = 0.0098225, size = 23, normalized size = 1.

$$\frac{3}{x+1} - \frac{5}{3(x+1)^3} + 2\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + 3*x^2 + 2*x^3)/(1 + x)^4,x]
```

```
[Out] -5/(3*(1 + x)^3) + 3/(1 + x) + 2*Log[1 + x]
```

Maple [A] time = 0.005, size = 22, normalized size = 1.

$$-\frac{5}{3\,(1+x)^{3}}+3\,(1+x)^{-1}+2\,\ln(1+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3+3*x^2+4)/(1+x)^4,x)
```

```
[Out] -5/3/(1+x)^3+3/(1+x)+2*ln(1+x)
```

Maxima [A] time = 1.0971, size = 46, normalized size = 2.

$$\frac{9\,x^{2}+18\,x+4}{3\left(x^{3}+3\,x^{2}+3\,x+1\right)}+2\,\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+4)/(1+x)^4,x, algorithm="maxima")
```

```
[Out] 1/3*(9*x^2 + 18*x + 4)/(x^3 + 3*x^2 + 3*x + 1) + 2*log(x + 1)
```

Fricas [B] time = 1.19698, size = 117, normalized size = 5.09

$$\frac{9\,x^{2}+6\left(x^{3}+3\,x^{2}+3\,x+1\right)\log(x+1)+18\,x+4}{3\left(x^{3}+3\,x^{2}+3\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+4)/(1+x)^4,x, algorithm="fricas")
```

```
[Out] 1/3*(9*x^2 + 6*(x^3 + 3*x^2 + 3*x + 1)*log(x + 1) + 18*x + 4)/(x^3 + 3*x^2
+ 3*x + 1)
```

Sympy [A] time = 0.104956, size = 31, normalized size = 1.35

$$\frac{9x^2 + 18x + 4}{3x^3 + 9x^2 + 9x + 3} + 2\log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**3+3*x**2+4)/(1+x)**4,x)
```

```
[Out] (9*x**2 + 18*x + 4)/(3*x**3 + 9*x**2 + 9*x + 3) + 2*log(x + 1)
```

Giac [A] time = 1.13363, size = 34, normalized size = 1.48

$$\frac{9\,x^2 + 18\,x + 4}{3\,(x + 1)^3} + 2\,\log(|x + 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+4)/(1+x)^4,x, algorithm="giac")
```

```
[Out] 1/3*(9*x^2 + 18*x + 4)/(x + 1)^3 + 2*log(abs(x + 1))
```

3.485 $\int \frac{x}{(1+x)^2(1+x^2)}\,dx$

Optimal. Leaf size=16

$$\frac{1}{2(x+1)}+\frac{1}{2}\tan^{-1}(x)$$

```
[Out] 1/(2*(1 + x)) + ArcTan[x]/2
```

Rubi [A] time = 0.0117033, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {801, 203}

$$\frac{1}{2(x+1)}+\frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 + x)^2*(1 + x^2)),x]
```

```
[Out] 1/(2*(1 + x)) + ArcTan[x]/2
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{x}{(1+x)^2(1+x^2)}\,dx = \int \left(-\frac{1}{2(1+x)^2}+\frac{1}{2(1+x^2)}\right)dx$$
$$= \frac{1}{2(1+x)}+\frac{1}{2}\int \frac{1}{1+x^2}\,dx$$
$$= \frac{1}{2(1+x)}+\frac{1}{2}\tan^{-1}(x)$$

Mathematica [A] time = 0.0064635, size = 12, normalized size = 0.75

$$\frac{1}{2}\left(\frac{1}{x+1}+\tan^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 + x)^2*(1 + x^2)),x]
```

```
[Out] ((1 + x)^(-1) + ArcTan[x])/2
```

Maple [A] time = 0.003, size = 13, normalized size = 0.8

$$\frac{1}{2+2x}+\frac{\arctan(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1+x)^2/(x^2+1),x)
```

```
[Out] 1/2/(1+x)+1/2*arctan(x)
```

Maxima [A] time = 1.69299, size = 16, normalized size = 1.

$$\frac{1}{2(x+1)}+\frac{1}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)^2/(x^2+1),x, algorithm="maxima")
```

```
[Out] 1/2/(x + 1) + 1/2*arctan(x)
```

Fricas [A] time = 1.2324, size = 50, normalized size = 3.12

$$\frac{(x+1)\arctan(x)+1}{2(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)^2/(x^2+1),x, algorithm="fricas")
```

```
[Out] 1/2*((x + 1)*arctan(x) + 1)/(x + 1)
```

Sympy [A] time = 0.102061, size = 10, normalized size = 0.62

$$\frac{\operatorname{atan}(x)}{2}+\frac{1}{2x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)**2/(x**2+1),x)
```

```
[Out] atan(x)/2 + 1/(2*x + 2)
```

Giac [B] time = 1.12361, size = 43, normalized size = 2.69

$$-\frac{1}{8}\pi-\frac{1}{2}\pi\left\lfloor-\frac{\pi-4\arctan(x)}{4\pi}+\frac{1}{2}\right\rfloor+\frac{1}{2(x+1)}+\frac{1}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)^2/(x^2+1),x, algorithm="giac")
```

```
[Out] -1/8*pi - 1/2*pi*floor(-1/4*(pi - 4*arctan(x))/pi + 1/2) + 1/2/(x + 1) + 1/
2*arctan(x)
```

3.486 $\int \frac{7-2x+3x^2-x^3+x^4}{2+x} dx$

Optimal. Leaf size=29

$$\frac{x^4}{4} - x^3 + \frac{9x^2}{2} - 20x + 47\log(x+2)$$

```
[Out] -20*x + (9*x^2)/2 - x^3 + x^4/4 + 47*Log[2 + x]
```

Rubi [A] time = 0.020162, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {1850}

$$\frac{x^4}{4} - x^3 + \frac{9x^2}{2} - 20x + 47\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[(7 - 2*x + 3*x^2 - x^3 + x^4)/(2 + x),x]
```

```
[Out] -20*x + (9*x^2)/2 - x^3 + x^4/4 + 47*Log[2 + x]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{7-2x+3x^2-x^3+x^4}{2+x}\,dx &= \int \left(-20+9x-3x^2+x^3+\frac{47}{2+x}\right)dx\\ &= -20x+\frac{9x^2}{2}-x^3+\frac{x^4}{4}+47\log(2+x)\end{aligned}$$

Mathematica [A] time = 0.0078275, size = 30, normalized size = 1.03

$$\frac{x^4}{4} - x^3 + \frac{9x^2}{2} - 20x + 47\log(x+2) - 70$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 - 2*x + 3*x^2 - x^3 + x^4)/(2 + x),x]
```

```
[Out] -70 - 20*x + (9*x^2)/2 - x^3 + x^4/4 + 47*Log[2 + x]
```

Maple [A] time = 0.003, size = 26, normalized size = 0.9

$$-20\,x+\frac{9\,x^{2}}{2}-x^{3}+\frac{x^{4}}{4}+47\,\ln\left(2+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-x^3+3*x^2-2*x+7)/(2+x),x)
```

```
[Out] -20*x+9/2*x^2-x^3+1/4*x^4+47*ln(2+x)
```

Maxima [A] time = 1.32159, size = 34, normalized size = 1.17

$$\frac{1}{4}\,x^{4}-x^{3}+\frac{9}{2}\,x^{2}-20\,x+47\,\log\left(x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^3+3*x^2-2*x+7)/(2+x),x, algorithm="maxima")
```

```
[Out] 1/4*x^4 - x^3 + 9/2*x^2 - 20*x + 47*log(x + 2)
```

Fricas [A] time = 0.943417, size = 65, normalized size = 2.24

$$\frac{1}{4}\,x^{4}-x^{3}+\frac{9}{2}\,x^{2}-20\,x+47\,\log\left(x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^3+3*x^2-2*x+7)/(2+x),x, algorithm="fricas")
```

```
[Out] 1/4*x^4 - x^3 + 9/2*x^2 - 20*x + 47*log(x + 2)
```

Sympy [A] time = 0.074734, size = 24, normalized size = 0.83

$$\frac{x^4}{4} - x^3 + \frac{9x^2}{2} - 20x + 47\log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-x**3+3*x**2-2*x+7)/(2+x),x)
```

```
[Out] x**4/4 - x**3 + 9*x**2/2 - 20*x + 47*log(x + 2)
```

Giac [A] time = 1.09244, size = 35, normalized size = 1.21

$$\frac{1}{4}x^4 - x^3 + \frac{9}{2}x^2 - 20x + 47\log(|x+2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^3+3*x^2-2*x+7)/(2+x),x, algorithm="giac")
```

```
[Out] 1/4*x^4 - x^3 + 9/2*x^2 - 20*x + 47*log(abs(x + 2))
```

3.487 $\int \frac{-1+x^3}{-1+x} dx$

Optimal. Leaf size=16

$$\frac{x^3}{3}+\frac{x^2}{2}+x$$

```
[Out] x + x^2/2 + x^3/3
```

Rubi [A] time = 0.0060318, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1586}

$$\frac{x^3}{3}+\frac{x^2}{2}+x$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^3)/(-1 + x),x]
```

```
[Out] x + x^2/2 + x^3/3
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{-1+x^3}{-1+x}\,dx &= \int \left(1+x+x^2\right)\,dx\\ &= x+\frac{x^2}{2}+\frac{x^3}{3} \end{aligned}$$

Mathematica [A] time = 0.0004014, size = 16, normalized size = 1.

$$\frac{x^3}{3}+\frac{x^2}{2}+x$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^3)/(-1 + x),x]
```

```
[Out] x + x^2/2 + x^3/3
```

Maple [A] time = 0.001, size = 13, normalized size = 0.8

$$x+\frac{x^2}{2}+\frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-1)/(x-1),x)
```

```
[Out] x+1/2*x^2+1/3*x^3
```

Maxima [A] time = 1.26357, size = 16, normalized size = 1.

$$\frac{1}{3}x^3+\frac{1}{2}x^2+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)/(-1+x),x, algorithm="maxima")
```

```
[Out] 1/3*x^3 + 1/2*x^2 + x
```

Fricas [A] time = 0.944745, size = 31, normalized size = 1.94

$$\frac{1}{3}x^3+\frac{1}{2}x^2+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)/(-1+x),x, algorithm="fricas")
```

```
[Out] 1/3*x^3 + 1/2*x^2 + x
```

Sympy [A] time = 0.055744, size = 10, normalized size = 0.62

$$\frac{x^3}{3} + \frac{x^2}{2} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-1)/(-1+x),x)
```

```
[Out] x**3/3 + x**2/2 + x
```

Giac [A] time = 1.0754, size = 16, normalized size = 1.

$$\frac{1}{3}\,x^3 + \frac{1}{2}\,x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)/(-1+x),x, algorithm="giac")
```

```
[Out] 1/3*x^3 + 1/2*x^2 + x
```

3.488 $\int \frac{2+2x}{(-1+x)^3(1+x^2)}\,dx$

Optimal. Leaf size=17

$$\frac{1}{x-1}-\frac{1}{(1-x)^2}+\tan^{-1}(x)$$

```
[Out] -(1 - x)^(-2) + (-1 + x)^(-1) + ArcTan[x]
```

Rubi [A] time = 0.0145243, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {801, 203}

$$\frac{1}{x-1}-\frac{1}{(1-x)^2}+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 2*x)/((-1 + x)^3*(1 + x^2)),x]
```

```
[Out] -(1 - x)^(-2) + (-1 + x)^(-1) + ArcTan[x]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{2+2x}{(-1+x)^3(1+x^2)}\,dx = \int \left(\frac{2}{(-1+x)^3} - \frac{1}{(-1+x)^2} + \frac{1}{1+x^2}\right) dx$$
$$= -\frac{1}{(1-x)^2} + \frac{1}{-1+x} + \int \frac{1}{1+x^2}\,dx$$
$$= -\frac{1}{(1-x)^2} + \frac{1}{-1+x} + \tan^{-1}(x)$$

Mathematica [A] time = 0.0120315, size = 17, normalized size = 1.

$$\frac{x+(x-1)^2\tan^{-1}(x)-2}{(x-1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 2*x)/((-1 + x)^3*(1 + x^2)),x]
```

```
[Out] (-2 + x + (-1 + x)^2*ArcTan[x])/(-1 + x)^2
```

Maple [A] time = 0.005, size = 16, normalized size = 0.9

$$\arctan(x) - (x-1)^{-2} + (x-1)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+2*x)/(x-1)^3/(x^2+1),x)
```

```
[Out] arctan(x)-1/(x-1)^2+1/(x-1)
```

Maxima [A] time = 1.83292, size = 23, normalized size = 1.35

$$\frac{x-2}{x^2-2\,x+1} + \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+2*x)/(-1+x)^3/(x^2+1),x, algorithm="maxima")
```

```
[Out] (x - 2)/(x^2 - 2*x + 1) + arctan(x)
```

Fricas [A] time = 0.935009, size = 72, normalized size = 4.24

$$\frac{\left(x^2-2\,x+1\right)\arctan\left(x\right)+x-2}{x^2-2\,x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+2*x)/(-1+x)^3/(x^2+1),x, algorithm="fricas")
```

```
[Out] ((x^2 - 2*x + 1)*arctan(x) + x - 2)/(x^2 - 2*x + 1)
```

Sympy [A] time = 0.115232, size = 14, normalized size = 0.82

$$\frac{x-2}{x^2-2x+1}+\operatorname{atan}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+2*x)/(-1+x)**3/(x**2+1),x)
```

```
[Out] (x - 2)/(x**2 - 2*x + 1) + atan(x)
```

Giac [A] time = 1.18602, size = 16, normalized size = 0.94

$$\frac{x-2}{\left(x-1\right)^2}+\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+2*x)/(-1+x)^3/(x^2+1),x, algorithm="giac")
```

```
[Out] (x - 2)/(x - 1)^2 + arctan(x)
```

3.489 $\int \frac{1}{bx+c(d+ex)^2} \, dx$

Optimal. Leaf size=47

$$-\frac{2\tanh^{-1}\left(\frac{b+2ce(d+ex)}{\sqrt{b}\sqrt{b+4cde}}\right)}{\sqrt{b}\sqrt{b+4cde}}$$

```
[Out] (-2*ArcTanh[(b + 2*c*e*(d + e*x))/(Sqrt[b]*Sqrt[b + 4*c*d*e])])/(Sqrt[b]*Sq
rt[b + 4*c*d*e])
```

Rubi [A] time = 0.0658729, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1981, 618, 206}

$$-\frac{2\tanh^{-1}\left(\frac{b+2ce(d+ex)}{\sqrt{b}\sqrt{b+4cde}}\right)}{\sqrt{b}\sqrt{b+4cde}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*(d + e*x)^2)^(-1),x]
```

```
[Out] (-2*ArcTanh[(b + 2*c*e*(d + e*x))/(Sqrt[b]*Sqrt[b + 4*c*d*e])])/(Sqrt[b]*Sq
rt[b + 4*c*d*e])
```

Rule 1981

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && Q
uadraticQ[u, x] &&  !QuadraticMatchQ[u, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{bx+c(d+ex)^2}\,dx &= \int \frac{1}{cd^2+(b+2cde)x+ce^2x^2}\,dx\\ &= -\left(2\,\mathrm{Subst}\left(\int \frac{1}{b(b+4cde)-x^2}\,dx,x,b+2cde+2ce^2x\right)\right)\\ &= -\frac{2\tanh^{-1}\left(\frac{b+2ce(d+ex)}{\sqrt{b}\sqrt{b+4cde}}\right)}{\sqrt{b}\sqrt{b+4cde}}\end{aligned}$$

Mathematica [A] time = 0.0253848, size = 47, normalized size = 1.

$$-\frac{2\tanh^{-1}\left(\frac{b+2ce(d+ex)}{\sqrt{b}\sqrt{b+4cde}}\right)}{\sqrt{b}\sqrt{b+4cde}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*(d + e*x)^2)^(-1),x]
```

```
[Out] (-2*ArcTanh[(b + 2*c*e*(d + e*x))/(Sqrt[b]*Sqrt[b + 4*c*d*e])])/(Sqrt[b]*Sq
rt[b + 4*c*d*e])
```

Maple [A] time = 0.006, size = 43, normalized size = 0.9

$$-2\,\frac{1}{\sqrt{4\,bcde+b^2}}\mathrm{Artanh}\left(\frac{2\,ce^2x+2\,cde+b}{\sqrt{4\,bcde+b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+c*(e*x+d)^2),x)
```

```
[Out] -2/(4*b*c*d*e+b^2)^(1/2)*arctanh((2*c*e^2*x+2*c*d*e+b)/(4*b*c*d*e+b^2)^(1/2
))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+c*(e*x+d)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.02551, size = 431, normalized size = 9.17

$$\left[\frac{\log\left(\frac{2\,c^2e^4x^2+2\,c^2d^2e^2+4\,bcde+b^2+2\left(2\,c^2de^3+bce^2\right)x-\sqrt{4\,bcde+b^2}\left(2\,ce^2x+2\,cde+b\right)}{ce^2x^2+cd^2+(2\,cde+b)x}\right)}{\sqrt{4\,bcde+b^2}}, \frac{2\sqrt{-4\,bcde-b^2}\arctan\left(\frac{\sqrt{-4\,bcde-b^2}\left(2\,ce^2x+2\,cde+b\right)}{4\,bcde+b^2}\right)}{4\,bcde+b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+c*(e*x+d)^2),x, algorithm="fricas")
```

```
[Out] [log((2*c^2*e^4*x^2 + 2*c^2*d^2*e^2 + 4*b*c*d*e + b^2 + 2*(2*c^2*d*e^3 + b*
c*e^2)*x - sqrt(4*b*c*d*e + b^2)*(2*c*e^2*x + 2*c*d*e + b))/(c*e^2*x^2 + c*
d^2 + (2*c*d*e + b)*x))/sqrt(4*b*c*d*e + b^2), 2*sqrt(-4*b*c*d*e - b^2)*arc
tan(sqrt(-4*b*c*d*e - b^2)*(2*c*e^2*x + 2*c*d*e + b)/(4*b*c*d*e + b^2))/(4*
b*c*d*e + b^2)]
```

Sympy [B] time = 0.268286, size = 151, normalized size = 3.21

$$\sqrt{\frac{1}{b\,(b+4cde)}}\log\left(x+\frac{-b^2\sqrt{\frac{1}{b(b+4cde)}}-4bcde\sqrt{\frac{1}{b(b+4cde)}}+b+2cde}{2ce^2}\right)-\sqrt{\frac{1}{b\,(b+4cde)}}\log\left(x+\frac{b^2\sqrt{\frac{1}{b(b+4cde)}}+4bcde\sqrt{\frac{1}{b(b}}}{2ce^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+c*(e*x+d)**2),x)
```

```
[Out] sqrt(1/(b*(b + 4*c*d*e)))*log(x + (-b**2*sqrt(1/(b*(b + 4*c*d*e))) - 4*b*c*
d*e*sqrt(1/(b*(b + 4*c*d*e))) + b + 2*c*d*e)/(2*c*e**2)) - sqrt(1/(b*(b + 4
```

```
*c*d*e)))*log(x + (b**2*sqrt(1/(b*(b + 4*c*d*e))) + 4*b*c*d*e*sqrt(1/(b*(b
+ 4*c*d*e))) + b + 2*c*d*e)/(2*c*e**2))
```

Giac [A] time = 1.15207, size = 65, normalized size = 1.38

$$\frac{2\,\arctan\left(\frac{2\,cxe^2+2\,cde+b}{\sqrt{-4\,bcde-b^2}}\right)}{\sqrt{-4\,bcde-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+c*(e*x+d)^2),x, algorithm="giac")
```

```
[Out] 2*arctan((2*c*x*e^2 + 2*c*d*e + b)/sqrt(-4*b*c*d*e - b^2))/sqrt(-4*b*c*d*e
- b^2)
```

3.490 $\int \frac{1}{a+bx+c(d+ex)^2} dx$

Optimal. Leaf size=57

$$-\frac{2\tanh^{-1}\left(\frac{b+2ce(d+ex)}{\sqrt{-4ace^2+b^2+4bcde}}\right)}{\sqrt{-4ace^2+b^2+4bcde}}$$

```
[Out] (-2*ArcTanh[(b + 2*c*e*(d + e*x))/Sqrt[b^2 + 4*b*c*d*e - 4*a*c*e^2]])/Sqrt[
b^2 + 4*b*c*d*e - 4*a*c*e^2]
```

Rubi [A] time = 0.0815414, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1981, 618, 206}

$$-\frac{2\tanh^{-1}\left(\frac{b+2ce(d+ex)}{\sqrt{-4ace^2+b^2+4bcde}}\right)}{\sqrt{-4ace^2+b^2+4bcde}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*(d + e*x)^2)^(-1),x]
```

```
[Out] (-2*ArcTanh[(b + 2*c*e*(d + e*x))/Sqrt[b^2 + 4*b*c*d*e - 4*a*c*e^2]])/Sqrt[
b^2 + 4*b*c*d*e - 4*a*c*e^2]
```

Rule 1981

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && Q
uadraticQ[u, x] &&  !QuadraticMatchQ[u, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a+bx+c(d+ex)^2}\,dx &= \int \frac{1}{a+cd^2+(b+2cde)x+ce^2x^2}\,dx\\ &= -\left(2\,\text{Subst}\left(\int \frac{1}{b^2+4bcde-4ace^2-x^2}\,dx, x, b+2cde+2ce^2x\right)\right)\\ &= -\frac{2\tanh^{-1}\left(\frac{b+2ce(d+ex)}{\sqrt{b^2+4bcde-4ace^2}}\right)}{\sqrt{b^2+4bcde-4ace^2}}\end{aligned}$$

Mathematica [A] time = 0.0279165, size = 61, normalized size = 1.07

$$\frac{2\tan^{-1}\left(\frac{b+2ce(d+ex)}{\sqrt{4ace^2-b^2-4bcde}}\right)}{\sqrt{4ace^2-b^2-4bcde}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*(d + e*x)^2)^(-1),x]
```

```
[Out] (2*ArcTan[(b + 2*c*e*(d + e*x))/Sqrt[-b^2 - 4*b*c*d*e + 4*a*c*e^2]])/Sqrt[-
b^2 - 4*b*c*d*e + 4*a*c*e^2]
```

Maple [A] time = 0.006, size = 61, normalized size = 1.1

$$2\,\frac{1}{\sqrt{4\,ace^2-4\,bcde-b^2}}\arctan\left(\frac{2\,ce^2x+2\,cde+b}{\sqrt{4\,ace^2-4\,bcde-b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*x+c*(e*x+d)^2),x)
```

```
[Out] 2/(4*a*c*e^2-4*b*c*d*e-b^2)^(1/2)*arctan((2*c*e^2*x+2*c*d*e+b)/(4*a*c*e^2-4
*b*c*d*e-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x+c*(e*x+d)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.02391, size = 547, normalized size = 9.6

$$\left[\frac{\log\left(\frac{2\,c^2e^4x^2+4\,bcde+2\,\left(c^2d^2-ac\right)e^2+b^2+2\,\left(2\,c^2de^3+bce^2\right)x-\sqrt{4\,bcde-4\,ace^2+b^2}\left(2\,ce^2x+2\,cde+b\right)}{ce^2x^2+cd^2+(2\,cde+b)x+a}\right)}{\sqrt{4\,bcde-4\,ace^2+b^2}}, -\frac{2\,\sqrt{-4\,bcde+4\,ace^2-b^2}\,\arctan\left(-\frac{\sqrt{-4\,bcd\ldots}}{}\right.}{4\,bcde-4\,ace^2+b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x+c*(e*x+d)^2),x, algorithm="fricas")
```

```
[Out] [log((2*c^2*e^4*x^2 + 4*b*c*d*e + 2*(c^2*d^2 - a*c)*e^2 + b^2 + 2*(2*c^2*d*
e^3 + b*c*e^2)*x - sqrt(4*b*c*d*e - 4*a*c*e^2 + b^2)*(2*c*e^2*x + 2*c*d*e +
 b))/(c*e^2*x^2 + c*d^2 + (2*c*d*e + b)*x + a))/sqrt(4*b*c*d*e - 4*a*c*e^2
+ b^2), -2*sqrt(-4*b*c*d*e + 4*a*c*e^2 - b^2)*arctan(-sqrt(-4*b*c*d*e + 4*a
*c*e^2 - b^2)*(2*c*e^2*x + 2*c*d*e + b)/(4*b*c*d*e - 4*a*c*e^2 + b^2))/(4*b
*c*d*e - 4*a*c*e^2 + b^2)]
```

Sympy [B] time = 0.323344, size = 294, normalized size = 5.16

$$-\sqrt{-\frac{1}{4ace^2-b^2-4bcde}}\log\left(x+\frac{-4ace^2\sqrt{-\frac{1}{4ace^2-b^2-4bcde}}+b^2\sqrt{-\frac{1}{4ace^2-b^2-4bcde}}+4bcde\sqrt{-\frac{1}{4ace^2-b^2-4bcde}}+b+2cde}{2ce^2}\right)+\sqrt{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x+c*(e*x+d)**2),x)
```

```
[Out] -sqrt(-1/(4*a*c*e**2 - b**2 - 4*b*c*d*e))*log(x + (-4*a*c*e**2*sqrt(-1/(4*a
*c*e**2 - b**2 - 4*b*c*d*e)) + b**2*sqrt(-1/(4*a*c*e**2 - b**2 - 4*b*c*d*e)
) + 4*b*c*d*e*sqrt(-1/(4*a*c*e**2 - b**2 - 4*b*c*d*e)) + b + 2*c*d*e)/(2*c*
e**2)) + sqrt(-1/(4*a*c*e**2 - b**2 - 4*b*c*d*e))*log(x + (4*a*c*e**2*sqrt(
-1/(4*a*c*e**2 - b**2 - 4*b*c*d*e)) - b**2*sqrt(-1/(4*a*c*e**2 - b**2 - 4*b
*c*d*e)) - 4*b*c*d*e*sqrt(-1/(4*a*c*e**2 - b**2 - 4*b*c*d*e)) + b + 2*c*d*e
)/(2*c*e**2))
```

Giac [A] time = 1.19572, size = 81, normalized size = 1.42

$$\frac{2\arctan\left(\frac{2\,cxe^2+2\,cde+b}{\sqrt{-4\,bcde+4\,ace^2-b^2}}\right)}{\sqrt{-4\,bcde+4\,ace^2-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x+c*(e*x+d)^2),x, algorithm="giac")
```

```
[Out] 2*arctan((2*c*x*e^2 + 2*c*d*e + b)/sqrt(-4*b*c*d*e + 4*a*c*e^2 - b^2))/sqrt
(-4*b*c*d*e + 4*a*c*e^2 - b^2)
```

3.491 $\int \frac{x^2}{1+\left(-1+x^2\right)^2}\,dx$

Optimal. Leaf size=188

$$\frac{\log\left(x^2-\sqrt{2\left(1+\sqrt{2}\right)}x+\sqrt{2}\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}}-\frac{\log\left(x^2+\sqrt{2\left(1+\sqrt{2}\right)}x+\sqrt{2}\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}}-\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2x}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)+\frac{1}{2}\sqrt{\frac{1}{2}(}$$

```
[Out] -(Sqrt[(1 + Sqrt[2])/2]*ArcTan[(Sqrt[2*(1 + Sqrt[2])] - 2*x)/Sqrt[2*(-1 + S
qrt[2])]])/2 + (Sqrt[(1 + Sqrt[2])/2]*ArcTan[(Sqrt[2*(1 + Sqrt[2])] + 2*x)/
Sqrt[2*(-1 + Sqrt[2])]])/2 + Log[Sqrt[2] - Sqrt[2*(1 + Sqrt[2])]*x + x^2]/(
4*Sqrt[2*(1 + Sqrt[2])]) - Log[Sqrt[2] + Sqrt[2*(1 + Sqrt[2])]*x + x^2]/(4*
Sqrt[2*(1 + Sqrt[2])])
```

Rubi [A] time = 0.188943, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {1989, 1127, 1161, 618, 204, 1164, 628}

$$\frac{\log\left(x^2-\sqrt{2\left(1+\sqrt{2}\right)}x+\sqrt{2}\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}}-\frac{\log\left(x^2+\sqrt{2\left(1+\sqrt{2}\right)}x+\sqrt{2}\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}}-\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2x}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)+\frac{1}{2}\sqrt{\frac{1}{2}(}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(1 + (-1 + x^2)^2),x]
```


```
[Out] -(Sqrt[(1 + Sqrt[2])/2]*ArcTan[(Sqrt[2*(1 + Sqrt[2])] - 2*x)/Sqrt[2*(-1 + S
qrt[2])]])/2 + (Sqrt[(1 + Sqrt[2])/2]*ArcTan[(Sqrt[2*(1 + Sqrt[2])] + 2*x)/
Sqrt[2*(-1 + Sqrt[2])]])/2 + Log[Sqrt[2] - Sqrt[2*(1 + Sqrt[2])]*x + x^2]/(
4*Sqrt[2*(1 + Sqrt[2])]) - Log[Sqrt[2] + Sqrt[2*(1 + Sqrt[2])]*x + x^2]/(4*
Sqrt[2*(1 + Sqrt[2])])
```


Rule 1989

```
Int[(u_)^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] :> Int[(d*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{d, m, p}, x] && TrinomialQ[u, x] && !TrinomialMatchQ[u,
 x]
```

Rule 1127

```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I
nt[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2
 - 4*a*c, 0] && PosQ[a*c]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^2}{1+\left(-1+x^2\right)^2}\,dx = \int \frac{x^2}{2-2x^2+x^4}\,dx$$
$$= -\left(\frac{1}{2}\int \frac{\sqrt{2}-x^2}{2-2x^2+x^4}\,dx\right) + \frac{1}{2}\int \frac{\sqrt{2}+x^2}{2-2x^2+x^4}\,dx$$
$$= \frac{1}{4}\int \frac{1}{\sqrt{2}-\sqrt{2\left(1+\sqrt{2}\right)}x+x^2}\,dx + \frac{1}{4}\int \frac{1}{\sqrt{2}+\sqrt{2\left(1+\sqrt{2}\right)}x+x^2}\,dx + \frac{\int \frac{\sqrt{2\left(1+\sqrt{2}\right)}+2x}{-\sqrt{2}-\sqrt{2\left(1+\sqrt{2}\right)}x-x^2}\,dx}{4\sqrt{2\left(1+\sqrt{2}\right)}} +$$
$$= \frac{\log\left(\sqrt{2}-\sqrt{2\left(1+\sqrt{2}\right)}x+x^2\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}} - \frac{\log\left(\sqrt{2}+\sqrt{2\left(1+\sqrt{2}\right)}x+x^2\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}} - \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{2\left(1-\sqrt{2}\right)-x^2}\right.$$
$$= -\frac{\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2x}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)}{2\sqrt{2\left(-1+\sqrt{2}\right)}} + \frac{\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}+2x}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)}{2\sqrt{2\left(-1+\sqrt{2}\right)}} + \frac{\log\left(\sqrt{2}-\sqrt{2\left(1+\sqrt{2}\right)}x+x^2\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}} - \frac{\log\left(\sqrt{2}+\sqrt{2\right.}\right.}{4\sqrt{2\left(\right.}}$$

Mathematica [C] time = 0.0307766, size = 39, normalized size = 0.21

$$-\frac{\tan^{-1}\left(\frac{x}{\sqrt{-1-i}}\right)}{(-1-i)^{3/2}} - \frac{\tan^{-1}\left(\frac{x}{\sqrt{-1+i}}\right)}{(-1+i)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(1 + (-1 + x^2)^2),x]
```

```
[Out] -(ArcTan[x/Sqrt[-1 - I]]/(-1 - I)^(3/2)) - ArcTan[x/Sqrt[-1 + I]]/(-1 + I)^
(3/2)
```

Maple [B] time = 0.077, size = 308, normalized size = 1.6

$$-\frac{\sqrt{2+2\sqrt{2}}\sqrt{2}\ln\left(x^2+\sqrt{2}+x\sqrt{2+2\sqrt{2}}\right)}{8} + \frac{\sqrt{2}\left(2+2\sqrt{2}\right)}{4\sqrt{-2+2\sqrt{2}}}\arctan\left(\frac{2x+\sqrt{2+2\sqrt{2}}}{\sqrt{-2+2\sqrt{2}}}\right) + \frac{\sqrt{2+2\sqrt{2}}\ln\left(x^2+\sqrt{2}+x\right.}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(1+(x^2-1)^2),x)
```

```
[Out] -1/8*(2+2*2^(1/2))^(1/2)*2^(1/2)*ln(x^2+2^(1/2)+x*(2+2*2^(1/2))^(1/2))+1/4*
2^(1/2)*(2+2*2^(1/2))/(-2+2*2^(1/2))^(1/2)*arctan((2*x+(2+2*2^(1/2))^(1/2))
/(-2+2*2^(1/2))^(1/2))+1/8*(2+2*2^(1/2))^(1/2)*ln(x^2+2^(1/2)+x*(2+2*2^(1/2
))^(1/2))-1/4*(2+2*2^(1/2))/(-2+2*2^(1/2))^(1/2)*arctan((2*x+(2+2*2^(1/2))^
(1/2))/(-2+2*2^(1/2))^(1/2))+1/8*(2+2*2^(1/2))^(1/2)*2^(1/2)*ln(x^2+2^(1/2)
-x*(2+2*2^(1/2))^(1/2))+1/4*2^(1/2)*(2+2*2^(1/2))/(-2+2*2^(1/2))^(1/2)*arct
an((2*x-(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))-1/8*(2+2*2^(1/2))^(1/2)*
ln(x^2+2^(1/2)-x*(2+2*2^(1/2))^(1/2))-1/4*(2+2*2^(1/2))/(-2+2*2^(1/2))^(1/2
)*arctan((2*x-(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(x^2-1\right)^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+(x^2-1)^2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((x^2 - 1)^2 + 1), x)
```

Fricas [A] time = 1.08493, size = 771, normalized size = 4.1

$$\frac{1}{16}\cdot 2^{\frac{1}{4}}\sqrt{2\sqrt{2}+4}\left(\sqrt{2}-2\right)\log\left(2^{\frac{3}{4}}x\sqrt{2\sqrt{2}+4}+2\,x^2+2\sqrt{2}\right)-\frac{1}{16}\cdot 2^{\frac{1}{4}}\sqrt{2\sqrt{2}+4}\left(\sqrt{2}-2\right)\log\left(-2^{\frac{3}{4}}x\sqrt{2\sqrt{2}+4}+2\,x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+(x^2-1)^2),x, algorithm="fricas")
```

```
[Out] 1/16*2^(1/4)*sqrt(2*sqrt(2) + 4)*(sqrt(2) - 2)*log(2^(3/4)*x*sqrt(2*sqrt(2)
 + 4) + 2*x^2 + 2*sqrt(2)) - 1/16*2^(1/4)*sqrt(2*sqrt(2) + 4)*(sqrt(2) - 2)
*log(-2^(3/4)*x*sqrt(2*sqrt(2) + 4) + 2*x^2 + 2*sqrt(2)) - 1/4*2^(3/4)*sqrt
(2*sqrt(2) + 4)*arctan(-1/2*2^(3/4)*x*sqrt(2*sqrt(2) + 4) + 1/2*2^(1/4)*sqr
t(2^(3/4)*x*sqrt(2*sqrt(2) + 4) + 2*x^2 + 2*sqrt(2))*sqrt(2*sqrt(2) + 4) -
sqrt(2) - 1) - 1/4*2^(3/4)*sqrt(2*sqrt(2) + 4)*arctan(-1/2*2^(3/4)*x*sqrt(2
```

```
*sqrt(2) + 4) + 1/2*2^(1/4)*sqrt(-2^(3/4)*x*sqrt(2*sqrt(2) + 4) + 2*x^2 + 2
*sqrt(2))*sqrt(2*sqrt(2) + 4) + sqrt(2) + 1)
```

Sympy [A] time = 0.457658, size = 24, normalized size = 0.13

$$\operatorname{RootSum}\left(128t^4+16t^2+1,\left(t\mapsto t\log\left(64t^3+4t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(1+(x**2-1)**2),x)
```

```
[Out] RootSum(128*_t**4 + 16*_t**2 + 1, Lambda(_t, _t*log(64*_t**3 + 4*_t + x)))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(x^2-1\right)^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+(x^2-1)^2),x, algorithm="giac")
```

```
[Out] integrate(x^2/((x^2 - 1)^2 + 1), x)
```

3.492 $\int -\frac{15-36x+5x^2+12x^3-34x^4+140x^5+15x^6+8x^7-30x^9}{(3+x+x^4)^4}\,dx$

Optimal. Leaf size=60

$$-\frac{5x^6}{(x^4+x+3)^3}+\frac{x^4}{(x^4+x+3)^3}+\frac{5x^2}{(x^4+x+3)^3}-\frac{3x}{(x^4+x+3)^3}+\frac{2}{(x^4+x+3)^3}$$

```
[Out] 2/(3 + x + x^4)^3 - (3*x)/(3 + x + x^4)^3 + (5*x^2)/(3 + x + x^4)^3 + x^4/(
3 + x + x^4)^3 - (5*x^6)/(3 + x + x^4)^3
```

Rubi [A] time = 0.135984, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 50, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {2102, 1588}

$$-\frac{5x^6}{(x^4+x+3)^3}+\frac{x^4}{(x^4+x+3)^3}+\frac{5x^2}{(x^4+x+3)^3}-\frac{3x}{(x^4+x+3)^3}+\frac{2}{(x^4+x+3)^3}$$

Antiderivative was successfully verified.

```
[In] Int[-((15 - 36*x + 5*x^2 + 12*x^3 - 34*x^4 + 140*x^5 + 15*x^6 + 8*x^7 - 30*
x^9)/(3 + x + x^4)^4),x]
```

```
[Out] 2/(3 + x + x^4)^3 - (3*x)/(3 + x + x^4)^3 + (5*x^2)/(3 + x + x^4)^3 + x^4/(
3 + x + x^4)^3 - (5*x^6)/(3 + x + x^4)^3
```

Rule 2102

```
Int[(Pm_)*(Qn_)^(p_.), x_Symbol] :> With[{m = Expon[Pm, x], n = Expon[Qn, x
]}, Simp[(Coeff[Pm, x, m]*x^(m - n + 1)*Qn^(p + 1))/((m + n*p + 1)*Coeff[Qn
, x, n]), x] + Dist[1/((m + n*p + 1)*Coeff[Qn, x, n]), Int[ExpandToSum[(m +
 n*p + 1)*Coeff[Qn, x, n]*Pm - Coeff[Pm, x, m]*x^(m - n)*((m - n + 1)*Qn +
(p + 1)*x*D[Qn, x]), x]*Qn^p, x], x] /; LtQ[1, n, m + 1] && m + n*p + 1 < 0
] /; FreeQ[p, x] && PolyQ[Pm, x] && PolyQ[Qn, x] && LtQ[p, -1]
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
```

```
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int -\frac{15-36x+5x^2+12x^3-34x^4+140x^5+15x^6+8x^7-30x^9}{\left(3+x+x^4\right)^4}\,dx = -\frac{5x^6}{\left(3+x+x^4\right)^3}+\frac{1}{6}\int \frac{-90+216x-30x^2-72x^3}{\left(3+x+\right.}$$

$$= \frac{x^4}{\left(3+x+x^4\right)^3}-\frac{5x^6}{\left(3+x+x^4\right)^3}-\frac{1}{48}\int \frac{720-1728}{}$$

$$= \frac{5x^2}{\left(3+x+x^4\right)^3}+\frac{x^4}{\left(3+x+x^4\right)^3}-\frac{5x^6}{\left(3+x+x^4\right)^3}+$$

$$= -\frac{3x}{\left(3+x+x^4\right)^3}+\frac{5x^2}{\left(3+x+x^4\right)^3}+\frac{x^4}{\left(3+x+x^4\right)^3}-$$

$$= \frac{2}{\left(3+x+x^4\right)^3}-\frac{3x}{\left(3+x+x^4\right)^3}+\frac{5x^2}{\left(3+x+x^4\right)^3}+$$

Mathematica [A] time = 0.0149316, size = 27, normalized size = 0.45

$$\frac{-5x^6+x^4+5x^2-3x+2}{\left(x^4+x+3\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[-((15 - 36*x + 5*x^2 + 12*x^3 - 34*x^4 + 140*x^5 + 15*x^6 + 8*x^7
 - 30*x^9)/(3 + x + x^4)^4),x]
```

```
[Out] (2 - 3*x + 5*x^2 + x^4 - 5*x^6)/(3 + x + x^4)^3
```

Maple [A] time = 0.009, size = 28, normalized size = 0.5

$$\frac{-5\,x^6+x^4+5\,x^2-3\,x+2}{\left(x^4+x+3\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((30*x^9-8*x^7-15*x^6-140*x^5+34*x^4-12*x^3-5*x^2+36*x-15)/(x^4+x+3)^4,x
)
```

```
[Out] (-5*x^6+x^4+5*x^2-3*x+2)/(x^4+x+3)^3
```

Maxima [A] time = 1.67129, size = 88, normalized size = 1.47

$$-\frac{5\,x^6-x^4-5\,x^2+3\,x-2}{x^{12}+3\,x^9+9\,x^8+3\,x^6+18\,x^5+27\,x^4+x^3+9\,x^2+27\,x+27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((30*x^9-8*x^7-15*x^6-140*x^5+34*x^4-12*x^3-5*x^2+36*x-15)/(x^4+x+
3)^4,x, algorithm="maxima")
```

```
[Out] -(5*x^6 - x^4 - 5*x^2 + 3*x - 2)/(x^12 + 3*x^9 + 9*x^8 + 3*x^6 + 18*x^5 + 2
7*x^4 + x^3 + 9*x^2 + 27*x + 27)
```

Fricas [A] time = 0.93845, size = 147, normalized size = 2.45

$$-\frac{5\,x^6-x^4-5\,x^2+3\,x-2}{x^{12}+3\,x^9+9\,x^8+3\,x^6+18\,x^5+27\,x^4+x^3+9\,x^2+27\,x+27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((30*x^9-8*x^7-15*x^6-140*x^5+34*x^4-12*x^3-5*x^2+36*x-15)/(x^4+x+
3)^4,x, algorithm="fricas")
```

```
[Out] -(5*x^6 - x^4 - 5*x^2 + 3*x - 2)/(x^12 + 3*x^9 + 9*x^8 + 3*x^6 + 18*x^5 + 2
7*x^4 + x^3 + 9*x^2 + 27*x + 27)
```

Sympy [A] time = 0.244175, size = 61, normalized size = 1.02

$$-\frac{5x^6-x^4-5x^2+3x-2}{x^{12}+3x^9+9x^8+3x^6+18x^5+27x^4+x^3+9x^2+27x+27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((30*x**9-8*x**7-15*x**6-140*x**5+34*x**4-12*x**3-5*x**2+36*x-15)/
(x**4+x+3)**4,x)
```

```
[Out] -(5*x**6 - x**4 - 5*x**2 + 3*x - 2)/(x**12 + 3*x**9 + 9*x**8 + 3*x**6 + 18*
x**5 + 27*x**4 + x**3 + 9*x**2 + 27*x + 27)
```

Giac [A] time = 1.12653, size = 41, normalized size = 0.68

$$-\frac{5\,x^6 - x^4 - 5\,x^2 + 3\,x - 2}{\left(x^4 + x + 3\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((30*x^9-8*x^7-15*x^6-140*x^5+34*x^4-12*x^3-5*x^2+36*x-15)/(x^4+x+
3)^4,x, algorithm="giac")
```

```
[Out] -(5*x^6 - x^4 - 5*x^2 + 3*x - 2)/(x^4 + x + 3)^3
```

3.493 $\int \left(\frac{3(-47+228x+120x^2+19x^3)}{(3+x+x^4)^4} + \frac{42-320x-75x^2-8x^3}{(3+x+x^4)^3} + \frac{30x}{(3+x+x^4)^2} \right) dx$

Optimal. Leaf size=27

$$\frac{-5x^6 + x^4 + 5x^2 - 3x + 2}{(x^4 + x + 3)^3}$$

```
[Out] (2 - 3*x + 5*x^2 + x^4 - 5*x^6)/(3 + x + x^4)^3
```

Rubi [F] time = 0.311277, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \left(\frac{3(-47 + 228x + 120x^2 + 19x^3)}{(3 + x + x^4)^4} + \frac{42 - 320x - 75x^2 - 8x^3}{(3 + x + x^4)^3} + \frac{30x}{(3 + x + x^4)^2} \right) dx$$

Verification is Not applicable to the result.

```
[In] Int[(3*(-47 + 228*x + 120*x^2 + 19*x^3))/(3 + x + x^4)^4 + (42 - 320*x - 75
*x^2 - 8*x^3)/(3 + x + x^4)^3 + (30*x)/(3 + x + x^4)^2,x]
```

```
[Out] -19/(4*(3 + x + x^4)^3) + (3 + x + x^4)^(-2) - (621*Defer[Int][(3 + x + x^4
)^(-4), x])/4 + 684*Defer[Int][x/(3 + x + x^4)^4, x] + 360*Defer[Int][x^2/(
3 + x + x^4)^4, x] + 44*Defer[Int][(3 + x + x^4)^(-3), x] - 320*Defer[Int][
x/(3 + x + x^4)^3, x] - 75*Defer[Int][x^2/(3 + x + x^4)^3, x] + 30*Defer[In
t][x/(3 + x + x^4)^2, x]
```

Rubi steps

$$\int \left(\frac{3\left(-47+228x+120x^2+19x^3\right)}{\left(3+x+x^4\right)^4}+\frac{42-320x-75x^2-8x^3}{\left(3+x+x^4\right)^3}+\frac{30x}{\left(3+x+x^4\right)^2}\right)dx = 3\int \frac{-47+228x+120x^2+19x^3}{\left(3+x+x^4\right)^4}$$

$$= -\frac{19}{4\left(3+x+x^4\right)^3}+\frac{1}{\left(3+x+x^4\right)^2}$$

$$= -\frac{19}{4\left(3+x+x^4\right)^3}+\frac{1}{\left(3+x+x^4\right)^2}$$

$$= -\frac{19}{4\left(3+x+x^4\right)^3}+\frac{1}{\left(3+x+x^4\right)^2}$$

Mathematica [A] time = 0.0094752, size = 27, normalized size = 1.

$$\frac{-5x^6+x^4+5x^2-3x+2}{\left(x^4+x+3\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3*(-47 + 228*x + 120*x^2 + 19*x^3))/(3 + x + x^4)^4 + (42 - 320*
x - 75*x^2 - 8*x^3)/(3 + x + x^4)^3 + (30*x)/(3 + x + x^4)^2,x]
```

```
[Out] (2 - 3*x + 5*x^2 + x^4 - 5*x^6)/(3 + x + x^4)^3
```

Maple [C] time = 0.024, size = 250, normalized size = 9.3

$$\frac{1}{\left(x^4+x+3\right)^2}\left(\frac{377432\,x^7}{195075}-\frac{1404328\,x^6}{195075}+\frac{234517\,x^5}{195075}+\frac{660506\,x^4}{195075}-\frac{208792\,x^3}{195075}-\frac{13339729\,x^2}{390150}+\frac{89881\,x}{13005}+\frac{121303}{21675}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(3*(19*x^3+120*x^2+228*x-47)/(x^4+x+3)^4+(-8*x^3-75*x^2-320*x+42)/(x^4+x
+3)^3+30*x/(x^4+x+3)^2,x)
```

```
[Out] (377432/195075*x^7-1404328/195075*x^6+234517/195075*x^5+660506/195075*x^4-2
08792/195075*x^3-13339729/390150*x^2+89881/13005*x+121303/21675)/(x^4+x+3)^
2+1/195075*sum((377432*_R^2-2808656*_R+703551)/(4*_R^3+1)*ln(x-_R),_R=RootO
```

```
f(_Z^4+_Z+3))+30*(-16/765*x^3+64/765*x^2-1/765*x-4/255)/(x^4+x+3)+2/51*sum(
(-16*_R^2+128*_R-3)/(4*_R^3+1)*ln(x-_R),_R=RootOf(_Z^4+_Z+3))+3*(-255032/58
5225*x^11+914728/585225*x^10-226867/585225*x^9-701338/585225*x^8+236024/585
225*x^7+13501313/1170450*x^6-2360372/585225*x^5-1873778/585225*x^4+10935781
/1170450*x^3+3415123/130050*x^2-62987/7225*x-76253/21675)/(x^4+x+3)^3+1/195
075*sum((-255032*_R^2+1829456*_R-680601)/(4*_R^3+1)*ln(x-_R),_R=RootOf(_Z^4
+_Z+3))
```

Maxima [B] time = 1.32357, size = 88, normalized size = 3.26

$$-\frac{5\,x^6 - x^4 - 5\,x^2 + 3\,x - 2}{x^{12} + 3\,x^9 + 9\,x^8 + 3\,x^6 + 18\,x^5 + 27\,x^4 + x^3 + 9\,x^2 + 27\,x + 27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3*(19*x^3+120*x^2+228*x-47)/(x^4+x+3)^4+(-8*x^3-75*x^2-320*x+42)/
(x^4+x+3)^3+30*x/(x^4+x+3)^2,x, algorithm="maxima")
```

```
[Out] -(5*x^6 - x^4 - 5*x^2 + 3*x - 2)/(x^12 + 3*x^9 + 9*x^8 + 3*x^6 + 18*x^5 + 2
7*x^4 + x^3 + 9*x^2 + 27*x + 27)
```

Fricas [B] time = 0.922794, size = 147, normalized size = 5.44

$$-\frac{5\,x^6 - x^4 - 5\,x^2 + 3\,x - 2}{x^{12} + 3\,x^9 + 9\,x^8 + 3\,x^6 + 18\,x^5 + 27\,x^4 + x^3 + 9\,x^2 + 27\,x + 27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3*(19*x^3+120*x^2+228*x-47)/(x^4+x+3)^4+(-8*x^3-75*x^2-320*x+42)/
(x^4+x+3)^3+30*x/(x^4+x+3)^2,x, algorithm="fricas")
```

```
[Out] -(5*x^6 - x^4 - 5*x^2 + 3*x - 2)/(x^12 + 3*x^9 + 9*x^8 + 3*x^6 + 18*x^5 + 2
7*x^4 + x^3 + 9*x^2 + 27*x + 27)
```

Sympy [B] time = 0.317374, size = 61, normalized size = 2.26

$$-\frac{5x^6 - x^4 - 5x^2 + 3x - 2}{x^{12} + 3x^9 + 9x^8 + 3x^6 + 18x^5 + 27x^4 + x^3 + 9x^2 + 27x + 27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3*(19*x**3+120*x**2+228*x-47)/(x**4+x+3)**4+(-8*x**3-75*x**2-320*
x+42)/(x**4+x+3)**3+30*x/(x**4+x+3)**2,x)
```

```
[Out] -(5*x**6 - x**4 - 5*x**2 + 3*x - 2)/(x**12 + 3*x**9 + 9*x**8 + 3*x**6 + 18*
x**5 + 27*x**4 + x**3 + 9*x**2 + 27*x + 27)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{30\,x}{\left(x^4+x+3\right)^2}-\frac{8\,x^3+75\,x^2+320\,x-42}{\left(x^4+x+3\right)^3}+\frac{3\left(19\,x^3+120\,x^2+228\,x-47\right)}{\left(x^4+x+3\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3*(19*x^3+120*x^2+228*x-47)/(x^4+x+3)^4+(-8*x^3-75*x^2-320*x+42)/
(x^4+x+3)^3+30*x/(x^4+x+3)^2,x, algorithm="giac")
```

```
[Out] integrate(30*x/(x^4 + x + 3)^2 - (8*x^3 + 75*x^2 + 320*x - 42)/(x^4 + x + 3
)^3 + 3*(19*x^3 + 120*x^2 + 228*x - 47)/(x^4 + x + 3)^4, x)
```

3.494 $\int \left(\frac{-3+10x+4x^3-30x^5}{\left(3+x+x^4\right)^3}-\frac{3\left(1+4x^3\right)\left(2-3x+5x^2+x^4-5x^6\right)}{\left(3+x+x^4\right)^4}\right) dx$

Optimal. Leaf size=27

$$\frac{-5x^6+x^4+5x^2-3x+2}{\left(x^4+x+3\right)^3}$$

```
[Out] (2 - 3*x + 5*x^2 + x^4 - 5*x^6)/(3 + x + x^4)^3
```

Rubi [F] time = 0.433148, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \left(\frac{-3+10x+4x^3-30x^5}{\left(3+x+x^4\right)^3}-\frac{3\left(1+4x^3\right)\left(2-3x+5x^2+x^4-5x^6\right)}{\left(3+x+x^4\right)^4}\right) dx$$

Verification is Not applicable to the result.

```
[In] Int[(-3 + 10*x + 4*x^3 - 30*x^5)/(3 + x + x^4)^3 - (3*(1 + 4*x^3)*(2 - 3*x
+ 5*x^2 + x^4 - 5*x^6))/(3 + x + x^4)^4,x]
```

```
[Out] 7/(2*(3 + x + x^4)^3) - (63*x)/(22*(3 + x + x^4)^3) - (12*x^2)/(3 + x + x^4
)^3 - (5*x^3)/(3 + x + x^4)^3 + (3*x^4)/(2*(3 + x + x^4)^3) - (10*x^6)/(3 +
 x + x^4)^3 - 1/(2*(3 + x + x^4)^2) + (5*x^2)/(3 + x + x^4)^2 + (144*Defer[
Int][(3 + x + x^4)^(-4), x])/11 + (828*Defer[Int][x/(3 + x + x^4)^4, x])/11
 + 18*Defer[Int][x^2/(3 + x + x^4)^4, x] - 4*Defer[Int][(3 + x + x^4)^(-3),
 x] - 20*Defer[Int][x/(3 + x + x^4)^3, x]
```

Rubi steps

$$\int \left(\frac{-3+10x+4x^3-30x^5}{\left(3+x+x^4\right)^3}-\frac{3\left(1+4x^3\right)\left(2-3x+5x^2+x^4-5x^6\right)}{\left(3+x+x^4\right)^4}\right)dx = -\left(3\int \frac{\left(1+4x^3\right)\left(2-3x+5x^2+x^4-5x^6\right)}{\left(3+x+x^4\right)^4}\,dx\right.$$

$$= -\frac{10x^6}{\left(3+x+x^4\right)^3}+\frac{5x^2}{\left(3+x+x^4\right)^2}-\frac{1}{6}\int \frac{18+1}{(3+}$$

$$= \frac{3x^4}{2\left(3+x+x^4\right)^3}-\frac{10x^6}{\left(3+x+x^4\right)^3}-\frac{1}{2\left(3+x+\right.}$$

$$= -\frac{5x^3}{\left(3+x+x^4\right)^3}+\frac{3x^4}{2\left(3+x+x^4\right)^3}-\frac{10x^6}{\left(3+x+\right.}$$

$$= -\frac{12x^2}{\left(3+x+x^4\right)^3}-\frac{5x^3}{\left(3+x+x^4\right)^3}+\frac{3x^4}{2\left(3+x+\right.}$$

$$= -\frac{63x}{22\left(3+x+x^4\right)^3}-\frac{12x^2}{\left(3+x+x^4\right)^3}-\frac{5x^3}{\left(3+x+\right.}$$

$$= \frac{7}{2\left(3+x+x^4\right)^3}-\frac{63x}{22\left(3+x+x^4\right)^3}-\frac{12x}{\left(3+x+\right.}$$

$$= \frac{7}{2\left(3+x+x^4\right)^3}-\frac{63x}{22\left(3+x+x^4\right)^3}-\frac{12x}{\left(3+x+\right.}$$

$$= \frac{7}{2\left(3+x+x^4\right)^3}-\frac{63x}{22\left(3+x+x^4\right)^3}-\frac{12x}{\left(3+x+\right.}$$

Mathematica [A] time = 0.0105844, size = 27, normalized size = 1.

$$\frac{-5x^6+x^4+5x^2-3x+2}{\left(x^4+x+3\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 + 10*x + 4*x^3 - 30*x^5)/(3 + x + x^4)^3 - (3*(1 + 4*x^3)*(2
- 3*x + 5*x^2 + x^4 - 5*x^6))/(3 + x + x^4)^4,x]
```

```
[Out] (2 - 3*x + 5*x^2 + x^4 - 5*x^6)/(3 + x + x^4)^3
```

Maple [B] time = 0.016, size = 112, normalized size = 4.2

$$-\frac{1}{\left(x^4+x+3\right)^2}\left(-\frac{34568\,x^7}{195075}+\frac{73672\,x^6}{195075}+\frac{15392\,x^5}{195075}-\frac{60494\,x^4}{195075}-\frac{68792\,x^3}{195075}-\frac{583927\,x^2}{195075}+\frac{3356\,x}{13005}-\frac{2069}{43350}\right)+3\,\frac{\cdots}{\left(x^4+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-30*x^5+4*x^3+10*x-3)/(x^4+x+3)^3-3*(4*x^3+1)*(-5*x^6+x^4+5*x^2-3*x+2)
/(x^4+x+3)^4,x)
```

```
[Out] -(-34568/195075*x^7+73672/195075*x^6+15392/195075*x^5-60494/195075*x^4-6879
2/195075*x^3-583927/195075*x^2+3356/13005*x-2069/43350)/(x^4+x+3)^2+3*(-345
68/585225*x^11+73672/585225*x^10+15392/585225*x^9-95062/585225*x^8-98824/58
5225*x^7-1322894/585225*x^6+36022/585225*x^5-129019/1170450*x^4-790303/5852
25*x^3-80674/65025*x^2-10951/14450*x+26831/43350)/(x^4+x+3)^3
```

Maxima [B] time = 1.31228, size = 88, normalized size = 3.26

$$-\frac{5\,x^6-x^4-5\,x^2+3\,x-2}{x^{12}+3\,x^9+9\,x^8+3\,x^6+18\,x^5+27\,x^4+x^3+9\,x^2+27\,x+27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-30*x^5+4*x^3+10*x-3)/(x^4+x+3)^3-3*(4*x^3+1)*(-5*x^6+x^4+5*x^2-
3*x+2)/(x^4+x+3)^4,x, algorithm="maxima")
```

```
[Out] -(5*x^6 - x^4 - 5*x^2 + 3*x - 2)/(x^12 + 3*x^9 + 9*x^8 + 3*x^6 + 18*x^5 + 2
7*x^4 + x^3 + 9*x^2 + 27*x + 27)
```

Fricas [B] time = 0.940147, size = 147, normalized size = 5.44

$$-\frac{5\,x^6-x^4-5\,x^2+3\,x-2}{x^{12}+3\,x^9+9\,x^8+3\,x^6+18\,x^5+27\,x^4+x^3+9\,x^2+27\,x+27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-30*x^5+4*x^3+10*x-3)/(x^4+x+3)^3-3*(4*x^3+1)*(-5*x^6+x^4+5*x^2-
3*x+2)/(x^4+x+3)^4,x, algorithm="fricas")
```

```
[Out] -(5*x^6 - x^4 - 5*x^2 + 3*x - 2)/(x^12 + 3*x^9 + 9*x^8 + 3*x^6 + 18*x^5 + 2
7*x^4 + x^3 + 9*x^2 + 27*x + 27)
```

Sympy [B] time = 0.285799, size = 61, normalized size = 2.26

$$-\frac{5x^6 - x^4 - 5x^2 + 3x - 2}{x^{12} + 3x^9 + 9x^8 + 3x^6 + 18x^5 + 27x^4 + x^3 + 9x^2 + 27x + 27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-30*x**5+4*x**3+10*x-3)/(x**4+x+3)**3-3*(4*x**3+1)*(-5*x**6+x**4
+5*x**2-3*x+2)/(x**4+x+3)**4,x)
```

```
[Out] -(5*x**6 - x**4 - 5*x**2 + 3*x - 2)/(x**12 + 3*x**9 + 9*x**8 + 3*x**6 + 18*
x**5 + 27*x**4 + x**3 + 9*x**2 + 27*x + 27)
```

Giac [B] time = 1.1459, size = 150, normalized size = 5.56

$$\frac{69136\,x^7 - 147344\,x^6 - 30784\,x^5 + 120988\,x^4 + 137584\,x^3 + 1167854\,x^2 - 100680\,x + 18621}{390150\,(x^4 + x + 3)^2} - \frac{69136\,x^{11} - 147344\,x^{10} -}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-30*x^5+4*x^3+10*x-3)/(x^4+x+3)^3-3*(4*x^3+1)*(-5*x^6+x^4+5*x^2-
3*x+2)/(x^4+x+3)^4,x, algorithm="giac")
```

```
[Out] 1/390150*(69136*x^7 - 147344*x^6 - 30784*x^5 + 120988*x^4 + 137584*x^3 + 11
67854*x^2 - 100680*x + 18621)/(x^4 + x + 3)^2 - 1/390150*(69136*x^11 - 1473
44*x^10 - 30784*x^9 + 190124*x^8 + 197648*x^7 + 2645788*x^6 - 72044*x^5 + 1
29019*x^4 + 1580606*x^3 + 1452132*x^2 + 887031*x - 724437)/(x^4 + x + 3)^3
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
```

```
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
```

```

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
```

```
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not
    as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
```

```
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
```

```
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
```

```
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
```

```
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
```

```
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(
    flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                     #since this estimate of leaf count is bit lower than
                     #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

```

```
def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
    ']

def is_appell_function(func):
```

```
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
```

```
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, " leaf_count_optimal=",leaf_count_optimal)
```

```
    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
   expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```