A look at centered difference approximation

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1 Introduction

To find centered difference approximation for $f'(x)$, we can processed as follows.

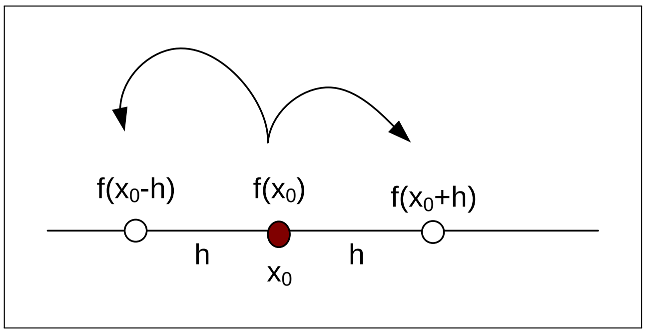

Figure 1: centered difference

2 approximation for $f'(x_o)$

Since

$$f(x_o+h)=f(x_o)+hf'(x_o)+O(h)$$

Then

$$f'(x_o)\approx\frac{1}{h}\left[f(x_o+h)-f(x_o)\right] \tag{1}$$

But we also know that

$$f(x_o-h)=f(x_o)-hf'(x_o)+O(h)$$

The trick is to find $f(x_o)$ from the above and plug it in (1). From the above we find

$$f(x_o)\approx f(x_o-h)+hf'(x_o)$$

substituting the above in (1) gives

$$\begin{aligned}f'(x_o)&\approx\frac{1}{h}\left[f(x_o+h)-\left(f(x_o-h)+hf'(x_o)\right)\right]\\&\approx\frac{f(x_o+h)-f(x_o-h)}{h}-f'(x_o)\end{aligned}$$

Hence

$$f'(x_o)\approx\frac{f(x_o+h)-f(x_o-h)}{2h}$$

3 approximation for $f''(x_o)$

We can do the same trick to find centered difference approximation for $f''(x_o)$. Since

$$f(x_o+h)=f(x_o)+hf'(x_o)+\frac{h^2}{2}f''(x_o)+O\left(h^2\right)$$

Then

$$f''(x_o)\approx\frac{2}{h^2}\left(f(x_o+h)-f(x_o)-hf'(x_o)\right) \tag{2}$$

But we also know that

$$\begin{aligned}f(x_o-h)&=f(x_o)-hf'(x_o)+\frac{h^2}{2}f''(x_o)+O\left(h^2\right)\\hf'(x_o)&\approx-f(x_o-h)+f(x_o)+\frac{h^2}{2}f''(x_o)\\f'(x_o)&\approx\frac{f(x_o)-f(x_o-h)}{h}+\frac{h}{2}f''(x_o)\end{aligned}$$

Sustituting the above into (2), we find

$$\begin{aligned}
f''(x_o) &\approx \frac{2}{h^2}\left[f(x_o+h)-f(x_o)-h\left(\frac{f(x_o)-f(x_o-h)}{h}+\frac{h}{2}f''(x_o)\right)\right]\\
&\approx \frac{2}{h^2}\left[f(x_o+h)-f(x_o)-\left(f(x_o)-f(x_o-h)+\frac{h^2}{2}f''(x_o)\right)\right]\\
&\approx \frac{2}{h^2}\left[f(x_o+h)-2f(x_o)+f(x_o-h)-\frac{h^2}{2}f''(x_o)\right]\\
&\approx \frac{2}{h^2}\left(f(x_o+h)-2f(x_o)+f(x_o-h)\right)-f''(x_o)\\
2f''(x_o) &\approx \frac{2}{h^2}\left(f(x_o+h)-2f(x_o)+f(x_o-h)\right)
\end{aligned}$$

Solving for $f''(x_0)$ from the above gives

$$f''(x_o) \approx \frac{f(x_o+h)-2f(x_o)+f(x_o-h)}{h^2}$$

This method can be used to find approximations for higher derivatives.