
Finite Volume Method (FVM)

■ Notes

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N := a + b ξ + c η + d ξ η + e ξ2 + f η2 + g ξ2 η + h ξ η2 + i ξ2 η2;
x := x1 + (x2 - x1) ξ + (x3 - x1) η; y := y1 + (y2 - y1) ξ + (y3 - y1) η; J =  $\begin{pmatrix} \partial_\xi x & \partial_\eta x \\ \partial_\xi y & \partial_\eta y \end{pmatrix}$ ;
Inverse[J]. $\left(\frac{\partial_\xi N}{\partial_\eta N}\right)$  // MatrixForm // FullSimplify

Clear[N]; N[ξ_, η_] := a + b ξ + c η + d ξ η + e ξ2 + f η2 + g ξ2 η + h ξ η2 + i ξ2 η2;

dNdx[ξ_, η_] :=

$$\frac{(-y1 + y3) (b + \eta (d + h \eta) + 2 (e + \eta (g + i \eta)) \xi) + (x1 - x3) (c + 2 f \eta + \xi (d + g \xi + 2 \eta (h + i \xi)))}{x3 (y1 - y2) + x1 (y2 - y3) + x2 (-y1 + y3)}$$
;
dNdy[ξ_, η_] :=

$$\frac{(y1 - y2) (b + \eta (d + h \eta) + 2 (e + \eta (g + i \eta)) \xi) + (-x1 + x2) (c + 2 f \eta + \xi (d + g \xi + 2 \eta (h + i \xi)))}{x3 (y1 - y2) + x1 (y2 - y3) + x2 (-y1 + y3)}$$
;

Solve[{N[0, 0] == q1, dNdx[0, 0] == q4, dNdy[0, 0] == q7, N[1, 0] == q2, dNdx[1, 0] == q5,
dNdy[1, 0] == q8, N[0, 1] == q3, dNdx[0, 1] == q6, dNdy[0, 1] == q9}, {a, b, c, d, e, f, g, h, i}]

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N = a + b ξ + c η + d ξ η + e ξ2 + f η2 + g ξ2 η + h ξ η2 + i ξ2 η2
a = q1
b = (-x1 + x2) q4 + (-x1 + x3) q7
c = (-y1 + y2) q4 + (-y1 + y3) q7
d = d
e = -q1 + q2 + (x1 - x2) q4 + (x1 - x3) q7
f = -q1 + q3 + (y1 - y2) q4 + (y1 - y3) q7
g = -d +  $\frac{1}{x1 - x3}$ 
(-2 (y1 - y3) q1 + 2 (y1 - y3) q2 - (x2 y1 + x3 y1 - x3 y2 - x2 y3 + x1 (-2 y1 + y2 + y3)) q4 +
(x3 y1 + x1 y2 - x3 y2 - x1 y3 + x2 (-y1 + y3)) q5) + 2 (y1 - y3) q7
h = -d +  $\frac{1}{y1 - y3}$  (-2 (x1 - x3) q1 + 2 (x1 - x3) q3 -
(x2 y1 + x3 y1 - x3 y2 - x2 y3 + x1 (-2 y1 + y2 + y3)) q4 +
(-x3 y1 - x1 y2 + x3 y2 + x2 (y1 - y3) + x1 y3) q6) + 2 (x1 - x3) q7
i =
i

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shape functions : $N_1 = \xi^3 = 1 - \xi - \eta$, $N_2 = \xi^1 = \xi$, $N_3 = \xi^2 = \eta$

$$\begin{aligned}
 u := & a \xi_1 + b \xi_2 + c \xi_3 + d \xi_1 \xi_2 + e \xi_2 \xi_3 + f \xi_3 \xi_1 + \\
 & g (\xi_1^2 \xi_2 + \xi_1 \xi_2 \xi_3 (3 (1 - \mu_3) \xi_1 - (1 + 3 \mu_3) \xi_2 + (1 + 3 \mu_3) \xi_3) / 2) + \\
 & h (\xi_2^2 \xi_3 + \xi_1 \xi_2 \xi_3 (3 (1 - \mu_1) \xi_2 - (1 + 3 \mu_1) \xi_3 + (1 + 3 \mu_1) \xi_1) / 2) + \\
 & i (\xi_3^3 \xi_1 + \xi_1 \xi_2 \xi_3 (3 (1 - \mu_2) \xi_3 - (1 + 3 \mu_2) \xi_1 + (1 + 3 \mu_2) \xi_2) / 2); \\
 \mu_1 = & (l k^2 - l j^2) / l i^2
 \end{aligned}$$

see Patankar, 1985
Malagasekara

- http://www.amazon.com/gp/offer-listing/0582218845/sr=8-1/qid=1157653744/ref=sr_1_1/002-2175139-6340059?ie=UTF8&s=books
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