In[271]:=
```
mMat = {{m, 0}, {0, m}}
```

Out[271]=
$$\begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix}$$

In[272]:=
```
kmat = {{2 k, -k}, {-k, 2 k}}
```

Out[272]=
$$\begin{pmatrix} 2k & -k \\ -k & 2k \end{pmatrix}$$

In[273]:=
```
MatrixForm[sys = mMat.{-A1 w^2, -A2 w^2} + kmat.{A1, A2}]
```

Out[273]//MatrixForm=
$$\begin{pmatrix} 2\,\mathrm{A1}\,k - \mathrm{A1}\,m\,w^2 - \mathrm{A2}\,k \\ -\mathrm{A1}\,k + 2\,\mathrm{A2}\,k - \mathrm{A2}\,m\,w^2 \end{pmatrix}$$

In[274]:=
```
syseq = sys[[1]] == 0
```

Out[274]=
$$2\,\mathrm{A1}\,k - \mathrm{A1}\,m\,w^2 - \mathrm{A2}\,k = 0$$

In[275]:=
```
eq = CoefficientArrays[sys, {A1, A2}] // Normal;
eq = eq[[2]]
```

Out[276]=
$$\begin{pmatrix} 2k - m\,w^2 & -k \\ -k & 2k - m\,w^2 \end{pmatrix}$$

In[277]:=
```
Det[eq]
```

Out[277]=
$$3k^2 - 4\,k\,m\,w^2 + m^2\,w^4$$

In[278]:=
```
sol = Solve[% == 0, w]
```

Out[278]=
$$\left\{\left\{w \to -\frac{\sqrt{k}}{\sqrt{m}}\right\}, \left\{w \to \frac{\sqrt{k}}{\sqrt{m}}\right\}, \left\{w \to -\frac{\sqrt{3}\ \sqrt{k}}{\sqrt{m}}\right\}, \left\{w \to \frac{\sqrt{3}\ \sqrt{k}}{\sqrt{m}}\right\}\right\}$$

In[279]:=
```
sol = {sol[[1]], sol[[3]]}
```

Out[279]=
$$\left\{\left\{w \to -\frac{\sqrt{k}}{\sqrt{m}}\right\}, \left\{w \to -\frac{\sqrt{3}\ \sqrt{k}}{\sqrt{m}}\right\}\right\}$$

In[280]:=
```
updatedSys = syseq /. sol[[1]]
```

Out[280]=
$$\mathrm{A1}\,k - \mathrm{A2}\,k = 0$$

In[281]:=
```
First[Solve[updatedSys, A1]]
```

Out[281]=
$$\{\mathrm{A1} \to \mathrm{A2}\}$$

In[282]:=
```
r1 = A2 / (A1 /. %)
```

Out[282]=
$$1$$

In[283]:=
```
updatedSys = syseq /. sol[[2]]
```

Out[283]=
$$-\mathrm{A1}\,k - \mathrm{A2}\,k = 0$$

In[284]:=
```
First[Solve[updatedSys, A1]]
```

Out[284]=
$$\{\mathrm{A1} \to -\mathrm{A2}\}$$

In[285]:=
```
r2 = A2 / (A1 /. %)
```

Out[285]= -1

In[286]:=
```
x1 = A11 Cos[(w /. sol[[1]]) t + θ1] + A12 Cos[(w /. sol[[2]]) t + θ2]
```

Out[286]= $\text{A11}\cos\left(\frac{\sqrt{k}\ t}{\sqrt{m}} - \theta 1\right) + \text{A12}\cos\left(\frac{\sqrt{3}\ \sqrt{k}\ t}{\sqrt{m}} - \theta 2\right)$

In[287]:=
```
x2 = A21 Cos[(w /. sol[[1]]) t + θ1] + A22 Cos[(w /. sol[[2]]) t + θ2]
```

Out[287]= $\text{A21}\cos\left(\frac{\sqrt{k}\ t}{\sqrt{m}} - \theta 1\right) + \text{A22}\cos\left(\frac{\sqrt{3}\ \sqrt{k}\ t}{\sqrt{m}} - \theta 2\right)$

In[288]:=
```
x2 = x2 /. {A21 → r1 A11, A22 → r2 A12}
```

Out[288]= $\text{A11}\cos\left(\frac{\sqrt{k}\ t}{\sqrt{m}} - \theta 1\right) - \text{A12}\cos\left(\frac{\sqrt{3}\ \sqrt{k}\ t}{\sqrt{m}} - \theta 2\right)$

In[289]:=
```
icx1 = {1, 0}
icx2 = {1, 0}
```

Out[289]= $\{1, 0\}$

Out[290]= $\{1, 0\}$

In[291]:=
```
eq1 = icx1[[1]] == x1 /. t → 0
```

Out[291]= $1 = \text{A11}\cos(\theta 1) + \text{A12}\cos(\theta 2)$

In[292]:=
```
eq2 = icx1[[2]] == D[x1, t] /. t → 0
```

Out[292]= $0 = \frac{\text{A11}\sqrt{k}\ \sin(\theta 1)}{\sqrt{m}} + \frac{\sqrt{3}\ \text{A12}\sqrt{k}\ \sin(\theta 2)}{\sqrt{m}}$

In[293]:=
```
eq3 = icx2[[1]] == x2 /. t → 0
```

Out[293]= $1 = \text{A11}\cos(\theta 1) - \text{A12}\cos(\theta 2)$

In[294]:=
```
eq4 = icx2[[2]] == D[x2, t] /. t → 0
```

Out[294]= $0 = \frac{\text{A11}\sqrt{k}\ \sin(\theta 1)}{\sqrt{m}} - \frac{\sqrt{3}\ \text{A12}\sqrt{k}\ \sin(\theta 2)}{\sqrt{m}}$

In[300]:=
```
MatrixForm[{eq1, eq2, eq3, eq4} /. {k → 1, m → 1}]
```

Out[300]//MatrixForm=

$$\begin{pmatrix} 1 = \text{A11}\cos(\theta 1) + \text{A12}\cos(\theta 2) \\ 0 = \text{A11}\sin(\theta 1) + \sqrt{3}\ \text{A12}\sin(\theta 2) \\ 1 = \text{A11}\cos(\theta 1) - \text{A12}\cos(\theta 2) \\ 0 = \text{A11}\sin(\theta 1) - \sqrt{3}\ \text{A12}\sin(\theta 2) \end{pmatrix}$$

In[301]:=
```
Solve[{eq1, eq2, eq3, eq4} /. {k → 1, m → 1}, {A11, A12, θ1, θ2}]
```

Solve::ifun :
Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. ≫

Solve::svars : Equations may not give solutions for all "solve" variables. ≫

Out[301]= $\{\{A11 \to -1,\ A12 \to 0,\ \theta 1 \to -\pi\},\ \{A11 \to -1,\ A12 \to 0,\ \theta 1 \to \pi\},\ \{A11 \to 1,\ A12 \to 0,\ \theta 1 \to 0\}\}$