$\int_a^b e^{c\tau}\sin(t-\tau)\,d\tau = \frac{c[e^{c\tau}\sin(t-\tau)]_a^b + [e^{c\tau}\cos(t-\tau)]_a^b}{(1+c^2)}$	$\int_a^b e^{c\tau}\cos(t-\tau)\,d\tau = \frac{c[e^{c\tau}\cos(t-\tau)]_a^b - [e^{c\tau}\sin(t-\tau)]_a^b}{1+c^2}$	$\int \cos at = \frac{\sin at}{a}$	$\int \sin at = \frac{-\cos at}{a}$

F(t)	Guess
ke^{bt}	Ae^{bt}
kt^n	$A_n t^n + \cdots + A_0$
$\cos\omega t$ or $\sin\omega$	$c_1\cos\omega t + c_2\sin\omega t$
$ke^{at}\cos\omega t$	$e^{at}(c_1\cos\omega t + c_2\sin\omega t)$
$x'' + x' + x = f(t)$	$\left[s^2X - sx(0) - x'(0)\right]$
$L(t) = \int_0^\infty x(t)e^{-st}dt$	$+[sX - x(0)] + X = F$

roots $\int udv = uv - \int vdu$	$x(t)$
real and distinct	$Ae^{\lambda_1 t} + Be^{\lambda_2 t}$
double real	$Ae^{\lambda t} + Bte^{\lambda t}$
complex $\alpha \pm j\beta$	$e^{\alpha t}\left(A\cos\beta t + B\sin\beta t\right)$
$x(t) = A\cos\omega_n t + B\sin\omega_n t$	$x(t) = C\sin(\omega_n t + \theta)$
$A = u(0)\, B = \frac{v(0)}{\omega_n}, \theta = \arctan\left(\frac{A}{B}\right)$	$C = \sqrt{A^2+B^2}$ $P = [v_1 v_2]$

$$\begin{pmatrix} \frac{\partial g}{\partial x_1} & \frac{\partial g}{\partial x_2} \\ \frac{\partial f}{\partial x_1} & \frac{\partial f}{\partial x_2} \end{pmatrix}$$

modal: $x = M^{-\frac{1}{2}}q$

Find eigenvalus of

$\tilde{k} = M^{-\frac{1}{2}}kM^{-\frac{1}{2}}$

$r'' = \Lambda r, q = P\ r,$

$P^T = P^{-1}(orthon.)$

$r(0) = P^T M^{\frac{1}{2}}x(0)$

$x(t) = M^{-\frac{1}{2}}P\ r(t)$

$mx''(t) + kx(t) = 0 \rightarrow x(t) = e^{-\zeta\omega_n t}(A\cos\omega_d t + B\sin\omega_d t)$ or $x(t) = Ce^{-\zeta\omega_n t}\sin(\omega_d t - \theta)$

$A = u(0), B = \frac{v(0)+x(0)\zeta\omega_n}{\omega_d}, C = \sqrt{A^2+B^2}, \theta = \tan^{-1}\left(\frac{B}{A}\right)$

$\omega_n^2 = \frac{k}{m}$
$r = \frac{\omega}{\omega_n}$
$\zeta = \frac{c}{c_{cr}}$
$c_{cr} = 2\sqrt{km} = 2\omega_n m$
$\omega_d = \omega_n\sqrt{1-\zeta^2}\quad \zeta < 1$

$L = T - U,\ \frac{d}{dt}\frac{\partial L}{\partial q_i'} - \frac{\partial L}{\partial q_i} = Q_i,$ Rayleigh $\frac{d}{dt}\frac{\partial L}{\partial q'} - \frac{\partial L}{\partial q} + \frac{\partial R}{\partial q'} = 0$ where $R = \frac{1}{2}c\left(q'\right)^2$ $\delta W = Q_i\delta q_i$

$mx''(t) + kx(t) = F_0\sin\omega t \rightarrow x(t) = A\cos\omega_n t + B\sin\omega_n t + \frac{F_0}{k}\frac{1}{1-r^2}\sin\omega t, A = x(0),\ B = \frac{v(0)}{\omega_n} - \frac{r}{1-r^2}$

$mx''(t) + cx'(t) + kx(t) = F_0\sin\omega t \rightarrow x(t) = e^{-\zeta\omega_n t}(A\cos\omega_d t + B\sin\omega_d t) + \frac{F_0}{k}\frac{1}{\sqrt{\left(1-r^2\right)^2+(2\zeta r)^2}}\sin(\omega t - \theta)$

$\theta = \arctan\left(\frac{2\zeta r}{1-r^2}\right),\ \lambda_{1,2} = \frac{-b}{2a} \pm \frac{\sqrt{b^2-4ac}}{2a}$ where $ax^2 + bx + c = 0,$

$\sin 2A$	$2\sin A\cos A$
$\cos 2A$	$2\cos^2 A - 1$
$\sin A\sin B$	$\frac{1}{2}(\cos(A-B) - \cos(A+B))$
$\cos A\cos B$	$\frac{1}{2}(\cos(A-B) + \cos(A+B))$
$\sin A\cos B$	$\frac{1}{2}(\sin(A-B) + \sin(A+B))$
$h = v_i t + \frac{1}{2}gt^2$	$h = \frac{v_i+v_f}{2}t$
$v_f^2 = v_i^2 + 2gh$	$v_f = v_i + gt$
speed is $\sqrt{2gh}$	$h_u(t) = \frac{\hat{F}}{m\omega_n}\sin\omega_n t$
	$\hat{F} = F\Delta t = mv$

phase roots λ_1 and $\lambda_2 > 0$	Unstable, repelling
phase roots λ_1 and $\lambda_2 < 0$	stable, attracting
both real, one >0 and one <0	unstable saddle point
equal roots and >0	unstable, degenrate
equal roots and <0	stable, degenrate
complex, real part>0	unstabe, spiral out
complex, real part<0	stable, spiral in
pure complex conjugrates	marginaly stable, cirlce

time betwen $\frac{x_2^2}{2} - \frac{g}{l}\cos$

$\frac{dx_1}{dt} = \pm\sqrt{c_1 + \frac{2g}{l}\cos x_1}$

$t = t_0 + \int_{x_1(t_0)}^{x_1(t)} \frac{dx_1}{\sqrt{c_1 + \frac{2g}{l}\cos}}$

convert: $x'' + kx = 0$

$\frac{dx_2}{dt} + kx_1 = 0, \frac{dx_2}{dx_1}\frac{dx_1}{dt} =$

$\frac{dx_2}{dx_1}x_2 = -kx_1$

$\frac{x_2^2}{2} = -k\frac{x_1^2}{2} + C$

$h_d(t) = \frac{1}{m\omega_d}e^{-\zeta\omega_n t}\sin\omega$

$c^2 = a^2 + b^2 - 2ab\cos$

$\lim_{t->0^+} f(t) = \lim_{s->\infty} sF(s)$

$\lim_{s->0} sF(s) = \lim_{t->\infty} f(t)$

$\sin(a \pm b)$	$\sin a\cos b \pm \cos a\sin b$
$\cos(a \pm b)$	$\cos a\cos b \mp \sin a\sin b$
$\sin^2 a$	$\frac{1}{2}(1 - \cos 2a)$

$\cos^2 a = \frac{1}{2}(1 + \cos 2a)$	$sin(A \pm 90) = \cos A$
$\cos\left(a \pm 90^0\right) = \mp\sin a$	$sin(A \pm 180) = \mp\sin A$
$\cos\left(a \pm 90^0\right) = \cos a$	$\cos(A \pm 180) = -\cos A$

$M\ddot{x} + kx = 0$, assume $x_i = A_i\cos\left(\omega t + \phi_i\right)$,Plug in, rewrite as $\left[sys\right][A] = 0$, find eigens of sys,each ω_i,find $r_1 = \left(\frac{A_2^{(1)}}{A_1^{(1)}}\right), r_2 = \left(\frac{A_2^{(2)}}{A_1^{(2)}}\right)$
$x_1 = A_1^{(1)}\cos\left(\omega_1 t + \phi_1\right) + A_1^{(2)}\cos\left(\omega_2 t + \phi_2\right), \quad x_2 = A_2^{(1)}\cos\left(\omega_1 t + \phi_1\right) + A_2^{(2)}\cos\left(\omega_2 t + \phi_2\right)$, use $A_2^{(1)} = r_1 A_1^{(1)}, A_2^{(2)} = r_2 A_1^{(2)}$
$x_1 = A_1^{(1)}\cos\left(\omega_1 t + \phi_1\right) + A_1^{(2)}\cos\left(\omega_2 t + \phi_2\right), \quad x_2 = r_1 A_1^{(1)}\cos\left(\omega_1 t + \phi_1\right) + r_2 A_1^{(2)}\cos\left(\omega_2 t + \phi_2\right)$

$g(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n\cos n\left(2\pi f\right)t + b_n\sin n\left(2\pi f\right)t$
$a_0 = \frac{1}{T/2}\int_0^T f(t) \qquad b_n = \frac{1}{T/2}\int_0^T f(t)\sin n\left(2\pi f\right)tdt$
$a_n = \frac{1}{T/2}\int_0^T f(t)\cos n\left(2\pi f\right)tdt \qquad T = period\ of\ f(t)$

$h_{over}(t) = \frac{1}{2m\omega_n\sqrt{\xi^2-1}}e^{-\xi\omega_n t}\left(e^{\omega_n\sqrt{\xi^2-1}t} - e^{-\omega_n\sqrt{\xi^2-1}t}\right)$

$h_{critical}(t) = \frac{1}{m}te^{-\xi\omega_n t}$

$f(t)$ =impulse= $F\Delta t = [mv(0^-) - mv(0^+)]\,\delta(t)$

solid disk, around center $I = \frac{mr^2}{2}$	thin loop, around center $I = mr^3$	solid sphere $I = \frac{2}{5}mr^2$	rod, axis at center of rod $I = \frac{ML^2}{12}$
rod, axis at end of rod $I = \frac{ML^2}{3}$	series: $\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}$ par $k = k_1 + k_2$	$\int_a^b \tau \sin \omega (t - \tau)\, d\tau = \frac{-\sin(\omega(t-a)) - a\omega \cos(\omega(t-a)) + \sin(\omega(t-b)) + b\omega \cos(\omega(t-b))}{\omega}$	

2 equations of motions for unbalanced: $(M - m)\ddot{x} + c\dot{x} + kx = F_r$ and $m(\ddot{x} + \ddot{x}_r) = -F_r$, where $x_r = e \sin \omega t$, eq for M is

$M\ddot{x} + c\dot{x} + kx = me\omega^2 \sin \omega t$, guess $X_p = X \sin(\omega t - \theta)$, we obtain $X = \frac{Me}{m} \frac{r^2}{\sqrt{(1-r^2)^2 + (2\xi r)^2}}, \theta = \tan^{-1} \frac{2\xi r}{1-r^2}$ perturbation:

$x'' + \omega_0^2 x + \alpha x^3 = 0 \rightarrow x = x_0 + \alpha x_1 + \alpha^2 x_2 + \cdots, \omega^2 = \omega_0^2 + \alpha\omega_1^2(A) + \alpha\omega_2^2(A) + \cdots$,hence $\omega_0^2 = \omega^2 - \alpha\omega_1^2(A)$. Sub in ODE, generate 2 ODE's and solve for x_0 and use result to find x_1.watch for IC and resonanse. For system ID, set up $\left|G(j\omega)\right| = \frac{1}{\sqrt{(c\omega)^2 + (k - m\omega^2)^2}}$ and from the spectrum, find m, c, k